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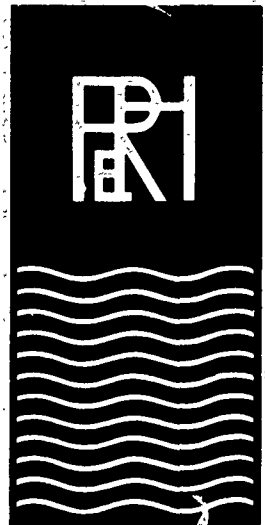
# ROHM & HAAS COMPANY

REDSTONE ARSENAL RESEARCH DIVISION  
HUNTSVILLE, ALABAMA

SPECIAL REPORT NO. S-68

HIGH PRESSURE SOLID PROPELLANT COMBUSTION  
STUDIES USING A CLOSED BOMB

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# ROHM & HAAS COMPANY

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HUNTSVILLE, ALABAMA

Report No. S-68

## HIGH PRESSURE SOLID PROPELLANT COMBUSTION STUDIES USING A CLOSED BOMB.

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# ROHM & HAAS COMPANY

REDSTONE ARSENAL RESEARCH DIVISION  
HUNTSVILLE, ALABAMA

## HIGH PRESSURE SOLID PROPELLANT COMBUSTION STUDIES USING A CLOSED BOMB

### ABSTRACT

✓ A self-pressurizing closed-bomb system is described for observing deflagration characteristics of solid propellants at pressures as high as 200,000 psi. Regression rates are obtained by microwave interferometry, a newly-adapted method which eliminates the need for introducing foreign bodies in the forms of wires, probes, etc., into the system. Pressure measurements are made with strain gauges mounted on the exterior of the bomb body, obviating many sealing problems associated with conventional transducers. The relative merits of the available instrumentation are discussed; the principles and limitations of the interferometry technique are treated in detail. Preliminary r-P data for a polybutadiene-acrylic acid composite propellant are reported up to 60,000 psi; results at higher pressures are indeterminate, owing to leakage and lack of high-pressure dynamic calibration equipment. ↗

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## HIGH PRESSURE SOLID PROPELLANT COMBUSTION STUDIES USING A CLOSED BOMB

### INTRODUCTION

During the last several years, considerable interest has been generated in the combustion of solid propellants at high pressures (greater than 2000 psi). The reasons are quite practical ones. "Intermediate" pressure combustion (at 2000 to 20,000 psi) is of current interest largely due to the value of increasing solid rocket chamber pressures into this range for certain specific missile system applications. Knowledge of burning rates, combustion efficiencies, etc., is necessary for feasibility studies of such systems. Combustion in the higher pressure region (up to 200,000 psi or higher) is also of interest but primarily with respect to hazard evaluation. This pressure regime approaches that involved both in high-order detonation and in less catastrophic but still destructive phenomena.

Since the mechanisms of detonation and similar explosive malfunctions are far from fully understood, research into the nature of solid propellant combustion processes at these high pressures offers the promise of providing information about the mechanisms of such malfunctions. In particular, high pressure, quasi-steady combustion may be viewed as a diagnostic technique, allowing insights into the possible nature of detonation, deflagration-to-detonation transition (DDT), etc. For example, in phenomena such as detonations, which involve high rates of chemical energy release, the question arises as to what influences promote such high rates.

In reactive gaseous systems it is well-known that shock compression can raise reactant temperatures to high values and thereby promote reaction rates which are much higher than those encountered in normal deflagration. The existence of such large temperature increases in shocked, void-free solid systems, such as propellants, is problematical, although local hot spots may be formed. It is important then to establish whether, in the absence of high temperatures, the high pressures and densities resulting

from shock compression in themselves promote rapid chemical energy release or whether the necessary release rate must be increased by other means. e.g., material fracture with an accompanying increase in solid area ("grain burning"). By systematic investigation of quasi-steady, high-pressure combustion it should be possible to diagnose the extent to which either of these two influences predominates in promoting high energy-release rates in solid propellant combustion malfunctions. This may be accomplished, for example, by observing the effects of propellant particle size, loading rate ( $dP/dt$  in closed bomb tests), etc., on burning rates as a function of pressure.

Although 200,000 psi (14 kbars) falls short of the 35 to 70 kbars required for shock initiation of detonable propellants and is an order of magnitude less than the detonation pressures of such propellants, successful combustion studies at pressures up to this level could bridge the gap between the usual deflagration and detonation regimes.

Several anomalies are apparent in the limited high pressure burning rate data currently available.<sup>1</sup> Whereas transitions to a high burning rate pressure exponent have been observed at intermediate combustion pressures (5,000 to 15,000 psi) in some closed bomb experiments, such transitions have not been apparent in other cases involving both vented and closed bomb tests at similar pressures. Extant explanations for this discrepancy relate to rate of pressure rise, physical nature of the burning surface, etc., but data are currently too scant to allow firm conclusions to be made. These burning rate transitions have been observed in a variety of practical propellant systems, thereby evidencing the need for a better understanding of the processes producing them. Further, the implications of these transitions with respect to high pressure combustion mechanism and consequently detonation and other explosion phenomena are not at all clear. If the observed transitions to high exponents are real and reproducible, as they appear to be

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<sup>1</sup> The literature for ~~unfilled~~ binder composite propellants is discussed in "Combustion of Solid Propellants at High Pressures--A Survey", Rohm & Haas Company, Special Report No. S-71, 39 pp. (May 20, 1965) (Confidential), by Richard B. Cole.

in some situations, the question arises as to whether such burning rates persist to very high pressure. Therefore, not only is the further investigation of the intermediate pressure transitions called for but also careful study of higher pressure effects. High pressure burning rate studies may also provide valuable information on the nature of the transition effect by indicating the extent of the high index regime at pressures well above those at which this transition is first observed. No comprehensive high pressure solid propellant combustion studies have been reported, although a few specific experiments have been carried out.

The measurement of solid propellant burning rates at pressures greater than about 2000 psi has for the most part been accomplished in closed bombs. Depending on the propellant loading density (i. e., propellant wt./bomb volume) used, such bombs, during firing, may undergo either little change in pressure following pre-pressurization to the desired level, or considerable change — from atmospheric pressure to several hundred thousand psi. In the former case, measurements of burning surface displacement with time have usually been used to determine burning rates; in the latter, burning rates have typically been deduced by analysis of pressure-time measurements for each firing (with concomitant difficulty due to necessary assumptions regarding equations of state, burning surface area, etc.).

Experimentally it is easier to allow the burning of a propellant sample to provide a given elevated test pressure rather than to pump the system to a high pressure level by external means. However, the difficulty of dynamic measurements, data reduction, and the implications of rapidly varying pressure on the burning itself usually make high-loading-density self-pressurizing firings difficult to interpret. Hence, they are not usually used except for very high pressure studies (<25,000 psi). Particularly awkward is the deduction of burning rates, since rapidly increasing pressures demand measurements of displacement vs. time by techniques of high spatial resolution. Common fuse-wire techniques are not suited to such measurements. Consequently, burning rates in high loading-density closed-bomb firings have typically been

deduced from pressure histories during firing. This alternate technique however, introduces other uncertainties, owing to the necessity of assuming equations of state for product gases and presuming relations for propellant burning area as a function of the fraction of propellant burned at a given time after ignition.

This report describes a preliminary study of high pressure propellant burning in a self-pressurizing closed bomb designed to operate at pressures up to 200,000 psi.<sup>1</sup> The purpose of the study was three-fold: (1) evaluation of the closed bomb technique, indication of future design areas, and construction of a closed bomb strand burner suitable for routine testing, (2) specification of suitable burning rate and pressure instrumentation techniques for routine firings, and (3) determination of preliminary burning rate and equation of state (covolume) data for typical propellants. This initial study was envisioned to be preparatory to initiation of a larger-scale program of high pressure combustion studies aimed at the measurement of burning rate—pressure relationships for a variety of propellant formulations.

#### EXPERIMENTAL PROGRAM

The experimental program involved (1) the design and fabrication of six "single-shot" strand burners capable of sustaining up to 200,000 psi, (2) the selection and development of suitable burning rate and pressure instrumentation, (3) the choice of suitable propellant formulations (4) the adaptation of firing range facilities to accommodate the strand burners and their instrumentation, and (5) low-pressure firings both of strands and rocket motors to obtain ignitability, strand restrictor, and low-pressure burning rate checks. Simultaneously, analytical studies of transient effects in burner firings (pressure waves and heat losses), total burning time scaling with sample size, high pressure equations of state, closed bomb ballistic equations, etc., were made.

<sup>1</sup> Portions of this study have been reported previously (J. Hester, Rohm & Haas Company, Quarterly Progress Report on ARPA Projects, P-62-26, p. 61 (January 14, 1962); R. B. Cole, Rohm & Haas Company, Quarterly Progress Report on Engineering Research, P-64-13, p. 33 (January 12, 1965); R. B. Cole, Rohm & Haas Company, Quarterly Progress Report on Engineering Research, P-64-20, p. 34 (March 8, 1965)).

## Apparatus

### 200,000 psi Strand Burner

A high pressure strand burner (HPSB) providing for the ignition, containment, and accessibility of a cylindrical propellant strand of  $\frac{1}{2}$ " diameter and 8" (max.) length was designed and fabricated of heat-treated<sup>1</sup> AISI 4340 steel. All of the steel burner parts including end plates, end plugs, and lead-through electrodes were of this same material. Figs. 1 and 2 show the HPSB and propellant sample arrangement.

End-Plug Design - One type of end plug incorporated two electrical lead-throughs and could be located at either end of the chamber. Another type contained a microwave window at the opposite end of the propellant strand from the igniter. Details of both end-plug types after modifications based on operating experience are shown in Figs. 1 and 3.

While the basic microwave-window end-plug design was not modified during this program, a change in the assembly procedure for the plug itself was necessary in order to eliminate leakage. The first two HPSB firings indicated that the truncated-cone-shaped, alumina window did not seal well in the tapered port of the end plug. In the third firing of the burner a layer of thick lead foil was wrapped around the window and the window was pressed (15,000-lb load) into the end plug before installation; but this remedy proved ineffective. Consequently, in the fourth and fifth firings, the window was lapped into the tapered seat and then sealed with two layers of lead foil.

Elimination of leakage at the electrodes of the igniter lead-through end plug was more complicated. The basic plug design was not altered, but the design of the electrode seal and its component parts underwent two stages of modification. The original lead-through seal design (Fig. 4a), though nominally of an "unsupported area" design<sup>2</sup> failed during the first HPSB firing, despite the fact that the seal had been statically tested at ambient temperature to the equivalent of 50 kpsi. Failure was attributed to deformation

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<sup>1</sup> Heat treatment consisted of normalizing at 1650°, soaking at 1525°F, quenching in oil, and drawing at 410°F for at least 4 hours.

<sup>2</sup> P. W. Bridgeman, The Physics of High Pressure, Macmillan, New York (1931).

of the seal washers in this design; lack of confinement allowed the extrusion of seal washer material into nearby clearances under the action of high pressures inside the burner. Hence, the seal design was "inverted" such that the seal washer could be confined by two ceramic insulators, one on either side (Fig. 4b).

Before use in the second HPSB, the inverted-seal design was static-tested with a load equivalent to a burner pressure of 100 kpsi. Only slight extrusion, compared with that found after static testing of the previous design, was observed in this test. Unfortunately, two forms of this seal design (two washers for the third HPSB firing and one for the fourth) also failed in use but in a different mode than previously. One of the lead-through electrodes was ejected from the end plug during the third firing and was recovered. It had fractured on the stem near the loading collar, apparently from strains imposed during pre-firing assembly of the end plug unit. A similar failure occurred in the fourth firing, from which both the electrode stem and load collar were recovered. The likelihood of tensile failure at the point of high stress concentration near the collar was verified by the static-loading of several similar electrodes to fracture ( $405^{+37}_{-20}$  lb). The theoretical tensile load required to produce fracture of the electrode stem without stress concentration was calculated as 504 lb, based on an ultimate tensile stress of 290,000 psi.

Failures of electrode stems were due either (1) to tensile loading by viscous forces resulting from seal extrusion between the outside diameter of the stem and the inside diameter of the ceramic insulation (Fig. 4) or (2) to wedging of an insulator against the electrode stem following angled, shear fracture of the insulator under compression. Consequently, the seal was redesigned to include a hard-rubber seal and smaller clearances between the electrode stem and the ceramic insulators, a larger fillet at the juncture of the electrode stem and loading collar (to relieve stress concentration), and two steel bearing washers, one for each insulator (to decrease the likelihood of insulation fracture due to bearing loads with this design) (Fig. 3). In the fifth HPSB firing, considerably less leakage occurred than in any previous firing at the same propellant loading density. Post-firing inspection revealed

that both electrodes had necked down but had not fractured. Thus, further seal redesign was indicated but was not carried out owing to discontinuation of the program.

Insulation - A phenolic-asbestos liner-tube of 0.690" outside diameter,  $1\frac{7}{32}$ " inside diameter, and  $8\frac{3}{8}$ " length (Fig. 2) was used to decrease heat losses from hot combustion products to the burner chamber wall during firing; such losses could influence both the actual pressure history during firing, including peak pressure, and that indicated by the thermal-strain-sensitive strain-gauge transducers which were used to measure burner chamber pressure. Check firings of instrumented HPSB bodies at ambient pressures, with an end plug removed, indicated that heat transfer through the phenolic-asbestos liner tube gave negligible thermal-strain-induced outputs from the pressure transducer strain gauges on the time scale of a typical HPSB firing (1 to 3 sec). The heat capacity of the mass of product gas in the annular volume between the liner and the burner cavity wall was assumed to be too small to influence pressure measurements by heat transfer to the burner wall, although this could not be readily checked experimentally.

#### Pressure Transducer Selection and Calibration

Besides being necessary to express pressure dependence of burning rate, pressure-time data are also desirable for the indirect determination of burning values from the rate-of-rise of pressure during a firing. These values may be used to corroborate direct regression rate measurements. Past high pressure combustion studies have relied solely on indirect measurement of regression rate; approximations and assumptions required for such determinations constituted prime sources of error in those studies.

Of the various possible techniques for closed bomb pressure measurement only two basic types of pressure-sensing elements were suitable for fast-response measurement at high pressures: (1) a mechanical element sensitive to elastic deformation (e.g., diaphragm), and (2) an electrical element with electrical properties sensitive to pressure (e.g., piezoresistive semi-conductor). Of

these, the latter was excluded owing to its characteristically-high temperature sensitivity, which implies considerable output distortion when used in rapidly-changing, high-pressure, high-temperature environments such as that of the HPSB.

Closed bomb pressure measurements have often been carried out using metal diaphragms or pistons as pressure-sensing or transmitting elements and piezoelectric materials as transducers.<sup>1,2,3</sup> Such complications were not, however, thought to be required for the purposes of this study. Since the pressure vessel to be used in these experiments could conveniently be a long, thick-walled cylinder, the small strains on the outside diameter of this cylinder should serve as a reasonably-accurate measurement of internal pressure. Such a technique requires careful pressure transducer calibration, but the commercial availability of accurate working standards for such calibration made this approach feasible. Since the strains to be measured in this fashion were planned to be small and since resistance strain gauges allow fast time response in strain measurements, strain gauge bridges were bonded to the outside diameter of the HPSB body. This technique had the advantage of being readily used with existing data recording facility designed for use with pressure and thrust transducers employed at this Division for routine rocket motor firings.

