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COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

ELECTRONICS RESEARCH LABORATORIES

THE THEORY AND PHYSICAL CONSTRAINTS OF AN ELECTRO-OPTICAL SIGNAL PROCESSOR FOR PHASED ARRAY ANTENNAS

TECHNICAL REPORT T-1/199

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ABSTRACT

Techniques are described for processing the signals obtained from the elements of a phased array antenna by means of electro-optical devices. These techniques employ one electro-optical device which simultaneously forms, as a continuum, all the beam patterns that a receiving planar array antenna is capable of forming. An analysis of an electro-optical processor for phased arrays is presented which shows that: (1) The target location angles are obtained unambiguously for each and every target that can be resolved, (2) Target angle resolution is identical to the resolution inherent in the antenna array, (3) Multiple targets produce multiple outputs with all angles properly associated, (4) The progressive envelope delay that is introduced at the antenna array does not result in an aperture-bandwith constraint.

Physical constraints on system bandwidth and the number of array elements that can be processed are considered. Typically, it should be possible to process electro-optically 10,000 array elements with a system bandwidth of 10 Mcs.

AUTHORIZATION

The work described in this report was performed at the Electronics Research Laboratories of the School of Engineering and Applied Science of Columbia University, and the report was prepared by L. Lambert and M. Arm.

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Director

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

An optical configuration is a three-dimensional circuit in the sense that a complete description of the characteristics of the "circuit" involves the specification of the light intensity, as a function of time, at each point in the object and image planes. This is different from a conventional two-port electric circuit which is a one-dimensional device and is completely specified by the temporal variations of the voltages at the input and output ports. An antenna aperture is also a three-dimensional circuit except that in conventional reflector antennas, a single feed point is used to illuminate the reflector surface with a time varying signal so that the combination of feed and reflector reduces to a one-dimensional circuit. In a typical phased array, many separate elements are positioned in a plane and the terminal of each element is available for purposes of processing and observation as a function of time. Thus, an array antenna and an optical configuration are both three-dimensional circuits and the use of optics to process the signals received by the elements of a phased array may have some advantages. This inference is made still more plausible when it is realized that the excitation of the elements of a phased array and the resulting radiation pattern form Fourier transform pairs, and that the image plane and object plane light intensity distributions for an optical configuration also form Fourier transform pairs.

Techniques will be described for processing the signals obtained from the elements of a phased array by means of electro-optical devices. These techniques employ one optical device which simultaneously forms, as a continuum, all the beam patterns that a receiving array antenna is capable of forming. This differs significantly from the case of a conventional phased array beam processor where simultaneous beam formation is quantized since each beam must be formed in a separate electrical circuit. This basic difference is a direct consequence of the three-dimensional nature of optical devices as compared to the one-dimensional nature of electric circuits. The unique beam forming capability inherent in certain electro-optical devices will be discussed.

On the assumption that electro-optical processing of phased array signals is possible, would it be of any practical value? In many array radar applications it has been found expedient to use separate arrays for transmission and reception. It is well known that in a search radar the product of transmitter average power and receiver area is fixed by the nature of the search problem. Therefore, without affecting radar performance, the transmitter array can consist of many low power elements or few high power elements. It is usually less expensive to build and maintain a few high power transmitting elements than many low power ones. However, if the transmitter array has fewer elements than the receiver array, it will have a broader beam pattern. If full use is to be made of the transmitted power, a number of receiver beams must be formed simultaneously in order to collect the energy from all parts of the transmitter beam. The additional cost and maintenance of the receiver beam forming networks and of their associated signal processing equipment usually offsets the potential saving in transmitter cost and maintenance that can be realized by the use of a small number of high power elements.

Now, since one optical device can form all the beams of the receiver array simultaneously, it becomes possible to use transmitter arrays with smaller numbers of high power elements without incurring additional receiver costs. It may also be practical to provide the electro-optical device with a time waveform processing capability so that, when the array transmits a suitable complex waveform, pulse compression can be simultaneously accomplished.

Furthermore, the beam forming capability of the electro-optical signal processor is not affected by the progressive envelope delay that is introduced at the receiving array. As a direct consequence of this inherent processor characteristic, the corresponding aperture-bandwidth constraint is relieved. The electro-optical signal processor does, however, introduce a constraint on the system bandwidth and the number of array elements that can be processed. Typically, it should be possible to process electro-optically 10,000 array elements with a system bandwidth in excess of 10 Mcs.

II. THEORETICAL CONCEPTS AND PAST RESULTS

The phased array signal processors that will be considered are based, in part, on certain well established electro-optical concepts and on previous experimental results that have been obtained at CUERL. This background is first reviewed so as to establish a firm and proven theoretical foundation. The applicable electro-optical processor techniques are then presented in succeeding sections along with a discussion of the typical system parameters that appear feasible at this time.

A. ELECTRO-OPTICAL SIGNAL PROCESSOR CONCEPTS

As was originally implied by Abbe[1] when he developed his theory of optical resolution for coherent illumination, optical systems are capable of processing information. Recently, optical systems have been described in the literature[2-6] in terms of filter and communication theories. It has been shown that, in addition to the independent time variable, optical systems provide two more degrees of freedom in the form of physical dimensions orthogonal to the direction of light propagation. Also, it has been shown that various planes in an optical system are related by two-dimensional Fourier transforms[7-12] so that integration and filtering can be accomplished.

In order to synthesize a signal processor by employing optical techniques, the input signal must first be converted to a signal which is capable of spatially modulating light. Conversions of this nature have been realized with photographic film[12-14] but this results in a considerable time delay due to the film processing time that is required, and serious constraints are imposed on the system due to the requirements of a stable film transport mechanism, a flat and uniform film, etc. For certain applications these disadvantages can be circumvented by employing transparent ultrasonic delay lines which act as "instantaneous" spatial light modulators.[15-19] Specifically, research efforts at CUERL have resulted in demonstrating the feasibility of employing "real-time" electro-optical techniques to obtain wideband, instantaneous spectrum analyzers, autocorrelators, crosscorrelators, pulse compression systems and two-dimensional filters. The basic electro-optical spectrum analyzer technique is discussed below

[1] For numbered references, see Sec. V, p. 41.

and is extended in concept in Section III to provide a signal processor which simultaneously forms and displays a continuum of the beams obtained from a planar phased array receiving antenna.

1. Optical Transfer Functions

Consider a plane, sinusoidal, electromagnetic (light) wave propagating in free space along the z-axis as shown in Fig. 1. Let the amplitude of this plane wave be E_c and let it be incident upon an aperture which contains an object located in the x,y plane (object plane). The object can, in general modify both the magnitude and phase of the incident plane wave of light, producing a light amplitude* distribution emanating from the object plane

$$E_o(x,y) = [E_c][T(x,y)] \quad . \tag{1}$$

$T(x,y)$ is defined as the complex transmission function of the object and it can be written as

$$T(x,y) \equiv \frac{E_o(x,y)}{E_c} = A(x,y)\ \exp[j\psi(x,y)] \tag{2}$$

where $A(x,y)$ is the relative amplitude, and $\psi(x,y)$ the relative phase of the light emanating from the object plane.

The light amplitude $E_i(x',y')$ at a point in the image plane (Fig. 1) can be shown[1,2,8,12] to be related to the light amplitude distribution emanating from the object plane $E_o(x,y)$ by

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*In this report, the term "amplitude" is defined as the complex amplitude (magnitude and phase) of an optical signal.

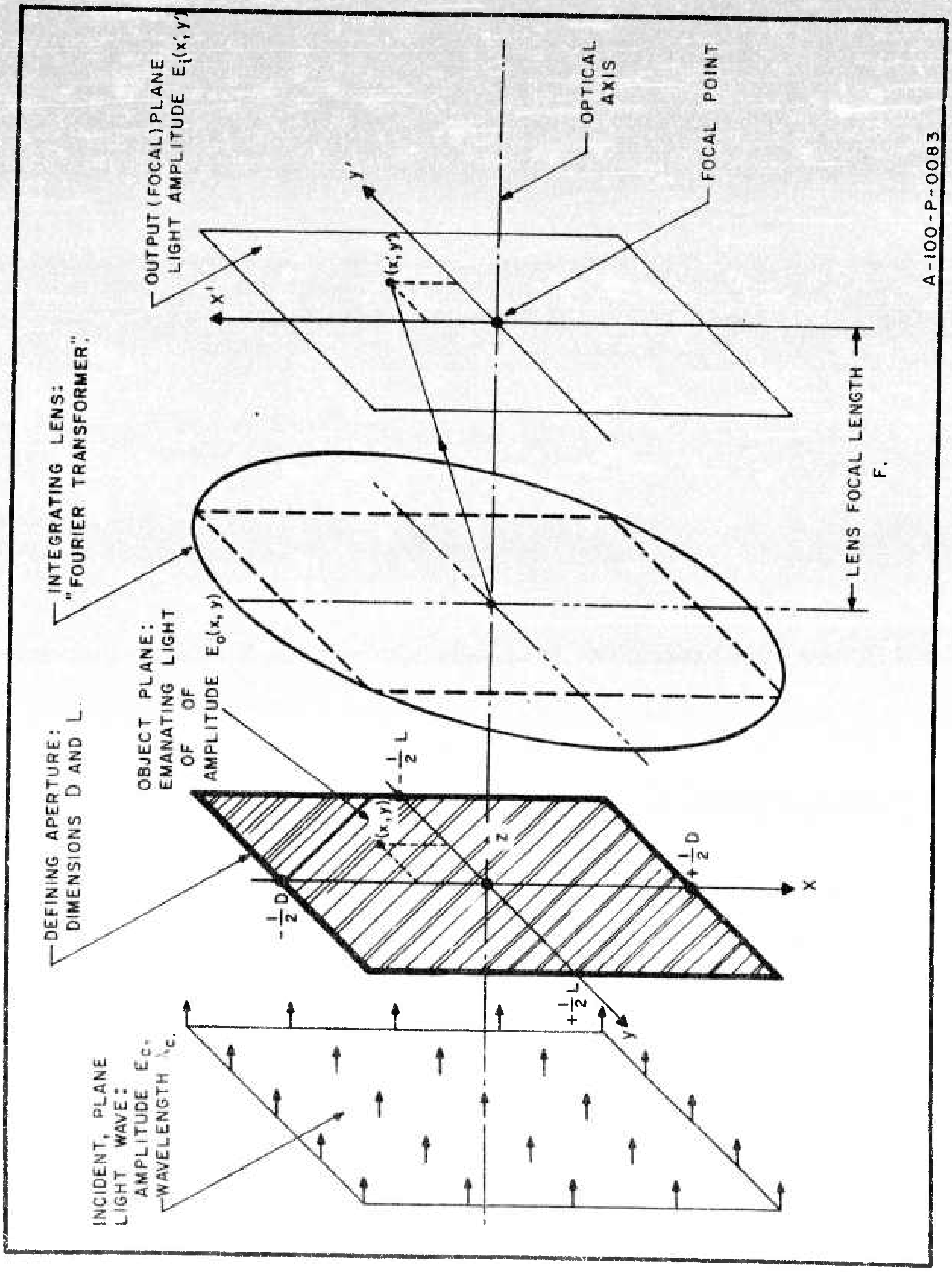

FIG. 1 SPATIAL MODULATION AND FOURIER TRANSFORMATION

$$E_i(x',y') = K_1 \int_{-\infty}^{\infty}\int E_o(x,y)e^{-jb(x'x+y'y)}dxdy \qquad (3)$$

where K_1 and b are constants. This is the well known Fraunhofer formula which relates the light distributions in the object and image planes by a Fourier transformation. A more convenient form for this relationship can be obtained by defining a relative (or normalized) light amplitude distribution in the image plane which is given by

$$E(u,v) = K \int_{-\infty}^{\infty}\int T(x,y)e^{-j2\pi(ux+vy)}dxdy \qquad (4a)$$

where u and v are normalized image plane dimensions given by

$$u \cong x'/F\lambda_L \quad , \quad v \cong y'/F\lambda_L \quad , \qquad (4b)$$

K is a normalizing constant, F is the focal length of the integrating lens and λ_L is the light (carrier) wavelength.

