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THE PROPERTIES OF A RECTANGULAR WAVEGUIDE WITH A DISSIPATIVE SLAB AT THE CENTER

by

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REPORT

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ABSTRACT

An exact solution is developed for the fields in a rectangular waveguide with a dielectric slab at the center. The surfaces of the slab are assumed to be parallel with the sidewalls of the waveguide, and the field is that of a TE mode with the electric vector parallel with the slab surfaces. The thickness of the slab is arbitrary, but the height of the slab is taken equal to that of the waveguide with the remaining space assumed to be free space. Although the equations are transcendental, it is possible to solve them to determine the guide wavelength and the guide attenuation constant if the slab parameters (dielectric constant, permeability, electric loss tangent and magnetic loss tangent and thickness) are known. Thus, the expressions are useful in the design of waveguide attenuators and phase shifters. The equations can be solved more readily for the parameters of the slab after the guide wavelength and attenuation constant have been measured, thus forming the basis of a promising technique for measuring dielectric constants, permeabilities, and loss tangents.

Simplified equations are derived for nonmagnetic slabs which are very thin in comparison with the wavelength and the waveguide width, and a set of curves are included to facilitate dielectric measurements and the design of waveguide attenuators and phase shifters.

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THE PROPERTIES OF A RECTANGULAR WAVEGUIDE WITH A DISSIPATIVE SLAB AT THE CENTER

I. INTRODUCTION

Microwave phase shifters and attenuators often consist of a rectangular waveguide with a dielectric slab at the center. The surfaces of the slab are parallel with the sidewalls of the waveguide as shown in Fig. 1, and the electric vector of the TE mode is parallel with the slab surfaces. Equations and graphs do not appear to be available in the literature for the design of these components.

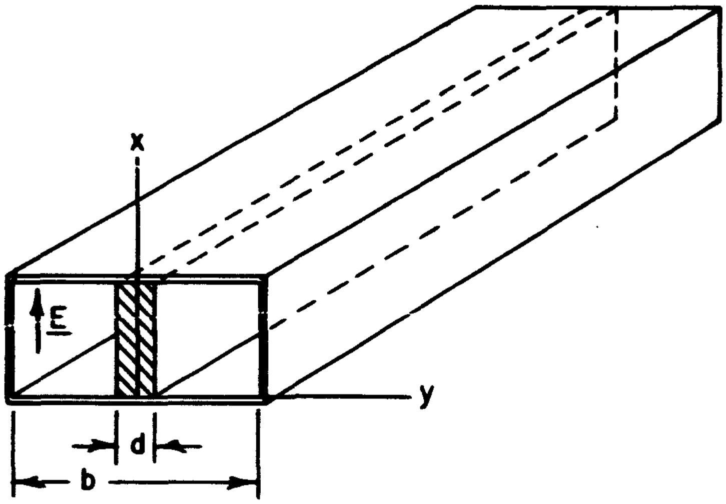

Fig. 1. Rectangular waveguide with dielectric slab at the center.

Although this configuration has several advantages for the measurement of dielectric constant, permeability, electric loss tangent and magnetic loss tangent, this application has evidently been overlooked in the past. These quantities can be determined accurately by placing a sample, in the form of a plane slab, at the center of a slotted waveguide and measuring the guide wavelength and the guide attenuation constant. This report derives equations which are useful for this purpose and presents graphs to facilitate dielectric measurements and the design of phase shifters and attenuators.

Some of the advantages of this arrangement are listed below.

1. The dielectric slab can have any thickness equal to or less than the waveguide width. This permits the use of stock sizes and reduces the machining costs.

2. No error arises from the air gap between the sample and the sidewalls of the waveguide, since this air gap is accounted for exactly in the equations.

3. A rigorous solution can be derived for the dielectric parameters of the sample (even when the loss tangents are very large) in terms of the waveguide attenuation constant and the guide wavelength which can be measured with the aid of a traveling probe.

4. The mathematical equations are simplified in the symmetrical case where the sample is at the center of the waveguide.

5. The sample is placed in a region of maximum electric field intensity, thus permitting more accurate measurements of the loss tangent.

