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Part IV

ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
Part IV: Ball Bearing Design Data

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TECHNICAL REPORT AFAPL-TR-65-45, PART IV

May 1965

Air Force Aero Propulsion Laboratory
Research and Technology Division
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

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FOREWORD

This report was prepared by Mechanical Technology Incorporated, 968 Albany-Shaker Road, Latham, New York 12110, under USAF Contract No. AF 33(615)-1895. The contract was initiated under Project No. 3145, "Dynamic Energy Conversion Technology", Task No. 314511, "Nuclear Mechanical Power Units". The work was administered under the direction of the Air Force Aero Propulsion Laboratory, Research and Technology Division, with Mr. John L. Morris (APFL) acting as project engineer.

This report covers work conducted from 1 April 1964 to 1 April 1965.

This report was submitted by the authors for review on 18 March 1965. It is Part IV of final documentation issued in multiple parts. This report also is identified by the contractor's designation MTI-65-TR-35.

This technical report has been reviewed and is approved.

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ABSTRACT

This Part IV of the Final Report presents design data for the stiffness characteristics of ball bearings for use in analyzing the dynamical performance of a rotor. The dynamic characteristics of fluid film bearings are given in Part III which also gives the methods for performing the analysis of the rotor-bearing system.

Design data are presented for the extra-light and light group of deep-grooved and angular contact bearings undergoing either a pure radial load, pure axial load, or combined radial load with axial preload. The data are given in graphical form and cover both radial stiffness and load-carrying capacity. A nominal damping value for ball bearings, obtained from experimentation, is suggested.

Some of the general guide rules for the selection of ball bearings are given. These are concerned with fatigue life, limiting speeds, design, and lubrication. Safe load levels are indicated.

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INTRODUCTION

The purpose of this report is to present design data for typical deep-groove and angular contact bearings of the commonly used light and extra-light series. The data may be used for various rotor-ball bearing system designs, in conjunction with our (MTI) critical speed and unbalance response programs.

The information is presented in graphical form. It consists of load carrying capacity, radial and axial stiffness, and load levels.

A complete description is given for all the variables used. A section entitled "Design Requirements" is written to describe various parameters and to present design considerations, guidelines and limitations.

A number of examples on the application of the curves to specific cases are included.

The analyses used are written in the Appendix, along with a computer program listing of the calculational procedure.

If any particular case is not covered by the included curves, more data may be generated by computer program PNO-182, IBM 1620-60K.

II

DESIGN REQUIREMENTS

A ball bearing schematic is shown as Fig. A to illustrate standard nomenclature.

Types of Bearings

The single row, deep groove ball bearing will sustain radial loads and in addition a substantial thrust load in either direction. When using this type of bearing, careful alignment between the shaft and housing is essential.

The angular contact ball bearing is designed to support a thrust load in one direction or a thrust load (preload) combined with a radial load. These bearings can be mounted singly or, when the side surfaces are flush ground, in multiple, either face-to-face or back-to-back for all combinations of thrust and radial loading. The basic difference between the two is the larger clearance and greater shoulder height of the angular contact bearing. Generally, this will permit operation with higher thrust loads and at higher speeds than the deep groove bearing.

Load Level

The load levels shown (C/P = 5 and C/P = 10) correspond to normally encountered Hertz Stress levels of 230,000 and 186,000 psi respectively.

Ball Bearing Damping

The only available damping information was from non-rotating tests on a grease packed ball bearing (A-2) system. The measured value was in the order of 15-20 pounds sec/in. This should be used only as a "ballpark" since it should be much higher in the rotating condition, and for larger bearing sizes.

Race Curvatures

Since the question of stiffness and rotor dynamics will be a major factor at high speeds, some design guidelines for this aspect are in order. Normally, more open curvatures, one piece machined retainers, and generous internal clearances are preferred. The normal curvatures are 51.6 percent for inner race and 53 percent for outer. Open curvatures, for high speed range, between 54 percent and 57 percent for both inner and outer are used. The 57 percent curvature is widely used and is included in the design discussion.

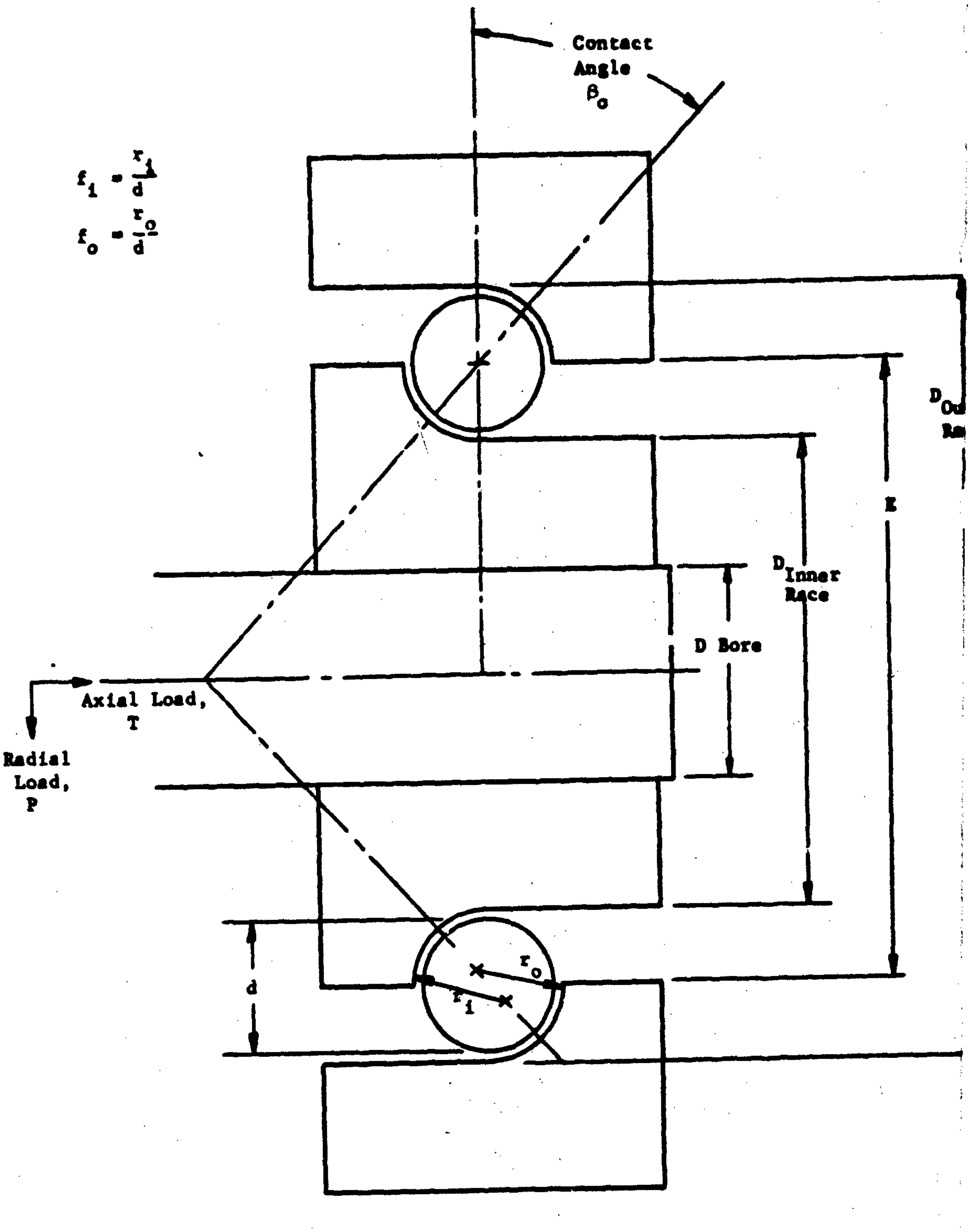


Fig. A   Ball Bearing Schematic

Ball Bearings - Life

The selection of ball bearings for various applications consider such factors as load, speed, temperature, environment, design, and lubrication. However, the initial sizing and selection is usually based upon the fatigue rating of the bearing.

Based upon a statistical distribution proposed by Weibull and the analytical and experimental work of Lundberg and Palmgren the life of the bearing for a given probability of survival has been found to vary inversely as the cube of the applied radial load. For other than radial loading, an equivalent radial load is defined. A Specific Dynamic Capacity (C) is defined as that radial load which will result in a life of one million inner race revolutions with a 90 percent probability of survival. The AFBMA (Anti Friction Bearing Manufacturers Association) has standardized on the following formula for (C):

$$C = f_c(i \cos \beta_o)^{0.7} Z^{2/3} d^{1.8} \tag{1}$$

where i = the number of rows of balls in any one bearing

Z = the number of balls per row

β_o = the angle of contact

d = the ball diameter, inch

f_c = a factor depending on oscillation and material

For normal bearing proportions $f_c \approx 4500$.

The life (90 percent probability of survival) at any other radial load or equivalent radial load (P) is related to the Specific Dynamic Capacity (C) as follows:

$$L = (C/P)^3 \quad \text{millions of inner race revolutions} \tag{2}$$

It is normally assumed that speed affects life in a linear fashion; that is, life varies inversely with speed.* For a given operating speed of N rpm, the number of revolutions which correspond to H hours of life is

$$L = 60\ NH \text{ revolutions}$$

---

* Based upon experimental data, equation (3) is too conservative at high speeds.

and

$$\left(\frac{C}{P}\right)^3 \times 10^6 = 60\ NH$$

$$H = \frac{(C/P)^3 \times 10^6}{60\ N}, \text{ hour} \tag{3}$$

Catalog ratings are generated in this fashion, usually for some given number of hours of life with a 90 percent probability of survival. Five hundred hours is a common catalog rating. Note that a rating at 33-1/3 rpm for 500 hours is the Specific Dynamic Capacity.

Since this is available in the catalogs of most of the bearing companies, no further explanation of this aspect is included in this report.

Bearing Centrifugal Loading

In some instances, it is desired to estimate the life of a ball bearing at extremely high speed with little or no externally applied loading. In this case, the fatigue life is determined by the centrifugal loading of the balls on the outer ring. (Ref. 4)

The outer ring capacity is given by

$$C_o = A\left(\frac{2f_o}{2f_o - 1}\right)^{0.41} \frac{(1+\gamma)^{1.39}}{(1-\gamma)^{1/3}} \gamma^{.3} d^{1.8} Z^{-1/3} \tag{4}$$

where

- A = material constant, usually 7140
- f_o = outer race curvature factor (ratio curvature radius to ball diameter)
- γ = ball diameter to pitch diameter ratio, d/E
- d = ball diameter - inch
- Z = number of balls
- E = pitch diameter - inch

The life of the outer ring is given as

$$\left(C_o/P_{c.f.}\right)^3 = 90\% \text{ life in } 10^6 \text{ revs.} \tag{5}$$

where

$$P_{c.f.} = \text{centrifugal ball loading}$$

$$P_{c.f.} = 5.257 \times 10^{-7}\ d^3\ EN_i^{\,2}\ (1-7)^2$$

Effects of Cyclic Loading on Bearing Life

As was previously shown, the fatigue life of a rolling element bearing is defined in terms of a 90 percent probability of survival. A specific dynamic capacity C is defined as that radial load which will result in a life of 10^6 inner race revolutions with 90 percent survival probability. The 90 percent life at any other load P is related to the specific dynamic capacity as follows:

$$L = (C/P)^3 \quad 10^6 \text{ Rev.} \tag{2}$$

When the load varies in a series of known steps, some equivalent or mean load is defined as follows:

$$P_m = \left(\frac{P_1^{\,3} N_1 + P_2^{\,3} N_2 \text{ ---- } + P_n^{\,3} N_n}{N_1 + N_2 \text{ ---- } + N_n} \right)^{1/3} \tag{6}$$

where P_1, P_2, P_n are loads applied for N_1, N_2, N_n cycles.

For the case of vibratory loading, an integral form of Equation (6) can be used:

$$P_m = \left(\frac{1}{N} \int P^3\ dN \right)^{1/3} \tag{7}$$

In the general case, the loading will consist of some steady load P_o and a sinusoidal load $P_1 \sin \omega t$.

The bearing loading P is given as

$$P = P_o + P_1 \sin \omega t \tag{8}$$

Using this in Equation (7) yields the following:

$$P_m = \left[\frac{1}{\pi} \int_o^{\pi} (P_o + P_1 \sin \omega t)^3\ d(\omega t) \right]^{1/3} \tag{9}$$

Expansion of Equation (9) gives

$$P_m = \left[\frac{1}{\pi} \int_0^\pi (P_o^3 + 3P_o^2P_1 \sin \omega t + 3P_oP_1^2 \sin^2 \omega t + P_1^3 \sin^3 \omega t)\, d\, \omega t)\right]^{1/3} \tag{10}$$

$$P_m = \left[\frac{1}{\pi} \left(P_o^3\, \omega t - 3\, P_o^2P_1 \cos \omega t + 3P_oP_1^2 \frac{\omega t}{2} - 3P_oP_1^2 \frac{\sin 2\omega t}{4} - P_1^3 \cos \omega t + P_1^3 \frac{\cos^3 \omega t}{3}\right)\Bigg|_0^\pi\right]^{1/3}$$

$$P_m = \left[\frac{1}{\pi} (P_o^3\pi + \frac{3}{2} P_oP_1^2\pi)\right]^{1/3} \tag{11}$$

$$= \left[P_o^3 \left(1 + \frac{3}{2} \frac{P_1^2}{P_o^2}\right)\right]^{1/3}$$

$$\frac{P_m}{P_o} = \left[1 + \frac{3}{2} \left(\frac{P_1}{P_o}\right)^2\right]^{1/3} \tag{12}$$

The results of Equation (12) are plotted in Figure B as a function of the cyclic load ratio.

The life may be found from Equation (2) using the equivalent load P_m.

Often the steady state load P_o is known and the effect of various cyclic loads is desired. The life due to the steady state load P_o is

$$L_o = (C/P_o)^3 \tag{13}$$

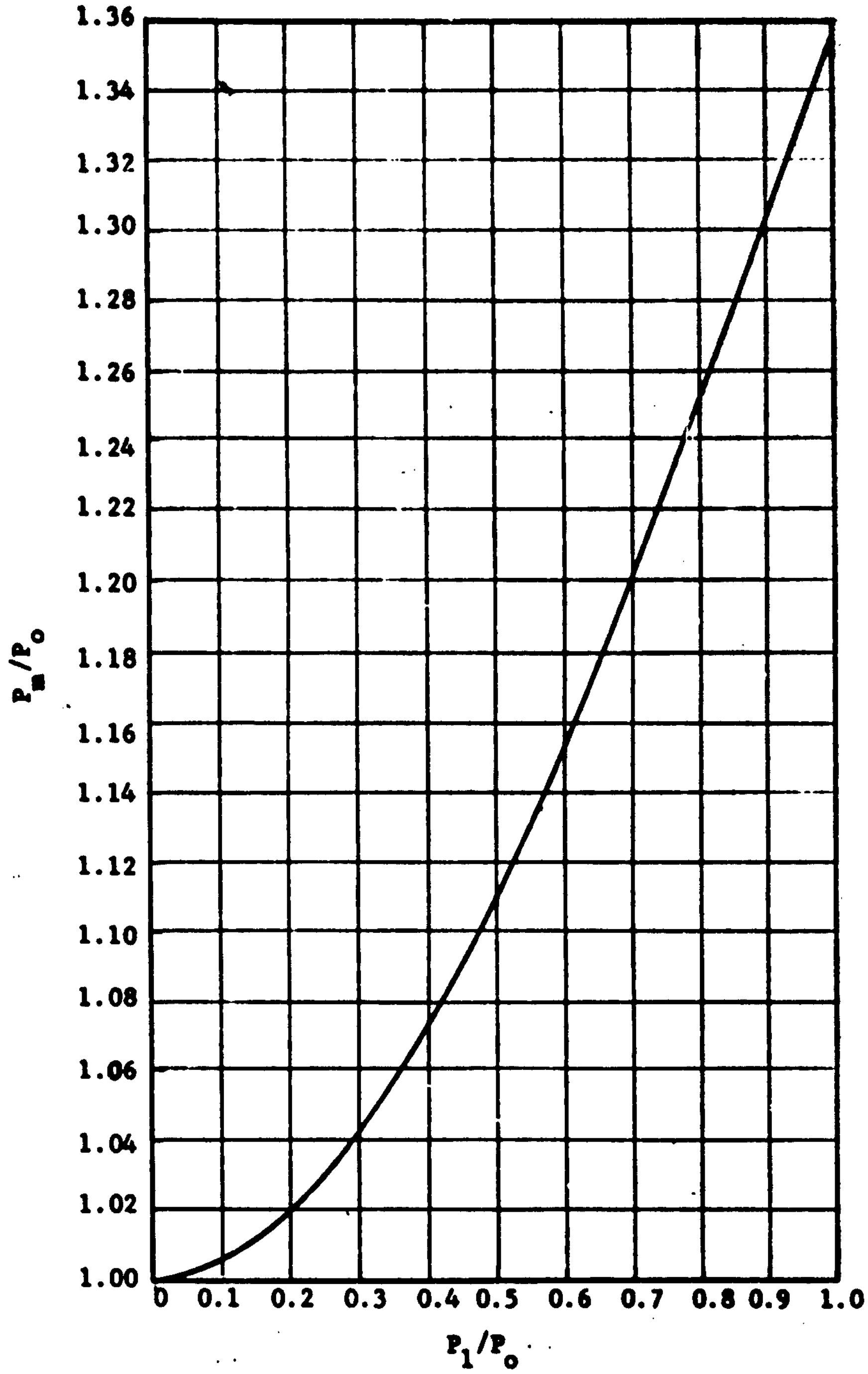


Fig. B Equivalent Load Ratio as a Function of Cyclic Load Ratio

While the life due to the equivalent load P_m is

$$L = (C/P_m)^3 \tag{14}$$

The ratio of the lives is

$$\frac{L}{L_o} = \frac{(C/P_m)^3}{(C/P_o)^3} = \left(\frac{1}{P_m/P_o}\right)^3 \tag{15}$$

The life ratio as a function of the equivalent load ratio is shown in Figure C.

In summary, the results apply to the following:

1. $P_1 \leq P_o$
2. Radial Loading
3. P_o is unidirectional
4. P_1 is the single amplitude of the cyclic disturbance.

The above can be used with manufacturers' catalog data by roting that these are set up for some given life (usually 500 hours) at various speeds. The corresponding load is tabulated.

The case where a cyclic load only is applied is sometimes encountered. The bearing load is then

$$P = P_1 \sin \omega t \tag{16}$$

Equation (7) now becomes

$$\begin{aligned} P_m &= \left[\frac{1}{\pi}\int_0^{\pi} (P_1 \ \sin\omega t)^3 \, d(\omega t)\right]^{1/3} \\ &= \left[\frac{1}{\pi} P_1{}^3 \left(-\cos \omega t + \frac{1}{3}\cos^3 \omega t\right)_o^{\pi}\right]^{1/3} \\ &= 0.752 \, P_1 \end{aligned} \tag{17}$$

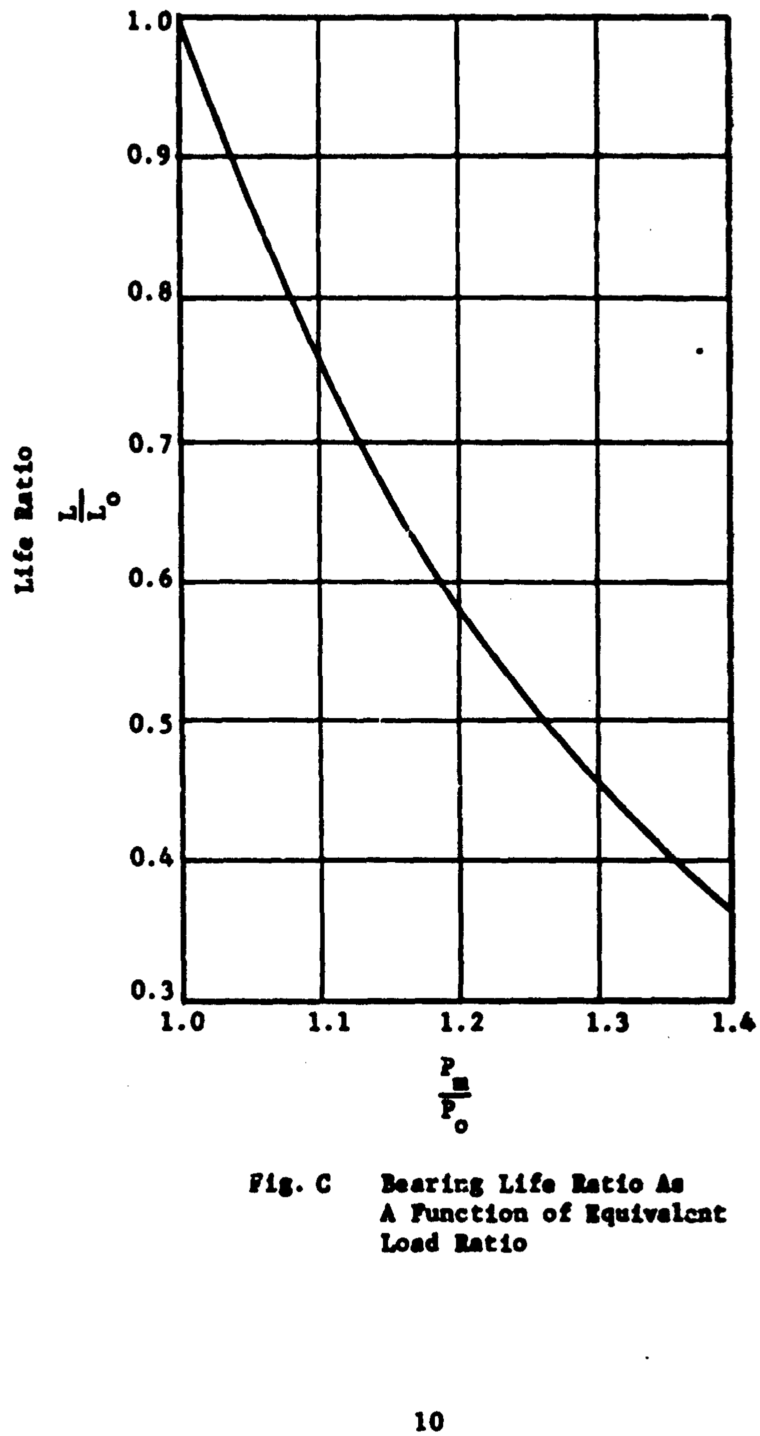


Fig. C Bearing Life Ratio As A Function of Equivalent Load Ratio

The ratio of equivalent loading to the single amplitude of the cyclic loading is

$$\frac{P_m}{P_1} = 0.752 \tag{18}$$

Bearing life is determined from P_m.

As was previously noted, Equation (12) and Figure B were derived for radial loading. However, the relations can be adapted for use with thrust loads, if the thrust load is represented by

$$T = T_o + T_1 \sin \omega t \ , \tag{19}$$

where T_o is the steady thrust load and T_1 is the single amplitude of the cyclic thrust loading. This gives a similar relation to Equation (12) as follows:

$$\frac{T_m}{T_o} = \left[1 + \frac{3}{2}\left(\frac{T_1}{T_o}\right)^2\right]^{1/3} \tag{20}$$

Figure B can be used to obtain either a mean radial or thrust load. However, in the case of thrust loading, bearing life must be calculated using an equivalent radial load with the specific dynamic capacity. For calculation purposes (preliminary engineering calculations) the equivalent radial load is given by:

$$\text{Equiv. Rad. Load} = 0.37\ P + 2T \tag{21}$$

where P is the radial load and T is the thrust load. Either T or P, or both are replaced by P_m and T_m where cyclic loading is involved. More accurate relationships for the various bearing types are found in the manufacturers' catalogs or the AFBMA standards.

Lubricant Life

In many instances, fatigue life is not the major consideration since the loading is light. The lubricant is usually the limiting item insofar as life is concerned. The first consideration is to be sure that lubricant and the lubrication system are adequate for the speed range.

Unfortunately, there are no exact guiderules that can be set. However, some generalizations are possible with respect to normal applications.

System	Speed Limit D x N (bore in mm x speed in RPM)	
Grease	250,000	(ribbon retainer)
Oil Level	300,000	(ribbon retainer)
Mist	700,000	(machined retainer)
Jet Oil	$>10^6$	(machined retainer)

Above 300,000 dN, the usual ribbon retainer would be replaced by a machined retainer of metal or phenolic. For normal temperatures, the phenolic retainer is commonly used. With special greases, retainer design, and light loading, grease lubrication has been used to speeds of 750,000 dN.

With respect to grease lubrication, there is evidence that life is reduced in some logarithmic fashion with increasing dN value. This is similar to the effect of temperature. Figure D shows a typical behavior of life with respect to temperature. A reasonable rule of thumb is that life is cut in half for each 10°C rise over 100°C.

