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NONLINEAR FEEDBACK SOLUTION FOR MINIMUM RENDEZVOUS WITH CONSTANT THRUST ACCELERATION

By

Arthur E. Bryson, Jr.

July 15, 1965

Technical Report No. 478

Cruft Laboratory
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Harvard University • Cambridge, Massachusetts

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NONLINEAR FEEDBACK SOLUTION FOR MINIMUM TIME RENDEZVOUS WITH CONSTANT THRUST ACCELERATION

Arthur E. Bryson, Jr.
Harvard University and the Boeing Co.*

Abstract

The instantaneous thrust-direction for a spacecraft to perform a minimum-time rendezvous with another (non-maneuvering) spacecraft is determined as a function of instantaneous relative velocity and position. The magnitude of the thrust acceleration is assumed constant and the acceleration due to external forces is neglected.

This ostensibly six-coordinate problem (three relative position coordinates and three relative velocity coordinates) can be reduced to a problem in two coordinates, namely $V^2/2ar$ and γ, where a is the magnitude of the thrust acceleration, V is the magnitude of the relative velocity, r is the distance between the two spacecraft, and γ is the angle between the relative velocity vector and the line-of-sight between the two spacecraft.

Let β be the angle between the thrust vector and the line-of-sight and let $T-t$ be the time-to-rendezvous. β and $\frac{a(T-t)}{V}$ for minimum-time rendezvous are given, both analytically and graphically, as functions of $V^2/2ar$ and γ. The analytic solution is in parametric form, namely $V^2/2ar$, γ, β, and $\frac{a(T-t)}{V}$ are expressed as functions of two parameters.

The open-loop solution (the bilinear tangent law) has been known for many years. The new contributions here are (1) showing that the solution depends on only two dimensionless coordinates and (2) putting the solution in the form of a feedback law depending on these two coordinates.

Natural quantities to measure during a rendezvous maneuver are r, $\dot{r}$, and

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* Professor, Division of Engineering and Applied Physics, and Consultant, respectively.

σ where $\dot{\sigma}$ is the rate of rotation of the line-of-sight relative to a fixed reference axis. The two dimensionless coordinates, in terms of these measured quantities, are:

$$\frac{V^2}{2ar} = \frac{(\dot{r})^2 + (r\dot{\sigma})^2}{2ar} \quad ; \quad \tan\gamma = \frac{r\dot{\sigma}}{(-\dot{r})}$$

For comparison, the minimum-time rendezvous maneuver using three constant-thrust-direction periods is presented. The time-to-rendezvous is found to be very close to that of the continuously variable thrust direction solution.

Introduction

The rendezvous maneuver consists of bringing the relative position and relative velocity of one spacecraft with respect to another to zero simultaneously. It is a difficult maneuver and feedback control will almost certainly be required to do it properly. In this paper we consider feedback control of rendezvous for the case where the target spacecraft is not maneuvering and the pursuing spacecraft has a thrust acceleration of constant magnitude, a , but controllable direction. External forces are neglected, or equivalently the external forces per unit mass (such as gravity) are assumed to be constant in magnitude and direction during the maneuver: this latter assumption is reasonably good for nearly-circular satellite orbits if the maneuver time is short enough that the angular distance traveled around the attracting center is smaller than 30° to 40°. (Reference 1)

Minimum Time Rendezvous Using Continuously Variable Thrust Direction

Take the origin in the target, which is assumed to be moving with constant velocity with respect to an inertial coordinate system. The rendezvous vehicle must then bring its position and velocity to zero in minimum time. The problem is two-dimensional since the target, the rendezvous vehicle, and the relative velocity vector determine a maneuvering plane. The equations of motion for the rendezvous vehicle are:

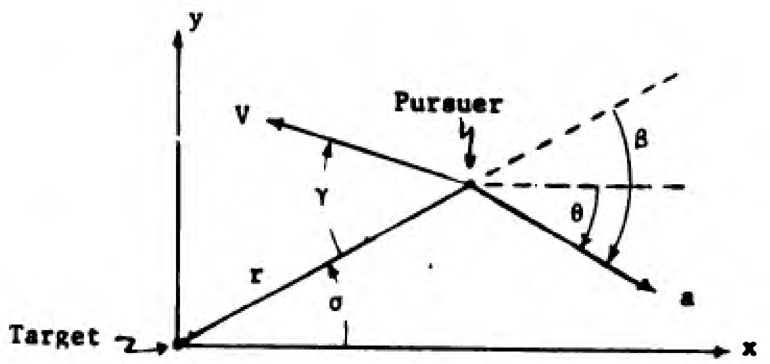
(1) $$\dot{u} = a \cos\theta \quad ;$$

(2) $$\dot{v} = -a \sin\theta \quad ;$$

(3) $$\dot{x} = u \quad ;$$

(4) $$\dot{y} = v \quad ;$$

where (u,v) are velocity components, (x,y) are position components, and the magnitude of the thrust acceleration, a, is assumed constant (see Sketch 1).



Sketch 1

The Hamiltonian of the system is

(5) $$H = \lambda_u a \cos\theta - \lambda_v a \sin\theta + \lambda_x u + \lambda_y v \quad ,$$

so the Euler-Lagrange equations are

(6) $$\dot{\lambda}_u = -\lambda_x \quad ,$$

(7) $$\dot{\lambda}_v = -\lambda_y \quad ,$$

(8) $$\dot{\lambda}_x = 0 \quad ,$$

(9) $$\dot{\lambda}_y = 0 \quad ,$$

(10) $$0 = a\,(\lambda_u \sin\theta + \lambda_v \cos\theta) \quad .$$

Equations (6)-(9) are easily integrated to yield

(11) $$\lambda_u = \lambda_{u_f} + \lambda_x(T-t) \quad ,$$

(12) $$\lambda_v = \lambda_{v_f} + \lambda_y(T-t) \quad ,$$

(13) $$\lambda_x = \text{constant} \quad ,$$

(14) $$\lambda_y = \text{constant} \quad ,$$

where t = time , T = final time , and λ_{u_f} , λ_{v_f} are final (constant) values of λ_u and λ_v . Combining these with (10) we obtain the "bilinear tangent law"

(15) $$-\tan\theta = \frac{\lambda_{v_f} + \lambda_y(T-t)}{\lambda_{u_f} + \lambda_x(T-t)} \quad .$$

