REPORT NO. RG-TR-65-7

AN ACTIVE LOW-PASS FILTER

by

H. V. White

May 1965

U S ARMY MISSILE COMMAND
REDSTONE ARSENAL, ALABAMA

1 May 1965

Report No. RG-TR-65-7

AN ACTIVE LOW-PASS FILTER

by

H. V. White

DA Project No. 1X279191D678

AMC Management Structure Code No. 5282.12.127

Inertial Systems Branch
Army Inertial Guidance and Control Laboratory
Directorate of Research and Development
U. S. Army Missile Command
Redstone Arsenal, Alabama

ABSTRACT

The analysis and design of an active low-pass filter for use in inertial platform systems (though not restricted to this application) are presented in this report. The filter features minimum size and weight and increased reliability through the use of a single micro-circuit, yet it maintains performance comparable to conventional types presently used for the same purpose. Little difficulty should be encountered in making the external circuitry an integral part of the operational amplifier in a finalized version of the unit.

INTRODUCTION

The alternating current to direct current signal conversion process usually performed in an inertial platform gimbal stabilization loop requires the elimination of carrier and other undesirable frequency components from the converted signal. The filtering process should, ideally, pass all frequencies of interest with zero attenuation and phase shift and suppress all others. Additionally, modern day requirements for small size and weight and increased reliability dictate the elimination of large bulky components with numerous discrete interconnections.

A low-pass filter with characteristics approaching those described above can be realized with a single microcircuit operational amplifier and a minimum number of discrete subminiature components. Such a filter is shown in Figure 1.

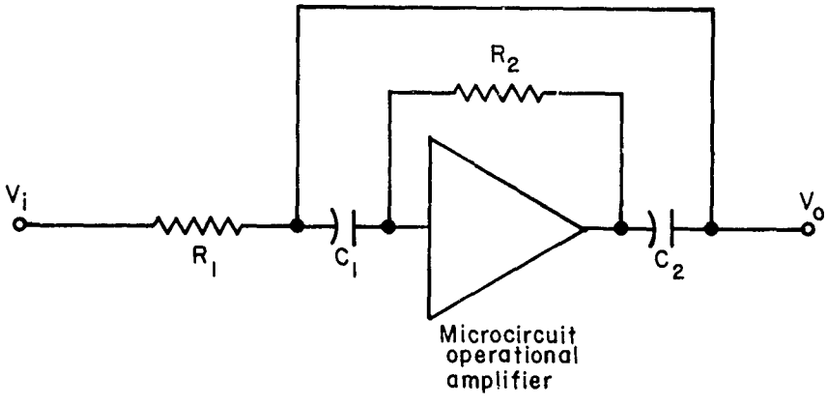

Figure 1. Active Low-Pass Filter

The filter characteristics for input frequencies of $\omega = 0$ and $\omega = \infty$ are immediately obvious. For the first case, the operational amplifier is isolated by capacitors C_1 and C_2 with the result that the input is connected directly to the output through resistor R_1. This gives $V_O = V_i$ if the filter is unloaded. For the second case, C_1 and C_2 are short circuits resulting in unity feedback from output to input such

that $V_o = 0$. The characteristics between these two extremes are best investigated by a frequency domain analysis of the filter.

ANALYSIS

The transfer function in Laplace transform notation for the system of Figure 1 is

$$\frac{V_o}{V_i} = \frac{1}{R_1R_2C_1C_2S^2 + R_1(C_1 + C_2)S^2 + 1} \tag{1}$$

The frequency response characteristics of such a second order system is well known and is shown in Figure 2 for reference.

From Figure 2, it is seen that minimum phase shift requires a small value of ζ. Corner peaking, however, increases with decreasing ζ. A compromise between these two characteristics will depend on a particular application. Figure 2 also indicates a rolloff rate of 12 decibels per octave.

If the parameters of Equation (1) are written in terms of the universal second order parameters of Figure 2,

$$R_1R_2C_1C_2 = \frac{1}{\omega_n^2} \tag{2}$$

and

$$R_1(C_1 + C_2) = \frac{2\zeta}{\omega_n} \tag{3}$$

From Equations (2) and (3), the value of C_2 is determined as

$$C_2 = \frac{\zeta}{R_1\omega_n} \pm \sqrt{\frac{\zeta^2}{R_1^2\omega_n^2} - \frac{1}{\omega_n^2 R_1R_2}} \tag{4}$$