6. When the sample is at or near the center of the waveguide, slight errors in its position have only a second-order effect on the measurements.

A variational solution by Berk[1] is available for the lossless slab, but its accuracy is somewhat limited. Although an exact transcendental solution is given by Montgomery, Dicke and Purcell,[2] for the lossless slab, an explicit solution for the dissipative slab does not appear to be available in the literature. Such a solution is developed in the Appendix of this paper in a form suitable for numerical calculations. The main body of this paper, however, is concerned with the special case where the slab has the same permeability as free space and its thickness is very small in comparison with the skin depth, the wavelength in the slab material, and the guide wavelength.

The thin slab is of considerable practical interest since the reflections at the ends of such a slab are small. Furthermore, the solution for the thin slab can be presented in the form of universal curves with fewer dimensionless parameters than are required for the thick slab.

The problem considered in this paper is of interest in the development of electronic polarization rotators for two reasons. It provides a convenient technique for the measurement of the dielectric constants of ferroelectric media which are believed to be useful in such devices. Furthermore, it provides a basis for the design of waveguide phase shifters which are employed in some of the polarization rotators.

II. THE BASIC THEORY

Consider a rectangular waveguide with a thin dissipative slab at the center, as shown in Fig. 1. The permeability of the slab is assumed to be the same as that of free space ($\mu = \mu_o$), and the slab material is assumed to be homogeneous and isotropic. For the harmonic traveling-wave case, the time dependence $e^{j\omega t}$ is understood and the electric field intensity of the dominant TE mode is given by

$$(1) \qquad E_x = \left[e^{(|y|-b/2)(-\alpha\cos\theta + j\beta\sin\theta)} - e^{(|y|-b/2)(\alpha\cos\theta - j\beta\sin\theta)}\right] \cdot e^{-\alpha z\sin\theta - j\beta z\cos\theta}$$

where b is the waveguide width, θ is the angle of propagation of the criss-crossing plane waves in the waveguide, and α and β are the attenuation constant and phase constant of each plane wave.

It may be noted that the above expression satisfies the boundary condition $E_x = 0$ at the waveguide walls at $y = b/2$ and $y = -b/2$. Furthermore, it satisfies the condition that tangential E be continuous across the boundaries of the card. To satisfy the wave equation in the free-space region on both sides of the slab, it is required that

$$(2) \qquad \beta^2 = \alpha^2 + k_o^2$$

where $k_o = \omega\sqrt{\mu_o\epsilon_o} = 2\pi/\lambda_o$ and λ_o denotes the free-space wavelength.

The thickness d of the slab is assumed to be much smaller than the wavelength and the waveguide width b. In this case, which is often of practical interest, the electric field intensity is essentially uniform (i.e., independent of y) through the slab. From Eq. (1), the electric field intensity in the slab is given by

$$\text{(3)} \qquad E_x = [e^{-(b/2)(-\alpha\cos\theta + j\beta\sin\theta)} - e^{-(b/2)(\alpha\cos\theta - j\beta\sin\theta)}]e^{-(\alpha\sin\theta + j\beta\cos\theta)z} .$$

From Maxwell's equations it is found that the magnetic field intensity is given by

$$\text{(4)} \qquad H_z = (1/j\omega\mu)\partial E_x/\partial y$$

$$= -\frac{(\alpha\cos\theta - j\beta\sin\theta)}{j\omega\mu}[e^{(|y|-b/2)(-\alpha\cos\theta + j\beta\sin\theta)} + e^{(|y|-b/2)(\alpha\cos\theta - j\beta\sin\theta)}]e^{-(\alpha\sin\theta + j\beta\cos\theta)z} \operatorname{sgn} y$$

where sgn y = 1 if y is positive and -1 if y is negative. The effect of the thin dielectric slab is the same as that of an equivalent sheet of electric surface current whose density is given by

$$\text{(5)} \qquad J_x = j\omega(\dot{\epsilon} - \epsilon_0)d\,E_x$$

where $\dot{\epsilon}$ represents the complex permittivity of the slab and E_x is the electric field intensity in the slab given by Eq. (3). Now the tangential component of the magnetic field intensity H_z must be discontinuous across this current sheet as follows:

