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THEORETICAL STUDY OF THE ROLLING RESPONSE OF AIRCRAFT TO TURBULENT AIR

by

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SUMMARY

A method is presented for assessing the lateral responses of aircraft to continuous turbulence. The method is applied to the study of rolling behaviour in turbulent air of a range of aircraft configurations.

It is shown that it is essential to include the pilot's control in this study.

Estimates are given of the aileron angle required to stabilize the aircraft in turbulent air and the effects of forward speed and aircraft size, and two simple autostabilization systems are briefly investigated.

A comparison is made between the continuous turbulence and discrete gust approaches.

It is pointed out that there is an absolute lack of experimental data against which to check the theory.

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THEORETICAL STUDY OF THE ROLLING RESPONSE OF AIRCRAFT TO TURBULENT AIR

J.K. Zbrozek*

1. INTRODUCTION

At present, there is no satisfactory method for estimating lateral aircraft responses and control requirements in turbulent air. It is very difficult, if not almost impossible, using a discrete gust approach to assess the lateral behaviour in turbulent air of an aircraft with a more advanced aerodynamic design for which the dynamic properties, including derivatives such as l_v , can be widely different from those of aircraft of the past generation. The difficulty is that the shape and magnitude of side gusts are unknown. The distribution of discrete vertical gusts, as derived from vertical acceleration measurements, cannot be used for lateral response studies without very serious reservations. First of all, the derived vertical gusts still contain some dynamic longitudinal properties of the aircraft which, on an assumption of dynamic longitudinal similarity, may not affect appreciably the estimates of vertical accelerations, but cannot reliably be used for lateral response studies, as the dynamic properties of the aircraft in longitudinal and in lateral planes might be widely different. Furthermore, although for longitudinal response the turbulence can often be treated satisfactorily as a one-dimensional phenomenon, this cannot be true for lateral responses, where the rolling effects of vertical gusts cannot a priori be neglected in comparison with the effects of side gusts.

A considerable amount of theoretical work has been done in the U.S.A. to study the transfer function of an aircraft in response to three-dimensional turbulence[1-5]. The object of this Report is to extend these studies to a form more suitable for practical application. Only rolling response is studied, but some thought is given to the problem of the aileron power required in turbulent air. Also, an attempt is made to compare the discrete gust and continuous turbulence approaches. In an attempt to study the effect of the dynamic properties of the aircraft in its response to turbulence, three types of aircraft are considered. The first, aircraft A, could represent a present-day subsonic fighter (Fig.3). The second, 4A, has exactly the same properties as aircraft A except that it is scaled up linearly four times, in order to study the effect of aircraft size in relation to turbulence scale. The third aircraft, B (Fig.8), represents an advanced aerodynamic shape and is based on a small experimental aircraft.

The present work can be readily applied to the studies of responses other than in roll and is particularly well suited for application to autostabilization systems.

2. ASSUMPTIONS

Regarding turbulence the assumptions made in this paper are basically those made in Reference 1:

**United Kingdom*

(1) The turbulence can be regarded as stationary;

(2) The turbulence is homogeneous and isotropic. With these assumptions, the cross-correlation between vertical, w_g, and lateral v_g, gust components is zero; thus the responses due to v_g and w_g are statistically independent and can be computed separately and added directly;

(3) Time correlation is equivalent to space correlation (Taylor's hypothesis);

(4) The space correlation (and corresponding spectral densities) of atmospheric turbulence are satisfactorily defined by[6]:

$$\left.\begin{aligned} f(r) &= e^{-r/L} \\ g(r) &= \left(1 - \tfrac{1}{2}\frac{r}{L}\right)e^{-r/L} \end{aligned}\right\} \qquad (1)$$

and the parameter L is taken as $L = 1000$ ft.

It should be mentioned that the above assumptions cannot be valid throughout the whole range of wavelengths of interest, and a few words of explanation may not be out of place.

(1) The assumption of stationarity is only valid within some finite body of air and thus, for this assumption to be valid, the wavelengths of the aircraft response must be small when compared with the dimensions of the body of air. This suggests that the theory may not be valid at very low frequencies.

(2) The assumption of isotropy can only be valid for wavelengths less than some given value. The available experimental evidence suggests that the atmospheric turbulence can be regarded as isotropic up to a wavelength of 2000 to 5000 ft and probably longer. There is also an indication that at least at low altitudes the lateral component of turbulence (with respect to wind axes) is somewhat more intense than the other components[7]. This violates the assumption of isotropy which requires that $\overline{u_g^2} = \overline{v_g^2} = \overline{w_g^2}$.

(3) There is a considerable body of experimental evidence which suggests that Taylor's hypothesis is valid, at least at reasonably high speeds. Nevertheless, the interpretation of Taylor's hypothesis at very low speeds (VTOL and STOL aircraft) is rather difficult at the present moment.

(4) Although the expressions for the correlation functions fit the available measurements reasonably well, they are only semi-empirical approximations. The theoretical objections to the non-vanishing slopes as $r \to 0$ from the expressions in Equation (1) are not very important in aircraft applications, as they refer to wavelengths much too short to affect aircraft responses noticeably.

The parameter L used in these functions, which is sometimes called the turbulence scale, is difficult to define. Its value depends on the balance between the energy input and energy transport of the turbulence. The assumed value of $L = 1000$ ft, which is commonly used, is open to doubt, as it appears that a better description of atmospheric turbulence can be obtained using the expressions in Equation (1) and a value of L larger than 1000 ft.

It should be noticed that all the above reservations about the assumptions refer to long wavelengths (or low frequencies), say $\lambda > 2000$ to 5000 ft. It is fortunate or perhaps unfortunate that at subsonic speeds, as considered in this paper, these wavelengths correspond to periods longer than say 3 - 10 secs. At these very low frequencies the input of the pilot is probably much more important than the input of the turbulence, and thus the computed responses neglecting pilot's action are of no real practical significance.

The assumptions regarding the aircraft are as follows:-

(1) The aircraft is rigid;

(2) There is no coupling between longitudinal and lateral modes of motion;

(3) No unsteady lift effects are taken into account. (This overestimates the response at high frequencies.);

(4) The rolling moment input due to a spanwise distribution of the vertical gust velocity w_g is defined by a rolling moment spectrum obtained from Reference 1;

(5) The rolling and yawing input moments due to a lateral (with respect to aircraft axes) gust v_g are defined by the instantaneous gust velocity and its first derivatives at the aircraft C.G. Thus the random distribution along the X-axes is neglected. This effect is believed to be small, at least for aircraft which are small in comparison with the turbulence scale (say with L);

(6) The side-force input is neglected. The side force has a negligible effect on the response, as is demonstrated in Reference 3.

(7) No effect of head-on u_g gust gradient is included but some consideration of this effect is given later. It is known[3] that this effect is small.

3. EQUATIONS OF MOTION

The conventional rigid-body lateral equations of motion are used. The equations, the nomenclature and the abbreviated lateral derivatives are those of Reference 8, (see also Table I).

The equations of motion with an input of a non-dimensional side gust, $\hat{v}_g = v_g/V$, are as follows:

$$\left.\begin{array}{lr}
(D + \bar{y}_v)\hat{v} \qquad\qquad + \hat{r} - \tfrac{1}{2}C_L\phi & = 0 \\
\omega_l\hat{v} + (D + \nu_l)\hat{p} - (e_A D + \nu_{lr})\hat{r} & = -\omega_l\hat{v}_g \\
-\omega_n\hat{v} + (-e_c D + \nu_{np})\hat{p} + (D + \nu_n)\hat{r} & = (\omega_n - \nu_n D)\hat{v}_g \\
\hat{p} & = D\phi
\end{array}\right\} \qquad (2)$$

The term $\nu_n D\bar{v}_g$ in the yawing equations represents the effect of the rate of change of the non-dimensional gust $\hat{v}_g$, which is approximately equal to a non-dimensional rate of yaw $D\hat{v}_g = \hat{r} = D\psi$.

The equations of motion with a non-dimensional rolling moment due to a vertical gust as input are similar except for the right hand side:

$$\left.\begin{array}{lr}
(D + \bar{y}_v)\hat{v} \qquad\qquad + \hat{r} - \tfrac{1}{2}C_L\phi & = 0 \\
\omega_l\hat{v} + (D + \nu_l)\hat{p} - (e_A D + \nu_{lr})\hat{r} & = l \\
-\omega_n\hat{v} + (-e_c D + \nu_{np})\hat{p} + (D + \nu_n)\hat{r} & = 0 \\
\hat{p} & = D\phi
\end{array}\right\} \qquad (3)$$

where the non-dimensional rolling moment l is defined as:

$$l = \frac{\text{rolling moment}}{\frac{i_A}{\mu_2}\,\rho\, S\, \frac{b}{2}\, V^2} \qquad (4)$$

By putting $D = i\omega$, the above equations are solved for the modulus squared of bank angle $|\phi|^2$, as needed for spectral calculations.

The squared moduli of the transfer functions are expressed as functions of the inverse of wavelength, $1/\lambda$, as this quantity is most suitable for the expressions of input spectral densities. Thus, we have two squared moduli of the transfer functions of the bank angle ϕ:

$$\left|\frac{\phi}{\hat{v}_g}\left(\frac{1}{\lambda}\right)\right|^2 \quad \text{due to a unit non-dimensional lateral gust } \hat{v}_g = v_g/V \qquad (5)$$

and

$$\left|\frac{\phi}{l}\left(\frac{1}{\lambda}\right)\right|^2 \quad \text{due to a unit non-dimensional rolling moment } l \text{ due to a vertical gust} \qquad (6)$$

To obtain the spectral densities of the angle of bank, ϕ, the transfer functions have to be multiplied by the appropriate spectral density of the inputs; thus

$$S_{\phi_v}\left(\frac{1}{\lambda}\right) = \left|\frac{\phi}{\hat{v}_g}\left(\frac{1}{\lambda}\right)\right|^2 \times G\left(\frac{1}{\lambda}\right) \times \frac{1}{V^2} \times \overline{v_g^2} \tag{7}$$

$$S_{\phi_w}\left(\frac{1}{\lambda}\right) = \left|\frac{\phi}{l}\left(\frac{1}{\lambda}\right)\right|^2 \times S_l\left(\frac{1}{\lambda}\right) \times 2L\mu_2^2\, \nu_l^2\, \frac{1}{V^2} \times \overline{w_g^2} \tag{8}$$

where the spectral densities of the angle of bank ϕ are in rad²/cycles per foot. The spectral density of the lateral gust $G(1/\lambda)$ for unit turbulence R.M.S. $\overline{v_g^2} = 1$ is given

$$\frac{G\left(\frac{1}{\lambda}\right)}{\overline{v_g^2}} = 2L\,\frac{1 + 3\left(2\pi\frac{L}{\lambda}\right)^2}{\left[1 + \left(2\pi\frac{L}{\lambda}\right)^2\right]^2} \tag{9}$$

The parameter L is assumed to be 1000 ft.

The term $1/V^2$ is required to change the input from the non-dimensional form $\hat{v}_g = v_g/V$ to the dimensional v_g in ft/sec.

The non-dimensional spectral density of the rolling moment $S_l(1/\lambda)$ is obtained from Reference 1 for the appropriate ratio of wing span to the turbulence parameter L.

The term $2L\mu_2^2\,\nu_l^2$ is necessary to convert the analysis from a non-dimensional to the dimensional form.

Figures 1 and 2 show the spectral density functions $G(1/\lambda)$ and $S_l(1/\lambda)$. $G(1/\lambda)$ is computed for $L = 1000$ ft and $S_l(1/\lambda)$ interpolated from Figure 7 of Reference 1 for three ratios of b/L as used in the present paper; $L = 1000$ ft is assumed. Figure 7 of Reference 1 shows spectra of rolling moment for a wing with rectangular span loading. This figure was chosen as the only one which gives small enough values of b/L for the present study. This choice is perfectly justified, as it has been shown that the span loading had only a very small effect on the rolling moment due to a vertical gust spectrum[1]. This is due to the non-dimensionalisation, where the spectrum is non-dimensionalised by dividing it by l_p, the rolling moment due to rate of roll derivative, and the response is multiplied again by l_p (ν_l in Equation (8)).

The final spectral density of the angle of bank, S_ϕ, is obtained by adding Equations (7) and (8), thus:

$$S_\phi\left(\frac{1}{\lambda}\right) = S_{\phi_v}\left(\frac{1}{\lambda}\right) + S_{\phi_w}\left(\frac{1}{\lambda}\right) \tag{10}$$

It should be stressed that in all computation it has been assumed, on the basis of Reference 7, that

$$\overline{v_g^2} = 1.4\,\overline{w_g^2} \qquad (11)$$

This assumption is appropriate for aircraft being flown along the mean wind axis at low altitude, which is of interest at low speeds corresponding to the landing or take-off phase. Furthermore, this assumption violates the condition of isotropy. It is felt that this violation will introduce some errors at low frequency, where some cross-correlation between v_g and w_g components has to be expected. It is thought that the present approach is the best that can be used in the light of existing knowledge, but more experimental work is needed before some of the reservations raised in this paper can be clarified.

4. BANK ANGLE RESPONSE OF AIRCRAFT A

A set of calculations have been made for a typical aircraft at three speeds: 150 knots, 200 knots and 580 knots respectively at sea level. The layout of the aircraft is shown in Figure 3 and the stability derivatives used in the calculations and the characteristic equations are given in Table I. This particular type of aircraft was chosen in the hope that some experimental verifications of the present study can be obtained from flight measurement on a similar aircraft.

Figure 4 shows the total spectrum density of the angle of bank S_ϕ and its two components S_{ϕ_w} and S_{ϕ_v} due to vertical and lateral components of atmospheric turbulence respectively.

The effect of the head on component u_g of turbulence is neglected in the present calculations. It can be shown that the ratio of the spectral density due to the vertical component S_{ϕ_w} to the spectrum due to the head on component S_{ϕ_u} is approximately

$$\frac{S_{\phi_w}}{S_{\phi_u}} \approx \frac{0.2}{\alpha_0^2} \qquad (12)$$

where α_0 is the incidence measured from the no-lift angle. It can be seen that the head-on gust component is significant only at high incidences, but even at $\alpha_0 = 10^\circ$ its contribution is about 15% of the vertical gust contribution which on its own is again only important at very low frequencies*.