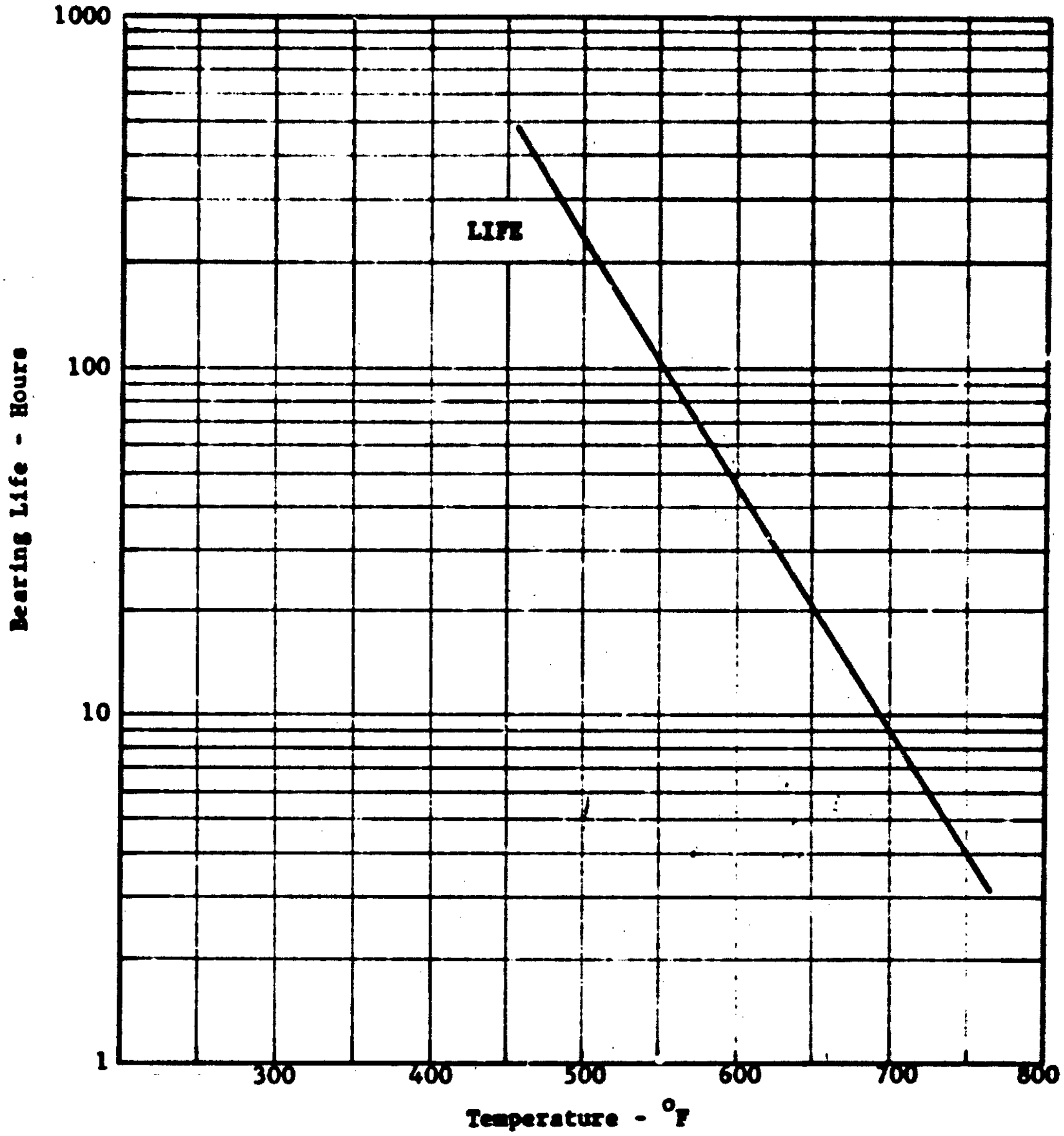


Fig. D Bearing Life Versus Temperature

III

DESIGN DATA

Description and Discussion of Charts

There are basically three separate sets of design charts included in this report, namely:

a. Pure Radial Loaded Bearings (Deep Groove) Contact Angle $\beta_o = 0^\circ$

b. Pure Thrust Loaded Bearings (Deep Groove) Contact Angle $\beta_o = 10^\circ$

c. Angular Contact Bearings with Axial Preload and Applied Radial Load $\beta_o = 25^\circ, 15^\circ$

Table I describes the dimensions and symbols used for the deep-grooved ball bearings. Table II contains information pertaining to the angular contact bearings.

The first set of four charts contains graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size and race curvatures are illustrated by these four charts. In general, a bearing with tighter raceway curvatures is a stiffer bearing. For example, a bearing with curvatures of $f_i = .516$, $f_o = .530$ is stiffer than the same bearing operating with curvatures of $f_i = f_o = .570$, for the same radial load. Radial stiffness is higher for a bearing with a larger bore diameter and/or a greater number of balls. Note, for pure radial load, the linear relationship between log S_R and log P.

The second set of eight charts contains graphs of axial stiffness and axial deflection versus axial thrust applied load. Load levels are tabulated in Table III for these particular bearings undergoing a pure thrust load since cross plots of C/P will add confusion when reading the curves. Deflection curves are included to aid in analyzing a double acting thrust bearing set. The deflection curve for a double acting thrust bearing set is constructed from the deflection curve of a single bearing by adding increments of deflection to one bearing and subtracting from the other. The corresponding load differences equal the externally applied load. A similar observation, as given above for radially loaded bearings, can be made for the thrust loaded bearing, i a bearing operating with curvatures of $f_i = .516$, $f_o = .530$ is stiffer than the same bearing operating with curvatures of $f_i = f_o = .570$, for the same axial load. For all practical purposes, however, an average curve may be drawn for axial stiffness versus axial load for all bearing sizes. In particular, the bearing with the smaller bore and less balls is less stiff at light loads and more stiff at heavy loads as compared to the larger bore bearing. There is an approximate linear relationship between log S_A and log T.

The third set of (24) charts contain graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size, race curvatures, initial contact angle, and axial preload are illustrated by these 24 charts. For the same radial load and axial preload, a bearing operati with curvatures of $f_i = .516$, $f_o = .530$ is stiffer than the same bearing operat with curvatures of $f_i = f_o = .570$. The radial stiffness level is higher for a bearing with a larger bore diameter and/or a greater number of balls, and the smaller initial contact angle. ($\beta_o = 15^o$). In general, the radial stiffness-radial load curve for an angular contact bearing is composed of three different behaving regions. One region shows the stiffness to be constant with varying radial load. (This is the light radial load region.) The middle, or moderate radial load region shows a minimum value for radial stiffness. The heavily radial loaded region shows a linear relationship between log S_R and log P. Th third region is similar in behavior to that of the characteristics of a pure ra loaded deep grooved bearing. The basic cause for this curve having three sepa regions is due to the axial preload. In region one, the axial preload has a great effect in holding the radial stiffness constant. In region two, where th applied radial load becomes equal in magnitude to the axial preload, the radial stiffness tends to decrease with increasing applied radial load to a minimum value. In the third region, the axial preload has little or no effect, and th angular contact bearing reflects the behavior of a pure radially loaded beari i.e., a linear log S_R versus log P relationship.

Thus another point one is led to observe is the role of axial preload magnitud on the three regions of a typical stiffness versus load curve. Three differen preloads are represented in these charts and are tabulated in Table II. These preloads are given the names selected light, moderate, and preferred heavy. The effect of increased preload is to increase the region one load range and decrease region three load range. Thus, the ultimate is a constant radial stiffness with varying radial load obtained with an infinite preload. The increased preload also has the effect of increasing the level of stiffness in regions one and two. However, it should be noted particularly that the level of stiffness in region three, for the same radial load, is the same for all preload values. This, as mentioned above, is because the axial preload effect is relieved entirely above a certain (radial load/axial preload) ratio. (Approximately R/T = 3 for $\beta_o = 25^o$ and R/T = 4 for $\beta_o = 15^o$.)

In general, the light and extra light deep grooved ball bearings examined here will have a radial stiffness ranging from 10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The angular contact bearings will have radial stiffness values from 2×10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The deep grooved ball bearings will have an axial stiffness per bearing of from 2×10^4 to 4×10^6 for thrust loads of from 10 to 10^4 lbs. As in the case of the preloaded radial bearing, preloading will increase these values of axial stiffness.

Table I Deep Groove Bearings

Bearing Symbol	Bore (Inch)	Bore mm	O.D. (Inch)	Ball Diameter, Inch	Number of Balls	f_i	f_o
A1	.5906	15	1.2598	.1875	9	.516	.530
A2	.5906	15	1.2598	.1875	9	.570	.570
B1	.9843	25	1.8504	.250	10	.516	.530
B2	.9843	25	1.8504	.250	10	.570	.570
C1	1.378	35	2.4409	.3125	11	.516	.530
C2	1.378	35	2.4409	.3125	11	.570	.570
D1	2.1654	55	3.5433	.40625	13	.516	.530
D2	2.1654	55	3.5433	.40625	13	.570	.570
E1	2.9528	75	4.5276	.46875	15	.516	.530
E2	2.9528	75	4.5276	.46875	15	.570	.570
AA1	.5906	15	1.378	.2345	8	.516	.530
AA2	.5906	15	1.378	.2345	8	.570	.570
BB1	.9843	25	2.0472	.3125	9	.516	.530
BB2	.9843	25	2.0472	.3125	9	.570	.570
CC1	1.378	35	2.8346	.4375	9	.516	.530
CC2	1.378	35	2.8346	.4375	9	.570	.570
DD1	2.1654	55	3.937	.5625	10	.516	.530
DD2	2.1654	55	3.937	.5625	10	.570	.570
EE1	2.9528	75	5.1181	.6875	11	.516	.530
EE2	2.9528	75	5.1181	.6875	11	.570	.570

Table II Angular Contact Bearings

$\beta_o = 15^o, 25^o$

Basic Static Load (Lb)	Bearing Number	Bore (Inch)	Bore mm	O.D. (Inch)	d (in)	Number Of Balls	f_i	f_o	Axial Preload (Lb) S.L.	M.	P.H.
630	PA 1 PA 2	.5906	15	1.2598	.1875	11	.516 .570	.530 .570	20	50	100
1400	PB 1 PB 2	.9843	25	1.8504	.2500	13	.516 .570	.530 .570	50	100	200
2600	PC 1 PC 2	1.3780	35	2.4409	.3125	15	.516 .570	.530 .570	50	100	200
5100	PD 1 PD 2	2.1654	55	3.5433	.40625	18	.516 .570	.530 .570	100	200	300
8600	PE 1 PE 2	2.9528	75	4.5276	.46925	21	.516 .570	.530 .570	100	200	300
760	PAA1 PAA2	.5906	15	1.3780	.23425	10	.516 .570	.530 .570	20	50	100
1640	PBB1 PBB2	.9843	25	2.0472	.3125	12	.516 .570	.530 .570	50	100	200
3750	PCC1 PCC2	1.378	35	2.8346	.4375	12	.516 .570	.530 .570	50	100	200
7300	PDD1 PDD2	2.1654	55	3.9370	.5625	14	.516 .570	.530 .570	100	200	300
12200	PEE1 PEE2	2.9528	75	5.1181	.6875	16	.516 .570	.530 .570	100	200	300

Table III Axial Loaded Deep-Grooved Bearings

Table of approximate load values corresponding to C/P = 5 and C/P = 10 load level

Bearing Symbol	Load (Lb)	
	C/P = 5	C/P = 10
A1	290	100
B1	600	250
C1	1100	450
D1	2250	950
E1	3650	1500
A2	70	30
B2	175	75
C2	300	100
D2	550	250
E2	950	350
AA1	380	155
BB1	800	300
CC1	1550	650
DD1	3000	1250
EE1	5050	2100
AA2	95	50
BB2	200	100
CC2	400	200
DD2	1500	350
EE2	2550	700