$$\text{(6)} \qquad H_z(y=0+) - H_z(y=0-) = J_x$$

Equations (3) through (6) lead to the following result:

$$(7) \quad 2(\alpha \cos\theta - j\beta \sin\theta)\left[1 + e^{(-\alpha\cos\theta + j\beta\sin\theta)b}\right] = k_o^2\, d(\dot{\epsilon}_r - 1)\left[1 - e^{(-\alpha\cos\theta + j\beta\sin\theta)b}\right]$$

where $\dot{\epsilon}_r = \dot{\epsilon}/\epsilon_o = \epsilon_r - j\,\epsilon_r \tan\delta$, ϵ_r is the relative dielectric constant of the slab and $\tan\delta$ is its loss tangent.

By equating the real parts on both sides of Eq. (7), and similarly the imaginary parts, it is possible to obtain two real equations which are simultaneous linear equations for $(d/\lambda_o)\epsilon_r \tan\delta$ and $(\epsilon_r-1)d/\lambda_o$. The solution is readily obtained and is given below.

$$(8) \quad (d/\lambda_o)\epsilon_r \tan\delta = \frac{(1-e^{-2\alpha b\cos\theta})(\beta/k_o)\sin\theta - (\lambda_o\alpha/\pi)e^{-\alpha b\cos\theta}\cos\theta\sin(\beta b\sin\theta)}{\pi\left[1 - 2e^{-\alpha b\cos\theta}\cos(\beta b\sin\theta) + e^{-2\alpha b\cos\theta}\right]}$$

$$(9) \quad (\epsilon_r-1)d/\lambda_o = \frac{(1-e^{-2\alpha b\cos\theta})(\alpha\lambda_o/2\pi)\cos\theta + 2(\beta/k_o)e^{-\alpha b\cos\theta}\sin\theta\sin(\beta b\sin\theta)}{\pi\left[1 - 2e^{-\alpha b\cos\theta}\cos(\beta b\sin\theta) + e^{-2\alpha b\cos\theta}\right]} .$$

If the dielectric constant and loss tangent of the slab are known, Eqs. (8) and (9) represent transcendental equations for $\alpha\cos\theta$ and $\beta\sin\theta$. A trial-and-error solution is possible, with the aid of Eq. (2). If, on the other hand, the guide wavelength and the guide attenuation constant are known (from experimental measurements, for example), Eqs. (8) and (9) represent explicit expressions for the loss tangent and the dielectric constant of the slab material and the solution is straightforward. The guide wavelength and the guide attenuation constant are given by

$$(10) \quad \lambda_g = 2\pi/(\beta\cos\theta)$$

$$(11) \quad \alpha_g = \alpha\sin\theta .$$

These quantities can readily be measured with the aid of a traveling probe which moves along the waveguide axis in the region containing the slab.

III. THE THIN LOSSLESS SLAB

In the lossless case, the attenuation constant vanishes and $\beta = k_o$. Thus Eq. (9) reduces to the following simplified form:

$$(12) \quad (\epsilon_r - 1)\, d/\lambda_o = \frac{\sin\theta \sin(k_o b \sin\theta)}{\pi(1 - \cos(k_o b \sin\theta))} \quad .$$

From Eq. (10) the guide wavelength in the lossless case is given by

$$(13) \quad \lambda_g = \lambda_o / \cos\theta \, .$$

If the slab thickness is zero or its dielectric constant is unity, Eqs. (12) and (13) lead to the well-known expression for the guide wavelength of a rectangular waveguide without a slab. Figure 2 is a set of graphs based on Eq. (12) for various waveguide dimensions such that $b/\lambda_o = 0.6,\ 0.7,\ 0.8$ and 0.9. These graphs can be employed in the design of waveguide phase shifters and in the measurement of the dielectric constant of lossless or low-loss slabs.