Three strain gauge bridges (four gauges each) were bonded to the HPSB exterior (Fig. 5). During firings, these three bridges provided various alternatives in recording chamber pressure. Typically, one bridge at each end of the burner was arranged to record the complete pressure history of the firing, while the second bridge at one end, instrumented for higher gain, was used to record low pressures. Standard equipment used to amplify and

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<sup>1</sup> J. M. Massey, Jr., "Measurement of Impetus, Covolume, and Burning Rate of Solid Propellants," Naval Weapons Laboratory Report No. 1862, 27 pp. (July 31, 1963).

<sup>2</sup> S. Wachtell, "Prediction of Detonation Hazard in Solid Propellants," American Chem. Soc., Div. of Fuel Chem., Preprints of Papers Presented at the Symposium on Explosives and Hazards and Testing of Explosives, pp. 83-107 (September 1963).

<sup>3</sup> J. Corner, Theory of the Internal Ballistics of Guns, John Wiley & Sons, New York (1950), p. 85.



record signals from the strain gauges included a CEC recording oscillograph equipped with Model 7-323 and 7-326 galvanometers.

Calibration of the pressure-transducer strain-gauge bridges was performed only statically. Hydrostatic calibration of the transducer was judged adequate considering the pressure histories expected and the characteristic time of elastic wave propagation in the burner body. No attempt was made to calibrate the system dynamically.

A 100,000-psi pressure balance (dead-weight tester) manufactured by the American Instrument Company was purchased for pressure calibration but damage to the unit during check-out required its return to the manufacturer for an extended period. Thus, only the transducers of the last (sixth) HPSB (not fired) could accurately be calibrated above 20,000 psi, the upper pressure limit of existing calibration standards available at this Division.

Meanwhile a Bourdon-tube dial gauge was used as a reference with a hydrostatic test apparatus for calibration of the first five HPSB transducers up to 50,000 psi. However, this gauge could not be compared with a primary standard over the whole range. Hence, the response of the gauge had to be assumed to be linear beyond the 20,000-psi limit of the available primary standard. Subsequent checking of the response of the Bourdon gauge after the Aminco pressure balance was functioning indicated that it did not, in fact, possess a linear response at the extrapolated pressure levels. As a result, transducer calibration data for the first five burners was re-examined as to the possibility of using a least-squares linear fit of only those data from below the 20,000-psi limit of the dial gauge to indicate transducer response. This procedure would have necessitated considerable extrapolation of calibration data and therefore would still have left appreciable uncertainty in the transducer response at high pressures. Time limitations precluded such an attempt at correcting the results of the original pressure transducer calibrations, but the small magnitude of the correction involved (relative to other anomalies in the data) is insignificant.

### Burning Rate Determination

Strand burning rates in the HPSB firings were measured by two techniques, pressure-rate-of-rise analysis and time-resolved microwave interferometry.

In an early phase, attempts at rate measurement were also made using an ionization (resistance) probe (IRP) which employed a spiral of fine nichrome wire wound on a small-diameter copper magnet wire core and which was cast axially in propellant strands. In tests of IRP-instrumented strands burning at atmospheric pressure, enlargement of the burning surface via "coning" of the burning surface around the IRP resulted in a higher apparent burning rate than that normal for the propellant. This effect, together with the additional problem of assuming the correspondence of probe-indicated regression with actual burning surface regression over a wide range of burning rates and pressures, prompted a decision to discontinue development of the IRP technique.

A "ladder" of transverse fuse wires located at different stations along the strand length was also tried and found unsatisfactory for rate measurement. This failure was caused at least in part by the fracture of fuse wires during displacement of the propellant by compression at the high pressures reached during firing.

Closed Bomb Pressure-Rate-of-Rise - As is indicated in Appendix A, this classic method of determining closed bomb burning rates necessitates several assumptions about the equation of state of the product gases and the variation of burning surface with the fraction of the propellant burned. Typical assumptions are (1) that an Abel equation of state (involving a constant covolume) applies to the product gases, (2) that the propellant "force" or, effectively, the adiabatic flame temperature of the propellant is a constant and independent of pressure, (3) that the initial propellant surface is effectively ignited uniformly and instantaneously, and (4) that propellant regression always occurs normal to the propellant surface (so-called "layer-burning"). The validity of the last two assumptions can only be inferred by the absence

of results which contradict them; they appear to have been satisfied by the experimental configuration and results of this study. The soundness of the first two assumptions must be established through theoretical thermochemical calculations and through empirical covolume determinations made using final pressure data obtained at different values of closed bomb loading density.

Theoretical thermochemical calculations predicting flame temperature and the mean molecular weight of product gases for the propellant studied did indicate a low sensitivity of propellant force to pressure variation (Table I). Unfortunately, leakage in all five firings of closed bombs with this propellant precluded the calculation of a meaningful empirical covolume. Hence, as is to be expected in closed bomb experiments, the validity of the assumptions necessary for burning rate deduction from the  $dP/dt$  measurements of this study is uncertain. For this reason, another burning rate measurement technique was used in this study.

Microwave Interferometry - The use of microwave Doppler-shift analysis or interferometry to determine burning surface displacement with time is subject to several sources of error; these are discussed in Appendix B. Two major sources of such error might be expected to evidence themselves in the dynamic pressure regimes of closed bomb firings; they arise from a change in the effective wave length of the burner cavity-propellant combination during firing. The first of these involves changes in the propellant strand environment with burning and changing chamber pressure; the second depends on changes in the dielectric constant of the propellant itself with changing pressure.

The cross section of the burner "seen" by the microwave beam consists of a core of propellant, an annular void partially-filled with restrictor, a phenolic-asbestos liner tube, a gas-filled annular void and the burner body. This composite structure may be considered to be equivalent to a simple, dielectric-filled, circular, microwave waveguide and to possess an effective waveguide diameter; the magnitude of this effective diameter depends upon the components of the real, composite waveguide structure.

It is well-known that both the effective wavelength and the microwave mode propagated in dielectric-filled waveguides are functions of waveguide dimensions as well as of the dielectric constants of the materials involved. Therefore, a change in any of the individual elements of the composite "waveguide" in these experiments may change the effective wavelength of the microwave radiation used to determine burning rate (surface displacement vs. time). Effective waveguide characteristics might be especially susceptible to changes in the annular spaces, owing to variations in product gas composition and pressure as deflagration proceeds. To minimize such effects (as well as to restrict burning to the end of the propellant strand), the inner annulus was filled with silicone grease and the outer annulus was made small.

As for the second major source of error — change in dielectric constant of the propellant itself — this dielectric property is the major factor (aside from configuration) which determines the microwave wavelength. Dielectric constants are known to change with hydrostatic pressure (compression)<sup>1</sup> but in a manner readily determinable only by experiment. Such experiments were not made in this study, however, owing to demands of other aspects of the technique. The propellant dielectric constant was assumed to be independent of pressure. Development of the techniques required for measurement of propellant dielectric properties as functions of pressure was judged unjustifiable within the envisioned scope of this program.

For all experimental firings, the quarter-wavelength of microwave radiation in the pressurized propellant (corresponding to a burning time between maxima and minima on the record of microwave detector output) was taken to be the same as that exhibited by the same propellant burning at atmospheric pressure in the same experimental configuration. In the atmospheric pressure experiments to determine this wavelength, the propellant sample length was measured carefully before firing; a count of the number of intervals between maxima and minima on the record of microwave detector output allowed determination of the effective microwave wavelength of the sample

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<sup>1</sup> R. S. Bradley, High Pressure Physics and Chemistry, Vol. 2, Academic Press, New York. (1963), pp. 328-329.

configuration. Table II summarizes the results of these firings as well as the results of several preliminary firings in which the propellant sample had been potted into the phenolic-asbestos liner using an epoxy cement.<sup>1</sup>

A schematic diagram of the instrumentation used for the microwave burning-rate measurements of this study is shown in Fig. 6. The microwave power and detection system was constructed from standard, commercial, K-band microwave components including rectangular and circular waveguides. The reflex klystron which was used generated approximately 250 mW at a frequency of 24.0 Gc/sec. Microwave radiation from the klystron was directed through a variable attenuator which was used to adjust the microwave power level and then through a frequency meter. A directional coupler followed the meter in the microwave waveguide and allowed separation of transmitted and reflected signals. The directional coupler passed reflected radiation with approximately 3-dB attenuation to a crystal detector which converted reflected signal level to an output voltage level. Detector output was recorded, after amplification, on an oscillograph. An E-H plane tuner was used in the waveguide between the directional coupler and the HPSB to adjust the detector signal level caused by microwave reflectors in the "downstream" transmission line. This tuner was added to the microwave circuit after the first HPSB firing which experienced an inconveniently-high signal level increase during the firing (see below).

In order to launch a single mode of microwave propagation into the propellant strand, a rectangular-to-circular waveguide transition section and subsequent circular waveguide sections were used following the EH-tuner (Fig. 7). The circular waveguide was joined with the alumina window in the HPSB end, plug through a polystyrene rod tapered at each end. One end of this rod, tapered to a point to reduce reflections, was inserted into the circular

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<sup>1</sup> Bearing on the previously-discussed influence of the annular volume between propellant and liner on effective microwave wavelength, it is noteworthy that the epoxy-resin-potted and the vacuum-grease-restricted samples did not exhibit significantly different microwave wavelengths at a K-band microwave frequency of 24.0 Gc/sec.

waveguide cavity; the other end of the rod, tapered to the diameter of the window, was mated to the window.

Immediately before each HPSE firing, the E-H tuner in the microwave system was adjusted to give a very small, reflected, reference signal at the detector. This level was made low enough that during firings the fixed-phase<sup>1</sup> signal level due to internal reflections from transmission lines, waveguides and waveguide transitions, microwave window surfaces, etc., would be smaller than the varying-phase signal from reflection from the burning surface as burning neared completion. The amplitude of this tuned, constant-phase signal therefore helped limit the amplitude of the oscillating signal observed at the detector toward the end of burning when varying-phase reflections from the surface were attenuated less because of decreased microwave path length in the propellant sample.

The trend to less signal attenuation toward the end of burning was a consequence of the fact that the phenolic-asbestos liner tube enclosing the propellant strand was a relatively high-loss component. Thus, the microwave energy reflected from the burning surface and received at the crystal detector during burning tended to increase slowly as the strand length, and hence the effective liner length, decreased. Though this increase was small over individual cycles, the large number of signal oscillations from start to end of burning (approximately 35 cycles) allowed the signal level to over-range the recorder during the last part of burning in the first HPSB firing. To minimize this problem in later shots, a non-linear amplifier (in addition to the E-H tuner) was used between the crystal detector and the recording oscillograph. The gain of this non-linear amplifier decreased with increasing output voltage from the detector and, hence, compensated partially for microwave signal level increases during burning.

The amplifier was also a.c. -coupled to the crystal detector. The large time constant so obtained served to attenuate signal level changes at frequencies below that of the interferometric or Doppler-shift "beat"

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<sup>1</sup> With respect to incident microwave radiation.

frequency used for burning rate measurement without affecting the interferometric signal. This coup'ng, therefore, in addition to the reference signal tuning to low amplitude, elped decrease recording difficulties with over-ranging resulting from d.c. and low-frequency signal-level changes during burning.

#### Propellant Selection and Sample Preparation

It was found early in the program that the use of microwave burning rate instrumentation limited the propellant types to those reasonably transparent to microwave radiation. In an attempt to identify highly absorptive materials, microwave attenuation tests were made on the several components of the plastisol nitrocellulose propellants, which propellants exhibited high absorptivity. Neither nitrocellulose-nitroglycerin (double-base) powder nor ammonium perchlorate alone was found in these tests to exhibit high microwave attenuation. However, the plasticizer, triethylene glycol dinitrate, and, to a lesser extent, the aluminum powder used in the propellant were highly absorbing of K-band microwave radiation. Another common plasticizer, trimethylolethane trinitrate, was checked and was also found to be highly attenuating.

Consequently, for HPSB firings, an unaluminized, PBAA propellant was selected which contained 75% ammonium perchlorate. This formulation was processed by this Division and vacuum cast in  $\frac{1}{2}$ "-I.D. polyethylene tubes. Samples for HPSB firing were prepared by stripping off the polyethylene tubing; they then fit within the phenolic-asbestos liner tube used in the HPSB. The strand was cut to an appropriate length which was then measured several times to the nearest 0.001". An average value of these measurements was used for burning rate calculations with Equation (A-20) of Appendix A.

Before being placed in the liner tube and prior to insertion into the HPSB body, the  $\frac{1}{2}$ "-diameter propellant strand was equipped on one end with an igniter of B&S No. 32 nichrome wire. This igniter was fashioned in a zig-zag pattern and sandwiched by cementing between the end of the strand and a thin disc ( $\frac{1}{2}$ " diam.  $\times$   $\frac{1}{32}$ " thick) of the test propellant (Fig. 2). Leads

to the igniter wire from the electrodes of the HPSB lead-through end plug were of B&S No. 34 copper magnet wire. This igniter configuration gave reliable ignition while releasing a minimum of extraneous product gas and igniter debris.

After being fitted with the igniter, the propellant strand was restricted on its cylindrical surface and non-ignition end with a  $\frac{1}{16}$ " layer of silicone vacuum grease. Special care was taken to eliminate air trapped between this layer and the propellant since compression of such trapped air during burning might prematurely ignite the propellant side. Since this restrictor coating was appreciably thicker than the  $\frac{1}{32}$ " clearance between the strand and the liner tube, excess restrictor accumulated at the mouth of the liner tube as the strand was inserted; there was thus a high probability that the annular space was filled with restrictor.

#### Procedure

##### Experimental

The following series of six HPSB firings was originally planned:

- 1) PBAA propellant with fine oxidizer ( $\text{NH}_4\text{ClO}_4$ ); 50,000-psi max. pressure; microwave burning rate measurement. Goals: (1) To check regularity of burning and the effectiveness of restrictor, (2) to check the strain gauge pressure measurement system and determine the shape of a typical pressure history during firing, (3) to check microwave technique, (4) to compare calculated and experimental final pressure and measure a propellant covolume, (5) to check heat loss to burner body via the effect of post-firing cooling on final pressure, and (6) to check a hot wire ignition technique.
- 2) Repeat 1 for reproducibility check.
- 3) Repeat 1 and 2 except max. pressure of 100,000 to 150,000psi. Goals: Same as 1 and 2 plus preliminary data on loading rate ( $dP/dt$ ) effects.
- 4) Repeat of 3 except for coarse oxidizer in the propellant. Goal: Preliminary data on oxidizer particle size effect.



- 5) Nitrocellulose plastisol propellant with fine oxidizer; max. pressure of 100,000 psi; no microwave measurements but modified (low heat conductivity) ionization resistance probe (if available). Goal: Same as 1 except for use of an aluminized, perchlorate-containing double-base propellant system.
- 6) Open for contingencies.

Experimental difficulty was encountered in achieving containment of propellant product gas in the HPSB beyond attainment of a peak pressure. This demanded more duplication than had been provided by the original firing plans above. Hence, a revised schedule of firings was undertaken (Table III).