This latter relation may be put into the form of a "linear tangent law" as follows:

(16) $$\tan(\theta-\alpha) = \tan(\theta_f-\alpha) + m(T-t)$$

where θ_f = final value of θ and

(17) $$\tan\alpha = \frac{\lambda_x}{\lambda_y} \quad ,$$

(18) $$m = \frac{\lambda_x^2 + \lambda_y^2}{\lambda_x\lambda_{v_f} - \lambda_y\lambda_{u_f}} \quad ,$$

(19) $$\tan(\theta_f-\alpha) = \frac{\lambda_x\lambda_{u_f} + \lambda_y\lambda_{v_f}}{\lambda_x\lambda_{v_f} - \lambda_y\lambda_{u_f}} \quad .$$

Differentiating (16) with respect to time yields

(20) $$\dot{\theta} = -m \cos^2(\theta-\alpha) \quad .$$

Using θ as the independent variable instead of t, we combine (1) and (2) with (20):

$$-\frac{du}{d\theta} = \frac{a}{m}\,\frac{\cos\theta}{\cos^2(\theta-\alpha)} = \frac{a}{m}\,\frac{\cos(\theta-\alpha)\cos\alpha - \sin(\theta-\alpha)\sin\alpha}{\cos^2(\theta-\alpha)}, \tag{21}$$

$$\frac{dv}{d\theta} = \frac{a}{m}\,\frac{\sin\theta}{\cos^2(\theta-\alpha)} = \frac{a}{m}\,\frac{\sin(\theta-\alpha)\cos\alpha + \cos(\theta-\alpha)\sin\alpha}{\cos^2(\theta-\alpha)}. \tag{22}$$

These relations are readily integrated, using $u(\theta_f) = v(\theta_f) = 0$:

$$\begin{matrix}(23)\\ \\(24)\end{matrix}\qquad \begin{bmatrix}-u\\ \\ -v\end{bmatrix} = \frac{a}{m}\begin{bmatrix}\cos\alpha, -\sin\alpha\\ \\ \sin\alpha,\ \cos\alpha\end{bmatrix}\begin{bmatrix}\sinh^{-1}(\tan(\theta-\alpha))-\sinh^{-1}(\tan(\theta_f-\alpha))\\ \\ \sec(\theta_f-\alpha)-\sec(\theta-\alpha)\end{bmatrix}$$

Again, using θ as independent variable, we combine (3) and (4) with (20) and (23)-(24), and integrate, using $x(\theta_f) = y(\theta_f) = 0$:

$$\begin{matrix}(25)\\ \\(26)\\ \\ \end{matrix}\qquad \begin{bmatrix}x\\ \\ y\end{bmatrix} = \frac{a}{m^2}\begin{bmatrix}\cos\alpha, -\sin\alpha\\ \\ \sin\alpha,\ \cos\alpha\end{bmatrix}\begin{bmatrix}\sec(\theta-\alpha)-\sec(\theta_f-\alpha)-\tan(\theta-\alpha)[\sinh^{-1}(\tan(\theta-\alpha))-\sinh^{-1}(\tan(\theta_f-\alpha))]\\ \\ \frac{1}{2}\{\sec(\theta_f-\alpha)[\tan(\theta_f-\alpha \quad an(\theta-\alpha)]+\tan(\theta-\alpha)[\sec(\theta-\alpha)-\sec(\theta_f-\alpha)]\\ \\ +\sinh^{-1}(\tan(\theta-\alpha)-\sinh^{-1}(\tan(\theta_f-\alpha))\}\end{bmatrix}$$

Equations (23)-(26) may be regarded as four equations in the four unknowns m, α, θ, and θ_f when u, v, x, y, and a are given. We can reduce this to two equations in the two unknowns, $\theta-\alpha$, $\theta_f-\alpha$ by introducing polar coordinates as follows (see Sketch 1):

$$\tan\sigma = \frac{y}{x}, \tag{27}$$

$$r = (x^2+y^2)^{1/2}, \tag{28}$$

$$\tan\gamma = \frac{r\dot{\sigma}}{(-\dot{r})}, \tag{29}$$

$$V = (u^2+v^2)^{1/2}. \tag{30}$$

Differentiating (27) and (28) with respect to time gives

(31) $$r\dot{\sigma} = \frac{xv - yu}{r} ,$$

(32) $$\dot{r} = \frac{xu + yv}{r} .$$

Substituting (31)-(32) into (29)-(30) gives

(33) $$\tan\gamma = \frac{uy - vx}{ux + vy} ,$$

(34) $$\frac{V^2}{2ar} = \frac{u^2 + v^2}{2a(x^2+y^2)^{1/2}} .$$

Let us write (23)-(26) in the form

(35) $$\begin{bmatrix} -u \\ -v \end{bmatrix} = \frac{a}{m}\begin{bmatrix} \cos\alpha, -\sin\alpha \\ \sin\alpha \quad \cos\alpha \end{bmatrix}\begin{bmatrix} f_u(\theta-\alpha,\theta_f-\alpha) \\ f_v(\theta-\alpha,\theta_f-\alpha) \end{bmatrix} ,$$

(36) $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{a}{m^2}\begin{bmatrix} \cos\alpha, -\sin\alpha \\ \sin\alpha, \cos\alpha \end{bmatrix}\begin{bmatrix} f_x(\theta-\alpha,\theta_f-\alpha) \\ f_y(\theta-\alpha,\theta_f-\alpha) \end{bmatrix} .$$

Using (35)-(36), we can write (16), (27), (33), and (34) in terms of f_x , f_y , f_u , f_v :

(37) $$\frac{a(T-t)}{V} = \frac{\tan(\theta_f-\alpha) - \tan(\theta-\alpha)}{\sqrt{f_u^2 + f_v^2}}$$

(38) $$\beta = \sigma-\theta = \sigma-\alpha-(\theta-\alpha) = \tan^{-1}\left(\frac{f_y}{f_x}\right) - (\theta-\alpha)$$

(39) $$\gamma = \tan^{-1}\frac{f_y f_u - f_x f_v}{f_x f_u + f_y f_v}$$

(40) $$\frac{V^2}{2ar} = \frac{f_u^2 + f_v^2}{2(f_x^2+f_y^2)^{1/2}}$$

Note that γ and $\frac{V^2}{2ar}$ determine $\theta-\alpha$, $\theta_f-\alpha$ through (39) and (40), and $\theta-\alpha$,