Thus, for the coherent optical configuration shown in Fig. 1, the relative light amplitude distribution in the image plane $E(x',y')$ is essentially equal to the <u>two-dimensional Fourier transform</u> of the complex transmission function $T(x,y)$ produced by the object (Eqs. (4)). The complex transmission function for any object is equal to the light amplitude distribution produced by the object when the object is illuminated with a plane wave of light having unit amplitude (Eq. (2)). We shall refer to the object as having <u>spatially modulated</u> the incident light wave.

By utilizing the Debye-Sears effect[15,28,29] in a transparent, ultrasonic delay line, we can obtain an "instantaneous" spatial modulation for a time varying input signal. As a simplified, one-dimensional example of this spatial modulation technique, consider the configuration in Fig. 2 where only modulation in the x dimension is employed. The input signal $v(t)$ excites the piezoelectric transducer which vibrates in a longitudinal (compressional) mode. These mechanical vibrations cause a pressure wave to propagate in the delay medium with a sonic speed S. The pressure wave causes a corresponding change in the refractive index of the delay medium as a function of x, the input space variable. As a result, the incident plane wave of light experiences (to a first approximation) a spatial phase modulation.

At a given instant, the complex transmission function $T(x)$ produced by an ultrasonic light modulator of length D and delay time $T = D/S$ can be written as

$$T(x) = p_D(x) \exp j\psi(x) \quad . \tag{5a}$$

Here, the finite aperture length is accounted for by

$$p_D(x) \equiv \begin{cases} 1 & \text{for} \quad |x| \leq D/2 \\ 0 & \text{for} \quad |x| > D/2 \end{cases} \quad , \tag{5b}$$

and the spatial phase modulation produced by the input signal $v(t)$ can be shown to be

$$\psi(x) = K_s v(x/S) \tag{5c}$$

K_s is a constant for a fixed delay medium, piezoelectric transducer and light wavelength.

Thus, at a given instant, the transparent ultrasonic delay line light modulator converts a time varying input signal to a proportional spatial phase modulation. As time progresses, the ultrasonic wave propagates in the delay

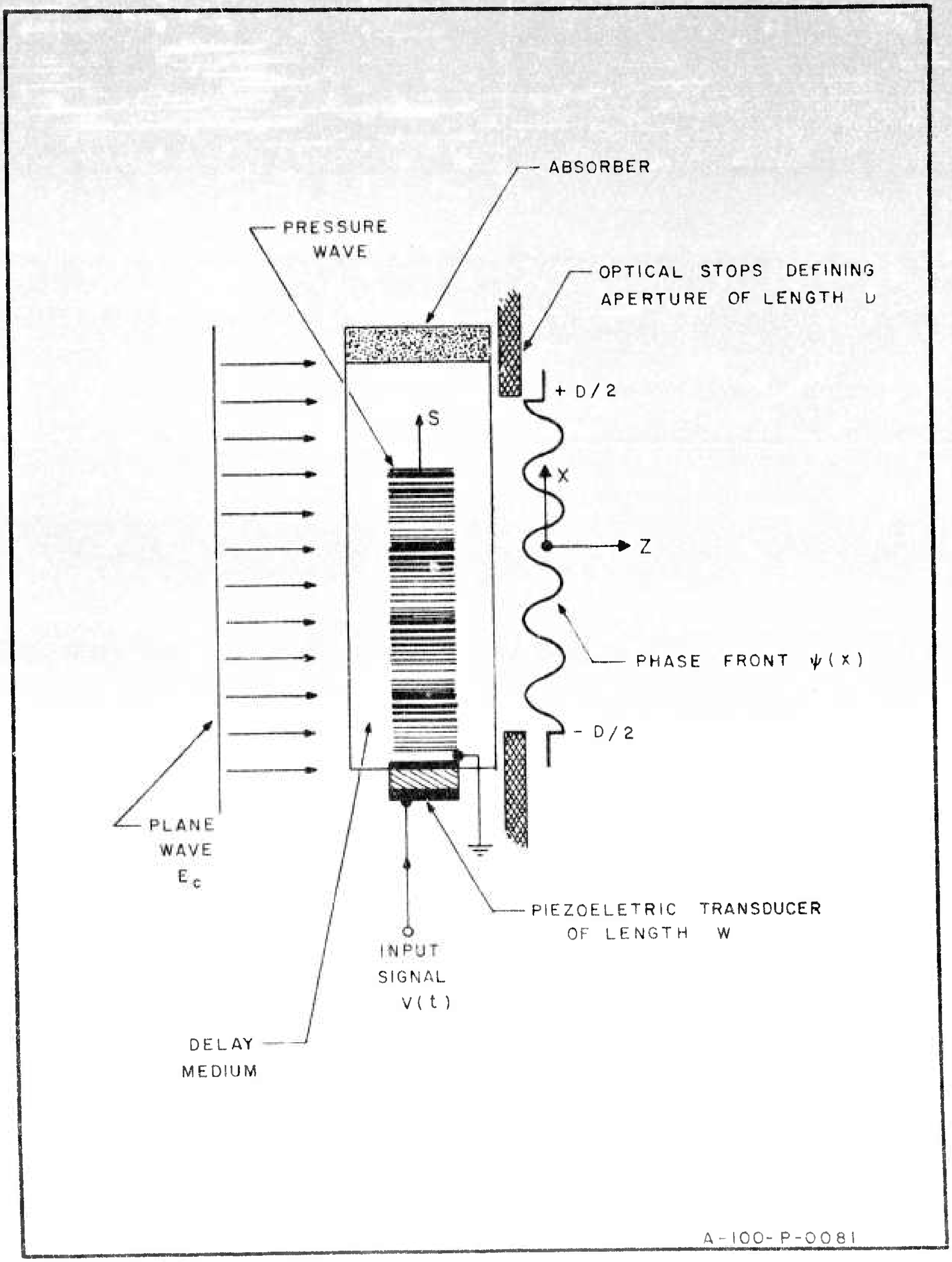

FIG. 2 A DEBYE-SEARS, SPATIAL LIGHT MODULATOR

line so that new portions of the input signal are continuously and instantaneously causing the incident light to be spatially modulated with a transmission function given by Eqs. (5).

2. Electro-Optical Spectrum Analyzers

The ultrasonic delay line light modulator (Fig. 2) when combined with a coherent optical configuration (Fig. 1) and a suitable output photodetector, form the basic device that will be applied to the processing of phased array signals. This device continuously produces, at its output, the Fourier transform (spectrum) of the input signal. A schematic diagram of such a spectrum analyzer is shown in Fig. 3.

In order to clearly visualize the output response of this device, consider the output that is obtained at the instant when a sinusoidal signal of arbitrary phase is contained in the light modulator. For an input signal

$$v(t) = V_o \cos(2\pi f_i t + \phi_o) \quad , \tag{6}$$

the complex transmission function $T(x)$ produced by the light modulator is most conveniently written in terms of a normalized input space variable defined by

$$\tau \equiv x/S \quad . \tag{7}$$

The complex transmission function for the delay line light modulator can now be written in the form

$$T(\tau) = p_T(\tau) e^{j\psi_m \cos(2\pi f_i \tau + \phi_o)} \tag{8a}$$

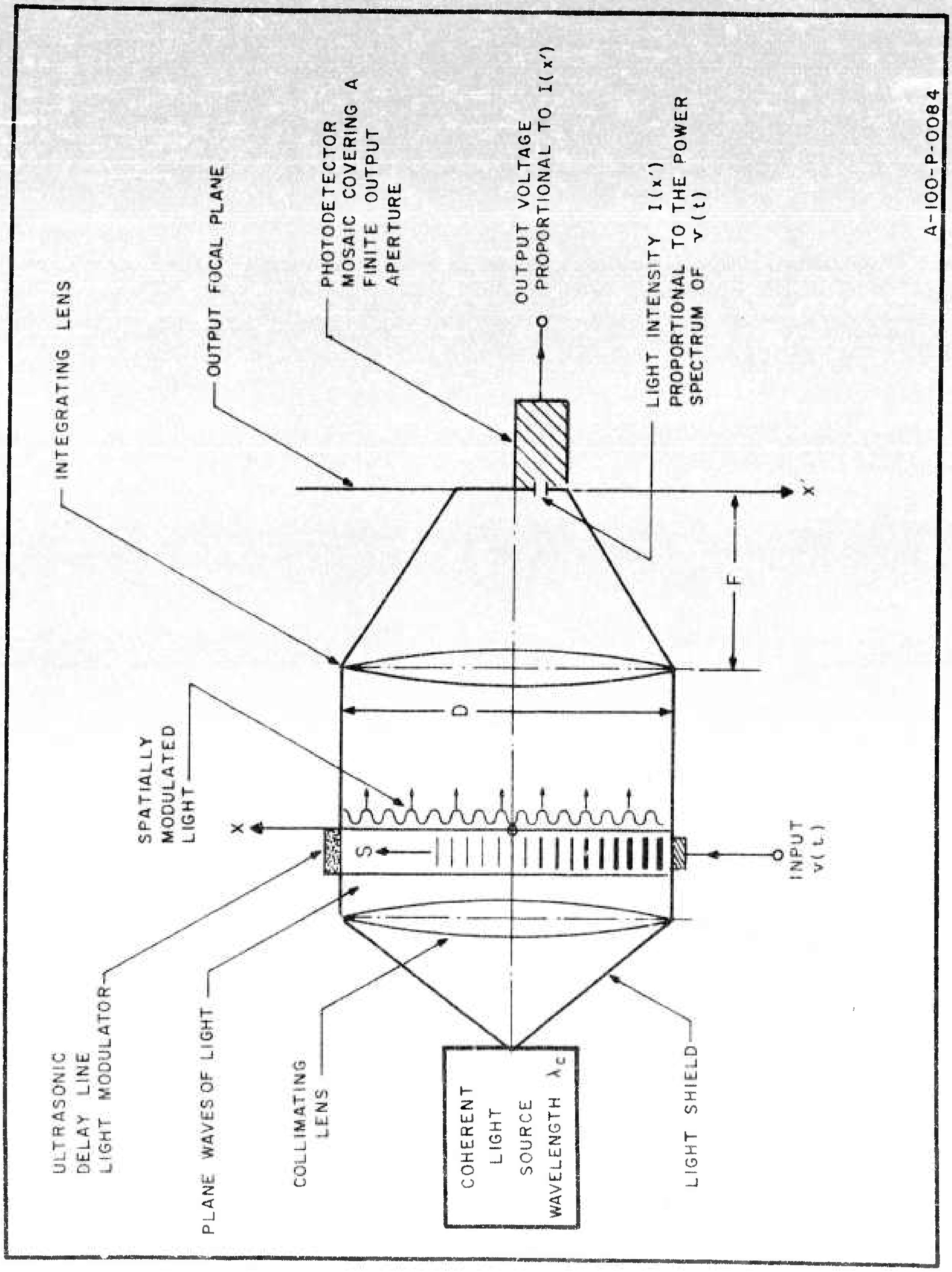

FIG. 3 ELECTRO-OPTICAL SPECTRUM ANALYZER (ONE-DIMENSIONAL)

where

$$p_T(\tau) \equiv \begin{cases} 1 & \text{for } |\tau| \leq \frac{1}{2}T \\ 0 & \text{for } |\tau| > \frac{1}{2}T \end{cases} , \tag{8b}$$

$$\psi_m = K_s V_o \equiv \text{modulation index} , \tag{8c}$$

and $T = D/S$ is the delay length of the ultrasonic light modulator for an aperture length D and a sonic velocity S.