#### 20-kpsi Strand Burner Firing

After the restricted propellant sample had been prepared and assembled with its hot-wire igniter and phenolic-asbestos liner tube, the igniter leads protruding from the open end of the liner tube were soldered to the terminals of a lead-through end plug for the HPSB. The sample assembly, including this end plug and a soft copper seal washer, were then inserted into the cylindrical HPSB cavity through one end and the microwave window end plug in the opposite end. The HPSB end plates were then secured to the burner body using six  $1\frac{1}{4}$ "-diameter Allen-head cap screws between each end and the center flange of the body; each of these screws was tightened to a final torque of 300 ft-lb. The burner was then transported to the firing range, pressure gauge cables and igniter wires were connected, and the microwave waveguide was positioned at the "window" end of the burner.

The burner was fired remotely at the central recording room by an igniter pulse which was time-sequenced to follow the recording of pressure transducer calibration steps on the four-channel oscillograph record produced for each firing. The ignition pulse triggered an igniter relay and thereby supplied 12-V d.c. power from a storage battery to the igniter. The HPSB was always fired in a horizontal position.

After firing, the HPSB was vented through a hole cut by an oxy-acetylene torch on a remote-controlled, movable arm. Although in practice all firings vented themselves owing to leakage, this torch technique for venting was still used to assure that depressurization was complete.

## RESULTS AND DISCUSSION

### r-P Data from Microwave Interferometry

With the exception of the second firing, those HPSB firings which yielded usable microwave interferometric records (fourth and fifth firings) gave r-P data in reasonably good agreement with independent r-P measurements from 2"-diameter rocket motor firings and a small number of  $\frac{1}{8}$ "-diameter strand burning experiments (Fig. 8). Unfortunately, for reasons indicated in Table III, only a limited pressure range was covered by the microwave r-P data of different firings. Nonetheless, the data for low combustion pressures ( $<10$  kpsi) did overlap and were near the level of reference data. The high pressure r-P data of the fourth and fifth firings do, however, evidence appreciably higher rates than the reference data. The low signal level and, hence, the high signal/noise ratio of the microwave record for the fourth firing may have contributed to error in some of these data, but the generally-apparent trend to rates which are higher than normal at high pressures may be due to one or both of the sources of error discussed above.

The microwave data of the second HPSB firing are particularly noteworthy. While they were reduced from a record of microwave signal which exhibited the most regular signal oscillations of any microwave record obtained and which covered a wide pressure range, these data are at considerable variance with both the other HPSB microwave data and any likely r-P trend considering the available reference data. The total number of maxima and minima in the microwave data of this HPSB firing was appreciably less (35%) than that required to account for the total propellant length on the basis of the microwave wavelength determined from atmospheric-pressure

firings. This apparent anomaly in r-P data and the accountability of total sample length was never successfully explained and has been attributed, in the final analysis, to an unidentified external occurrence, since a similar result was never again obtained in later firings.

#### r-P Data from Pressure-Rate-Of-Rise

In those cases where leakage, as evidenced by unusual trends in the slope of the P-t record, was not apparent (first, second, fifth, and possibly fourth firings), r-P values were calculated from  $dP/dt$  data by the method outlined in Appendix A. r-P values from different firings were fairly consistent among themselves except toward the end of each firing where leakage is expected to have been significant. The values are, however, at variance with both microwave r-P data and independent r-P data (Fig. 9). Except for data from the end of each firing, the r-P data deduced from pressure-rate-of-rise never displayed a pressure index less than about 1.5, even for the various low pressure extremes (1 to 5 kpsi) of the data from the several firings involved.

The equations used to calculate r from  $dP/dt$  should properly account for imperfect-gas effects through the use, for example, of a covolume-included equation of state for the closed bomb chamber gases. Analytical determination of such imperfect-gas equations of state for closed bomb combustion is generally laid aside in favor of empirical covolume determinations. Covolumes may be determined from the values of maximum pressure achieved in closed bomb firings of different propellant loading densities.<sup>1</sup> Since appreciable leakage occurred in all the firings of this study (except for the fifth, for which an accurate maximum pressure could not be obtained owing to recording difficulties), no empirical covolume data could be deduced. Hence, a perfect-gas equation of state was used to calculate the data.

Since leakage precluded usable maximum pressure determinations, the maximum pressures used in calculating r-P data were obtained by loading-density-scaling of the maximum indicated pressure of the fifth HPSB firing

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<sup>1</sup> J. M. Massey, Jr., op. cit.

(175 kpsi), during which little leakage was apparent. Attempts to calculate r-P data in better agreement with microwave and reference r-P data by use of a reasonable propellant covolume ( $1.0 \text{ cm}^3/\text{gm}$ ) were not successful; the discrepancies between the various r-P data were too large to be accounted for in this fashion. The original (zero-covolume) r-P data were such, however, that discrepancies were somewhat decreased when reasonable assumed covolume effects were introduced.

At pressures below 5 kpsi, all r-P data calculated from pressure-rate-of-rise show consistently high indexes and unexpectedly low burning rates. Scatter of the data in this pressure range is high, and suggests that errors in pressure measurement at very low pressure-transducer output levels might be a cause. This possibility is not a strong one, however, since the microwave r-P data at these low measured pressure levels are reasonable, so that the very low rates calculated at low pressures must be due to some other cause or causes.

One reason for low calculated burning rates at low pressures may be the result of neglect of the amount of original chamber gas in the closed bomb before firing. Appendix A indicates, however, that in the calculation of burning rate values from P-t data, the initial mass of gas in the chamber free volume may be considered of negligible effect if  $m_o/m_b \leq 1$ . Since the pressure for which  $m_o/m_b = 1$  is only about 500 psi in the firings of this study, or 10% to 25% of the operating pressure, it does not appear likely that initial mass effects introduced sufficient error in calculation to account for the low-rate, high-index data at pressures below 5 kpsi.

The fact that closed bomb pressures below 5 kpsi correspond to very small distances burned introduces another possibility. The calculation procedure assumes that the burning-surface area is constant. This assumption is certainly not valid immediately after ignition and for the first, small mass of propellant burned. Visual observation of strand ignition in the atmosphere, with the same igniter configuration as was used in HPSB firings, indicated that a near-planar, constant-area burning surface was usually

not achieved until burning had proceeded  $\frac{1}{8}$ " to  $\frac{1}{4}$ ". Since, in this study, a pressure of 5 kpsi corresponded to burning over about  $\frac{1}{8}$ " of the strand length, it is not unlikely that ignition transients were responsible for observed r-P anomalies at pressures below 5 kpsi.

### CONCLUSIONS

It may be concluded from the results of this experimental program that microwave interferometry holds singularly good promise as an instrumentation technique for high pressure burning rate determinations using closed bombs. While this technique would require further development before its capability for accurate burning surface displacement measurements is fully utilized, its convenience and other advantages relative to the classic pressure-rate-of-rise technique for determining closed bomb burning rates seem assured.

The major source of error likely in this type of interferometry appears to be the determination of effective microwave wavelengths in the sample configuration used and the extent of the possible change in this wavelength during burning. The results of this preliminary study involving a single propellant suggest that such error may not be so large as to present insuperable difficulties. This possibility must be clarified by further microwave experimentation before its pertinence to high pressure closed bomb tests may be judged.

The major limitation of applying the microwave interferometry technique to propellant burning rate measurement is the microwave absorption characteristics of the propellant formulations themselves. Strongly absorbing propellants may result in attenuation of microwave power to an impractically low level. Thus nitrocellulose plastisol propellants with common plasticizers may not be amenable to study by this technique. Satisfactory measurements of the burning rates of highly-attenuating propellants might be accomplished through higher incident power levels than the 250 mW used in this study. However, the heating of propellants produced by higher power levels might make r-P data observed under such conditions difficult to interpret.

Strain gauges bonded to the outside of the pressure vessel may be used effectively as indicators of closed bombs internal pressures. The results of this study confirm that the body of the bomb itself is a convenient pressure sensor and that the convenience, low cost, and reliability of strain gauges makes their use as pressure transducers advantageous.

Continuation of self-pressurizing closed bomb combustion studies and development of instrumentation techniques is contingent upon the urgency of the need for very high pressure data. On the basis of the experience of this study and the relative need for such data, further development of a capability for solid-propellant burning rate measurement above 50 kpsi is not recommended. The current program status at this Division does not justify further effort at this time. Accordingly, high pressure studies have been suspended.

Table I

Calculated Thermochemical Data<sup>a</sup> for the PBAA  
Propellant Used in HPSB Firings<sup>b</sup>

Pressure psia	Flame Temp. $T_f$ , °K	Mean Molecular Wt. of Product Gas ( $\bar{M}$ , gm-mole <sup>-1</sup> )	$T/\bar{M}$ (°K-gm <sup>-1</sup> -mole <sup>-1</sup> )
1000	1646.3	20.023	82.22
10,000	1649.2	20.051	82.25
25,000	1654.3	20.112	82.25
100,000	1778.1	20.593	86.34

<sup>a</sup>Calculation procedure assumes ideal-gas equation of state

<sup>b</sup>25% PBAA binder, 75%  $\text{NH}_4\text{ClO}_4$  (1% TCP coating)

Table II

K-Band Microwave Wavelength Determinations for PBAA  
Propellant<sup>a</sup> Samples With and Without 1/2% Added KCl

Propellant and Batch (PBAA-75% $\text{NH}_4\text{ClO}_4$ )	Sample Type	Microwave (24.0 Gc/sec) Wave Length (inches)
RH-B-8-1003(ce)	Potted in liner	0.252 ± 0.004 <sup>c</sup>
RH-B-8-1006(ce)	Restricted, loose in liner	0.250 ± 0.004 <sup>d</sup>
RH-B-8-1006(ce)	Restricted, loose in liner	0.250 ± 0.010 <sup>b</sup> -0.009
RH-B-8-1008(ce) (with 1/2% KCl)	Restricted, loose in liner	0.253 ± 0.009 <sup>d</sup> ± 0.011

<sup>a</sup>All 25.0% PBAA, 75.0% (15 $\mu$ )  $\text{NH}_4\text{ClO}_4$

<sup>b</sup>Average of two firings, 3 mos. after original Batch -1006 firings

<sup>c</sup>Average of three firings

<sup>d</sup>Average of four firings

Table III  
Summary of High Pressure Strand Burner Firings

HPSB Firing No.	Propellant <sup>a</sup>		Pressure Instrumentation		End Plug Seals	
	Sample Length (inches)	HPSB Loading Density (gm/cc)	No. and Sensitivity of Pressure-Recording Channels	Max. Recorded Pressure (kpsi)	Microwave Window	Igniter Lead-Through
1	4.36	0.50	one channel (125 kpsi full scale)	95	as machined, no lead foil	original, unconfined-seal- washer design
2	2.21	0.248	three channels (125 kpsi full scale)	30	as machined, no lead foil, window pre- pressed into port with 10,000 lbf.	"inverted," confined-seal washer design (two Teflon seal washers)
3	4.33	0.509	three channels (2-125 kpsi full scale) (1-31 kpsi full scale)	94	as machined, 1 lead foil layer between window and port, pre- pressed with 15,000 lbf	"inverted," confined-seal washer design, (two Teflon seal washers)
4	4.38	0.510	three channels (2-125 kpsi full scale) (1-31 kpsi full scale)	145	hand-fitted (by polishing), 2 lead foil layers, 20,000 lb pre- pressing	"inverted," confined-seal washer design (one Teflon seal washer)
5	4.37	0.505	three channels (2-125 kpsi full scale) (1-31 kpsi full scale)	>175	hand-fitted (by polishing), 2 lead foil layers, 20,000 lb pre- pressing	"inverted," confined-seal- washer design (one hard-rubber seal washer), two steel bearing washers
6	Not fired					

<sup>a</sup> Composition: PBAA-75% (15μ) AP

<sup>b</sup> Based on 42.55 cc burner volume (excluding liner) and measured weight of strand plus igniter disc



Table III

## Summary of High Pressure Strand Burner Firings

Experimentation	End Plug Seals			Comments on Firing
	Microwave Window	Igniter Lead-Through	Leakage	
Max. Recorded Pressure (kpsi)				
95	as machined, no lead foil	original, unconfined-seal-washer design	At both end plugs, around microwave window and lead-through electrodes	Abrupt change in microwave signal level at about 7 kpsi; last usable microwave signal (maximum) at about 2 kpsi; a. c. igniter voltage supply caused high 60-cps noise level in pressure record resulting in low accuracy, low-pressure data for use with 3 usable rate measurements from microwave record.
30	as machined, no lead foil, window pre-pressed into port with 10,000 lbf.	"inverted," confined-seal washer design (two Teflon seal washers)	None	Usable microwave signal for indicated pressures between 2 and 30 kpsi but with large d. c. level change above 3 kpsi; Pressure records suffered from low signal level due to over-estimation of final pressure and decreased accuracy owing to reversed transducer polarity.
94	as machined, 1 lead foil layer between window and port, pre-pressed with 15,000 lbf	"inverted," confined-seal washer design (two Teflon seal washers)	Considerable at both microwave window and lead-through electrodes (one electrode ejected)	Leakage at microwave window apparently made trace useless; very abnormal two-peaked pressure trace apparently due to early leakage and subsequent blockage.
145	hand-fitted (by polishing), 2 lead foil layers, 20,000 lb pre-pressing	"inverted," confined-seal washer design (one Teflon seal washer)	Considerable, at lead-through end plug copper washer seat (corroded in prev. atmos. pressure tests)	Error in set-up resulted in very low microwave signal level; irregular microwave signal amplitude; pressure trace suggested appreciable leakage below 40 kpsi.
>175	hand-fitted (by polishing), 2 lead foil layers, 20,000 lb pre-pressing	"inverted," confined-seal-washer design (one hard-rubber seal washer), two steel bearing washers	Slight, at one lead-through electrode seal	Pressure channel amplifiers saturated, abrupt increases in $dP/dt$ and microwave signal at 50 kpsi and in $dP/dt$ at 137 kpsi; high noise level and abrupt level change at 50 kpsi restricted useful microwave record to <10 kpsi.

