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CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91109

HYDRODYNAMICS LABORATORY

KARMAN LABORATORY OF
FLUID MECHANICS AND JET PROPULSION

May 26, 1965

Recipients of California
Institute of Technology
Hydrodynamics Laboratory
Report No. 111.3

SUBJECT Errata for Report No. 111.3

Gentlemen:

It is requested that the following changes be made in your copy of California Institute of Technology Hydrodynamics Laboratory Report No. 111.3, entitled "The Wall Effect In Cavity Flow," by D. K. Ai and Z. L. Harrison, dated April 1965:

The information in the caption of Fig. 10 should appear with Fig. A-2.

The information in the caption of Fig. A-2 should appear with Fig. 10.

Very truly yours,

Mary E. Goodwin

Mary E. Goodwin

Office of Naval Research
Department of the Navy
Contract Nonr 220(41)

THE WALL EFFECT IN CAVITY FLOW

by
D.K. Ai
and
Z. L. Harrrison

Hydrodynamics Laboratory
Kármán Laboratory of Fluid Mechanics and Jet Propulsion
California Institute of Technology
Pasadena, California

Report No. 111.3 APRIL 1965

Office of Naval Research
Department of the Navy
Contract Nonr-220(41)

THE WALL EFFECT IN CAVITY FLOW

by

Daniel K. Ai

and

Z. L. Harrison

Hydrodynamics Laboratory
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California Institute of Technology
Pasadena, California

Report No. 111.3

Approved by: T. Y. Wu

April 1965

ABSTRACT

A non-linear theory for the calculation of the flow field of an oblique flat plate under blockage condition is given using the techniques of integral equations. Numerical results are obtained with the aid of a high speed digital computer for the plate situated mid-channel at values of the angle of attack from 5° to 90° and the channel width-chord ratio from 3 to 20. Also obtained are results for the plate situated at two different off-center positions for a channel width-chord ratio 5 and angles of attack less than 30°.

NOMENCLATURE

A	=	leading edge of the plate; origin of the coordinate system.
B	=	trailing edge of the plate; $x = 1$.
C	=	scale factor.
C_D	=	drag coefficient.
C_L	=	lift coefficient.
D	=	stagnation point on the plate.
F_D	=	drag force.
f	=	complex potential.
H_A	=	normal distance from A to the upper channel wall.
H_1, H_2	=	widths of the flow above and below the dividing streamline at upstream infinity.
h_1, h_2	=	widths of the flow above and below the cavity at downstream infinity.
I	=	flow at upstream infinity.
J	=	flow at downstream infinity between the upper cavity and channel wall.
K	=	flow at downstream infinity between the lower channel and cavity wall.
k_o, k_1, k_2	=	parameters in the transformation.
p_∞	=	free stream pressure.
p_c	=	cavity pressure.
U, V	=	uniform up- and downstream velocities respectively.
W	=	channel width $= H_1 + H_2$.
w	=	complex velocity.
z	=	$x + iy$, the physical plane.
α	=	angle of attack.
σ	=	$(p_\infty - p_c)/\frac{1}{2}\rho U^2$, cavitation number.
ρ	=	density of the liquid.
θ	=	direction of the flow.

Introduction

The two-dimensional cavity theory of an unbounded fluid, with the recently published works(1), (2) on the non-linear solution of bodies of general shape, can be considered a well established field. For an experimentalist who has to perform the tests, however, the existing theories cannot always be applied directly and have to be modified mainly because the flow one creates for experiment is often bounded by different types of boundaries. These boundaries can be free surfaces of constant pressure if the equipment in use is a jet, rigid walls if it is a water tunnel, or both if it is a free surface channel. The essential problem is to determine the effect of the boundaries. In the past, many papers have been published on this problem. A small portion of these are listed here as references(3), (4), (5), (6). We shall not repeat what they have done since these references are available and well known. This paper also deals with this problem, but the interest is focused only on the unsymmetric flow for arbitrary angle of attack and the boundaries considered are rigid walls. In a subsequent report, we shall treat the cases of the free jet and the free surface channel.

It is a well known fact that the cavity length behind a body depends essentially on σ, an important parameter known as the cavitation number and defined as

$$\sigma = (p_\infty - p_c)/\tfrac{1}{2}\rho U^2. \tag{1}$$

Here p_∞ is the free stream pressure, U the free stream velocity, and p_c is the cavity pressure presumably nearly constant. In general, the length would increase as σ decreases and with the presence of free surfaces, e.g. in a jet, σ would become zero when the length approaches infinity. However, this phenomenon is not observed in a water tunnel with rigid walls. In this case σ would reach a finite positive limit σ_c as the cavity length increases indefinitely hence cavitation numbers below σ_c are not attainable. Thus not all cavity flow conditions can be modeled in a water tunnel. The phenomenon corresponding to $\sigma = \sigma_c$ in a water tunnel is called blockage or choking and the determination of σ_c is therefore one of the central problems in water tunnel testing of cavity flows.

In this paper, the two-dimensional non-linear theory of a choked unsymmetrical flow over a flat plate at an arbitrary angle of attack is worked out using the techniques of integral equations. Numerical results are obtained on a high speed digital computer (IBM 7094).

Formulation of the problem and preliminary calculations.

Consider the idealized cavity flow in a water tunnel with rigid walls, depicted in Fig. 1. We assume a uniform upstream flow with velocity U, and a uniform downstream velocity V as the cavity, which has a stationary interface, approaches its maximum cross-section. The flat plate, set at an arbitrary angle of attack, can be located anywhere in the tunnel and the stream after impinging on the frontal side of the plate separates smoothly at both its leading and trailing edges. The plate is of length unity or one may say all dimensions are normalized by the chord. The coordinate axes are set normal and parallel to the plate with the origin at the leading edge.

If we call $\overline{w}$, the velocity vector, with magnitude $|w|$ and direction θ, $\overline{w} = |w| e^{i\theta}$, then in our coordinate system, the uniform velocities at up- and downstream infinities are $U e^{i\alpha}$ and $V e^{i\alpha}$ respectively.

The boundary conditions for the problem are:

$$\begin{aligned} \theta &= \alpha, && \text{on the channel walls} \\ \theta &= 0, && \text{on the plate from D to B} \\ \theta &= \pi, && \text{on the plate from D to A} \\ |w| &= V, && \text{on the cavity walls.} \end{aligned}$$

The force coefficients at the choking condition can be obtained from momentum consideration in terms of the channel width, the angle of attack and the critical cavitation number;

$$\rho V^2 (h_1 + h_2) - \rho U^2 W = (p_\infty - p_c) W - F_D \tag{2}$$

where F_D is the drag, by conservation of volume, $V(h_1 + h_2) = WU$ and by definition $\sigma^* = (p_\infty - p_c)/\frac{1}{2}\rho U^2$. Substituting

$$F_D = (p_\infty - p_c) W + \rho U^2 W - \rho VUW \tag{3}$$

---

* The subscript is dropped since the cavitation number we refer to from now on is always the choked one.

and the drag coefficient is

$$C_D = \frac{2F_D}{\rho U^2 A^+} = 2W\left[(1 + \frac{\sigma}{2}) - \sqrt{1 + \sigma}\right]. \tag{4}$$

A^+, the area per unit span, is equal to one for a plate of unit length. The lift coefficient is

$$C_L = C_D \cot \alpha . \tag{5}$$

Theory

Assuming the flow to be irrotational, there exists a complex potential $f = \varphi + i\psi$. The flow field in the f-plane is shown in Fig. 2. The two channel walls are represented by the streamlines ψ_1 and $-\psi_2$ while the dividing streamline coincides with the negative φ-axis. The stagnation point D is chosen as the origin and the cut along the positive φ-axis represents the two branches of the streamline split at D. The leading and trailing edges of the plate are located somewhere on the upper and lower branches respectively. With reference to Fig. 1, the values of ψ_1 and ψ_2 are given as

$$\psi_1 = UH_1 = Vh_1$$
$$\psi_2 = UH_2 = Vh_2 . \tag{6}$$

We proceed to solve the problem by introducing a new variable Ω, defined by

$$\Omega = \log \frac{V}{w} = \tau + i\theta$$
$$\tau = \log \frac{V}{|w|}, \quad \theta = -\arg w. \tag{7}$$

The choice of using Ω as the dependent variable for the problem is suggested by the existing boundary conditions.

The flow fields in the f and the Ω-planes are to be connected through a parametric variable $\zeta = \xi + i\eta$. Consider the transformation

$$\frac{df}{d\zeta} = \frac{C}{(\zeta - k_o)(\zeta - k_1)(\zeta - k_2)} \tag{8a}$$

or

$$f(\zeta) = \frac{1}{\pi}\left[(h_1 + h_2)\log(\zeta - k_o) - h_1 \log(\zeta - k_1) - h_2 \log(\zeta - k_2)\right]. \quad (8b)$$

The flow in the f-plane is mapped into the upper half ζ-plane with the boundaries on the real ζ or ξ-axis as shown in Fig. 3. The upstream I behaves like a source at $\xi = k_o$ while the downstreams J and K behave like sinks at $\xi = k_1$ and k_2 respectively. The net strength of these singularities is certainly zero. The jump conditions on the f-plane further furnish the relations;

$$h_1 = \frac{-\pi C}{V} \frac{1}{(k_2 - k_1)(k_o - k_1)} \quad (9)$$

and

$$h_2 = \frac{-\pi C}{V} \frac{1}{(k_2 - k_1)(k_2 - k_o)} \quad . \quad (10)$$

Combining Eqs. (9) and (10) yields the ratio

$$\frac{h_1}{h_2} = \frac{k_2 - k_o}{k_o - k_1} \quad . \quad (11)$$

The constant C is a scale factor to be determined later in the theory.