For this one-dimensional case, a convenient output variable can be defined by

$$f \equiv uS \cong x'(S/F\lambda_L) \tag{8d}$$

so that the relative light amplitude distribution in the image plane is

$$E(f) = K \int_{-\infty}^{\infty} T(\tau) e^{-j2\pi f\tau} d\tau \quad . \tag{9a}$$

Note that τ is in units of time, f is in units of frequency and that f is proportional to actual distance x' in the image plane. Also, the output photodetector mosaic actually detects light intensity $I(f)$ so that the detected output signal $V(x')$ in Fig. 3 is proportional to

$$I(f) \equiv |E(f)|^2 \tag{9b}$$

at any instant in time.

By combining Eqs. (8) and (9) with $K = 1/T_s$ we obtain the output signal shown in Fig. 4, where $I(f)$ is plotted as a function of the output frequency variable f. This output waveform is recognized as the power spectrum of the input signal when the modulation index ψ_m permits approximating the Bessel functions $J_n(\psi_m)$ with the terms shown. In fact, the condition that $\psi_m \leq 0.3$ radian is always met for large dynamic range systems so that the output signal $I(f)$ may be considered as being the power spectrum of the input signal.

Note that the spectrum is symmetrical about $f = 0$ and that no usable information is contained in the zero order fringe which occurs at the focal point $(f = x' = 0)$. We now see that it is only necessary to detect those output signals which fall within a frequency band B. This coverage band is shown in Fig. 4 as being centered at f_o, the nominal carrier frequency of the ultrasonic light modulator. Of course, the power spectrum is not a function of the phase angle ϕ_o.

In practice, the frequency coverage B is limited by the bandwidth of the light modular. Special transducer matching techniques have been developed at CUERL[20-26,33] which produce a 50 per cent linear phase bandwidth $(B = 1/2\ f_o)$.

The integration time T and the output frequency resolution $\Delta f \cong 1/T$ are limited by the characteristics of the delay medium and/or the maximum optical aperture length D that can be used. Experimental and theoretical investigations at CUERL[20-26,33,34] has resulted in data which shows that water is a near optimum, low sonic velocity delay medium. With available diffraction-limited three-inch optics and a water delay medium, experimental results were obtained at CUERL for a spectrum analyzer configuration. These results were essentially the same as those predicted theoretically. Typically, for an integration time (T) of 50 μsec (3-inch apertures), the measured frequency resolution was $\Delta f = 20$ kcps across a phase-linear bandwidth (B) of 10 mcps centered at $f_o = 20$ mcps. This corresponds to a time-bandwidth product* of 500 with an integration efficiency close to 100 per cent.

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*The time bandwidth product of a spectrum analyzer is equal to the number of frequency resolution elements N contained within the frequency coverage, i.e., $N = B/\Delta f = TB$.

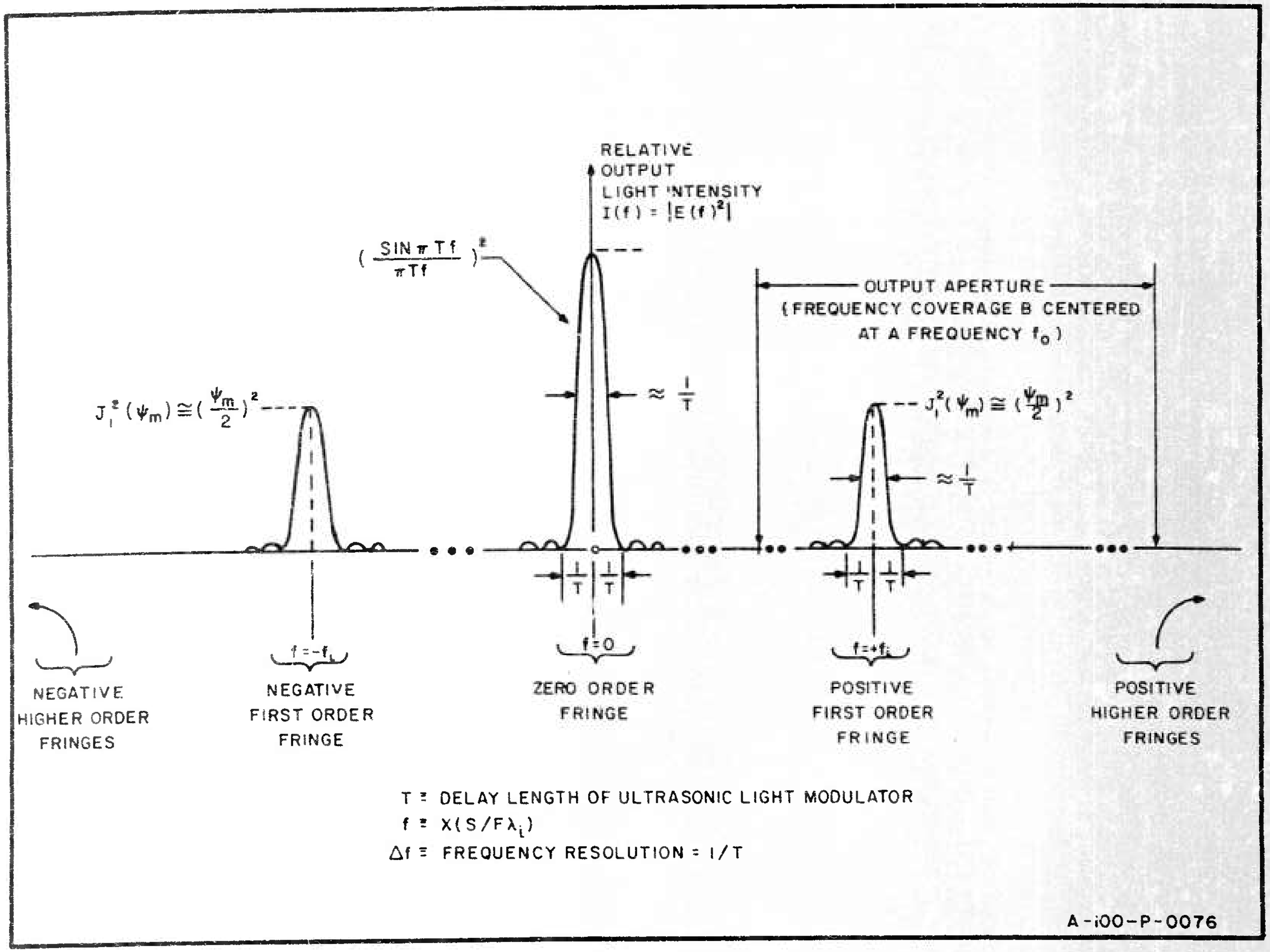


FIG. 4 LIGHT INTENSITY DISTRIBUTION IN THE OUTPUT PLANE FOR A SINUSOIDALLY EXCITED, ULTRASONIC DELAY LINE LIGHT MODULATOR

Thus, the ultrasonic delay line light modulator, incorporated into a coherent configuration with an appropriate light source and photodetector is a realizable spectrum analyzer. The entire frequency coverage appears simultaneously as a continuum at the output. Also, the relative amplitudes of the output signals within the finite frequency coverage B are closely predicted by

$$I_1(f) \equiv |E_1(f)|^2 \tag{10a}$$

with

$$E_1(f) \equiv K \int_{-\infty}^{\infty} T_1(\tau) e^{-j2\pi f\tau} d\tau \quad . \tag{10b}$$

For an output signal of the general form

$$v(t) = \sum_n V_n \cos(2\pi f_n t + \phi_n) \quad , \tag{10c}$$

the pertinent complex transmission function can be written as

$$T_1(\tau) = p_T(\tau) \sum_n a_n e^{j(2\pi f_n \tau + \phi_n)} \tag{10d}$$

where a_n is the normalized modulation index, i.e., $a_n = V_n/V_o$ and the normalizing constant is $K = 1/T$. Note that $p_T(\tau)$ specifies the integration limits (the delay length as given in Eq. (8b) for a pulse of duration T .

This electro-optical spectrum analyzer and its corresponding set of defining equations will now be applied to the problem of processing signals obtained from a linear phased array antenna. In later sections, the optical system will be extended so as to utilize its second dimension and provide processing capability applicable to planar array antennas.

B. ARRAY ANTENNA WAVEFORMS AND INHERENT CHARACTERISTICS

The waveforms that are typically obtained at the elements of a radar receiving array will now be derived. Consider a reflecting target in space which is illuminated by a transmitted signal $v_T(t)$ having unit amplitude, a duration T and a carrier frequency f_c so that

$$v_T(t) = p_T(t) \cos(2\pi f_c t) \tag{11a}$$

where

$$p_T(t) \equiv \begin{cases} 1 & \text{for} \quad |t| < \frac{1}{2} T \\ 0 & \text{for} \quad |t| > \frac{1}{2} T \end{cases} . \tag{11b}$$

The signal that is reflected by the target reaches the receiving antenna in the form of a traveling plane wave which is delayed by the round trip propagation time and shifted in frequency by the rate of change of this propagation path length. For simplicity, we first consider the linear, uniform array shown in Fig. 5.

The incident wavefront is not received simultaneously by all array elements for targets having a finite location angle θ . The signal received by the n^{th} array element is delayed (relative to element zero in Fig. 5) by an additional amount given by