2

measured weight of strand plus igniter disc

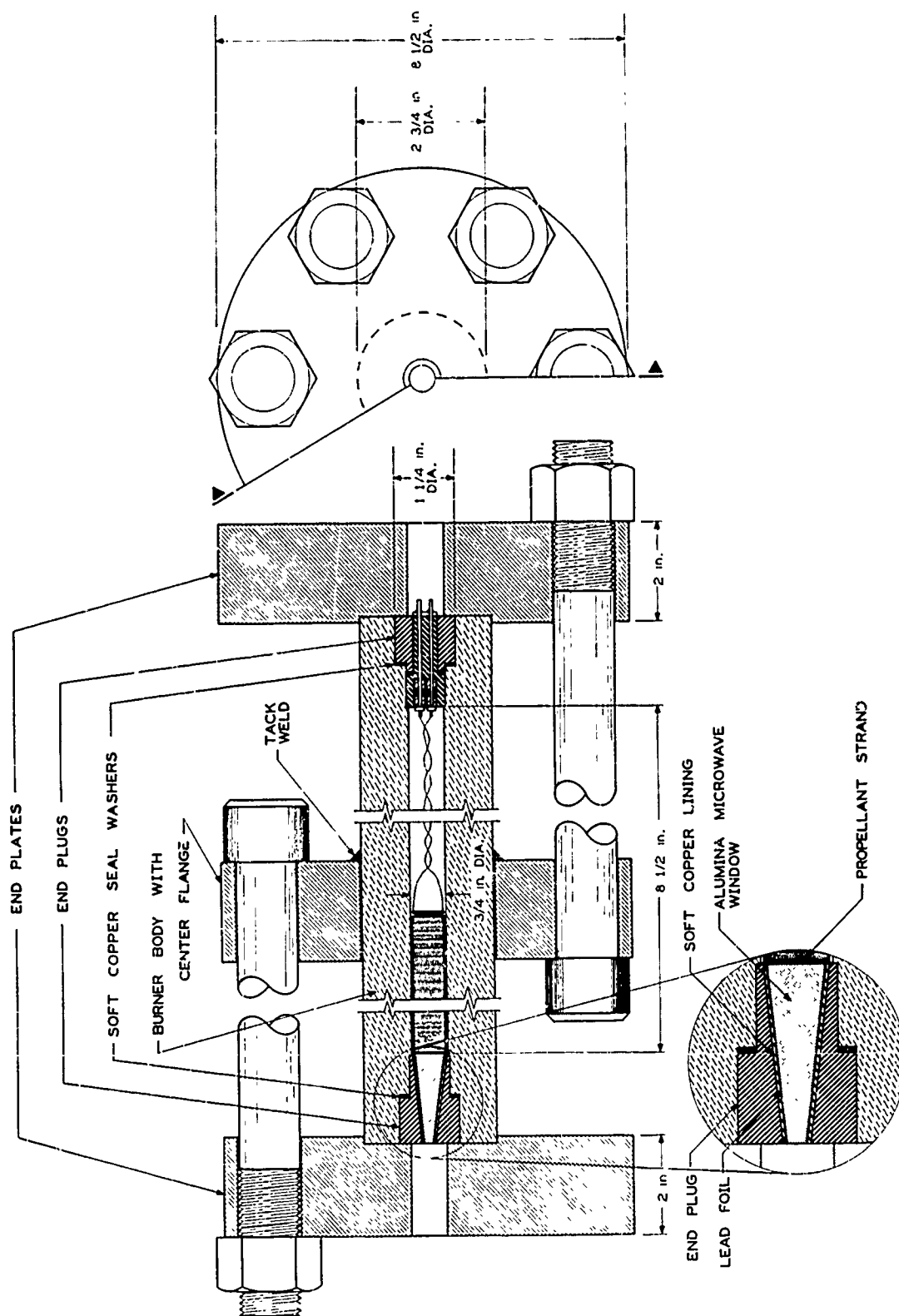


FIG. 1 HIGH PRESSURE STRAND BURNER WITH PROPELLANT STRAND, OVERALL VIEW

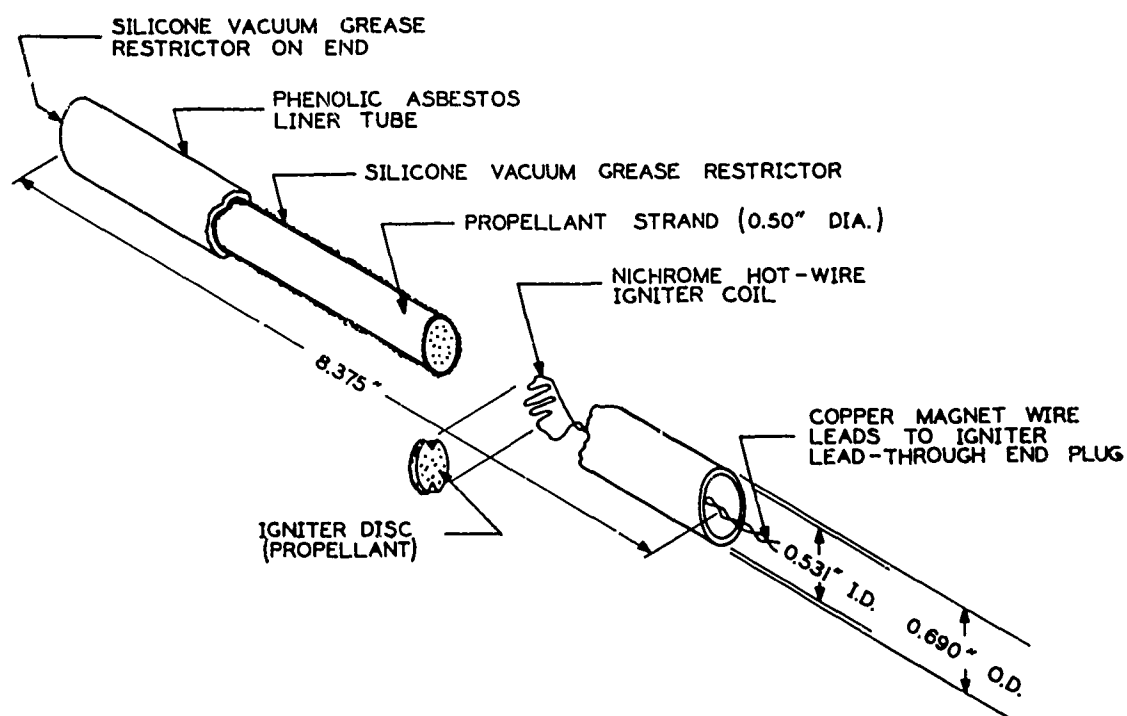


FIG. 2 PROPELLANT STRAND, IGNITER, AND LINER TUBE, EXPLODED VIEW

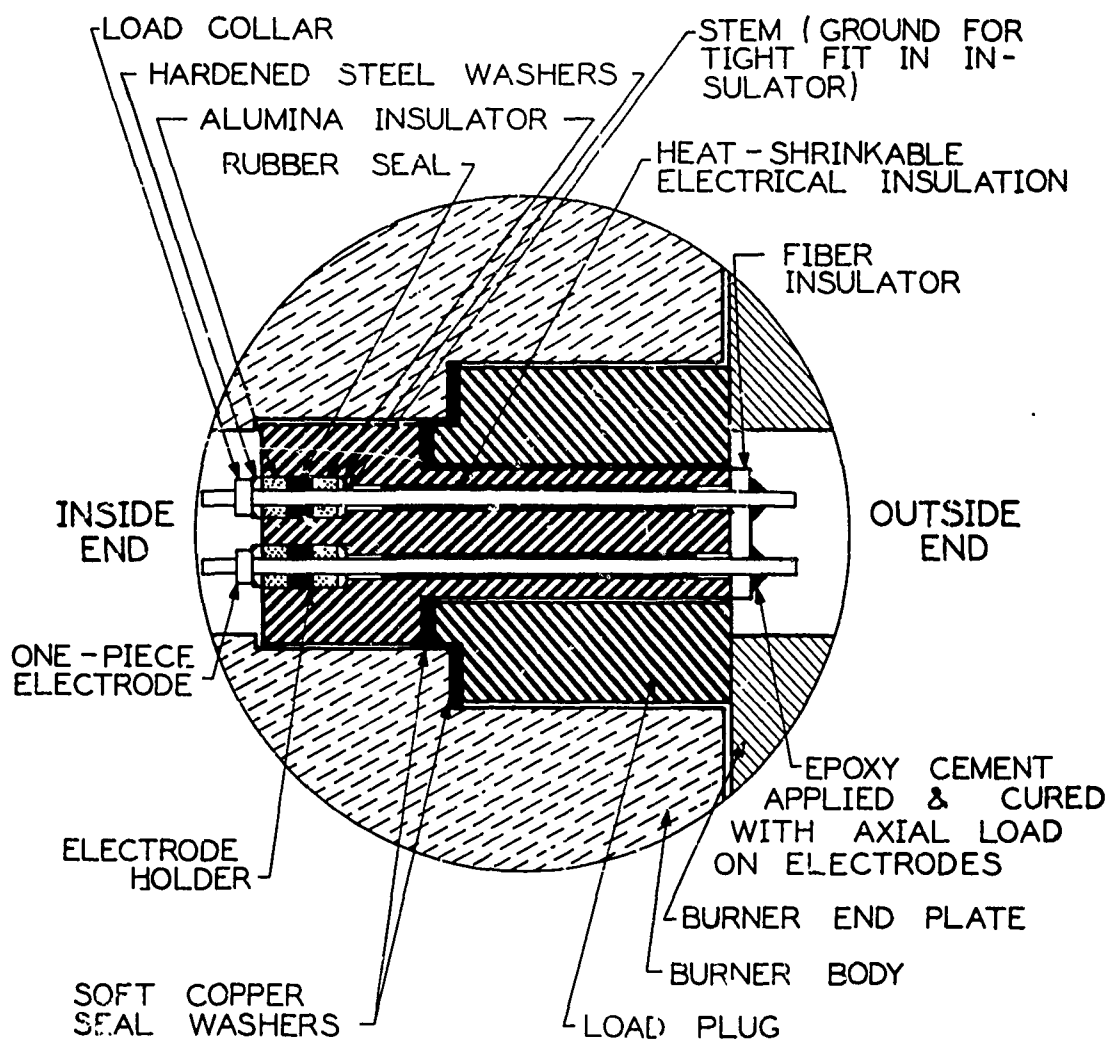


FIG. 3 LEAD-THROUGH END PLUG ASSEMBLY (FINAL)

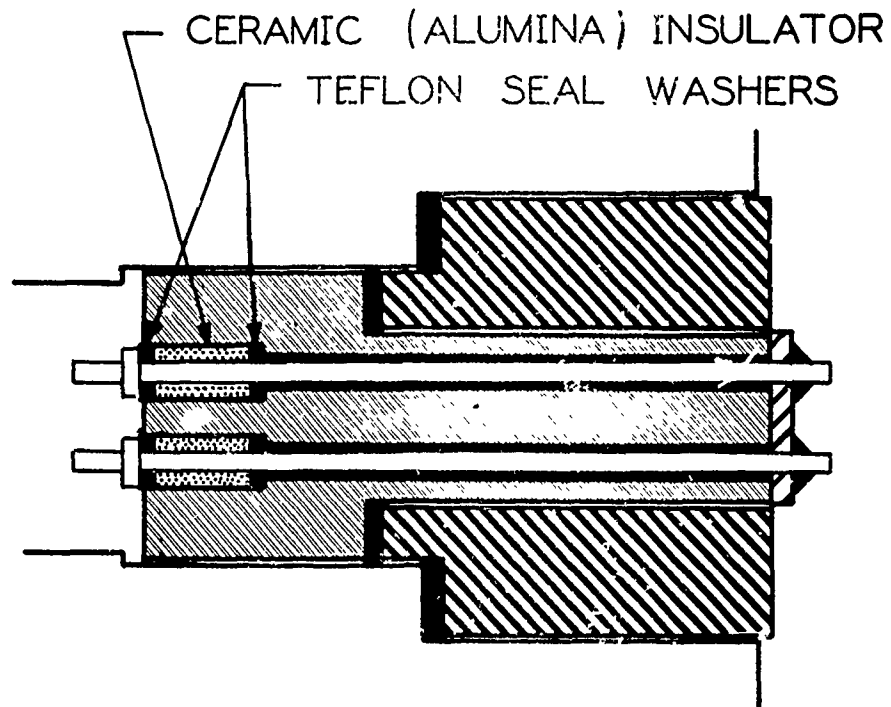


FIG. 4a LEAD-THROUGH END PLUG ASSEMBLY (ORIGINAL)

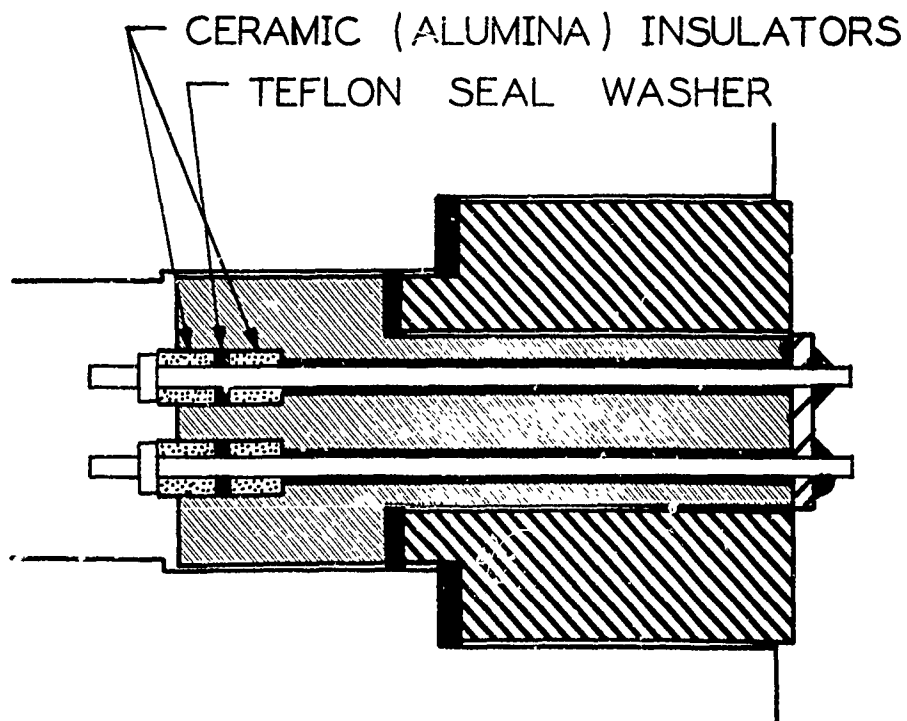


FIG. 4b LEAD-THROUGH END PLUG ASSEMBLY (MODIFIED)