For a real solution,

$$\frac{\zeta^2}{R_1^2\omega_n^2} \geq \frac{1}{\omega_n^2R_1R_2} \tag{5}$$

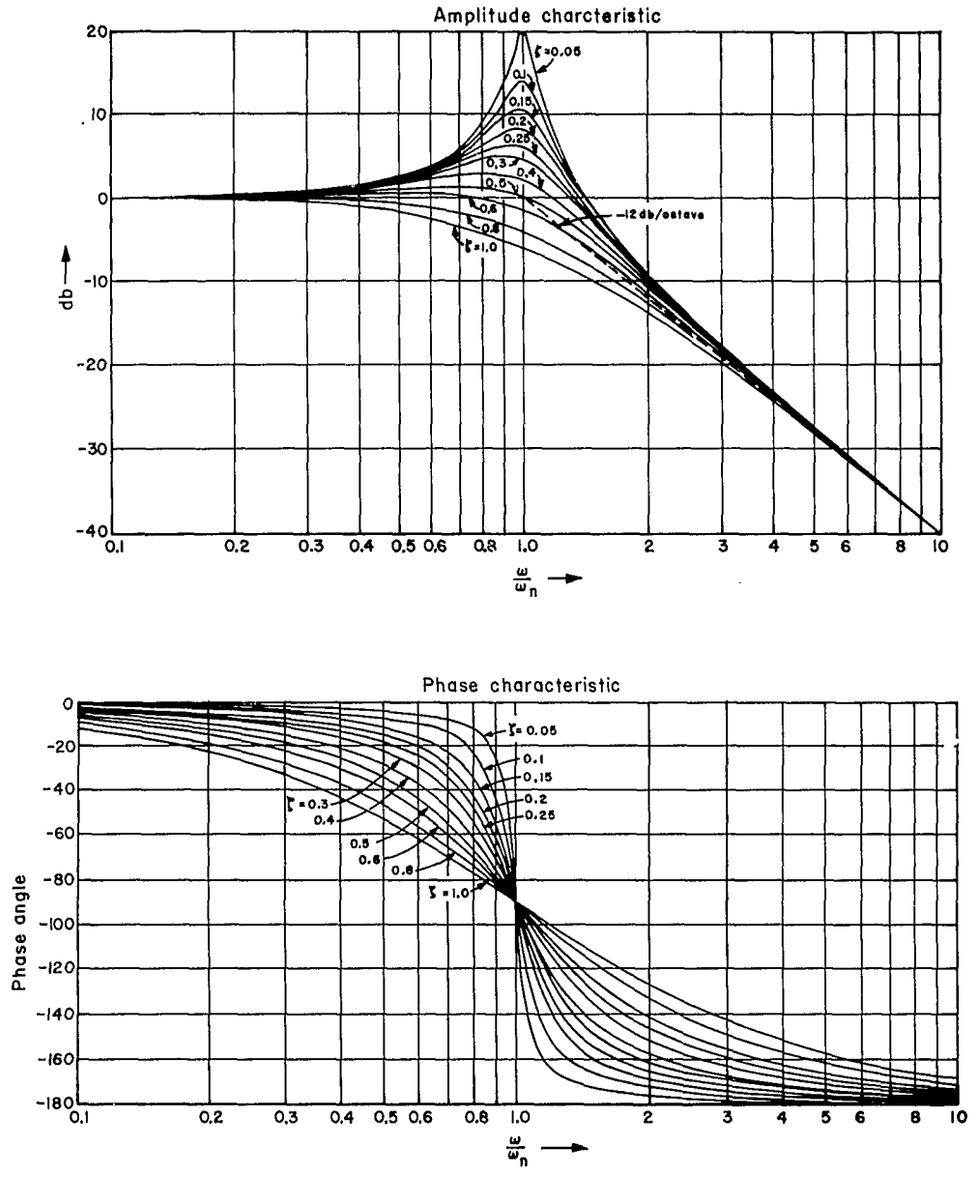

Figure 2. Normalized Frequency Response of a Second Order System

$$F(S) = \frac{1}{\frac{S^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} S + 1} \quad \text{where } S = j\omega$$

from which

$$R_1 \leq \zeta^2 R_2 \tag{6}$$

A minimum spread in resistance values is obviously obtained when

$$R_1 = \zeta^2 R_2 \tag{7}$$

For this condition,

$$C_1 = C_2 = \frac{\zeta}{R_1 \omega_n} \tag{8}$$

with no spread in capacitance values.

