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DYNAMIC STABILITY OF A SYSTEM CONSISTING OF A STABLE PARACHUTE AND AN UNSTABLE LOAD

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FOREWORD

This report was prepared by the Department of Aeronautics and Engineering Mechanics of the University of Minnesota, Minneapolis, Minnesota, in accordance with Air Force Contract AF33(657)-11184, Project 6065, Task 606503, "Parachute Aerodynamics and Structures."

The work being accomplished under this contract is jointly sponsored by the U. S. Army Natick Laboratories, Department of the Army; Bureau of Naval Weapons, Department of the Navy; and Air Force Systems Command, United States Air Force. The contract was administered under the direction of the Recovery and Crew Station Branch, Air Force Flight Dynamics Laboratory, Research and Technology Division, with Mr. James H. DeWeese acting as Project Engineer.

The work efforts accomplished in support of the particular investigation reported herein were initiated on 15 April 1963 and completed in May 1964.

The authors wish to express their appreciation to their associates and to the students of Aerospace Engineering who assisted in the preparation of this report.

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This technical report has been reviewed and is approved.

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ABSTRACT

The several equations of motion governing the dynamic stability of a parachute-load system, in which the parachute as well as the load possesses aerodynamic drag and stability characteristics, are established. The general equations are linearized, which process provides satisfactory results for relatively small deflections. A further simplification is accomplished under the assumption of a vertical descent. A numerical example is used to illustrate the application of the analytical methods.

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SYMBOLS

a_c $\left(\frac{\partial C_{N_c}}{\partial \alpha}\right)_s$ = slope of the canopy normal force coefficient under static conditions

a_ℓ $\left(\frac{\partial C_{N_\ell}}{\partial \alpha}\right)_s$ = slope of the load normal force coefficient under static conditions

$A_1, A_2, A_3,$ B_1, B_2, B_3 constants of integration

C_{N_c} normal force coefficient of the canopy

C_{N_ℓ} normal force coefficient of the load

C_{T_c} tangent force coefficient of the canopy

C_{T_ℓ} tangent force coefficient of the load

I_a apparent moment of inertia of the entire system, including the effect of the enclosed air mass, about its center of mass (slug-ft^2)

$I_{c.g.}$ moment of inertia of the canopy and load about the center of mass of the system (slug-ft^2)

I_{TOT} total moment of inertia of system (slug-ft^2)

$\bar{I}$ dimensionless moment of inertia, $\frac{I}{\frac{1}{2}\rho S_c^{5/2}}$

L_1 distance between the center of pressure of the canopy and center of mass of the system (ft)

L_2 distance between the center of volume of the canopy and center of mass of the system (ft)

L_3 distance between the center of mass of the load and center of mass of the system (ft)

L_4 distance between the center of mass of the canopy material and center of mass of the system (ft)

L_5 distance between the center of pressure of the load and center of mass of the system (ft)

$\bar{L}$ dimensionless length, $\frac{L}{S_c^{1/2}}$

m_{ax_c} apparent mass of the canopy including the effect of the enclosed air mass in the x direction (slug)

m_{ax_ℓ}	apparent mass of the load in the x direction (slug)
m_{ay_c}	apparent mass of the canopy including the effect of the enclosed air mass in the y direction (slug)
m_{ay_ℓ}	apparent mass of the load in the y direction
m_c	mass of the canopy material (slug)
m_ℓ	mass of the suspended load (slug)
$\overline{m}$	dimensionless mass, $\frac{m}{\frac{1}{2}\rho S_c^{3/2}}$
N_c	normal force acting on the canopy (lb)
N_ℓ	normal force acting on the load (lb)
r	projected radius of the canopy (ft)
S_c	characteristic area of canopy (ft^2)
S_ℓ	characteristic area of load (ft^2)
T_c	tangent force acting on the canopy (lb)
T_ℓ	tangent force acting on the load (lb)
V	velocity of the center of mass of the system (ft/sec)
V_c	velocity of the center of volume of the canopy (ft/sec)
V_e	equilibrium velocity of the system (ft/sec)
V_ℓ	velocity of the center of mass of the load (ft/sec)
V_x	velocity component in the direction of the system axis (ft/sec)
V_y	velocity component perpendicular to the system axis (ft/sec)
$\overline{V}$	dimensionless velocity, $\frac{V}{V_e}$
W_c	weight of the parachute material (lb)
W_ℓ	weight of the load (lb)
α	angle between the velocity vector of the center of mass of the system and the canopy axis (radians)
α_c	angle of attack of the center of volume of the parachute (radians)

α_ℓ	angle of attack of the center of mass of the load (radians)
β	angle between the velocity vector of the center of mass of the system and the vertical (radians)
γ	area ratio = S_ℓ / S_c
θ	angle between the canopy axis and the vertical (radians)
$\lambda_1, \lambda_2 \lambda_3$	roots of the frequency equation
ρ	air density (slug/ft^3)
τ	dimensionless time = $\dfrac{V_e t}{S_c^{\frac{1}{2}}}$
ω	angular velocity of the system (radians/sec)

Subscripts

$(\ )_o$	initial value of a quantity (at $\tau = 0$)

Superscripts

$(\dot{\ })$	differentiation with respect to real time, t
$(\ )'$	differentiation with respect to non-dimensional time, τ
$(\bar{\ })$	dimensionless term

I. INTRODUCTION

The present report is concerned with an extension of the dynamic stability study of a parachute-point mass system considered in Ref 1. Reference 1 considered a simplified system composed of a point mass, possessing neutral stability characteristics, and a statically stable canopy. One must realize, however, that for most practical applications, the load will possess a degree of stability (or instability) of its own. Thus, it appears advantageous to attempt an analysis of such a parachute-load system.

Therefore, this report presents an analysis in which the load exhibits aerodynamic characteristics of its own. In addition to these characteristics, the motion of the system depends upon the aerodynamic coefficients, their derivatives, and upon the physical, as well as apparent mass and moment of inertia of the parachute.

The general governing equations are ultimately simplified for vertical or near vertical descent.

II. EQUATIONS OF MOTION

The motion of a parachute-load system involves, in general, six degrees of freedom. In order to obtain an analytical solution with a reasonable amount of effort, one must consider a simplified physical model. In view of these circumstances, the following assumptions shall be utilized.

1. The entire system constitutes a rigid body.
2. The mass and aerodynamic forces of the suspension lines are neglected.
3. The effects of the apparent mass of the parachute canopy and that of the load act at the canopy center of volume and load center of mass, respectively.
4. The motion is restricted to the x-y plane.

This physical model, with the acting forces, is shown in Fig 1.