From an inspection of Figure 4 it is evident that the contribution of the vertical gust to the bank angle spectrum is dominant at low frequencies, but from frequencies about half of the Dutch roll frequency this contribution becomes negligible. The vertical gust contribution will depend upon the size of the aircraft measured by the ratio b/L, span to turbulence scale, but, as will be shown in Section 5 even for an aircraft with $b = 140$ ft (4 times the size of aircraft A), the contribution of

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*For further discussion of this problem, and for the discussion of response quantities other than ϕ, see Reference 3.

w_g is roughly of the same proportions. Figure 5 shows the total spectral densities of the angle of bank, S_ϕ, for aircraft A at three speeds. For comparison, a similar spectrum is replotted from Figure 3(a) of Reference 3. The results of Reference 3 refer to an aircraft of similar dynamic characteristics as the present aircraft A, but flying at 412 knots at 30,000 ft. It should be noticed that all the spectra of Figure 5 show a remarkable degree of consistency. There is a large peak corresponding to the lightly damped mode of the Dutch roll and a very large increase in spectrum value at low frequencies due to the presence of the spiral mode, which is usually very lightly damped.

Due to the presence of the large contribution at low frequencies to the total variance of the angle of bank, σ_ϕ^2, the realistic estimates of aircraft behaviour in turbulent air need some discussion. It is known that at very low frequencies, say below 0.02 c.p.s. the aircraft banking will be corrected automatically by the pilot, without his being even conscious of it. This low-frequency banking will appear as a lack of lateral trim and will be obscured entirely in turbulent air by the aircraft behaviour at higher frequencies. It will be shown later that, to remove the low-frequency contribution, very little aileron angle is required, demonstrating again that this is a problem of trimming.

It is felt that more realistic comparisons between the responses to turbulent air of different aircraft and of the same aircraft but flying at different speeds can be made by removing the low-frequency components. It is proposed to introduce the term 'SIMPLY CONTROLLED' response, where all the responses at frequencies below 0.2 c.p.s. are removed by the pilot. It means then, e.g., that the estimates of the angle of bank R.M.S., $(\sigma_\phi)_c$, are obtained by integration of the spectrum from a frequency $f = 0.2$ up to infinity. This is a somewhat arbitrary limit, but it is felt that at periods longer than 5 seconds the pilot can control the non-violent (no peaks in spectrum) motion of the aircraft quite successfully, and for periods longer than say 10 seconds he can almost entirely stabilise the aircraft motion without special conscious effort.

It might be mentioned that we are facing a similar problem when estimating the vertical gust loads. In the estimates of the aircraft longitudinal responses to turbulent air we must account for the pilot reaction, if the contribution of the low frequencies cannot be regarded as negligible. This situation will arise when a large aircraft is flown with a small static margin.

For comparison with 'SIMPLY CONTROLLED' responses, some estimates of aircraft response are given, including the whole spectrum. This condition corresponds to a flight with controls fixed, and thus will be termed 'FIXED' responses. There is little physical meaning in the 'FIXED' responses, except maybe for free flight models, as at these low frequencies not only can the pilot's behaviour not be neglected, but also our knowledge of the atmospheric turbulence spectrum is very limited.

It should be noted (Fig.4) that, under 'CONTROLLED' conditions, the banking response of the aircraft is almost entirely determined by the lateral component, v_g, of the atmospheric turbulence, at least for this particular aircraft configuration.

From the response spectra of Figure 5, the following quantities have been estimated: R.M.S. of the angle of bank, controls fixed $(\sigma_\phi)_F$ and under pilot's control $(\sigma_\phi)_C$, the R.M.S. of the rate of roll $(\sigma_p)_F$ and $(\sigma_p)_c$, for controls fixed and under pilot's control, and the number per second of zero crossings with positive slope $(N_0)_F$ and $(N_0)_c$ (Ref.7). The results are shown in Table II, and are computed for the turbulence intensity, measured by the R.M.S. of the vertical component, $\overline{w_g^2} = 1$ (f.p.s.)^2. It can be seen from Table II that the contribution of low frequencies to the value of σ_ϕ is very large, especially at 580 knots, as the result of a poor spiral stability. The effect on the rate of roll R.M.S., σ_p, is small, as could be expected.

It can be seen from Table II that, under 'CONTROLLED' conditions, the bank angle response to the turbulent air increases with decreasing speed, which is a well known fact. A just opposite effect would be obtained if the whole spectrum were taken into account in the estimates of σ_ϕ; this is again the effect of spiral mode stability, which deteriorates with increasing speed (decreasing C_L).

To provide a better illustration of the numbers in Table II, Figure 6 has been prepared where the angle of bank expected to be reached on average within a given time is shown for a turbulence intensity $\overline{w_g^2} = 1$ (f.p.s.)^2. It can be seen that for this particular aircraft no large angles of bank under controlled conditions are to be expected even in a very severe turbulence, say $\left(\overline{w_g^2}\right)^{1/2} = 10$ f.p.s. Under these turbulence conditions no angle of bank greater than 30° can be expected within 10 minutes when flying at 150 knots. The bank response at high speed is apparently about three times as small as at low speeds. Under 'controls fixed' conditions the aircraft shows very large bank angle responses, which indicates that at high speed constant attention to controls is required; this, however, as will be shown later, does not demand large aileron power.

5. EFFECT OF AIRCRAFT SIZE; THE BANK ANGLE RESPONSE OF AIRCRAFT OF CONFIGURATION A

It is expected that aircraft size will have an effect on its response to turbulent air. To investigate this effect, the response calculations have been repeated for an aircraft similar to aircraft A but with a span four times as large, viz. b = 140 ft. The general layout of this larger aircraft, designated now 4A, is shown in Figure 3, and all the non-dimensional derivatives like n_v, l_v, l_p, etc. are identical with aircraft A. The forward speed used for comparison was 200 knots. The relevant data pertaining to the aircraft 4A are shown in Table I. It can be noticed that the spiral mode and roll mode time constants are almost identical with aircraft A, but the Dutch roll frequency is lower and damping higher, as one would expect. The estimates of the R.M.S.'s of angle of bank and of rate of roll are presented in Table II, for simply controlled flight (below 0.2 c.p.s.) and for control-fixed conditions. It is immediately apparent that the response, as measured in terms of bank angle and of rate of roll, is much smaller for the large aircraft than for the small one, even with controls fixed. Under 'controlled' flight conditions the difference is even bigger and, in terms of the angle of bank attained within a given time, the response of the large aircraft is of one order smaller than the response of the small aircraft.

It might be noted (Fig.7) that due to the definition of 'controlled' flight (cut-off frequency 0.2 c.p.s.) a large part of the Dutch roll (0.158 c.p.s.) contribution to response is removed by the pilot.

The contribution of the vertical gusts to the overall response of the large aircraft is mainly at periods longer than 10 seconds and thus apparently within the frequency range under the pilot's control.

6. BANK ANGLE RESPONSE OF AIRCRAFT B

It is of interest to study the response to turbulent air of an aircraft with dynamic characteristics similar to those of possible future aircraft shapes. For that purpose an aircraft B was chosen, the layout of which is shown in Figure 8. This is a small experimental aircraft representative both aerodynamically and inertially of slender-wing designs. There is some concern about the lateral behaviour of this type of aircraft in turbulent air and it was thought worthwhile to extend the spectral study to this configuration.

The stability derivatives used in the calculations and the roots of the characteristic equation are shown in Table I. It should be noticed that the time constant of the spiral mode is quite small, indicating reasonable stability in this mode and easy control at low frequency. However, the damping of the Dutch roll is rather poor, being only 4% of critical at $C_L = 0.2$. This lack of damping may lead to undesirable responses in turbulent air. Figures 9(a) to 9(c) show the spectral-densities of bank angle and its components for three values of C_L (three forward speeds) at sea level. It is interesting to notice that at low frequencies (long wavelength) the response due to the lateral component of turbulence v_g is always appreciable in comparison with the response due to the vertical component w_g , and thus in that respect aircraft B is very different from the more conventional aircraft A. It is thought that this is due to two effects, good spiral stability and small span reducing the response to w_g , and a larger value of l_v increasing response to v_g .

Figure 10 shows the spectra of the total response in angle of bank for the three values of C_L . This figure should be compared with Figure 5 for aircraft A. It can be seen that, although at low frequencies the response of aircraft B is somewhat smaller than that of aircraft A, in the vicinity of the Dutch roll frequency the response of aircraft B is considerably larger than that of aircraft A. This is not entirely due to small damping of the Dutch mode as, e.g., at $C_L = 0.4$ the damping of aircraft B is 10% of critical, which compares with 9.2% at $C_L = 0.412$ (200 knots) for aircraft A, and still the peak in the response spectrum of aircraft B is about 4 times as large as for aircraft A. This is probably due to the larger excitation of aircraft B, resulting from a smaller value of μ_2 (inertia) and a larger value of l_v .

It is apparent from Figure 10 that the 'cut off' frequency of 0.2 c.p.s. defining 'CONTROLLED' flight lies well below the peak at the Dutch roll frequency, which suggests that the efficient stabilisation of the aircraft B by the pilot, when flown in turbulent air, may prove to be difficult.

From the spectra of Figure 10, the values of the angle of bank and rate of roll R.M.S.'s were estimated and are shown in Table II. It can be seen that the responses to turbulent air of aircraft B are larger than the response of aircraft A or 4A.

For further comparison Figure 11 was prepared where the angles of bank the aircraft B is expected to reach on average within a given time in a turbulence of 1 f.p.s.

R.M.S. are plotted for three values of C_L. Figure 11 should be compared with Figure 6. It can be seen that with controls fixed there is no fundamental difference between aircraft B and A. In controlled conditions at lower speeds, however, aircraft B may experience angles of bank almost three times as large as aircraft A. It should be remembered that the 'simply controlled flight' conditions are somewhat arbitrarily defined in this Report, but, nevertheless, in order to decrease the angles of bank from those indicated by Figure 11, the pilot would have to control the aircraft at frequencies higher than 0.2 c.p.s.*

7. RESPONSE TO DISCRETE GUSTS AND COMPARISON WITH RESPONSE TO CONTINUOUS TURBULENCE

There is no accepted and satisfactory method of calculation of the lateral response to discrete gusts. Some drastic assumptions must be made with regard to both the discrete gust shape and the interpretation of the computed time history of the output.

For the present purpose of comparing discrete gusts and continuous turbulence techniques, the following approach has been used.

It is assumed that only side gusts, v_g, are of interest. Each discrete gust is regarded as a separate entity, and gust shape is assumed to be a step in v_g. The response level is defined as the value of the first peak in the response time history, the following oscillations being assumed to be suppressed by the pilot's action.

For the comparison two aircraft have been used, aircraft A, representing a dynamically satisfactory aircraft, and aircraft B, with dynamic characteristics which can be regarded as poor and somewhat unusual (large l_v, poor damping).

The computed time histories of angle of bank due to a 1 f.p.s. step input in v_g are shown in Figure 12. It is assumed that the aircraft response to unit gust is measured by the value of ϕ_{max}. Thus the response of aircraft A to a 1 f.p.s. side gust is 0.638° of angle of bank, and of aircraft B is 1.52° of angle of bank.

Reference 7 proposes a model of an average low-altitude turbulence in terms of the distribution of the variances $\overline{w_g^2}$ of the vertical components of the turbulence; it also suggests that the lateral component of turbulence is 1.4 times as large as the vertical one, $\overline{v_g^2} = 1.4\, \overline{w_g^2}$. Figure 5 of the same reference shows a distribution (in miles per gust) of the derived, discrete vertical gusts. These gusts were obtained by computing the longitudinal response of an aircraft to continuous turbulence and then converting these responses via a single gust alleviation factor into discrete gusts. Thus, although the distribution of the discrete gusts in Reference 7 is compatible with the spectral representation of turbulence, it retains in some degree the dynamic longitudinal properties of the aircraft used in the analysis and cannot be regarded strictly as a true gust distribution. However, the discrete gusts distribution of Reference 7 was used here to study lateral responses as the best information

*Frequencies higher than this value may be difficult to control in normal operational flying but it is to be expected that higher values could be assumed in test flying for which this aircraft is intended.

available. The values of w_g were multiplied by $\sqrt{1.4}$ to convert them into distribution of v_g. From that distribution and the response value of the angle of bank, the distribution of the angle of bank expected to be reached within a given time was computed and is shown in Figure 13.

In Figure 13 the results of the spectral calculations are shown also. In spectral calculations the values of the R.M.S. and N_0 response are those of Table II, and the model of continuous turbulence is taken from Reference 7. Two distributions of the angle of bank ϕ response due to continuous turbulence are shown. The first distribution, due to response with controls fixed, includes the contribution of all frequencies of the response from zero to infinity. The second distribution, flight under control, includes the contribution of frequencies from 0.2 c.p.s. to infinity. It is assumed, in similarity with the discrete gust response, that the pilot control will remove all the low-frequency contributions from 0 to 0.2 c.p.s.

It is interesting to notice that for aircraft A (Fig.13(a)) both approaches, the continuous turbulence and the discrete gusts, yield very similar answers, when both include, however vaguely, the pilot's control. Neglecting the pilot's control the continuous turbulence approach gives much larger values which are due to the contribution of the responses at very low frequencies, the response in the spiral mode being predominant. The inclusion of the very low frequencies in the spectral calculation has very little physical justification, except in the case of free-flight models, when there is no pilot on board. It appears that for more conventional aircraft the discrete gusts approach, used sensibly, may give a reasonable answer.

To check if this tentative conclusion still holds, when the method is applied to a less conventional aircraft, similar calculations have been made for aircraft B and the results are shown in Figure 13(b). It can be seen that in this case the discrete gust and continuous turbulence approaches do not yield similar answers, although the difference is not dramatically large. What is interesting is the small difference between controls fixed and controlled flight; this has already been pointed out in Section 6 and indicates a large contribution of higher frequencies to the total responses, that is, frequencies too high to be readily controlled by the pilot.

Summing up, it might be very tentatively concluded that, if the aircraft has a satisfactory dynamic characteristic, the much simpler, discrete gust approach may be used in the study of lateral responses, if only to establish the order of magnitude of the responses. This requirement of 'satisfactory dynamic characteristics' is the same requirement as in the studies of longitudinal aircraft response to turbulence, when using discrete gust technique.