PURE RADIAL LOAD

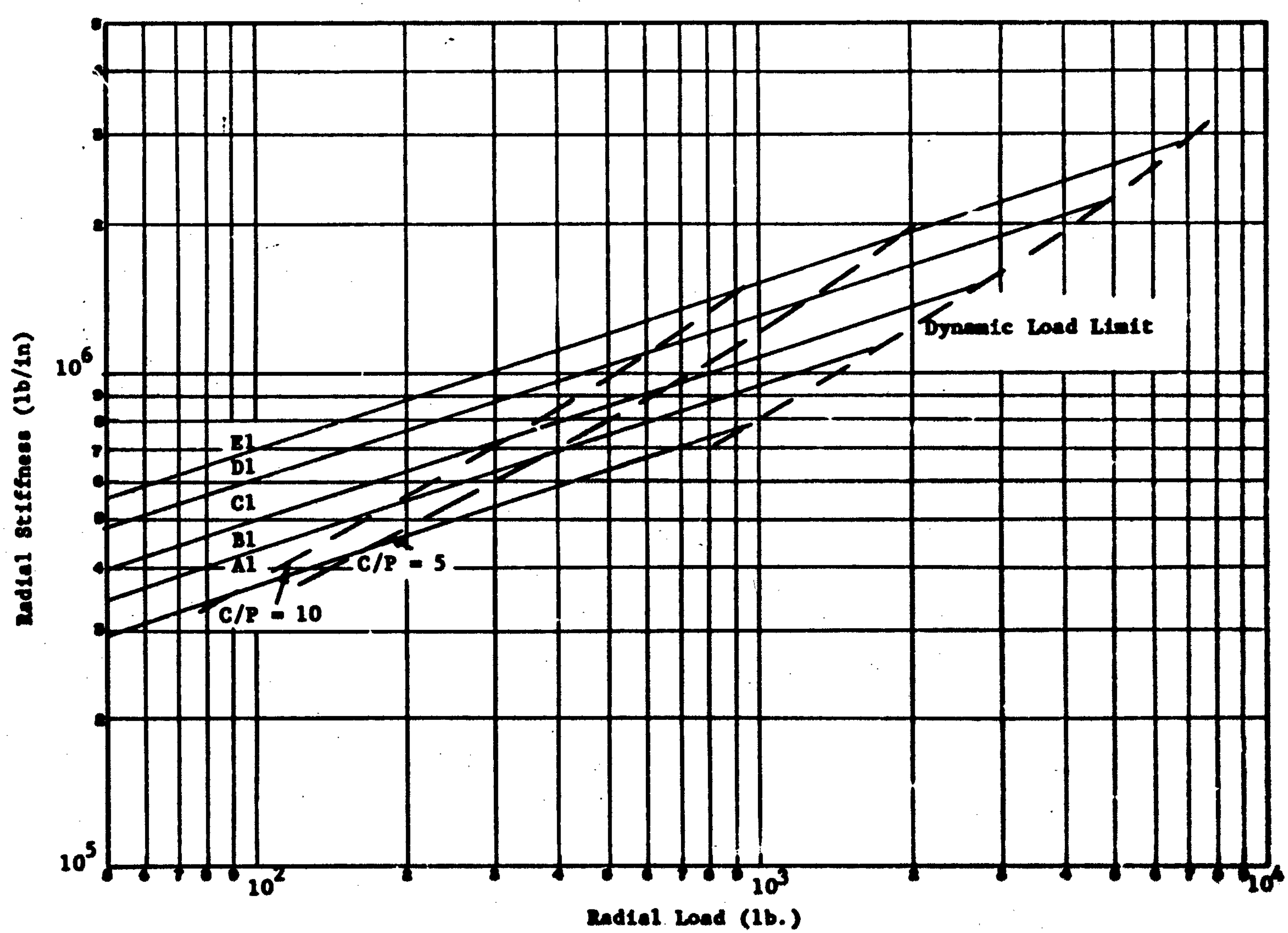


Fig. 1 Radial Stiffness for Deep Groove Ball Bearing-Pure Radial Load

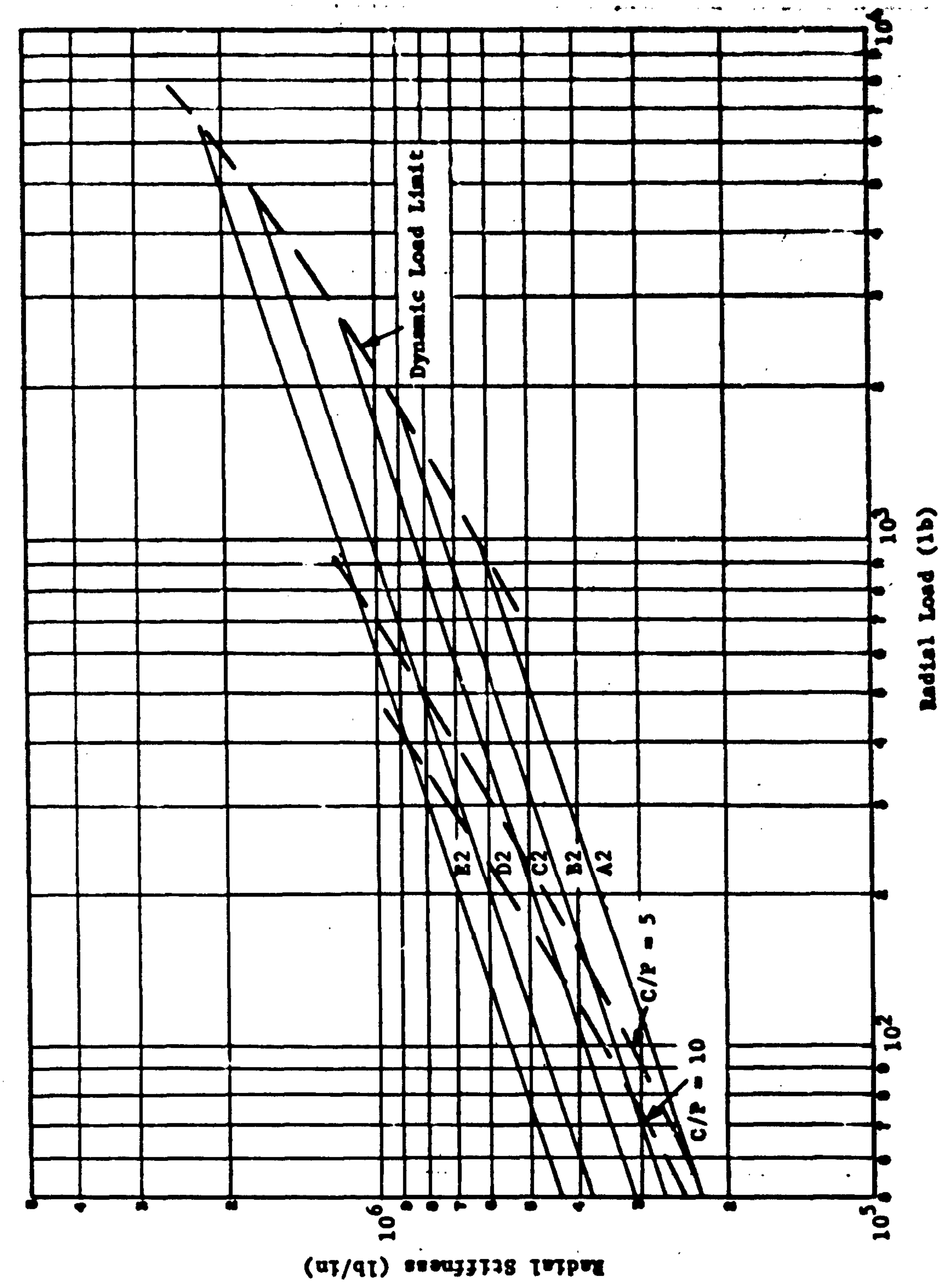


Fig. 2 Radial Stiffness for Deep Groove Ball Bearings-Pure Radial Load

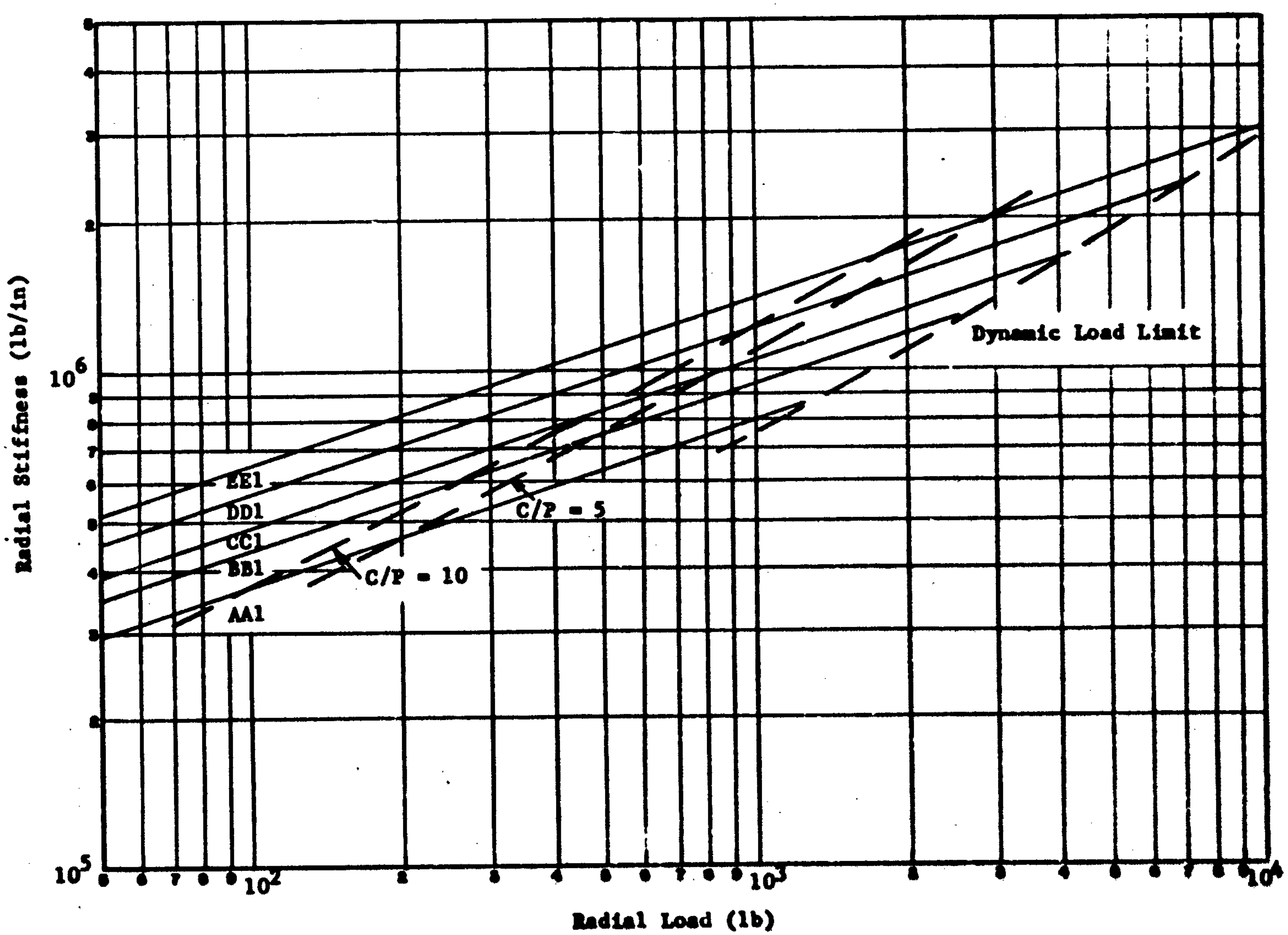


Fig. 3 Radial Stiffness for Deep Groove Ball Bearings-Pure Radial Load

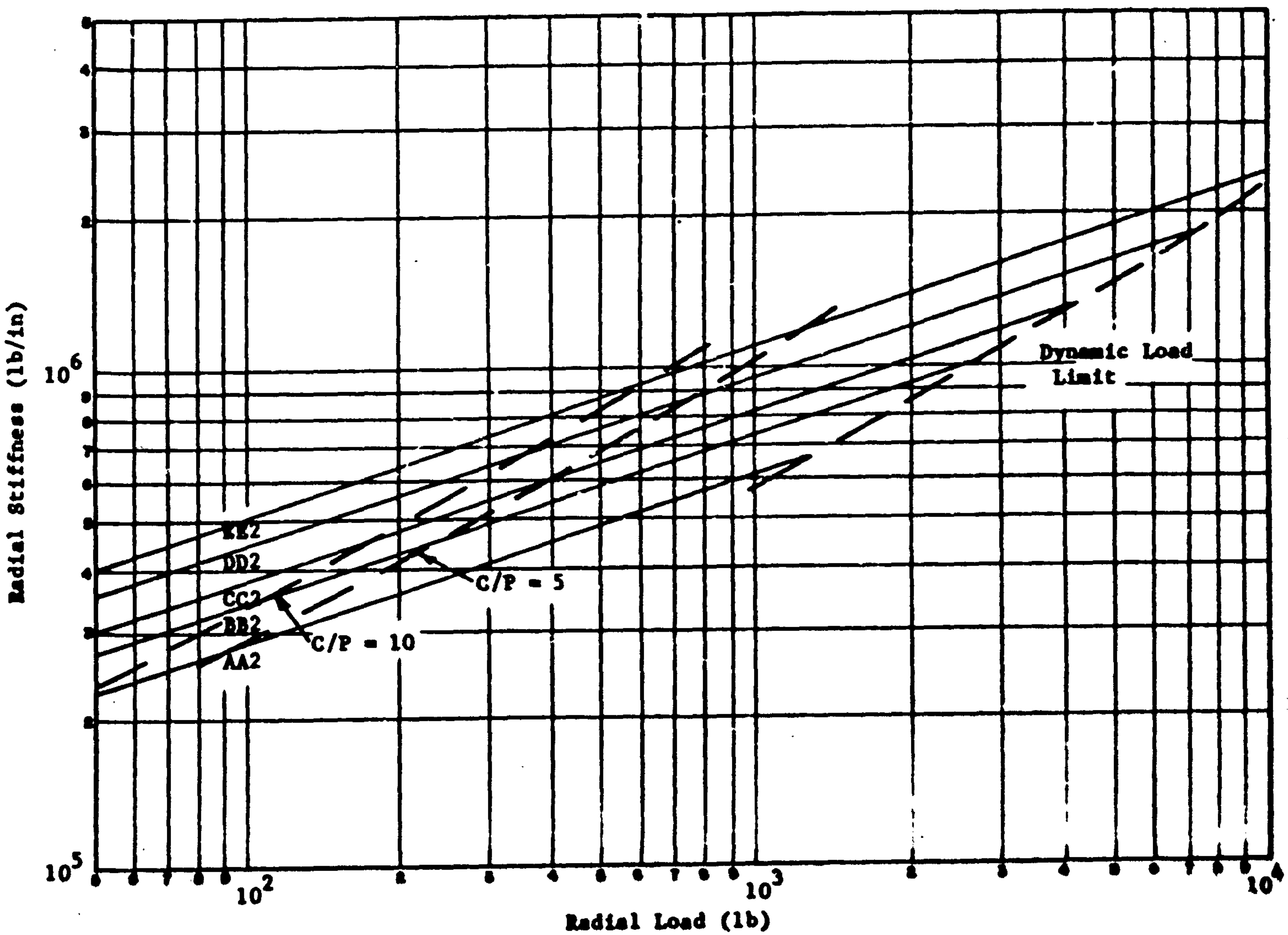


Fig. 4 Radial Stiffness for Deep Groove Ball Bearings-Pure Radial Load

PURE THRUST LOAD

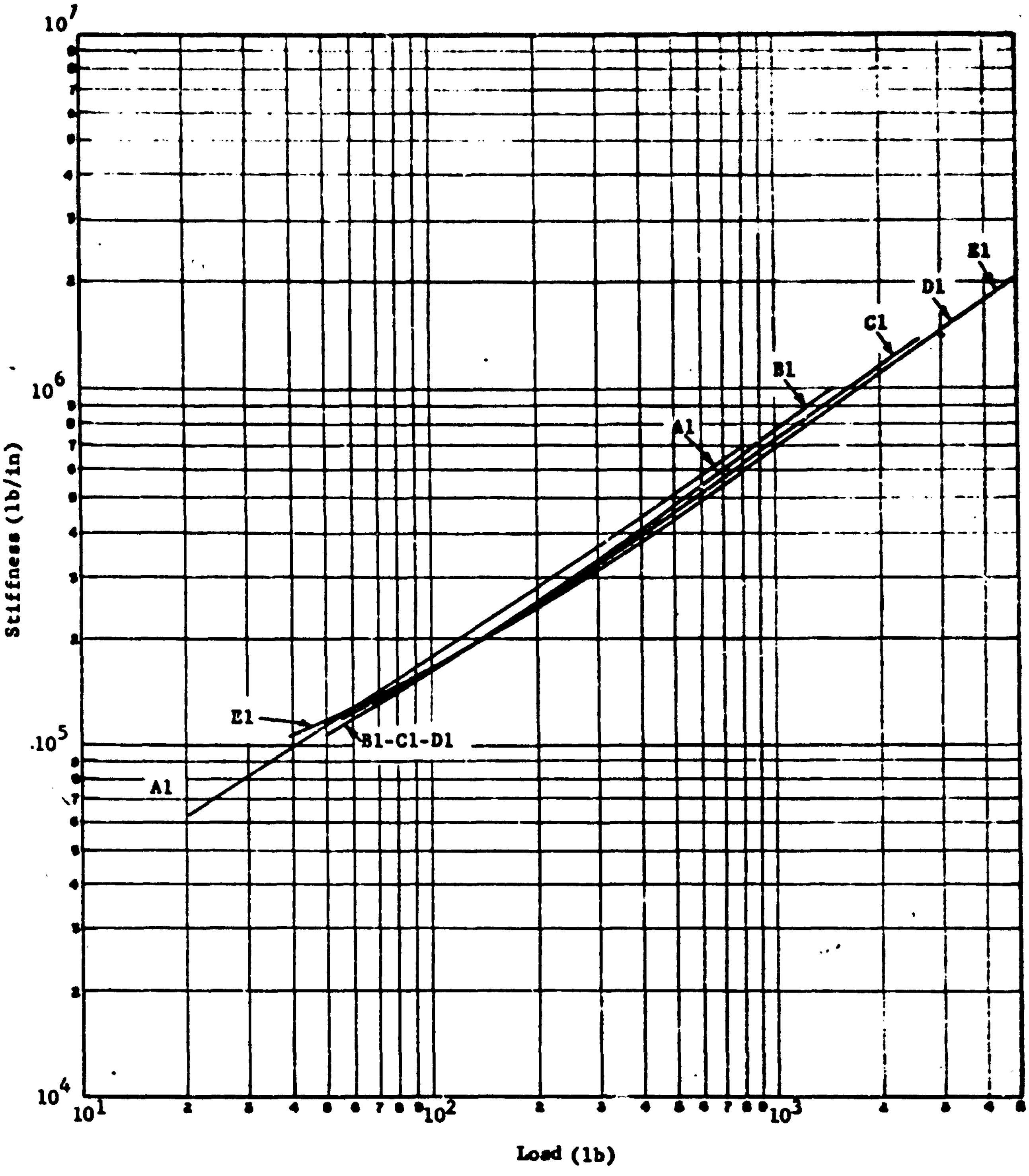


Fig. 5 Axial Stiffness versus Axial Load No Radial Load

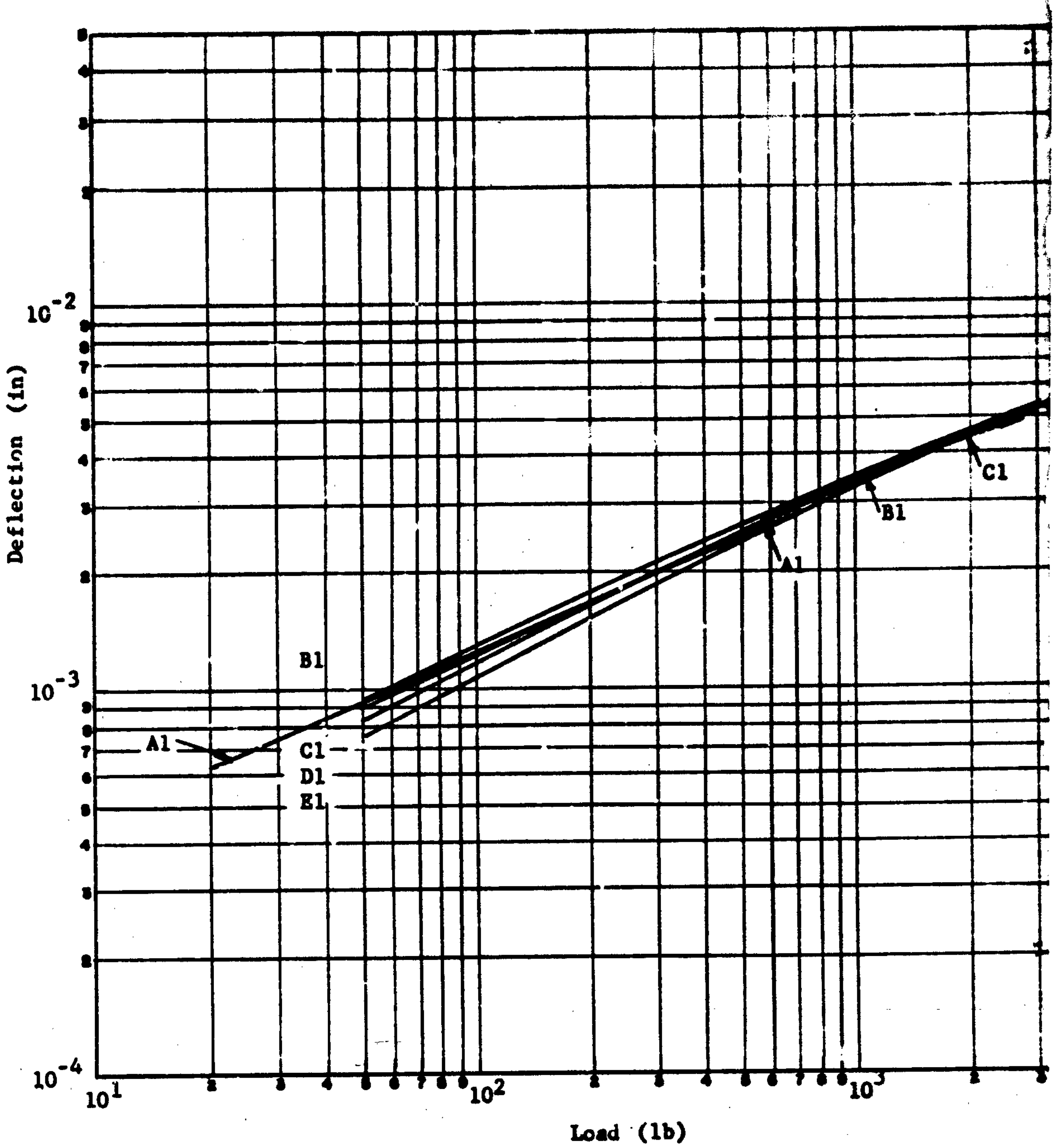


Fig. 6 Axial Deflection versus Axial Load
No Radial Load

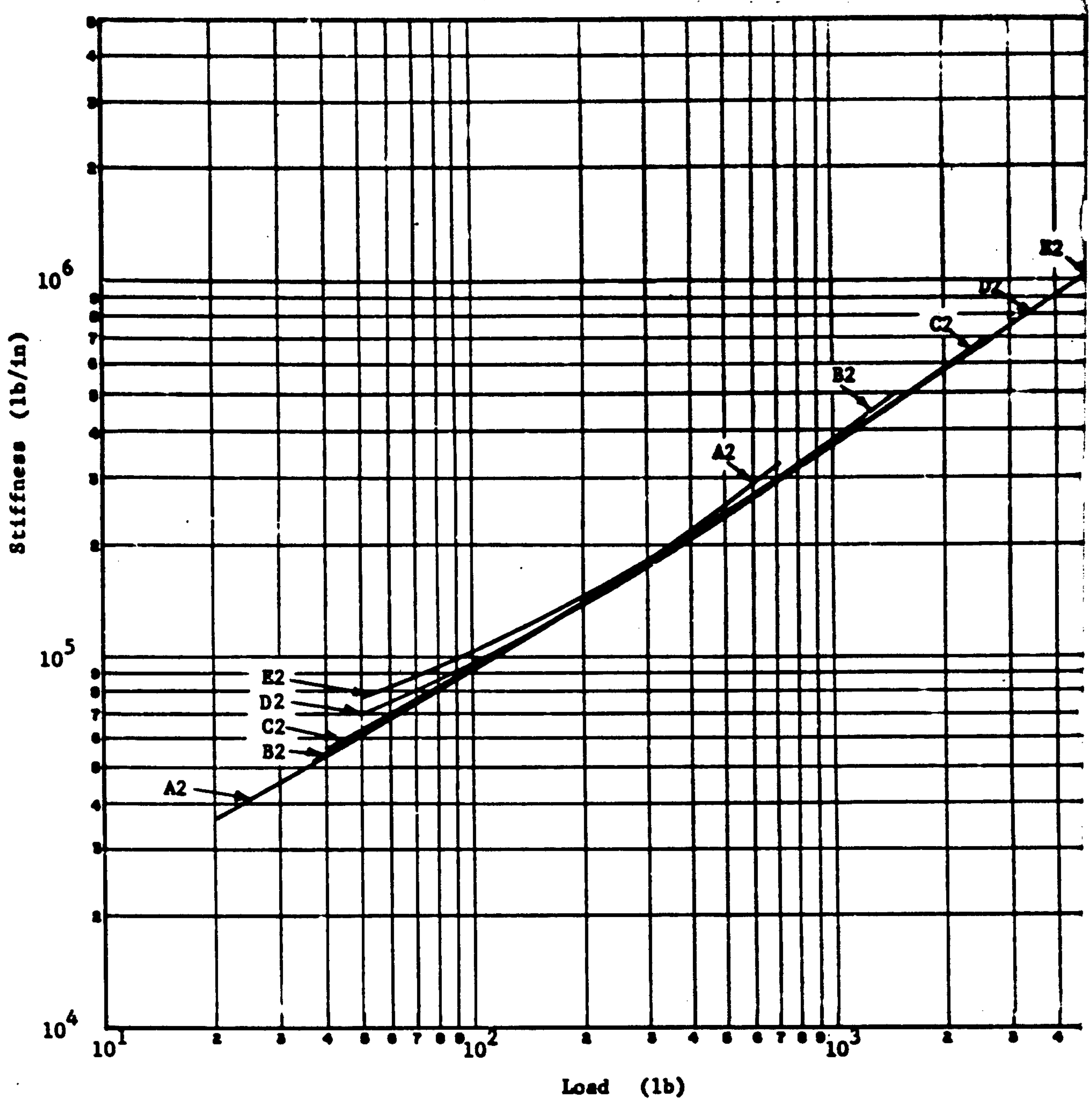


Fig. 7 Axial Stiffness versus Axial Load
No Radial Load

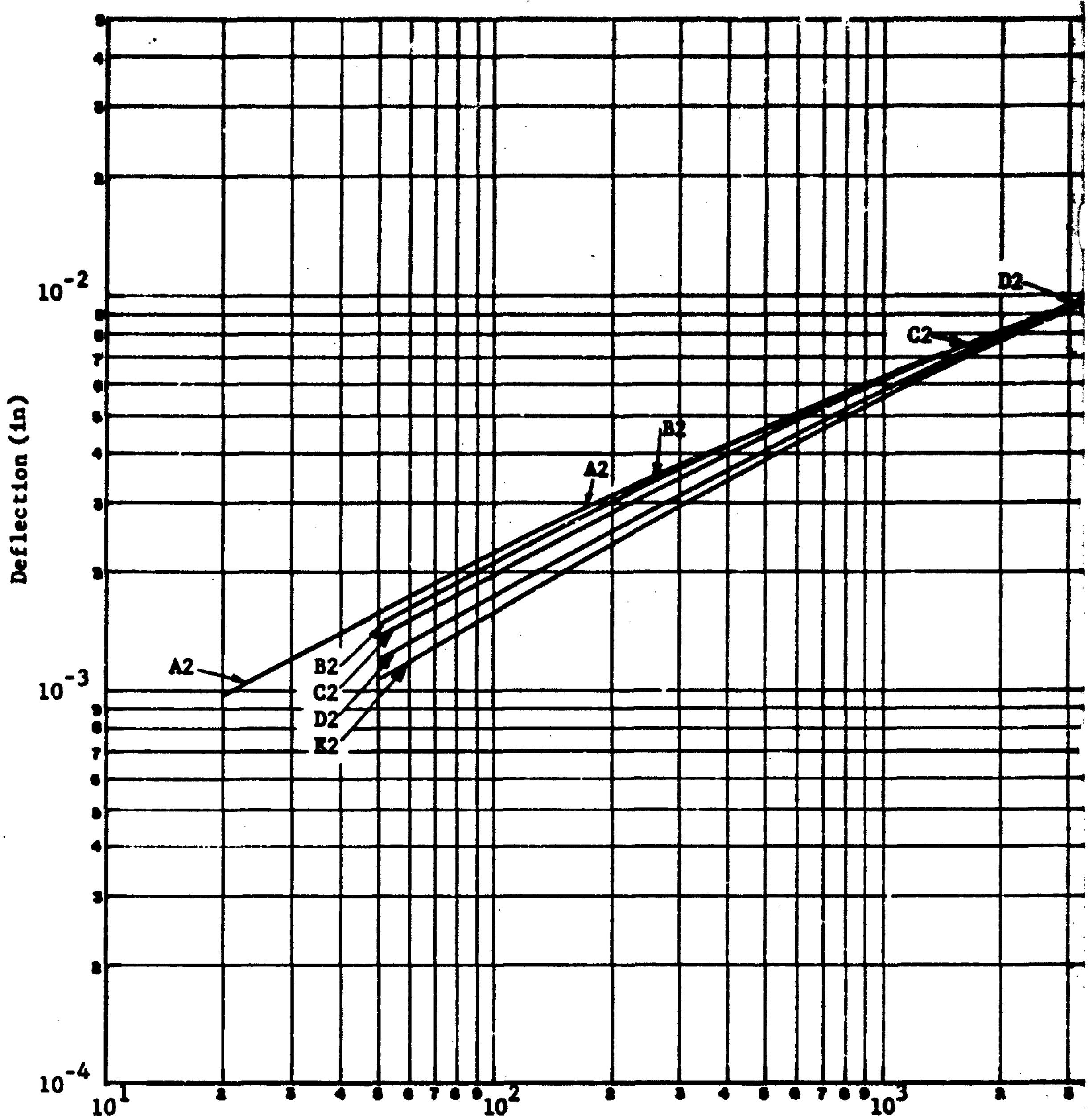


Fig. 8 Axial Deflection versus Axial Load
No Radial Load

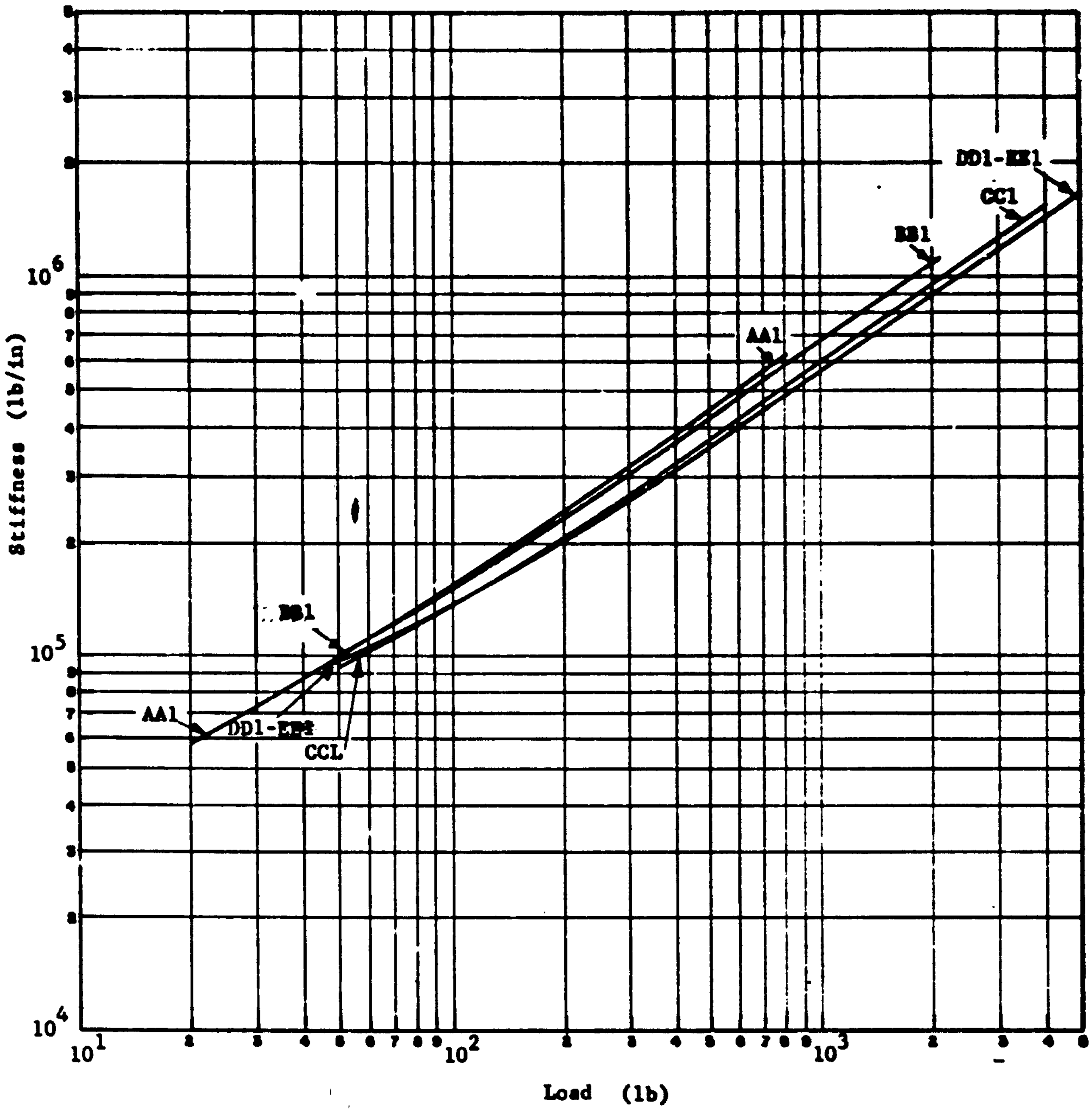