$$n\tau_\theta = n(d \sin \theta / c) \tag{12a}$$

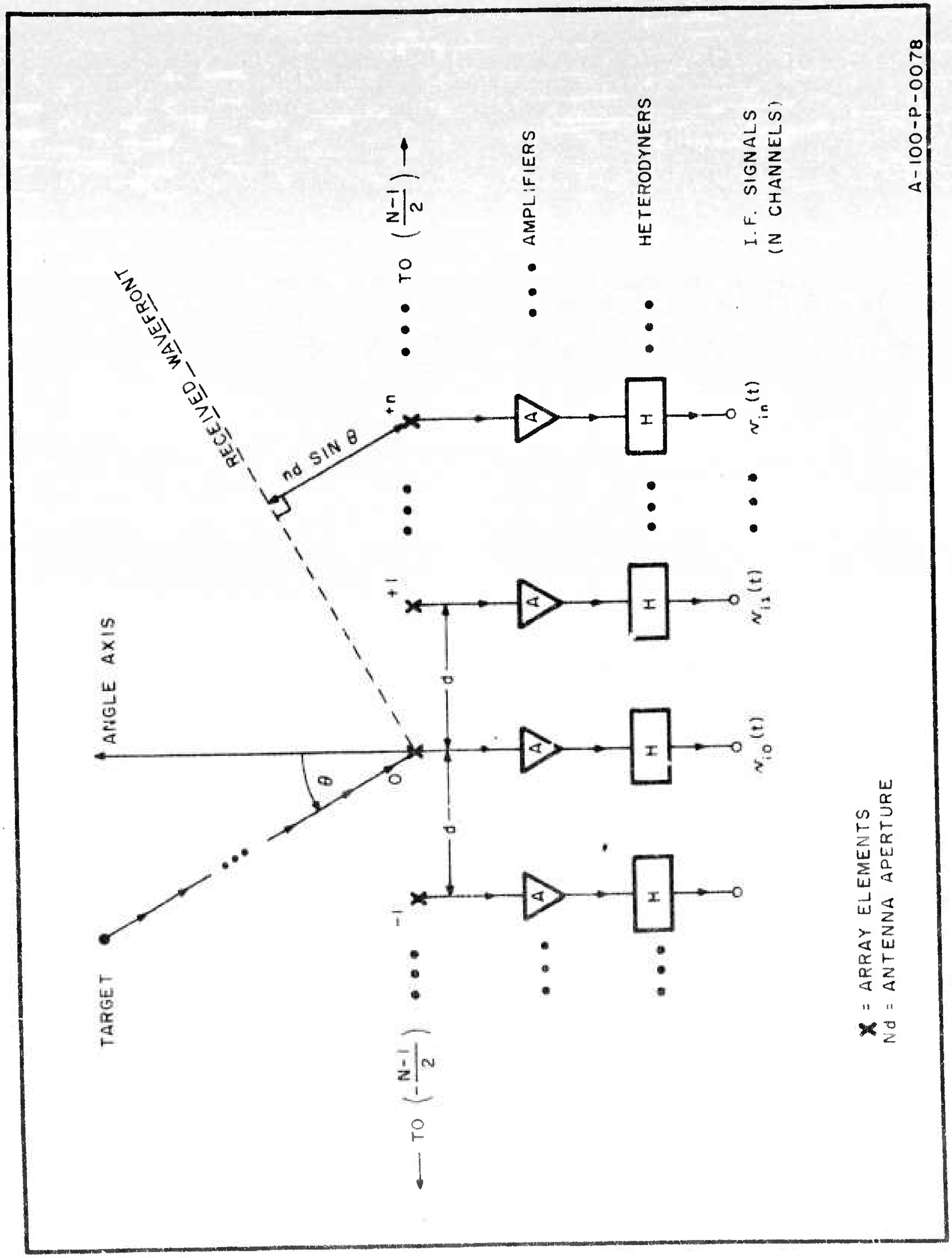


FIG. 5 LINEAR ARRAY ANTENNA SIGNALS

where d is the spacing between array elements and $C = \lambda_c f_c$ is the speed of electromagnetic wave propagation.

In addition, the received signals are amplified and then heterodyned to a convenient intermediate frequency as indicated in Fig. 5. The resulting signal obtained at the n^{th} intermediate frequency channel can be written in the form

$$v_{in}(t') = p_T(t')\cos 2\pi[(f_o+f_d)t' - n\tau_\theta(f_c-f_o) + \phi_i/2\pi] \quad (12b)$$

where
$t' \equiv t - \tau_R - n\tau_\theta$,
$\tau_R \equiv$ range delay $= 2R/C$,
$f_o \equiv$ intermediate carrier frequency ,
$f_d \equiv$ Doppler frequency $= -2\dot{R}f_c/c$,
$2R \equiv$ propagation path length ,
$R \equiv$ target range for a monostatic system, and
$\phi_i \equiv$ phase angle which is independent of θ , n , and t .

Note that the envelope (pulse) function is given by

$$p_T(t') \equiv \begin{cases} 1 & \text{for} \quad |t'| < \frac{1}{2}T \\ 0 & \text{for} \quad |t'| > \frac{1}{2}T \end{cases} \quad (12c)$$

with $t' = t - \tau_R - n\tau_\theta$. This means that the time of occurrence of any one signal is a function of target range, target location angle θ and the relative position of the array element which produced the signal. Thus, all signals do not occur simultaneously and for systems having a large aperture-bandwidth product, signals at separated elements may occur during entirely separate time intervals.

The electro-optical processing techniques described below determine the location angle θ , range R and range-rate $\dot{R}$ without loss in resolution or signal-to-noise ratio due to the progressive envelope delay τ_θ . In fact, it is shown that these processors are capable of producing all location angles θ as a continuum with the theoretically optimum resolution $\Delta\theta$ that is obtainable from the array.

For a uniform array the theoretical angle resolution (unity signal-to-noise ratio) is approximately equal to the peak-to-first null width of the beam pattern, i.e.,

$$\Delta\theta \cong \frac{1}{\frac{Nd}{\lambda_c} (\cos \theta)} \quad . \tag{13}$$

Here, (Nd/λ_c) is the antenna aperture length measured in wavelengths and $Nd \cos \theta$ is the projected antenna aperture, as seen by the received wavefront. Thus, the theoretical resolution width increases as θ increases so that, as a practical matter, θ is restricted to about $\pm$ 45 deg.

It can be shown that in order to properly form the receiving beams, $d \cong 1/2 \lambda_c$ and a large number of receiving elements N must be employed if low sidelobe levels are desired. Also, the sidelobe characteristics of the antenna can be shaped by employing either a nonuniform illumination pattern or by omitting selected receiver elements is a pseudo-random manner. Again, the electro-optical processors considered here will produce a replica of the inherent beam forming capability of the antenna including its modified sidelobe characteristics.

III. ELECTRO-OPTICAL PROCESSORS FOR LINEAR ARRAY ANTENNAS

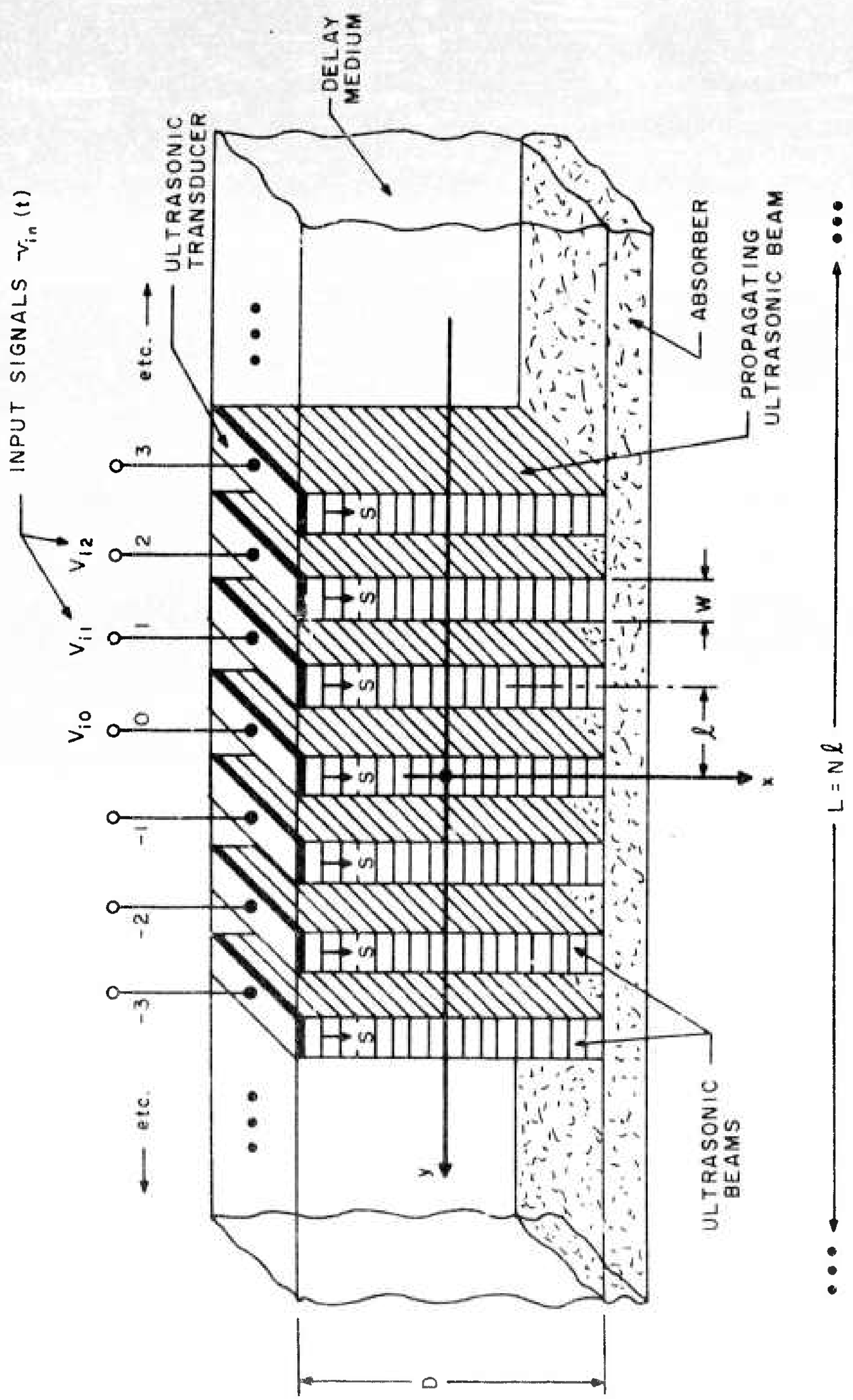
The ensemble of waveforms obtained from the elements of a linear array can be processed by employing either one of the two electro-optical techniques described below. As shown in Sec. IV, these two techniques can be advantageously combined in a single optical configuration to provide a unique processing capability for planar array antennas. In all cases, the inherent characteristics of the array antennas are realized directly as output signals from the processors.

A. SPATIAL MULTIPLEXING TECHNIQUE

A linear array antenna consisting of N elements produces N waveforms of the type described by Eq. (12b). In order to properly process these signals to obtain the location angles θ, ranges R and range rates $\dot{R}$ of each of a multitude of targets, the basic electro-optical spectrum analyzer technique described in Sec. II-A will be employed. Since the array produces an ensemble of N waveforms and since three-dimensional information (θ, R and $\dot{R}$) is desired, all dimensions of the electro-optical processor must be utilized.

The ultrasonic light modulator (Fig. 2) is essentially a one-dimensional light modulator since no spatial variations were introduced in the y dimension. This second dimension will now be exploited to significantly enhance the signal processing capability of these electro-optical systems. The basic technique that appears particularly applicable to phased array processors consists of filling the y dimension of the object plane with separate and parallel ultransonic channels as shown in Fig. 6. Each channel now consists of a piezoelectric transducer of width W which produces its own distinct ultrasonic traveling wave. The separation between adjacent channels is ℓ and the optical aperture dimensions are L and D as shown. Note that the signal duration contained in the light modulator is $T = D/S$ and the number of phased array elements that are processed is $N = L/\ell$.

The spatially multiplexed, ultrasonic light modulator is placed in the optical configuration shown in Fig. 7. This configuration can be seen to be the two-dimensional extension of the spectrum analyzer shown in Fig. 3. Also, since we are only concerned with a finite (detected) area in the image plane which contains the desired radar information, the defining