Comparing Figures 13(a) and 13(b) it appears that under control (as defined in this paper) the aircraft B will roll in turbulent air much more than aircraft A. On the basis of discrete gusts calculation, aircraft B will roll twice as much as aircraft A, and on the basis of spectral calculations three times as much.

8. AILERON POWER REQUIRED TO MINIMISE AIRCRAFT ROLLING IN TURBULENT AIR

8.1 General Remarks

The ultimate object of this study would be to provide a rational method by which the required aileron power to counteract gusts could be established. It should be mentioned that from the lateral control point of view, the most critical regime of flight is at low speed, and thus in landing and take-off. The required aileron power in these conditions is determined, first by the amount of aileron needed to counteract the disturbances due to gusts, and second by the amount to keep the wings level in the presence of sideslip, which may arise during landing with a cross-wind. These two requirements are additive and should be less, by some agreed safety margin, than the available control. The present paper deals with the first aspect only, that of aileron control required in turbulent air; the second aspect is very easy to estimate once the cross-wind requirements are known. It should be remebered that, at low altitudes, the magnitude of turbulence is apparently proportional to wind speed[7] and thus the cross-wind strength is related to the intensity of turbulence.

In order to establish the amount of aileron required by the pilot in turbulent air we have to know the law by which the pilot controls his aircraft, or in other words we have to know the pilot's transfer function. Quite a lot of work has been done in this field and in some applications we can establish an approximate transfer function of the human pilot and in the opinion of the present writer a simple lag function, with a time constant of the order of 0.3 sec, is as good as any. However, the fundamental question is not answered yet: to what stimuli the human-pilot reacts? In tracking tests for example, this is a comparatively straightforward matter; the tracking error can be regarded as the input to the pilot, visual cues being the most important.

In the case of lateral control during the landing and take-off phase, when the pilot's attention is focussed on other things than the lateral trim only, it is impossible to establish with any certainty what is the 'input' to the pilot. He will perceive the angle of bank when amplitude is large enough, but he will also react to the rate of roll and most definitely to the rolling acceleration, and thus we may expect that the pilot will react not only to the bank but to its first two derivatives and probably to the third one also (rate of acceleration). Thus it appears futile to try to include the pilot's dynamics in the present investigation.

Some preliminary calculations have shown that the estimates of the aileron requirements are not very sensitive to the form of the pilot's transfer function, but depend on the gain and the type of input assumed.

In view of these remarks, the transfer function of the pilot has been neglected and simple control laws have been assumed. Two control laws have been studied. First, the aileron deflection is assumed proportional to the angle of bank:

$$\xi = k \times \phi \tag{13}$$

and second, the aileron deflection is assumed proportional to the rate of roll $p = d\phi/dt$:

$$\xi = k \times p \tag{14}$$

Neglecting the yawing moment due to ailerons, which is the case throughout this study, the second law is equivalent to increasing the l_p derivative.

The aileron effectiveness used in the calculations has been based on the measured values of the rate of roll for full aileron deflection, and hence the aileron rolling power l_ξ, given in Table I, is marked 'effective'.

8.2 Aileron Control in Turbulent Air, Aileron Gearing $\xi = k \times \phi$

The computed spectral densities of the angle of bank for aircraft A and for five aileron gearings, k, are shown in Figure 14. The response parameters computed from these curves are presented in Table III. It can be seen that this particular law of aileron control alleviates considerably the aircraft rolling response to turbulent air at very low frequencies, decreases the response somewhat in the neighbourhood of the Dutch Roll frequency and slightly increases the response at higher frequencies. To stabilise the very low frequencies, corresponding to the spiral mode, extremely small aileron angles are sufficient, as shown in Table III.

This points again to the fact that stabilisation at low frequencies, being a problem of trimming, does not require large aileron angles.

To gain some more insight into the effectiveness of the aileron control law $\xi = k \times \phi$, Figure 15 has been prepared. In this figure the R.M.S.'s of bank angle ϕ, rate of roll p and of aileron angle in the turbulence of unit intensity ($\overline{w_g^2} = 1$) are plotted against the aileron gearing, $k = \xi/\phi$ (Fig.15(a)). Figure 15(b)) shows the 'rate of exchange' between R.M.S.'s of angle of bank and of aileron angle. It can be seen very clearly that very little aileron is required to decrease the value of the R.M.S. of the bank angle from its 'control fixed' value. Roughly speaking, this first small amount of aileron is required to stabilise the low frequencies, and with further increase in gain the ailerons are used to stabilise a wider frequency band. Curves similar to those of Figure 15 can be used for the estimation of the required gearing and aileron power, given the maximum permissible angle of bank and the intensity of turbulence.

Figure 16 shows the angle of bank and aileron angle expected to be reached within a given time for three aileron gearings, with turbulence intensity, and $\overline{w_g^2} = 1$.

Let us assume, for the sake of illustration, that we have a flight lasting 1 hour in extremely heavy, continuous turbulence with $\sqrt{(\overline{w_g^2})} = 10$ f.p.s., and do not wish to exceed an angle of bank of 15°. It can be seen that the required gearing would then be about $\xi/\phi \approx 0.7$, and the maximum aileron angle expected during this flight about $\xi \approx 12^\circ$. These values appear to be reasonable.

Also in Figure 16 the expected angle of bank is plotted for 'simply controlled' flight as defined in the previous section. It appears that the assumption of 'simple control' is reasonable, as it seems to correspond to a mild gearing and thus a 'simple' demand on the pilot.

Similar calculations of the response of the aircraft with aileron gearing $\xi = k\phi$ have been made for the large aircraft, with span b = 140 ft. As the results are very similar to the results for the small aircraft (aircraft A, b = 35 ft), only the final results are given in Table III and plotted in Figure 17. This figure should be compared with Figure 15 for the small aircraft. The noticeable difference is that although the basic response to turbulence of the large aircraft (controls fixed) is smaller than that of the small aircraft, the ailerons are noticeably less effective in stabilising the aircraft in turbulent air. As a measure of the aileron effectiveness a value of $\Delta\sigma_\phi/\Delta\sigma_\xi$ can be used in the approximately linear range of Figures 15(b) and 17(b). These values are approximately 4.1 and 1.8 for small and large aircraft respectively. These results could be expected on the basis that for similar aircraft the aileron effectiveness in terms of the steady-state rate of roll is inversely proportional to the span.

8.3 Aileron Control in Turbulent Air, Aileron Gearing $\xi = k \times p$

In this set of calculations of aircraft and aileron responses in turbulent air, it has been assumed that the aileron deflection is proportional to the rate of roll, i.e. $\xi = k \times p$. The calculations have been made for the same aircraft and conditions as in Section 8.2, viz. aircraft A, 200 knots, and $\overline{w_g^2} = 1\ (1\ \text{f.p.s.})^2$.

The spectral densities of the aircraft response in terms of the angle of bank for three aileron gearings are shown in Figure 18. The aileron gearings were chosen to represent doubling and quadrupling the value of the basic derivative l_p . It can be seen from Figure 18 that the effect of the control law $\xi = k \times p$ on the spectrum of response is almost uniform throughout the frequency range. Thus, this particular law is not expected to represent the pilot's action, but only the effect of an idealised roll damper.

The final results in terms of response R.M.S.'s are given in Table III and are plotted in Figure 19. It is apparent that stabilisation by the aileron deflection proportional to the rate of roll (Fig.19) is more effective than stabilisation by aileron deflection proportional to the angle of bank. By comparison of Figures 19 and 15 and using as a measure the parameter $\Delta\sigma_\phi/\Delta\sigma_\xi$, the aileron law $\xi = kp$ is roughly four times as effective as the law $\xi = k\phi$.

9. CONCLUDING REMARKS

A method is presented which makes possible the rational study of the lateral behaviour of an aircraft in turbulent air.

The method is used for an assessment of the study of rolling responses of a few aircraft when flown in turbulent air.

It is shown in the analysis that the pilot's reaction cannot be neglected and a very simple approach to the problem is suggested. It is proposed to cut off the low-frequency range of the output spectrum below some frequency which is defined by the pilot's dynamic characteristics. In the present work the cut off frequency is assumed to be 0.2 c.p.s. It is felt that more experimental information regarding the pilot's behaviour in this particular application is urgently required.

It is shown by numerical examples that increasing the aircraft size diminishes the aircraft responses to the turbulent air but does not necessarily decrease the requirements of aileron power.

Two simple systems of autostabilisation are investigated. In one the aileron deflection is proportional to the angle of bank; in the second it is proportional to the rate of roll. The second system appears to be much more efficient in terms of reducing the angle of bank. It is suggested that the stabilisation law in which ailerons are proportional to the angle of bank may represent approximately the human pilot's behaviour.

A comparison between the bank responses to continuous turbulence and to discrete gusts has been made. It appears that, for aircraft with satisfactory dynamic characteristics, both approaches yield similar answers.

The numerical results obtained in the present study appear to be reasonable. However, there are no experimental data against which the validity and accuracy of the present theory can be checked. It is proposed to initiate some experimental work to supplement the present theoretical work.

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TABLE I

Stability Characteristics of Three Aircraft Used in Analysis

	Aircraft A, W = 19,000 lb, b = 35 ft; W/S = 56 lb ft^{-2}, $\mu_2 = V\hat{t}/b/2 = 41.7$		
	V = 580 Kts = 980 ft sec^{-1}; C_L = 0.049; $\hat{t} = V\,C_L/2g$ = 0.745 sec	V = 200 Kts = 338 ft sec^{-1}; C_L = 0.412; $\hat{t}$ = 2.16 sec	V = 150 Kts = 254 ft sec^{-1}; C_L = 0.73; $\hat{t}$ = 2.89 sec
$\bar{y}_v = -y_v$	0.207	0.207	0.200
$\omega_l = -\mu_2 l_v/i_A$	34.0	83.5	112
$\omega_n = \mu_2 n_v/i_c$	12.7	13.75	15.3
$\nu_l = -l_p/i_A$	5.0	4.8	4.37
$\nu_n = -n_r/i_c$	0.608	0.80	0.593
$\nu_{np} = -n_p/i_c$	-0.06	0.053	0.147
$\nu_{lr} = l_r/i_A$	0.74	1.51	2.07
$\delta_{l_\xi} = -\mu_2 l_\xi/i_A$	$(28.9)_{effective}$	$(87.5)_{effective}$	$(77.5)_{effective}$
i_A	0.054	0.055	0.058
i_c	0.282	0.282	0.278
i_e	0	-0.014	-0.036
Characteristic equation	$(s + 0.00418)(s + 5.085) \times (s^2 + 0.711s + 13.16)$	$0.987\,(s + 0.089) \times (s + 4.9) \times (s^2 + 0.78s + 18.09)$	$0.92\,(s + 0.1012)(s + 4.435) \times (s^2 + 1.244s + 30.6)$
Dutch roll, f_0, c.p.s.	0.777	0.313	0.306
ζ, damping ratio	0.098	0.092	0.112
Roll time const, sec	0.146	0.44	0.051
Spiral time const, sec	178	31.3	28.6

TABLE I (Contd)

	Aircraft 4A $W = 304,000$ lb, $b = 140$ ft $W/S = 56$, $\mu_2 = 10.425$	Aircraft B, $W = 5050$ lb, $b = 20$ ft $W/S = 11.7$; $\mu_2 = 15.3$		
	$V = 200$ Kts $= 338$ ft sec^{-1} $C_L = 0.412$ $\hat{t} = 2.16$ sec	$V = 186$ Kts $= 314$ ft sec^{-1} $C_L = 0.1$ $\hat{t} = 0.487$ sec	$V = 132$ Kts $= 222$ ft sec^{-1} $C_L = 0.2$ $\hat{t} = 0.69$ sec	$V = 93$ Kts $= 157$ ft sec^{-1} $C_L = 0.4$ $\hat{t} = 0.975$ sec
$\bar{y}_v = -y_v$	0.207	0.185	0.180	0.180
$\omega_1 = -\mu_2 l_v/i_A$	20.8	6.00	12.75	22.00
$\omega_n = \mu_2 n_v/i_c$	3.44	0.965	1.02	1.29
$\nu_l = -l_p/i_A$	4.8	1.12	1.36	1.42
$\nu_n = -n_r/i_c$	0.60	0.31	0.316	0.351
$\nu_{np} = -n_p/i_c$	0.053	0	0.015	0.046
$\nu_{lr} = l_r/i_A$	1.51	1.11	1.67	2.18
$\delta_{l_\xi} = -\mu_2 l_\xi/i_A$	22.2	-	-	-
i_A	0.055	0.092	0.090	0.107
i_c	0.282	1.14	1.14	1.121
i_E	-0.014	0.042	-0.020	-0.126
Characteristic equation	$0.987\,(s + 0.0684)(s + 4.92)$ $\times\,(s^2 + 0.76s + 4.53)$	$0.963\,(s + 0.028)(s + 1.362)$ $\times\,(s^2\ 0.21s + 1.051)$	$0.996\,(s + 0.0838)(s + 1.69)$ $\times\,(s^2 + 0.111s + 1.67)$	$0.868\,(s + 0.139)(s + 1.885)$ $\times\,(s^2 + 0.415s + 4.3)$
Dutch roll, f_0, c.p.s.	0.158	0.333	0.30	0.34
ζ, damping ratio	0.177	0.1025	0.043	0.10
Roll time const, sec	0.44	0.36	0.41	0.52
Spiral time const, sec	31.7	17.4	8.25	7.0

TABLE II

Rolling Response Parameters of Three Aircraft

Turbulence Intensity, $\overline{w_g^2} = 1(\text{f.p.s.})^2$

	Aircraft A b = 35 ft			Aircraft 4A b = 140 ft	Aircraft B b = 20 ft		
V	580 Kts	200 Kts	150 Kts	200 Kts	186 Kts	132 Kts	93 Kts
C_L	0.049	0.412	0.73	0.412	0.1	0.2	0.4
Controls fixed $0 \leqslant f \leqslant \infty$ c.p.s.							
$(\sigma_\phi)_F$ °	~3.5	1.93	1.18	0.935	1.95	2.9	2.35
$(\sigma_p)_F$ °/sec	~0.63	1.6	1.6	0.43	1.85	4.36	3.81
$(N_0)_F$ per sec	~0.03	0.132	0.216	0.073	0.151	0.240	0.259
Controlled $0.2 \leqslant f \leqslant \infty$ c.p.s.							
$(\sigma_\phi)_c$ °	0.145	0.783	0.835	0.136	0.986	2.42	2.04
$(\sigma_p)_c$ °/sec	0.625	1.53	1.56	0.24	1.78	4.30	3.73
$(N_0)_c$ per sec	0.690	0.312	0.300	0.282	0.288	0.284	0.292