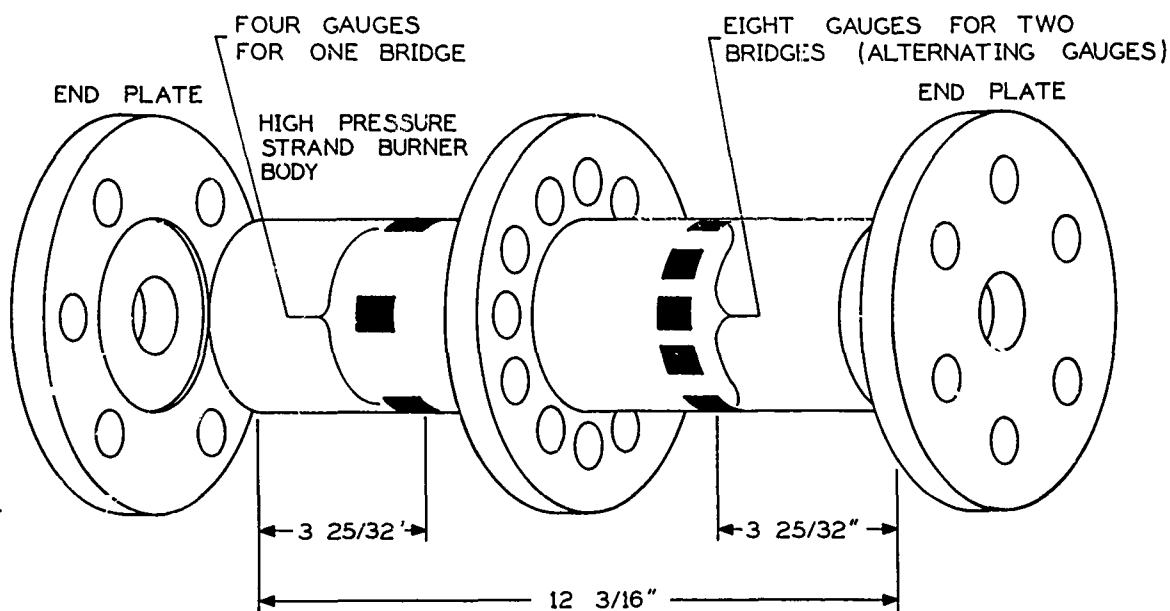


FIG. 5 PRESSURE TRANSDUCER STRAIN GAUGE LOCATIONS

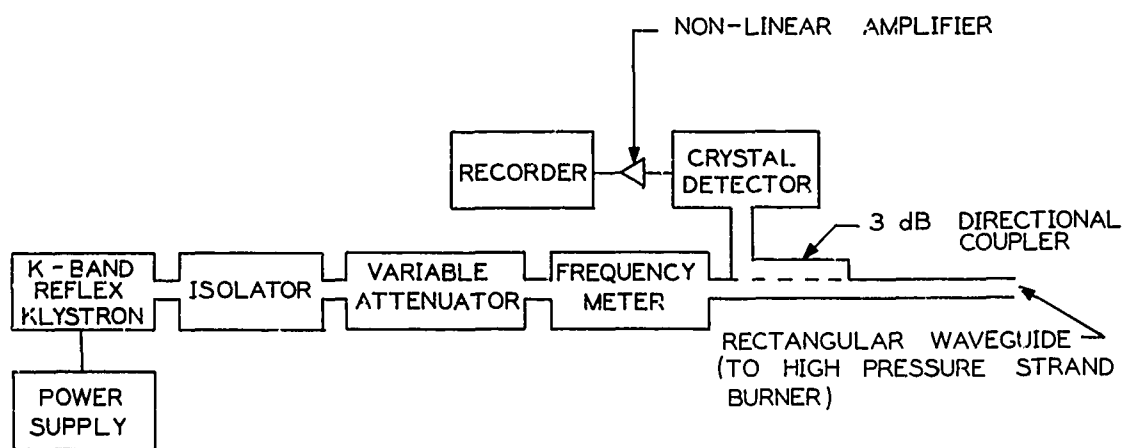


FIG. 6 BASIC MICROWAVE LAYOUT FOR BURNING RATE MEASUREMENT BY MICROWAVE INTERFEROMETRY

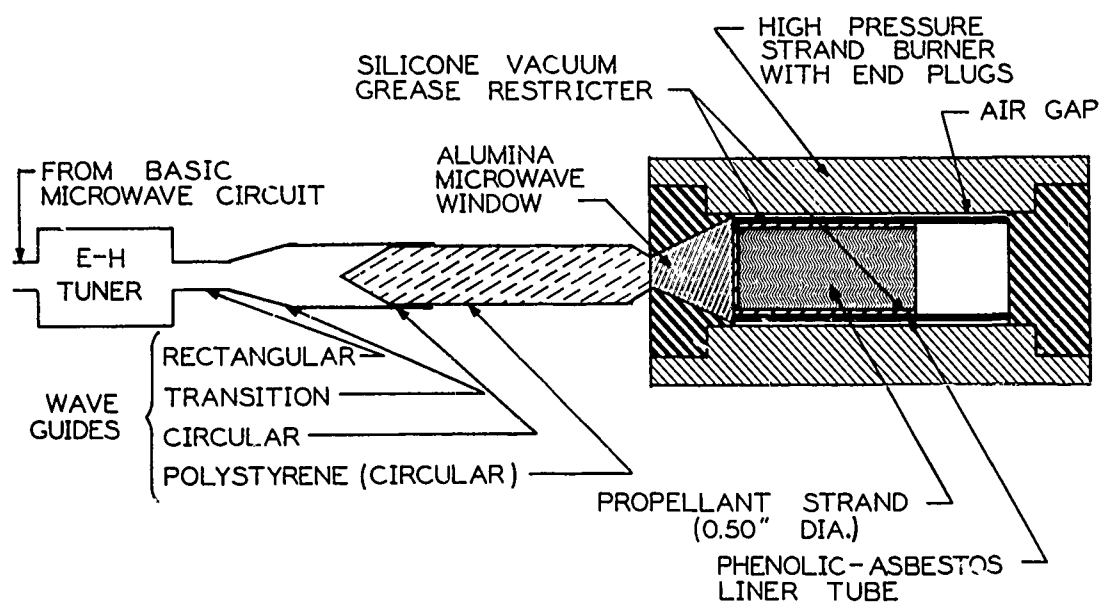


FIG. 7 ARRANGEMENT OF MICROWAVE COMPONENTS FOR HIGH PRESSURE STRAND BURNER BURNING RATE MEASUREMENT (NOT TO SCALE)

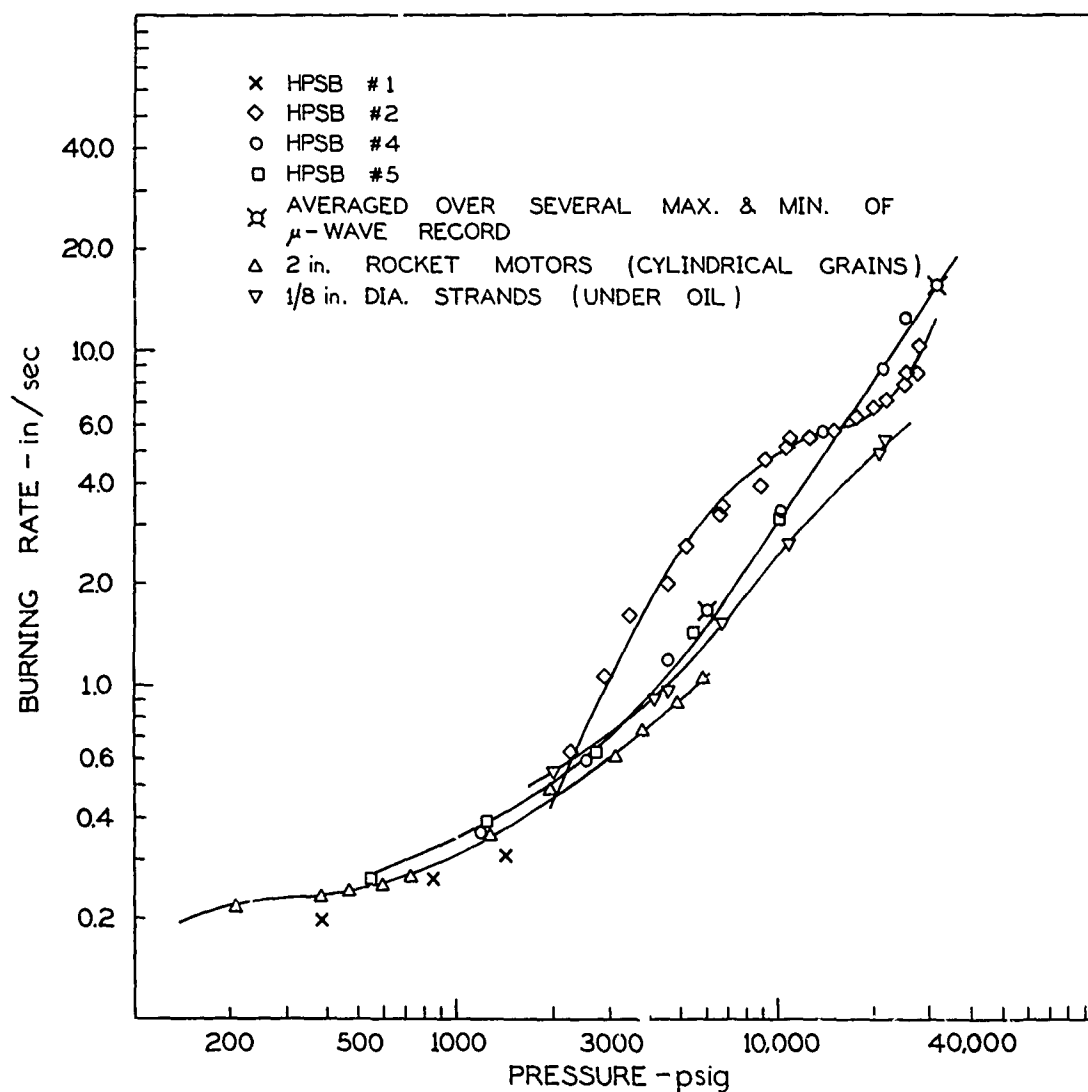


FIG. 8 r-P DATA FOR 75% (15 $\mu$ )AP PBAA-BINDER PROPELLANT FROM HIGH PRESSURE STRAND BURNER FIRINGS WITH r MEASURED BY MICROWAVE INTERFEROMETRY



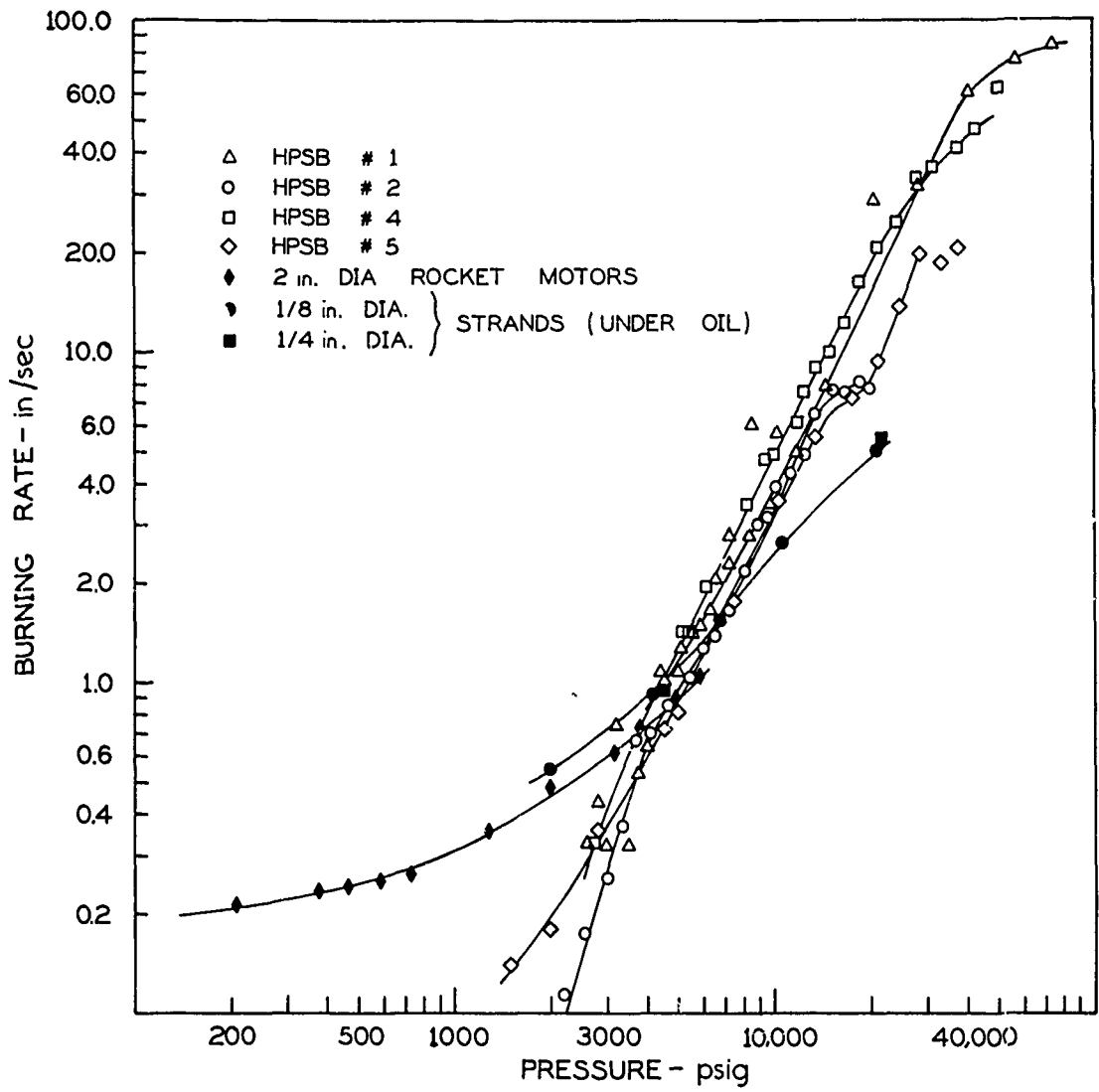


FIG. 9 r-P DATA FOR 75% (15 $\mu$ ) AP PBAA-BINDER PROPELLANT FROM HIGH PRESSURE STRAND BURNER FIRINGS WITH r CALCULATED FROM  $dP/dt$

## Appendix A

SOLID PROPELLANT BURNING RATE DETERMINATIONS FROM  
CLOSED BOMB PRESSURE HISTORY

To determine propellant burning rates from closed bomb pressure history, it is necessary, first, to predict the dependence of bomb pressure on the mass of propellant burned (and released as gas into the bomb cavity) and, second, to express the mass release rate term of this dependence in terms of the linear burning rate of the propellant. The first section of this appendix derives relations between bomb pressure and propellant mass (fraction) burned, and the second section relates the time rate-of-change of burned mass to a linear burning rate and hence results in a relation between burning rate and pressure rate-of-rise.

Pressure vs. Mass Fraction Burned<sup>1</sup>

Assume the validity of an Abel equation of state for the gaseous products of combustion

$$P(V-m\eta) = m \frac{R}{\bar{M}} T \quad (A-1)$$

where  $\eta$  is the covolume.

The changing pressure in the bomb during burning is then described by

$$\frac{dP}{P} = \frac{dm}{m} + \frac{dT}{T} - \frac{d\bar{M}}{\bar{M}} - \frac{d(V-m\eta)}{V-m\eta} \quad (A-2)$$

For negligible initial gas mass ( $m_o/m \ll 1$ )

Early in closed bomb burning the gas in the bomb chamber is a mixture of initial and burned (product) gases while after a short period of burning, the total gas mass present is composed almost entirely of combustion products. This section presents an analysis of the latter period of burning, i.e., that during which a negligible effect on pressure

<sup>1</sup>See Key to Nomenclature for symbols.

or pressure history is caused by the mass of gas originally (before ignition) in the "free volume" of the bomb. The next section considers the additional influences of this initial gas mass and shows them to be negligible even for moderate values ( $\leq 1$ ) of  $m_o/m$ .