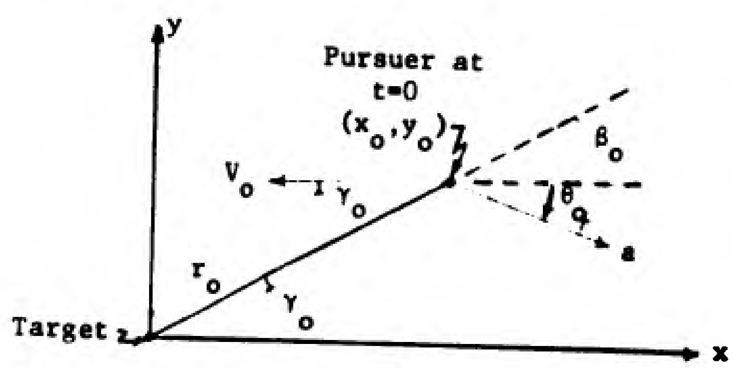
$\theta_f - \alpha$ in turn determine $a(T-t)/V$ and β through (37) and (38). Thus the optimum thrust direction angle with respect to the line-of-sight, β, is given as a function of γ and $V^2/2ar$; this is the feedback law for minimum time rendezvous. Also, the dimensionless time-to-rendezvous, $\frac{a(T-t)}{V}$, is given as a function of γ and $V^2/2ar$.

Figure 1 shows several minimum-time paths on a $V^2/2ar$ vs. γ plot and also shows contours of constant thrust-direction angle, β .

Figure 2 shows the same minimum-time paths as Figure 1 and also shows contours of constant dimensionless time-to-rendezvous, $\frac{a(T-t)}{V}$.

Minimum Time Rendezvous Using Three Constant Thrust-Direction Periods

Using the same assumptions as in the previous section, we use the further convention that, at the beginning of the rendezvous maneuver, $v = \dot{y} = 0$. This simply determines the direction of the x,y axes; x_o , y_o , and V_o are arbitrary, except that we can always choose coordinate axes so that y_o and V_o are positive or zero (see Sketch 2).



Sketch 2

For given a , y_o , and V_o , there is an $(x_o)_{opt}$ that produces minimum maneuver time to rendezvous, hereafter called the "minimum fuel path." If we allow only <u>three constant thrust-direction periods</u> then there are only two different types of minimum-time paths, separated by the minimum fuel path, as shown in Figure 3. These correspond to $x_o < (x_o)_{opt}$ or $x_o > (x_o)_{opt}$. Figures 4 and 5 show the $u(t)$, $v(t)$ histories for these two types of path, and the thrust direction histories $\theta(t)$; the thrust directions are shown as arrows on Figure 3.

Note there <u>is</u> one switch in $\dot{u}$ (at time t_u) and one switch in $\dot{v}$ (at time $\frac{T}{2}$), and the minimum fuel path is the case where $t_u = T$ (or equivalently $t_u = 0$). The velocity components are given by

(41) $$u = \begin{cases} -V_o + (a\cos\theta_o)t & ; \quad 0 < t < t_u \\ -V_o + (a\cos\theta_o)(2t_u - t) & ; \quad t_u < t < T \end{cases}$$

(42) $$v = \begin{cases} -(a\sin\theta_o)t & ; \quad 0 < t < \frac{T}{2} \\ -(a\sin\theta_o)(T-t) & ; \quad \frac{T}{2} < t < T \end{cases}$$

Integrating these two relations we obtain the position coordinates:

(43) $$x = \begin{cases} x_o - V_o t + (a\cos\theta_o)\frac{t^2}{2} & ; \quad 0 < t < t_u \\ x_o - V_o t + (a\cos\theta_o)(\frac{t^2}{2} - 2t_u t + t_u^2) & ; \quad t_u < t < T \end{cases}$$

(44) $$y = \begin{cases} y_o - (a\sin\theta_o)\frac{t^2}{2} & ; \quad 0 < t < \frac{T}{2} \\ y_o - (a\sin\theta_o)[\frac{T^2}{4} - \frac{(T-t)^2}{2}] & ; \quad \frac{T}{2} < t < T \end{cases}$$

Putting $u(T) = x(T) = y(T) = 0$ in (41), (43), (44), we obtain three simultaneous equations for the three unknowns, θ_o, t_u, and T:

(45) $$V_o = a\cos\theta_o(2t_u - T)$$

(46) $$x_o = a\cos\theta_o(t_u^2 - \frac{T^2}{2})$$

(47) $$y_o = a\sin\theta_o\,\frac{T^2}{4}$$

where (45) was used to eliminate V_o from (43).

From (45), solve for $\frac{t_u}{T}$:

(48) $$\frac{t_u}{T} = \frac{1}{2}(1+ \frac{V_o}{aT} \sec\theta_o)$$

Note that $0 \leq \frac{t_u}{T} \leq 1$ implies that

(49) $$|\cos\theta_o| \leq \frac{V_o}{aT}$$

Let $x_o = r_o\cos\gamma_o$, $y_o = r_o\sin\gamma_o$ and use (48) in (46) and (47) to obtain:

(50) $$\tan\gamma_o = \frac{\tan\theta_o}{(1+ \frac{V_o}{aT} \sec\theta_o)^2 - 2}$$

(5i) $$\frac{V_o^2}{2ar_o} = \frac{2\sin\gamma_o}{(\frac{aT}{V_o})^2 \sin\theta_o}$$

These latter equations are simultaneous equations for θ_o and $\frac{aT}{V_o}$ as functions of $V_o^2/2ar_o$ and γ_o .

Figure 6 shows contours of constant $\beta_o = \theta_o + \gamma_o$ on a $V_o^2/2ar_o$ vs. γ_o plot. Note that the locus of initial conditions for minimum fuel, $x_o = (x_o)_{opt}$, corresponds to a discontinuity in the contours and to equality in (49).

Figure 7 shows contours of const ·· $\frac{aT}{V_o}$ on a $V_o^2/2ar_o$ vs. γ_o plot.