TABLE III

Rolling Response Parameters for Different Aileron Gearings

Turbulence Intensity, $\overline{w_g^2} = 1(\text{f.p.s.})^2$

	Aircraft A, b = 35 ft 200 Kts. S.L.					Aircraft 4A b = 140 ft 200 Kts. S.L.				Aircraft A 200 Kts. S.L.	
k = ξ/ϕ Aileron gearing	0	0.0057	0.0114	0.5	1.0	0	0.5	1.0	k = ξ/p Aileron gearing	0.118 ($l_p \times 2$)	0.354 ($l_p \times 4$)
σ_ϕ °	1.93	1.50	1.40	0.47	0.275	0.935	0.502	0.311	σ_ϕ °	1.00	0.575
σ_p °/sec	1.6	-	-	0.882	0.597	-	-	-	σ_p °/sec	0.565	0.225
N_0 per sec	0.132	-	-	0.298	0.346	-	-	-	N_0 per sec	0.09	0.0625
σ_ξ °	0	0.0086	0.016	0.235	0.275	0	0.251	0.311	σ_ξ °	0.0666	0.0796

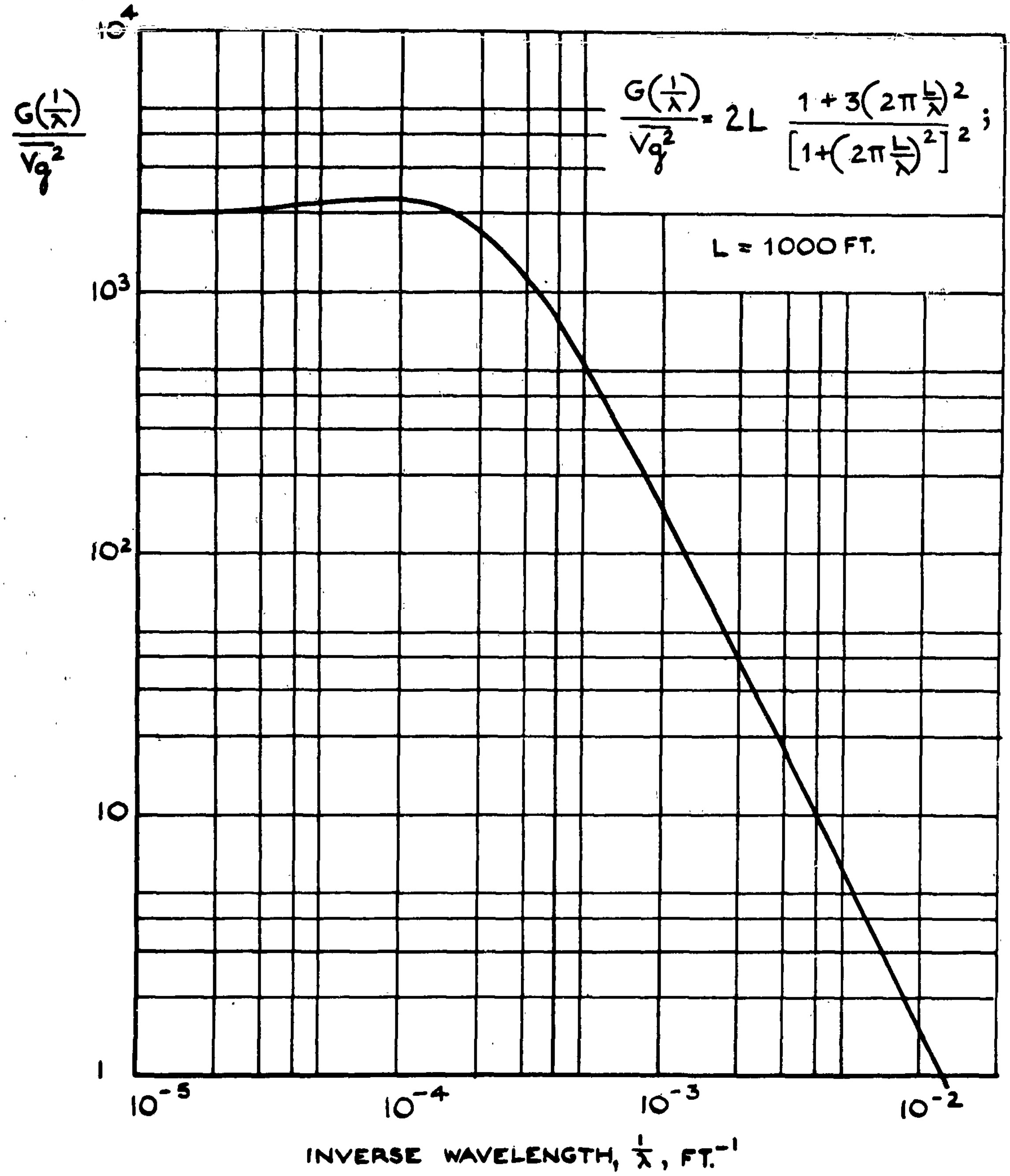


Fig.1 Spectral density of the lateral component, v_g, of the atmospheric turbulence

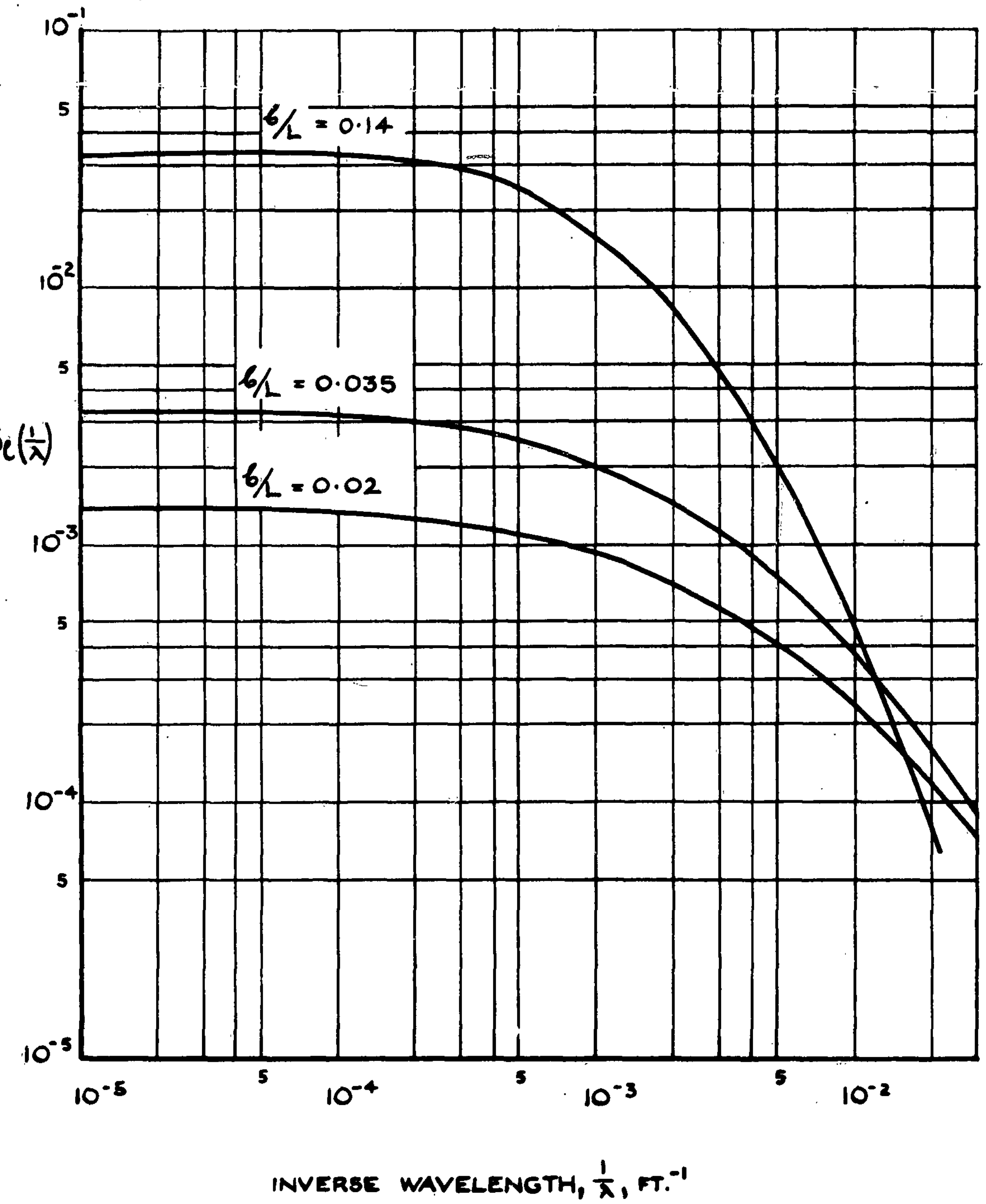


Fig.2 Non-dimensional spectral densities of rolling moment due to vertical component, w_g, of atmospheric turbulence for three wing span to turbulence scale ratios (from Ref.1)

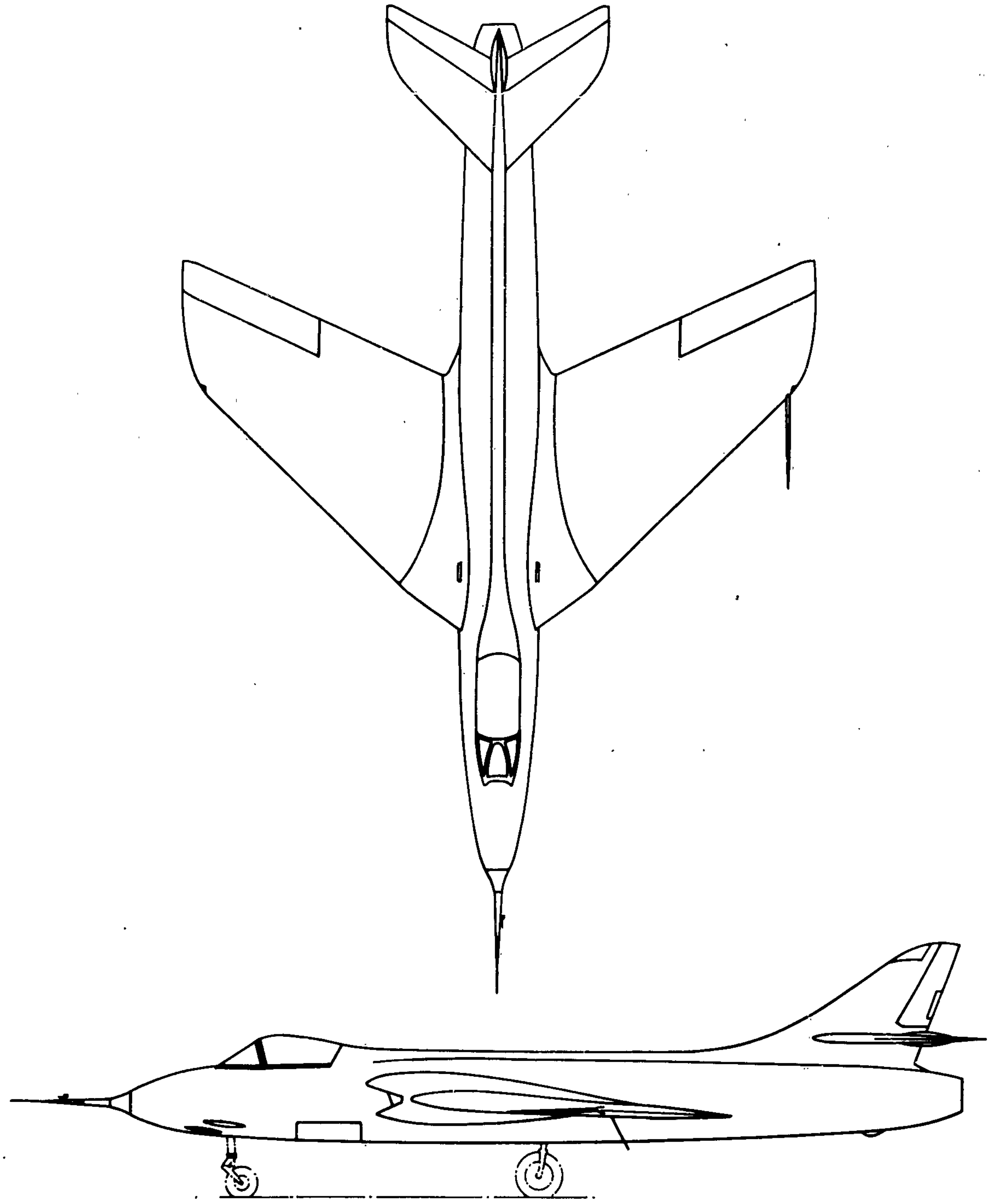


Fig. 3 General arrangement of aircraft A, span 35 ft (aircraft 4A, span 140 ft)