Ω has the following values along the ξ-axis and $\eta = 0_+$.

$$\begin{aligned} &\mathrm{Im}\,\Omega = \pi, && \xi < -1 \\ &\mathrm{Im}\,\Omega = 0, && \xi > 1 \\ &\mathrm{Re}\,\Omega = 0, && -1 < \xi < k_1,\ k_2 < \xi < 1 \\ &\mathrm{Im}\,\Omega = \alpha, && k_1 < \xi < k_2 . \end{aligned} \quad (12)$$

We further define

$$\Omega = \Omega_o + \Omega_1 , \quad (13)$$

where Ω_o is associated with the unbounded fluid. Ω is split up in this way to make the solution readily obtainable. The term Ω_1, which represents the effect of the walls, is added to the unbounded fluid transcendentally to maintain the total exact solution. When the channel width-chord ratio is very large, Ω_1 then can be considered merely as a perturbation of the unbounded flow.

With reference to Figures 4a and 4b, Ω_o has the form

$$\Omega_o = \cosh^{-1}\zeta = \log(\zeta + \sqrt{\zeta^2 - 1}) . \tag{14}$$

Along the ξ-axis,

$$\begin{aligned} \operatorname{Im}\Omega_o &= \pi, & \xi &< -1 \\ \operatorname{Im}\Omega_o &= 0, & \xi &> 1 \\ \operatorname{Re}\Omega_o &= 0, & -1 < \xi &< 1. \end{aligned} \tag{15}$$

For $|\xi| < 1$, Ω_o is given by

$$\operatorname{Im}\Omega_o = \beta(\xi) = \arctan(\sqrt{1 - \xi^2}/\xi). \tag{16}$$

The remaining part, Ω_1 which is caused by the presence of the channel walls, then takes the boundary values;

$$\begin{aligned} \operatorname{Im}\Omega_1 &= 0, & \xi &< -1 \\ \operatorname{Im}\Omega_1 &= 0, & \xi &> 1 \\ \operatorname{Re}\Omega_1 &= 0, & -1 < \xi &< k_1, \quad k_2 < \xi < 1 \\ \operatorname{Im}\Omega_1 &= \alpha - \beta(\xi), & k_1 < \xi &< k_2 . \end{aligned} \tag{17}$$

The boundary condition for Ω_1 on the ξ-axis is shown in Fig. 5. Three branch cuts appear on the ξ-axis; these are from $\xi = -\infty$ to $\xi = -1$, from $\xi = 1$ to $\xi = +\infty$ and from $\xi = k_1$ to $\xi = k_2$. The first two cuts represent the plate and the last one the channel walls in the z-plane.

We now have a boundary value problem involving the unknown function, $\Omega_1(\zeta)$, which can be formulated and solved as a Hilbert problem. By analytic continuation

$$\Omega_1(\bar{\zeta}) = -\bar{\Omega}_1(\zeta). \tag{18}$$

The region is extended to the lower half ζ-plane with the $\operatorname{Re}\Omega_1$ odd and the $\operatorname{Im}\Omega_1$ even with respect to the ξ-axis. We now introduce the auxiliary function

$$H(\zeta) = \frac{1}{\sqrt{(\zeta - k_1)(\zeta - k_2)(\zeta^2 - 1)}} , \tag{19}$$

which has the following properties;

i) $H(\zeta)$ has the proper branch cuts ($\zeta = -1$, k_1, k_2, and $+1$ are branch points).

ii) On the Re ζ axis:

$$
\begin{array}{lll}
\operatorname{Im} H = 0, & & -\infty < \xi < -1 \\
\operatorname{Re} H = 0, & & -1 < \xi < k_1 \\
\operatorname{Re} H = -\dfrac{1}{\sqrt{(\xi - k_1)(k_2 - \xi)(1-\xi^2)}}, & \operatorname{Im} H = 0, & k_1 < \xi < k_2 \\
\operatorname{Re} H = 0, & & k_2 < \xi < 1 \\
\operatorname{Im} H = 0, & & 1 < \xi < \infty
\end{array}
$$

A new function

$$G(\zeta) = \Omega_1(\zeta)\, H(\zeta)$$

is then formed which has on the Re ζ axis;

$$
\begin{array}{ll}
\operatorname{Im} G = 0, & -\infty < \xi < k_1 \\
\operatorname{Im} G = -\dfrac{\alpha - \beta(\xi)}{\sqrt{(\xi - k_1)(k_2 - \xi)(1-\xi^2)}}, & k_1 < \xi_1 < k_2 \\
\operatorname{Im} G = 0, & k_2 < \xi < +\infty
\end{array}
$$

With the aid of Plemelj's formula[7], the solution of the Hilbert problem is given immediately by

$$\Omega_1(\zeta) = -\frac{1}{\pi}\sqrt{(\zeta - k_1)(\zeta - k_2)(\zeta^2 - 1)} \int_{k_1}^{k_2} \frac{[\alpha - \beta(t)]\, dt}{(t - \zeta)\sqrt{(t-k_1)(k_2 - t)(1 - t^2)}} \tag{20}$$

However, the constants k_1 and k_2 cannot be arbitrarily chosen for a fixed α because of a physical condition that must be imposed on the solution. Specifically, the solution Ω must behave locally like a stagnation flow at the stagnation point. In the ζ-plane, the stagnation point corresponds to $\zeta = \infty$. The local stagnation flow can be shown to behave like $\log \zeta$ as ζ approaches infinity; this behavior is already incorporated into the function Ω_0. We require, therefore that Ω_1 be finite at infinity. If Eq. (20) is expanded in inverse powers of ζ, this condition is seen to be given by the requirement that

$$\int_{k_1}^{k_2} \frac{\alpha - \beta(t)}{\sqrt{(t-k_1)(k_2-t)(1-t^2)}}\, dt = 0 \,. \tag{21}$$

Thus for a given value of α, Eq. (21) determines the relation between k_1 and k_2.

We are now able to determine σ from Ω_1 in Eq. (20). At upstream infinity in the physical plane, $|w| = U$ and $\zeta = k_o$.

$$\Omega_1 = \tau_1 = \log \frac{V}{U} = \frac{1}{\pi}\sqrt{(k_2 - k_o)(k_o - k_1)(1-k_o^2)} \int_{k_1}^{k_2} \frac{[\alpha - \beta(t)]\ dt}{(t-k_o)\sqrt{(t-k_1)(k_2-t)(1-t^2)}} \,. \tag{22}$$

From our previous discussion, $\sigma = V^2/U^2 - 1$, therefore Eq. (22) enables us to calculate σ when the integral on the right hand side is evaluated.

In order to complete the solution, we shall relate the geometry of the flow to the undetermined constants. Utilizing the definition of the complex velocity, we have the relation

$$dz = \frac{1}{w}\, df = \frac{1}{w}\, \frac{df}{d\zeta}\, d\zeta = \frac{1}{V}\, e^{\Omega(\zeta)}\, \frac{df}{d\zeta}\, d\zeta \,. \tag{23}$$

With the aid of Eqs. (8a), (13) and (14) and upon integration,

$$\int dz = \frac{C}{V} \int \frac{\zeta + \sqrt{\zeta^2 - 1}}{(\zeta - k_o)(\zeta - k_1)(\zeta - k_2)}\, e^{\Omega_1(\zeta)}\, d\zeta \,. \tag{24}$$

When we apply Eq. (24) along the plate from A to B, the normalized chord is obtained;

$$z_B - z_A = 1 = \frac{C}{V}\left[\int_{\infty}^{1} \frac{\xi + \sqrt{\xi^2 - 1}}{(\xi - k_o)(\xi - k_1)(\xi - k_2)}\, e^{\Omega_1(\xi)}\, d\xi - \int_{-\infty}^{-1} \frac{\xi - \sqrt{\xi^2 - 1}}{(\xi - k_o)(\xi - k_1)(\xi - k_2)}\, e^{\Omega_1(\xi)}\, d\xi\right]. \tag{25}$$

The first integral on the right hand side represents the distance from the stagnation point to the trailing edge, we therefore find the location of the stagnation point.

H_A, the height of A below the upper channel wall can be determined in a similar way. With reference again to Fig. 1,

$$H_A = h_1 + \mathrm{Im}\left[e^{-i\alpha}(z_J - z_A)\right]$$

$$= h_1 + \frac{C}{V}\,\mathrm{Im}\left[e^{-i\alpha}\int_{-1}^{k_1} \frac{e^{i\beta(\xi)}\,e^{\Omega_1(\xi)}}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\,d\xi\right]$$

or

$$= h_1 + \frac{C}{V}\int_{-1}^{k_1} \frac{\sin(\theta_1 + \beta - \alpha)}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\,d\xi\,, \qquad (26)$$

since Ω_1 is purely imaginary and equal to $i\theta_1$ in the interval $-1 \le \xi \le k_1$.