Fig. 9 Axial Stiffness versus Axial Load
No Radial Load

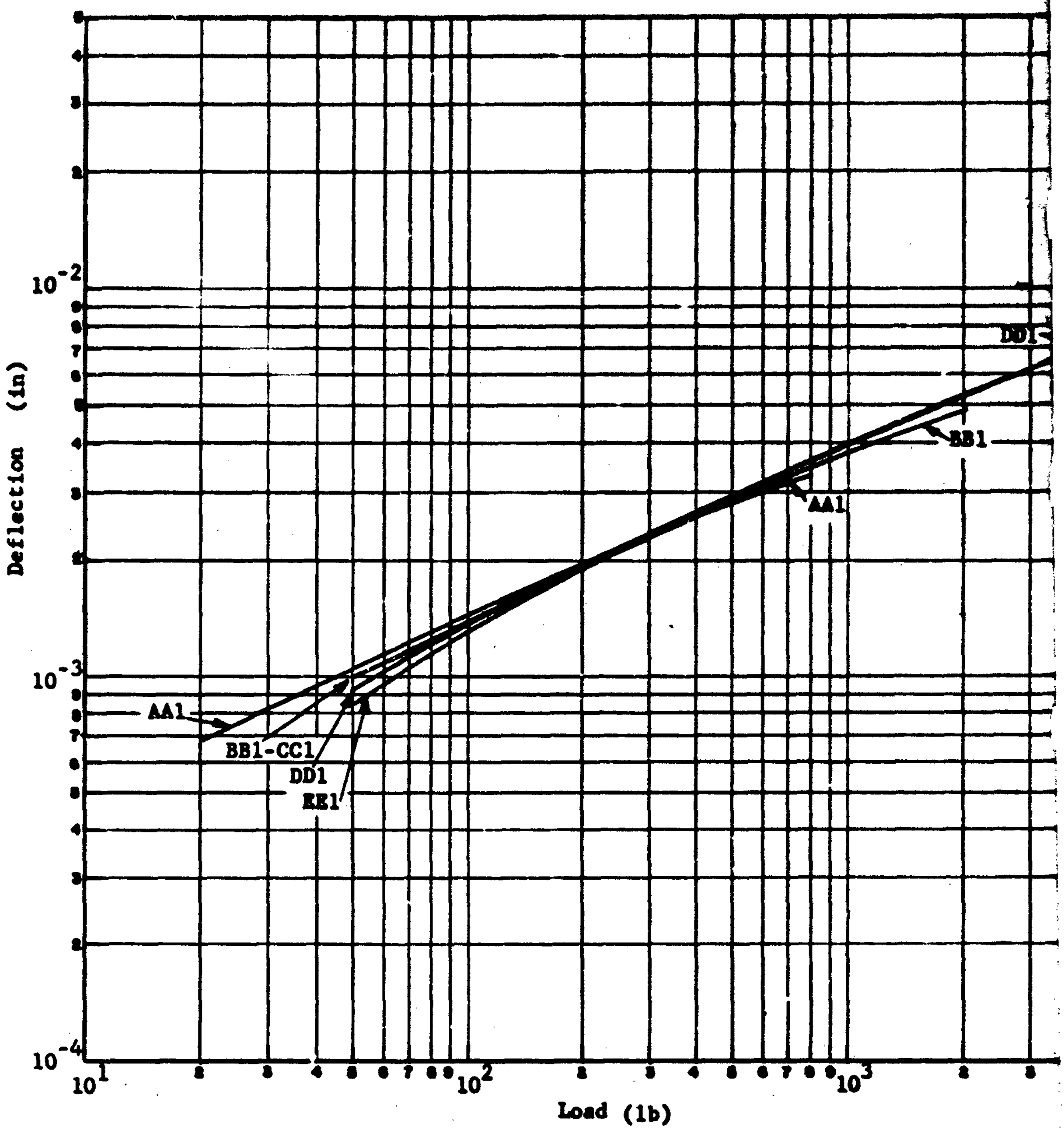


Fig. 10 Axial Deflection versus Axial Load
No Radial Load

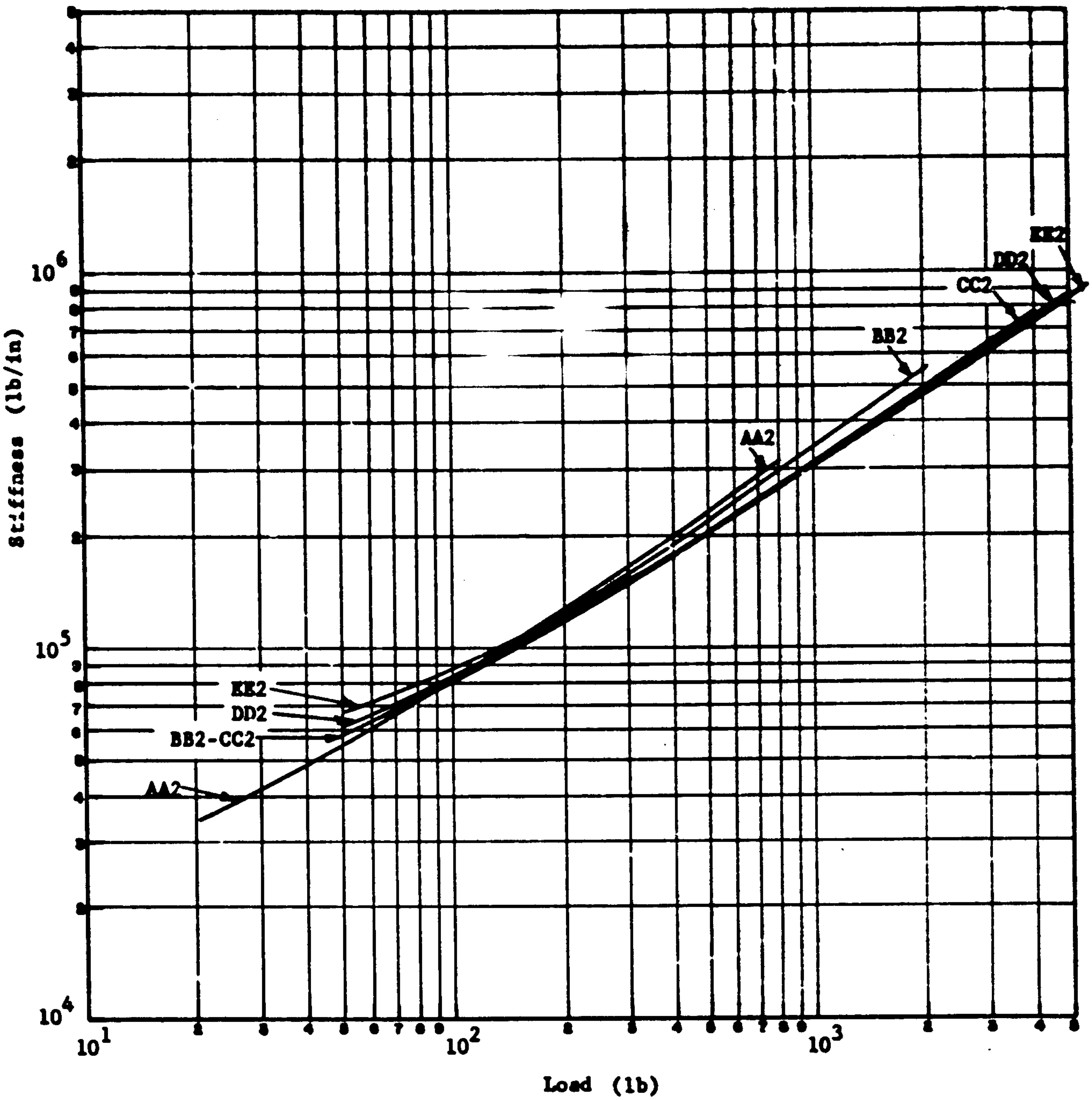


Fig. 11 Axial Stiffness versus Axial Load
No Radial Load

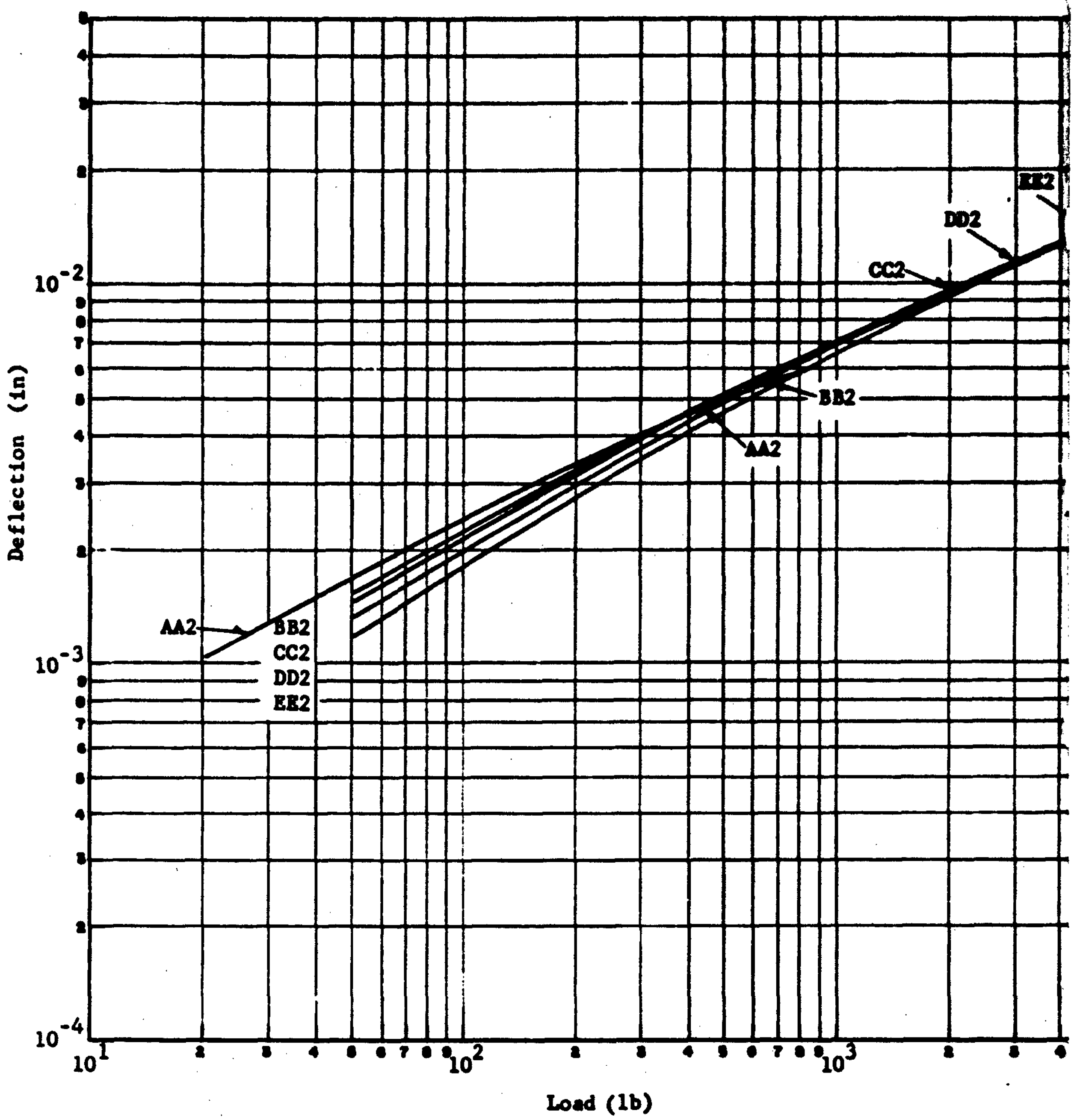


Fig. 12 Axial Deflection versus Axial Load
No Radial Load

RADIAL LOAD WITH AXIAL PRELOAD

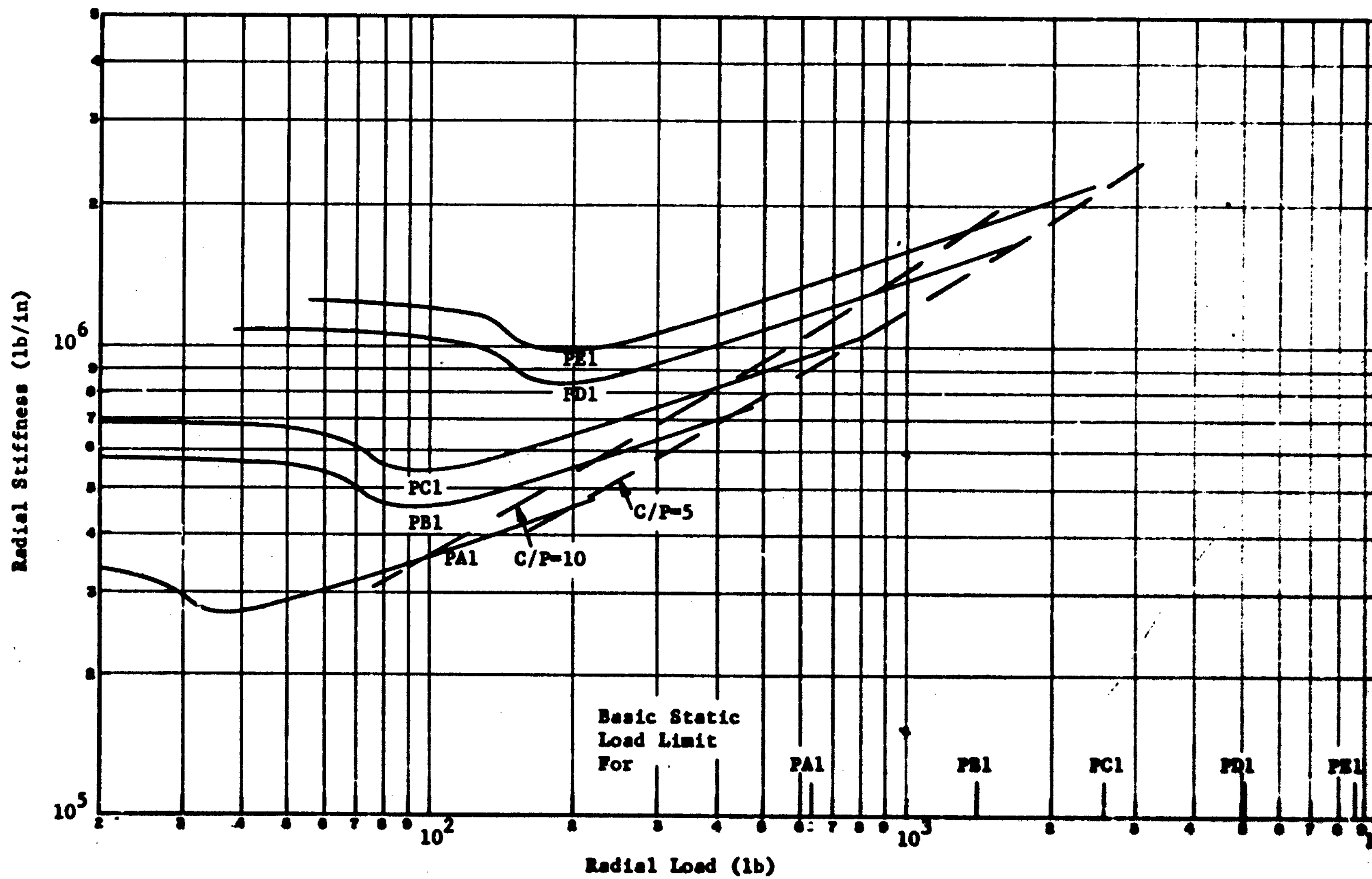


Fig. 13 Radial Stiffness for Angular Contact Bearing
Pre Load - Selected Light
$\beta = 25^{\circ}$

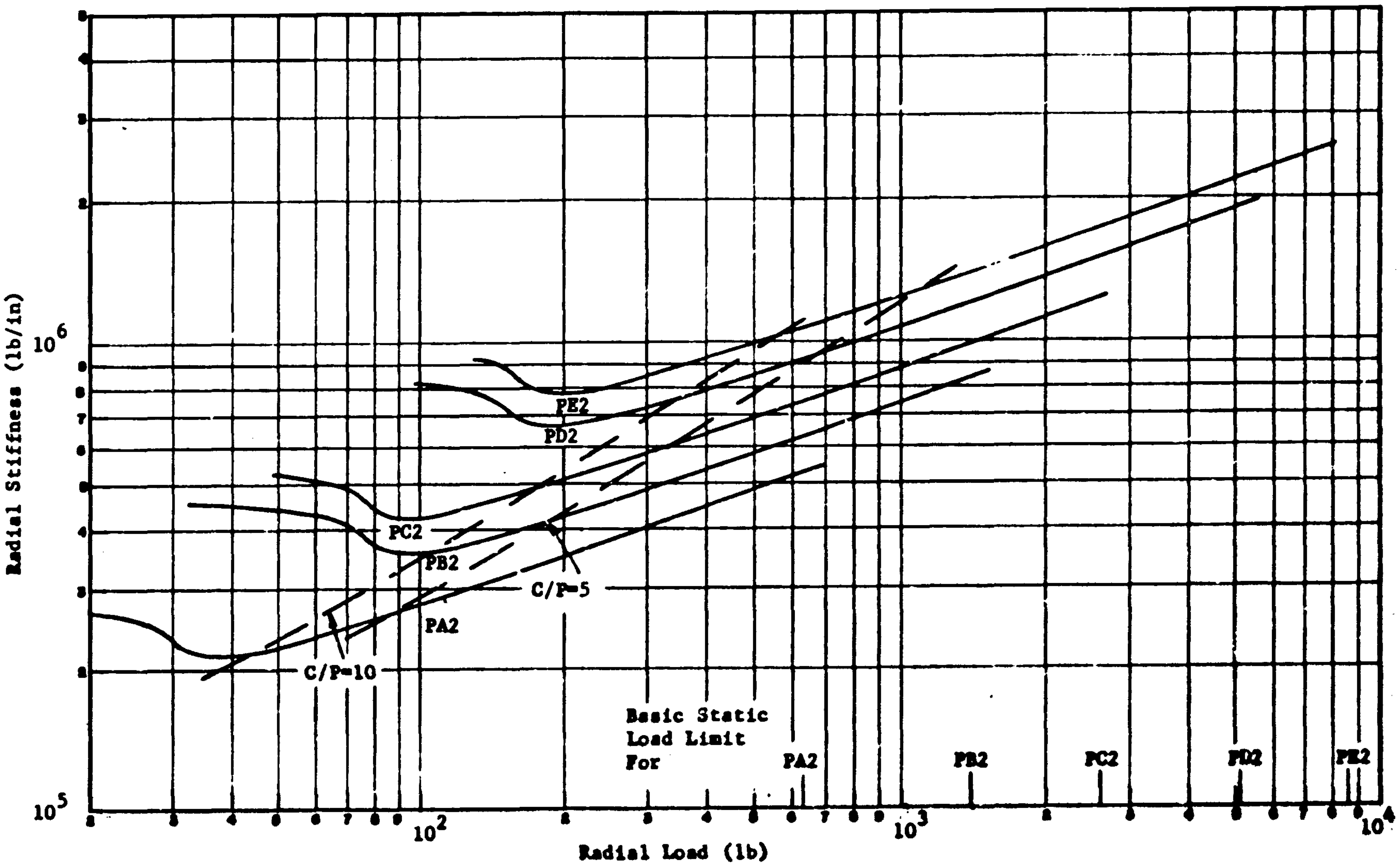


Fig. 14 Radial Stiffness for Angular Contact Bearing
Pre Load - Selected Light
$\beta = 25^{o}$

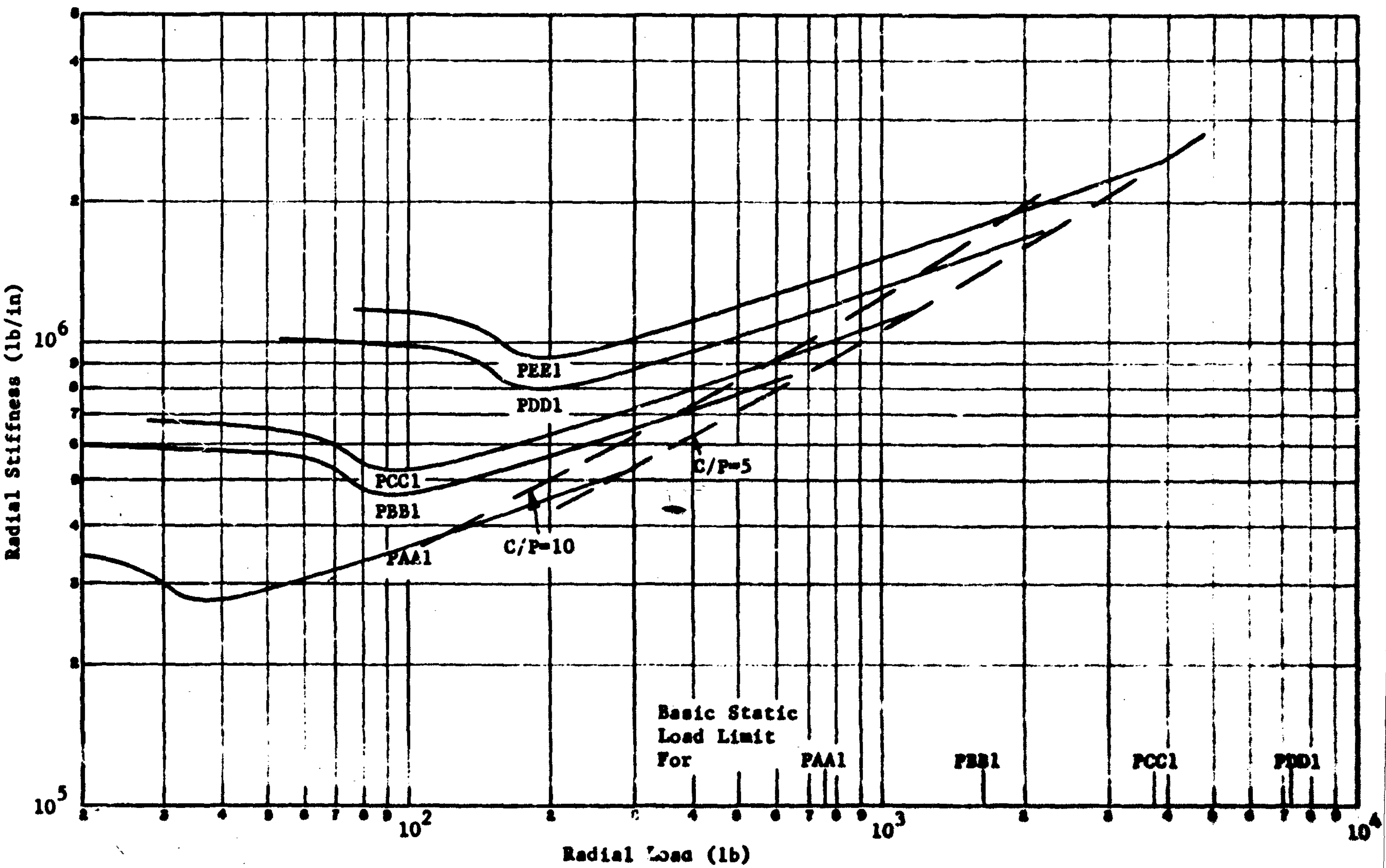


Fig. 15 Radial Stiffness for Angular Contact Bearing
Pre Load - Selected Light
$\beta = 25^{o}$

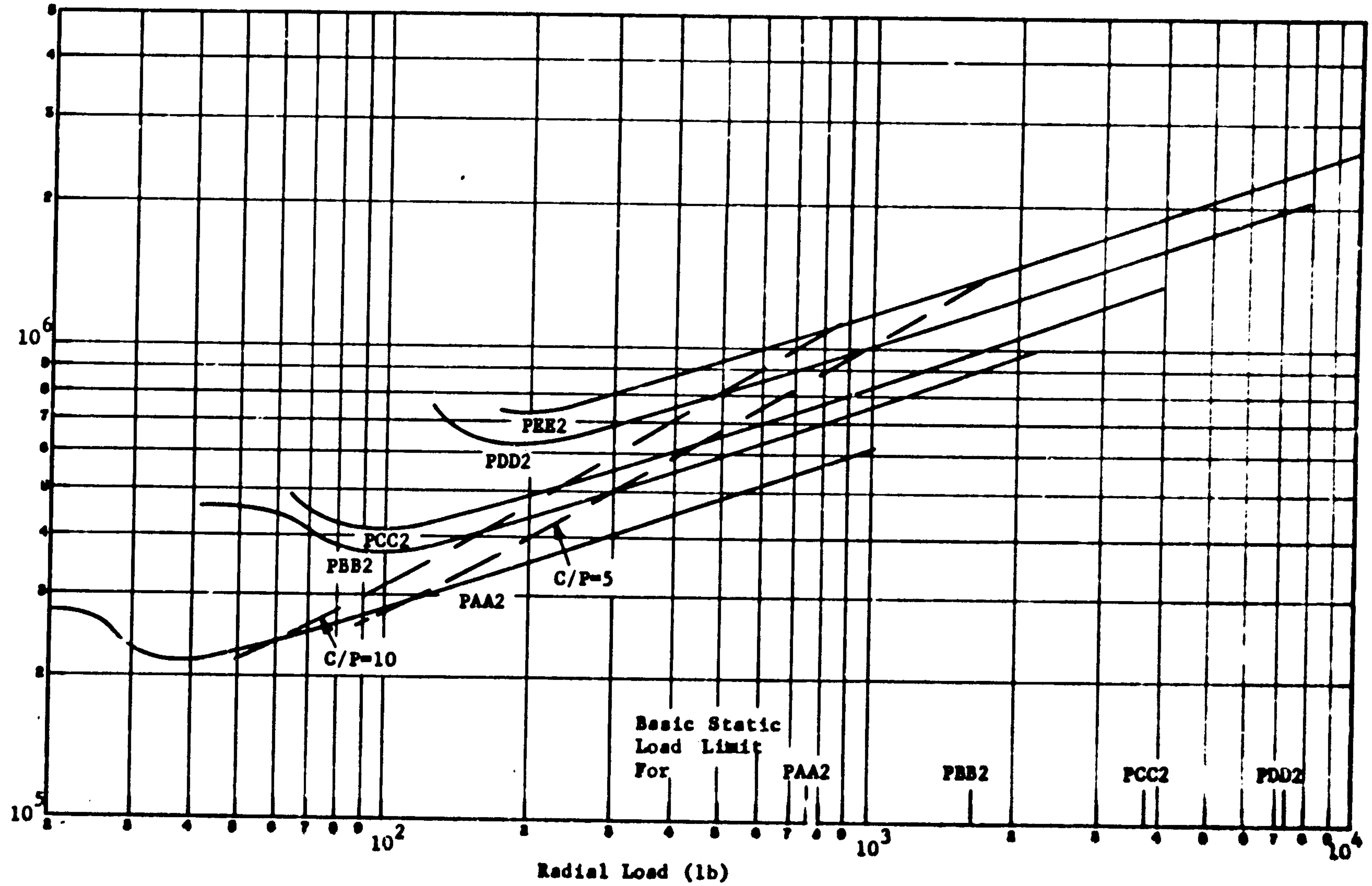
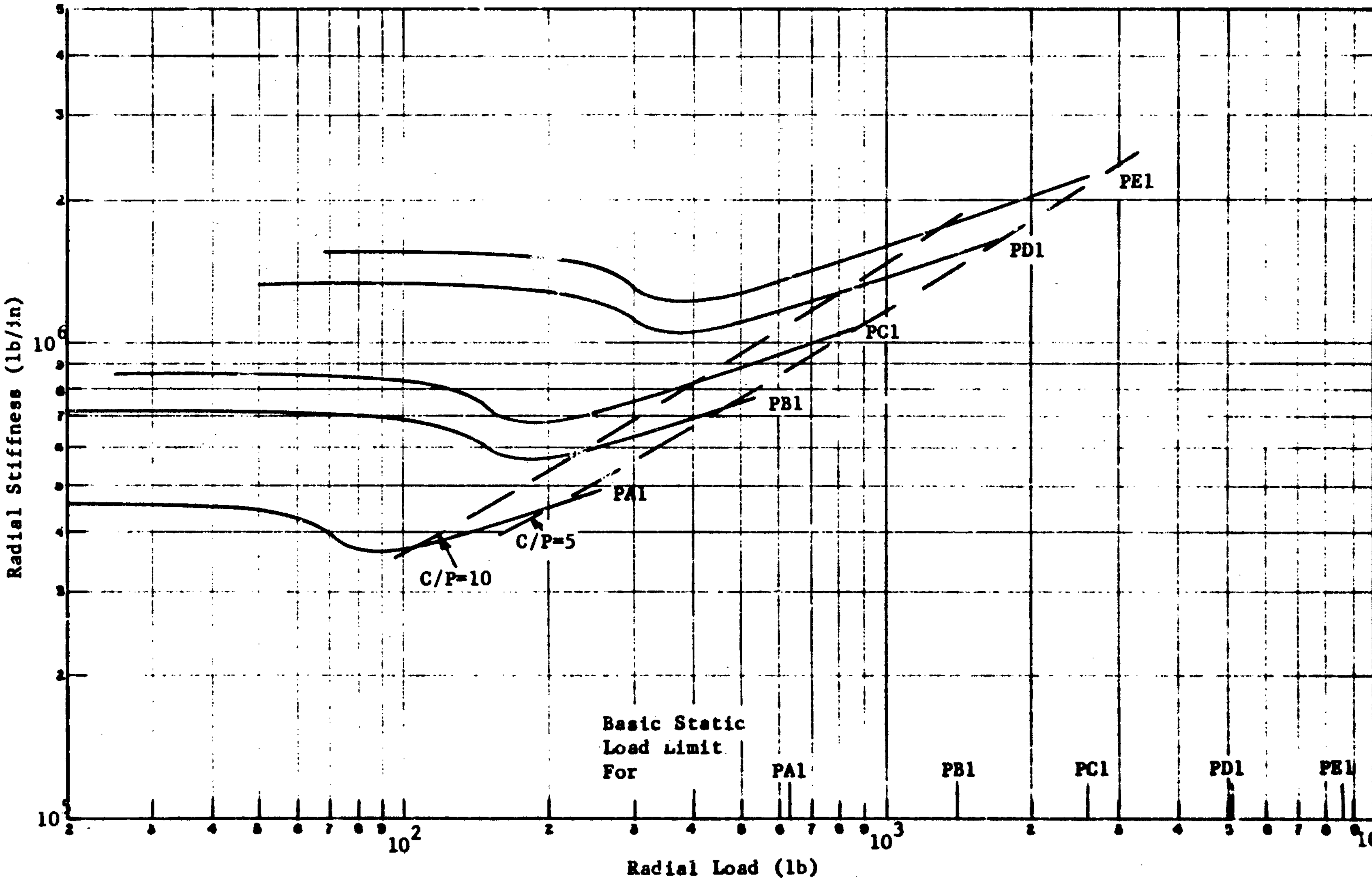