OPTICAL APERTURE: L x D
ULTRASONIC BEAMS: WIDTH W; SPACING ℓ; PROPAGATION SPEED S

FIG 6 A SPATIALLY MULTIPLEXED LIGHT MODULATOR

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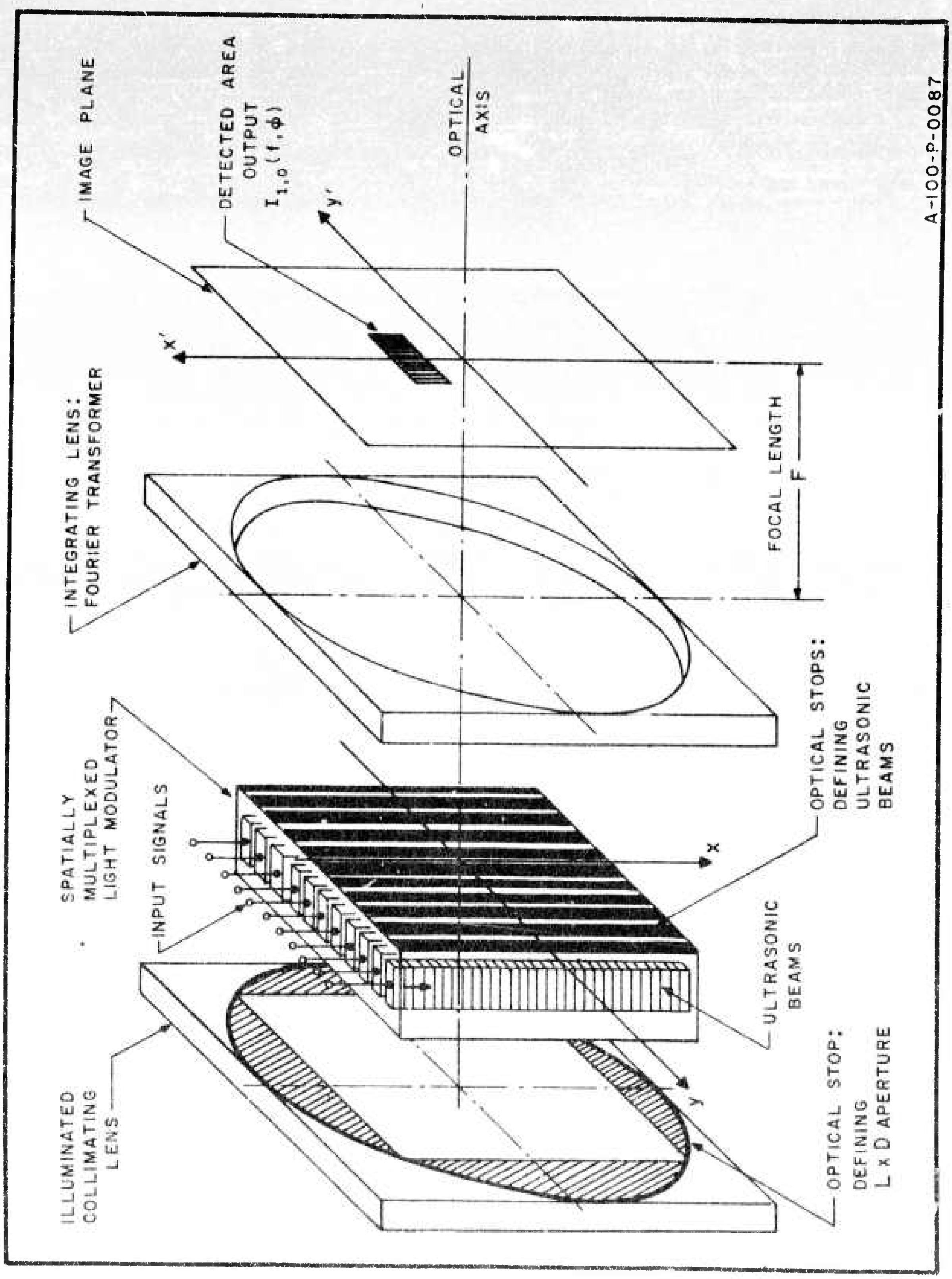


FIG. 7 A SPATIALLY MULTIPLEXED, ELECTRO-OPTICAL PROCESSOR

transfer functions contained in Eqs. (10) can be applied. A direct extension of Eqs. (10) to two dimensions, leads us to define two normalized input parameters;

$$\tau \equiv x/S \ , \ \gamma \equiv y/\ell \tag{14}$$

and two normalized output parameters;

$$f \equiv x'(S/F\lambda_L) \ , \ \phi \equiv y'(\ell/F\lambda_L) \tag{15}$$

where (x, y) are object plane dimensions, (x', y') are image plane dimensions, S and ℓ are ultrasonic beam parameters and F and λ_L are optical parameters as defined in Figs. 6 and 7.

Each of the signals $v_{in}(t)$, as given by Eqs. (12), acts as an input to a corresponding ultrasonic transducer. The resulting output signals obtained from a detector, such as a photodetector mosaic, placed at the image plane are proportional to

$$I_1(f, \phi) = |K \int_{-\infty}^{\infty}\int T_1(\tau, \gamma) e^{-j2\pi(f\tau+\phi\gamma)} d\tau\, d\gamma|^2 \tag{16}$$

where $T_1(\tau, \gamma)$ is the complex transmission function produced by the spatially multiplexed light modulator. This transmission function essentially consists of the sum of the transmission functions produced by the signals exciting the separated ultrasonic channels.

Again, Eqs. (10c) and (10d) are applicable, and for the phased array waveforms of Eqs. (12), we obtain

$$T_1(\tau, \gamma) = \sum_{n=-\frac{N-1}{2}}^{+\frac{N-1}{2}} \left\{ p_{\frac{W}{\ell}}(\gamma-n) p_T(\tau-n\tau_\theta) \right. \tag{17a}$$

$$\left. \cdot\, e^{j2\pi[(f_o+f_d)(\tau-n\tau_\theta)-n\tau_\theta(f_c-f_o)]} \right\}$$

where the pulse (envelope) functions are defined by

$$p_A(b) \equiv \begin{cases} 1 & \text{for } |b| < \frac{1}{2}A \\ 0 & \text{for } |b| > \frac{1}{2}A \end{cases} \quad . \tag{17b}$$

In Eqs. (17), $p_{W/\ell}(\gamma-n)$ defines the physical location and extent of the nth ultrasonic channel in the y dimension and $p_T(\tau-n\tau_\theta)$ defines the location and extent of the same channel in the x dimension. Note that this last pulse function describes the differential delay τ_θ that is introduced by the phased array.

Combining Eqs. (16) and (17), and performing the indicated mathematical operations with the normalizing constant $K = \ell/WTN$, we obtain

$$I_1(f, \phi) = \Big| \operatorname{sinc}[f-(f_o+f_d)]T \operatorname{sinc} \phi W/\ell \tag{18}$$

$$\cdot \sum_{n=-\infty}^{\infty} \operatorname{sinc}[\phi+(f+f_c-f_o)\tau_\theta-n]N \Big|^2$$

where

$$\operatorname{sinc} z \equiv \frac{\sin \pi z}{\pi z} \quad .$$

Equation (18) describes the two-dimensional distribution of the output light intensity in the vicinity of the first order fringe which occurs at $f = f_o + f_d$. It can be shown that all possible target location angles $|\theta| < 90$ deg and all possible Doppler frequencies f_d are contained unambiguously within an output area for which $n = 0$.

Thus, all signals are present within a finite detected output area and each target produces a signal of the form

$$I_{1,o}(f, \phi) = \left| \operatorname{sinc} [f-(f_o+f_d)]T \operatorname{sinc} \phi W/\ell \right. \tag{19}$$

$$\left. \cdot \operatorname{sinc} [\phi+(f+f_c-f_o)\tau_\theta]N \right|^2 .$$

This output signal has a single intensity peak which is located at $f = f_i$, $\phi = \phi_M$ where

$$f_i = f_o + f_d \tag{20a}$$

and

$$\phi_M = -\tau_\theta(f_c + f_d) \simeq -(\sin\theta)\left(\frac{d}{\lambda_c}\right) . \tag{20b}$$

A representation of the output signal that appears in the image plane is shown in Figs. 8 and 9. By measuring the coordinates of the output signal we obtain ϕ_M and f_i. From these two measurements, we immediately obtain the location angle θ and the Doppler frequency f_d from Eqs. (20) since f_o, d, λ_c and f_c are known system parameters. Also, target range R is obtained since this peak signal only occurs at a time $t = \tau_R = 2R/C$.

The angle resolution $\Delta\theta$, Doppler resolution Δf_d and range delay resolution $\Delta\tau_R$ can be obtained by examining the fine structure of the light intensity distribution within the detected output area. The cuts A - A' and B - B' in Fig. 8 can be most conveniently used to plot light intensity as a function of ϕ and f respectively as shown in Fig. 9. This figure shows the output response for a single target located at $\theta = 45$ deg which has a finite Doppler frequency f_d. From Eqs. (19) and (20), we obtain the resolution width $\Delta\phi_M$ parallel to the ϕ axis as

$$\Delta\phi_M = \frac{1}{N} . \tag{21a}$$

However, by differentiating Eq. (20b) we obtain

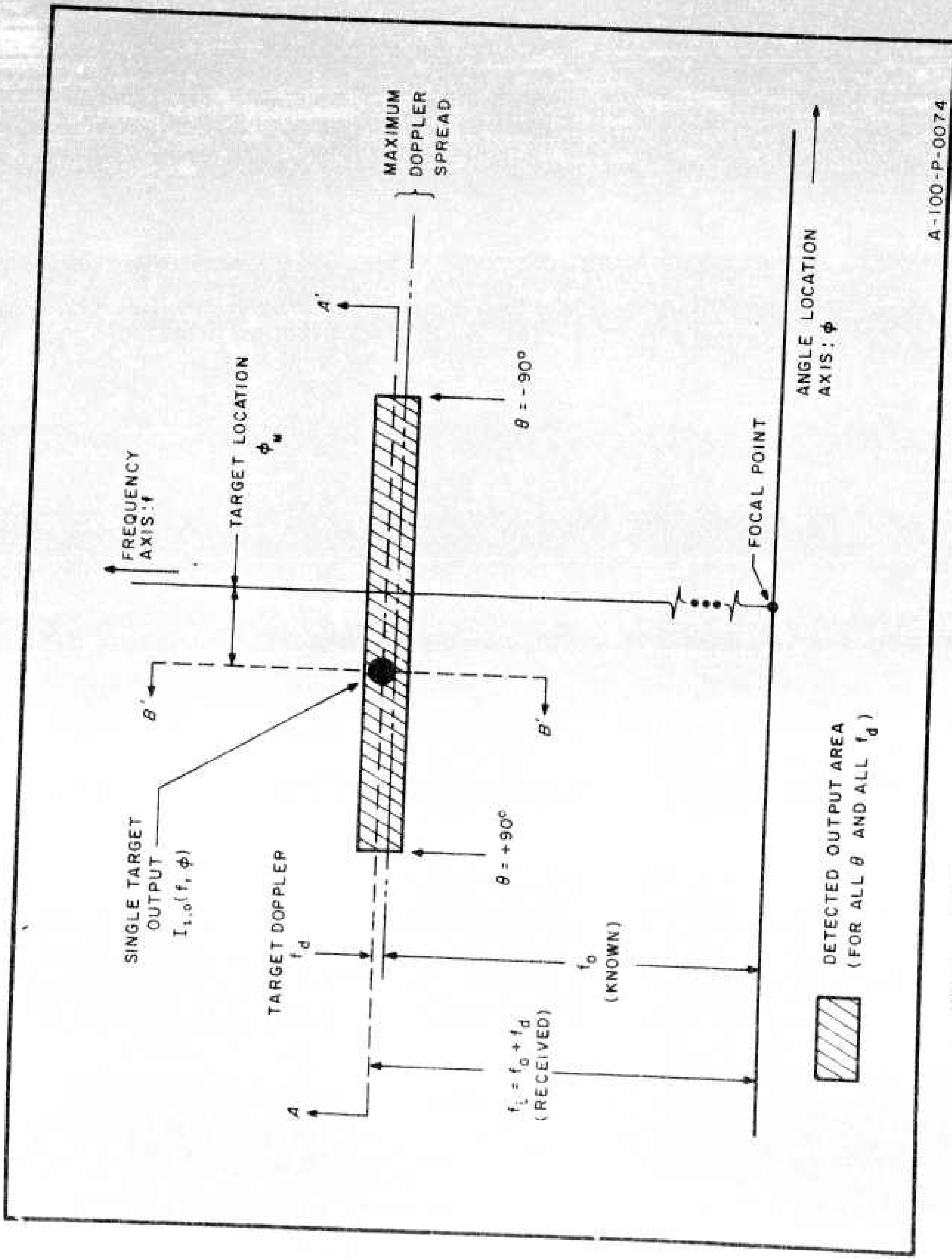
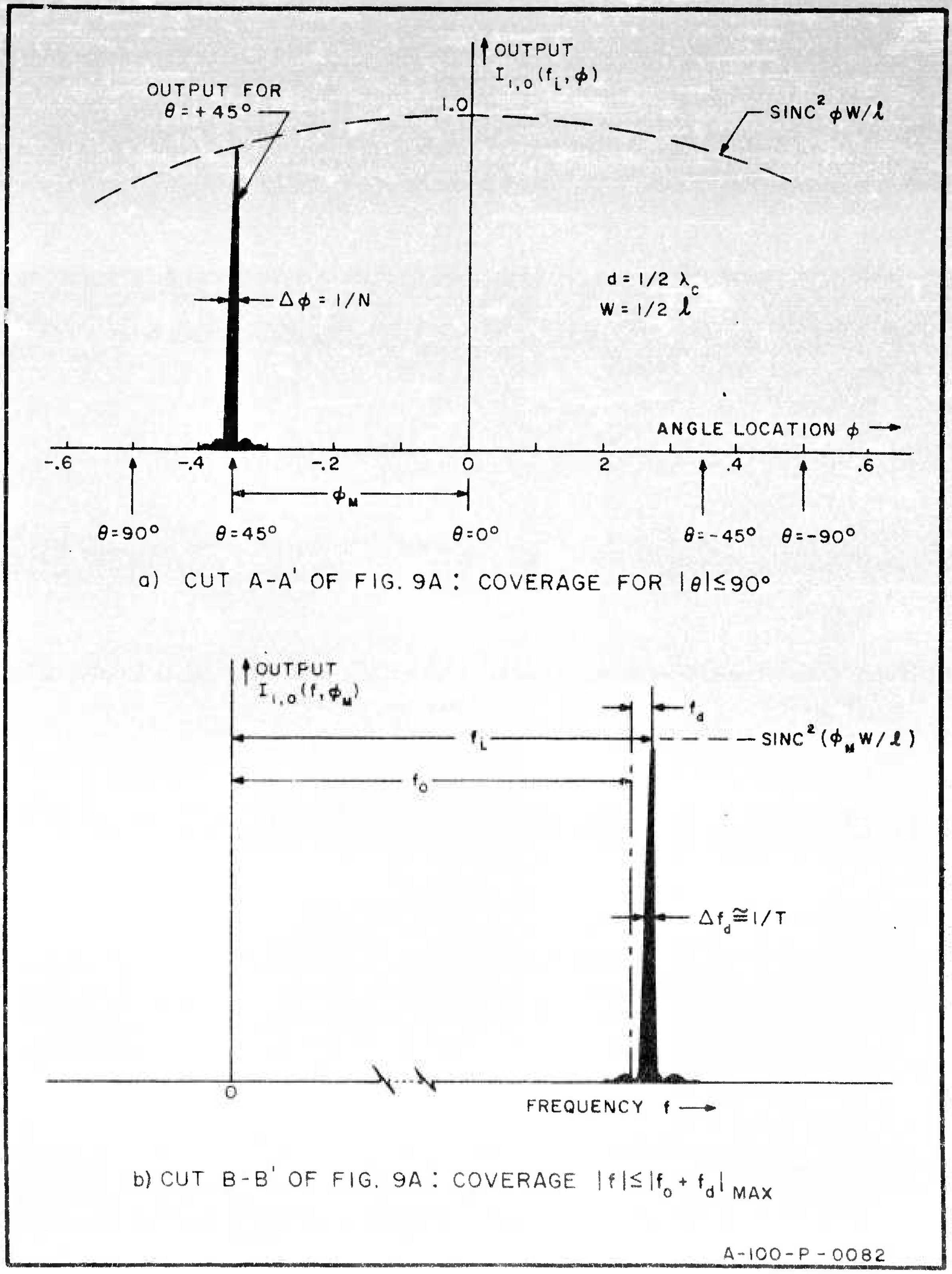