The channel width W is expressed through continuity as

$$W = (h_1 + h_2)\,\frac{V}{U} = \frac{(h_1 + h_2)}{\sqrt{1+\sigma}}\;. \qquad (27)$$

<u>Final Results.</u>

We have applied the theory to calculate the case of a flat plate positioned mid-channel for various values of α and W. The range in α is from 5° to 90° and in W from 3 chords to 20 chords. We also obtained results for two off-center positions of the plate, $H_A = 1.0$ and 4.0 for $W = 5$ chords and values of α up to 30°.

The lift coefficient, drag coefficient and choking cavitation number are plotted versus α for $W = 5$ chords and the plate in mid-channel in Fig. 6. The dash-dot curve in Fig. 6 is the lift coefficient given by Wu's unbounded fluid theory[1].

In Fig. 7, we show the enlarged portion of Fig. 6 for $\alpha \le 30^\circ$ with experimental values from Wade[8] corresponding to the choking σ determined by the present theory. Wade's experiment was performed with $W \approx 5$ and the experimental cavitation numbers were obtained from measured cavity pressure. Since it is difficult to operate the tunnel near choking condition, Wade was not able to obtain data at such low cavitation numbers: Therefore, we show values extrapolated from his data curves. One notices that all points are enveloped by the two theoretical curves.

Cohen, Sutherland and Tu[5] have worked out a linearized theory for a flat plate at small angles of attack and calculated σ for mid-channel position. However, their definition of mid-channel is different from ours as we define mid-channel when the center of the plate lies on the centerline while they locate the leading edge of the plate on the centerline. As a result, it is not possible to make a direct comparison between the two theories. For very small angles of attack, however, the discrepancy caused by the difference in definition becomes small and for $\alpha = 6^\circ$, $W = 5$, the present theory gives $\sigma = 0.133$, $C_L = 0.198$ for $H_A/(W-H_A) = 0.959$ and their theory gives $\sigma \approx 0.13$ and $C_L \approx 0.22$ for $H_A/(W-H_A) = 1$. The values of their calculations are read from Fig. 10 of Ref. 3, hence are only approximate.

Figure 8 shows the effect of plate position for $W = 5$ chords. In this figure C_L, C_D and σ are plotted versus α for $\alpha \leq 30^\circ$ and two values of H_A. The dashed curves are $H_A = 1.0$ and the solid ones are $H_A = 4.0$. As would be expected C_L, C_D and σ all increase as the plate is lowered in the channel.

We show C_L and σ versus α for $W = 8$ with experimental values from Parkin[9] in Fig. 9. Again the experimental cavitation numbers were obtained from measured cavity pressure, however, in this case only the data for $\alpha = 8^\circ$ and 10° were obtained from extrapolated curves.

Figures 10a and b show drag coefficient, C_D, and choking cavitation number, σ, versus channel width, W, for a flat plate located mid-channel. The range in angle of attack is 5° to 90° and in W from 3 to 20 chords. C_L may easily be obtained from Fig. 10a and Eq. (5) and therefore is not presented here.

In order to show the effect of the channel walls, we have plotted RC_D, the ratio of C_D bounded to C_D unbounded [1], ($RC_N = RC_L = RC_D$) versus the channel width W in Fig. 11. The curves are for the same values of α and W as Figs. 10a and b except that for $\alpha = 90^\circ$ RC_D is shown for W as low as 1.224. As would be expected when $W = 20$ there is very little wall effect and as the channel becomes smaller the ratio drops until the walls approach the plate where the ratio increases. This effect is shown for $\beta = 90^\circ$ where the ratio tends to infinity at $W = 1$.

The curve of RC_D for each α will have a similar asymptote at $W = \sin\alpha$.

In Fig. 12, we have shown the distance of the stagnation point from the leading edge as a function of α. At $\alpha = 15^{o}$, it is already difficult to tell the difference between the two points and at $\alpha = \frac{\pi}{2}$ the stagnation point is at mid-plate as one would expect.

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APPENDIX

Procedures for numerical computations

In a direct problem one would specify the channel width, W, and the location of the plate (of chord unity), H_A, for a given angle of attack, then calculate σ, h_1, h_2 and the force coefficients. However, in view of the form of our solution it is necessary to do the inverse problem of choosing the transformation parameters k_2, k_1, and k_o (the downstream conditions, h_1 and h_2, when the flow is choked) and calculating W and H_A. The values of h_1 and h_2 are related to the k's through the transformations in Eqs. (8a), (8b) and the jump conditions in Eqs. (9) and (10). The choice of k_2 defines the geometry so that the scaling factor, C, or more precisely, the ratio V/C is determined from Eq. (25).

Given a value of k_2, k_1 is easily determined by iteration of the integral condition of Eq. (21). The integral in this equation cannot be numerically integrated in its present form so we divide it into two integrals

$$\int_{k_1}^{k_2} \frac{a-\beta(t)}{\sqrt{(t-k_1)(k_2-t)(1-t^2)}}\,dt = \int_{k_1}^{\frac{k_1+k_2}{2}} \frac{a-\beta(t)}{\sqrt{(t-k_1)(k_2-t)(1-t^2)}}\,dt + \int_{\frac{k_1+k_2}{2}}^{k_2} \frac{a-\beta(t)}{\sqrt{(t-k_1)(k_2-t)(1-t^2)}}\,dt; \tag{A-1}$$

make the transformations

$$p = x^2 + k_1 \qquad \text{in the first integral,}$$

$$s = k_2 - x^2 \qquad \text{in the second integral,}$$

and

$$x_o^2 = \tfrac{1}{2}(k_2 - k_1)$$

and Eq. (21) becomes

$$\int_0^{x_o} \frac{a-\beta(p)}{p^2\sqrt{2x_o^2-x^2}}\,dx + \int_0^{x_o} \frac{a-\beta(s)}{s^2\sqrt{2x_o^2-x^2}}\,dx = 0. \tag{A-2}$$

This integral iteration presents no problem since all calculations are preformed on a high speed digital computer (IBM 7094). We now can calculate the remaining transformation parameter, k_o, from the specified ratio h_2/h_1 and Eq. (11).

To aid in the trial and error solution of this problem the behavior of k_2 and k_1 for the plate in mid-channel are shown in Fig. A-1 and for the off center positions in Fig. A-2. The remaining parameter to be guessed is h_2/h_1 which may be approximated by $(W-H_A-\sin\alpha)/H_A$. For the mid-channel case this quantity is roughly unity.

The choking cavitation number, σ, the channel width, W, the drag coefficient, C_D, and the lift coefficient, C_L, are readily determined from Eqs. (22), (27), (4) and (5) respectively. The integral in Eq. (22) is treated in the same way as Eq. (21).

To determine H_A we must first evaluate the ratio V/C. From Eq. (25)

$$\frac{V}{C} = -\int_1^\infty \frac{\xi+\sqrt{\xi^2-1}}{(\xi-k_0)(\xi-k_1)(\xi-k_2)}\, e^{\Omega_1(\xi)} d\xi - \int_1^\infty \frac{\xi+\sqrt{\xi^2-1}}{(\xi+k_0)(\xi+k_1)(\xi+k_2)}\, e^{\Omega_1'(\xi)} d\xi, \qquad \text{(A-3)}$$

where $\Omega_1(\xi)$ is given in Eq. (20) and

$$\Omega_1'(\xi) = -\frac{1}{\pi}\sqrt{(\xi+k_1)(\xi+k_2)(\xi^2-1)} \int_{k_1}^{k_2} \frac{\alpha-\beta(t)}{(t+\xi)\sqrt{(t-k_1)(k_2-t)(1-t^2)}}\, dt. \qquad \text{(A-4)}$$