Fig. 16 Radial Stiffness for Angular Contact Bearing Pre-Load - Selected Light
$\beta = 25^{\circ}$


PE1
PD1
PC1
PB1
PA1
C/P=5
C/P=10
Basic Static
Load Limit
For
PA1
PB1
PC1
PD1
PE1
Radial Stiffness (lb/in)
Radial Load (lb)
10^5
10^6
10^2
10^3
10^4

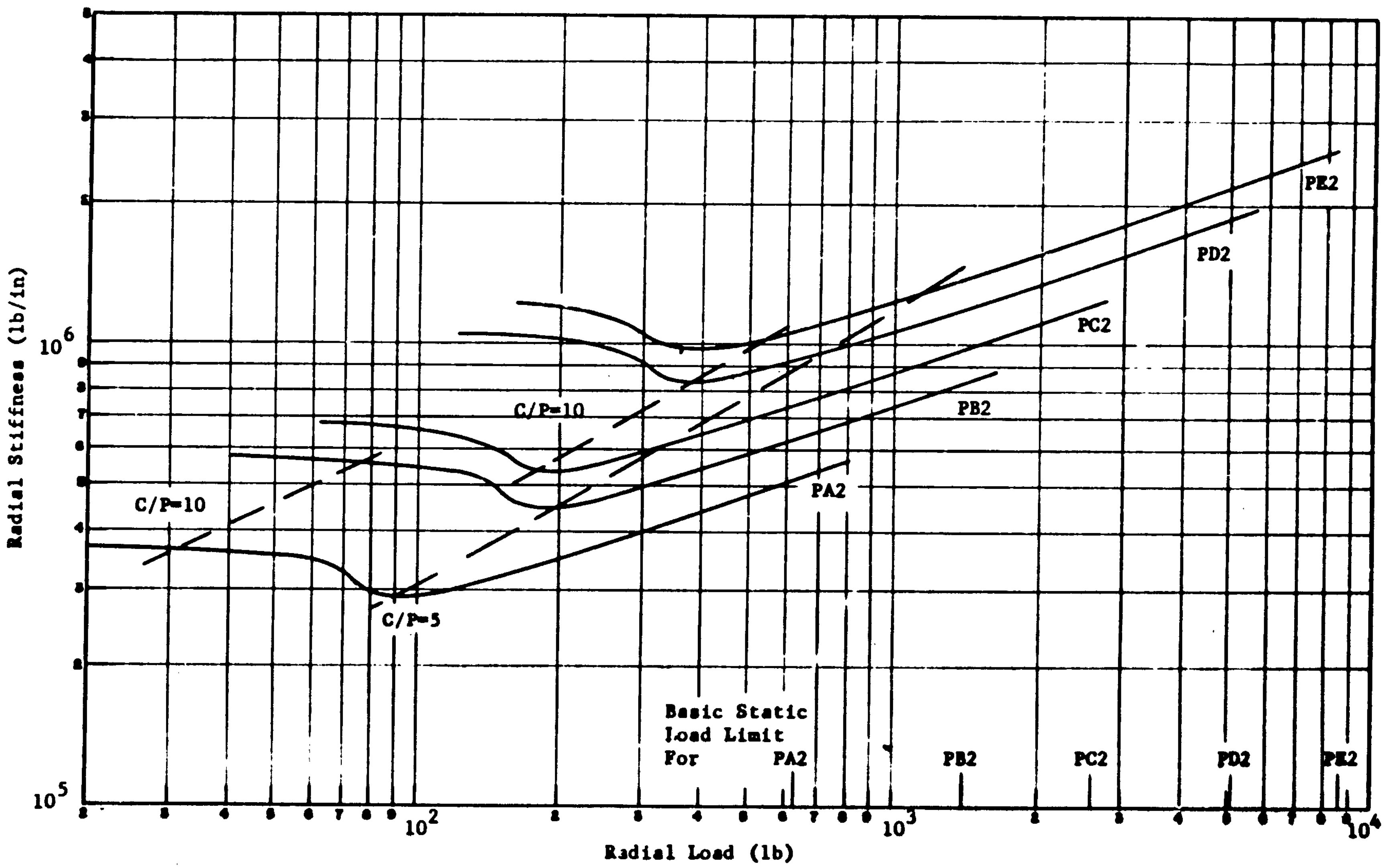


Fig. 18 Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
$\beta = 25°$

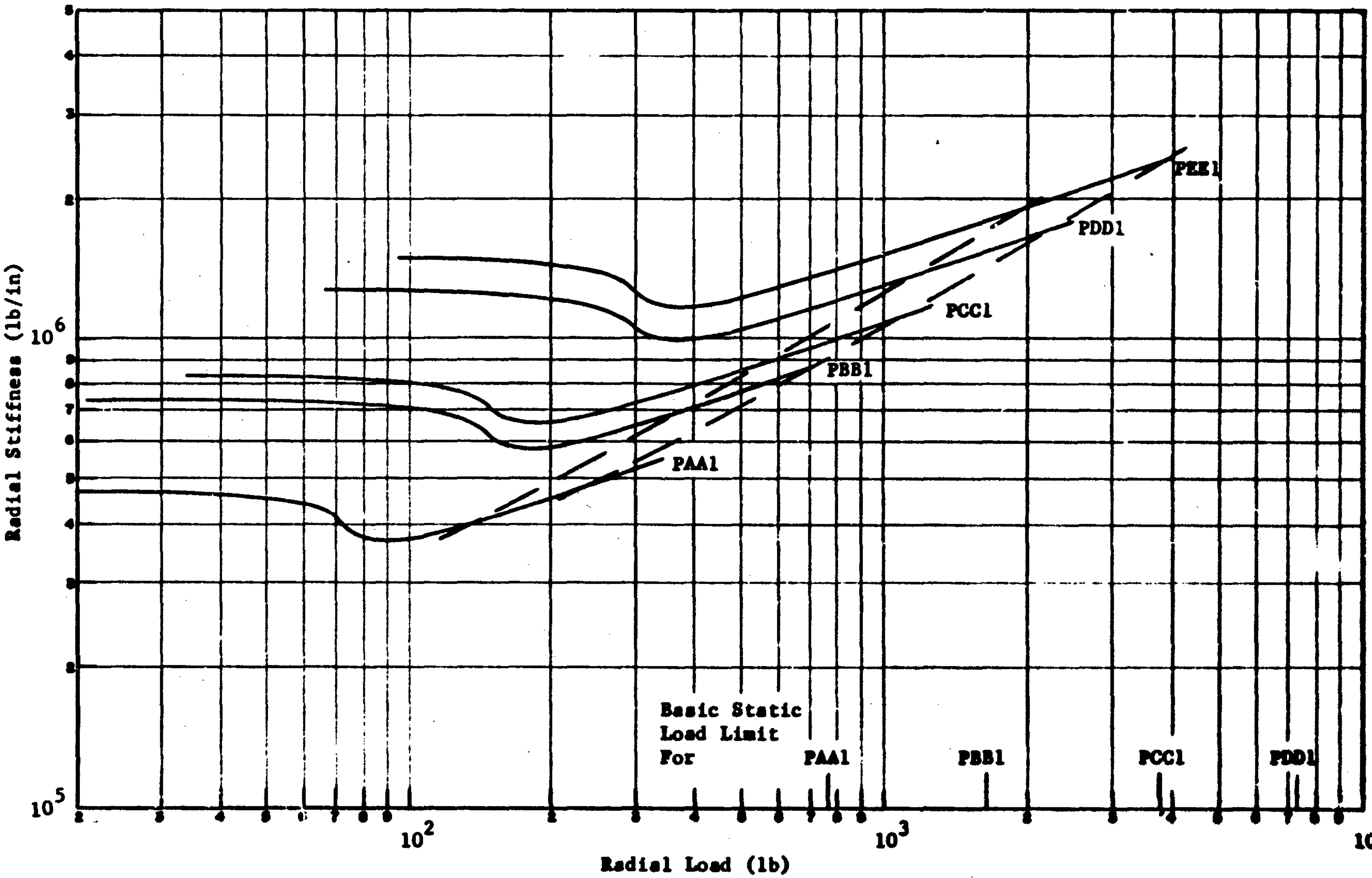


Fig. 19 Radial Stiffness for Angular Contact Bearings

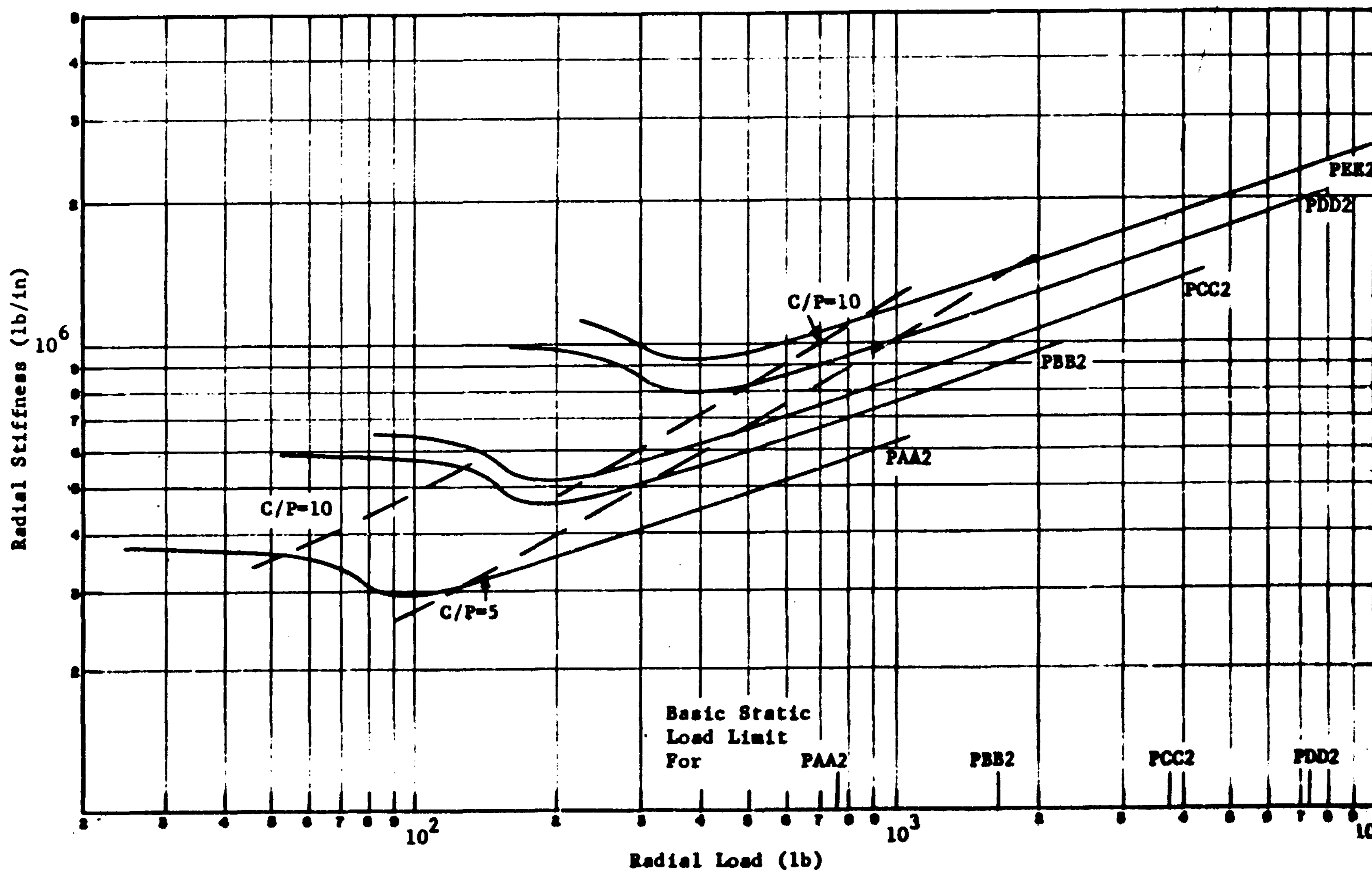


Fig. 20 Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
$\beta = 25^{o}$

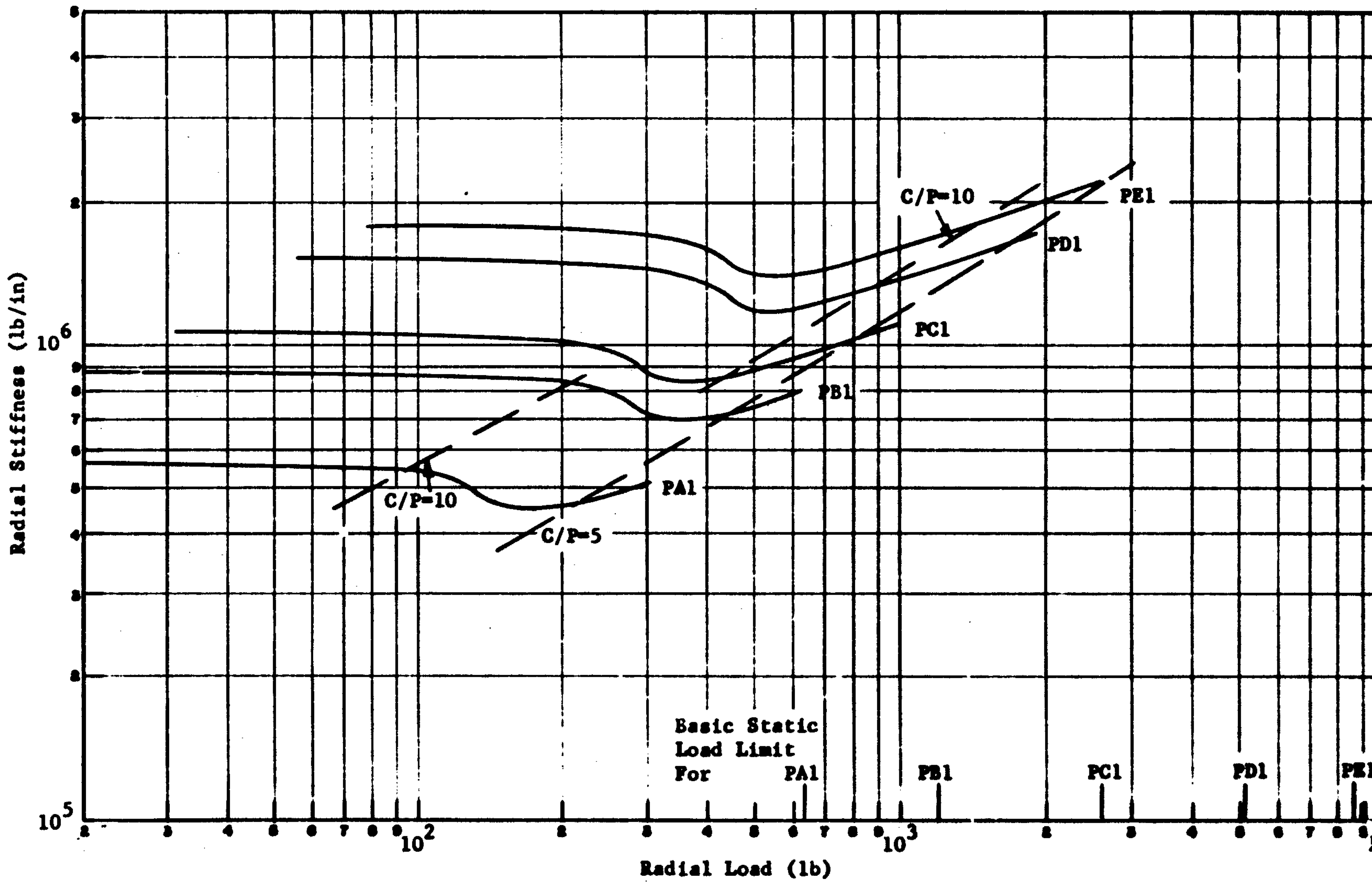


Fig. 21 Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
α = 25°

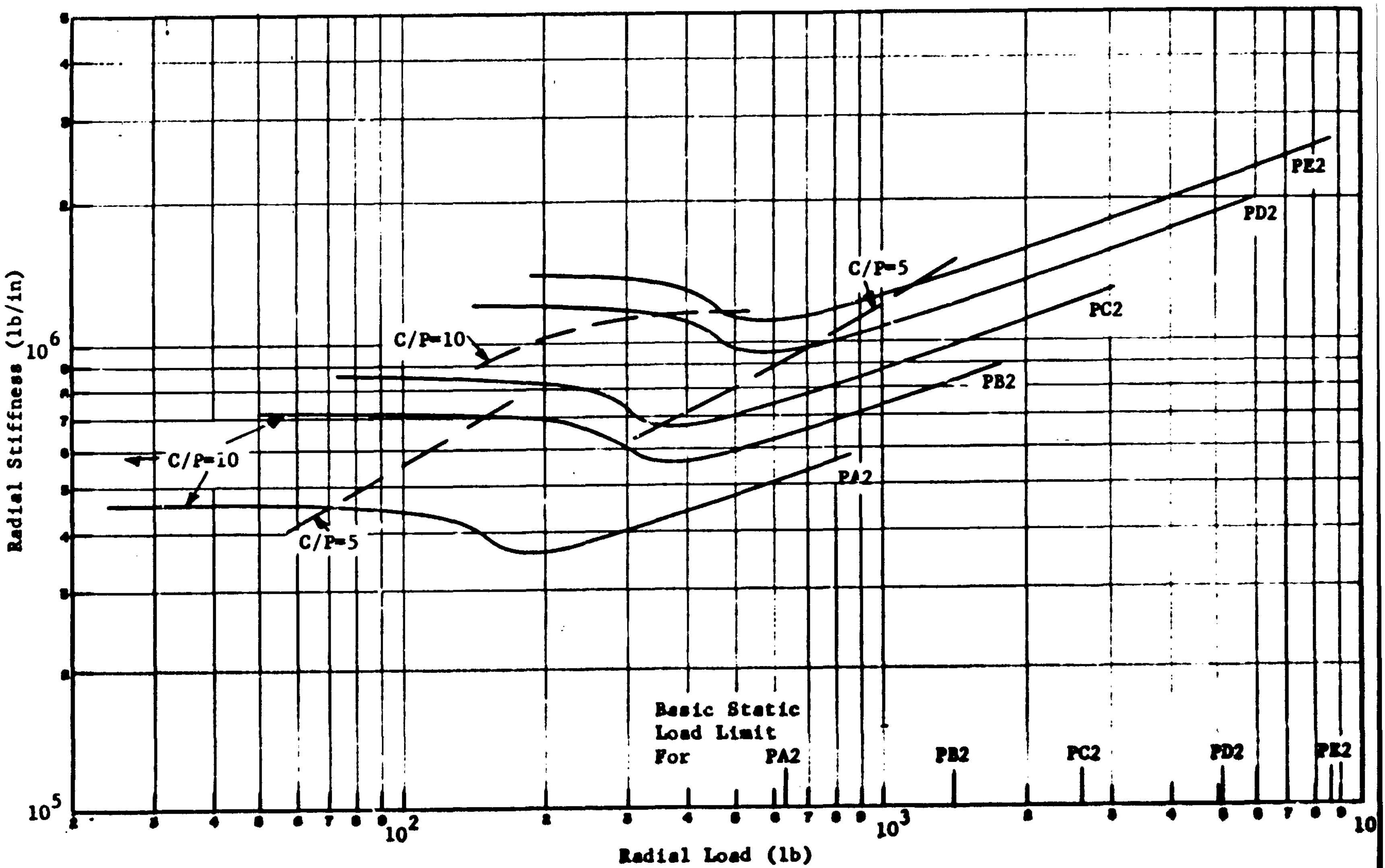


Fig. 22 Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 25^{o}$

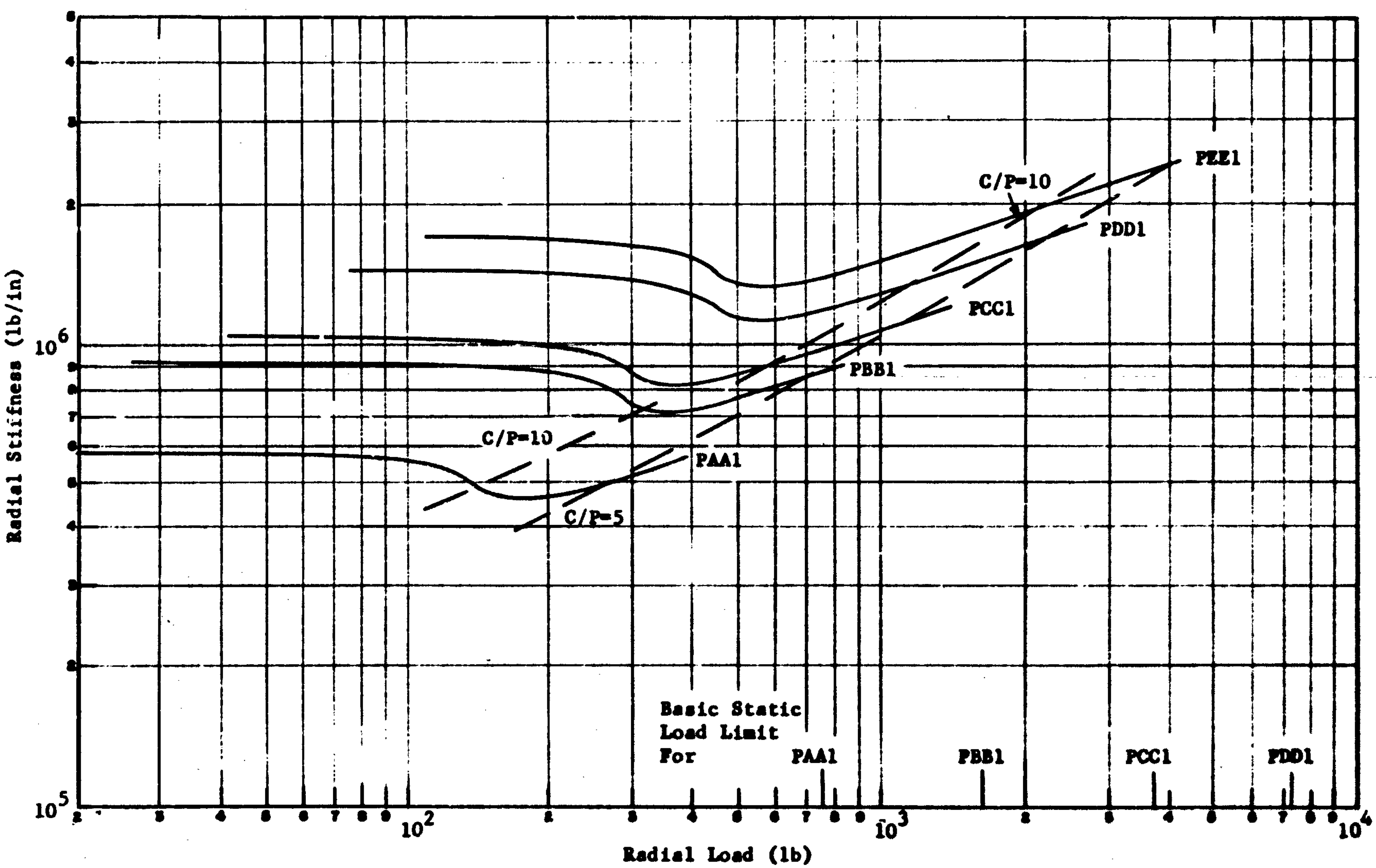


Fig. 23 Radial Stiffness for Angular Contact Bearing Pre-Load - Preferred Heavy
$\beta = 25^o$