<u>Comparison of Exact and Approximate Solutions</u>

Comparing Figure 2 with Figure 7, it is apparent that minimum time using three constant thrust-direction periods is only slightly longer than minimum time using continuously variable thrust-direction. This result agrees well with the results of Reference 2 which considers the more complicated problem of rendezvous and fly-by trajectories of a spacecraft with Mars using two or more constant thrust-direction periods.

Comparison of Figure 1 with Figure 6 is more difficult since Figure 6 shows only <u>initial</u> thrust-direction angle, β_o , whereas Figure 1 shows thrust-direction angle, β , throughout the rendezvous maneuver. In other words, Figure 1 displays a "closed-loop" continuous feedback solution whereas Figure 6 displays an "open-loop" program based only on initial conditions.

Minimum Fuel Solution

In many cases the minimum-fuel solution (which corresponds to minimum maneuver time in this problem) will be of interest and hence the time to begin thrusting must be determined. Figure 3 shows the situation where the pursuer is coasting toward the target with constant relative velocity V_o , and, if no thrust were used, the pursuer would miss the target by a distance y_o . This straight-line coasting path shows on Figure 2 as a sine curve since

$$\frac{V^2}{2ar} = \frac{V_o^2}{2ay_o} \sin\gamma_o \tag{52}$$

and $V_o^2/2ay_o$ is constant during coast. Along this sine curve there will be a minimum value of $\frac{aT}{V}$. The locus of such points is shown in Figures 1 and 2 and, for the three constant thrust-direction period solution, in Figures 6 and 7. These minimum-fuel paths correspond with the case $\lambda_x = 0$ in Equations (11)-(19); note this gives a "linear tangent law" in Equation (15).

Conclusion

A continuous nonlinear feedback law has been obtained for controlling thrust direction to produce minimum time rendezvous of a spacecraft with a non-maneuvering target. This feedback law depends on only two dimensionless quantities which can be determined by measurements of three physical quantities: (1) distance to the target, (2) closing velocity along the line-of-sight, and (3) rate of rotation of the line-of-sight with respect to an inertial axis in the maneuver plane.

An approximate solution using only three constant thrust-direction periods was presented and shown to increase the time-to-rendezvous by only a few percent over the minimum time.