For numerical integration, it is more convenient to introduce the new integration variable $\tau = \xi^{-1}$. The first integral on the right hand side of Eq. (A-3) now may be written

$$\int_1^\infty \frac{\xi+\sqrt{\xi^2-1}}{(\xi-k_0)(\xi-k_1)(\xi-k_2)}\, e^{\Omega_1(\xi)} d\xi =$$

$$\int_0^{\tau_0} \frac{1+\sqrt{1-\tau^2}}{(1-k_0\tau)(1-k_1\tau)(1-k_2\tau)}\, e^{\Omega_1^\dagger} d\tau + \int_{\tau_0}^1 \frac{1+\sqrt{1-\tau^2}}{(1-k_0\tau)(1-k_1\tau)(1-k_2\tau)}\, e^{\Omega_1^*} d\tau, \qquad \text{(A-5)}$$

and the second integral

$$\int_1^\infty \frac{\xi + \sqrt{\xi^2 - 1}}{(\xi + k_0)(\xi + k_1)(\xi + k_2)}\, e^{\Omega_1'(\xi)}\, d\xi =$$

$$\int_0^{\tau_0} \frac{1 + \sqrt{1 - \tau^2}}{(1 + k_0\tau)(1 + k_1\tau)(1 + k_2\tau)}\, e^{\Omega_1'^{+}}\, d\tau + \int_{\tau_0}^1 \frac{1 + \sqrt{1 - \tau^2}}{(1 + k_0\tau)(1 + k_1\tau)(1 + k_2\tau)}\, e^{\Omega_1'^{*}}\, d\tau, \quad \text{(A-6)}$$

where

$$\Omega_1^{+} = \frac{1}{\pi}\sqrt{(1 - k_1\tau)(1 - k_2\tau)(1 - \tau^2)}\left[g_1 + \tau g_2 + \tau^2 g_3 + \ldots + \tau^6 g_7\right], \quad \text{(A-7)}$$

$$\Omega_1'^{+} = \frac{1}{\pi}\sqrt{(1 + k_1\tau)(1 + k_2\tau)(1 - \tau^2)}\left[g_1 - \tau g_2 + \tau^2 g_3 - \ldots + \tau^6 g_7\right], \quad \text{(A-8)}$$

$$\Omega_1^{*} = \frac{1}{\pi\tau}\sqrt{(1 - k_1\tau)(1 - k_2\tau)(1 - \tau^2)}\int_{k_1}^{k_2} \frac{a - \beta(t)}{(1 - t\tau)\sqrt{(t - k_1)(k_2 - t)(1 - t^2)}}\, dt, \quad \text{(A-9)}$$

$$\Omega_1'^{*} = -\frac{1}{\pi\tau}\sqrt{(1 + k_1\tau)(1 + k_2\tau)(1 - \tau^2)}\int_{k_1}^{k_2} \frac{a - \beta(t)}{(1 + t\tau)\sqrt{(t - k_1)(k_2 - t)(1 - t^2)}}\, dt, \quad \text{(A-10)}$$

and

$$g_n = \int_{k_1}^{k_2} \frac{t^n[a - \beta(t)]\, dt}{\sqrt{(t - k_1)(k_2 - t)(1 - t^2)}}. \quad \text{(A-11)}$$

Ω^{+} and Ω'^{+} are obtained from the power series expansion of τ and therefore are suitable for the calculation of small values of τ or large values of ξ. For a properly chosen τ_0, say $\tau_0 = 0.1$, a truncated series of seven terms is sufficient to give $\Omega_1^{+}(\tau_0)$ identical to $\Omega_1^{*}(\tau_0)$ to five significant figures.

The remaining quantity to be evaluated is the integral in Eq. (26)

$$\int_{\cdot_1}^{k_1} \frac{\sin(\theta_1 + \beta - \alpha)}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\, d\xi.$$

Some analysis is needed before a straightforward numerical integration can be performed. One notices that the denominator of the integrand has a factor $(\xi - k_1)$ which causes the integral to be logarithmically singular unless the numerator also vanishes at the upper limit. It is therefore necessary to examine the behavior of the numerator as ξ approaches k_1. Let us study the behavior of

$$\theta_1 = \frac{1}{\pi}\sqrt{(k_1 - \xi)(k_2 - \xi)(1 - \xi^2)} \int_{k_1}^{k_2} \frac{\alpha - \beta(t)}{(t - \xi)\sqrt{(t - k_1)(k_2 - t)(1 - t^2)}}\, dt \tag{A-12}$$

as ξ approaches k_1. We denote the regular part of the integrand in the neighborhood of $t = k_1$ as

$$F(t) = \frac{\alpha - \beta(t)}{\sqrt{(k_2 - t)(1 - t^2)}}\ . \tag{A-13}$$

$F(t)$ has the Taylor expansion

$$F(t) = F(k_1) + F'(k_1)(t - k_1) + \frac{F''(k_1)}{2}(t - k_1)^2 + \ldots$$

where the coefficients $F(k_1)$, $F'(k_1)$ etc. are finite numbers which can be evaluated without any difficulty. If we now subtract from $F(t)$ the first terms of its Taylor expansion then add them on later, the integral in θ_1 can be written

$$\int_{k_1}^{k_2} \frac{[\alpha - \beta(t)]\, dt}{(t-\xi)\sqrt{(t-k_1)(k_2-t)(1-t^2)}} =$$

$$\int_{k_1}^{k_2} \frac{dt}{(t-\xi)\sqrt{t-k_1}} \left[F(t) - F(k_1) - F'(k_1)(t-k_1) - \frac{F''(k_1)}{2}(t-k_1)^2\right]$$

$$+ F(k_1)\int_{k_1}^{k_2} \frac{(t-k_1)^{-\frac{1}{2}}}{t-\xi}\, dt + F'(k_1)\int_{k_1}^{k_2} \frac{(t-k_1)^{\frac{1}{2}}}{t-\xi}\, dt + \frac{F''(k_1)}{2}\int_{k_1}^{k_2} \frac{(t-k_1)^{\frac{3}{2}}}{t-\xi}\, dt. \tag{A-14}$$

The integrand of the first integral of Eq. (A-14) is finite and tends to zero as t approaches k_1 because the terms in the bracket are of the order $(t-k_1)^3$. The remaining integrals of Eq. (A-14) are integrated in closed form:

$$\int_{k_1}^{k_2} \frac{1}{t-\xi} \frac{dt}{\sqrt{t-k_1}} = \frac{2}{\sqrt{k_1-\xi}} \left[\frac{\pi}{2} - \tan^{-1} \sqrt{\frac{k_1-\xi}{k_2-k_1}} \right]$$

$$\int_{k_1}^{k_2} \frac{\sqrt{t-k_1}}{t-\xi}\, dt = 2\sqrt{k_2-k_1} - 2\sqrt{k_1-\xi} \left[\frac{\pi}{2} - \tan^{-1} \sqrt{\frac{k_1-\xi}{k_2-k_1}} \right]$$

$$\int_{k_1}^{k_2} \frac{(t-k_1)^{\frac{3}{2}}}{t-\xi}\, dt = \frac{2}{3}(k_2-k_1)^{\frac{3}{2}} - 2(k_2-k_1)\sqrt{k_1-\xi} + 2(k_1-\xi)^{\frac{3}{2}} \left[\frac{\pi}{2} - \tan^{-1} \sqrt{\frac{k_1-\xi}{k_2-k_1}} \right].$$

From Eqs. (A-13) and (A-14) it can be shown that in the neighborhood of $\xi = k_1$, θ_1 has the asymptotic expression

$$\theta_1 = \alpha - \beta + M\sqrt{k_1-\xi} + O(k_1-\xi)$$

where

$$M = \frac{\sqrt{(k_2-k_1)(1-k_1^2)}}{\pi} \int_{k_1}^{k_2} \frac{dt}{(t-k_1)^{\frac{3}{2}}} \left[F(t) - F(k_1) - F'(k_1)(t-k_1) - \frac{F''(k_1)}{2}(t-k_1)^2 \right]$$

$$- \frac{2}{\pi} \frac{\alpha - \beta(k_1)}{\sqrt{k_2-k_1}} + \frac{2F'(k_1)}{\pi} \sqrt{1-k_1^2}\,(k_2-k_1) + \frac{F''(k_1)}{3\pi} \sqrt{1-k_1^2}\,(k_2-k_1)^2 . \qquad \text{(A-15)}$$

In our final form for numerical computation, the range of integration of the integral in Eq. (26) is split into two intervals;

$$\int_{-1}^{k_1} \frac{\sin(\theta_1 + \beta - \alpha)}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\, d\xi = \int_{-1}^{k_1 - \epsilon} \frac{\sin(\theta_1 + \beta - \alpha)}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\, d\xi + \int_{k_1 - \epsilon}^{k_1} \frac{(\theta_1 + \beta - \alpha)\, d\xi}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}, \qquad \text{(A-16)}$$

where ϵ, small and positive, is chosen so that at $\xi = k_1 - \epsilon$, $\sin(\theta_1 + \beta - \alpha)$ can be safely approximated by $\theta_1 + \beta - \alpha$. If we replace θ_1, in the second integral of Eq. (A-16) by its asymptotic expression, Eq. (26) becomes

$$H_A = h_1 + \frac{C}{V}\left[\int_{-1}^{k_1 - \epsilon} \frac{\sin(\theta_1 + \beta - \alpha)}{(\xi - k_0)(\xi - k_1)(\xi - k_2)}\, d\xi - 2M \int_0^{\sqrt{\epsilon}} \frac{d\lambda}{(k_0 - \xi)(k_2 - \xi)} \right], \qquad \text{(A-17)}$$

where

$$\lambda^2 = k_1 - \xi$$

The integrals in Eq. (A-17) can be easily evaluated since their integrands are regular everywhere.