Equation (8) indicates that for a given ζ and ω_n small valued capacitors may be used if R_1 is made large. Large R_1, however, is not compatible with the requirement for minimum attenuation when the filter is loaded, since the requirement on R_1 relative to R_L is

$$R_1 \ll R_L \tag{9}$$

Thus, a compromise must be made between capacitor size and the amount of allowable signal attenuation.

DESIGN EXAMPLE

The following specifications will serve to illustrate the design of a practical low-pass filter:

1) Frequencies of interest = direct current to 100 cycles per second.
2) Resonant rise $\leq$ 10 decibels.
3) Phase shift at 100 cycles per second < 60 degrees.
4) Passband attenuation < −3 decibels.
5) Four hundred cycles per second attenuation $\geq$ −20 decibels.
6) R_L = 15,000 ohms.

Figure 2 indicates specifications 1, 2, 3, and 5 can be approximated by choosing $\zeta = 0.2$ and $\omega_n = 2\pi(120)$ radians per second.

Under these conditions

$$C_1 = \frac{265 \times 10^{-6}}{R_1} \tag{10}$$

Since R_L = 15,000 ohms, R_1 = 1,000 ohms is chosen, thus arriving at a compromise between component size and signal attenuation, A, such that C = 0.265 microfarad and

$$A \approx \frac{R_L}{R_1 + R_L} = \frac{15,000}{16,000} = 0.94 = -0.54 \text{ decibel} \tag{11}$$

over the flat portion of the amplitude characteristic. From Equation (7), R_2 = 25,000 ohms, and the design is complete.

It is noted that more attenuation is allowable (specification 4), and therefore the value of C_1 and C_2 may be reduced if desired. Finally, it is noted that the phase shift at 100 cycles per second can be decreased by increasing the filter resonant frequency, but at the expense of decreased attenuation at 400 cycles per second.

CONCLUSIONS

The foregoing design was built and tested in this laboratory, using an operational amplifier and components specified in the Appendix. The frequency response characteristic was not significantly different from the ζ = 0.2 curves of Figure 2 and is therefore not repeated here.

The filter performed as well or better than the conventional passive type presently used for the same purpose. The conventional filter is a higher order system of considerable complexity containing both capacitors and large inductors. Size and weight of the active filter are significantly reduced through the use of the microcircuits and through the the absence of the large inductors. Reliability is enhanced by a minimum number of interconnections among the external discrete components and the operational amplifier. Little difficulty should be encountered in making the external circuitry an integral part of the operational amplifier in a finalized version of the unit.

Appendix

OPERATIONAL AMPLIFIER AND COMPONENT CHARACTERISTICS

1. Operational Amplifier Characteristics

Size — 0.675 by 0.375 by 0.1 inch flatpack.

Supply voltage — +12 volts, −12 volts.

Voltage gain — 73 decibels.

Input impedance — 200,000 ohms.

Output impedance — 200 ohms.

2. Capacitor Characteristics

Capacitors used were standard 0.27-microfarad units, ±20 percent tolerance. Subminiature versions suitable for this application are available.

3. Resistor Characteristics

Resistors used were standard $\frac{1}{10}$-watt units, ±10 percent tolerance. Subminiature resistors suitable for this application are available.

DISTRIBUTION

	No. of Copies
U.S. Army Missile Command Distribution List A	84
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
AMSMI-R, Mr. McDaniel	1
-RF	1
-RR	1
-RD	1
-RE	1
-RG, Mr. Rowe	1
-RGL, Mr. Grunwald	1
-RGL, Mr. White	10
-RBL	5
-RAP	1

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
Army Inertial Guidance and Control Laboratory
Directorate of Research and Development
U.S. Army Missile Command
Redstone Arsenal, Alabama

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE

AN ACTIVE LOW-PASS FILTER

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(Last name, first name, initial)*

White, H. V.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
1 May 1965	8	

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. (DA) 1X279191D678	RG-TR-65-7
c. AMC Management Structure Code No. 5282.12.127	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	AD _____

10. AVAILABILITY/LIMITATION NOTICES

Qualified requesters may obtain copies of this report from DDC.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Same as No. 1

13. ABSTRACT

The analysis and design of an active low-pass filter for use in inertial platform systems (though not restricted to this application) are presented in this report. The filter features minimum size and weight and increased reliability through the use of a single microcircuit, yet it maintains performance comparable to conventional types presently used for the same purpose. Little difficulty should be encountered in making the external circuitry an integral part of the operational amplifier in a finalized version of the unit.

DD FORM 1 JAN 64 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Low-pass filter Amplifier Attenuation						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through _____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through _____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through _____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U).*

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.