FIG. 8 OUTPUT FROM A SPATIALLY MULTIPEXED PROCESSOR



a) CUT A-A' OF FIG. 9A : COVERAGE FOR $|\theta| \leq 90°$

b) CUT B-B' OF FIG. 9A : COVERAGE $|f| \leq |f_0 + f_d|_{MAX}$

FIG. 9 OUTPUT FINE STRUCTURE FOR A SPATIALLY MULTIPLEXED, ELECTRO-OPTICAL PROCESSOR

$$\Delta\phi_M \cong (\cos\,\theta)(\frac{d}{\lambda_c})\Delta\theta \tag{21b}$$

so that the angle resolution is

$$\Delta\theta \cong \frac{1}{(\frac{Nd}{\lambda_c})(\cos\,\theta)} \quad . \tag{21c}$$

Thus, the output angle resolution is the same as the inherent resolution of the antenna. Eq. (13). Similarly, the output side-lobe characteristics can be shown to be a replica of the side-lobe characteristics of the antenna. The output shown in Fig. 9 is for a uniform array, resulting in the characteristic sinc-function response. For any other aperture weighting function, the output will be modified and remain a replica of the true side-lobe characteristics of the antenna. In fact, aperture weighting can be directly incorporated into the electro-optical processor by introducing shading masks to shape the side-lobes characteristics.

Doppler resolution is obtained in a similar fashion as shown in Fig. 9b. Again, from Eqs. (19) and (20) the output resolution width Δf_d parallel to the f axis is given directly by

$$\Delta f_d \cong \frac{1}{T} \tag{22}$$

which is the optimum frequency resolution obtainable after coherently integrating a signal of duration T. In addition, it can be shown that the range (delay) resolution, as contained in the time response of the output light intensity, is given by

$$\Delta\tau_R \cong T \quad . \tag{23}$$

This is the same as the range (delay) resolution inherent in a finite duration pulse.

The following conclusions can be formed from these derived results: (1) The target location angle θ is obtained unambiguously for each and every target that can be resolved. (2) Target angle resolution is identical to the resolution inherent in the antenna array. (3) All antenna beam angles appear simultaneously as a continuum in a form which is identical to the beam forming capability and side-lobe response of the antenna array. (4) In addition to angle measurement, the second dimension provides Doppler frequency. (5) Doppler frequency is obtained unambiguously with a resolution which is identical to that obtained from an ideal coherent integrator. (6) Angle and Doppler appear at the output completely associated in orthogonal dimensions. (6) The time of occurrence of each output signal is proportional to the range of each target.

B. TIME-DELAY MULTIPLEXING TECHNIQUE

The second electro-optical processing technique utilizes only one dimension of the light modulator. This is accomplished by forming a single waveform from the ensemble of waveforms obtained from a linear array as shown in Fig. 10.

Each of the signals of finite duration T obtained from an N element linear array (Eqs. (12)) is passed through a delay line and the delayed signals are then summed. The delay lines are designed to provide a constant incremental delay difference T_D between adjacent channels, i.e., $T_{n+1} - T_n \equiv T_D$. The incremental delay T_D is chosen so that the summed waveform $v_s(t)$ consists of a series of separated pulsed carriers, the nth pulse representing the delayed signal from the nth array element. This is accomplished when

$$T_D = T + |\tau_\theta|_{max} = T + \left(\frac{d}{c}\right) |\sin\theta|_{max} \quad . \qquad (24)$$

Typically, for $|\theta| \leq 90$ deg and $d/\lambda_c = 1/2$, we have $T_D \simeq T + 1/2f_c \simeq T$ since the radar carrier frequency f_c is much greater than the radar bandwidth $1/T$.

The summed waveform $v_s(t)$ now consists of N pulsed carriers, each of duration T and separation $T_s \equiv T_D + \tau_\theta$ so that the total signal duration is NT_s. The form of this signal is obtained from Eq. (12b) after the indicated delay and summation is performed, i.e.,

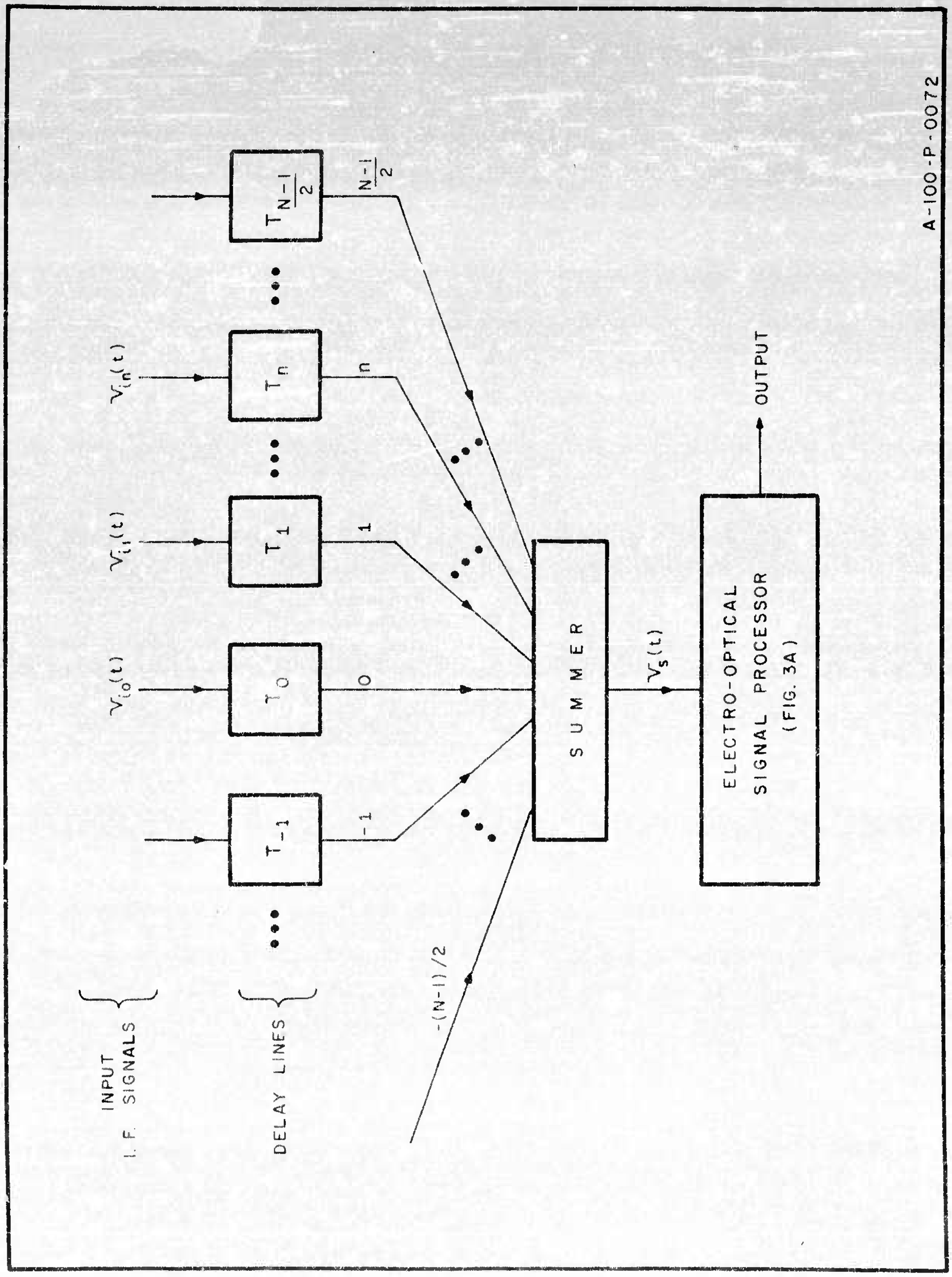


FIG. 10 TIME-DELAY MULTIPLEXED ELECTRO-OPTICAL SIGNAL PROCESSOR

$$v_s(t) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left\{ p_T(t-\tau_R-n\tau_\theta-T_n) \right. \tag{25}$$

$$\left. \cdot \cos 2\pi[(f_o+f_d)(t-\tau_R-n\tau_\theta-T_n)-n\tau_\theta(f_c-f_o)+\phi_i/2\pi] \right\} .$$

This waveform now acts as the input signal to a single channel, ultrasonic light modulator and coherent optical configuration as was shown in Fig. 3. In this case, Eqs. (10) are directly applicable since integration occurs in only one dimension. The contributing complex tran.mission function of interest can be shown to be given by

$$T_1(\tau) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left\{ p_T(\tau-nT_s) \right. \tag{26}$$

$$\left. \cdot\ e^{j2\pi[(f_o+f_d)(\tau-nT_s)-n\tau_\theta(f_c-f_o)]} \right\}$$

where τ is the normalized object plane variable $(\tau = x/S)$ at the time instant when the entire signal is present in the light modulator and $p_T(\tau-nT_s)$ represents the duration T of the nth pulse which is located at $\tau = nT_s$.