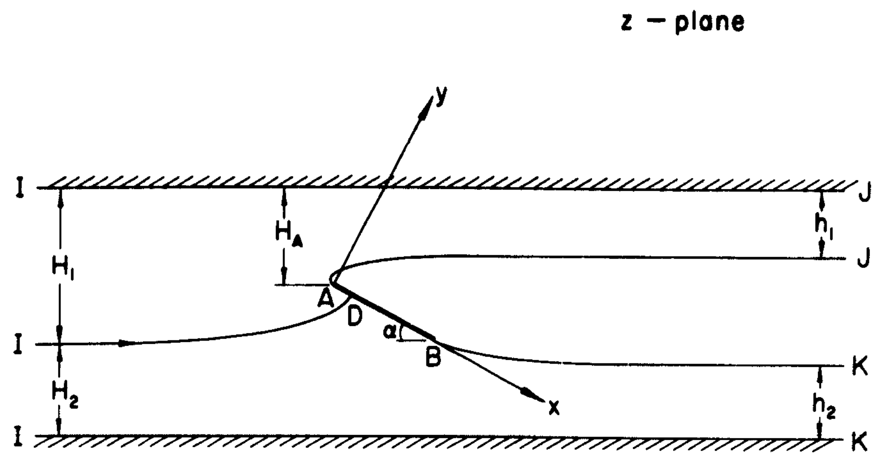

Fig. 1 Sketch showing a flat plate with an infinite cavity in a water tunnel.

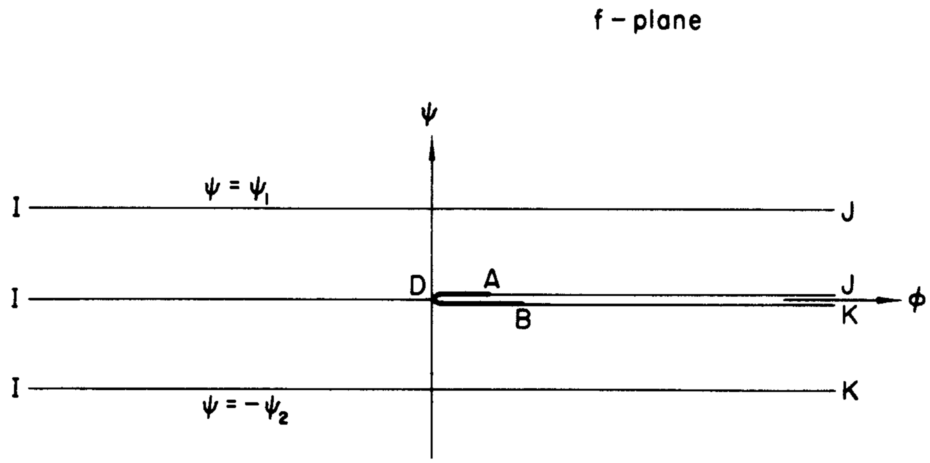

Fig. 2 The complex potential plane of the flow.

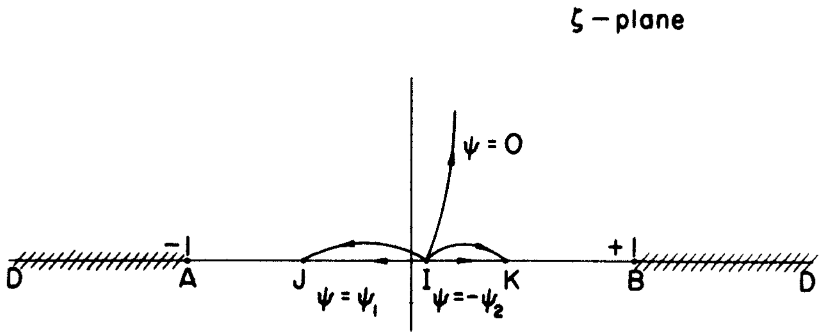

Fig. 3 The parametric ζ-plane.

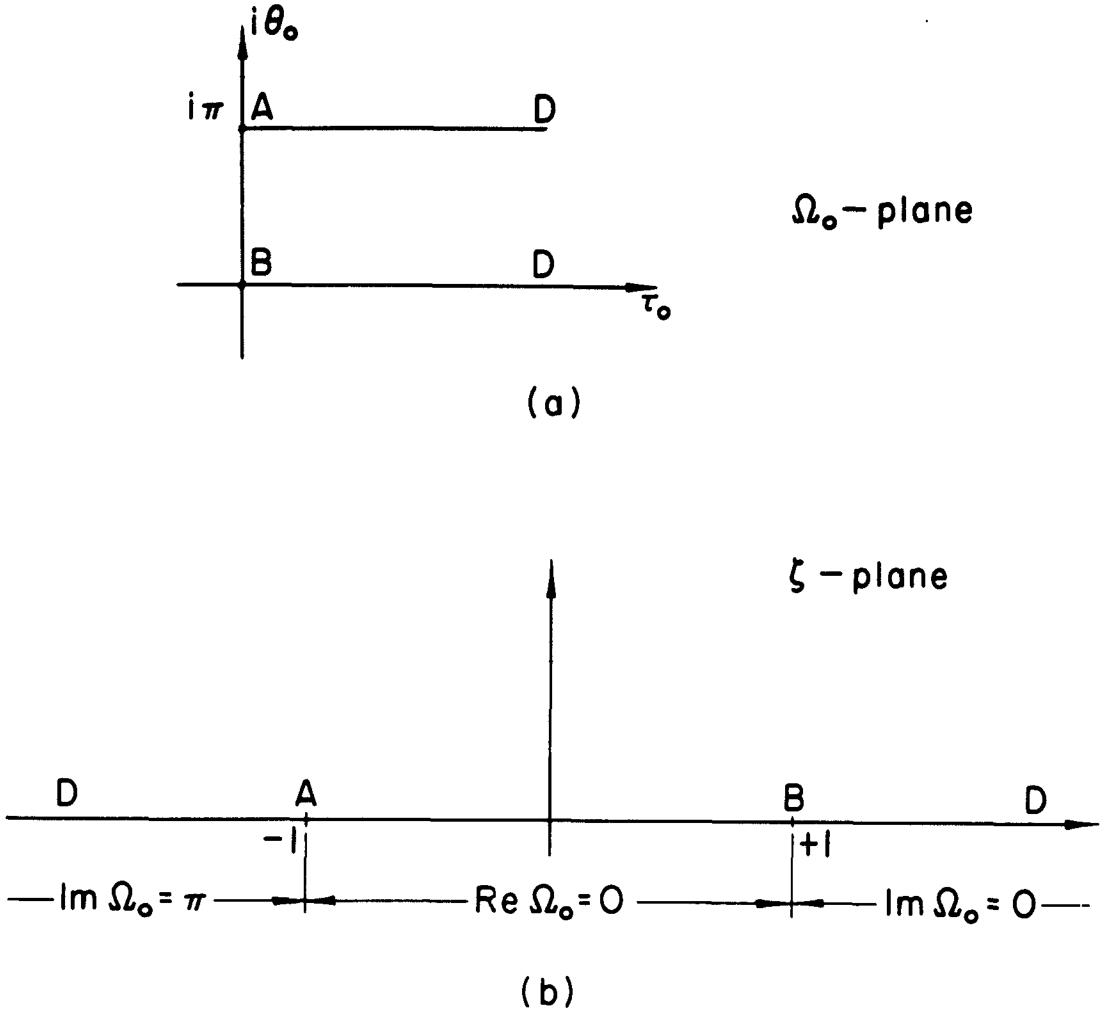

Fig. 4 The Ω_o- and ζ-planes. $\Omega_o = \cosh^{-1} \zeta = \log(\zeta+\sqrt{\zeta^2-1})$.

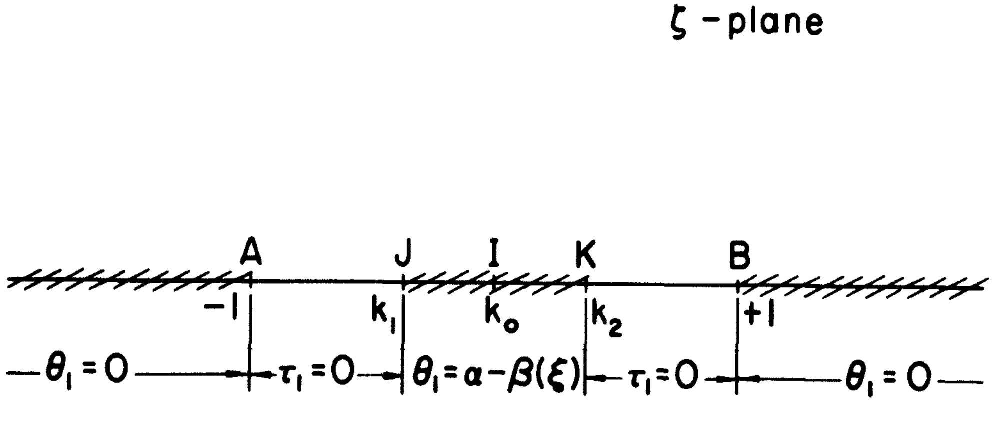

Fig. 5 Boundary values of Ω_1 on Re ζ-axis.