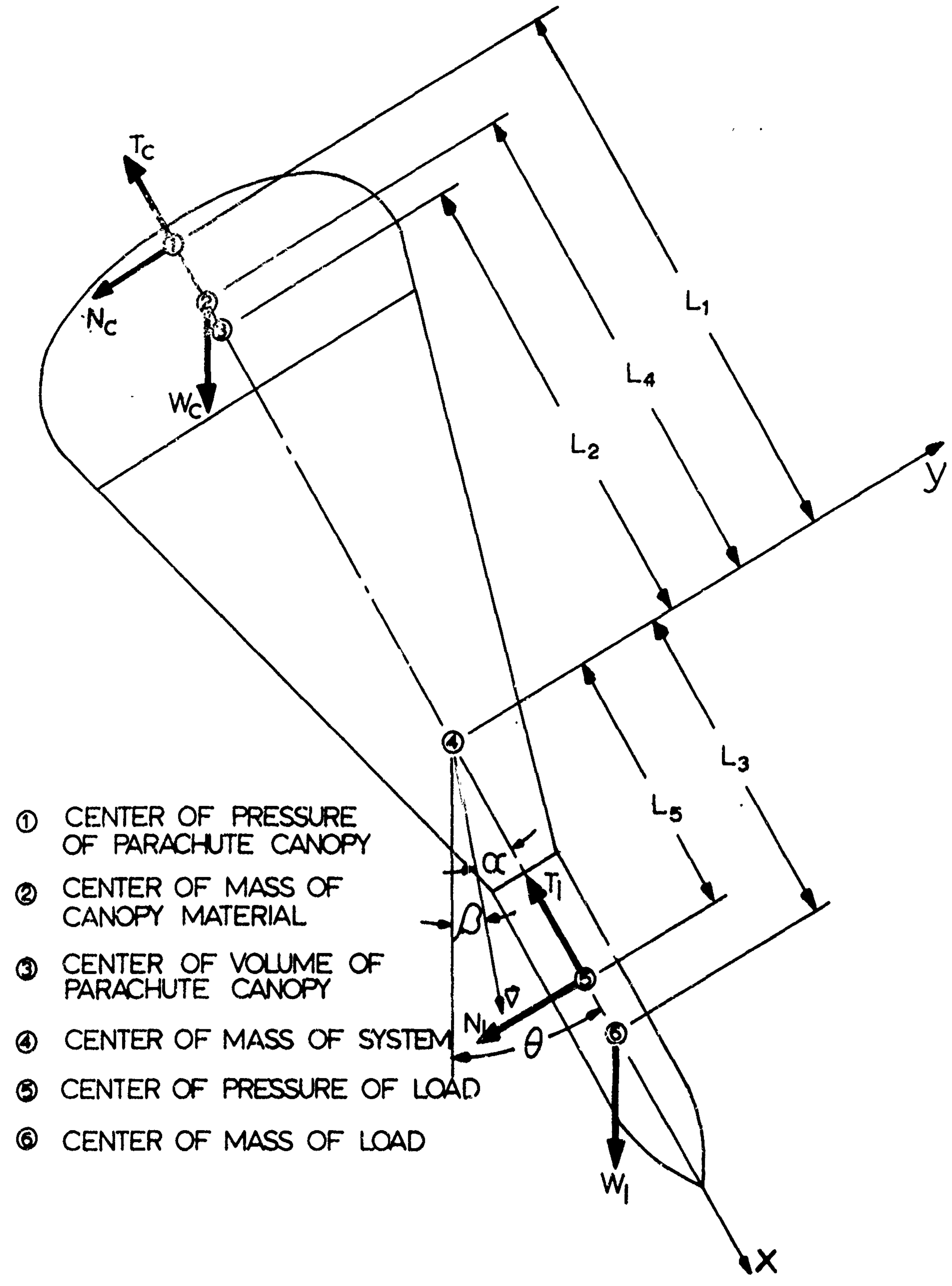

Fig 1. The Parachute - Load System

One may write the velocity of the center of mass of the system in the canopy fixed reference frame as:

$$\vec{V} = V_x\hat{i} + V_y\hat{j} \tag{1}$$

where: V_x = velocity of the center of mass in the direction of the canopy axis

V_y = velocity of the center of mass perpendicular to the canopy axis.

To determine the equations of motion of the parachute-load system, one may use Newton's second law, which may be expressed symbolically as:

$$\sum\vec{F} = m\frac{d^{(1)}\vec{V}}{dt} \tag{2}$$

where: $\sum\vec{F}$ = sum of all external forces acting on the system

m = physical mass of the system

$\frac{d^{(1)}\vec{V}}{dt}$ = acceleration of the center of mass of the system in an inertial reference frame.

The absolute (total) acceleration may be expressed as (Ref 1):

$$\frac{d^{(1)}\vec{V}}{dt} = \frac{d^{(2)}\vec{V}}{dt} + \vec{\omega}\times\vec{V} \tag{3}$$

where: $\frac{d^{(2)}\vec{V}}{dt}$ = acceleration with respect to reference frame 2

$\vec{\omega}$ = angular velocity of reference frame 2 with respect to reference frame 1.

In the present analysis, the canopy fixed reference frame is chosen to be reference frame 2 and $\vec{\omega}$ may be expressed as $\vec{\omega} = \dot{\theta}\hat{k}$. Using Eqns 3 and 1, one finds:

$$\frac{d^{(1)}\vec{V}}{dt} = (\dot{V}_x - \dot{\theta}V_y)\hat{i} + (\dot{V}_y + \dot{\theta}V_x)\hat{j}. \tag{4}$$

Using this relation in Eqn 2, one obtains:

$$(m_p + m_l)\left[(\dot{V}_x - \dot{\theta}V_y)\hat{i} + (\dot{V}_y + \dot{\theta}V_x)\hat{j}\right] = \sum\vec{F}. \tag{5}$$

where: m_c = mass of the canopy material

m_ℓ = mass of the suspended load

The various external forces acting on the system will now be considered. The aerodynamic forces, as shown in Fig 1, may be expressed as:

$$\vec{F}_a = -(T_\ell + T_c)\hat{i} - (N_\ell + N_c)\hat{j} \,, \qquad (6)$$

where: T_ℓ = tangent force acting on the load

T_c = tangent force acting on the canopy

N_ℓ = normal force acting on the load

N_c = normal force acting on the canopy.

One may express the gravity forces as:

$$\vec{F}_g = (W_\ell + W_c)\cos\theta\hat{i} - (W_\ell + W_c)\sin\theta\hat{j} \qquad (7)$$

The apparent mass forces can be expressed as:

$$\vec{F}_{am} = -\left[m_{ax_\ell} a_{x_\ell} + m_{ax_c} a_{x_c}\right]\hat{i} - \left[m_{ay_\ell} a_{y_\ell} + m_{ay_c} a_{y_c}\right]\hat{j} \,, \qquad (8)$$

where: m_{ax_ℓ} = apparent mass of the load in the x direction

m_{ay_ℓ} = apparent mass of the load in the y direction

m_{ax_c} = apparent mass of the canopy including the effect of the enclosed air mass in the x direction (Ref 2)

m_{ay_c} = apparent mass of the canopy including the effect of the enclosed air mass in the y direction (Ref 2)

a_{x_ℓ} = acceleration of the center of mass of the load in the x direction

a_{y_ℓ} = acceleration of the center of mass of the load in the y direction

a_{x_c} = acceleration of the center of volume of the canopy in the x direction

a_{y_c} = acceleration of the center of volume of the canopy in the y direction.