An exact transcendental equation for the lossless slab of arbitrary thickness is given by Harrington.[3] In terms of the notation employed here, the exact expression is

$$(14) \quad \sqrt{\epsilon_r - \cos^2\theta}\, \tan\left(k_o \sqrt{\epsilon_r - \cos^2\theta}\, d/2\right) = \sin\theta \cot\left(k_o \frac{b-d}{2} \sin\theta\right) .$$

The data given in Fig. 2, based on Eq. (12), are found to agree accurately with the exact equation (Eq. (14)) if the slab thickness d is much smaller than the wavelength.

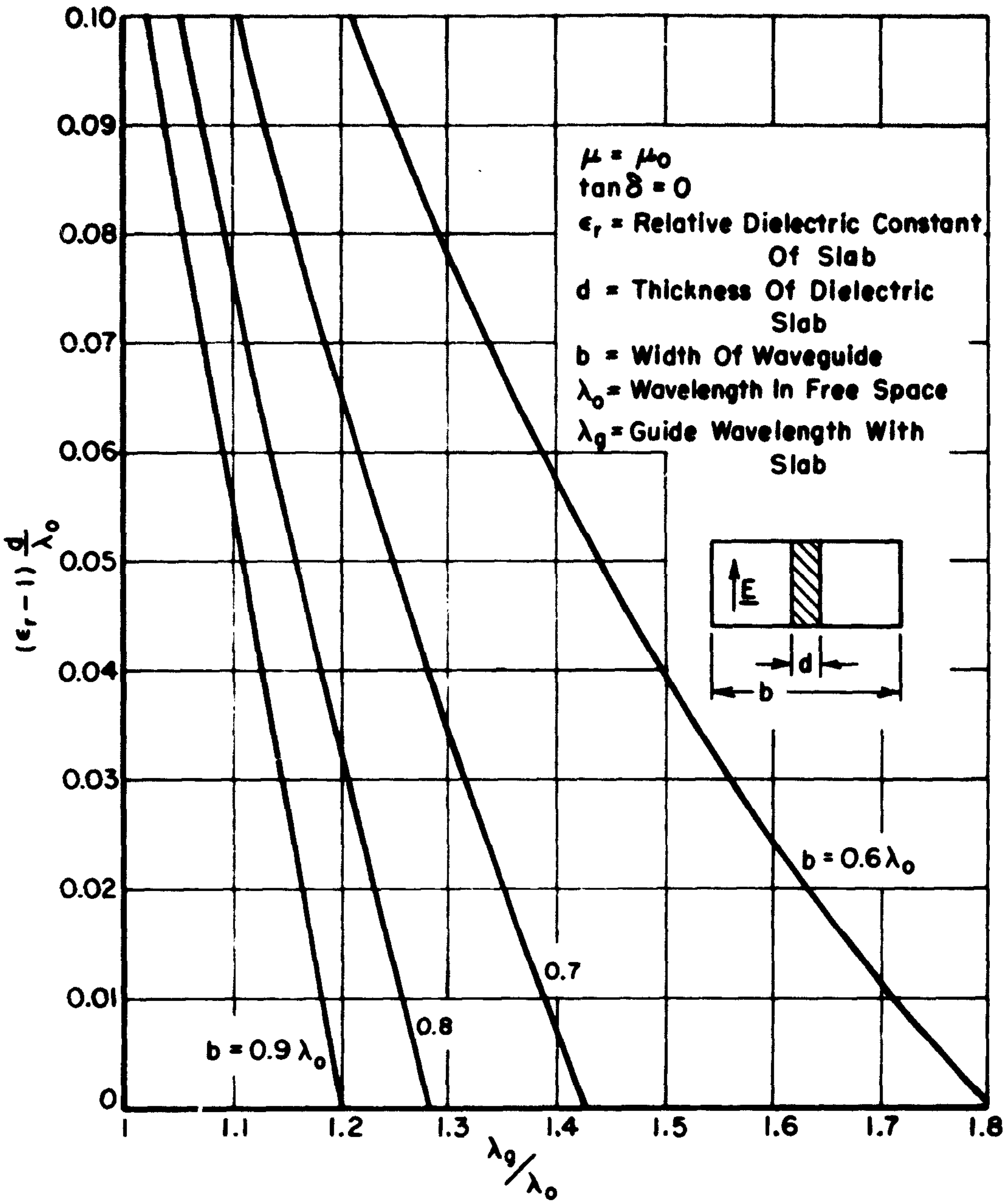

Fig. 2. Effect of thin lossless dielectric slab on guide wavelength of TE_{10} mode in rectangular waveguide.