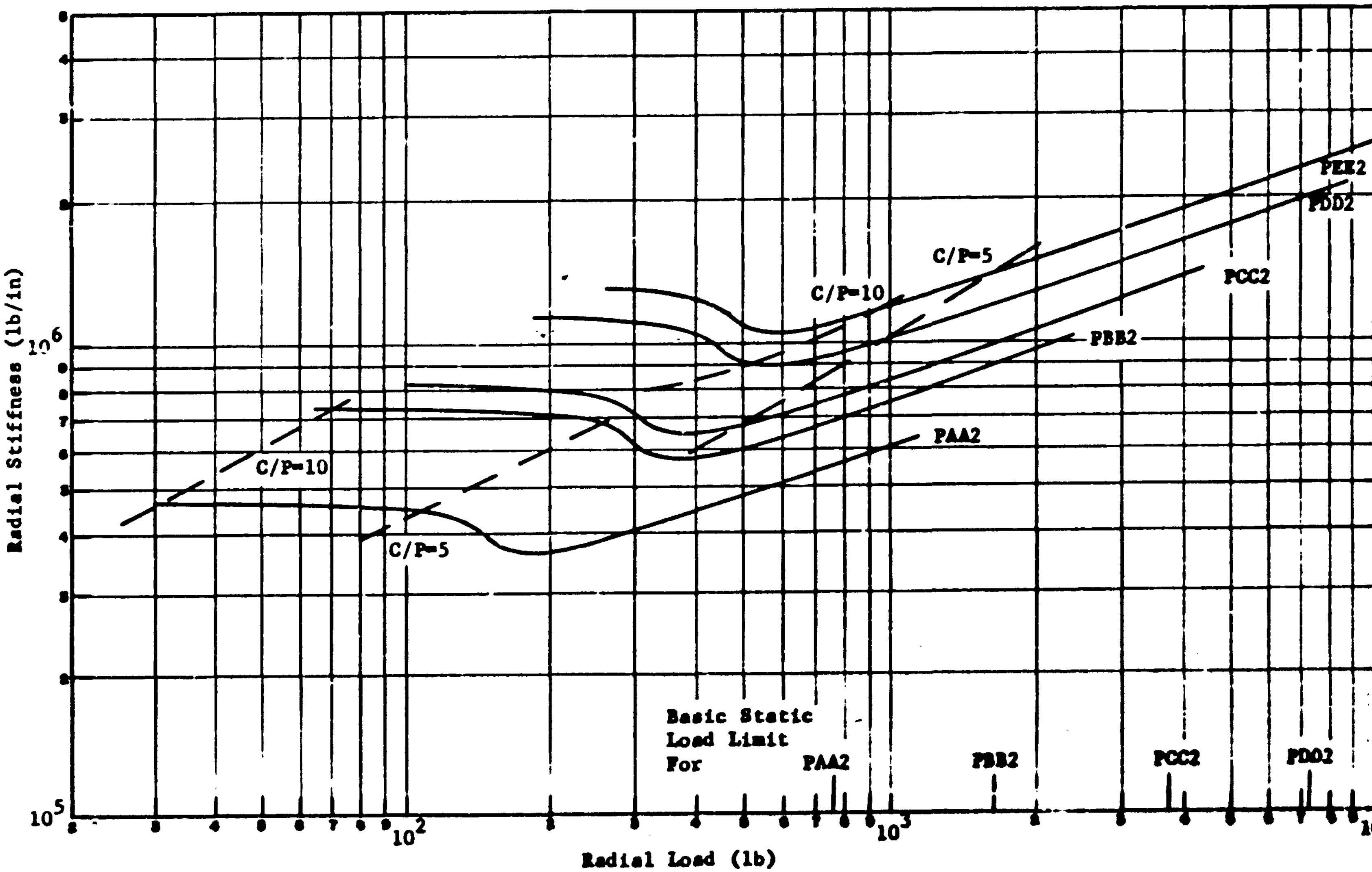


Fig. 24. Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 25^{\circ}$

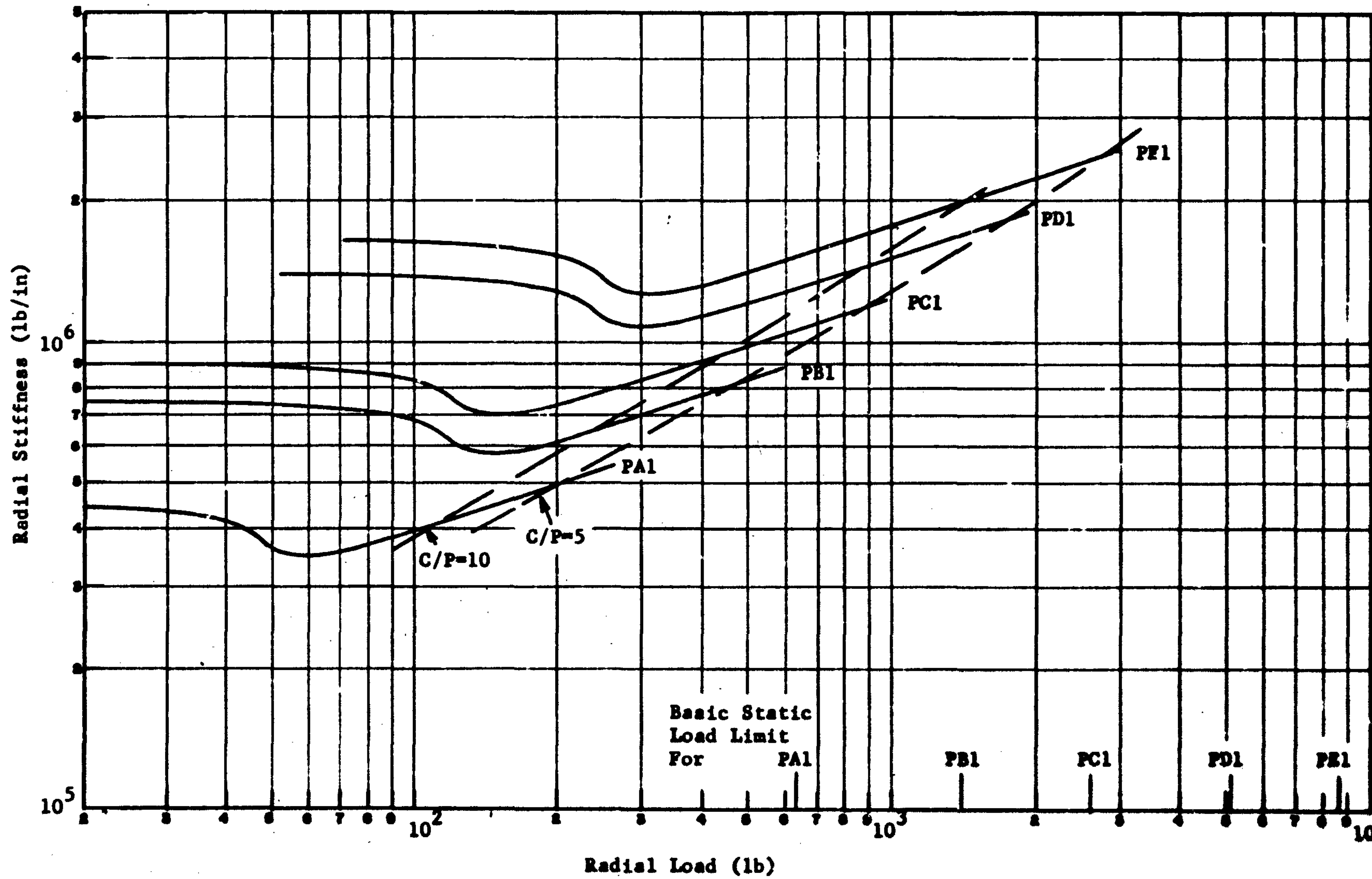


Fig. 25 Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
$\beta = 15^{\circ}$

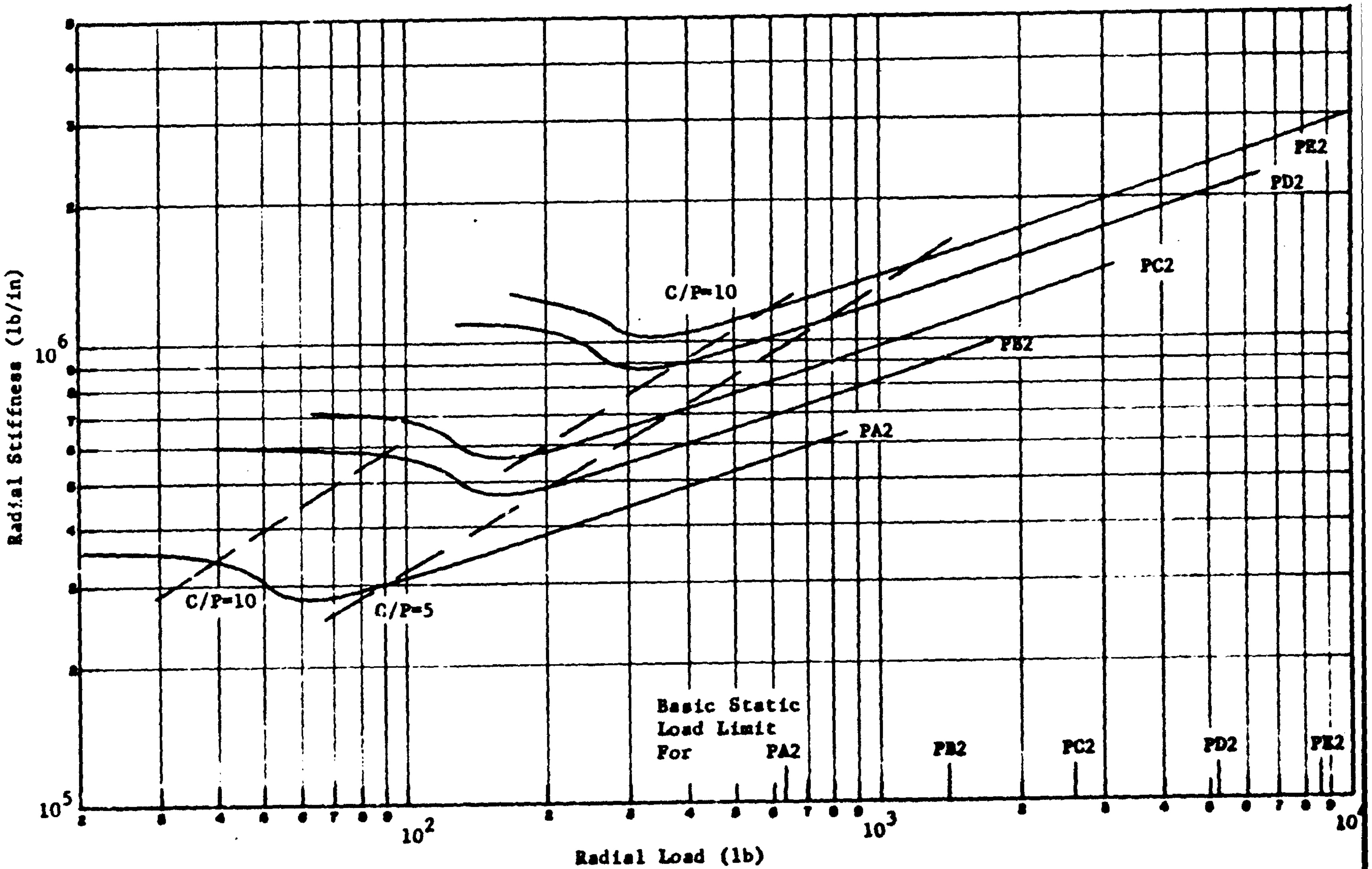


Fig. 26 Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
β = 15°

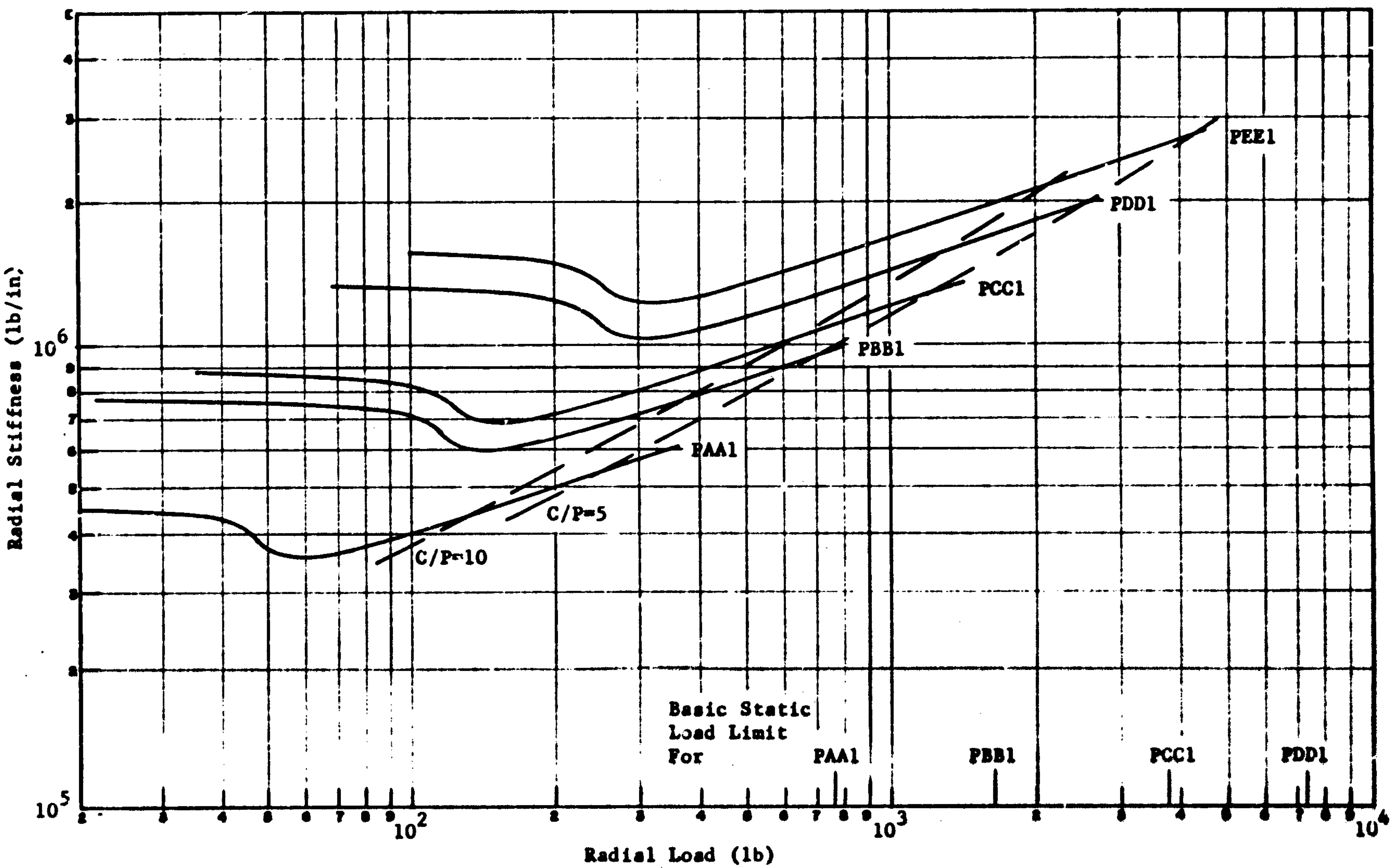


Fig. 27 Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
β = 15°

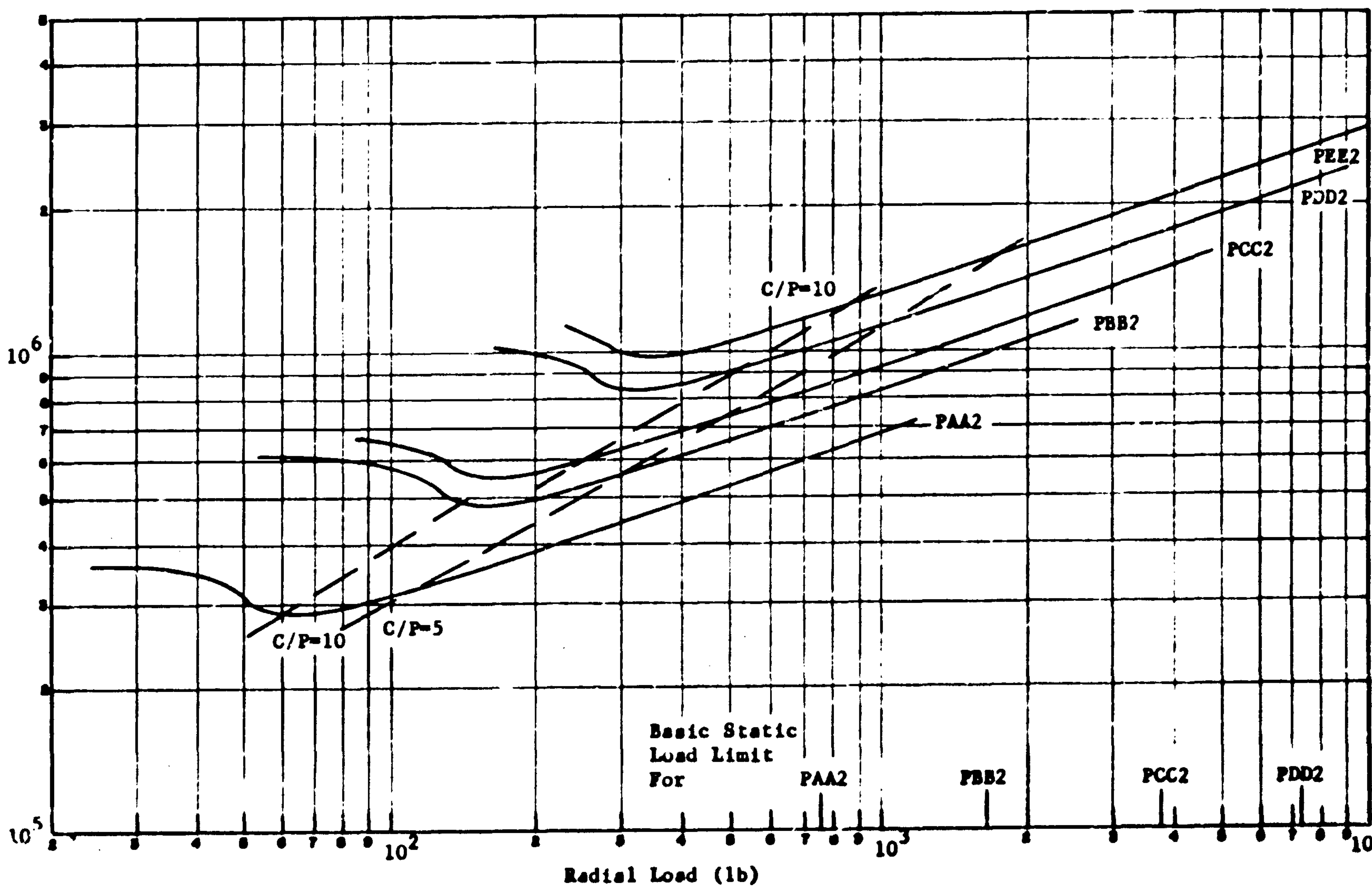


Fig. 28 Radial Stiffness for Angular Contact Bearing
Pre-Load - Selected Light
$\beta = 15^{\circ}$

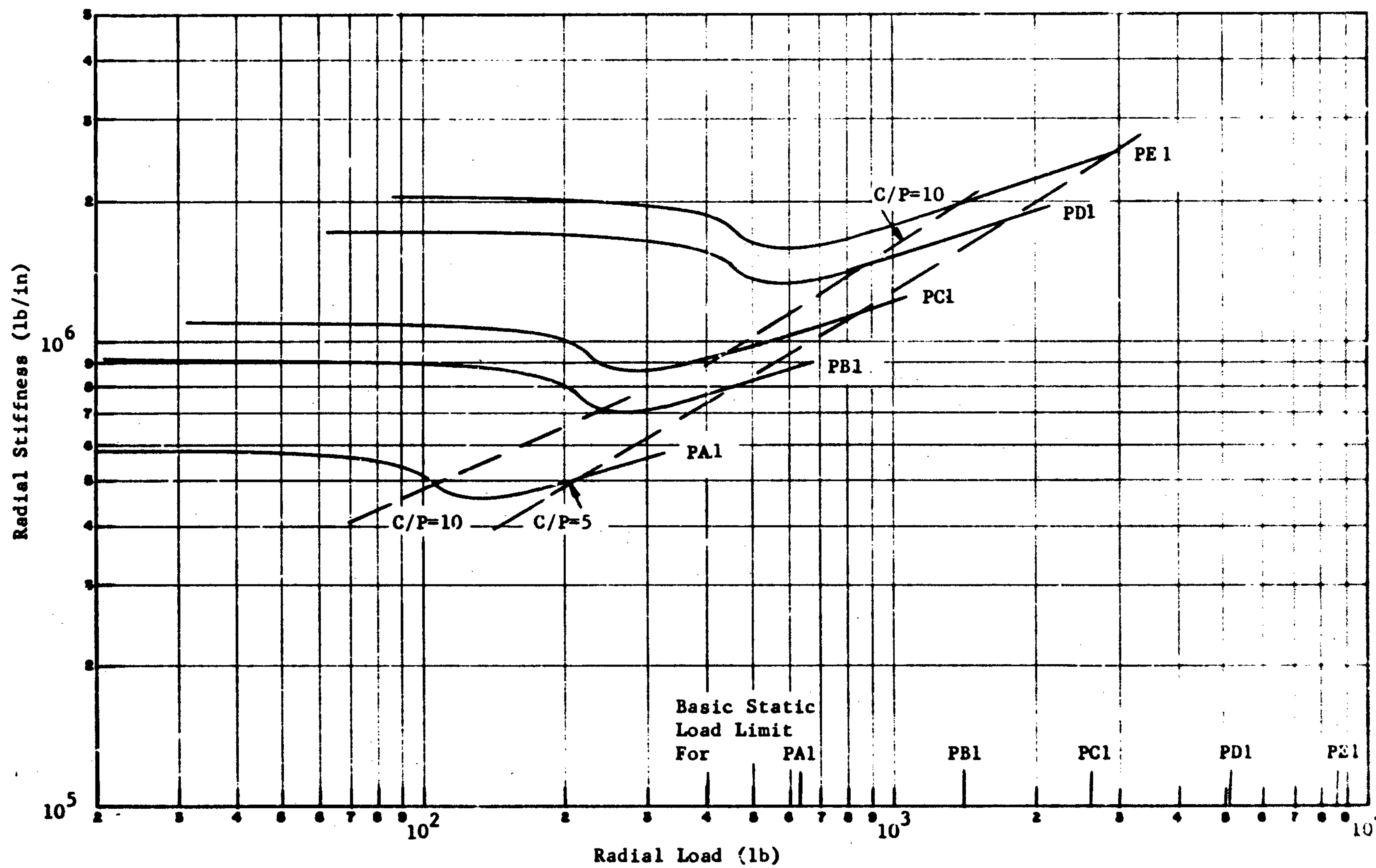


Fig. 29 **Radial Stiffness for Angular Contact Bearing**
Pre-Load - Moderate
$\beta = 15^o$