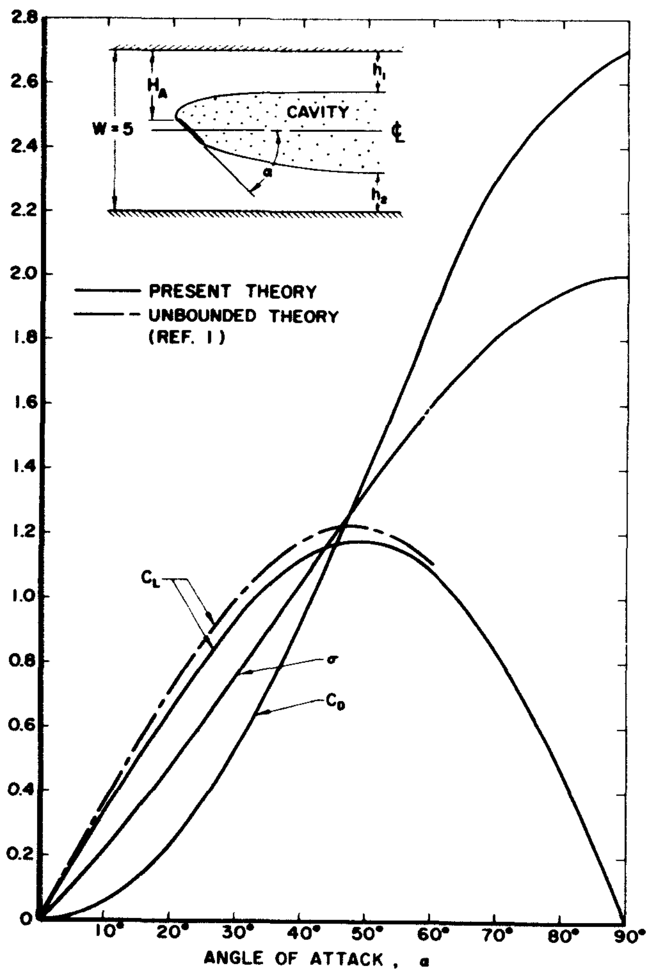

Fig. 6 The dependence of C_L, C_D and σ on the angle of attack, α, for a flat plate located midway in a channel of width W = 5.

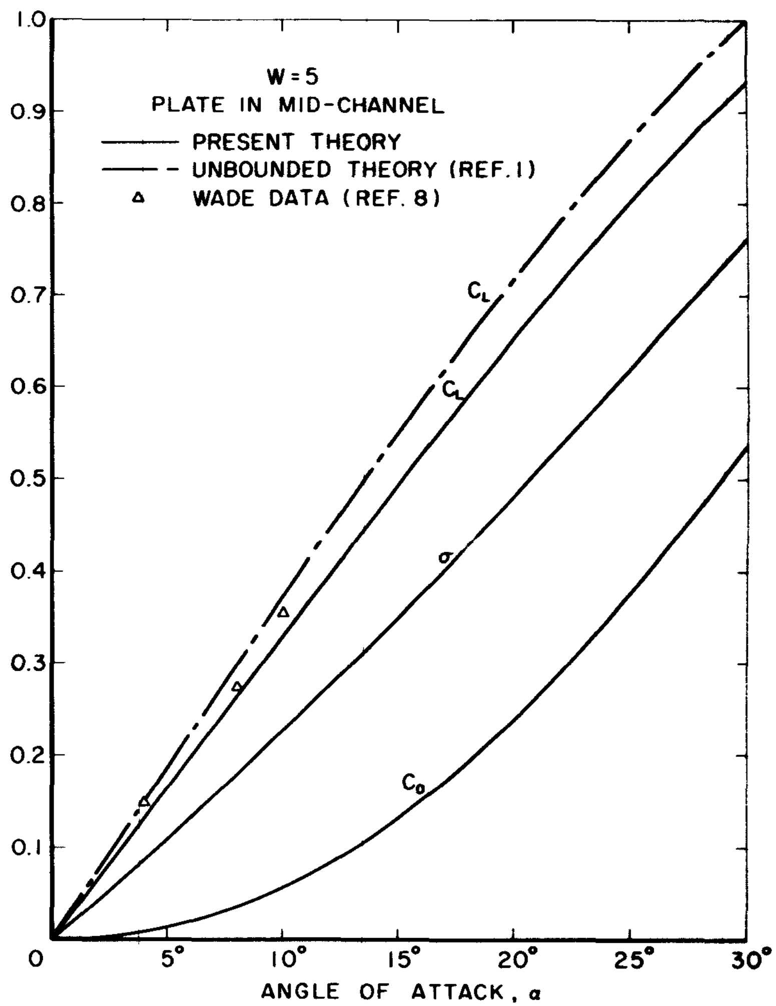

Fig. 7 Enlarged section of Fig 6 for small angles of attack with experimental data extrapolated from Wade's data curves.

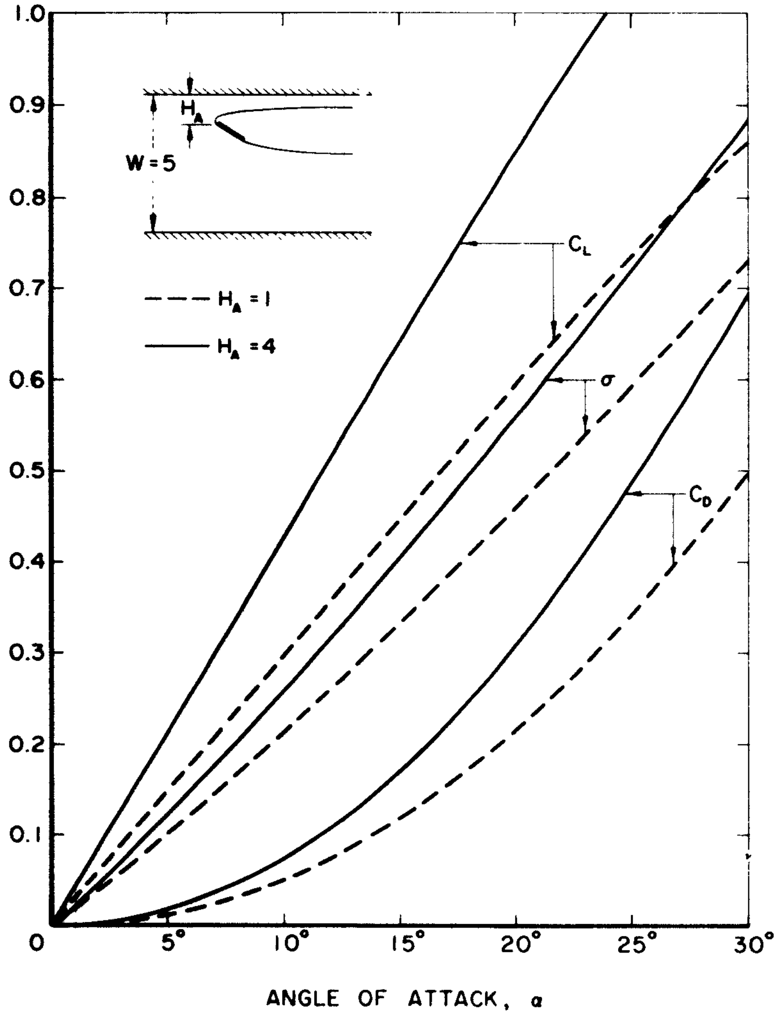

Fig. 8 The effect of angle of attack on C_L, C_D and σ for a flat plate located in two off-center positions in a channel of W = 5.

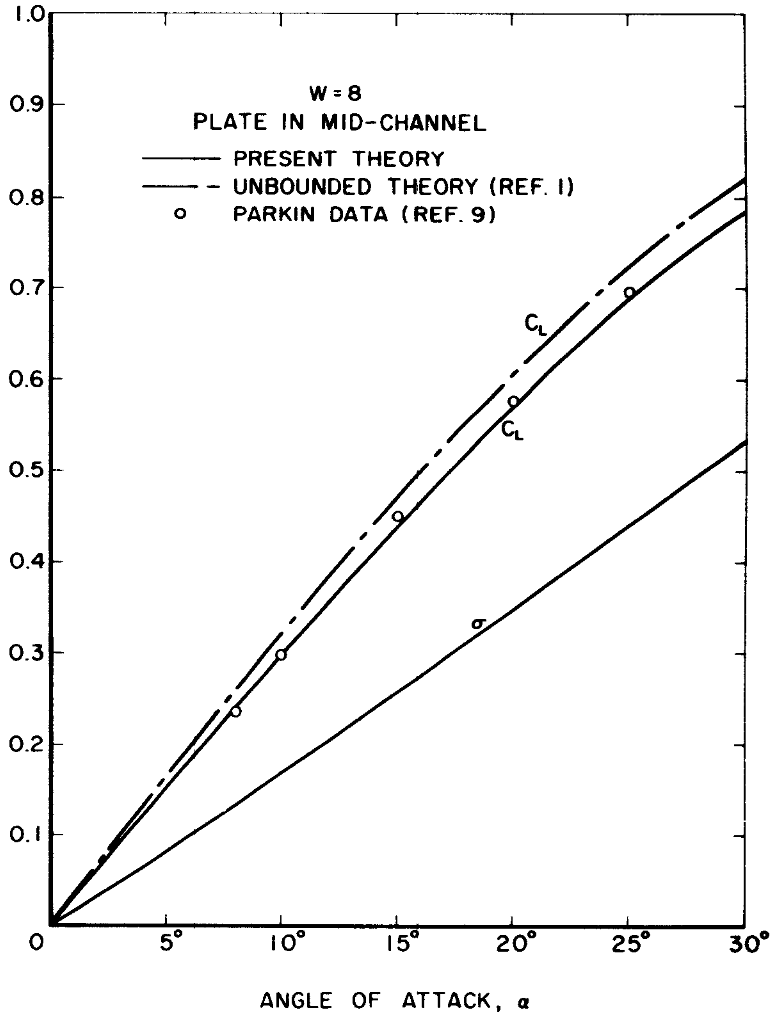

Fig. 9 Comparison of C_L from the present theory with the unbounded theory and experimental data from Parkin for a plate located mid-channel in a channel of W = 8.