Acknowledgement

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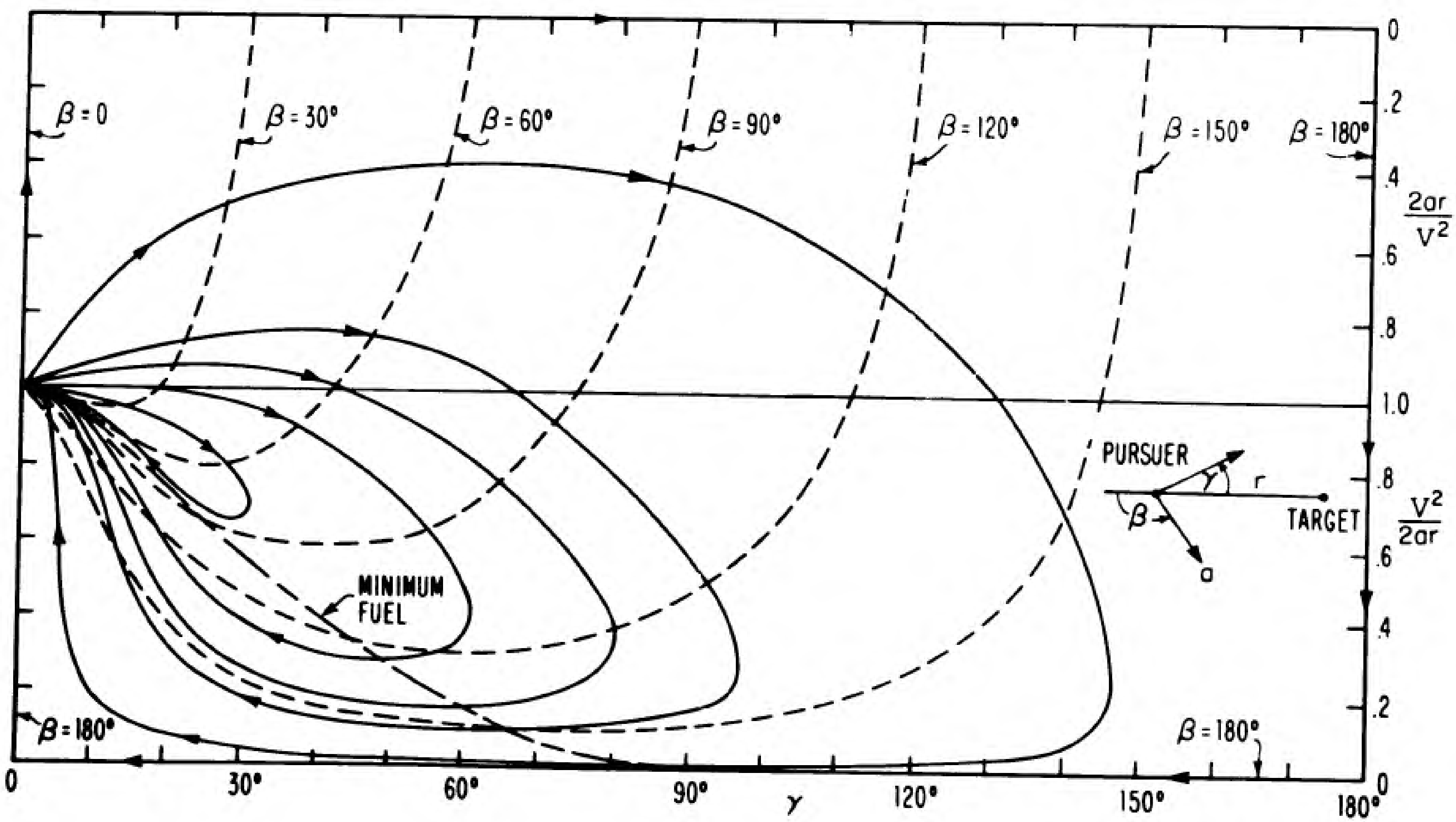
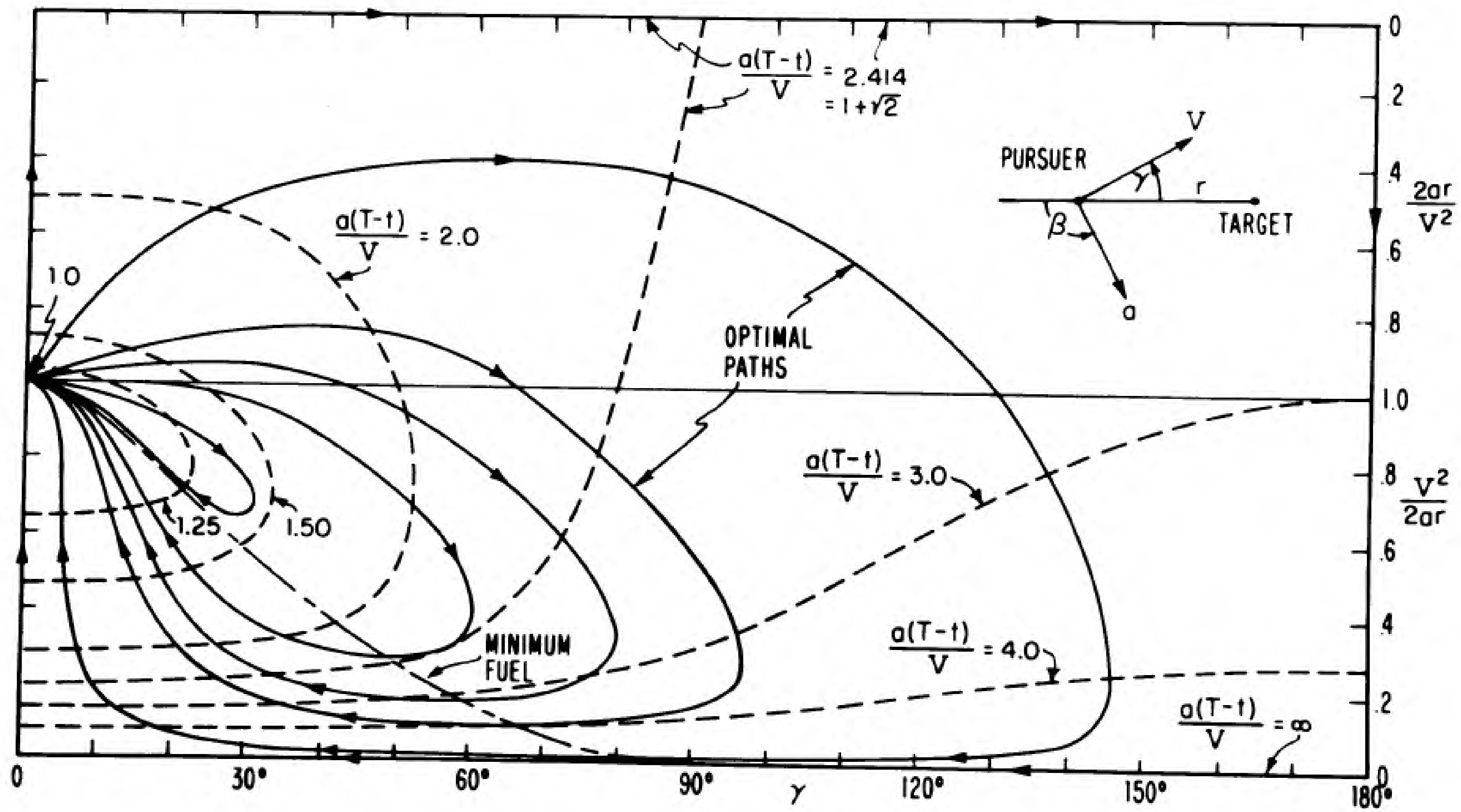
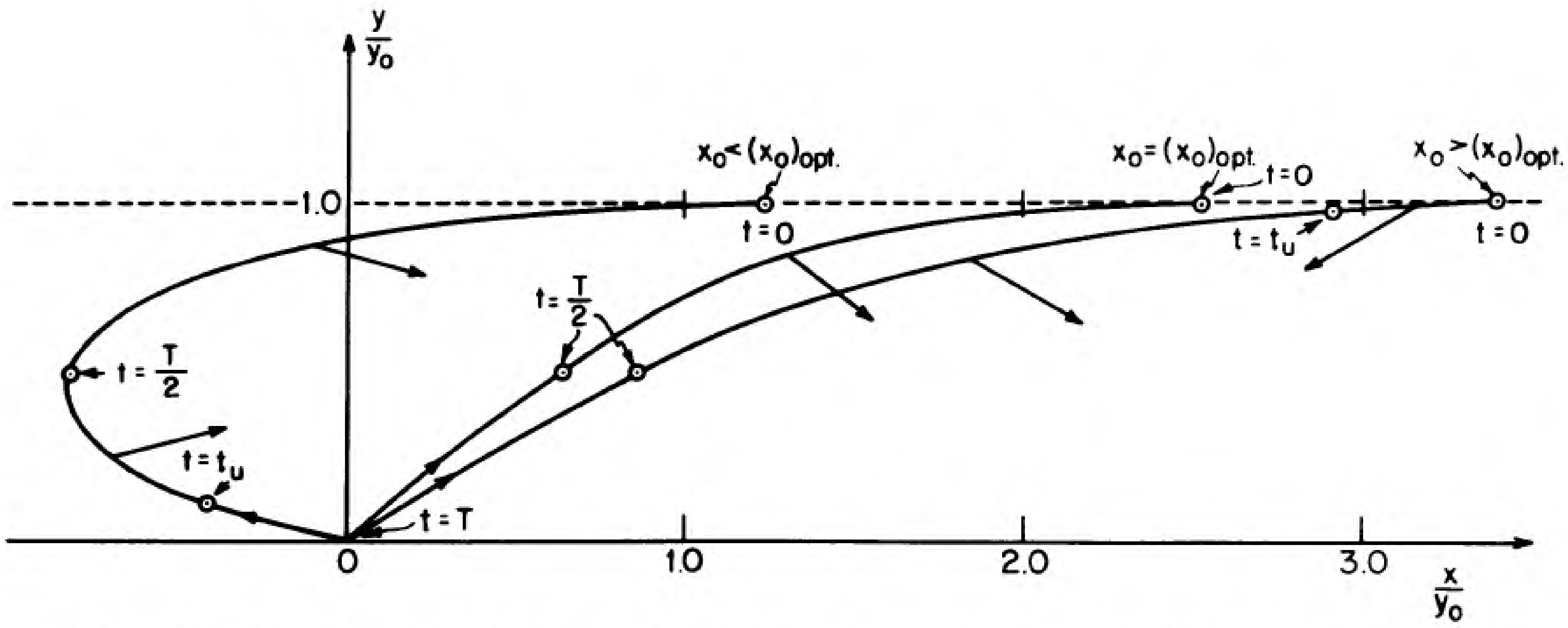