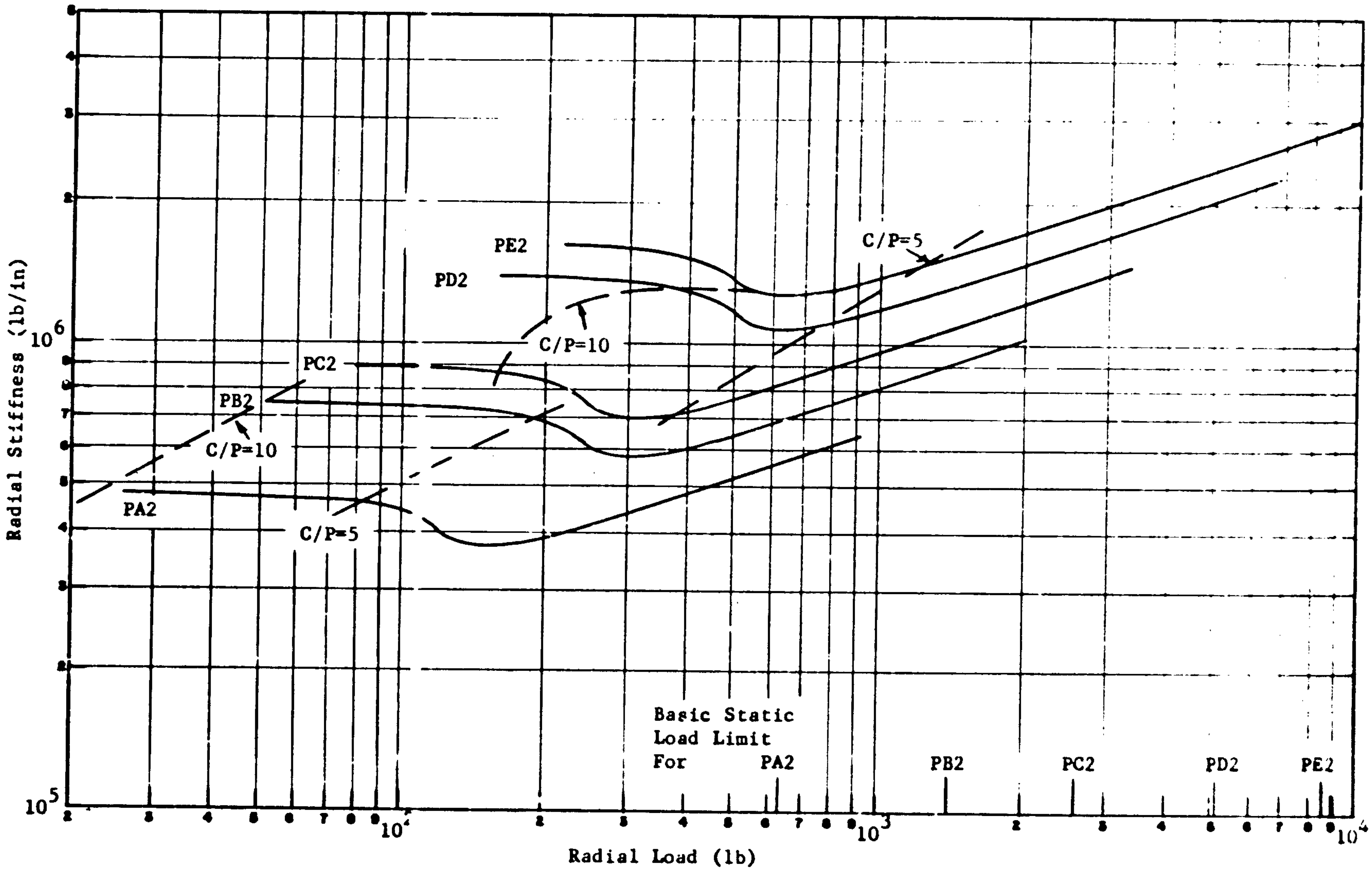


Fig. 30 Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
$\beta = 15^{o}$

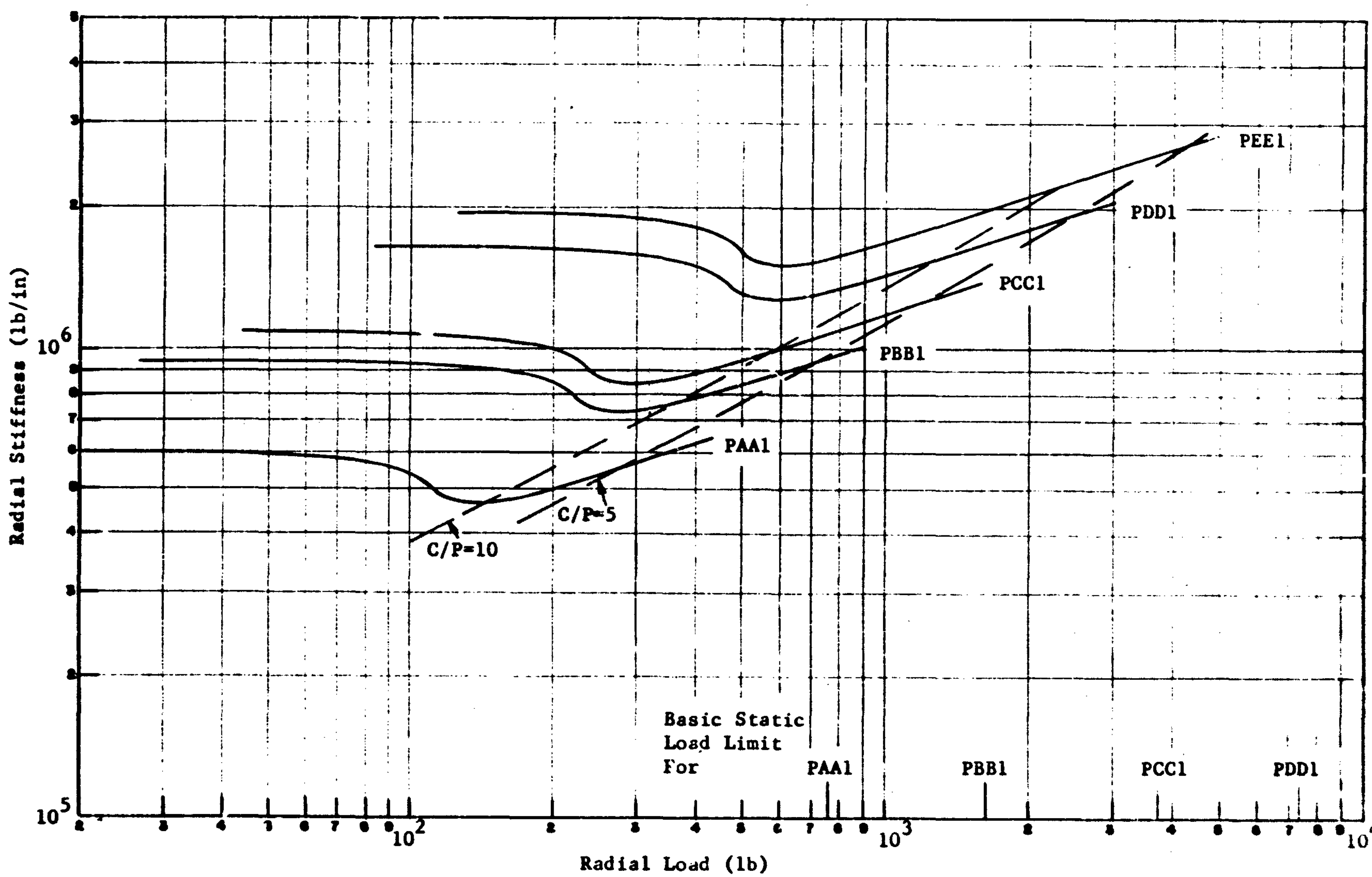


Fig. 31 Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
β = 15°

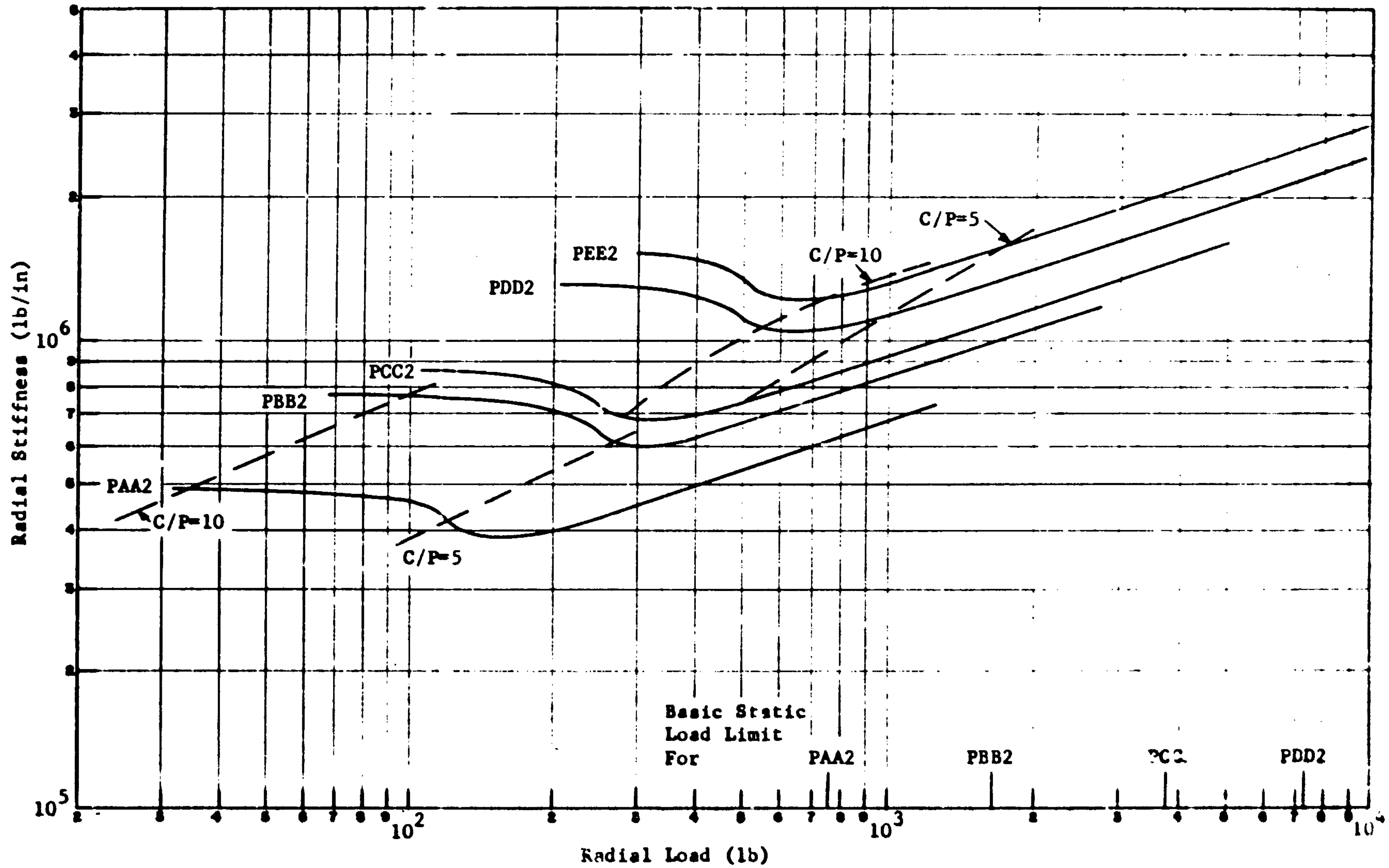


Fig. 32 Radial Stiffness for Angular Contact Bearing
Pre-Load - Moderate
$\beta = 15^{o}$

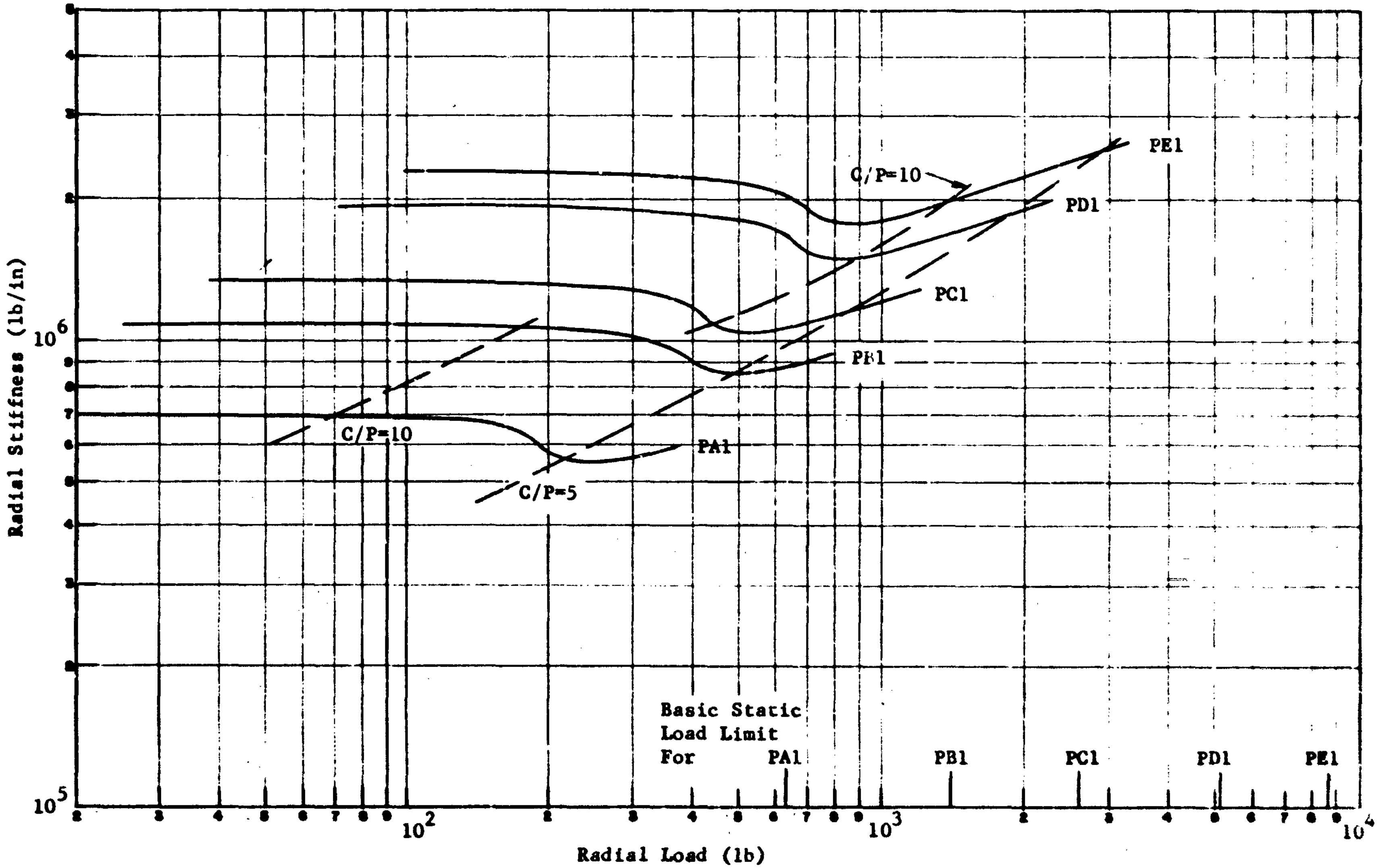


Fig. 33 Racial Stiffness for Angular Contact Be ring
Pre-Load - Preferred Heavy
$\beta = 15^{o}$

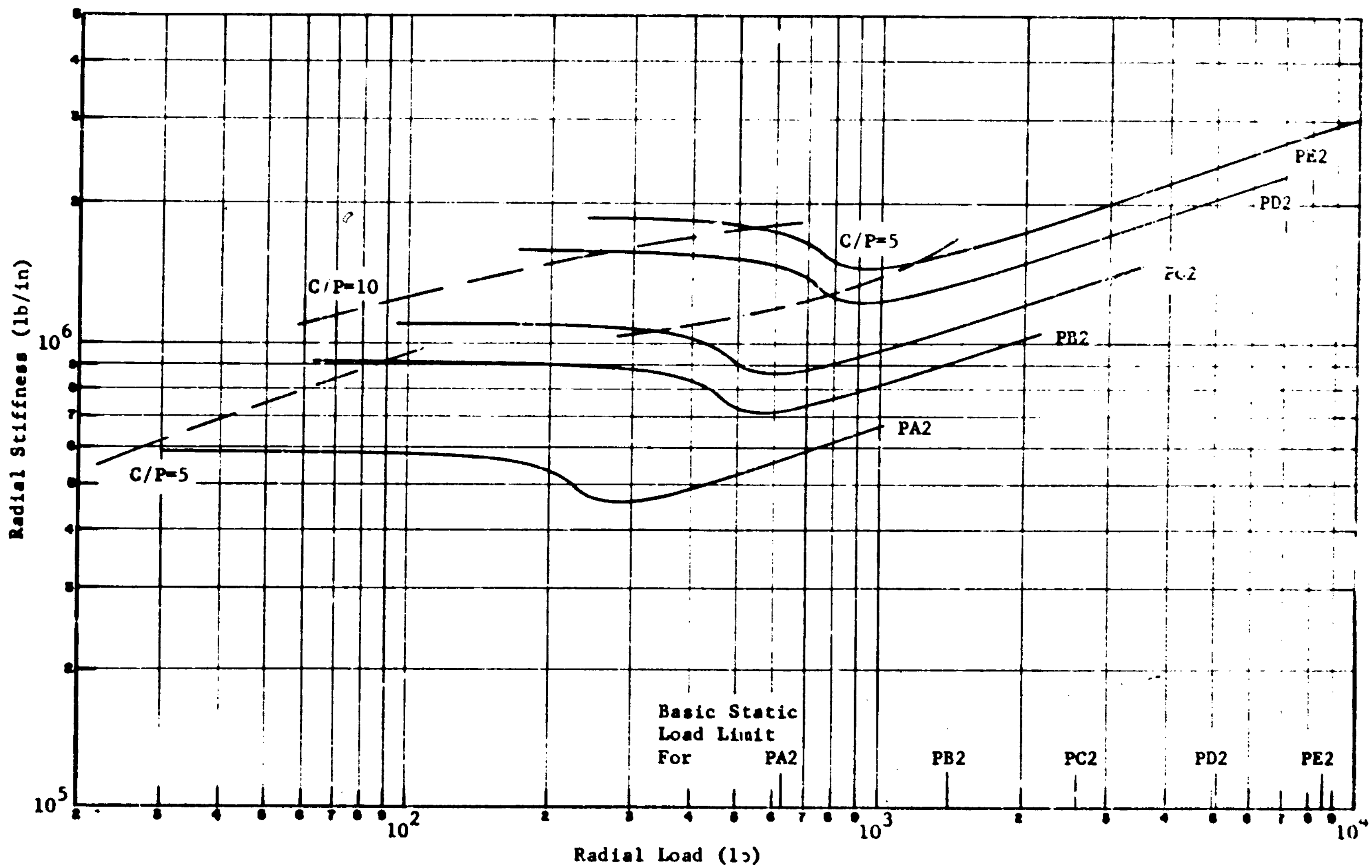


Fig. 34 Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 15^{\circ}$

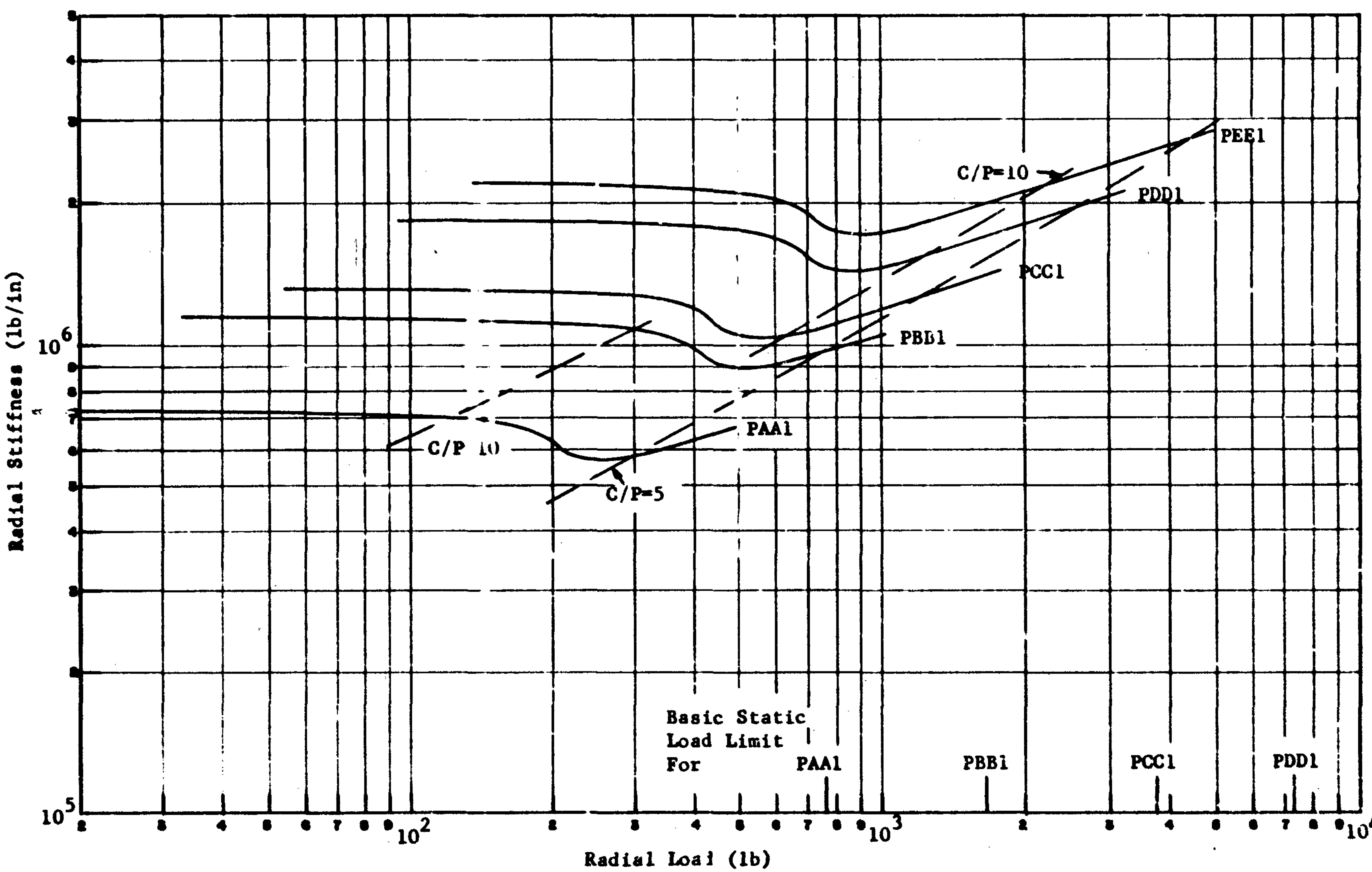


Fig. 35 Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 15^{\circ}$

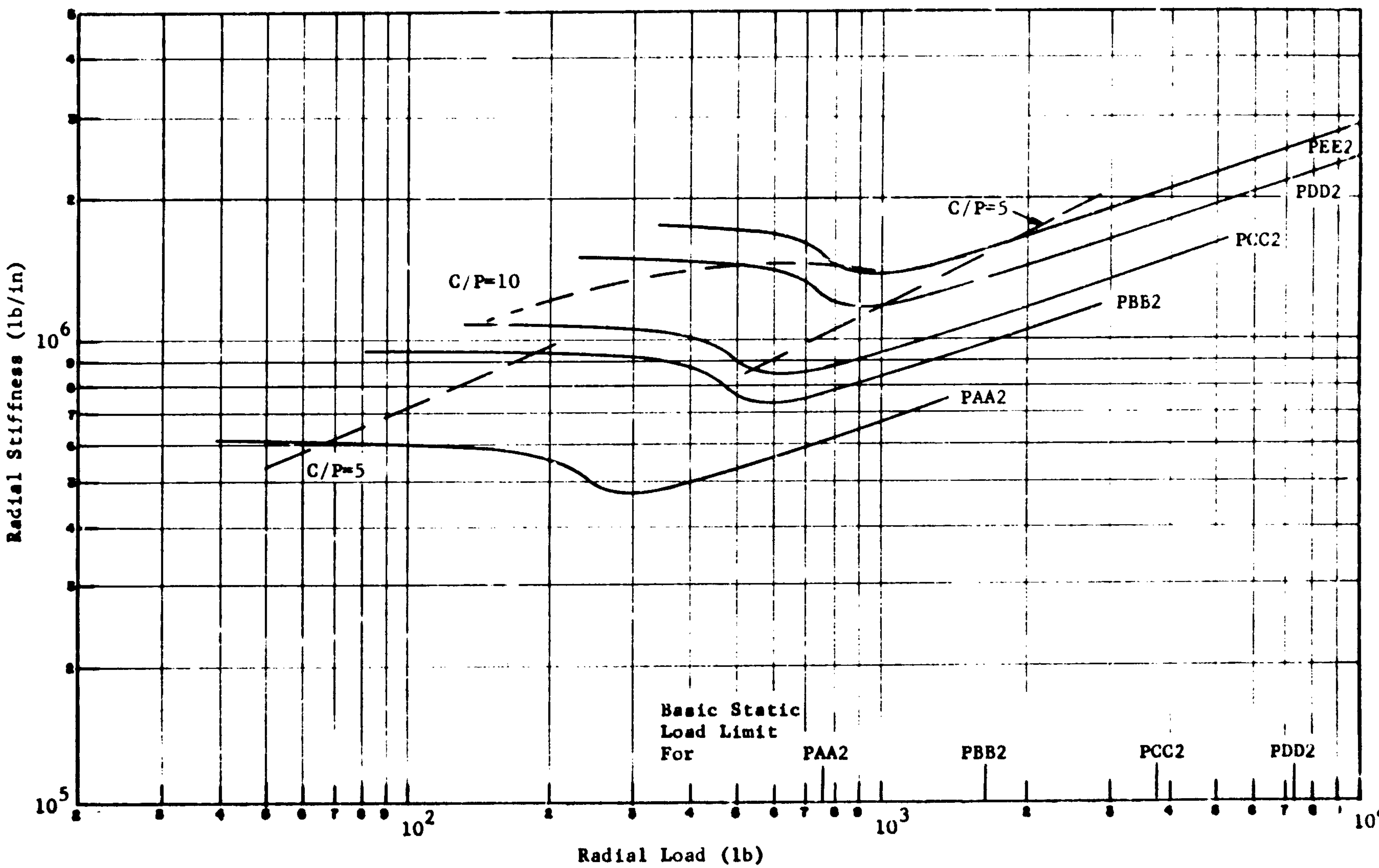


Fig. 36 Radial Stiffness for Angular Contact Bearing
Pre-Load - Preferred Heavy
$\beta = 15^o$

IV

SAMPLE PROBLEMS TO ILLUSTRATE USE OF DESIGN CHARTS

Four particular examples are included in this section:

1. Pure Radial Loaded Bearing
2. Unidirectional Thrust Loaded Bearing
3. Double-Acting Thrust Loaded Bearing
4. Radial Loaded, Axial Preloaded Angular Contact Bearing

1. Pure Radial Loaded Bearing

What are the radial stiffness values corresponding to radial loads of 100 and 700 pounds for a deep-grooved ball bearing with a BORE = .5906 inch and $f_i = f_o = .570$?

From Table 1, this bearing corresponds to A2.

From Figure 2:

Radial Load (lb)	Radial Stiffness (lb/in)
100	2.88×10^5
700	5.55×10^5

2. Unidirectional Thrust Loaded Bearing

What are the axial stiffness values corresponding to axial loads of 100 and 700 pounds for the same deep-grooved ball bearing as used in sample problem 1, above?

From Figure 7:

Thrust Load (lb)	Thrust Stiffness (lb/in)
100	9.10×10^4
700	3.22×10^5

3. Double-Acting Thrust Loaded Bearing

What is the axial stiffness for a double-acting, deep-grooved ball bearing, thrust bearing set, preloaded to 200 pounds? The bearings are type A-2.

From Figure 8: Read off loads corresponding to equal deflections around preload of 200 pounds.