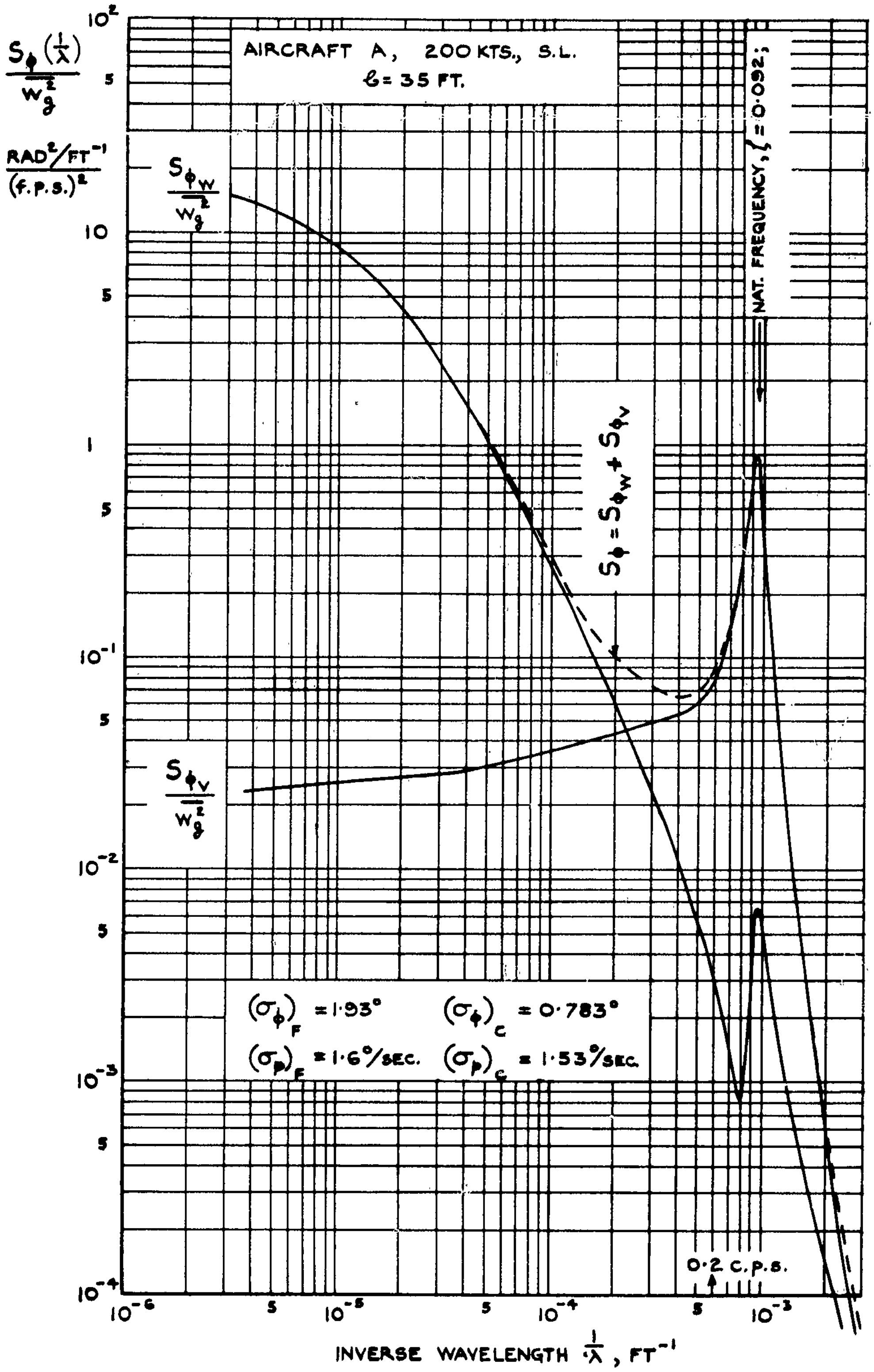


Fig.4 Spectral density of angle of bank and its two components; aircraft A, 200 Kts., S.L.

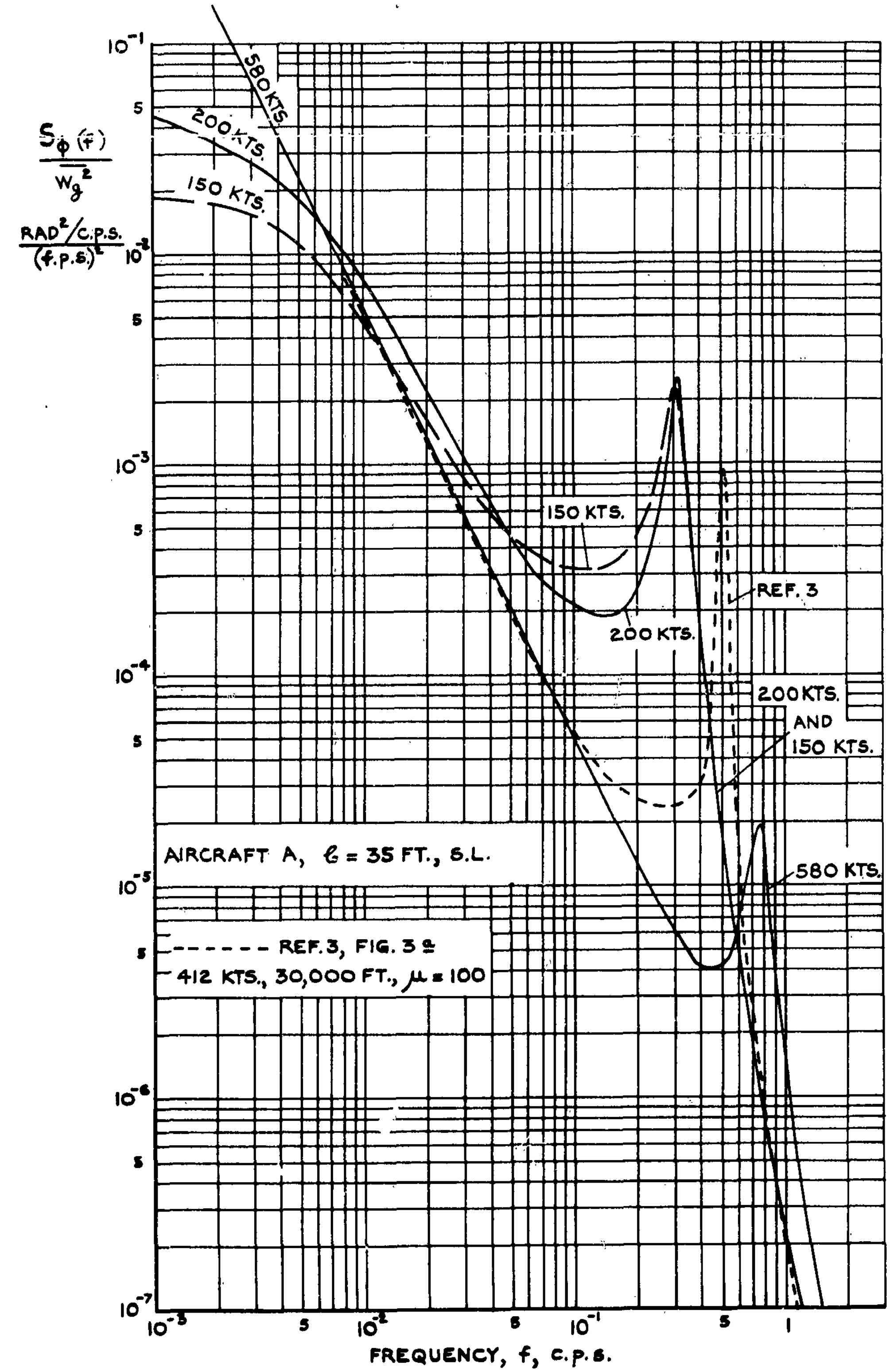
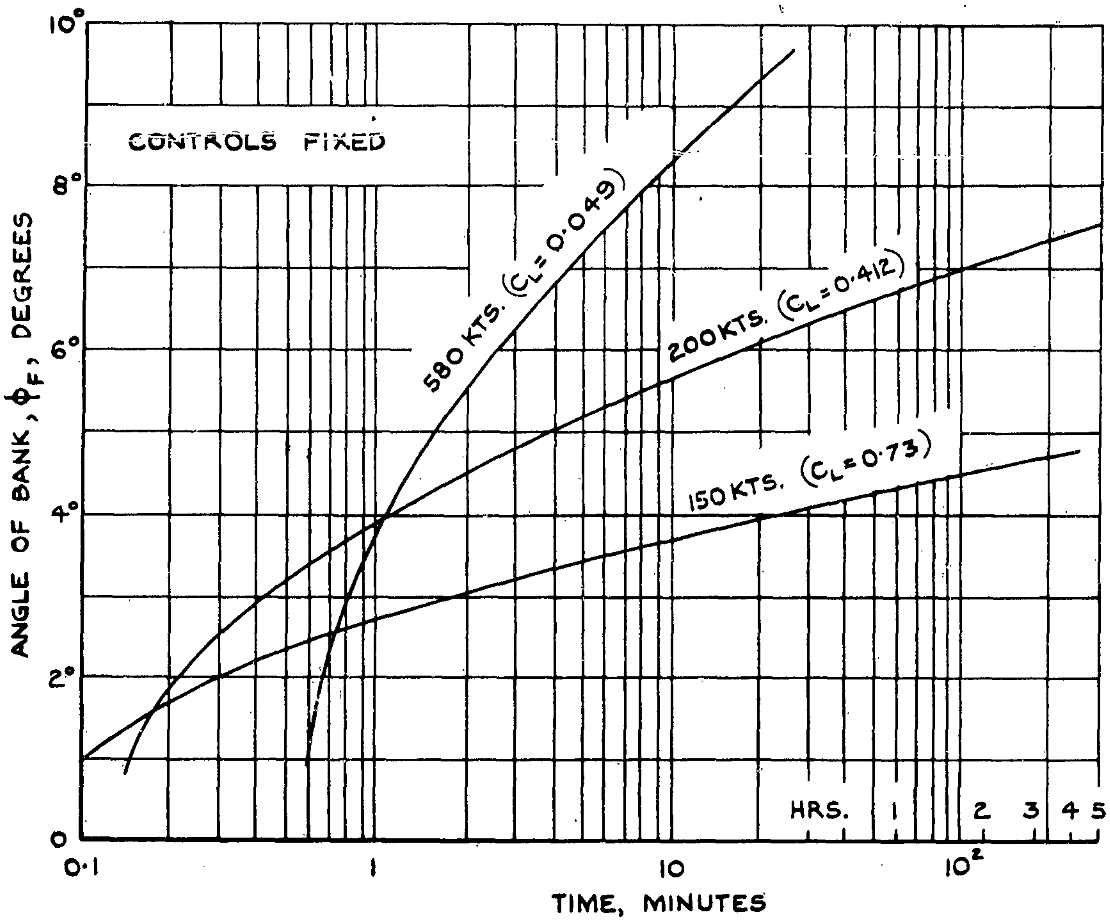


Fig.5 Spectral density of angle of bank for three forward speeds aircraft A, sea level



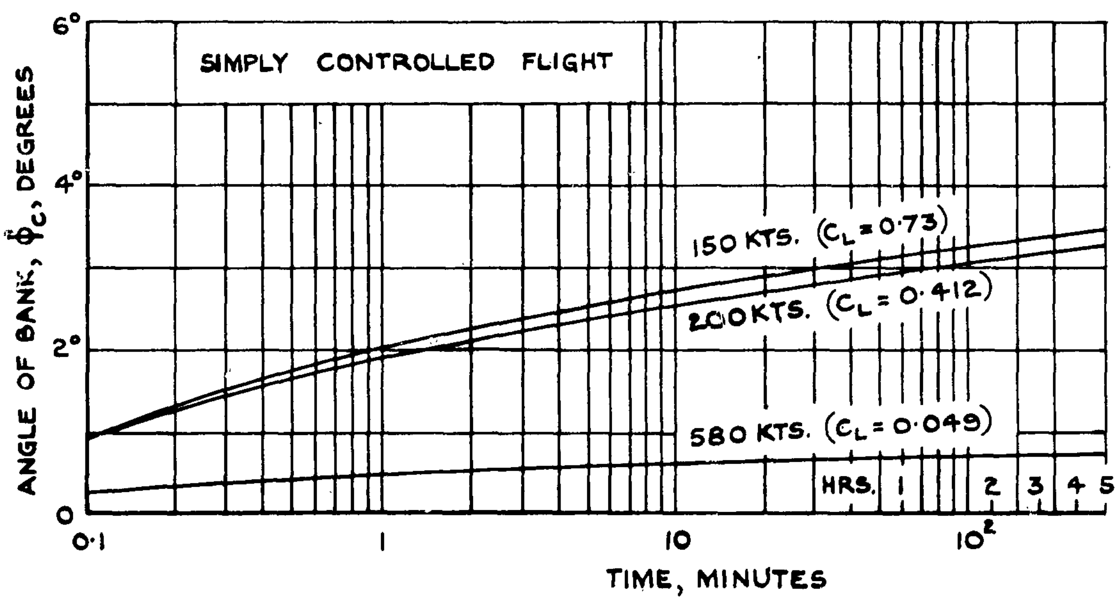


Fig. 6 Angle of bank expected within a given time; aircraft A, S.L., continuous turbulence $\overline{w_g^2} = 1$ (f.p.s.)^2

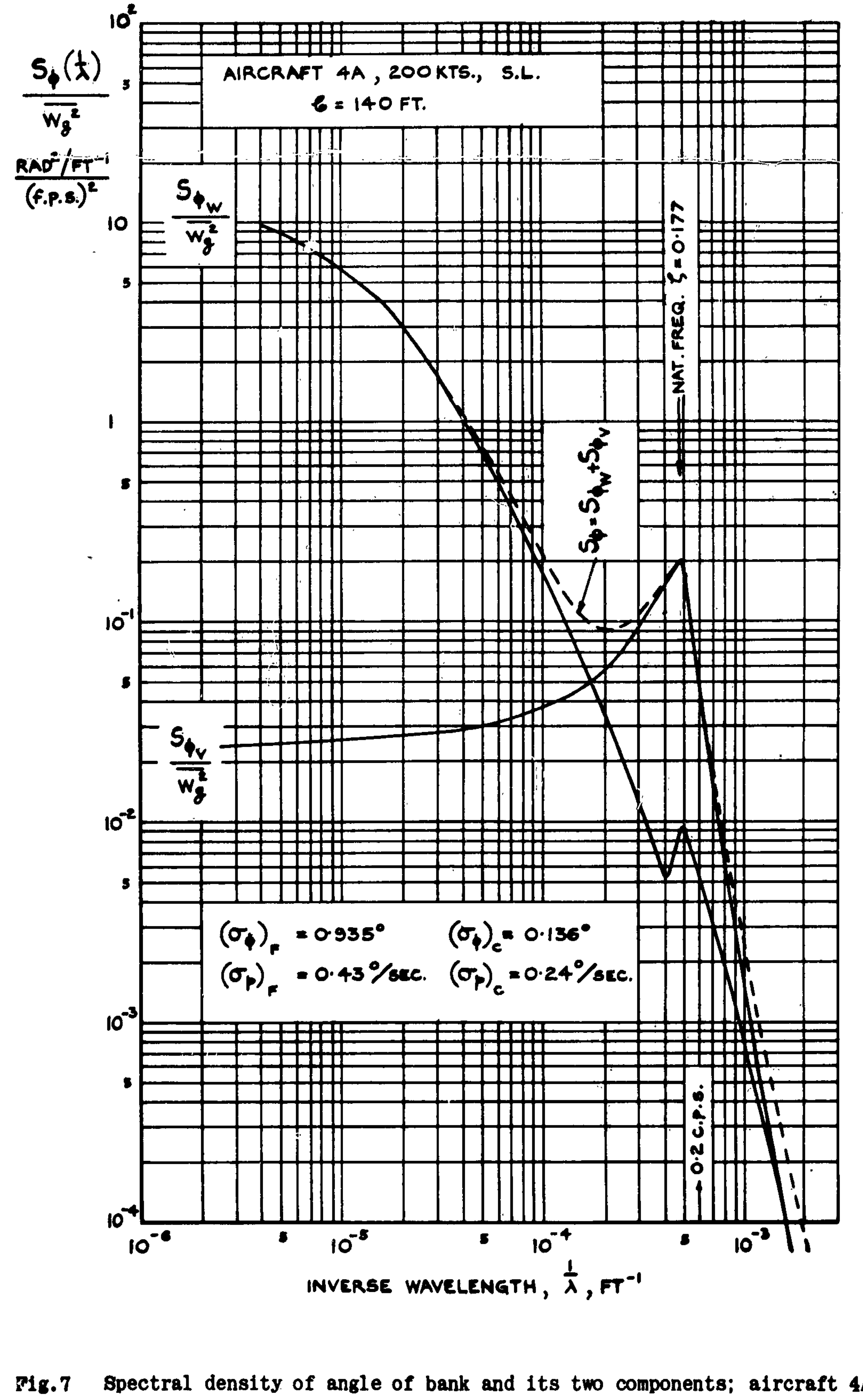


Fig.7 Spectral density of angle of bank and its two components; aircraft 4A, 200 Kts., S.L.