The velocity of the center of mass of the load and center of volume of the canopy consists of the velocity of the center of mass of the system $\vec{V}$ plus a rotational velocity about the center of mass of the parachute-load system. Thus, one may write:

$$\begin{aligned} \vec{V}_\ell &= \vec{V}+\vec{\omega}\times L_3\hat{i} = V_x\hat{i} + (V_y+L_3\dot{\theta})\hat{j} \\ \vec{V}_c &= \vec{V}+\vec{\omega}\times(-L_2\hat{i}) = V_x\hat{i} + (V_y-L_2\dot{\theta})\hat{j} \end{aligned} \tag{9}$$

The acceleration of the center of mass of the load and center of volume of the canopy may be written as:

$$\vec{a}_\ell = \frac{d^{(1)}\vec{V}_\ell}{dt}$$
$$\vec{a}_c = \frac{d^{(1)}\vec{V}_c}{dt}$$

and, using relations 3 and 9, the above equations become:

$$\begin{aligned} \vec{a}_\ell &= \left[\dot{V}_x - \dot{\theta}V_y - L_3\dot{\theta}^2\right]\hat{i} + \left[\dot{V}_y + \dot{\theta}V_x + L_3\ddot{\theta}\right]\hat{j} \\ \vec{a}_c &= \left[\dot{V}_x - \dot{\theta}V_y + L_2\dot{\theta}^2\right]\hat{i} + \left[\dot{V}_y + \dot{\theta}V_x - L_2\ddot{\theta}\right]\hat{j} \end{aligned} \tag{10}$$

Or, in scalar form, one may write the corresponding acceleration components as:

$$a_{x_\ell} = \dot{V}_x - \dot{\theta}V_y - L_3\dot{\theta}^2$$

$$a_{y_\ell} = \dot{V}_y + \dot{\theta}V_x + L_3\ddot{\theta}$$

$$a_{x_c} = \dot{V}_x - \dot{\theta}V_y + L_2\dot{\theta}^2$$

$$a_{y_c} = \dot{V}_y + \dot{\theta}V_x - L_2\ddot{\theta}$$

and the apparent mass forces, in accordance with Eqn 8, become:

$$\begin{aligned} \vec{F}_{am} = &-\left[m_{ax_\ell}(\dot{V}_x - \dot{\theta}V_y - L_3\dot{\theta}^2) + m_{ax_c}(\dot{V}_x - \dot{\theta}V_y + L_2\dot{\theta}^2)\right]\hat{i} \\ &-\left[m_{ay_\ell}(\dot{V}_y + \dot{\theta}V_x + L_3\ddot{\theta}) + m_{ay_c}(\dot{V}_y + \dot{\theta}V_x - L_2\ddot{\theta})\right]\hat{j} \end{aligned} \tag{11}$$

Utilizing relations 6, 7 and 11 in Eqn 5 yields two scalar equations of motion in the x and y directions, respectively:

$$(m_c + m_\ell + m_{ax_\ell} + m_{ax_c})(\dot{V}_x - \dot{\theta}V_y) = -(T_\ell + T_c) + (W_\ell + W_c)\cos\theta + (m_{ax_\ell}L_3 - m_{ax_c}L_2)\dot{\theta}^2 \tag{12}$$

$$(m_c + m_\ell + m_{ay_\ell} + m_{ay_c})(\dot{V}_y + \dot{\theta}V_x) = -N_\ell - N_c - (W_\ell + W_c)\sin\theta + (m_{ay_c}L_2 - m_{ay_\ell}L_3)\ddot{\theta} \tag{13}$$

A third equation may be written which governs the rotational motion of the entire system. This equation, the angular momentum equation, states that the sum of the external moments acting about the center of mass of the system equals the time rate of change of the angular momentum of the system about its center of mass. This statement may be expressed as:

$$(I_{c.g.} + I_a)\ddot{\theta} = N_c L_1 - N_\ell L_5 \tag{14}$$

where: $I_{c.g.}$ = moment of inertia of the canopy and load about the center of mass of the system

I_a = apparent moment of inertia of the entire system including the effect of the enclosed air mass about its center of mass

L_1, L_5 = lengths (see Fig 1).

The various aerodynamic forces are conventionally expressed as:

$$N_c = C_{N_c}\frac{1}{2}\rho V_c^2 S_c \qquad T_c = C_{T_c}\frac{1}{2}\rho V_c^2 S_c$$
$$N_\ell = C_{N_\ell}\frac{1}{2}\rho V_\ell^2 S_\ell \qquad T_\ell = C_{T_\ell}\frac{1}{2}\rho V_\ell^2 S_\ell \tag{15}$$

In addition to these definitions, it is convenient to make the equations of motion dimensionless in the following manner:

$$\overline{m} = \frac{m}{\frac{1}{2}\rho S_c^{3/2}} \qquad \overline{I} = \frac{I}{\frac{1}{2}\rho S_c^{5/2}}$$

$$\overline{L} = \frac{L}{S_c^{1/2}} \qquad \overline{V} = \frac{V}{V_e} \qquad \tau = \frac{V_e t}{S_c^{1/2}} \tag{16}$$

V_e = equilibrium velocity.

Introducing these relations, along with Eqn 15, into Eqns 12 through 14, one obtains after some algebraic manipulations:

$$\begin{aligned}(\overline{m}_c + \overline{m}_\ell + \overline{m}_{ax_\ell} + \overline{m}_{ax_c})(\overline{V}_x' - \theta'\overline{V}_y) = &-C_{T_\ell}\overline{V}_\ell^2 \frac{S_\ell}{S_c} - C_{T_c}\overline{V}_c^2 \\ &+ \frac{(W_\ell + W_c)}{\frac{1}{2}\rho V_e^2 S_c}\cos\theta \\ &+ (\overline{m}_{ax_\ell}\overline{L}_3 - \overline{m}_{ax_c}\overline{L}_2)\theta'^2\end{aligned} \tag{12a}$$

$$\begin{aligned}(\overline{m}_c + \overline{m}_\ell + \overline{m}_{ay_\ell} + \overline{m}_{ay_c})(\overline{V}_y' + \theta'\overline{V}_x) = &-C_{N_\ell}\overline{V}_\ell^2 \frac{S_\ell}{S_c} - C_{N_c}\overline{V}_c^2 \\ &- \frac{(W_\ell + W_c)}{\frac{1}{2}\rho V_e^2 S_c}\sin\theta \\ &+ (\overline{m}_{ay_c}\overline{L}_2 - \overline{m}_{ay_\ell}\overline{L}_3)\theta''\end{aligned} \tag{13a}$$

$$(\overline{I}_{cg} + \overline{I}_a)\theta'' = -C_{N_\ell}\overline{V}_\ell^2 \frac{S_\ell}{S_c}\overline{L}_5 + C_{N_c}\overline{V}_c^2\overline{L}_1 \ , \tag{14a}$$

where the prime (') indicates differentiation with respect to the dimensionless time τ .

The equilibrium velocity is defined by:

$$W_\ell + W_c = (C_{T_\ell} S_\ell + C_{T_c} S_c)\frac{1}{2}\rho V_e^2 \ ,$$

which may be written as:

$$\frac{W_\ell + W_c}{\frac{1}{2}\rho V_c^2 S_c} = C_{T_c} + \frac{S_\ell}{S_c} C_{T_\ell} \tag{17}$$

To abbreviate the form of Eqns 12a through 14a, let us write:

$$\begin{aligned}
\bar{m}_x &= \bar{m}_c + \bar{m}_\ell + \bar{m}_{ax_\ell} + \bar{m}_{ax_c} \\
\bar{m}_y &= \bar{m}_c + \bar{m}_\ell + \bar{m}_{ay_\ell} + \bar{m}_{ay_c} \\
\bar{I}_{TOT.} &= \bar{I}_{cg} + \bar{I}_a \\
\gamma &= \frac{S_\ell}{S_c} \\
A_x &= \bar{m}_{ax_\ell}\bar{L}_3 - \bar{m}_{ax_c}\bar{L}_2 \\
A_y &= \bar{m}_{ay_c}\bar{L}_2 - \bar{m}_{ay_\ell}\bar{L}_3
\end{aligned} \tag{18}$$