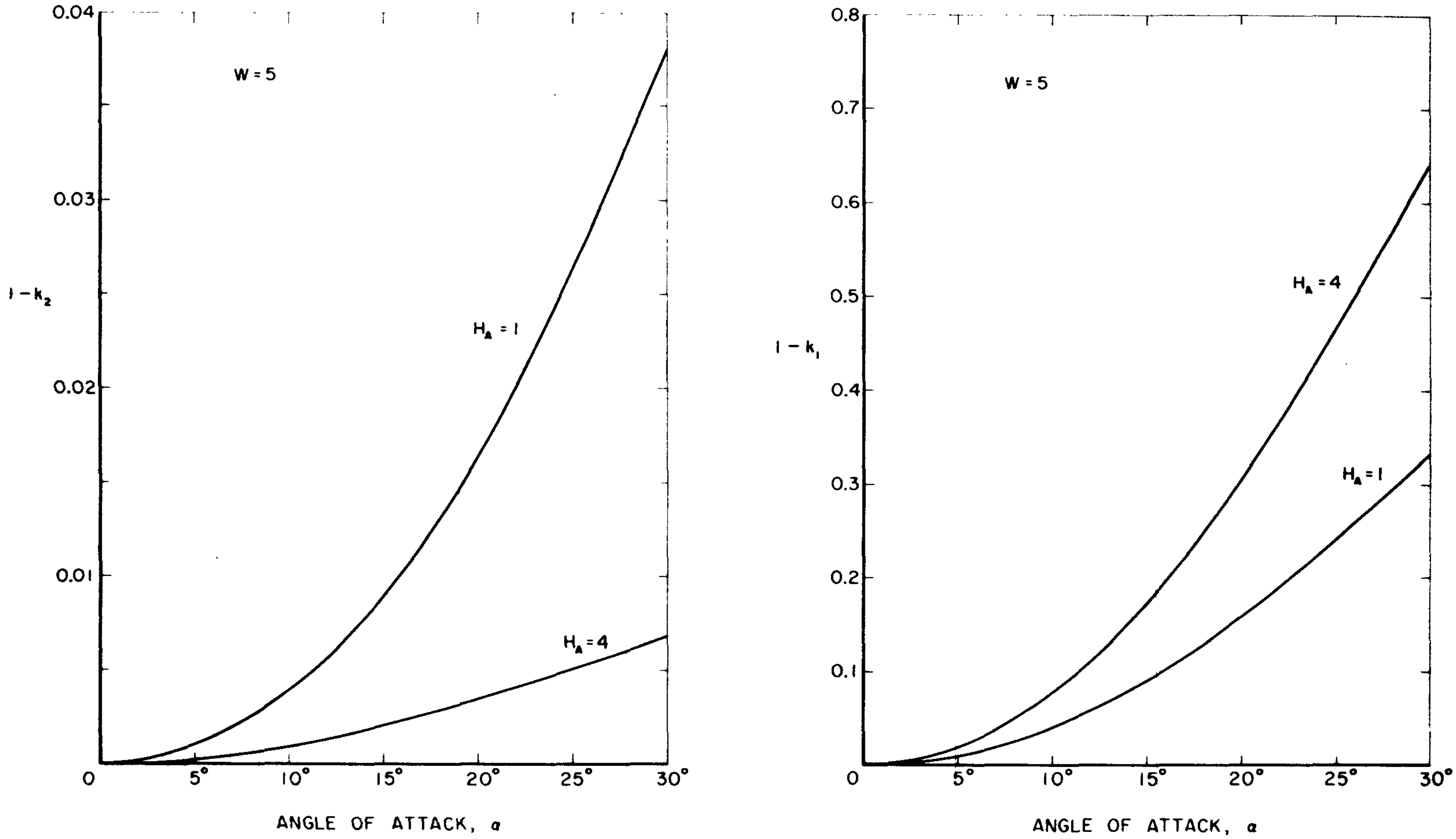

Fig. 10 Effect of channel width on (a) the drag coefficient and (b) the choking cavitation number for a flat plate located mid-channel at various angles of attack.

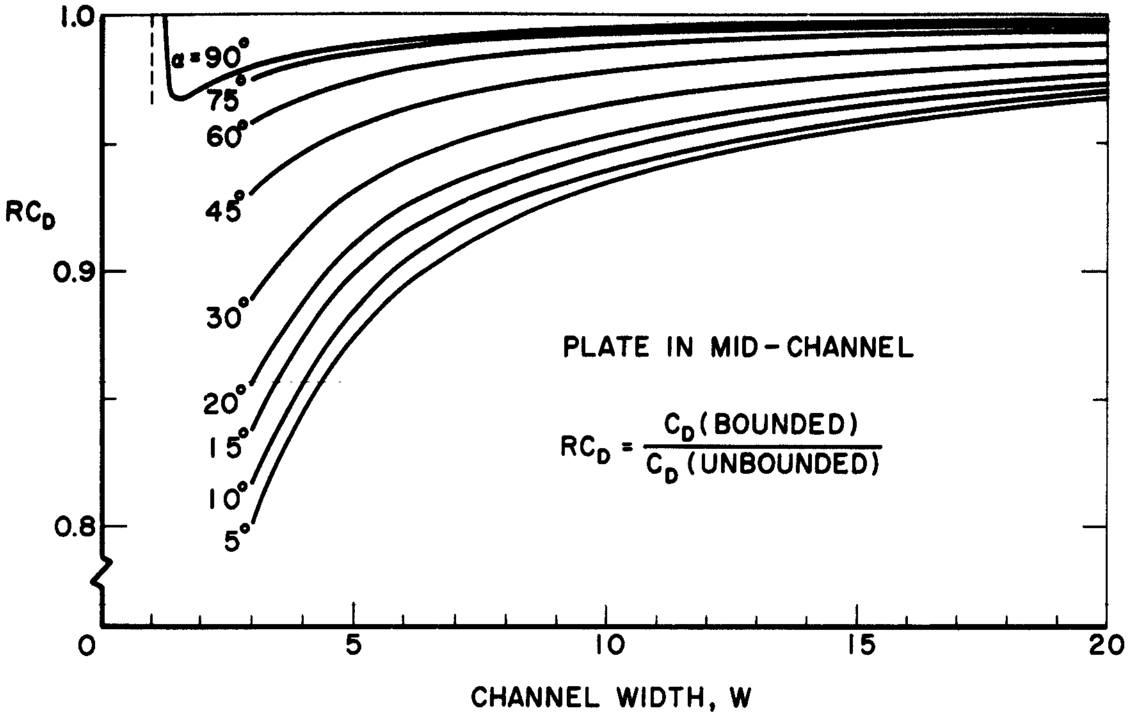

Fig. 11 The ratio of C_D (bounded) to C_D (unbounded) versus W for a flat plate located mid-channel.

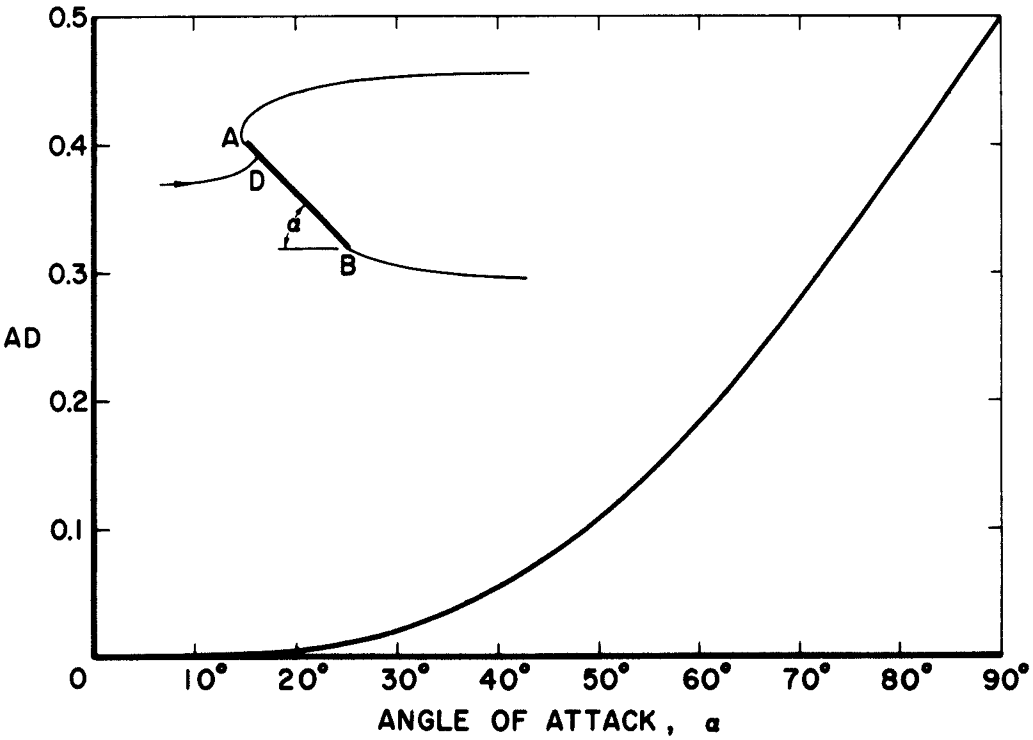

Fig. 12 Distance of stagnation point from the leading edge for W = 5 and the plate in mid-channel.