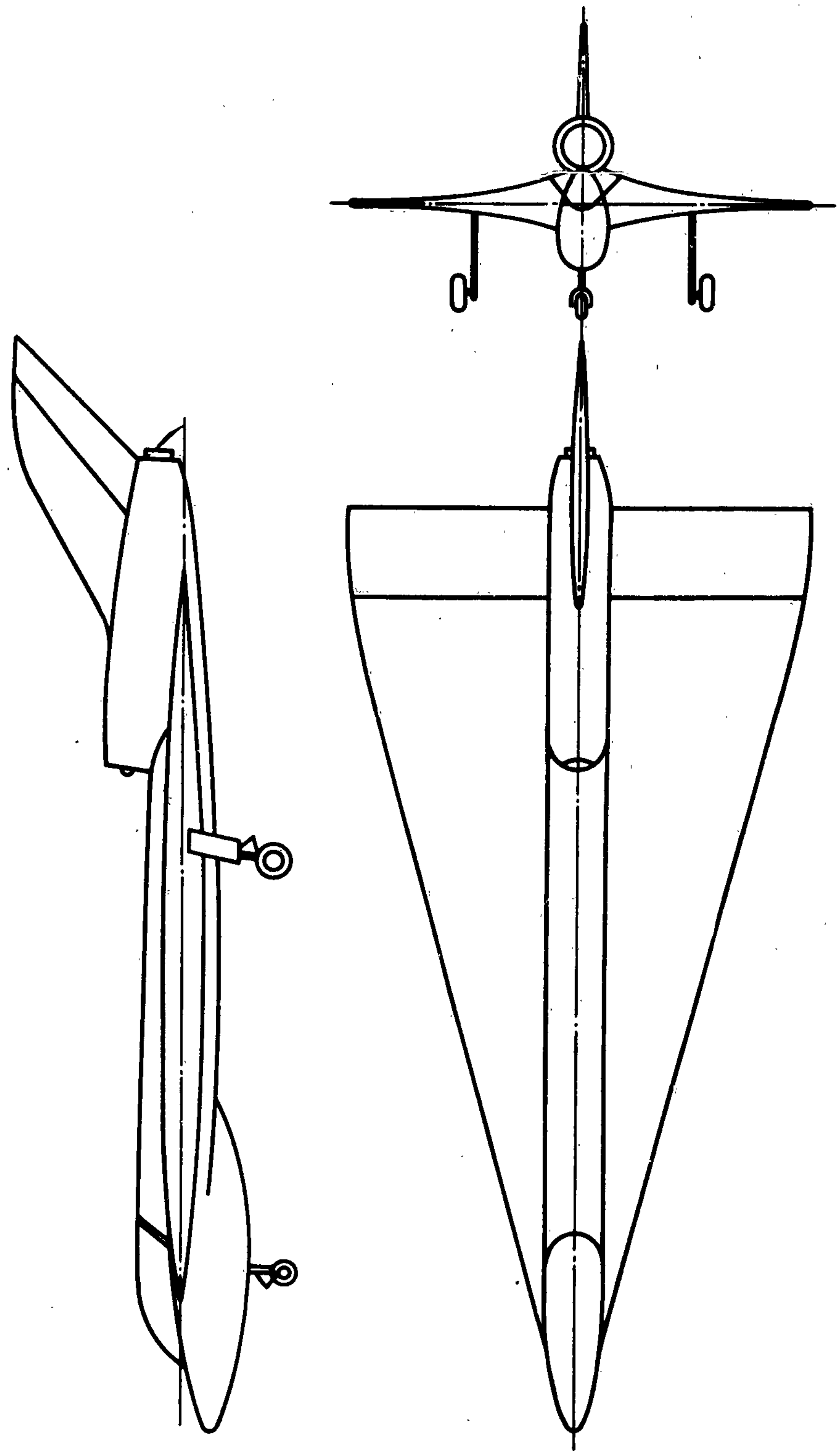
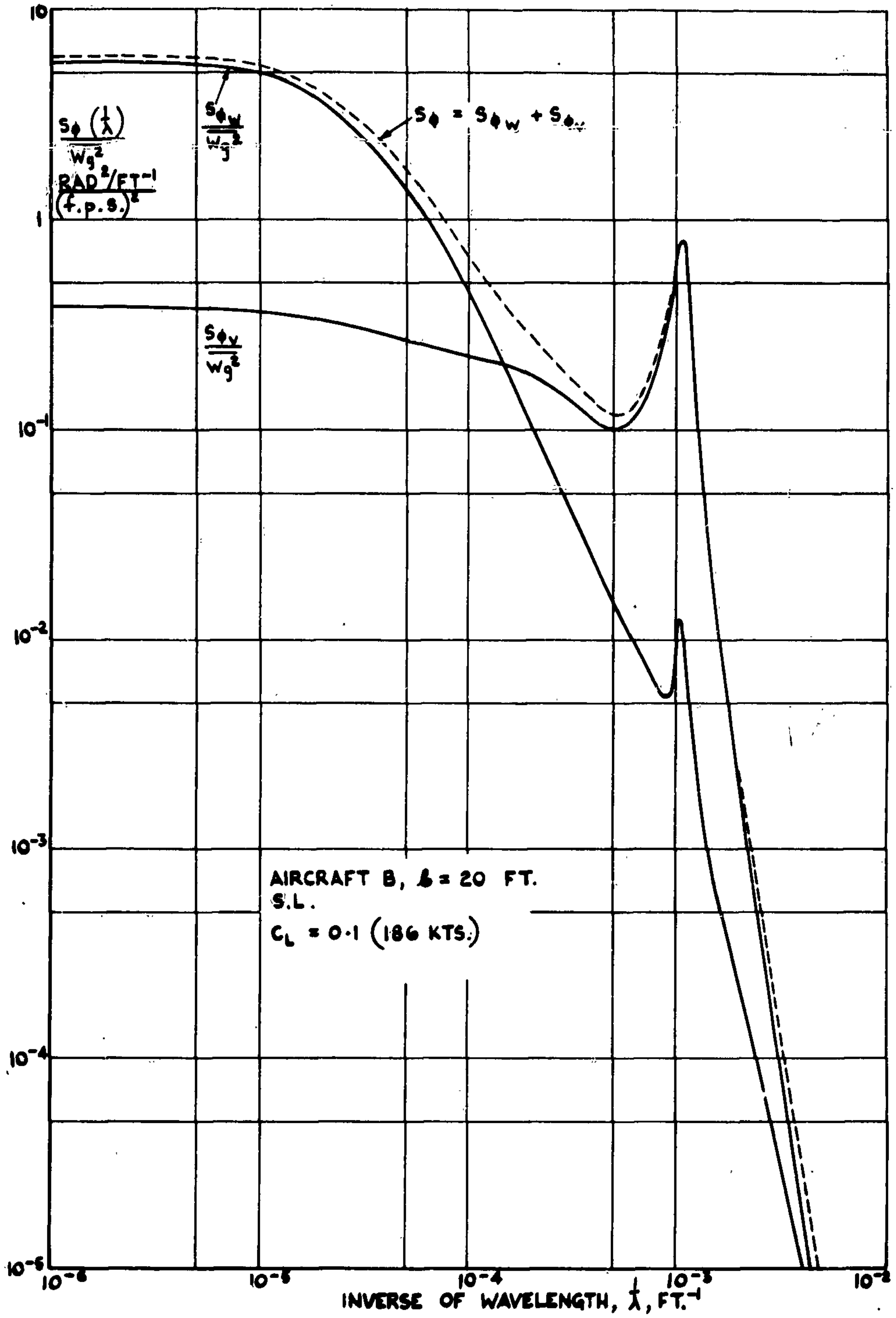
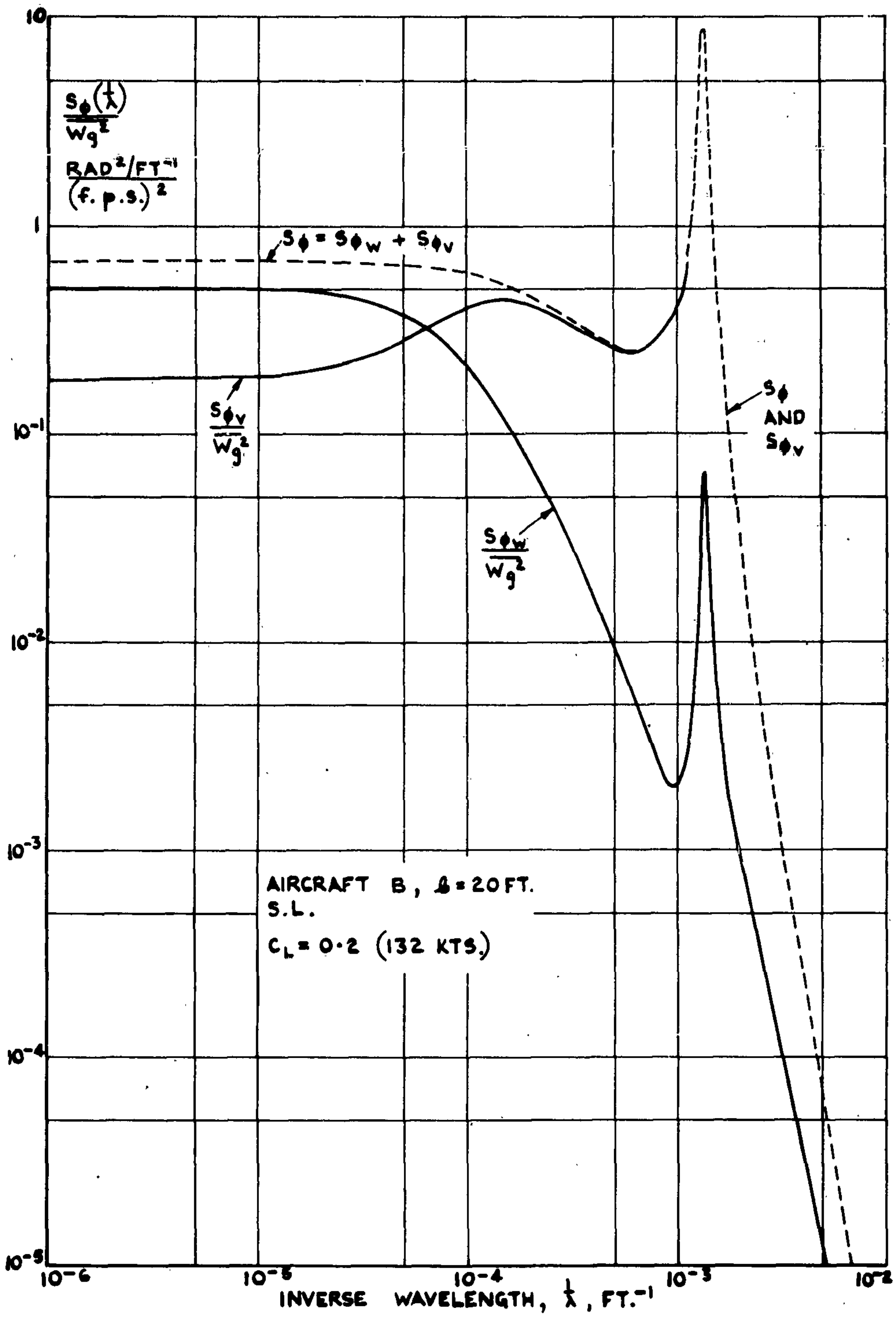