Substituting the definitions 17 and 18 into Eqns 12a through 14a yields:

$$\bar{m}_x(\bar{V}_x' - \theta'\bar{V}_y) = -\gamma C_{T_\ell}\bar{V}_\ell^2 - C_{T_c}\bar{V}_c^2 + (C_{T_c} + \gamma C_{T_\ell})\cos\theta + A_x\theta'^2 \tag{12b}$$

$$\bar{m}_y(\bar{V}_y' + \theta'\bar{V}_x) = -\gamma C_{N_\ell}\bar{V}_\ell^2 - C_{N_c}\bar{V}_c^2 - (C_{T_c} + \gamma C_{T_\ell})\sin\theta + A_y\theta'' \tag{13b}$$

$$\bar{I}_{TOT}\theta'' = -\gamma C_{N_\ell}\bar{V}_\ell^2\bar{L}_5 + C_{N_c}\bar{V}_c^2\bar{L}_1 \, . \tag{14b}$$

In order to obtain the final equations of motion in a convenient form, the following relationships may be deduced from Eqn 9:

$$\bar{V}_{\ell}^2 = \bar{V}_x^2 + (\bar{V}_y + \bar{L}_3\theta')^2$$
$$\bar{V}_c^2 = \bar{V}_x^2 + (\bar{V}_y - \bar{L}_2\theta')^2 \qquad (19)$$

The angle of attack of the center of mass of the load α_{ℓ} and center of volume of the canopy α_c will eventually be required to evaluate the normal force (C_N) coefficients. One may determine these angles by means of Eqn 9 as:

$$\tan \alpha_c = -\left(\frac{V_y}{V_x}\right)_c = -\frac{V_y - L_2\dot{\theta}}{V_x}$$

$$\tan \alpha_{\ell} = -\left(\frac{V_y}{V_x}\right)_{\ell} = -\frac{V_y + L_3\dot{\theta}}{V_x}$$

But, from Fig 1 one notes that the angle of attack of the centerline of the system amounts to:

$$\tan \alpha = -\frac{V_y}{V_x}$$

and, therefore, using the dimensionless notation:

$$\tan \alpha_c = \tan \alpha + \frac{\bar{L}_2\theta'}{\bar{V}_x}$$

$$\tan \alpha_{\ell} = \tan \alpha - \frac{\bar{L}_3\theta'}{\bar{V}_x} \qquad (20)$$

Noting from the geometry of Fig 1 that:

$$\bar{V}_x = \bar{V} \cos \alpha$$
$$\bar{V}_y = -\bar{V} \sin \alpha \qquad (21)$$

Eqns 19 and 20 become now:

$$\bar{V}_{\ell}^2 = \bar{V}^2 - 2\bar{L}_3\bar{V}\theta' \sin \alpha + \bar{L}_3^2\theta'^2$$
$$\bar{V}_c^2 = \bar{V}^2 + 2\bar{L}_2\bar{V}\theta' \sin \alpha + \bar{L}_2^2\theta'^2 \qquad (22)$$

$$\tan\alpha_{\ell} = \tan\alpha - \frac{L_3\theta'}{\bar V\cos\alpha}$$

$$\tan\alpha_c = \tan\alpha + \frac{L_2\theta'}{\bar V\cos\alpha} \tag{23}$$

Utilizing relations 21 and 22, the equations of motion 12b through 14b assume the following form:

$$\begin{aligned}\bar m_x\left[\bar V'\cos\alpha - \bar V\alpha'\sin\alpha + \bar V\theta'\sin\alpha\right] = &-\gamma C_{T\ell}\left[\bar V^2 - 2L_3\bar V\theta'\sin\alpha + L_3^2\theta'^2\right]\\ &- C_{T_c}\left[\bar V^2 + 2L_2\bar V\theta'\sin\alpha + L_2^2\theta'^2\right]\\ &+ (C_{T_c}+\gamma C_{T\ell})\cos\theta + A_x\theta'^2\end{aligned} \tag{24}$$

$$\begin{aligned}m_y\left[-\bar V'\sin\alpha - \bar V\alpha'\cos\alpha + \bar V\theta'\cos\alpha\right] = &-\gamma C_{N\ell}\left[\bar V^2 - 2L_3\bar V\theta'\sin\alpha + L_3^2\theta'^2\right]\\ &- C_{N_c}\left[\bar V^2 + 2L_2\bar V\theta'\sin\alpha + L_2^2\theta'^2\right]\\ &- (C_{T_c}+\gamma C_{T\ell})\sin\theta + A_y\theta'^2\end{aligned} \tag{25}$$

$$\begin{aligned}I_{TOT}\theta'' = &-\gamma C_{N\ell}L_5\left[\bar V^2 - 2L_3\bar V\theta'\sin\alpha + L_3^2\theta'^2\right]\\ &+ C_{N_c}L_1\left[\bar V^2 + 2L_2\bar V\theta'\sin\alpha + L_2^2\theta'^2\right]\end{aligned} \tag{26}$$

III. A FIRST ORDER METHOD OF OBTAINING NUMERICAL SOLUTIONS

A simplified numerical method of solution will now be outlined. In the most general situation, certain initial conditions must be given. A typical set of initial conditons is given by:

$$\text{at } \tau = 0, \quad \Theta = \Theta_o, \quad \alpha = \alpha_o, \quad \Theta' = \Theta_o', \quad \bar{V} = \bar{V}_o$$

Let us first consider Eqn 23. One observes that these initial conditons are sufficient to determine the initial angle of attack of the load and canopy, α_{ℓ_o} and α_{c_o}. It may be assumed that the aerodynamic coefficients C_{N_c}, C_{T_c}, C_{N_ℓ} and C_{T_ℓ} are known functions of α_c and α_ℓ respectively. Consequently, α_{ℓ_o} and α_{c_o} determine the initial values of the coefficients. Substituting the initial coefficient values and the initial conditions from above into Eqns 24 through 26 yields three equations of the form:

$$A_1\bar{V}_o' + B_1\alpha_o' = C_1 \tag{24a}$$

$$A_2\bar{V}_o' + B_2\alpha_o' = C_2 + D_2\Theta_o'' \tag{25a}$$

$$A_3\Theta_o'' = C_3 \; . \tag{26a}$$

The constants A_1, A_2, . . . are determined from Eqns 24 through 26. For example, by comparison, one finds that $A_1 = \bar{m}_x \cos \alpha_o$, $A_2 = -\bar{m}_x V_o \sin \alpha_o$, etc. Thus Eqns 24a through 26a represent three linear equations from which one can determine $\bar{V}_o'$, α_o', and Θ_o''.

One next selects a small dimensionless time interval and calculates:

$$\delta\bar{V} = \bar{V}_o'\,\delta\tau$$
$$\delta\Theta' = \Theta_o''\,\delta\tau$$
$$\delta\alpha = \alpha_o'\,\delta\tau$$
$$\delta\Theta = \Theta_o'\,\delta\tau$$

Thus, after the time interval $\delta\tau$, the new values of the variables become:

$$\bar{V}_1 = \bar{V}_0 + \delta\bar{V}$$

$$\theta_1' = \theta_0' + \delta\theta'$$

$$\alpha_1 = \alpha_0 + \delta\alpha$$

$$\theta_1 = \theta_0 + \delta\theta$$

One now has a new set of conditions $\bar{V}_1$, θ_1', α_1, θ_1, and the preceding steps can be repeated to determine new values of $\bar{V}$, θ', θ, and α. This procedure may be repeated indefinitely until the entire trajectory is determined as:

$$\bar{V} = f_1(\tau)$$
$$\theta = f_2(\tau)$$
$$\alpha = f_3(\tau)$$

The above procedure is equivalent to expanding V, θ, and α in a Taylor series and retaining only the first two terms. In this manner, the nonlinear differential equations, 24 through 26, including a nonlinear C_N, C_T, $-\alpha$ relationship, can be solved.