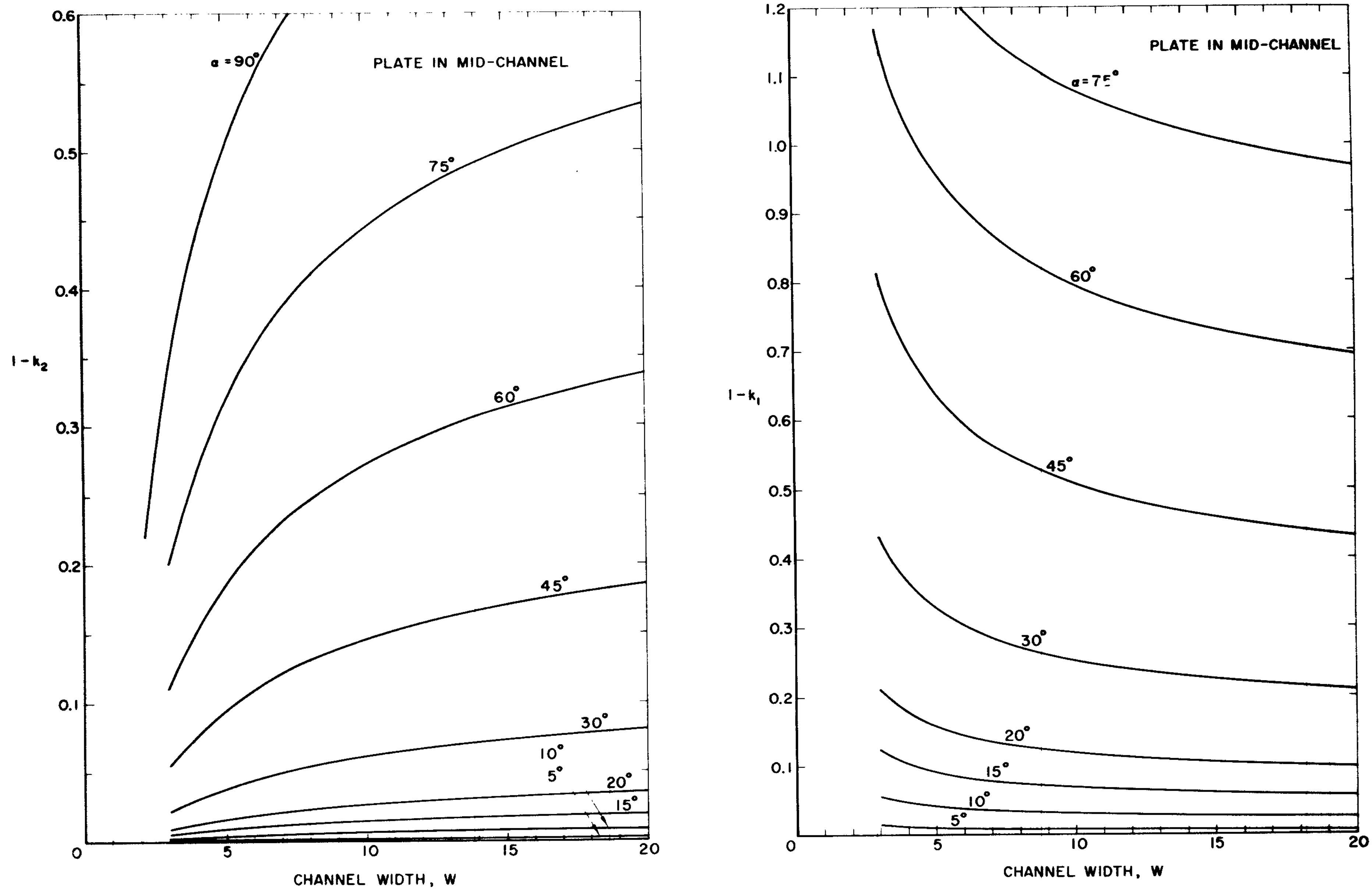

Fig. A-1 Effect of channel width and angle of attack on the transformation parameters (a) k_2 and (b) k_1 for a flat plate located mid-channel.

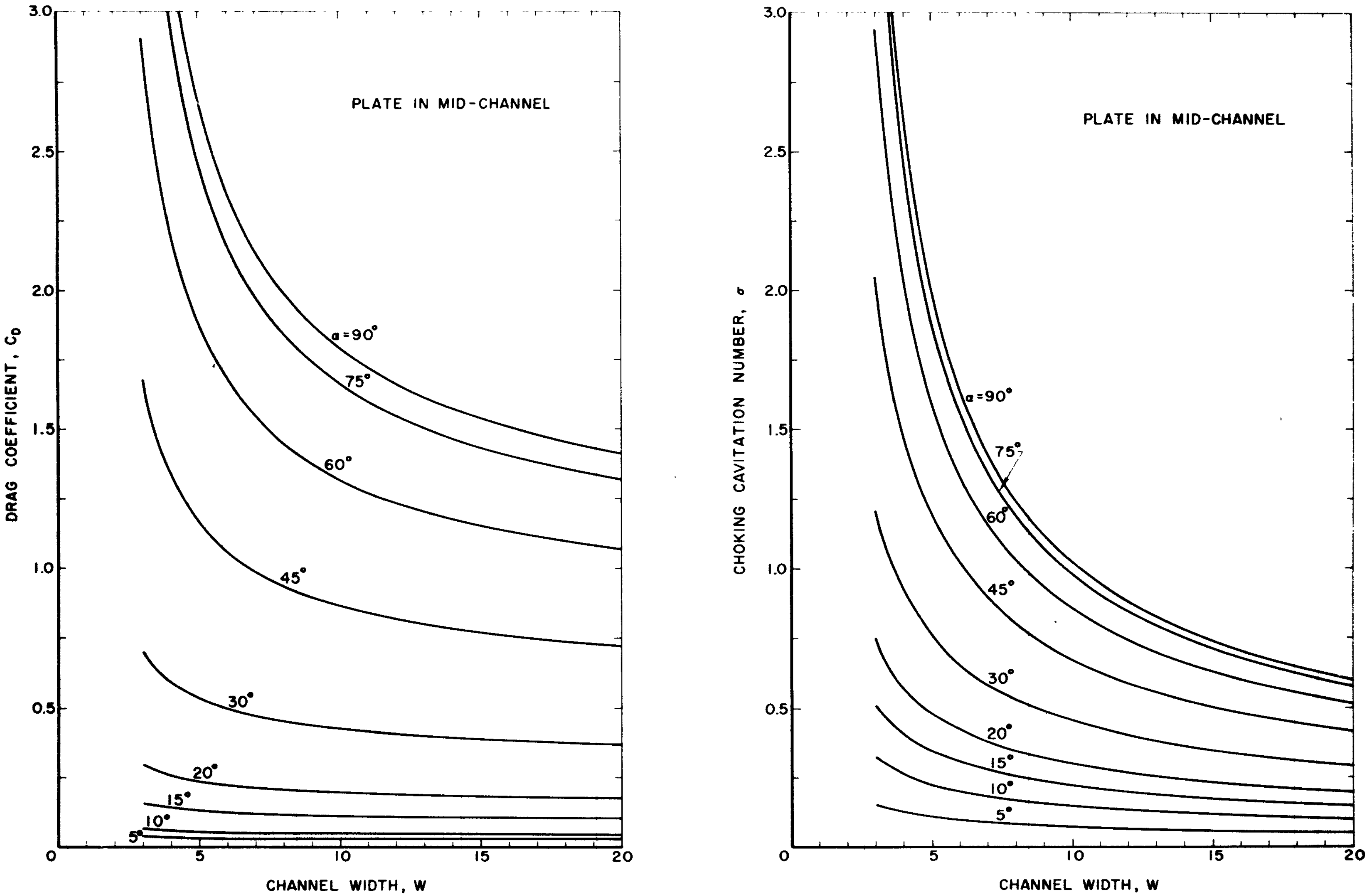

Fig. A-2 Effect of angle of attack and plate position on the transformation parameters (a) k_2 and (b) k_1 for a flat plate in two off-center positions $H_A = 1$ and 4, in a channel of $W = 5$.

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A nonlinear theory for the calculation of the flow field of an oblique flat plate under blockage condition is given using the techniques of integral equations. Numerical results are obtained with the aid of a high speed digital computer for the plate situated mid-channel at values of the angle of attack from 5° to 90° and the channel-width-chord ratio from 3 to 20. Also obtained are results for the plate situated at two different off-center positions for a channel width-chord ratio of 5 and angles of attack less than 30°.

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