The resulting output signal $I_1(f)$ is now given by Eqs. (10a) and (10b) as the power spectrum of $v_s(t)$. Performing the indicating operations and normalizing, we obtain

$$I_1(f) = \left| \operatorname{sinc}(f-f_o-f_d)T \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left[f + \frac{(f_c-f_o)\tau_\theta}{T_s} - \frac{n}{T_s} \right] NT_s \right|^2 . \tag{27}$$

The first sinc function is located at $f \equiv f_i = f_o + f_d$ and it has a peak-to-null width of $1/T$. The remaining sinc functions are located at

$$f_n = \frac{[n - (f_c - f_o)\tau_\theta]}{T_s} \tag{28}$$

$$= \frac{[n - (f_c - f_o)\tau_\theta]}{[T_D + \tau_\theta]}$$

and they have a peak-to-null width of

$$\Delta f_M \equiv \frac{1}{NT_s} = \frac{1}{N(T_D + \tau_\theta)} \tag{29}$$

as shown in Fig. 11.

Since f_o is the (known) intermediate frequency, the frequency difference $f_M = f_o - f_n$ can be measured directly in the image plane and from Eq. (28) we obtain

$$f_M = \frac{(f_c\tau_\theta + f_o T_D - k)}{(T_D + \tau_\theta)} \quad . \tag{30a}$$

where k is an integer such that $f_M \leq 1/2T$. For convenience, we can select f_o so that $f_o T_D = k$ and Eq. (30a) simplifies to

$$f_M = \tau_\theta \left(\frac{f_c}{T_D + \tau_\theta}\right) \tag{30b}$$

so that

$$\sin\theta = \frac{f_M\left(\frac{\lambda_c}{d}\right) T_D}{\left(\frac{1 - f_M}{f_c}\right)} \quad . \tag{30c}$$

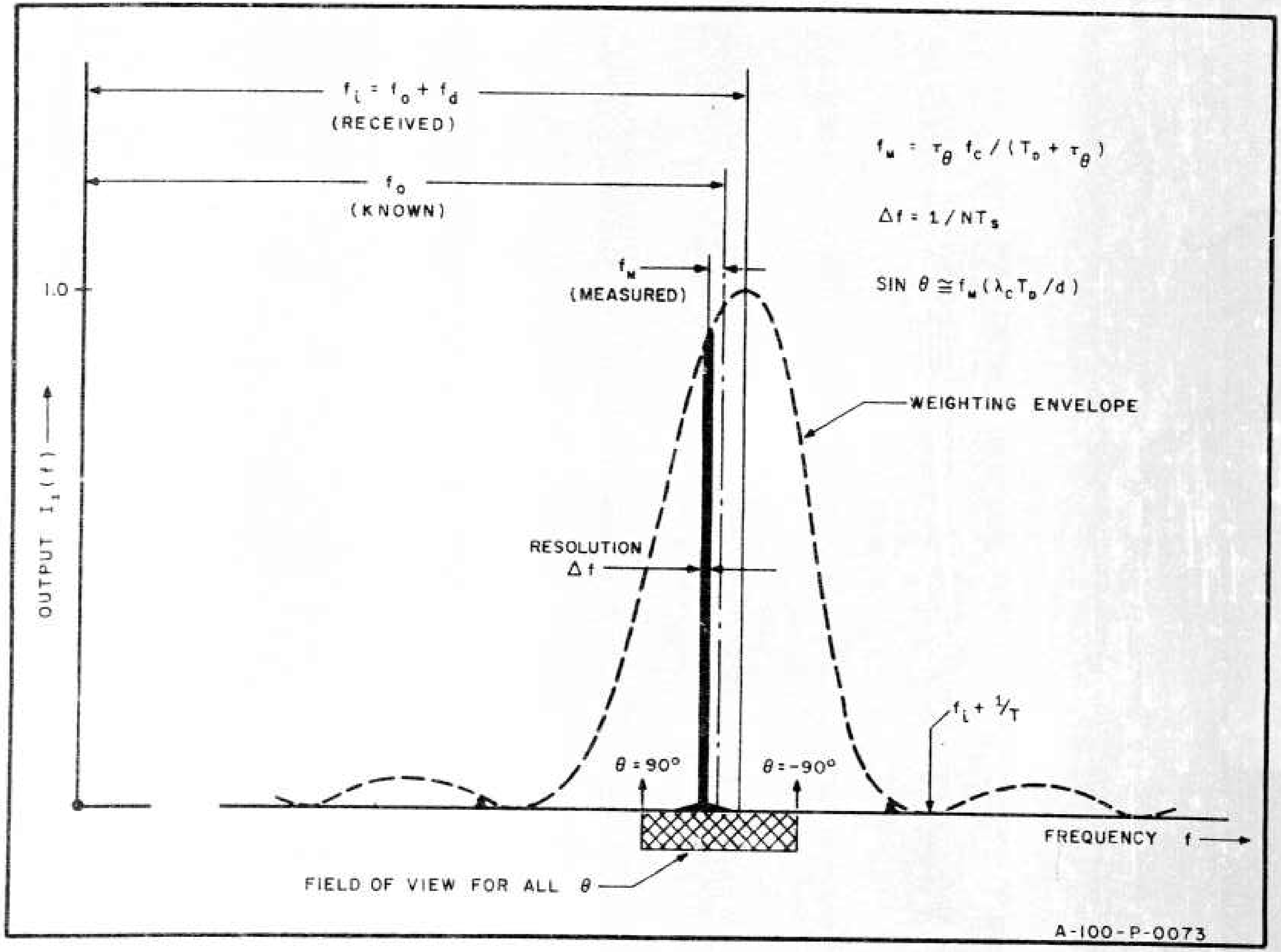


FIG. 11 OUTPUT FROM A TIME-DELAY MULTIPLEXED ELECTRO-OPTICAL SIGNAL PROCESSOR

Thus, the target location angle θ is uniquely determined by measuring the fringe displacement f_M in the image plane.

It should be noted that since $T_D = T + |\tau_\theta|_{max}$, the denominator of Eq. (30b) can never be larger than $T + 2|\tau_\theta|_{max}$. However, $2|\tau_\theta|_{max} < 1/f_c$ for $|\theta| < 90$ deg and for practical radar transmitters, $f_c \gg 1/T$. Thus, $T_D + \tau_\theta \cong T_D$ and we can write

$$f_M \cong \tau_\theta\left(\frac{f_c}{T_D}\right) \tag{30d}$$

so that

$$\sin\theta \cong f_M\left(\frac{\lambda_c T_D}{d}\right) \quad . \tag{30e}$$

This approximation is valid independent of the number of array elements N and the array aperture Nd that is employed.

The angle resolution $\Delta\theta$ is obtained by differentiating Eq. (30e) with respect to θ and combining this result with Eq. (29). This gives

$$\Delta\theta = \frac{1}{\left(\frac{Nd}{\lambda_c}\right)\cos\theta}$$

which is the theoretical angle resolution obtainable from an array antenna as was previously derived in Eq. (13).

The following conclusions can be drawn from these derived results: (1) Within the field of view required to include all possible location angles ($|\theta| \leq 90$ deg), no ambiguities appear. (2) Within this field of view, all antenna beam angles appear simultaneously as a continuum in a form which is identical to the beam forming capability of the antenna array. (3) By measuring the location "frequency" f_M of the output signal, the physical location angle θ of the target is obtained. (4) The output resolution width $\Delta f_M = 1/NT_s$ corresponds to an angle resolution $\Delta\theta$ which is the same as that obtained directly from the phased array. (5) All responses are weighted with an amplitude function centered at the frequency of the input signals ($f_i \equiv f_o + f_d$). (6) The frequency location of the output signal is not a function of Doppler frequency.

IV. ELECTRO-OPTICAL PROCESSOR FOR PLANAR PHASED ARRAYS

By combining the time-delay and the spatial multiplexing techniques in a single electro-optical device, two-dimensional angle information can be obtained. Typically, a planar array consisting of N columns and M rows is processed by employing an N channel light modulator which has each channel time-delay multiplexed M times. Such a configuration is shown schematically in Fig. 12.

A RESPONSE CHARACTERISTICS

Each element in the array has its signal amplified, heterodyned and delayed. The resulting signals from each column are then summed (hence, time-multiplexed) in exactly the same manner as was previously discussed (see Fig. 10). The signal derived from the n^{th} column of the array is fed to the n^{th} ultrasonic transducer of the spatially multiplexed, electro-optical spectrum analyzer (see Figs. 6 and 7). The two-dimensional output response from this processor is similar to the output shown in Fig. 8 except that each ultrasonic channel is now time multiplexed. As a result, it can be seen that the output obtained in the f dimension is similar to that shown in Fig. 11. Thus, if the narrow beam dimension formed by the rows of elements in the phased array is called θ and the orthogonal angle is called ψ (formed by columns), then these angles appear at the output of the spectrum analyzer as measured values ϕ_M and f_M as shown in Fig. 13.

As a direct consequence of the results presented in previous sections, we can conclude that: (1) All possible two-dimensional beam angles are obtained simultaneously as a continuum. (2) The angle resolutions are identical to the resolutions which are theoretically available from the antenna. (3) Multiple targets will produce multiple outputs with all angles properly associated. (4) The incremental delay between signals (τ_0) that is introduced by the phased array is processed so as not to result in an aperture-bandwidth constraint. (5) A two-dimensional matrix of delay lines or phase shifting networks is not required. (6) The number of delay lines required for time multiplexing, equals the number of array elements (M different delay lines repeated N times). (7) No ambiguities are contained within the output coverage region and the side lobes of any output signal are identical to the side lobes produced by the antenna. (8) This processing technique permits