IV. LINEARIZED THEORY

If one wishes to consider only the class of motions where the oscillations are small (α small), the following equations are applicable:

$$\sin\alpha = \alpha \qquad \cos\alpha = 1$$

In addition, if the trajectory is almost vertical (β small), one may write:

$$\sin\beta = \beta \qquad \cos\beta = 1$$

Utilizing these assumptions in Eqns 24 through 26 and neglecting second order terms such as $\alpha\alpha'$, one obtains:

$$\bar{m}_X \bar{V}' = -(\gamma C_{T_\ell} + C_{T_c})(\bar{V}^2 - 1) \qquad (27)$$

$$\overline{m}_y\overline{V}(\theta'-\alpha') = -(\gamma C_{N_\ell}+C_{N_c})\overline{V}^2-(C_{T_c}+\gamma C_{T_\ell})\theta + A_y\theta' \tag{28}$$

$$\overline{I}_{TOT}\theta' = (C_{N_c}\overline{L}_1-\gamma C_{N_\ell}\overline{L}_5)\overline{V}^2 \tag{29}$$

Equation 27 may be integrated directly to give the variation of V with τ.

$$\overline{V} = \frac{\frac{1+\overline{V}_0}{1-\overline{V}_0}\exp\left[\frac{2(\gamma C_{T_\ell}+C_{T_c})\tau}{\overline{m}_x}-1\right]}{\frac{1+\overline{V}_0}{1-\overline{V}_0}\exp\left[\frac{2(\gamma C_{T_\ell}+C_{T_c})\tau}{\overline{m}_x}+1\right]} \tag{30}$$

where $\overline{V}_0$ is the value of $\overline{V}$ at $\tau = 0$.

Equations 28 and 29 can be presented in a more explicit form by realizing that for small oscillations, C_{T_ℓ} and C_{T_c} are constant while C_{N_ℓ} and C_{N_c} are linear functions of α_ℓ and α_c respectively. Thus, one may write:

$$C_{N_\ell} = a_\ell\alpha_\ell \tag{31}$$
$$C_{N_c} = a_c\alpha_c$$

where: a_ℓ = slope of C_{N_ℓ} versus α_ℓ under static conditions

a_c = slope of C_{N_c} versus α_c under static conditions.

Introducing these relations into Eqns 28 and 29 and assuming that the system is descending at approximately its equilibrium speed (i.e., $\overline{V} \cong 1$), one finds:

$$\overline{m}_y(\theta'-\alpha') = -\gamma a_\ell\alpha_\ell-(C_{T_c}+\gamma C_{T_\ell})\theta + A_y\theta' - a_c\alpha_c \tag{28a}$$

$$\overline{I}_{TOT}\theta' = (a_c\alpha_c\overline{L}_1 - \gamma a_\ell\alpha_\ell\overline{L}_5) \tag{29a}$$

Again utilizing the assumption of small oscillations, Eqn 23 assumes the form:

$$\alpha_\ell = \alpha - \bar{L}_3\theta' \qquad\qquad (23a)$$
$$\alpha_c = \alpha + \bar{L}_2\theta'$$

Introducing these relations into Eqns 28a and 29a yields, after rearranging:

$$A_y\theta'' + \left[\gamma\bar{L}_3a_\ell - \bar{L}_2a_c - \bar{m}_y\right]\theta' - \left[C_{T_c} + \gamma C_{T_\ell}\right]\theta \qquad (28b)$$
$$+ \bar{m}_y\alpha' - \left[\gamma a_\ell + a_c\right]\alpha = 0$$

$$\bar{I}_{TOT}\theta'' - \left[\gamma\bar{L}_3\bar{L}_5a_\ell + \bar{L}_1\bar{L}_2a_c\right]\theta' - (\bar{L}_1a_c - \gamma\bar{L}_5a_\ell)\alpha = 0 \qquad (29b)$$

Equations 28b and 29b are coupled but linear, and one may assume a solution of the form:

$$\alpha = Ae^{\lambda\tau} \qquad\qquad (32)$$
$$\theta = Be^{\lambda\tau}$$

Substituting these functions into Eqn 28b and 29b yields, after rearranging:

$$a_{11}A + a_{12}B = 0 \qquad\qquad (33)$$
$$a_{21}A + a_{22}B = 0$$

where:

$$a_{11} = \bar{m}_y\lambda - \left[\gamma a_\ell + a_c\right]$$
$$a_{12} = A_y\lambda^2 + \left[-\bar{L}_2a_c + \gamma\bar{L}_3a_\ell - m_y\right]\lambda - \left[C_{T_c} + \gamma C_{T_\ell}\right]$$
$$a_{21} = \gamma\bar{L}_5a_\ell - \bar{L}_1a_c \qquad\qquad (33a)$$
$$a_{22} = \bar{I}_{TOT}\lambda^2 - \left[\gamma\bar{L}_3\bar{L}_5a_\ell + \bar{L}_1\bar{L}_2a_c\right]\lambda$$

A nontrivial solution of Eqn 33 for A and B exists if and only if the determinant of the coefficients is identically zero. That is, if:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$$

Expanding the determinant, one finds after rearranging:

$$a\lambda^3 + b\lambda^2 + c\lambda + d = 0 \tag{34}$$

The coefficients, a, b, c, and d, obtained by a direct expansion of the determinant, can be written with the aid of relation 18 and 33a as:

$$\begin{aligned} a &= \bar{m}_y \bar{I}_{TOT} \\ b &= -(\bar{m}_c + \bar{m}_\ell)(\bar{L}_1\bar{L}_2 a_c + \gamma \bar{L}_3\bar{L}_5 a_\ell) \\ &\quad -(\bar{L}_2 + \bar{L}_3)(\bar{m}_{ay\ell}\bar{L}_1 a_c + \bar{m}_{ayc}\bar{L}_5 a_\ell) - \bar{I}_{TOT}(a_c + \gamma a_\ell) \\ c &= \gamma(\bar{L}_1 + \bar{L}_5)(\bar{L}_2 + \bar{L}_3) a_\ell a_c - \bar{m}_y(\bar{L}_1 a_c - \gamma \bar{L}_5 a_\ell) \\ d &= -(C_{T_c} + \gamma C_{T_\ell})(\bar{L}_1 a_c - \gamma \bar{L}_5 a_\ell) \end{aligned} \tag{35}$$

Equation 34 is referred to as the frequency equation of the system. In general, it yields three distinct values of λ.

Routh's criteria (Ref 3) requires that for a dynamically stable system, the following inequalities be satisfied:

$$\begin{array}{ll} a > 0 & d > 0 \\ b > 0 & bc > d \\ c > 0 & \end{array} \tag{36}$$

In essence, the angle of attack α and the related angle Θ (Eqn 32) decay with time if the real part of the roots of the frequency equation (34) are negative. This is the case if Routh's criteria is satisfied.

Now, details of the solution of the governing equations shall be discussed. Since three roots of the frequency equation (λ_1, λ_2, λ_3) can be found, the general solution of the problem may be written as:

$$\begin{aligned} \alpha &= A_1 e^{\lambda_1 \tau} + A_2 e^{\lambda_2 \tau} + A_3 e^{\lambda_3 \tau} \\ \theta &= B_1 e^{\lambda_1 \tau} + B_2 e^{\lambda_2 \tau} + B_3 e^{\lambda_3 \tau} \end{aligned} \tag{37}$$

where A_1, A_2, A_3, B_1, B_2, and B_3 are the constants of integration. Three relations for these constants can be found through the use of the initial conditions expressed as $\alpha = \alpha_o$, $\theta = \theta_o$, and $\theta' = \theta_o'$ at $\tau = 0$. Applications of these conditions yield:

$$\begin{aligned} A_1 + A_2 + A_3 &= \alpha_o \\ B_1 + B_2 + B_3 &= \theta_o \\ \lambda_1 B_1 + \lambda_2 B_2 + \lambda_3 B_3 &= \theta_o' \end{aligned} \tag{38}$$

Additional equations can be obtained from either of Eqns 33. Utilizing the second of these equations, one finds, with the use of 33a, the three relations:

$$\frac{A_i}{B_i} = -\frac{\bar{I}_{IOT}\lambda_i^2 - (\delta \bar{C}_3 \bar{C}_5 a_\ell + \bar{C}_1 \bar{C}_2 a_c)\lambda_i}{-\bar{C}_1 a_c + \delta \bar{C}_5 a_\ell} \tag{39}$$

where i assumes the values of 1, 2, and 3. Thus, using Eqns 38 and 39, all of the constants are specified and the solution established.