Assuming, as an approximation, that the product gas composition is invariant with changing combustion pressure, and that the chamber gases are always well-mixed, then

$$T = T_{f, V} = \text{constant} \quad (\text{A-3})$$

$$\bar{M} = \text{constant}$$

and, therefore, from Eq. A-2

$$\frac{dP}{P} = \frac{dm}{m} - \frac{d(V-m\eta)}{V-m\eta} \quad (\text{A-4})$$

Differentiating the volume term of Eq. A-4 with the covolume,  $\eta$ , assumed constant and rearranging,

$$\frac{dP}{dm} = \frac{P}{m} - \frac{P}{V-m\eta} \left( \frac{dV}{dm} - \eta \right)$$

However, from Eq. A-1 and the definition of  $\rho_s$ , then

$$\frac{dP}{dm} = \frac{RT_{f, V}}{\bar{M}(V-m\eta)} - \frac{P}{V-m\eta} \left( \frac{1}{\rho_s} - \eta \right)$$

and since  $D \equiv \frac{m}{V_m}$  and  $\frac{RT_{f, V}}{\bar{M}} = P_m \left( \frac{V_m - m_m \eta}{m_m} \right) = P_m \left( \frac{1}{D} - \eta \right)$

$$\frac{dP}{dm} = \frac{P_m}{D(V-m\eta)} \left[ 1 - D\eta - D \left( \frac{1}{\rho_s} - \eta \right) \frac{P}{P_m} \right]$$

Rearranging and introducing  $m_m$  and  $Z \equiv \frac{m}{m_m}$ ,

$$\frac{d\left(\frac{P}{P_m}\right)}{1 - D\eta - D\left(\frac{1}{\rho_s} - \eta\right) \frac{P}{P_m}} = \frac{dZ}{1 - \frac{D}{\rho_s} + D\left(\frac{1}{\rho_s} - \eta\right)Z} \quad (\text{A-5})$$

If Eq. A-5 is integrated,

$$\frac{1}{1 - D \left( \frac{\frac{1}{\rho_s} - \eta}{1 - D\eta} \right) \frac{P}{P_m}} = 1 + \left( \frac{\frac{1}{\rho_s} - \eta}{1 - \frac{D}{\rho_s}} \right) DZ \quad (A-6)$$

which is the required relation between  $P$  and  $Z$  where  $P/P_m = 1$ ,  $Z = 1$  and  $P/P_m = 0$ ,  $Z = 0$  correspond to final and initial states, respectively. Further,

$$\frac{1}{1 - \frac{D}{\rho_s} \frac{P}{P_m}} = 1 + \frac{\frac{D}{\rho_s}}{1 - \frac{D}{\rho_s}} Z \quad \text{for } \eta = 0 \quad (A-7)$$

For appreciable initial gas mass ( $m_o/m \geq 0.1$ )

In the beginning stages of closed bomb firings, the initial gas mass of the original "free volume" of the bomb may be appreciable compared with the product gas mass present. Assuming that the chamber gas is near-perfect,<sup>1</sup>

$$\frac{dP}{P} = \frac{dm}{m} + \frac{dT}{T} - \frac{d\bar{M}}{\bar{M}} - \frac{dV}{V} \quad (A-8)$$

Since the initial gas is usually not at  $T_{f,V}$ , the temperature of the gas mixture in the bomb is variable, and, since internal energy must be conserved,

$$\begin{aligned} \bar{c}_{V,p} T_p dm &= \bar{c}_{V,p} T_{f,V} dm = d(\bar{c}_V m T) \\ &= \bar{c}_V T dm + m \bar{c}_V dT + m T d\bar{c}_V \end{aligned} \quad (A-9)$$

<sup>1</sup>Since the bomb pressure will typically be low when the initial gas mass is appreciable.

Therefore, 
$$\frac{dT}{T} = \left( \frac{\bar{c}_{V,p}}{\bar{c}_V} \frac{T_{f,V}}{T} - 1 \right) \frac{dm}{m} - \frac{d\bar{C}_V}{\bar{C}_V} + \frac{d\bar{M}}{\bar{M}} \quad (\text{A-10})$$

Further 
$$\frac{dV}{V} = \frac{dm}{\rho_s(V_o + V_b)} = \frac{\rho_o}{\rho_s} \frac{1 + \frac{m_o}{m_b}}{\frac{m_o}{m_b}} \left( \frac{1}{1 - \frac{V_b}{V_o}} \right) \frac{dm}{m} \quad (\text{A-11})$$

and since, typically,<sup>1</sup> 
$$\frac{m_o}{m_b} \cong O[10^{-1}]$$

$$\frac{\rho_o}{\rho_s} = O[10^{-3}]$$

and, therefore 
$$\frac{V_b}{V_o} = \frac{\rho_o}{\rho_s} \frac{m_b}{m_o} \leq O[10^{-2}]$$

then 
$$\frac{dV}{V} \cong \frac{\rho_o}{\rho_s} \frac{1 + \frac{m_o}{m_b}}{\frac{m_o}{m_b}} \frac{dm}{m} \quad (\text{A-12})$$

Substituting Eqs. A-12 and A-9 into Eq. A-8,

$$\frac{dP}{P} = \left( \frac{\bar{c}_{V,p}}{\bar{c}_V} \frac{T_{f,V}}{T} - \frac{\rho_o}{\rho_s} \frac{1 + \frac{m_o}{m_b}}{\frac{m_o}{m_b}} \right) \frac{dm}{m} - \frac{d\bar{C}_V}{\bar{C}_V} \quad (\text{A-13})$$

Considering the orders of magnitude of the terms of Eq. A-13

$$\frac{\bar{c}_{V,p}}{\bar{c}_V} \frac{T_{f,V}}{T} \geq O[1]$$

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<sup>1</sup>Where  $O[\dots]$  means "of the order of".

$$\frac{\rho_o}{\rho_s} = \frac{1 + \frac{m_o}{m_b}}{\frac{m_o}{m_b}} = O[10^{-2}]$$

That is, the volume change term,  $dV/V$ , in Eq. A-8 is typically negligible under the conditions considered here, and, therefore,

$$\frac{dP}{P} \approx \frac{\bar{c}_{V,p}}{\bar{c}_V} \frac{T_{f,V}}{T} \frac{dm}{m} - \frac{d\bar{c}_V}{\bar{c}_V} \quad (A-14)$$

$$\frac{dP}{P} \approx \frac{\bar{c}_{V,p}}{\bar{c}_V} \frac{\bar{M}}{\bar{M}_p} \frac{T_{f,V}}{T} \frac{dm}{m} - \frac{d\bar{c}_V}{\bar{c}_V}$$

where the variables  $\bar{c}_V$  are dependent on  $T$ .

Since the terms of the right-hand-side of Eq. A-14 contain the temperature-dependent variables  $\bar{c}_V$  and  $\bar{c}_{V,p}$ , the values of these terms depend on  $m_o/m_b$  in two ways. First  $\bar{c}_V$  depends on the composition of the mixture of  $m_o$  and  $m_b$ , independent of temperature, and, second,  $\bar{c}_V$  depends on the mixture temperature independent of composition. It is notable that, since a propellant isochoric flame temperature,  $T_{f,V}$ , is typically much higher than that of the initial gas, temperatures near  $T_{f,V}$  are achieved in closed bomb firings well before  $m_o/m_b$  is small. Therefore, it is reasonable, as an approximation, to consider the possible effects of  $m_o$  on composition and, consequently, on specific heats while chamber temperature effects on specific heats are neglected.

Since the major product gas components from under-oxidized or near-stoichiometric solid propellants are diatomic (e.g.,  $H_2$ ,  $CO$ ,  $N_2$ ,  $HCl$ ) as are the usual initial gases (air or  $N_2$ ), the molar specific heats of product and initial gases do not differ greatly at elevated temperatures. At  $1800^\circ K$ , for example, the isochoric, molar specific heat,  $\bar{c}_{V,p}$ , calculated for the

propellant used in this study, is about 6.86 cal/gm-mole-°K. At the same temperature, the isochoric specific heat of air, a typical initial gas, is about 6.58 cal/gm-mole-°K.<sup>1</sup> Therefore, as long as  $m_o/m_b$  is small enough that  $T/T_{f,V}$  is near unity (say,  $m_o/m_b \leq 1$ ), the value of  $\bar{C}_{V,p}/\bar{C}_V$  is also near unity (within 10%). Therefore, Eq. A-14 becomes

$$\frac{dP}{P} \approx \frac{\bar{M}}{\bar{M}_p} \frac{T_{f,V}}{T} \frac{dm}{m}$$

or

$$dP \approx \frac{T_{f,V}}{\bar{M}_p V} dm \quad (A-15)$$

Thus, as a valid first approximation for moderate as well as small values of  $m_o/m_b$ , it is permissible to neglect the effect of  $m_o$  on chamber pressure rise during burning, i.e.,  $m_o$  does not greatly affect the pressure rise,  $dP$ , associated with the addition of a mass,  $dm$ , to the chamber. Therefore, near the beginning of burning in a closed bomb ( $0.1 < m_o/m_b < 1$ ) the pressure rise  $dP/dm$  does not depend strongly on  $m_o/m_b$  or  $m_o$ , though the actual pressure level and its history,  $P(t)$  or  $P(m_b)$ , may.

Since Eq. A-15 can be shown to be a limiting form of Eq. A-5 for negligible covolume effects and negligible chamber volume changes during burning, it is apparent from the above discussion that the use of Eq. A-5 is reasonably valid even for  $m_o/m_b$  as high as unity.

#### Burning Rate vs. $dP/dt$

From Eqs. A-5 and A-6

$$dZ = \frac{(1 - \frac{D}{\rho_s})(1 - D\eta)}{[1 - D\eta - D(\frac{1}{\rho_s} - \eta)\frac{P}{P_m}]^2} d\left(\frac{P}{P_m}\right) \quad (A-16)$$

<sup>1</sup>J. H. Keenan and J. Kaye, Gas Tables, John Wiley & Sons, New York (1948), Table 2.

and, therefore,

$$\frac{dZ}{dt} = \frac{(1 - \frac{D}{\rho_s})(1 - D\eta)}{[1 - D\eta - D(\frac{1}{\rho_s} - \eta)\frac{P}{P_m}]^2} \frac{d(\frac{P}{P_m})}{dt} \quad (A-17)$$

But, for a cylindrical strand burning planarly in the axial direction,

$$Z = \frac{x}{L} = \frac{m_b}{m_p}$$

$$\text{and, thus } r \equiv \frac{dx}{dt} = L \frac{dZ}{dt} \quad (A-18)$$

Substituting Eq. A-18 into Eq. A-17 and rearranging,

$$r = \frac{L(1 - \frac{D}{\rho_s})(1 - D\eta)}{[1 - D\eta - D(\frac{1}{\rho_s} - \eta)\frac{P}{P_m}]^2} \frac{d(\frac{P}{P_m})}{dt} \quad (A-19)$$

which is suitable for determining  $r$  from  $P(t)$ .

If  $\eta = 0$ , Eq. A-19 reduces to

$$r = \frac{L(1 - \frac{D}{\rho_s})}{[1 - \frac{D}{\rho_s}(\frac{P}{P_m})]^2} \frac{d(\frac{P}{P_m})}{dt} \quad (A-20)$$

In the light of the discussion of the section above, Eqs. A-19 and A-20 are reasonably valid not only for  $m_o/m_b \ll 1$ , but also for  $m_o/m_b \leq 1$ .



## KEY TO NOMENCLATURE

$\eta$	=	propellant product gas covolume
$c_v$	=	isochoric specific heat capacity
$C_v$	=	isochoric, molar heat capacity
$D$	=	closed bomb loading densit, (mass of propellant/total bomb volume)
$L$	=	original length of propellant strand
$m$	=	mass
$\overline{M}$	=	mean molecular weight of gas mixture
$P$	=	pressure
$R$	=	ideal gas constant
$t$	=	time
$T$	=	temperature
$V$	=	volume
$x$	=	distance burned along propellant strand
$Z$	=	propellant mass fraction burned
$\rho$	=	mass density

Subscripts

$b$	=	property of burned propellant (solid)
$f$	=	property of propellant flame
$m$	=	property of maximum pressure closed bomb state
$o$	=	property of original "free volume" gas
$p$	=	property of propellant product gas
$s$	=	property of solid propellant
$V$	=	property or result of isochoric process

## Appendix B

### BURNING RATE MEASUREMENT BY MICROWAVE INTERFEROMETRY

With respect to interactions with electro magnetic radiation, typical solid propellants can be considered as dielectric materials. A propellant can be characterized in electromagnetic terms by its complex dielectric constant,

$$\epsilon = \epsilon' - i\epsilon'' \quad (\text{B-1})$$

The real part,  $\epsilon'$ , of this dielectric constant determines the wavelength of electromagnetic radiation in the material, the wavelength being independent of  $\epsilon''$  to the first order.<sup>1</sup> Similarly the imaginary part,  $\epsilon''$ , determines, in the first approximation, the electromagnetic attenuation or transmission loss of the material.

As with any dielectric material, interfaces between the propellant and other materials (solid, liquid, or gas) of different dielectric properties will reflect, transmit, and refract electromagnetic waves; the burning surface of a solid propellant sample is such an interface. A propellant burning surface can, however, be considered as a geometrical (infinitesimally thin) interface only if irregularities on it are less than the electromagnetic wavelength considered. This is the case for the typical propellant burning surface if microwave-frequency electromagnetic radiation is considered; wavelengths of order 1 centimeter are typical for this radiation, while burning surface irregularities are, in contrast, expected to be of order 0.01 centimeters or less.

With respect to microwave radiation the burning surface of a solid propellant may be modeled not only as a geometrical interface but also, as an interface between two uniform dielectrics, one the unburned solid propellant, the other, the hot combustion product gases. This is a consequence of the fact that the thermal wave in the solid phase of a burning propellant as well as

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<sup>1</sup> C. G. Montgomery, R. H. Dicke, E. M. Purcell, Principles of Microwave Circuits, McGraw-Hill, New York (1948), p. 366.

the gas-phase reaction zone are, at combustion pressures above atmospheric, usually much smaller than common micro-wave wavelengths. The dielectric regions used to model a burning solid propellant may also be considered as homogeneous and isotropic since the scale of heterogeneity in both solid and gas phases is much less than the microwave wavelength in the propellant.

Since a propellant burning surface may be considered to act as a dielectric interface and hence as a reflector of microwave radiation, it is possible to determine burning surface displacement as a function of time (propellant regression rate) using microwaves reflected from this surface. The following sections describe several of the concepts and techniques applicable to such determinations with stress on applications to strand burning rate measurement in closed bombs.

#### Conceptualizations of Regression Rate Measurement Using Microwaves

Consider, for simplicity, a fixed, semi-infinite region of dielectric material (solid propellant) and a moving boundary (the burning surface). This boundary is the demarcation between the solid propellant and the region of combustion products which is presumed to be of a different, homogeneous dielectric material. If plane-polarized, coherent, monochromatic electromagnetic radiation is normally incident on the boundary, some of this radiative power will be reflected and propagate in a direction opposite to that of the incident beam; the remainder will be transmitted.

It is reasonable to concentrate attention on radiation incident from the direction of the solid phase, since the dielectric region of hot combustion products in the model may be expected to attenuate incident microwave radiation more than the solid region, owing to the presence of ionized combustion product species in the product gas phase. Since solid propellant materials are frequently of low attenuation, microwave radiation incident on the surface from the propellant side and reflected back through the propellant is often readily detectible, analyzable, etc. Attenuation by the solid phase is not negligible, however, and must eventually be considered but discussion will be postponed until later. For the present solid propellants will be regarded as ideal, loss-free dielectrics.