Fig.8 Sketch of aircraft B span 20 ft



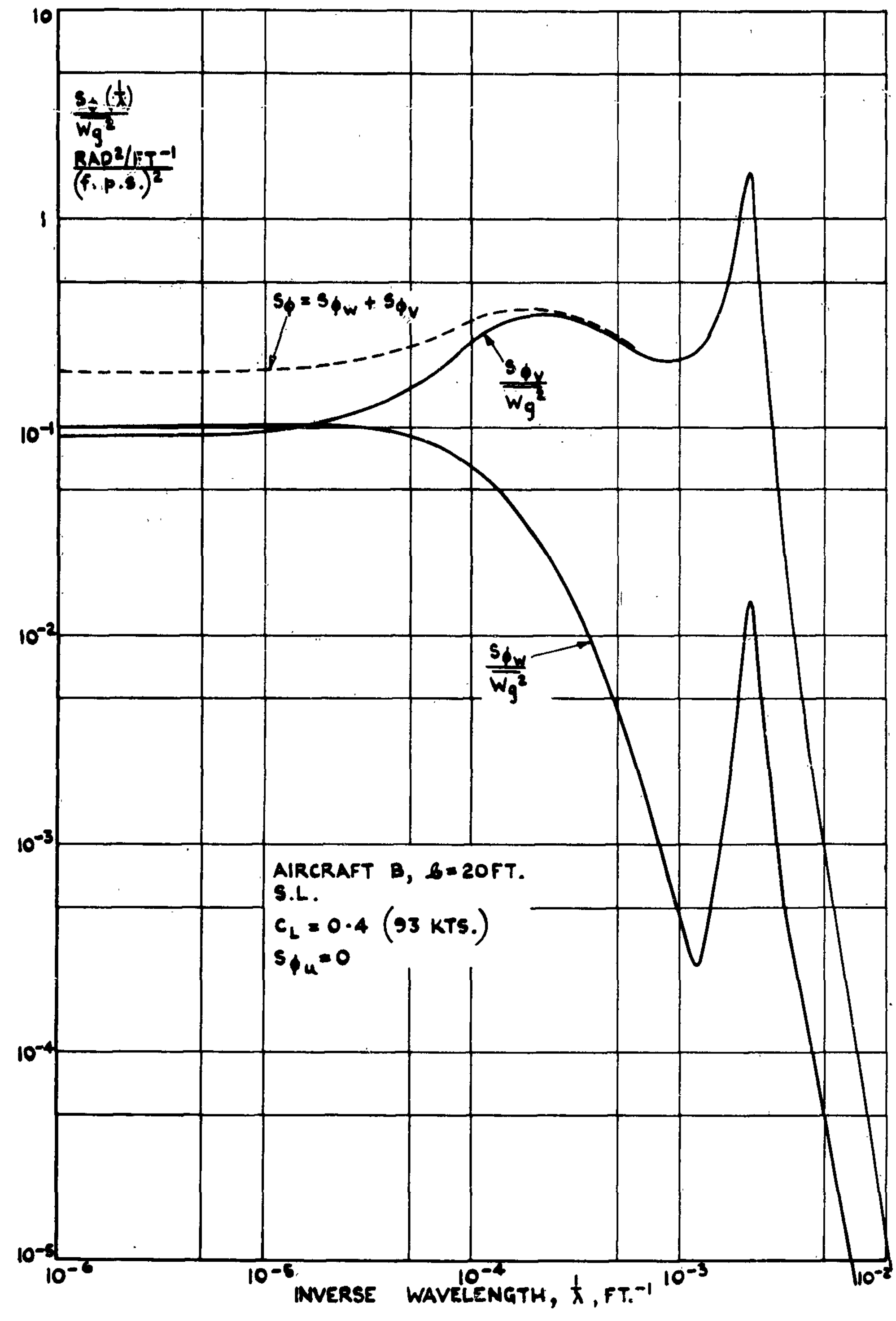
(a) $C_L = 0.1$

Fig.9 Spectral density of bank angle and its two components, aircraft B S.L. $\overline{w_g^2} = 1$ (f.p.s.)^2



(b) C_L = 0.2

Fig.9 Spectral density of bank angle and its two components, aircraft B S.L. $\overline{w_g^2}$ = 1 (f.p.s.)^2



(c) $C_L = 0.4$

Fig.9 Spectral density of bank angle and its two components, aircraft B S.L. $\overline{w_g^2} = 1\ (f.p.s.)^2$

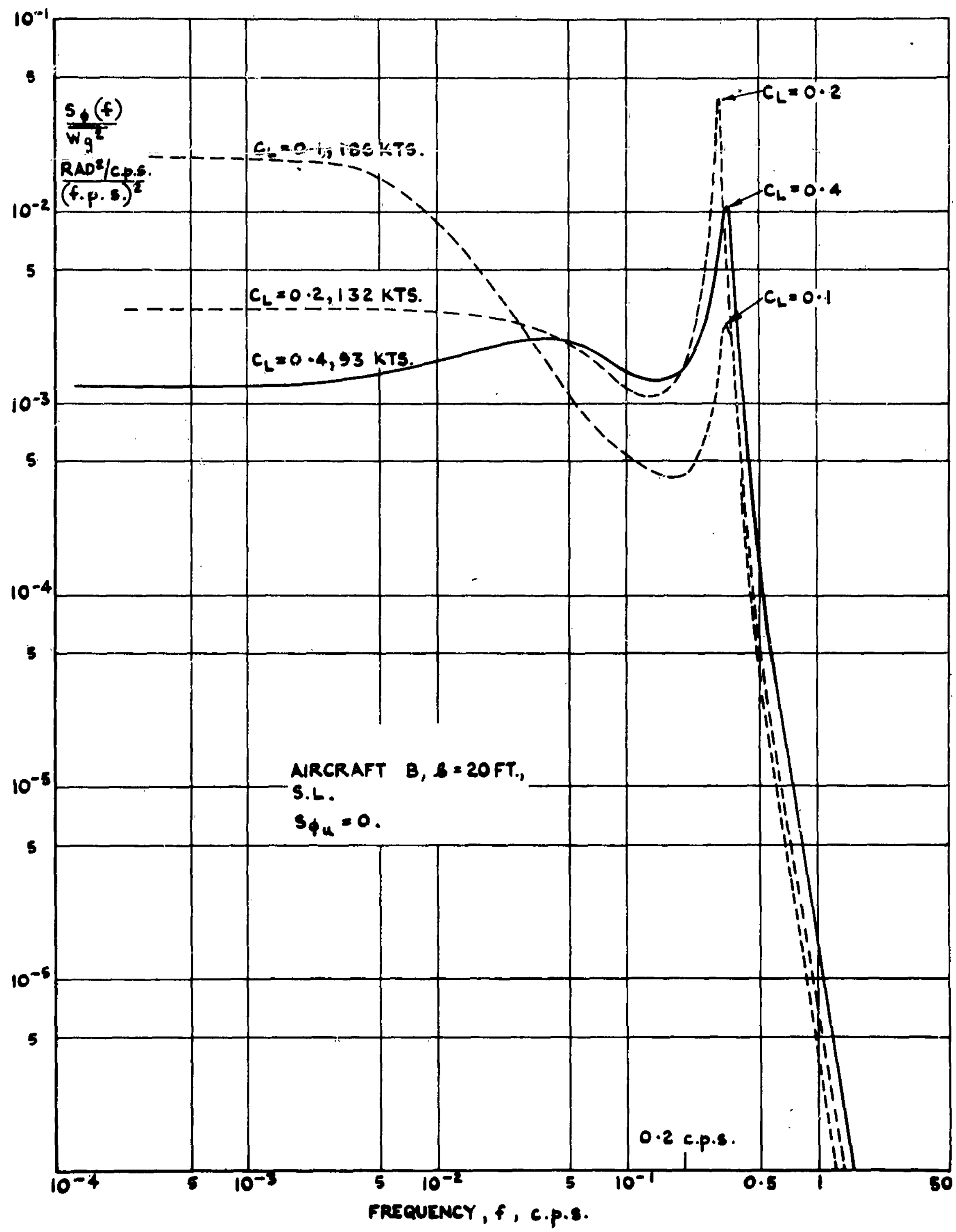
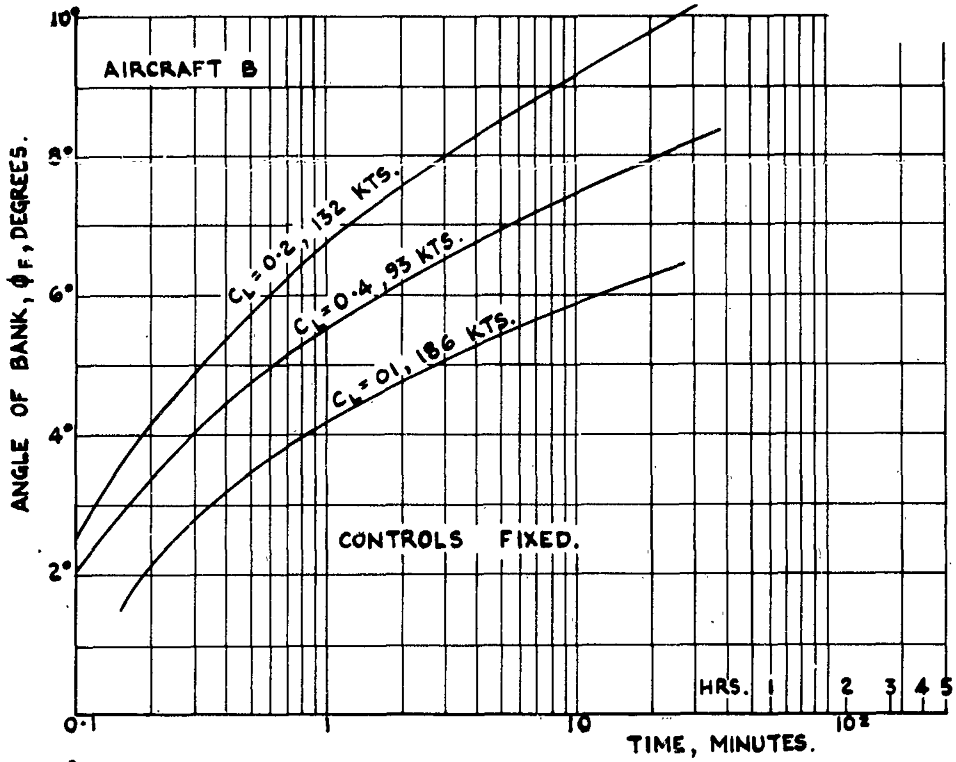


Fig.10 Spectral density of angle of bank for three forward speeds aircraft B, sea level



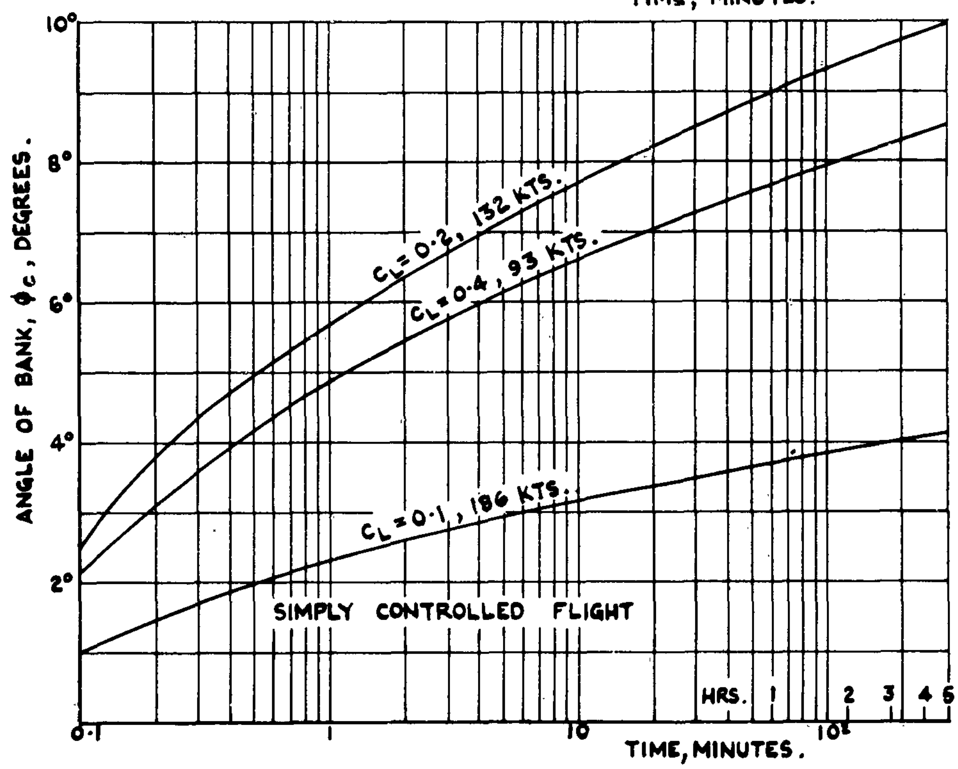


Fig.11 Angle of bank expected within a given time; aircraft B, S.L., turbulence, $\overline{w_g^2} = 1$ (f.p.s.)^2