A particular numerical example of a system comprised of a stable parachute and unstable load is treated in Section 7.

V. AN APPROXIMATE SOLUTION FOR VERTICAL DESCENT ($\beta = 0$)

Linearized equations governing the linear and rotational motion of a parachute-load system were presented in the preceding section. For the case where the descent is vertical, Fig 1 indicates that β is zero and therefore the angles α and Θ must be identical. Because of this fact, one may replace α by Θ in the last term of Eqn 29b and obtain:

$$\bar{I}_{TOT}\Theta'' - (\bar{C}_1\bar{C}_2 a_c + \gamma\bar{C}_3\bar{C}_5 a_1)\Theta' - (\bar{C}_1 a_c - \gamma\bar{C}_5 a_1)\Theta = 0$$

which may be rewritten as:

$$\Theta'' + m\Theta' + \frac{n^2}{4}\Theta = 0 \tag{40}$$

where:

$$m = \frac{-\bar{C}_1\bar{C}_2 a_c - \gamma\bar{C}_3\bar{C}_5 a_1}{\bar{I}_{TOT}}$$

$$\frac{n^2}{4} = \frac{-\bar{C}_1 a_c + \gamma\bar{C}_5 a_1}{\bar{I}_{TOT}} \tag{41}$$

The solution of this linear equation can be written as:

$$\Theta = Ae^{\lambda t} \tag{42}$$

Substitution of this relation into Eqn 40 yields:

$$\lambda^2 + m\lambda + \frac{n^2}{4} = 0 \tag{43}$$

which yields for λ:

$$\lambda = -\frac{m}{2}\left(1 \pm \sqrt{1 - \left(\frac{n}{m}\right)^2}\right)$$

Denoting these two roots by λ_1 and λ_2, one has:

$$\lambda_1 = -\frac{m}{2}\left(1 + \sqrt{1-\left(\frac{n}{m}\right)^2}\right)$$

$$\lambda_2 = -\frac{m}{2}\left(1 - \sqrt{1-\left(\frac{n}{m}\right)^2}\right) \tag{44}$$

The general solution of Eqn 40 may now be written in one of several ways, depending on the relative values and signs of m and n^2. The case of a statically unstable parachute, where $a_c > 0$, is certainly not capable of stabilizing an unstable load. Therefore, this case may be disregarded.

Thus, for all physically realistic systems, one sees from their definitions that $\overline{L}_1$, $\overline{L}_2$, $\overline{L}_3$, γ , and $\overline{I}_{TOT}$ are always positive and a_c is negative. Also, for most practical cases, the slope of the normal force coefficient versus the angle of attack for the load is negative. However, $\overline{L}_5$ can be either positive or negative, depending on the relative sizes of the parachute and load as well as on the stability of the load by itself.

The above considerations thus show that both m and n^2 can be either positive or negative quantities. However, for most practical systems, both m and n^2 will be positive and as a result, only this case will be considered in detail in this report.

In view of Eqn 44, it is advantageous to consider the following three cases of the term $(n/m)^2$:

$$0 < \left(\frac{n}{m}\right)^2 < 1$$

For this case, examination of Eqn 44 shows that both λ_1 and λ_2 are negative and the general solution can be written as:

$$\Theta = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

Since λ_1 and λ_2 are negative, the angle (Θ) decays with time and the system is dynamically stable.

$$\left(\frac{n}{m}\right)^2 = 1$$

In this case one finds that $\lambda_1 = \lambda_2 = -\frac{m}{2}$, thus giving the solution:

$$\Theta = e^{-\frac{m\tau}{2}}(A_1 + A_2\tau)$$

Once again this solution indicates a dynamically stable system.

$$(\frac{n}{m})^2 > 1$$

In this case, the two roots λ_1 and λ_2 are complex numbers and one can write:

$$\lambda_1 = -\frac{m}{2} - i\frac{m}{2}\sqrt{(\frac{n}{m})^2 - 1}$$

$$\lambda_2 = -\frac{m}{2} + i\frac{m}{2}\sqrt{(\frac{n}{m})^2 - 1} \qquad (45)$$

and the general solution can be written as:

$$\Theta = e^{-\frac{m\tau}{2}}\left[A_1 \sin\frac{m}{2}\sqrt{(\frac{n}{m})^2 - 1}\,\tau + A_2 \cos\frac{m}{2}\sqrt{(\frac{n}{m})^2 - 1}\,\tau\right] \qquad (46)$$

which represents a stable system. This case represents the most common situation for a stable system and one can deduce certain characteristics of the motion of the system from this relation.

The damping factor, $e^{-\frac{m}{2}\tau}$, indicates the rate at which the oscillations are damped. Large values of m correspond to a rapid damping. One observes from Eqn 41 that large values of m (rapid damping) correspond to a combination of a very stable parachute (large negative a_c), long suspension lines, and a small moment of inertia, I.

In order to solve Eqn 46, one must determine the constants of integration, preferably from the initial conditions $\Theta = \Theta_o$, $\Theta' = \Theta_o'$ at $\tau = 0$. One finds the constants to be:

$$A_1 = \frac{\Theta_o' + \frac{m}{2}\Theta_o}{\frac{m}{2}\left[(\frac{n}{m})^2 - 1\right]^{1/2}}$$

$$A_2 = \Theta_o$$

and the solution of Eqn 46 amounts to:

$$\Theta = e^{-\frac{m}{2}\tau} \left\{ \left[\frac{\Theta_o' + \frac{m}{2}\Theta_o}{\frac{m}{2}\sqrt{\left(\frac{n}{m}\right)^2 - 1}} \right] \sin \frac{m}{2} \sqrt{\left(\frac{n}{m}\right)^2 - 1}\ \tau + \Theta_o \cos \frac{m}{2} \sqrt{\left(\frac{n}{m}\right)^2 - 1}\ \tau \right\} \tag{47}$$

VI. CONCLUSIONS

With the assumption of small oscillations and near vertical descent, one may completely specify the motion of a parachute-load system in which the load possesses distinct aerodynamic stability properties. This rigorous solution requires lengthy calculations. To alleviate this problem, one may choose to solve the simplified equation presented in Section 5. This approach is justified for vertical descent ($\beta = 0$).

A substantial portion of the calculations can be eliminated if one merely wishes to determine whether or not the system is dynamically stable. The answer to this question is furnished by Routh's criteria (Section 4). One must, in this case, only determine the value of the coefficients of the frequency equation (Eqns 34 and 35) and check to see if they satisfy relations 36. Because of the number of parameters involved and the way in which they enter the frequency equation, it is difficult to draw general conclusions pertaining to the stability of a general system. Therefore, for any practical case, Routh's criteria must be satisfied in its various aspects.