IV. THE THIN DISSIPATIVE SLAB

If the attenuation constant is small, β is nearly equal to k_o. In particular, if $\alpha\lambda_o$ is less than 0.6, it is found from Eq. (2) that β differs from k_o by less than one-half of one percent. Over this same range of attenuation constants it can be shown that the guide wavelength is essentially the same as that given in Fig. 2 for the lossless slab.

Figure 3 shows the quantity $(d/\lambda_o)\epsilon_r \tan\delta$ versus the guide attenuation constant for the case where the waveguide width is $b = 0.6\lambda_o$. The curves in Fig. 3 are based on Eq. (8) with β taken equal to k_o and with the guide wavelength assumed to be equal to that for the lossless case. This was found to be an accurate assumption for the range of parameters involved in Fig. 3. Similar graphs can readily by plotted for other waveguide widths by means of Eq. (8).

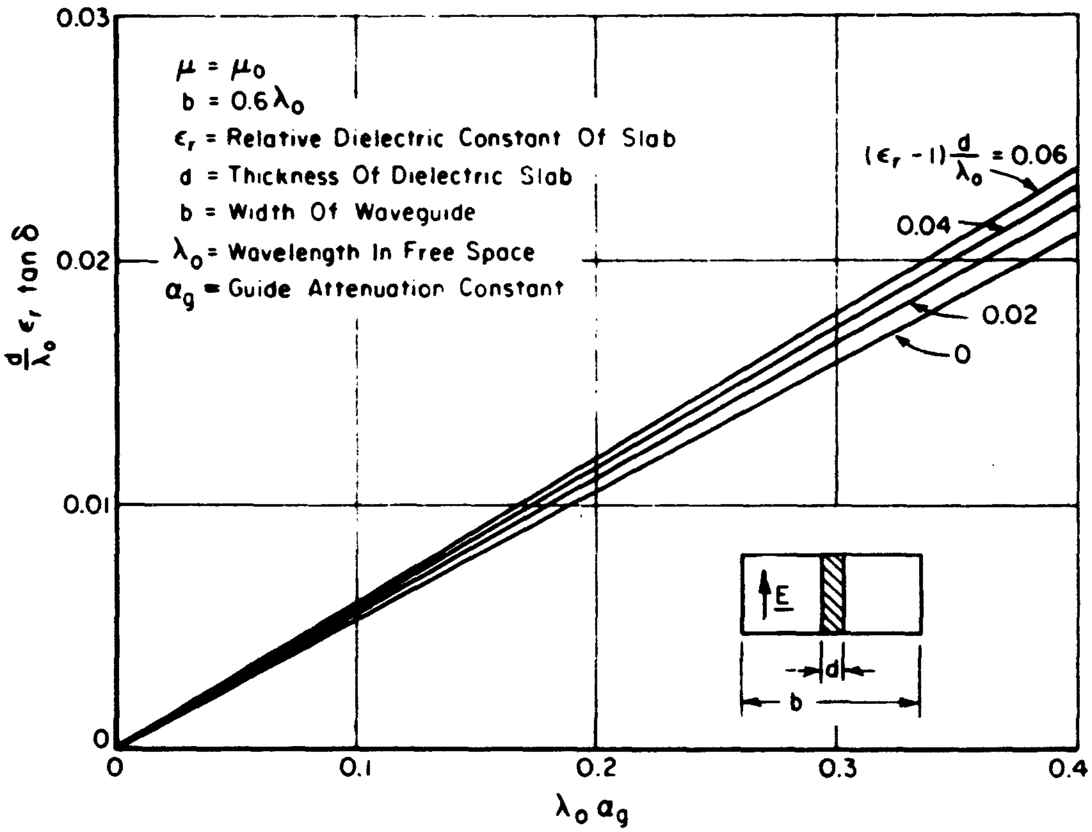

Fig. 3. Relation between guide attenuation constant and loss tangent of thin slab at center of rectangular waveguide with TE_{10} mode.