Thus,

Load (lb)	Deflection (in)
160	2.8×10^{-3}
*200	3.1×10^{-3}
240	3.4×10^{-3}

The axial stiffness is $S_A = \frac{\Delta L}{\delta} = \frac{240 - 160}{.3 \times 10^{-3}}$

$$S_A = 267{,}000 \text{ lb/in}$$

This problem may also be solved using Figure 7 in conjunction with Figure 8.

Thus,

Load (lb)	Deflection (in)	Stiffness (lb/in)
160	2.8×10^{-3}	1.22×10^{5}
200	3.1×10^{-3}	1.4×10^{5}
240	3.4×10^{-3}	1.58×10^{5}

$$S_A = \Sigma S = (1.58 + 1.22) \times 10^5 = 2.8 \times 10^5 \text{ lb/in}$$

or $$S_A = 2 \times 1.4 \times 10^5 = 2.8 \times 10^5 \text{ lb/in}$$

For light loads, i.e., loads less than the axial preload, the load-deflection characteristics are essentially linear.

4. Radial Loaded, Axial Preloaded Angular Contact Bearing

What are the radial stiffness values corresponding to radial loads of 100 and 700 pounds for an angular contact bearing with a BORE = .5906 inch and $f_i = f_o = .570$. The contact angle is 15 degrees, and it has a medium axial preload.

From Table 2, this bearing corresponds to PA2.

From Figure 30,

Radial Load (lb)	Radial Stiffness (lb/in)
100	4.45×10^5
700	5.90×10^5

Note: One must be careful in using the charts, in particular for the angular contact bearing, that the design contact angle and axial preloading design value correspond to the values given in the Figure legend.

V

REFERENCES

1. New Departure Engineering Data, Analysis of Stresses and Deflections, Vol. 1, 2, 1946, New Departure, Division of General Motors Corporation, Bristol, Connecticut.

2. SKF General Catalog, No. 425, 1958, SKF Industries, Inc., Philadelphia, Pennsylvania.

3. New Departure Handbook - Ball Bearing Catalog, Twenty-third Edition, April 1955, New Departure, Division of GMC, Bristol, Connecticut.

4. Jones, A. B., "The Life of High-Speed Ball Bearings", Transactions of the ASME, July 1952, pp. 695-703.

VI

APPENDIX

A. Analysis

The theory used for predicting the load carrying capacity, deflections, and stresses of deep-grooved and angular contact bearings is that of Reference 1.* The equations relating load and deflection were differentiated to obtain the equation for stiffness.

A calculational procedure was devised for predicting the maximum ball load, deflection, stiffness, and inner and outer race stresses, as a function of total applied load, preload and bearing geometry. This procedure was programmed as computer program PNO182, IBM 1620-60K.

Three separate cases are treated:

1) Pure Radial Load, deep grooved bearing (Ref. 2,3)
2) Pure Thrust Load, deep grooved bearing
3) Combined Radial Load with Axial Preload, angular contact bearing (Ref. 2,3)

Pure Radial Load

Maximum Ball Load, P_o

$$P_o = 4.37 * P/n \qquad \text{A-1}$$

Radial Deflection, δ_N

$$\delta_N = C_o \, P_o^{2/3} \qquad \text{A-2}$$

where

$$C_o = 7.8107 \times 10^{-6} \, (C\delta_o + C\delta_i)/d^{1/3} \qquad \text{A-3}$$

and

$$C\delta_o, \, C\delta_i = f(f_i, \, f_o, \, E, \, d, \, \beta_o) \qquad \text{A-4}$$

---

* See Reference Section

Radial Stiffness, S_R

$$S_R = 3/2 \; P/\delta_N \qquad \text{A-5}$$

Compressive Stresses, S_o, S_i

$$S_m = f_{sm}(15079)\left(\frac{P_o}{d^2}\right)^{1/3} \qquad \text{A-6}$$

where $f_{sm} = f(f_i,\ f_o,\ E,\ d,\ \beta_o)$ A-7

and $m = i$, or o.

Pure Thrust Load

Maximum Ball Load, P_o

$$P_o = \frac{T}{n \sin\beta_1} \qquad \text{A-8}$$

Axial Deflection, δ_H

$$\delta_H = Bd \; * \; \frac{\sin(\beta_1 - \beta_o')}{\cos \beta_1} \qquad \text{A-9}$$

where

$$B = f_i + f_o - 1 \qquad \text{A-10}$$

and

β_1 is found by an iteration scheme, written below.

Applied Thrust (Axial) Load, T

$$T = nd^2K \cdot \sin \beta_1 \left(\frac{\cos \beta_o'}{\cos \beta_1} - 1\right)^{3/2} \qquad \text{A-11}$$

where

$$K = \left[\frac{B \times 10^{+6}}{7.8107(C\delta_o + C\delta_i)}\right]^{3/2} \qquad \text{A-12}$$

and

β_1 is found as follows:

Define the following quantities:

$$\frac{T}{nd^2 K} = b \qquad \cos \beta_o' = a \qquad \cos \beta_1 = x$$

Then Equation (A-11) may be written as

$$(1 - x^2)^{1/3} \left(\frac{a}{x} - 1\right) = b$$

Let $y = \frac{a}{x} - 1$, A-13

Then $y = b(1 - x^2)^{-1/3}$. A-14

It is a known fact that $b \ll 1$. Therefore, as a good guess to start the iteration scheme for solving (A-14), let

$$y = by_1 + b^2 y_2 + \ldots.. \qquad \text{A-15}$$

where

$$y_1 = 1 + \frac{a^2}{3}$$

and

$$y_2 = -\frac{2}{3} a^2 y_1 \quad . \qquad \text{A-16}$$

The procedure is:

1) Calculate y_1, y_2 from (A-16) and y from (A-15), knowing a and b.
2) Calculate x from (a-13) knowing y.
3) Calculate y from (A-14).
4) Check y from step (3) with y from step (2).
5) If the values are equal

$$\beta_1 = \cos^{-1}(x),$$

otherwise use an average value for y and repeat steps (2) through (5) until agreement is obtained.

Axial Stiffness, S_A

$$S_A = \frac{B}{nd_B K} * \left\{ \frac{3}{2} \sin^2 \beta_1 \left[\frac{\cos \beta_o'}{\cos \beta_1} - 1 \right]^{1/2} + \frac{\cos^3 \beta_1}{\cos \beta_o} \left[\frac{\cos \beta_o'}{\cos \beta_1} - 1 \right]^{3/2} \right\} \quad \text{A-17}$$

Compressive Stresses, S_o, S_i

$$S_m = f_{sm} (15079) \left(\frac{P_o}{d^2} \right)^{1/3} \quad \text{A-6}$$

where

$$f_{sm} = f(f_i, f_o, E, d, \beta_1) \quad \text{A-7}$$

$m = i$, or o

and

P_o is calculated from (A-8)

Combined Radial Load and Axial Preload

The same procedure for finding β_1 as applied in the pure thrust load case is applied in this case in order to find δ_H. A value of radial deflection, δ_V, is assumed, then the radial force, ΣV, is calculated as a function of δ_H and δ_V.

The following definitions are written:

$$k' = \frac{\delta_V}{Bd} \quad , \quad h' = \frac{\delta_H}{Bd} \qquad \text{and}$$

$$\phi' = \cos^{-1} \frac{\left[1 - (\sin \beta_o' + h')^2\right]^{1/2} - \cos \beta_o'}{k'} \quad , \quad \cos \phi' > -1 \quad \text{A-18}$$

$$\phi' = \pi \quad , \quad \cos \phi' < -1$$

Maximum Ball Load, P_o

$$P_o = Kd \left[\sqrt{(\sin \beta_o' + h')^2 + (\cos \beta_o' + k' \cos \phi)^2} - 1 \right]^{3/2} \quad \text{A-19}$$

where $\phi = 0^\circ$

<u>Radial Deflection</u>, δ_V; <u>Axial Deflection</u>, δ_H

$$\delta_V = k' \; Bd \qquad\qquad \delta_H = h' \; Bd \tag{A-20}$$

<u>Axial Preload</u>, T

Calculate from (A-11) and (A-12).

<u>Radial Load</u>, ΣV

$$\Sigma V = nd^2 \, K \, \frac{1}{\pi} \int_{\phi'}^{o} A d\phi$$

where

$$A = \frac{\left[\sqrt{(\sin \beta_o' + h')^2 + (\cos \beta_o' + K' \cos \phi)^2} - 1\right]^{3/2} (\cos \beta_o' + k' \cos\phi) \cos \phi}{\left[(\sin \beta_o' + h')^2 + (\cos \beta_o' + k' \cos \phi)^2\right]^{1/2}}$$

and

$$0 \leq \phi \leq \phi' \tag{A-21}$$

<u>Radial Stiffness</u>, S_R

$$S_R = \frac{B}{nd \; K} * \frac{1}{\pi} \int_{\phi'}^{o} A \left\{ \frac{\cos \phi}{(\cos \beta_o' + k' \cos \phi)} + \right.$$

$$\left. A \left[\frac{\frac{1}{2} \left[(\sin \beta_o' + h')^2 + (\cos \beta_o' + k' \cos \phi)^2 \right]^{1/2} + 1}{\left[\sqrt{(\sin\beta_o' + h')^2 + (\cos\beta_o' + k' \cos\phi)^2} - 1\right]^{5/2} \left[(\sin\beta_o' + h')^2 \; (\cos\beta_o' + k' \cos\phi)^2\right]^{1/2}} \right] \right\} d\phi$$

A-22

<u>Compressive Stresses</u>, S_o, S_1

$$S_m = f_{sm}(15079) \left(\frac{P_o}{d^2} \right)^{1/3} \tag{A-6}$$

where

$$f_{sm} = f(f_i, f_o, E, d, \beta_1) \qquad \text{A-7}$$

m = i, or o, and P_o is calculated from (A-19).

B. Computer Program

A Fortran II computer program listing is included in this memorandum.

The Input Format written below should be followed when using this program, PN0182 IBM 1620-60K.

Input Format

Card 1 Identification Card

Anything may be punched in columns 2-72.

Card 2 (6 F10. 6, 3I4)

Item

1. BORE, Bore diameter, in.
2. ØD, Extreme outer diameter, in.
3. DB, Ball diameter, in.
4. FI, Radius of Curvature of Inner Race
5. FØ, Radius of Curvature of Outer Race
6. BETA, Contact Angle, deg ($\beta = 0^{o}$ for pure radial load)
7. N, Total number of balls
8. IND, An indicator used to specify either one of three different types of calculations.
   - IND: 0 Pure Radial Load
   - IND: 1 Pure Thrust Load
   - IND: ≥ 2 Combined Radial Load - Axial Preload
9. LC, An indicator used to stop calculation procedure
   - LC: 0 Program returns to Card 1 for more input.
   - LC: 1 Program stops after computation is completed

Card 3 (3F10.6, I5)

IND = 0

Item

1. RI, Initial Radial Load, lb.
2. RD, Radial Load Increment, lb
3. RF, Final Radial Load, lb. (Not used in calculation. RF = 0.0)
4. M, Total number of radial loads

IND = 1

Item

1. TI, Initial Thrust Load, lb
2. TD, Thrust Load Increments, lb
3. CØNV, A radius of convergence used in the iteration process for calculating β_1. CØNV = .0005 is a typical value.
4. M, Total number of thrust loads

IND $\geq$ 2

Item

1. TI, Axial Preload, lb
2. TD, Axial Preload Increment, lb (Not used in calculation T' = 0.0)
3. CØNV, a radius of convergence used in the iteration process for calculating β_1.
4. M, Total number of preloads (must be set equal to one; i.e. M = 1)

Note: The total number of radial loads which will be calculated as a function of axial preload and radial deflection is equal to the value of IND. Thus, IND = 20, the calculational procedure will solve for '20' consecutive radial loads. A maximum of IND = 24 is allowed.

Output Format

The output is self explanatory. All linear dimensions are in inches. All loads are in pounds. The stiffness units are in lb/in. The stresses are measured in psi.

```
C      BALL BEARING STIFFNESS AND STRESS CALCULATION ROUTINE FOR
C      PURE RADIAL,PURE THRUST,OR COMBINED LOADING INCLUDING CENTRIFUGAL
C      PN0182  3JUNE(64) SBM AND P(ABJ)L  FOR JL
       DIMENSION  FF(12),BB(12),CC(12),DD(12),BS(12),CS(12),DS(12)
       DIMENSION AKI(24)
       KOP=0
       FF(1)=.506
       FF(2)=.510
       FF(3)=.516
       FF(4)=.520
       FF(5)=.530
       FF(6)=.540
       FF(7)=.550
       FF(8)=.560
       FF(9)=.570
       FF(10)=.580
       FF(11)=.590
       FF(12)=.600
       BB(1)=.816
       BB(2)=.928
       BB(3)=1.037
       BB(4)=1.092
       BB(5)=1.197
       BB(6)=1.275
       BB(7)=1.335
       BB(8)=1.385
       BB(9)=1.428
       BB(10)=1.465
       BB(11)=1.498
       BB(12)=1.525
       CC(1)=.790
       CC(2)=.895
       CC(3)=1.0
       CC(4)=1.05
       CC(5)=1.15
       CC(6)=1.22
       CC(7)=1.278
       CC(8)=1.321
       CC(9)=1.36
       CC(10)=1.395
       CC(11)=1.425
       CC(12)=1.45
       DD(1)=.850
       DD(2)=.968
       DD(3)=1.085
       DD(4)=1.145
       DD(5)=1.260
       DD(6)=1.341
       DD(7)=1.410
       DD(8)=1.462
       DD(9)=1.510
       DD(10)=1.55
       DD(11)=1.585
       DD(12)=1.62
       BS(1)=.85
       BS(2)=.94
       BS(3)=1.03
       BS(4)=1.075
       BS(5)=1.17
       BS(6)=1.24
       BS(7)=1.30
```

```
      BS(8)=1.35
      BS(9)=1.40
      BS(10)=1.44
      BS(11)=1.47
      BS(12)=1.51
      CS(1)=.70
      CS(2)=.78
      CS(3)=.86
      CS(4)=.90
      CS(5)=.98
      CS(6)=1.04
      CS(7)=1.09
      CS(8)=1.15
      CS(9)=1.18
      CS(10)=1.21
      CS(11)=1.25
      CS(12)=1.275
      DS(1)=1.15
      DS(2)=1.25
      DS(3)=1.40
      DS(4)=1.45
      DS(5)=1.58
      DS(6)=1.65
      DS(7)=1.75
      DS(8)=1.80
      DS(9)=1.85
      DS(10)=1.92
      DS(11)=1.95
      DS(12)=2.00
    6 READ 100
      READ 102, BORE,OD,DB,FI,FO,BETA,N,IND,LC
      PUNCH 111
      PUNCH 100
      PUNCH 103
      PUNCH 104, BORE,OD,DB,FI,FO,BETA,N
      IF (IND-1) 1,2,3
    1 READ 107, RI,RD,RF,M
      PUNCH 106
      GO TO 4
    2 READ 107, TI,TD,CONV,M
      PUNCH 108
      GO TO 4
    3 READ 107,TI,TD,CONV,M
      KOP=KOP+1
      IF (KOP-1) 18,18,4
   18 AK1(1)=.002
      AK1(2)=.003
      AK1(3)=.004
      AK1(4)=.005
      AK1(5)=.006
      AK1(6)=.007
      AK1(7)=.008
      AK1(8)=.009
      AK1(9)=.010
      AK1(10)=.012
      AK1(11)=.014
      AK1(12)=.016
      AK1(13)=.018
      AK1(14)=.020
      AK1(15)=.022
      AK1(16)=.024
```

```
      AK1(17)=.026
      AK1(18)=.028
      AK1(19)=.030
      AK1(20)=.035
      AK1(21)=.040
      AK1(22)=.050
      AK1(23)=.060
      AK1(24)=.070
      GO TO 4
C     PRELIMINARY CALCULATIONS
    4 BET=0.0174533*BETA
      E=(BORE+OD)/2.0
      AN=N
      COSB=COSF(BET)
      SINB=SINF(BET)
      Z=DB*COSB/E
      CALL TLU (FI,B,FF,BB,12)
      CALL TLU (FI,D,FF,DD,12)
      CDI=-(E-D)*Z*2.5+B
      CALL TLU (FO,B,FF,BB,12)
      CALL TLU (FO,C,FF,CC,12)
      CDO=-(B-C)*Z*2.0+B
      CALL TLU (FO,BZ,FF,BS,12)
      CALL TLU (FO,CZ,FF,CS,12)
      FSO=BZ-(BZ-CZ)*2.0*Z
      CALL TLU (FI,ZB,FF,BS,12)
      CALL TLU (FI,ZD,FF,DS,12)
      FSI=ZB-(ZB-ZD)*2.0*Z
      DB3=DB**0.333333
      C=7.8107E-06*(CDO+CDI)/DB3
      HCON=15079./DB3/DB3
      IF (IND-1) 8,9,9
    8 R=RI-RD
      DO 5 I=1,M
      R=R+RD
      PO=4.37*R/AN
      PO3=PO**0.333333
      DN=C*PO3*PO3
      DRDDN=1.5*R/DN
      SMI=PO3*HCON
      SMO=FSO*SMI
      SMI=FSI*SMI
    5 PUNCH 105, R,PO,DN,DRDDN,SMI,SMO
   20 IF (LC) 6,6,15
    9 B=FI+FO-1.0
      AK=(D/(CDO+CDI)/7.8107E-06)**3
      D=AN*DB*DB
      AK=SQRTF(AK)
      AK=AK*D
      T=TI-TD
      A=COSB
      Y1=1.0+A*A/3.0
      Y2=-0.666667*A*A*Y1
      DO 10 I=1,M
      T=T+TD
      B1=T/AK
      B2=B1**0.333333
      B2=B2*B2
      Y=B2*Y1+B2*B2*Y2
   13 X=A/(Y+1.0)
      YY=(1.0-X*X)**0.333333
```

```
      YY=B2/YY
      ET=SQRTF(Y*Y+YY*YY)
      ETA=ABSF(Y)-ABSF(YY)
      ETA=ABSF(ETA)/ET-CONV
      IF (ETA) 11,11,12
   12 Y=(Y+YY)/2.0
      GO TO 13
   11 XS=SQRTF(1.0-X*X)
      BET1=ATANF(XS/X)
      DH1=1.0/X*SINF(BET1-BET)
      DH=B*DB*DH1
      Z=DB/E*X
      FSO=BZ-(BZ-CZ)*2.0*Z
      FSI=ZB-(ZB-ZD)*2.0*Z
      DT=A/X-1.0
      DTS=SQRTF(DT)
      DTDDH=DTS*AK/B/DB*(1.5*XS*XS+DT*X*X*X/A)
      IF(IND-1) 16,16,17
   16 PO=T/AN/XS
      PO3=PO**0.333333
      SMI=PO3*HCON
      SMO=FSO*SMI
      SMI=FSI*SMI
   10 PUNCH 105, T,PO,DH,DTDDH,SMI,SMO
      GO TO 20
   17 S1=(SINB+DH1)**2
      PUNCH 109
      PUNCH 105,T,B1,DH,DH1,DTDDH
      PUNCH 106
      DO 19 I=1,IND
      FK=AKI(I)
      PHC=(SQRTF(1.0-S1)-COSB)/FK
      PHCA=ABSF(PHC)
      IF(PHCA-1.0) 40,41,41
   41 PHI=3.1415927
      GO TO 31
   40 PHS=SQRTF(1.0-PHC*PHC)
      PHI=ATANF(PHS/PHC)
      IF(PHC) 30,31,31
   30 PHI=3.1415927+PHI
   31 DPHI=PHI/30.0
      PHID=PHI*57.29578
      X=-DPHI
      SUM=0.0
      SUMS=0.0
      DO 32 J=1,31
      X=X+DPHI
      CP=COSF(X)
      S2=COSB+FK*CP
      IF (J -1) 35,35,33
   35 YX=0.5
      GO TO 34
   33 YX=1.0
   34 P=S1+S2**2
      P=SQRTF(P)
      IF (P-1.0) 53,53,54
   53 PM1=1.0E-06
      GO TO 55
   54 PM1=SQRTF(P-1.0)
   55 PM2=PM1**3
      PM3=PM2*PM1*PM1
```

```
      SST=PM2*S2*CP/P
      SUM=SUM+SST*YX
      S3=CP/S2
      ST=S3    +SST*(0.5*P+1.0)/P/PM3
      IF(SENSE SWITCH 2) 52,32
   52  PUNCH 105,S3,ST,P,PM1,SST,S2
   32 SUMS=SUMS+ST*SST*YX
      SUM=(SUM-0.5*SST)*DPHI/3.1415927
      SUMS=(SUMS-0.5*SST*ST)*DPHI/3.1415927
      IF(SENSE SWITCH 2) 50,51
   50  PUNCH 105, SUM,SUMS,PHID,FK
   51 PO=SQRTF(S1+(COSB+FK)**2)
      PO=SQRTF(PO-1.0)
      PO=PO**3*AK/AN
      PO3=PO**0.333333
      SMI=PO3*HCON
      SMO=FSO*SMI
      SMI=FSI*SMI
      R=SUM*AK
      DRDDN=SUMS*AK/B/DB
      DN=FK*B*DB
   19 PUNCH 105,R,PO,DN,DRDDN,SMI,SMO
      GO TO 20
   15 STOP
C     FORMAT STATEMENTS
  100 FORMAT(72H0
     1                                    )
  102 FORMAT(6F10.6,3I4)
  104 FORMAT(6F10.5,I8)
  103 FORMAT(72H0     BORE      O.D.      BALL DIA.     F(I)      F(O) CONT.A
     1NGLE NO.OF BALLS)
  105 FORMAT(1X,E11.4,1X,E11.4,1X,E11.4,1X,E11.4,1X,E11.4,1X,E11.4)
  106 FORMAT(72H0TOT.RAD.LOAD BALL LOAD   DEFLECTION   STIFFNESS   I.R.ST
     1RESS   O.R.STRESS)
  107 FORMAT(3F10.6,I5)
  108 FORMAT(72H0THRUST LOAD  BALL LOAD    DEFLECTION   STIFFNESS   I.R.ST
     1RESS   O.R.STRESS)
  109 FORMAT(62H0THRUST LOAD     T/NDDK   AXIAL DEFL.     H/BD      AXIAL
     1STIFF.)
  110 FORMAT(2F10.6,I5)
  111 FORMAT(1H1)
      END
```

```
      SUBROUTINE TLU (A,B,C,D,N)
C                 A   INDEPENDENT VARIABLE
C                 B      DEPENDENT VARIABLE       ANSWER)
C                 C INDEPENDENT TABLE
C                 D      DEPENDENT TABLE
C                 N   NO OF ENTRIES IN TABLE
      DIMENSION  C(50),D(50)
      NTX2=0
      I=1
      NTXI=N
      M=N/2
   13 IF (M-1) 39,39,42
   39 B=D(1)+(A-C(1))*(D(1)-D(2))/(C(1)-C(2))
      GO TO 99
   42 IF (C(I)-C(I+1)) 45,43,44
   43 I=I+1
      GO TO 42
   44 IF (C(M)-A) 6,7,8
   45 IF (A-C(M)) 6,7,8
    7 B=D(M)
      GO TO 99
    6 IF (M-NTX2-1)9,10,15
   15 NTXI=M
      M=M-(M-NTX2)/2
      GO TO 13
    9 NTXI= M
      GO TO 14
    8 NTX2=M
   14 M=(NTXI-M)/2+M
      IF (NTX2-NTXI+1)13,18,13
   18 M= NTXI
   10 DENO= C(M)-C(M-1)
      DIFF= A-C(M)
      B= DIFF/DENO*(D(M)-D(M-1))+D(M)
   99 RETURN
       END
```

C. Nomenclature for Analysis

Symbol	Definition	Units
B	Total Curvature, Eq. A-10	
C_o	Radial Deflection Constant, Eq. A-3	
$C_{\delta o}$	Deflection Constant of Outer Race, Eq. A-4	
$C_{\delta i}$	Deflection Constant of Inner Race, Eq. A-4	
d	Ball Diameter	in.
E	Pitch Circle Diameter	in.
f_i	Inner Race Curvature	
f_o	Outer Race Curvature	
f_{si}	Stress Factor for Inner Race, Eq. A-7	
f_{so}	Stress Factor for Outer Race, Eq. A-7	
h'	Relative Displacement of Races in Axial Direction Eq. A-18	in.
K	Axial Deflection Constant, Eq. A-12	
k'	Relative Displacement of Races in Radial Direction Eq. A-18	in.
n	Number of Balls	
P_o	Maximum Ball Load	lb.
P	Magnitude of Radial Load for Deep Grooved Bearing	lb.
S_A	Stiffness/Bearing in Axial Direction Due to Load in Axial Direction	lb./in.
S_R	Stiffness in Radial Direction	lb./in.
S_i	Compressive Stress in Inner Race	psi
S_o	Compressive Stress in Outer Race	psi
T	Axial Load, or Preload	lb.
β_o	Initial Contact Angle	deg.
β_o'	Contact Angle after Preload	deg.
β_1	Operating Contact Angle	deg.
δ_H	Deflection in Axial Direction	in.
δ_N	Deflection in Radial Direction	in.
δ_V	Deflection in Vertical or Radial Direction	in.
ΣV	Magnitude of Radial Load for Angular Contact Brg.	in.
ϕ	Angle Measured to a Load Vector within Loaded Zone of Ball	deg.
ϕ'	Half Angular Extent of Loaded Zone of Ball $(0 \leq \phi' \leq \pi)$	deg.

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Lubrication						
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Rolling Element						
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