OPTIMAL PATHS FOR MINIMUM-TIME RENDEZVOUS AND THRUST DIRECTION, β, AS A FUNCTION OF $V^2/2ar$ AND γ

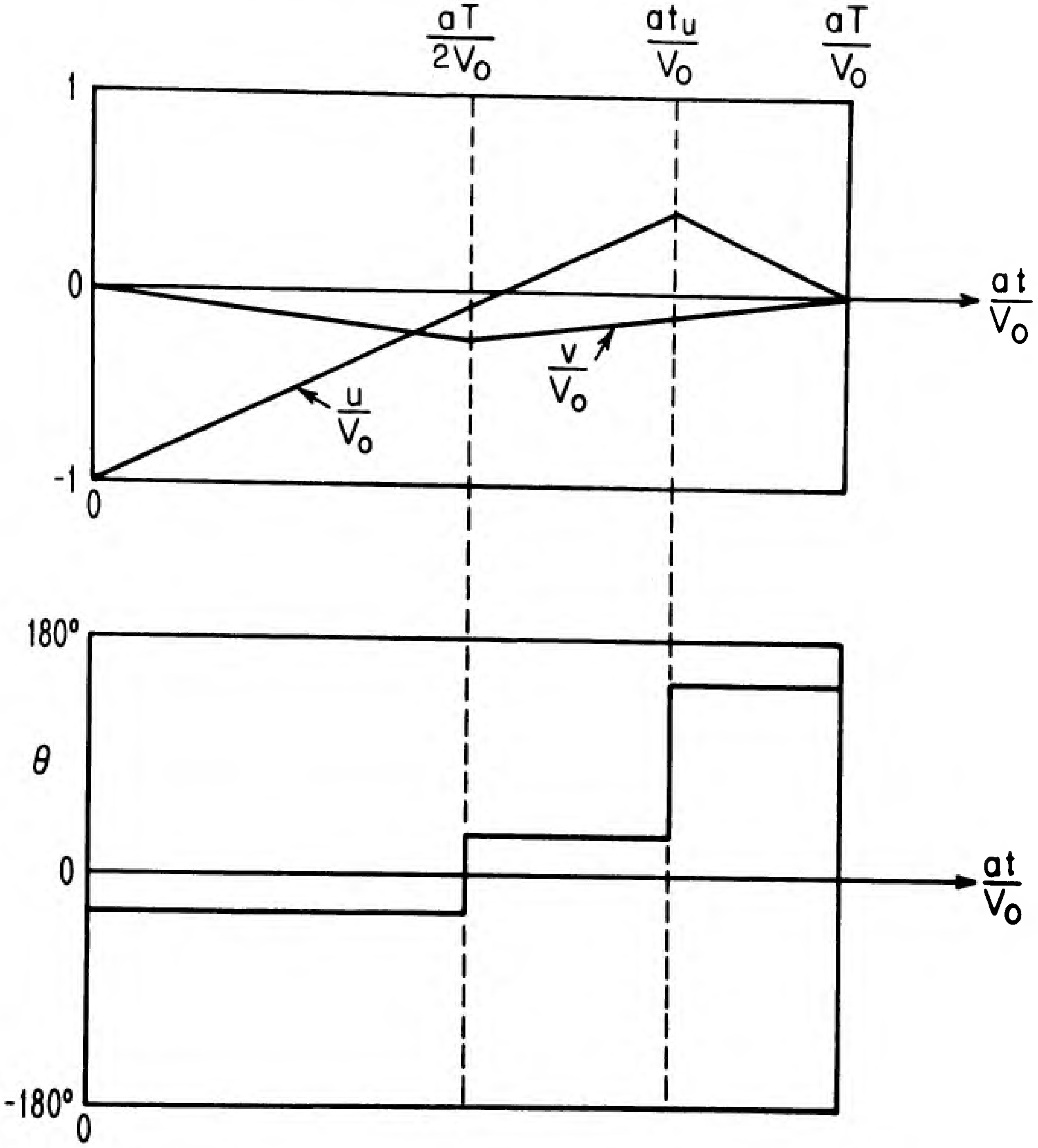


OPTIMAL PATHS AND MINIMUM TIME-TO-RENDEZVOUS AS A FUNCTION OF $V^2/2ar$ AND γ

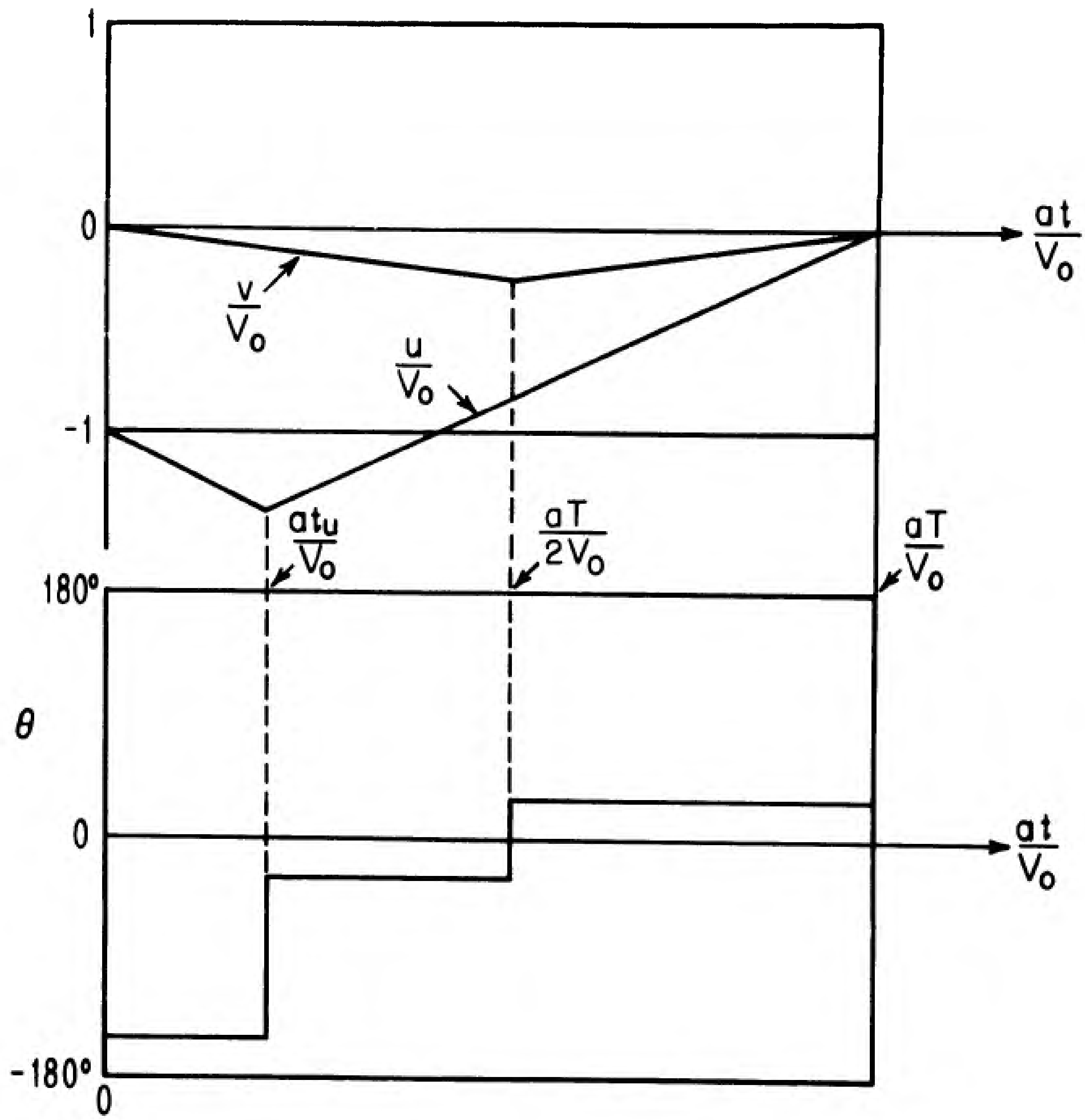


MINIMUM-TIME RENDEZVOUS PATHS USING 3 CONSTANT THRUST-DIRECTION PERIODS

$V_0^2 / 2ay_0 = 2.0$

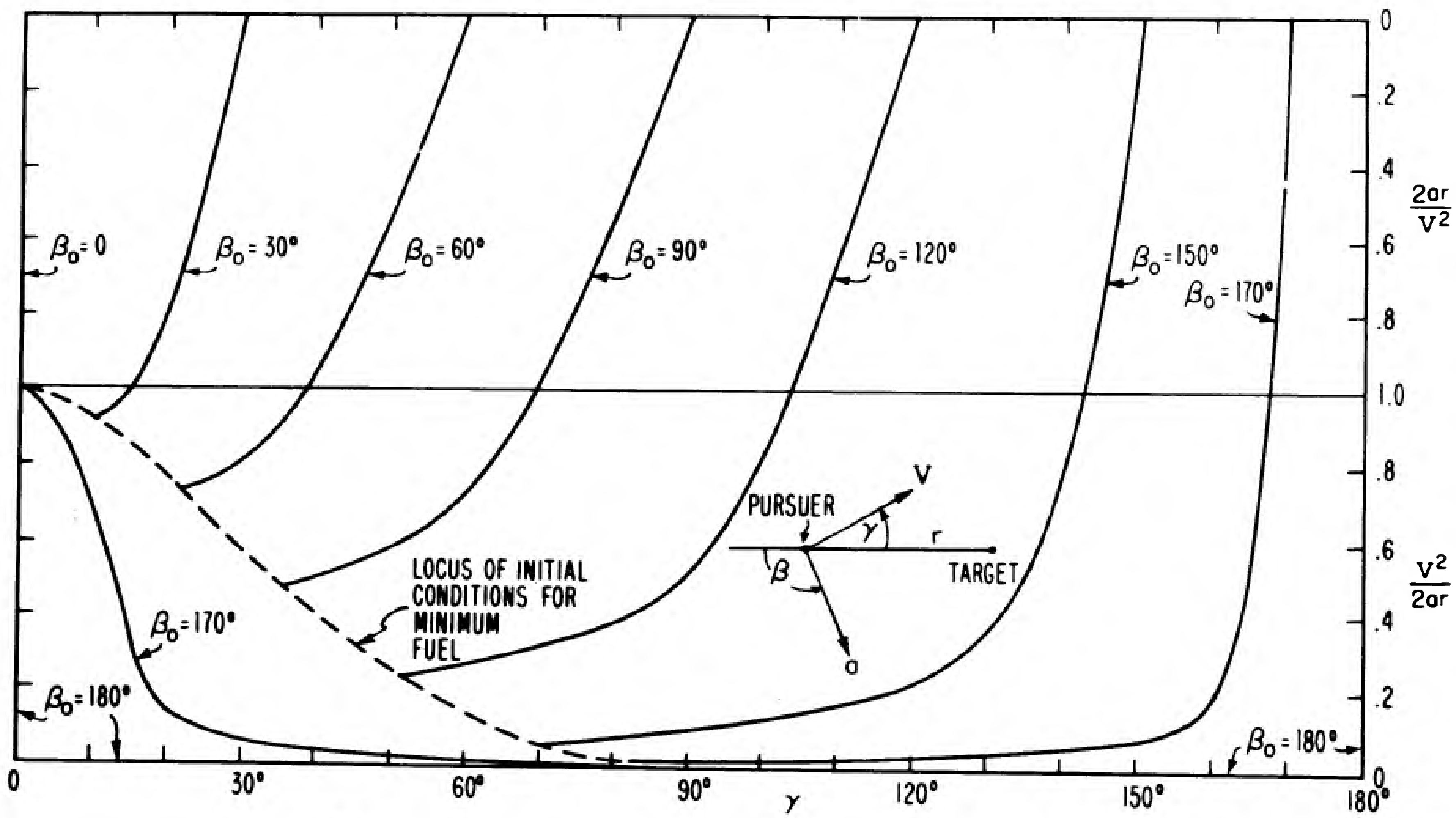


VELOCITY AND THRUST ANGLE HISTORIES
$x_o \quad (x_o)_{opt.}$

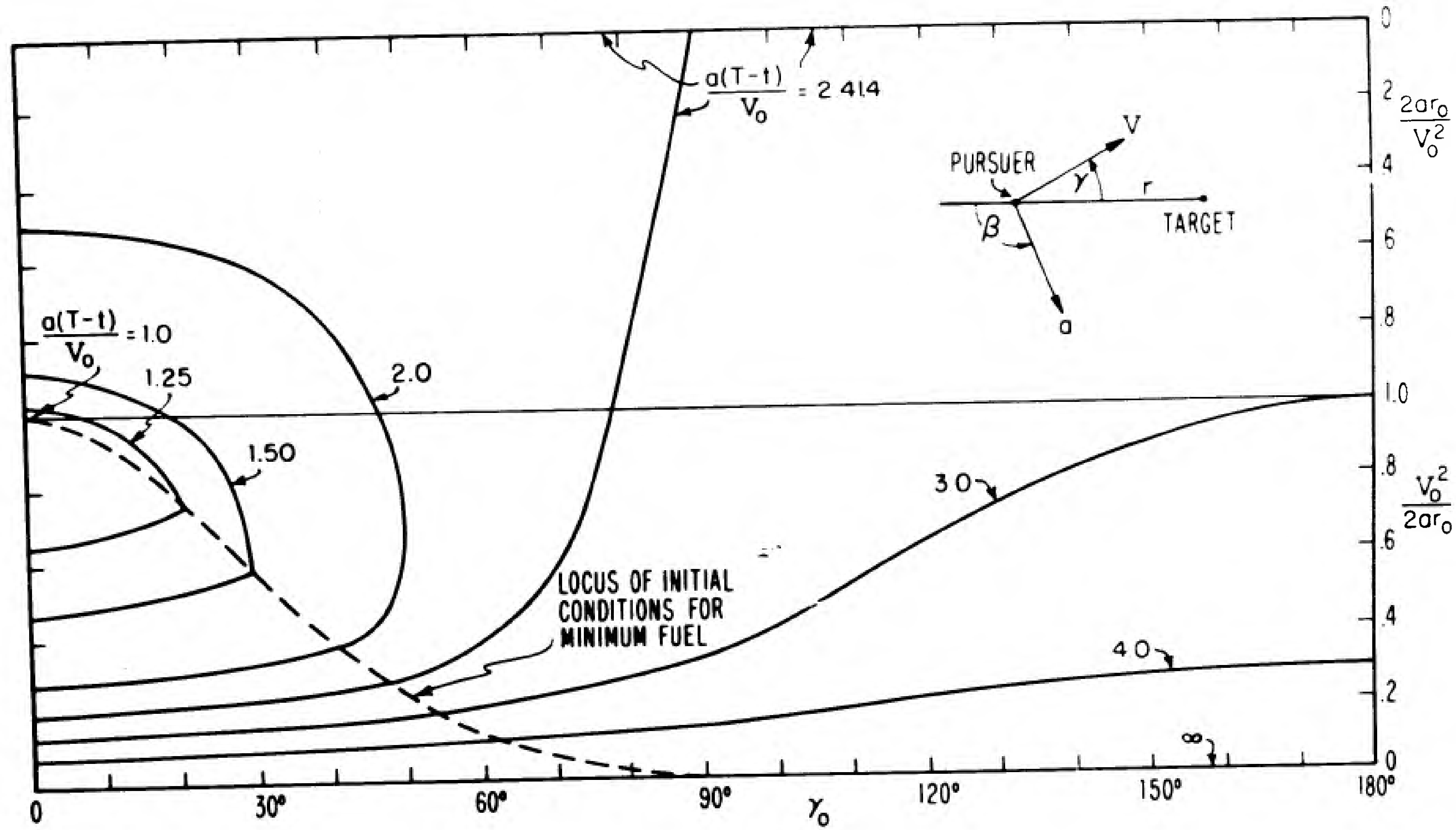


VELOCITY AND THRUST ANGLE HISTORIES

$x_o > (x_o)_{opt.}$



INITIAL THRUST DIRECTION ANGLE AS A FUNCTION OF $V^2/2ar$ AND γ FOR MINIMUM TIME RENDEZVOUS USING THREE CONSTANT THRUST-DIRECTION PERIODS



MINIMUM TIME-TO-RENDEZVOUS AS A FUNCTION OF $V_o^2/2ar_o$ AND γ_o FOR THREE CONSTANT THRUST-DIRECTION PERIODS