Curves such as those plotted in Fig. 3 are useful in the design of waveguide attenuators and in the measurement of the loss tangent of a thin slab. In the measurement of dielectric parameters, it is suggested that one first measure the guide wavelength in the region containing the slab. Then the quantity $(\epsilon_r-1)\, d/\lambda_o$ can be determined from Eq. (9) or Fig. 2. Knowing the slab thickness and the free-space wavelength, it is then a simple matter to determine the relative dielectric constant of the slab material. Next, the guide attenuation constant may be measured in the region containing the slab, and the loss tangent is easily determined with the aid of Eq. (8) or Fig. 3. To obtain accurate results, the slab should be at least several wavelengths long and the ends should be tapered to reduce reflections. The slab thickness should be great enough to obtain an easily measurable attenuation much greater than that of the waveguide itself. Furthermore, the thickness should be sufficiently great to produce a measurable change in the guide wavelength while keeping $(\epsilon_r-1)d/\lambda_o$ less than about 0.1.

It will be noticed that the graphs shown in Fig. 3 are essentially straight lines. This suggests the existence of an approximate linear relation between the guide attenuation constant and the quantity $(d/\lambda_o)\epsilon_r \tan\delta$. The linear approximation can be obtained from Eq. (8). It is given as follows:

$$(d/\lambda_o)\epsilon_r \tan\delta \approx \lambda_o \alpha_g \frac{(X-\sin X)\cot\theta}{2\pi^2\,(1-\cos X)} \tag{15}$$

where

$$X = k_o b \sin\theta\,. \tag{16}$$

Equation (15) agrees accurately with Eq. (8) for the cases illustrated in Fig. 3.

Figure 4 shows graphs of the quantity $(d/\lambda_o)\epsilon_r \tan\delta/(\lambda_o\alpha_g)$ versus the waveguide width b/λ_o for $(\epsilon-1)\, d/\lambda_o = 0$ and 0.05. These curves, based on Eq. (15) and the data given in Fig. 2, are useful in calculating the loss tangent of the slab after the guide attenuation constant has been measured.

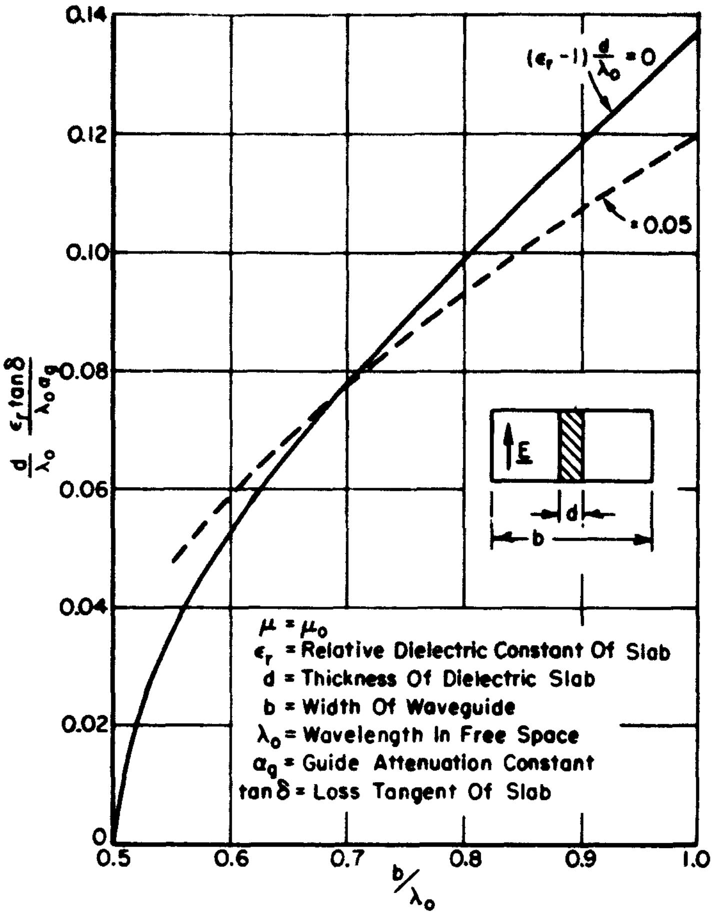

Fig. 4. Relation between loss tangent and guide attenuation constant as a function of waveguide width for thin slab in rectangular waveguide.