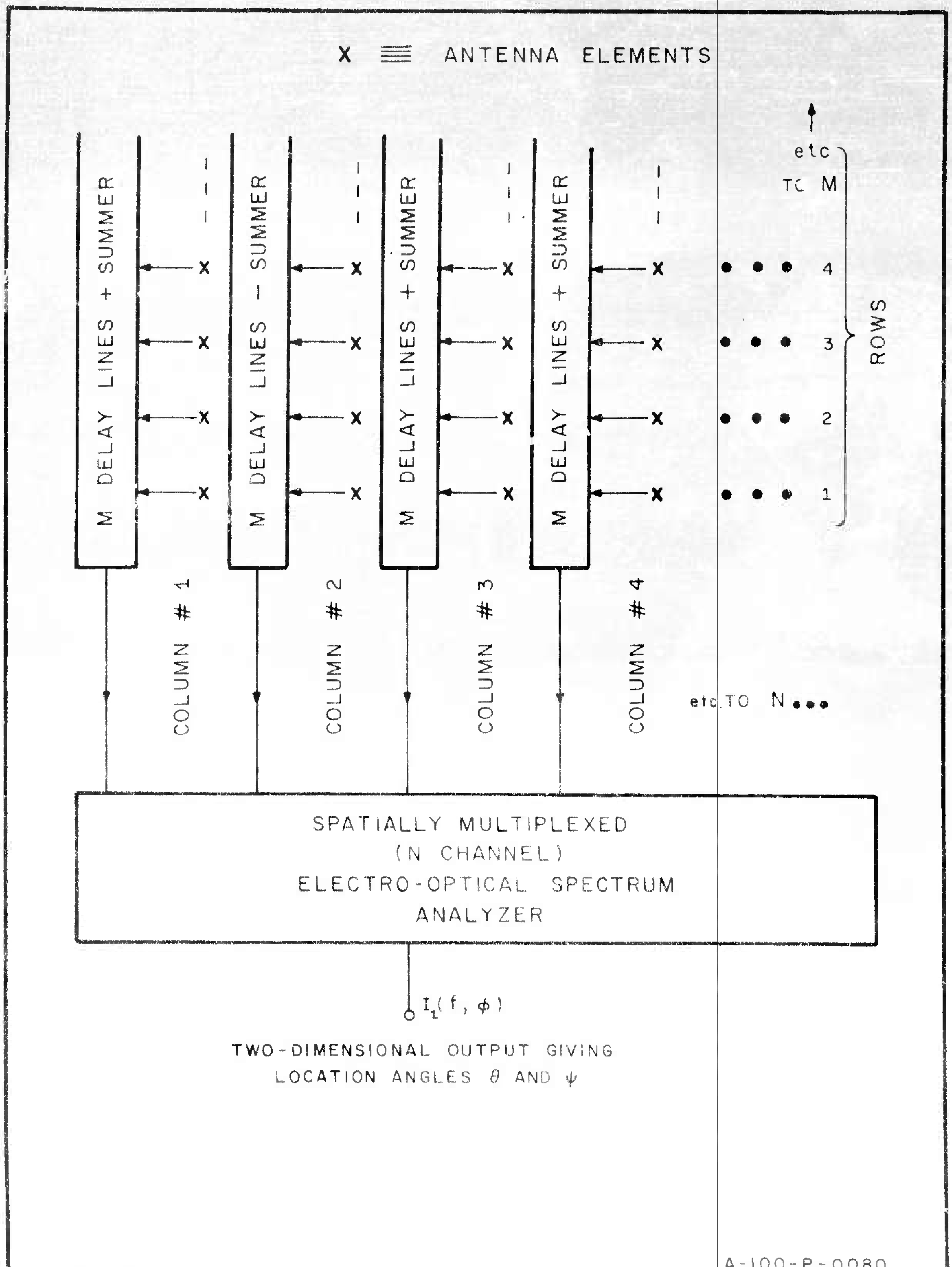


FIG. 12 PLANAR ARRAY PROCESSOR

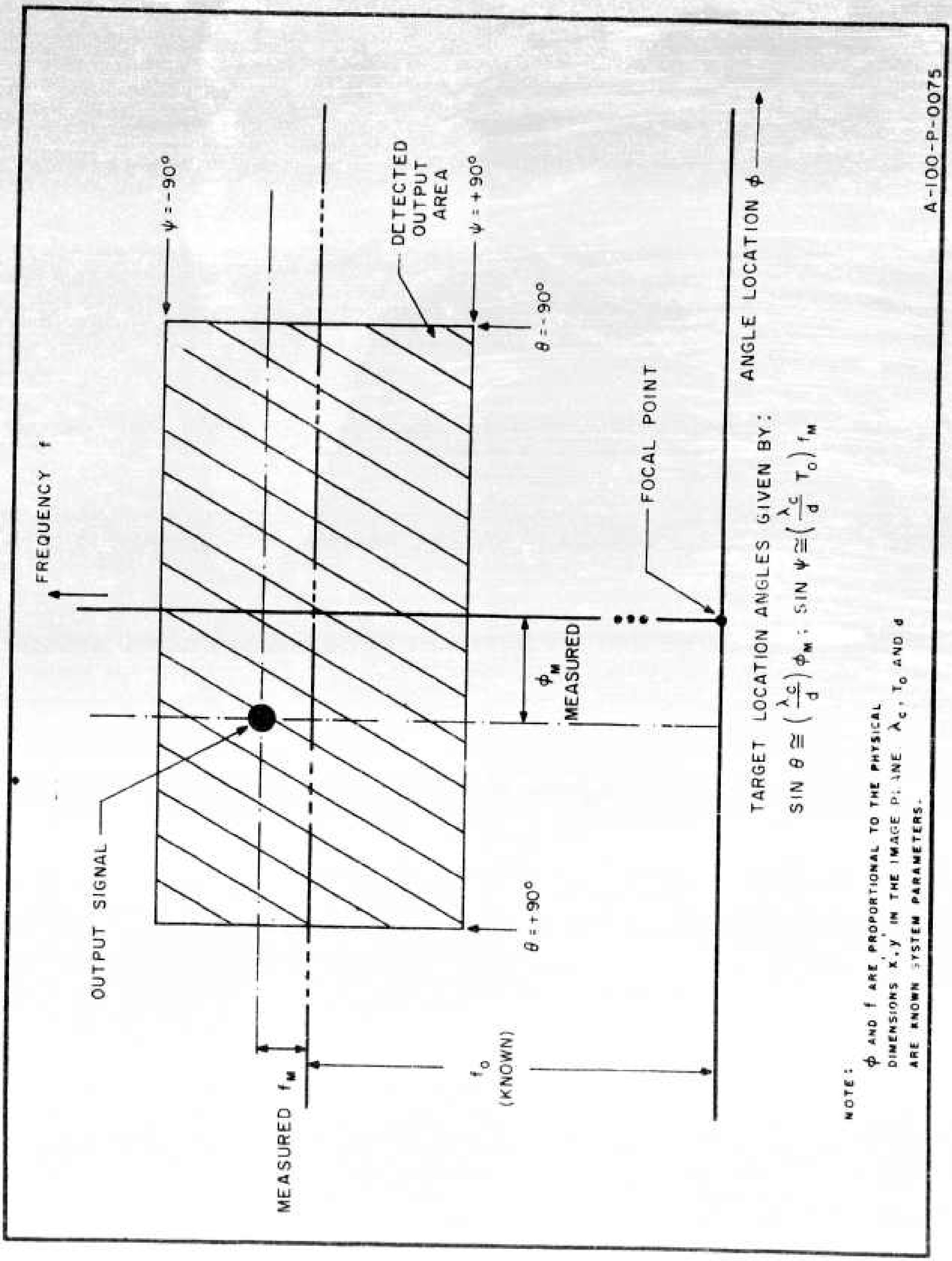


FIG. 13 TWO-DIMENSIONAL OUTPUT FROM A PLANAR ARRAY PROCESSOR

full use of antenna beam shaping functions (amplitude taper, phase taper, random element distributions, etc.) (9) Additional beam shaping and side-lobe suppression can be directly introduced into the optical aperture in the form of shading masks and stops. (10) Pulse compression can be employed by transmitting a frequency modulated carrier and introducing variable pitch gratings into the optical subsystem of the processor.[20-27]

B. TYPICAL SYSTEM PARAMETERS FOR SQUARE ARRAYS

Consider a square, planar array antenna consisting of N^2 elements with an element spacing of one-half wavelength. Let this antenna operate as a radar receiver, obtaining signals of finite duration (T) from a multitude of reflecting targets which are located at various ranges and at various location angles in space. The signal processor for this antenna would consist of a time-delay multiplexing network which accepts the N^2 signals from the array and produces N new time waveforms. These signals are then applied as inputs to an N channels, spatially multiplexed, electro-optical signal processor. The output of this processor is a direct measure of the range and the (two-dimensional) location angles of each target.

The signal pulse duration (T) and the number of array elements (N^2) that can realistically be processed may be constrained by the following parameters of the electro-optical processor: (1) The maximum optical aperture dimension that can be utilized while maintaining a diffraction limited condition, (2) the maximum obtainable bandwidth and carrier frequency of the light modulator (3) the effect of ultrasonic attenuation on output resolution and side-lobe response, and (4) the maximum number of ultrasonic transducers that can be accommodated in the optical aperture.

The aforementioned experimental results obtained at CUERL indicate that processor parameters are governed by the following considerations: (1) Optical configurations with a 3-inch (7.5 cm) clear aperture can be maintained diffration limited. (2) Light modulator carrier frequencies up to 30 mcps in a water delay medium have been successfully used. (3) Light modulator bandwidths from 5 per cent (airbacked) to 100 per cent (mercury backed) of the carrier frequency have been measured. (4) Immersion of the ultrasonic transducer in the water delay medium (water backing) produces a 10-per cent bandwidth. (5) Ultrasonic matching-section techniques have been developed for single transducer configurations, producing a 50 per cent linear phase bandwidth with low insertion loss.

A realistic set of initial parameters for a multiplexed electro-optical processor would, therefore consist of the following: (1) a 3-inch optical aperture, (2) at light modulator employing a water delay medium, (3) a carrier frequency no greater than 30 mcps and, (4) a minimum bandwidth of 2 mcps. In this case, ultrasonic attenuation is negligible and signal durations from 0.5 μsec to 50 μsec can be processed. The number of ultrasonic transducers that can be employed is essentially determined by the acoustic beam spreading which occurs when transducers of small width W are employed. A typical design criterion requires that adjacent ultrasonic beams do not overlap and that beam spreading does not result in an amplitude taper which significantly degrades the system response.

The design equations which result from these considerations can be shown to be given by

$$N^2 \cong (f_o L^2/4S^2T)^{2/3} \quad , \qquad (31a)$$

$$D = NTS \leq L \quad , \qquad (31b)$$

and

$$W \cong L/2N = \ell/2 \qquad (31c)$$

where N^2 is the total number of square array elements that can be processed, f_o is the ultrasonic carrier frequency, L and D are the optical aperture dimensions, S is the sonic propagation speed in the delay medium, T is the (received) signal pulse duration, W is the ultrasonic transducer width and ℓ is the spacing between transducers.

The aforementioned initial design parameters specify $f_o \cong 25$ mcps, L = 7.5 cm = 3 inches, S = 15×10^4 cm/sec and a 2-mcps bandwidth. For a signal duration T = 0.5 μsec, we obtain the result that $N^2 \cong (32)^2 = 1024$, D = 2.5 cm, W $\cong$ 1.15 mm and $\ell \cong 2.3$ mm. Furthermore, by adapting the aforementioned ultrasonic transducer matching technique to the spatially multiplexed configuration, the signal pulse duration could be reduced to 0.1 μsec (a 10-mcps bandwidth). This would extend the processing capability to $N^2 \cong (55)^2 = 3025$ elements.

To extend the processing capability further, the carrier frequency f_o and/or the optical aperture dimension L must be increased. In either case, ultrasonic attenuation in water can be shown to introduce an additional constraint expressed by

$$f_o{}^2 NT \leq 4 \times 10^{10} \quad . \tag{32}$$

By meeting this constraint, it can be shown that the system response will not be significantly different from that obtained with no acoustic attenuation. Typically, to process a 10,000 element square array ($N = 100$) with a 3-inch optical aperture and a 50 per cent ultrasonic bandwidth, acoustic attenuation affects (Eq. (32)) require that $T \geq 0.01$ µsec. Now, from Eq. (31a) we obtain $T \cong 0.07$ µsec so that $f_o \cong 28$ mcps and the system response is not degraded by acoustic attenuation.

Thus, electro-optical processor considered is capable of producing the location angles of multiple targets with optimum resolution and side-lobe response. Starting with an inherent processing capability for 1000 array elements with a system bandwidth of 2 mcps, the ultrasonic light modulator can be further developed to accommodate 10,000 array elements with a system bandwidth of 14 mcps.

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