It also can be seen that a system consisting of a stable or unstable load combined with a stable parachute can be analyzed in the same manner as a system with a point load and a stable parachute (Ref 2). For the case of the point load, the governing equations of this study may be reduced to the same form as given in Ref 2 when the aerodynamic coefficients of the load approach zero.

VII. NUMERICAL EXAMPLE

A numerical example, concerning a parachute-load system (Fig 2) which consists of a five-foot ribless guide surface parachute canopy, having a nominal porosity of 70 ft^3/ft^2-min, and a one-foot diameter ogive cylinder weighing 350 lbs shall be used to illustrate the presented theory.

The mass of the parachute, m_c, shall be the same as in Ref 2, namely:

$$m_c = 2.82\,\pi\, r^2 \times 10^{-3}\ \text{slugs}$$

which, in dimensionless terms is:

$$\overline{m}_c = 0.537$$

From the same source, the mass of the enclosed air amounts to:

$$m_i = 0.419$$

and in dimensionless form:

$$\overline{m}_i = 0.468$$

From Ref 4 one finds:

$$\frac{\overline{m}_{ax_c}}{\overline{m}_i} = 0.3$$

and therefore:

$$\overline{m}_{ax_c} = 0.1404$$

The dimensionless mass of the load is:

$$\overline{m}_\ell = 105.17$$

From the geometry and various physical masses of the components of the system, one finds:

$$\overline{L}_1 = 2.2166$$

$$\overline{L}_2 = 2.1665$$

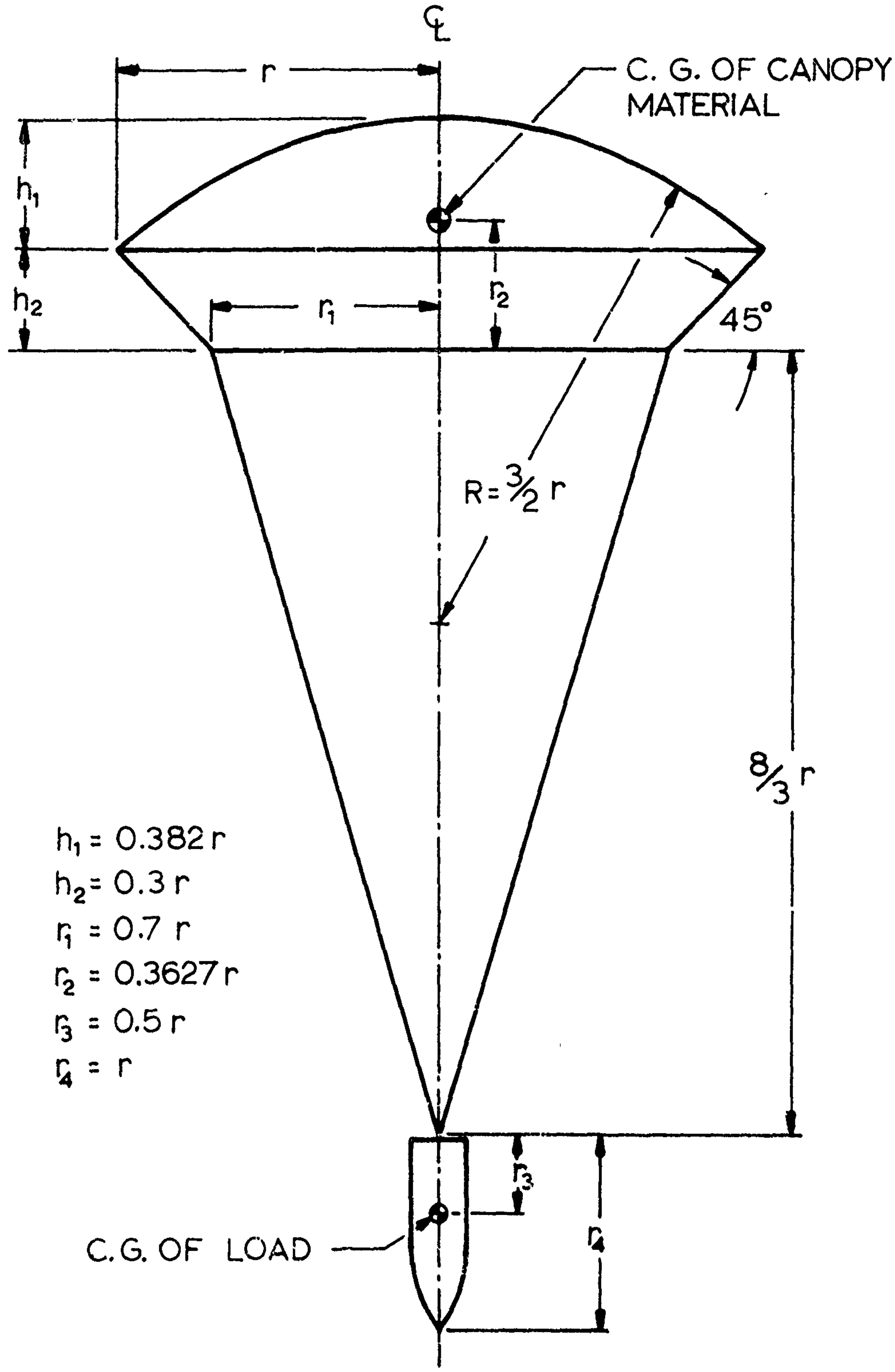

Fig 2. Geometry of the Ribless Guide Surface Canopy and Load

$\bar{L}_3 = 0.0064$

$\bar{L}_4 = 2.2105$

A...ing that the distribution of mass in the ogive is uniform, the moment of inertia amounts to:

$$\bar{I}_{C.G.} = 10.8536$$

Again, from Ref 2 one finds the apparent moment of inertia of the canopy about the center of mass of the system to be:

$$I_a = 0.187 \frac{4}{3} \pi r^3 \rho L_2^2$$

and for this particular configuration:

$$\bar{I}_a = 1.3204$$

thus giving a total inertia of:

$$\bar{I}_{TOT} = \bar{I}_{CG} + \bar{I}_a = 12.1740$$

The center of pressure of the ogive cylinder has been experimentally determined through wind tunnel tests and it is found to be located a distance of 0.3ℓ behind the tip of the ogive (Note: ℓ represents the length of the ogive cylinder). With the preceding geometry, one can determine the distance L_5, in dimensionless form, as:

$$\bar{L}_5 = 0.2095$$

Also, the above-mentioned experiments have shown that:

$$C_{T_\ell} = 0.23$$

$$a_\ell = -2.55 \text{ per radian}$$

and from Ref 2 for the parachute:

$$C_{T_c} = 1.08$$

$$a_c = -0.676 \text{ per radian}$$

All of the preceding results may be summarized as:

$$\bar{m}_c = 0.537$$
$$\bar{m}_{ax_c} = \bar{m}_{ay_c} = 0.1404$$
$$\bar{m}_\ell = 105.17$$
$$\bar{L}_1 = 2.2166$$
$$\bar{L}_2 = 2.1665$$
$$\bar{L}_3 = 0.0064$$
$$\bar{L}_4 = 2.2105$$
$$\bar{L}_5 = 0.2095$$
$$\bar{I}_{TOT} = 12.1740$$
$$C_{T_\ell} = 0.23$$
$$C_{T_c} = 1.08$$
$$a_\ell = -2.55 \text{ per radian}$$
$$a_c = -0.676 \text{ per radian}$$
$$\gamma = 0.04$$

In addition to these values, it will here be assumed that the apparent mass of the load is negligible and $\bar{m}_{ax_\ell} = \bar{m}_{ay_\ell} = 0$.