If the semi-infinite propellant configuration is made more realistic with regard to configurations suitable for surface regression rate measurement, a propellant slab of finite thickness must be considered. Radiation may now be considered to be normally incident on one, fixed surface of the slab (the entrance or base surface), the combustion zone boundary surface moving relative to it (Fig. B-1). Some of the incident radiation will be reflected (directly) by the entrance surface, and some will be transmitted through the propellant, reflected from the burning surface, and eventually will propagate back toward the source, and be superimposed on the directly-reflected radiation from the entrance surface. For initial simplicity, it will be assumed that the burning surface is totally reflecting and that the propellant is an ideal, nonattenuating dielectric ( $\epsilon_2'' = 0$ ). Region 1 of Fig. B-1 is presumed to be free space.

Several alternative routes exist leading to understanding the use of the various incident and reflected microwaves to determine burning surface regression rates. Such alternatives are:

- (1) Microwave interferometry
- (2) Microwave amplitude modulation via "beat" phenomena
- (3) Microwave standing wave displacement via a moving reflector.

Each can be illustrated by considering the output of a crystal, "square-law" detector located in the region from which microwaves are incident on the stationary face of the propellant sample (Region 1 in Fig. B-1). All the alternative conceptualizations are equivalent, and differ only in the points of view taken in visualizing them.

Analogous to optical interferometry, two reflected waves, the one from the stationary propellant surface and the other from the moving (burning) surface, may be superimposed and their resultant found to be dependent on the relative magnitudes and phases of the two individual waves. When the microwave path length in the propellant (twice the slab thickness) changes by the microwave wavelength in the propellant,  $\lambda_p$ , the phase angle between the two reflected waves goes through a shift of  $2\pi$ . Thus in this process the directly and indirectly

reflected waves pass, for example, from reinforcement to interference and back to reinforcement. The output of a detector sensing the resultant electric field of these two waves will, therefore, undergo one oscillation for each change in path length of  $\lambda_p$  i.e., for each displacement of the burning surface by  $\lambda_p/2$ . Therefore, the frequency of the oscillating signal from the detector,  $\nu_S$ , is related to the regression rate,  $r$ , by

$$\nu_S = \frac{r}{\lambda_p/2} \quad (\text{B-2})$$

or

$$r = \frac{\lambda_p}{2} \nu_S \quad (\text{B-3})$$

The regression rate of the surface may, therefore, be determined from the frequency of such an oscillating signal. The relationship between these variables is illustrated in Fig. B-2.

Alternatively, one may consider the "beat" frequency resulting from the superposition of the two reflected waves, the one reflected from the regressing surface being at a slightly different frequency from the wave reflected from the stationary face of the propellant. This is a consequence of the Doppler shift experienced by waves reflected from a moving reflector. This frequency shift,  $\Delta\nu$ , can be expressed by

$$\nu_R - \nu_O \equiv \Delta\nu = \nu_O \left( \frac{1 + \frac{r}{c}}{1 - \frac{r}{c}} \right) - \nu_O = \nu_O \left( \frac{2 \frac{r}{c}}{1 - \frac{r}{c}} \right) \quad (\text{B-4})$$

where  $\nu_O$  is the frequency of the first, directly reflected wave,  $\nu_R$  is the frequency of the wave reflected from the moving boundary, and  $c$  is the speed of light in a vacuum.<sup>1</sup> However, for non-relativistic regression rates ( $r/c \ll 1$ ), and since  $\nu_O = \frac{c}{\lambda_p}$ , then,

$$\Delta\nu = \frac{2r}{\lambda_p} \quad (\text{B-5})$$

<sup>1</sup> This expression is obtained by considering the reflector first as a moving "observer" with respect to the microwave source and then, as a moving "source" with respect to the detector. See G. Joos, Theoretical Physics, Hafner, New York (1950), p. 243.

Therefore, the resultant of the superimposed reflected waves will be a single wave at the microwave frequency,  $\nu_O$ , modulated at a beat frequency,  $\Delta\nu$ . A crystal detector sensing this resultant will show a signal frequency,  $\nu_S$ , equal to  $\Delta\nu$ , and this situation is equivalent to that attributed above to a simple interferometric effect.

A more complicated but tenable view of the process is that involving the displacement of standing waves in response to motion of the burning surface. This view is a consequence of the consideration of standing waves and standing-wave power ratios common in the microwave literature<sup>1</sup> though standing waves are also well-known optical reflector phenomena.<sup>2</sup> If the combustion zone boundary were not moving, a standing wave resulting from superposition of the resultant of the two reflected waves and the single incident wave could be observed in Region 1 of Fig. B-1. Such an observation might be made by traversing a detector probe in a direction normal to the propellant surface. Different relative positions of the reflecting surface (different slab thicknesses) would result in different phases of the waves reflected from them and, hence, in different phases and amplitudes for the resultant reflected wave relative to the incident one. Therefore, the standing waves resulting from superposition of incident and resultant reflected waves would vary in amplitude and in location, being both displaced and modulated as the movable, reflecting propellant surface is displaced. The signal from a detector fixed in Region 1 and sensing the superimposed waves would, therefore, exhibit a periodicity equal to that of the resultant of the two reflected waves, i. e., equal to the frequency of displacement of the moving reflector by increments of  $\frac{\lambda_p}{2}$  or,

$$\nu_S = \frac{r}{\lambda_p/2} \quad (B-6)$$

This view is obviously equivalent to the interferometric or Doppler shift views.

<sup>1</sup> C. G. Montgomery et al., op. cit., p. 61.

<sup>2</sup> M. Born and E. Wolf, Principles of Optics, Pergamon, New York (1st Edition, 1959), p. 276.

In summary, therefore, it has been demonstrated that a crystal detector in the free space (Region 1, Fig. B-1) preceding a finite propellant slab (Region 2) will produce a signal of frequency  $\nu_S = 2r/\lambda_p$  if the slab is bounded by a perfect reflector moving at velocity  $r$ . Hence, the regression rate of a propellant burning surface may be determined from measurements of the detector signal frequency if the microwave wavelength in the propellant is known. Inherent in the technique is the determination of regression rate in dynamic systems, e.g., deflagration under varying combustion pressure. In these cases, since  $r$  (and likewise  $\nu_S$ ) changes with time, average values of  $r$  are readily obtained. These average values correspond to the regression rate at some state intermediate to those corresponding to the successive maxima and minima of the oscillating detector signal, the maxima and minima being used to measure signal frequency. Theoretically, if the shape of the oscillating signal is predictable, instantaneous burning rates may also be determined.

If the moving combustion zone interface is not a perfect reflector, the microwave detector signal may be complicated by additional reflections from the region beyond the second slab surface (Region 3). There are additional complications if the propellant and microwave system cannot be modeled properly by the one-dimensional configuration illustrated in Fig. B-1.

#### Propellant Absorptivity Effects

The complex dielectric constant of a material (say, propellant), while characterizing the electrical properties of the material, is not a convenient representation of its properties for interferometric purposes. Specification of the material properties in terms of a complex refractive index,

$$n = n' + in'' \quad (B-7)$$

is more convenient for optical purposes such as interferometry. This complex refractive index, which is directly relatable to the complex dielectric constant,<sup>1</sup> is composed of a real part which specifies the speed of propagation,  $v$ , and the wavelength,  $\lambda$ , of electromagnetic radiation in the material ( $n' = v/c = 2\pi c/\omega\lambda$ )

<sup>1</sup> M. Born and E. Wolf, op. cit., p. 610.

and an imaginary part which is related to the Beer's Law absorption coefficient,  $\chi$ , of the material ( $n'' = c\chi/2\omega$ ). On this basis, it may be shown that the attenuation of microwaves by a dielectric such as a propellant is suitably expressed by an exponential attenuation factor of the Beer's Law type. Thus, the oscillating signal amplitude,  $W$ , from a crystal detector in an interferometric configuration situation as described previously, contains a factor involving the absorption coefficient,  $\chi$ ,<sup>1</sup>

$$W = W^0 \exp(-2\chi x) \quad (B-8)$$

where  $x$  is the distance between the moving and stationary interfaces and  $W^0$  is the amplitude of the oscillating signal in the absence of absorption loss (Fig. B-2). Therefore, in a case where the dielectric propellant used for microwave interferometry is absorptive, the crystal detector output would display an amplitude increasing with time (Fig. B-3).

The extent to which absorption losses affect interferometric signal level must be accounted for in measuring propellant regression rate. This extent is indicated by the change in signal level during one oscillation of the signal (one half-wavelength displacement of the burning surface). This change must be small if attenuation is to be considered of negligible effect on the phase of the observed signal, i. e., on the positions of maxima and minima of the time-varying signal. Such was the case with the propellant formulation used in the studies of this report, and therefore, attenuation effects were neglected. This is not to say that detector signal outputs were of constant amplitude during high pressure strand burner experiments but only whatever changes did occur could not be attributed to propellant absorptivity. Neglect of absorptive effects was justified by the observation that signal level changes were negligible during low (constant) pressure firings in the same configuration as was used for high pressure strand burner firings.<sup>2</sup>

Aside from the question of absorption effects on the accuracy of interferometric regression rate measurement, another aspect of microwave

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<sup>1</sup>ibid., p. 611.

<sup>2</sup>See page 12 of the main body of this report.



absorption is important: If propellant absorptivity of microwave radiation is too high, it is impossible to obtain sufficiently high signal levels during the regression of the propellant over more than one or two wavelengths. Conceivably, microwave power might be increased to very high levels to overcome signal attenuation by the propellant, but the cost, inconvenience, and effect of absorbed power on the propellant are likely to be excessive in many such cases. During this study no investigation of these trade-offs was made. It was, however, determined that common plastisol-nitrocellulose propellants could not be studied since practical reasonable concentration of common nitrate-ester plasticizers resulted in attenuations too high<sup>1</sup> to allow use of 250 mW of microwave power which was available.

#### Influences of System Configuration

The previous discussion has shown that plane-polarized, coherent, monochromatic microwave radiation can be used interferometrically to determine the displacement of a microwave-reflecting interface between regions of different dielectric constants. The approach will now be extended to include the measurement of interface displacements for confined, two-dimensional dielectric samples -- specifically a burning surface moving inside a propellant-filled metal tube (Fig. B-4). This configuration is a practical one for propellant strand burning experiments, having been used with some modification for the experiments described in the main body of this report. The equivalent microwave configuration, the dielectric-filled waveguide, is a configuration widely-discussed in the microwave literature.

The complex electromagnetic waves which propagate in two- and three-dimensional confined regions (along waveguides) may be synthesized by a superposition of simple, plane electromagnetic waves. This superposition of waves is required in order to satisfy the differential equations and associated boundary conditions for electromagnetic wave propagation. These complex waves may, even when produced by a monochromatic microwave source, be

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<sup>1</sup> See page 15 of the main body of this report.

of different types or modes. The dimensional scales (wavelengths) and the shapes of the electric and magnetic fields associated with these modes depend not only on the electromagnetic properties of the medium through which the waves propagate but also on the physical dimensions of the waveguide. Several such complex waves of different types and wavelength may propagate simultaneously in the same waveguide.

Many aspects of the individual modes of microwave propagation in waveguides have been described using "equivalent" one-dimensional waves, and by employing a different simple wave for each mode. Such equivalent waves are, for example, used in the classic transmission-line approach to the analysis of microwave power transmission by waveguides.<sup>1</sup> Analogous to the description of these effects in the preceding section, equivalent waves may also be used to interpret interferometric, Doppler-shift, or standing-wave effects. Treatment of these effects in terms of equivalent, one-dimensional waves necessitates accounting for the influences of waveguide shape and size in addition to that of the substance filling the waveguide. Eq. B-3 expresses the relation between the regression rate of a propellant burning surface, the signal frequency from a square-law detector, and the microwave wavelength in the propellant for a one-dimensional propellant and microwave configuration. If propellant-filled waveguides are considered, a similar expression may be derived.

$$r = \frac{\lambda_{p,g}}{2} \nu_S \quad (B-9)$$

with the one-dimensional wavelength,  $\lambda_p$ , replaced by the effective wavelength  $\lambda_{p,g}$ . This latter wavelength — of the complex, modal radiation pattern in dielectric-filled waveguides — is a function not only of the dielectric properties of the propellant but also of the dimensions and shape of the guide as well. The magnitude of  $\lambda_{p,g}$  also depends on the mode of propagation being considered. Values have been determined for a variety of guide shapes and

<sup>1</sup> C. G. Montgomery et. al., op. cit., p. 60.

and for waveguides filled with either uniform or composite dielectrics;<sup>1</sup> the various modes to which these wavelengths correspond have been analyzed in detail. Since monochromatic radiation is convenient for interferometry and since a knowledge of its wavelength is essential to determination of quantitative interferometric data, the possibility of several coexisting modes of propagation presents two problems in propellant regression rate measurement. First, it is almost essential to achieve single-mode microwave operation in such experiments, and second, it is necessary to establish the wavelength of the microwave mode used.

The problem of launching a single mode of propagating microwaves in the sample is eased by a fundamental characteristic of the various modes propagated in dielectric-filled waveguides, namely, that each mode exhibits a "cut-off" frequency (or wavelength) in a given waveguide configuration and transmission medium. Propagation in a given mode is possible only if the frequency of microwaves in the dielectric within the waveguide is greater than the waveguide cut-off frequency of that mode.<sup>2</sup> Conversely, consideration of microwave energy of a given frequency (but with the waveguide allowed to vary in dimension) shows a minimum waveguide dimension for which microwave energy of a given mode can be propagated in the guide. As the dimension of a waveguide is increased, first one and then more and more propagating modes can appear. For any one waveguide shape, there is one so-called dominant mode, which exhibits, at a given frequency, a minimum waveguide dimension at cut-off. It is, therefore, possible in practice to achieve a single mode of microwave propagation by judicious adjustment of waveguide dimensions. Although waveguide considerations did not greatly enter in to the

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<sup>1</sup> A. F. Harvey, Microwave Engineering, Academic Press, New York (1963), p. 11.

<sup>2</sup> ibid., p. 13.

closed-bomb design in advance, the appearance of regular single wave forms in the course of experiments was considered satisfactory evidence of single-mode microwave propagation.

The problem of determining the wavelength of the propagating mode can be solved in a simple, dielectric-filled waveguide configuration by calculations based on a measurement of the dielectric constant. In complex configurations, such as those involving a composite dielectric, such calculations are not easily carried out. Multi-component composite dielectric structures are, however, necessary in propellant combustion studies owing to the necessity of coatings of restrictor on strand sides, insulating liners to minimize heat loss, etc. This makes direct experimental wavelength determination mandatory; such determinations are accomplished by observations during burning over a measured length of propellant.

#### Effect of a Microwave-Transparent Burning Surface

In the discussion up to this point the combustion zone boundary has been treated as totally reflecting. However, since the combustion products constitute a dielectric gas rather than a perfect electrical conductor, it is possible that not all incident power is reflected from the surface; appreciable power may be transmitted through it. Thus, for example, in the high pressure strand burner used in this study, microwave power can, in principle, be transmitted by the burning surface, passed through the hot product gases, be reflected from the closed end of the burner, returned through the product gas, burning surface, and propellant, and be superimposed on the directedly reflected signal (Fig. B-4). Unless the combustion gas is highly absorptive, these secondary reflections can seriously complicate attempts at interferometric measurements of the burning surface; the desired signal could be completely obscured. Since it is difficult to predict the dielectric properties of propellant product gases and thereby to determine the magnitude of such an effect, the extent of burning surface transmission was clarified empirically.