V. CONCLUSIONS

A rigorous solution is developed for the dominant TE mode in a rectangular waveguide with a dissipative dielectric slab at the center. The slab may have any width equal to or less than that of the waveguide. The equations are in a form suitable for numerical calculations. They are useful in the design of microwave attenuators and phase shifters, and they form the basis of a promising technique for the measurement of dielectric constant, permeability, electric loss tangent, and magnetic loss tangent.

Simplified equations are developed for the special case where the slab is nonmagnetic and its thickness is small in comparison with the skin depth, the wavelength in the slab material, and the waveguide width. A set of graphs are included to facilitate the design of microwave components and the measurement of dielectric parameters.

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APPENDIX. RIGOROUS SOLUTION FOR SLAB OF ARBITRARY THICKNESS

Consider a dielectric slab of arbitrary thickness d, real permittivity ϵ, real permeability μ, electric loss tangent $\tan \delta$ and magnetic loss tangent $\tan \delta'$. If the slab is centered in a rectangular waveguide as shown in Fig. 1, the fields of the dominant TE mode are given as follows:

(17) $$E_x = \cosh[(\alpha_1 \cos \theta' - j\beta_1 \sin \theta_1)y]\, e^{-\alpha_1 z \sin \theta' - j\beta_1 z \cos \theta_1},$$

for $y < d/2$

(18) $$E_x = A \sinh\left[(\alpha_o \cos \theta_o - j\beta_o \sin \theta_o)\left(\frac{b}{2} - y\right)\right] e^{-\alpha_o z \sin \theta_o - j\beta_o z \cos \theta_o},$$

for $y > d/2$

where θ_o and θ_1 are the angles of propagation, with respect to the waveguide axis, of the criss-crossing plane waves in the free-space region and the slab, respectively. The angle θ' is that of the planes of constant amplitude of each plane wave in the slab, with respect to the waveguide axis. If the slab is lossless, the angles θ_1 and θ' will be equal; in the dissipative slab they will differ.

To satisfy the boundary conditions at the slab surfaces, the z dependence must be the same in both regions. This leads to the following equations.

(19) $$\alpha_1 \sin \theta' = \alpha_o \sin \theta_o$$

(20) $$\beta_1 \cos \theta_1 = \beta_o \cos \theta_o \ .$$

The following relations are obtained by enforcing the wave equation in each region:

(21) $$\beta_o^2 = \alpha_o^2 + k_o^2$$

(22) $\beta_1^2 = \alpha_1^2 + \omega^2\mu\epsilon(1-\tan\delta\tan\delta')$

(23) $2\alpha_1\beta_1 \sin(\theta' - \theta_1) = \omega^2\mu\epsilon(\tan\delta + \tan\delta')$.

It may be noted that Eq. (18) satisfies the boundary condition that the tangential electric field intensity must vanish at the surfaces of the perfectly conducting waveguide walls. The following result is obtained by making the tangential electric field intensity continuous across the surfaces of the slab:

(24) $$A \sinh\left[(\alpha_0 \cos\theta_0 - j\beta_0 \sin\theta_0)\left(\frac{b}{2} - \frac{d}{2}\right)\right]$$

$$= \cosh[(\alpha_1 \cos\theta' - j\beta_1 \sin\theta_1)d/2] \ .$$

To complete the solution it is necessary to make the tangential magnetic field intensity continuous across the surfaces of the slab. From Maxwell's equations the tangential magnetic field intensity is found as follows:

(25) $$H_z = (1/j\omega\dot{\mu})(\partial E_x/\partial y) = (1/j\omega\dot{\mu})(\alpha_1 \cos\theta' - j\beta_1 \sin\theta_1)$$

$$\cdot \sinh[(\alpha_1 \cos\theta' - j\beta_1 \sin\theta_1)y]e^{-\alpha_1 z \sin\theta' - j\beta_1 z \cos\theta_1}$$

for $y < d/2$

(26) $$H_z = (1/j\omega\mu_0)(\partial E_x/\partial y) = (1/j\omega\mu_0)(-\alpha_0 \cos\theta_0 + j\beta_0 \sin\theta_0)$$

$$\cdot A \cosh\left[(\alpha_0 \cos\theta_0 - j\beta_0 \sin\theta_0)\left(\frac{b}{2} - y\right)\right]e^{-\alpha_0 z \sin\theta_0 - j\beta_0 z \cos\theta_0}$$

for $y > d/2$.

The following transcendental equation is obtained by making the tangential magnetic field intensity continuous across the boundaries of the slab:

$$X \tanh(Xd/2) = -\mu_r(1 - j \tan \delta') \, Y \coth [Y(b-d)/2] \tag{27}$$

where

$$X = \alpha_1 \cos \theta' - j\beta_1 \sin \theta_1 \tag{28}$$

$$Y = \alpha_o \cos \theta_o - j\beta_o \sin \theta_o \, . \tag{29}$$

The guide attenuation constant and the guide wavelength are given by

$$\alpha_g = \alpha_o \sin \theta_o = \alpha_1 \sin \theta' \tag{30}$$

$$\lambda_g = 2\pi/(\beta_o \cos \theta_o) = 2\pi/(\beta_1 \cos \theta_1) \, . \tag{31}$$

After the guide attenuation constant and the guide wavelength have been measured, it is possible to calculate the angle θ_o by means of the following expression which is based on Eqs. (21), (30), and (31):

$$\sin^2\theta_o = 0.5 \left[1 - \frac{\alpha_g^2}{k_o^2} - \frac{\lambda_o^2}{\lambda_g^2} + \sqrt{\left(1 - \frac{\alpha_g^2}{k_o^2} - \frac{\lambda_o^2}{\lambda_g^2}\right)^2 + \frac{4\alpha_g^2}{k_o^2}} \right] . \tag{32}$$

Now it is possible to calculate the quantity Y defined in Eq. (29). If the slab is nonmagnetic, one can proceed to solve the transcendental Eq. (27) to determine the quantity X defined in Eq. (28). The dielectric constant and the loss tangent can then be found by means of Eqs. (22), (23), (28), (30) and (31).

If the slab has a permeability differing from that of free space (as in the case of a ferrite slab), a unique solution cannot be obtained for μ, ϵ, $\tan \delta$ and $\tan \delta'$ from measurements of the guide wavelength and guide attenuation constant at a single frequency with one slab. However, these parameters can be calculated with the aid of the above equations if measurements are made at a single frequency with two slabs of the same material with different thicknesses.

If the slab is lossless, it is easy to verify that the above solution reduces to the one given by Harrington.[3] The rigorous equations for the lossless case are listed below.

(33) $$\cos\theta_o = \lambda_o/\lambda_g$$

(34) $$\sqrt{\mu_r\epsilon_r - \cos^2\theta_o}\,\tan(0.5k_o d\sqrt{\mu_r\epsilon_r - \cos^2\theta_o}) = \mu_r \sin\theta_o \cot[0.5k_o(b-d)\sin\theta_o].$$

If a dissipative slab completely fills the waveguide, the general solution simplifies as follows:

(35) $$\tan\theta_1 = \lambda_g/2b$$

(36) $$\beta_1 = \pi/(b\sin\theta_1)$$

(37) $$\omega^2\mu\epsilon(1 - \tan\delta\tan\delta') = \beta_1^2 - \alpha_g^2$$

(38) $$\omega^2\mu\epsilon(\tan\delta + \tan\delta') = 4\pi\alpha_g/\lambda_g\ .$$

A very brief discussion is given by Marcuvitz[4] of the rectangular waveguide with a resistive film at the center, and a waveguide with a dielectric slab of arbitrary thickness at the center.