Utilizing the above values in the frequency equation (Eqn 34), one finds:

$$1285\,\lambda^3 + 352.47\,\lambda^2 + 156.7314\,\lambda + 1.6088 = 0$$

of, dividing by the coefficient of λ^3:

$$\lambda^3 + 0.2742\,\lambda^2 + 0.1219\,\lambda + .001251 = 0 \qquad (48)$$

Solving this relation by the method presented in Ref 5, one finds:

$$\lambda_1 = -0.010535$$

$$\lambda_2 = -0.1286 + 0.315554i$$

$$\lambda_3 = -0.1286 - 0.315554i \tag{49}$$

As initial conditions, we choose at $\tau = 0$:

$$\Theta_o = 10^\circ = 0.1745$$

$$\Theta_o' = 0$$

$$\alpha_o = 10^\circ = 0.1745$$

Using Relations 38 and 39, one finds the values of the constants A_1, A_2, A_3, B_1, B_2, B_3 to be:

$$\begin{aligned}
A_1 &= -0.00008294 \\
A_2 &= 0.09460\, e^{-0.39561i} \\
A_3 &= 0.09460\, e^{0.39561i} \\
B_1 &= -0.0037315 \\
B_2 &= 0.09621\, e^{-0.38630i} \\
B_3 &= 0.09621\, e^{0.38630i}
\end{aligned} \tag{50}$$

After several algebraic manipulations, the linearized general theory provides the angles Θ and α as:

$$\Theta = -0.003731\, e^{-0.01054\tau} + 0.19242\, e^{-0.1286\tau} \cos(0.3156\,\tau - 0.38630) \tag{51}$$

$$\alpha = -0.00008294\, e^{-0.01054\tau} + 0.1892\, e^{-0.1286\tau} \cos(0.3156\,\tau - 0.39561) \tag{52}$$

Using the simplified analysis for the vertical descent ($\beta = 0$), Eqn 47 gives:

$$\Theta = 0.1885\, e^{-0.1304 t} \cos(0.3187 t - 0.38834) \qquad (53)$$

The relations 51 and 53 are graphically presented in Fig 3 and tabulated in Table 1. It can be seen that both approaches provide nearly identical results and it appears that, for many practical cases, the simplified method would be entirely satisfactory. However, for a case involving a trajectory with a strongly changing inclination angle or a system with nonlinear aerodynamic coefficients, a numerical solution of the nonlinearized equations may be required.

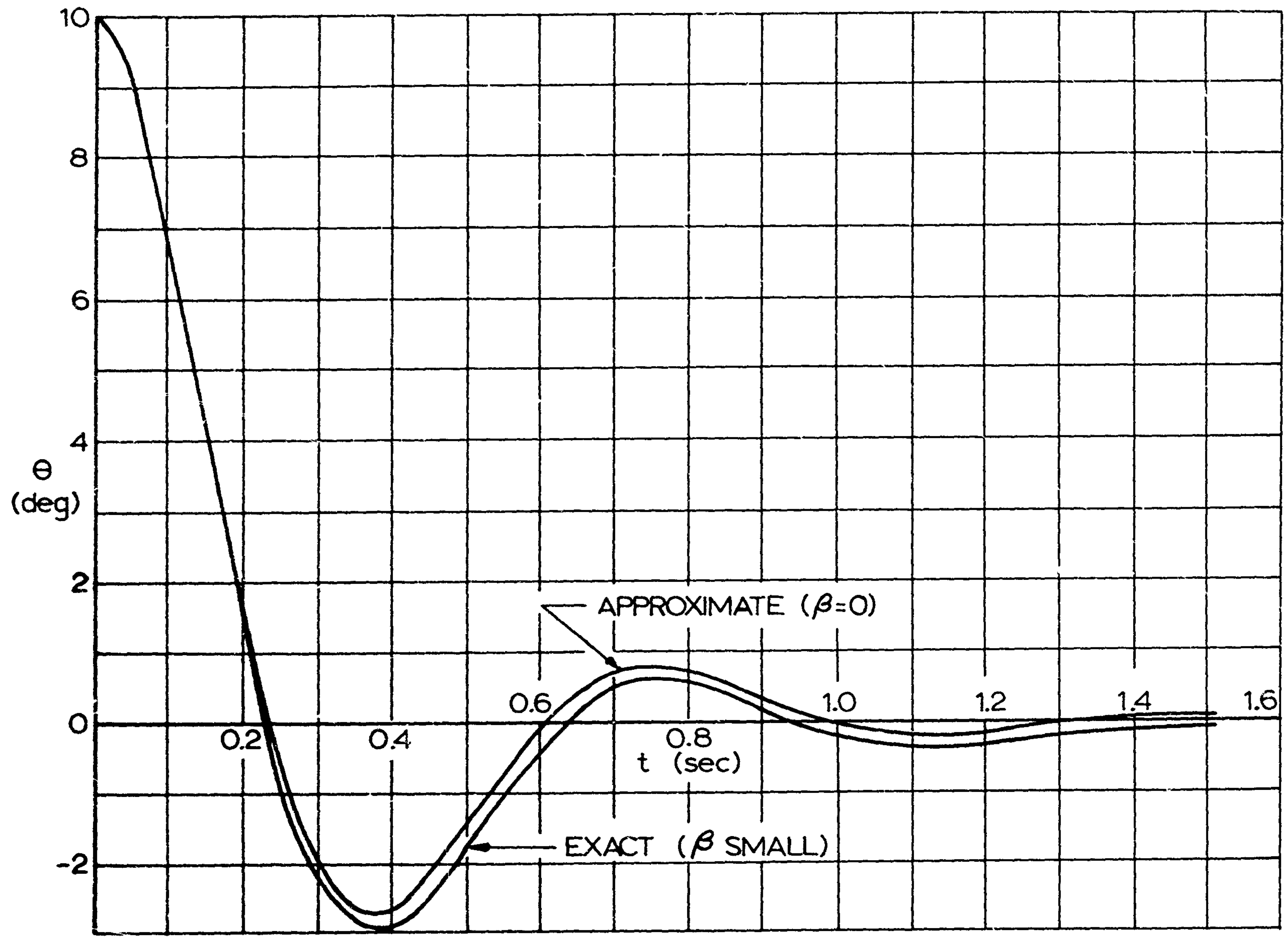

Fig 3 Θ as a Function of Time for the Exact and Approximate Solutions

Table 1. Values of the Angle Θ for the Exact and Approximate Solutions

τ	t (sec)	θ (deg) exact	θ (deg) approx.
2	.0755	8.06	8.06
4	.1511	4.02	4.05
6	.2266	0.12	0.23
8	.3022	-2.31	-2.11
10	.3777	-3.04	-2.77
12	.4532	-2.47	-2.16
14	.5288	-1.34	-1.04
16	.6043	-0.25	0
18	.6799	0.42	0.62
20	.7554	0.62	0.76
22	.8309	0.46	0.58
24	.9065	0.14	0.27
26	.9820	-0.15	-0.02
28	1.0567	-0.33	-0.18
30	1.1331	-0.38	-0.21
32	1.2086	-0.33	-0.16
34	1.2842	-0.23	-0.07
36	1.3597	-0.14	0.01
38	1.4353	-0.09	0.05
40	1.5108	-0.08	0.06

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The several equations of motion governing the dynamic stability of a parachute-load system, in which the parachute as well as the load possesses aerodynamic drag and stability characteristics, are established. The general equations are linearized, which process provides satisfactory results for relatively small deflections. A further simplification is accomplished under the assumption of a vertical descent. A numerical example is used to illustrate the application of the analytical methods.

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There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

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