The fact that only regular, single-frequency microwave signals have been observed during the closed bomb firings of this study implies either (1) almost total reflection from the burning surface or (2) nearly complete

transmission by the surface with subsequent reflection from the end plate; but no appreciable degree of superposition is indicated. The possible sources of the observed interferometric microwave signal then become: (I) direct reflection from the burning surface with negligible transmission through the boundary, (II) appreciable transmission by the surface followed by attenuation to negligible levels, owing to high-loss properties of the product gases, (III) negligible reflection and appreciable transmission at the burning surface, followed by little attenuation in the gas phase and ultimate reflection from the end of the closed bomb. Consideration of the extent of transmission effects can be limited to deciding which of these alternatives is effective, and the possibility of simultaneous effects of both types was not considered.

Either (I) or (II) is acceptable for regression rate determinations using an effective microwave wavelength characteristic of the propellant-filled waveguide and nearly constant with changing pressure. The third alternative would result, especially at high pressures, in an erroneous burning rate if the wavelength were assumed pressure-independent. Therefore, only the last was investigated further.

Two types of experiments were carried out to assure that observed interferometric signals were not a consequence of burning surface transmission via III. In the first experiments, compound strands of propellant were fired at atmospheric pressure, one half of each strand length was of the normal propellant formulation; the other half was of the same formulation with  $\frac{1}{2}\%$  KCl added, this concentration calculated to give a product-gas electron concentration  $10^5$  greater than that of the untreated propellant. Except for a slight disruption of the microwave record when the burning surface passed through the cement layer between the strands, there was no apparent change in the oscillation frequency of the signal or in the signal amplitude, which varied only slowly during the burning over the entire 4-inch length.

These results implied that, if microwave transmission by the burning surface were significant, gas-phase attenuation was so low that a change in electron density of the product gases of  $10^5$  had little effect. Hence, for the product gases to affect the interference signal, they would have to behave

almost as a loss-free, "ideal" dielectric ( $\epsilon'' \ll \epsilon'$ ). All that remains, then, is to determine the magnitude of  $\epsilon'$  or  $n'$ , the real parts of the dielectric constant and the refractive index of the product gases.

The real part of the dielectric constant or refractive index of gases, at moderate pressures and in the absence of absorption, is near unity,<sup>1</sup> unless the frequencies are high enough to result in resonant absorption by rotational energy levels of the gas molecules. In view of the results of the firings, resonant absorption would have to be either so strong as to preclude transmission effects (II) or negligible, as indicated by the low changes in signal level during firing at atmospheric pressure. Therefore, the propellant product gas could only be interferometrically "active" by III if its refractive index or dielectric constant were near unity. Accordingly, an experiment was carried out to determine what the effective wavelength in the high pressure strand burner would have been if the interferometric record observed had been due to a changing pathlength in product gases of refractive index near 1. In this experiment a movable circular reflector was inserted in the open end of a normal closed bomb body which had been loaded with propellant in the normal configuration. This reflector was displaced various distances within the cavity while the output of a crystal detector in the usual location was observed. The wavelength of the burner cavity filled by a material (air) of microwave-frequency refractive index very near unity was found to be  $0.674'' \pm_{-0.010}^{+0.008}''$  based on five measurements with reflector motion over 5 to 11 half-wavelengths. Since this wavelength is much greater than that which was measured in atmospheric-pressure burning experiments and that which gave reasonable low-pressure r-P data in closed bomb firings (0.252 inches), it was established that interferometric effects via burning surface transmission of microwaves and consequent reflection back through the burning surface (III) were negligible. This view was reinforced by the observation that detector output, while oscillating with reflector motion, decreased considerably as the reflector was moved away from the propellant sample inside the cavity. This

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<sup>1</sup> C. J. F. Boettcher, Theory of Electric Polarization, Elsevier, New York (1952), p. 183.

implied that the phenolic-asbestos liner tube was highly-attenuating and also could be expected to decrease the possible influence of transmitted radiation.

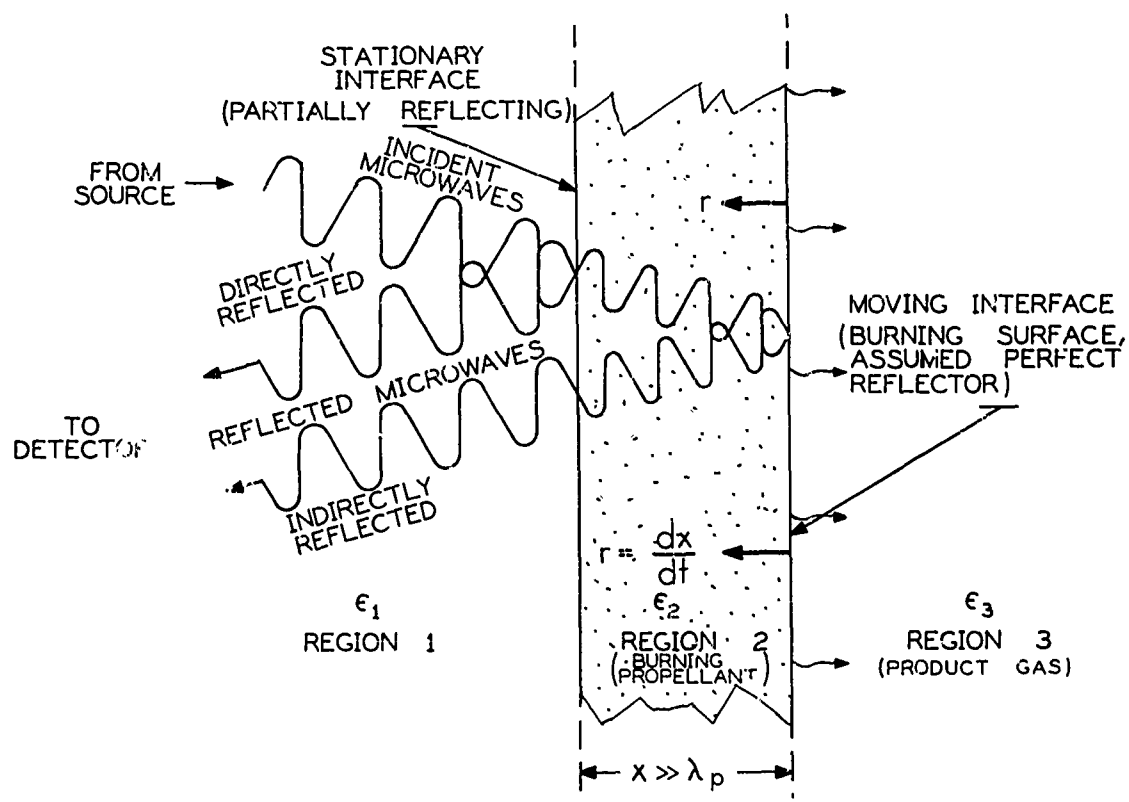


FIG. B-1 SCHEMATIC OF MICROWAVE INTERACTION WITH SLAB OF BURNING SOLID PROPELLANT



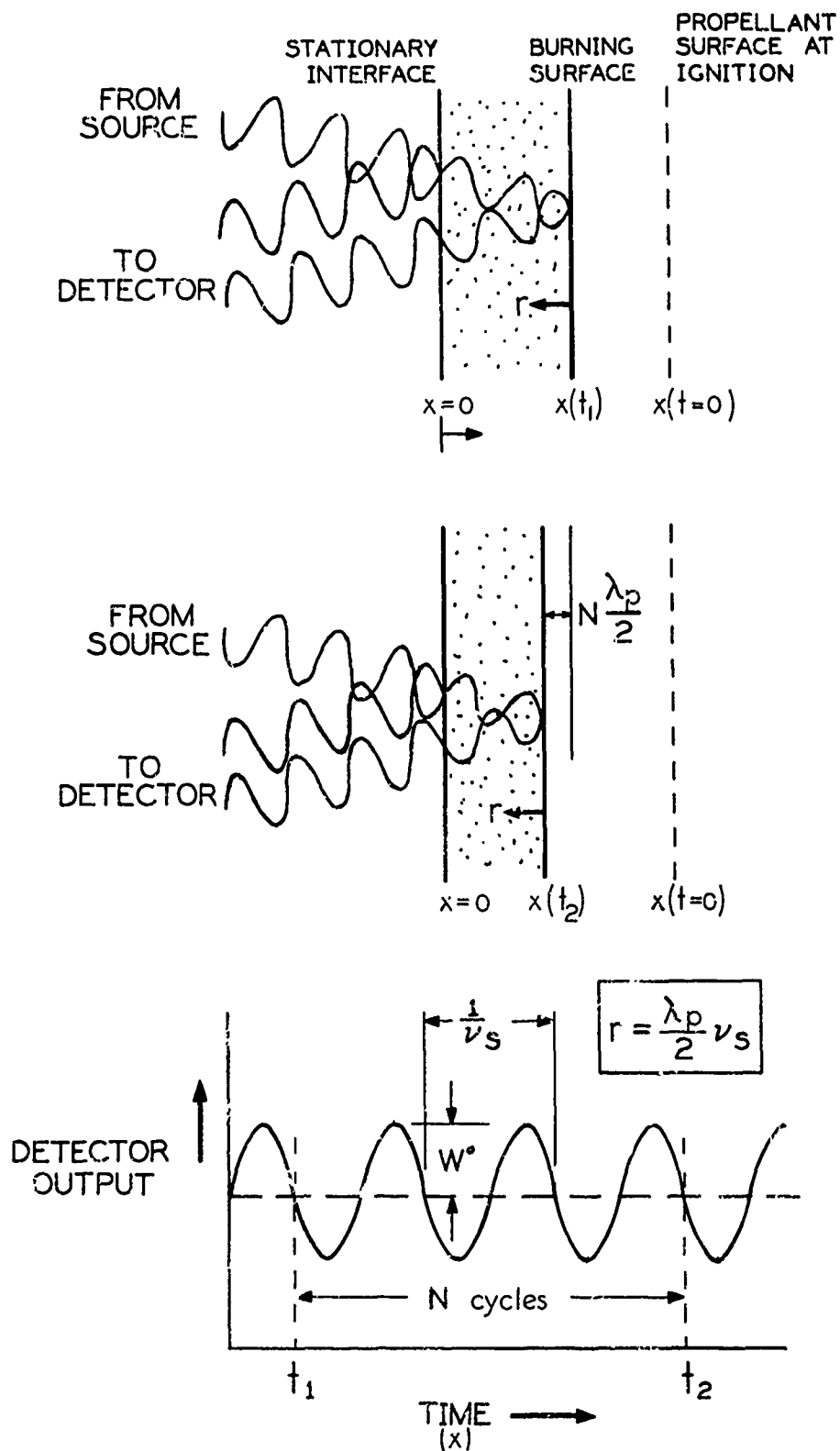


FIG. B-2 SCHEMATIC INDICATING BURNING RATE DETERMINATION FROM OSCILLATING MICROWAVE SIGNAL

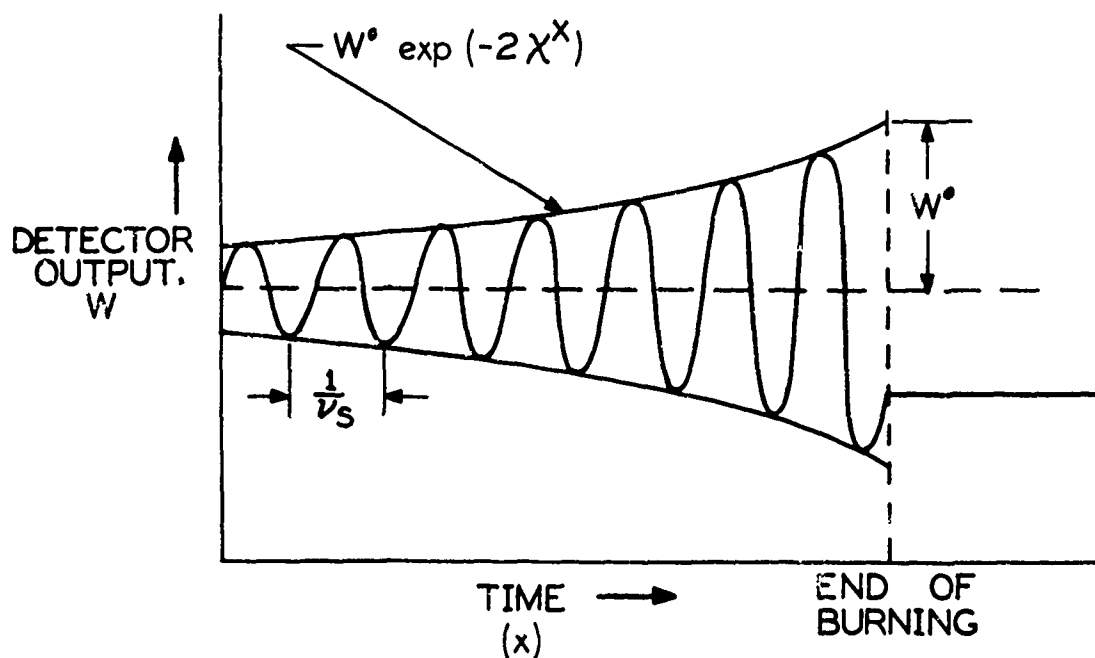


FIG. B-3 SCHEMATIC OF INFLUENCE OF PROPELLANT ABSORPTIVITY,  $\chi$ , ON OSCILLATING MICROWAVE SIGNAL

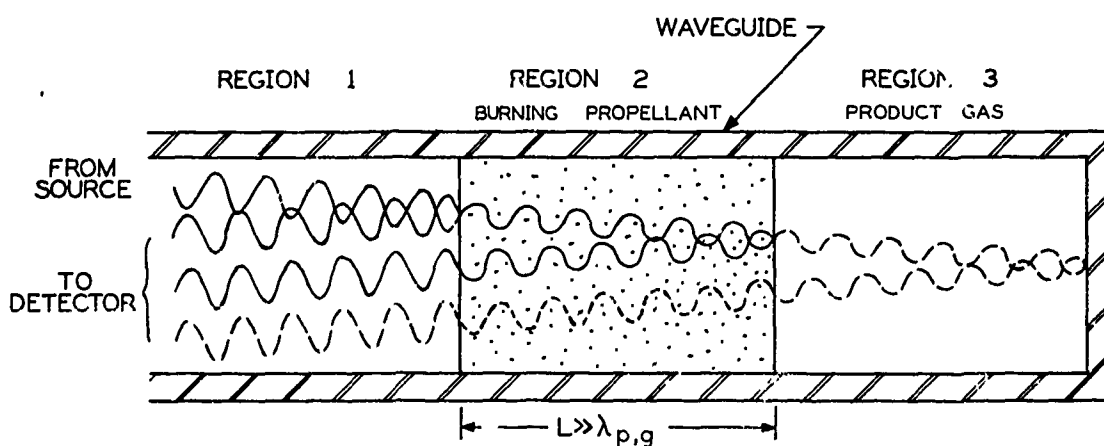


FIG. B-4 SCHEMATIC OF MICROWAVE INTERACTION WITH WAVEGUIDE PARTIALLY-FILLED BY DIELECTRIC PROPELLANT

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The above referenced report was mailed to you on 29 August 1965. Please add the date, August 1965, which was inadvertently left off this report. Thank you.

AD-471575