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13 ABSTRACT

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This ostensibly six-coordinate problem (three relative position coordinates and three relative velocity coordinates) can be reduced to a problem in two coordinates, namely, $V^2/2ar$ and γ, where a is the magnitude of the thrust acceleration, V is the magnitude of the relative velocity, r is the distance between the two spacecraft, and γ is the angle between the relative velocity vector and the line-of-sight between the two spacecraft.

Let β be the angle between the thrust vector and the line-of-sight, and let $T-t$ be the time-to-rendezvous. β and $a(T-t)/V$ for minimum-time rendezvous are given, both analytically and graphically, as functions of $V^2/2ar$ and γ. The analytic solution is in parametric form, namely, $V^2/2ar$, γ, β, and $a(T-t)/V$ are expressed as functions of two parameters.

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Natural quantities to measure during a rendezvous maneuver are r, $\dot{r}$, and $\dot{\sigma}$ where $\dot{\sigma}$ is the rate of rotation of the line-of-sight relative to a fixed reference

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13. ABSTRACT, continued

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$$\frac{V^2}{2ar} = \frac{(\dot{r})^2 + (r\dot{\sigma})^2}{2ar} \; ; \; \tan\gamma = \frac{r\dot{\sigma}}{(-\dot{r})}$$

For comparison, the minimum-time rendezvous maneuver using three constant-thrust-direction periods is presented. The time-to-rendezvous is found to be very close to that of the continuously variable direction solution.

14. KEY WORDS

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