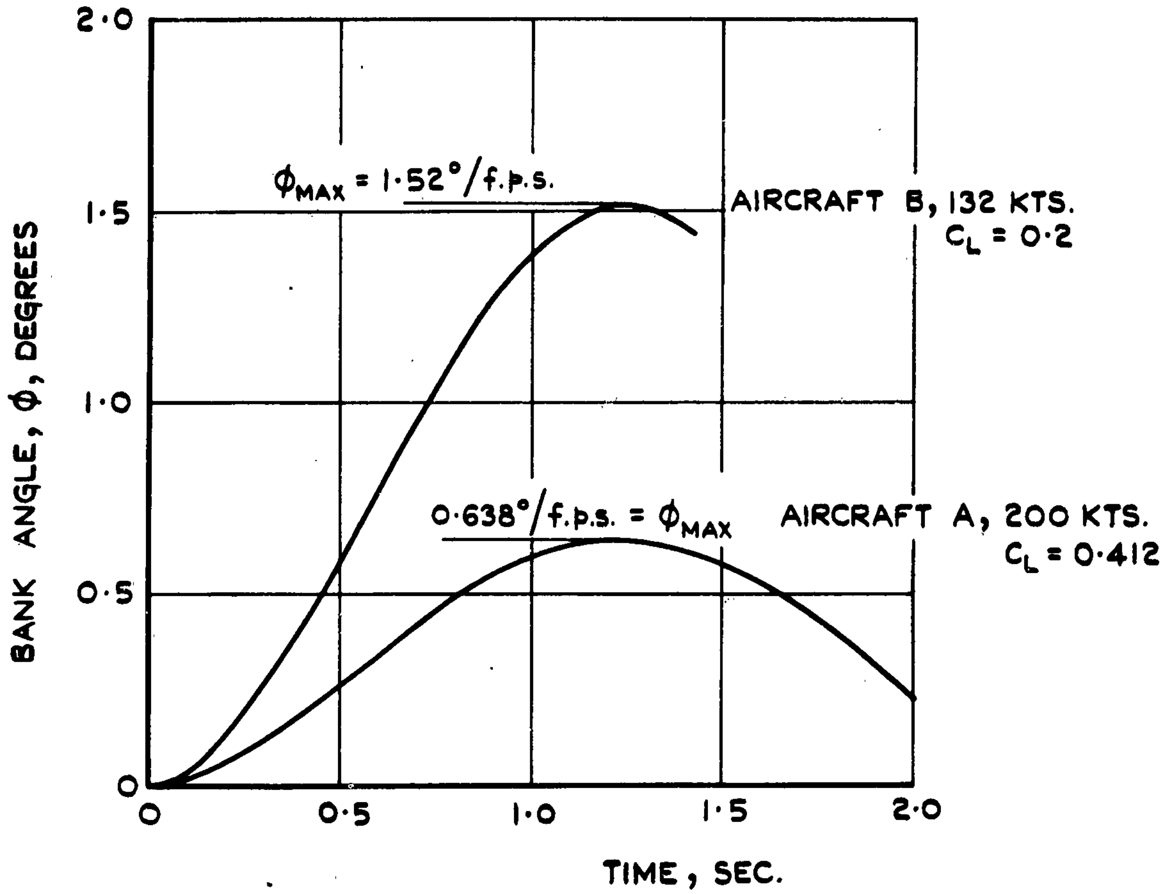
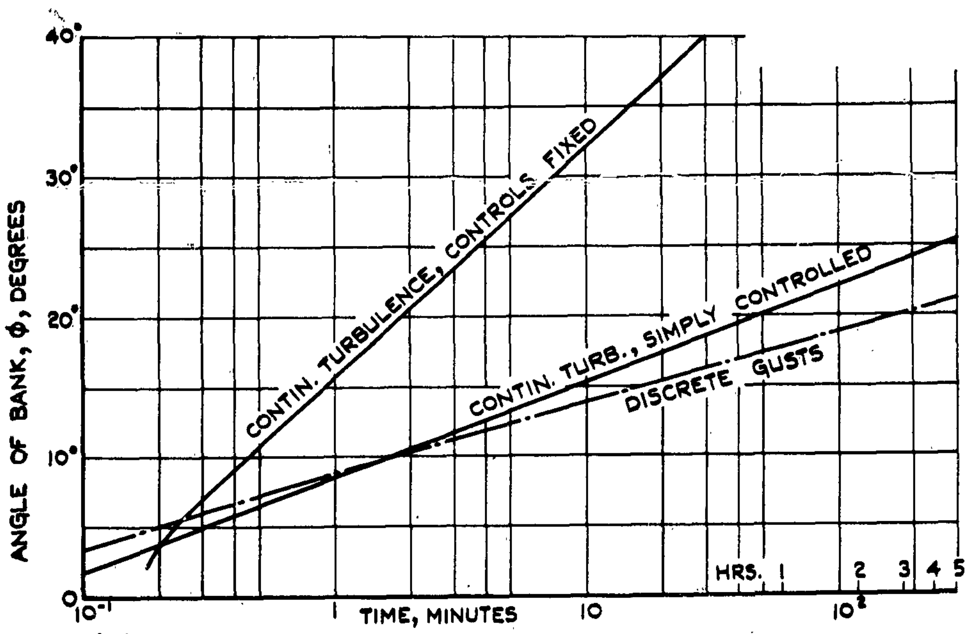
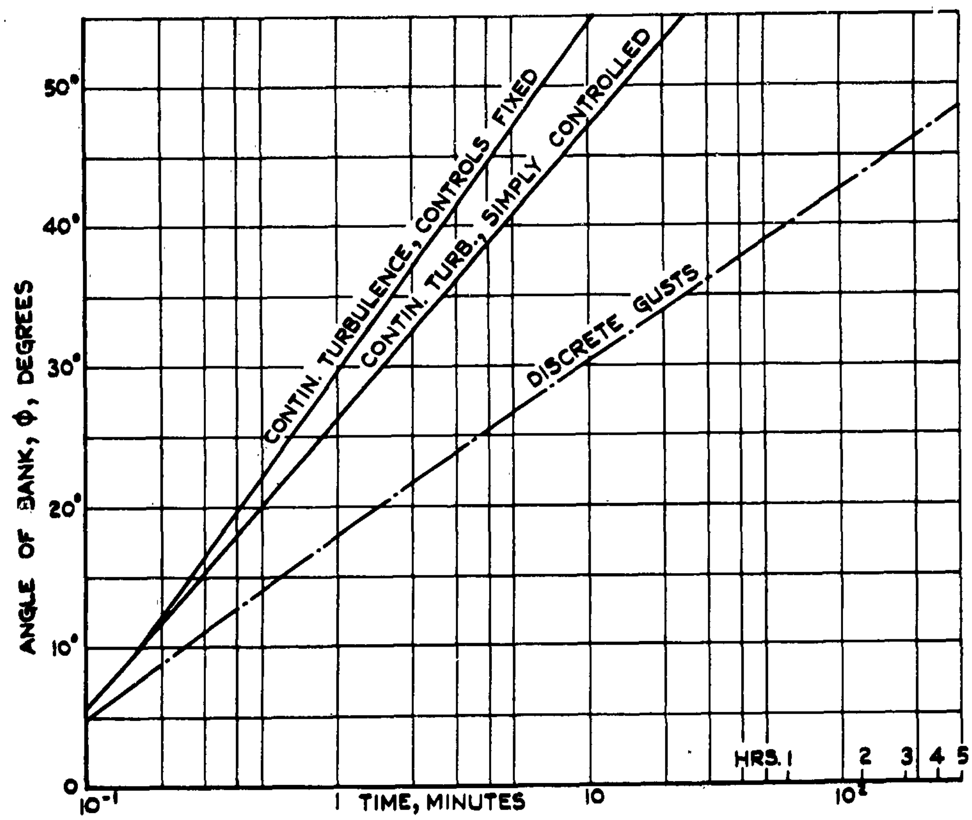


Fig.12 Response in angle of bank due to unit step input in lateral velocity, v_g



(a) AIRCRAFT A, 200 KTS., $C_L = 0{\cdot}412$.



(b) AIRCRAFT B, 132 KTS., $C_L = 0{\cdot}2$.

Fig.13 Angle of bank expected within a given time; comparison of estimates based on continuous turbulence and on discrete gusts techniques; low-altitude turbulence model (Ref.7)

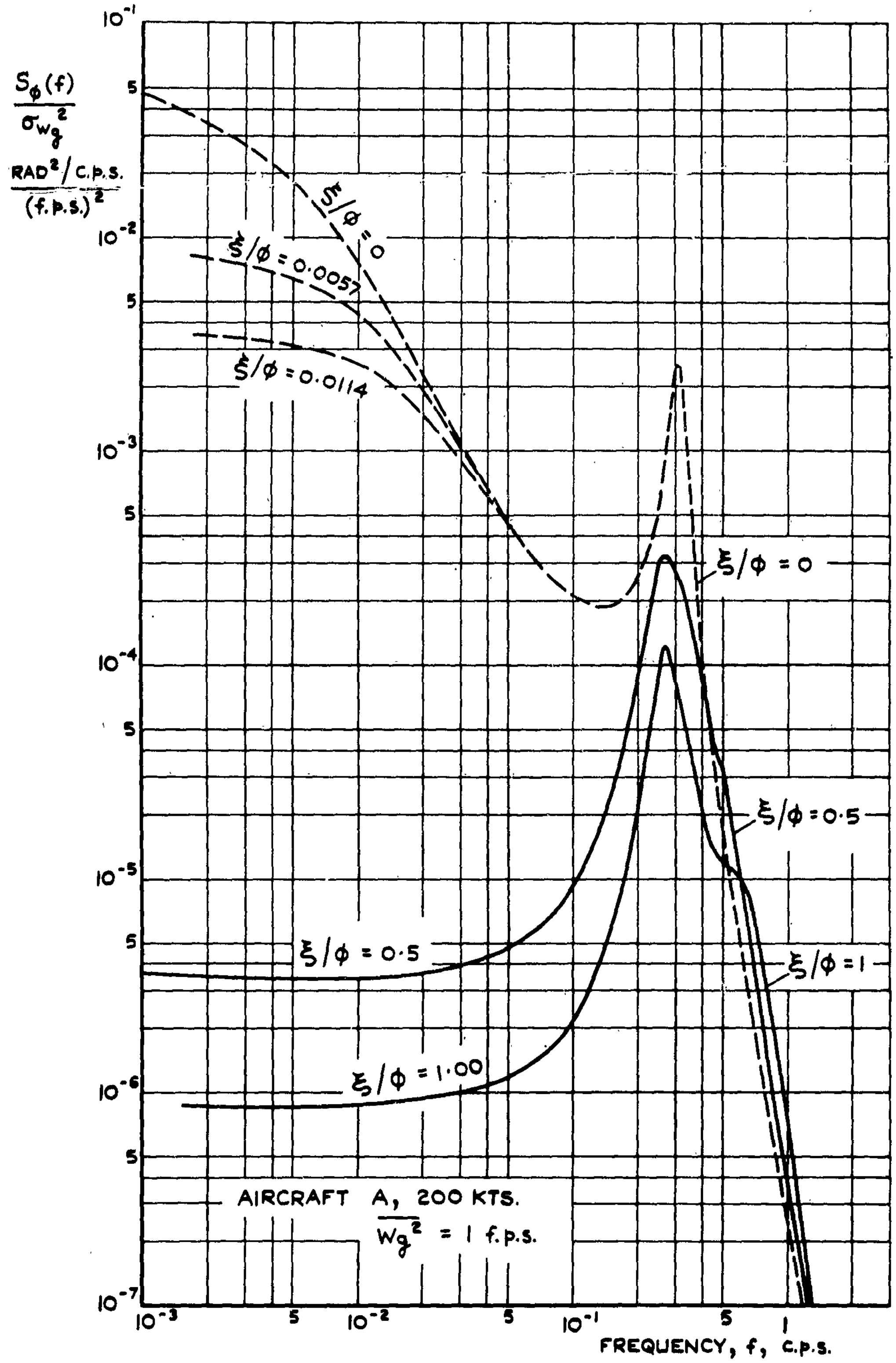
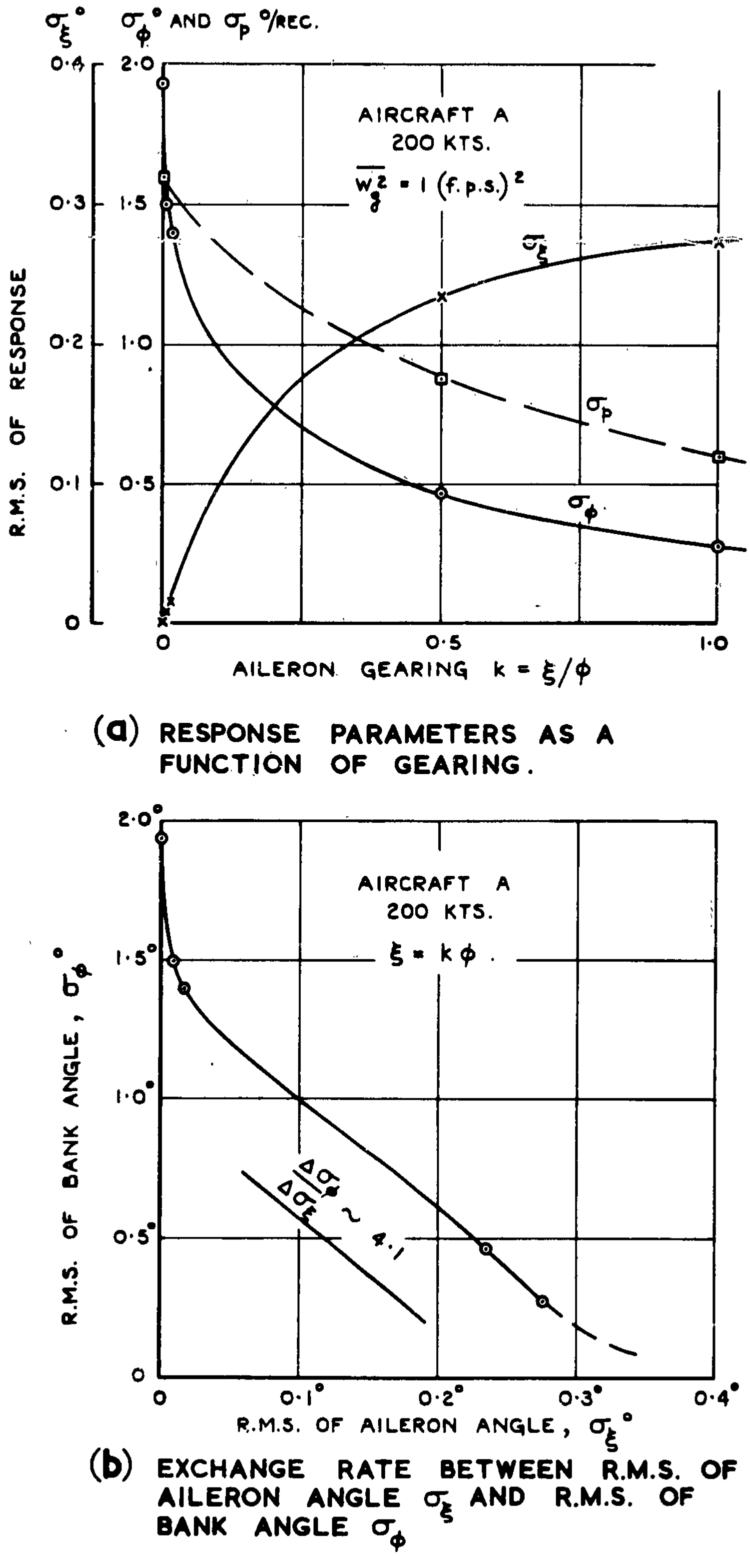


Fig.14 Spectral density of bank angle for five ailerons gearings (aileron angle proportional to bank angle)



(a) RESPONSE PARAMETERS AS A FUNCTION OF GEARING.

(b) EXCHANGE RATE BETWEEN R.M.S. OF AILERON ANGLE σ_ξ AND R.M.S. OF BANK ANGLE σ_ϕ

Fig.15 Effect of aileron gearing on the response to continuous turbulence of aircraft A, b = 35 ft

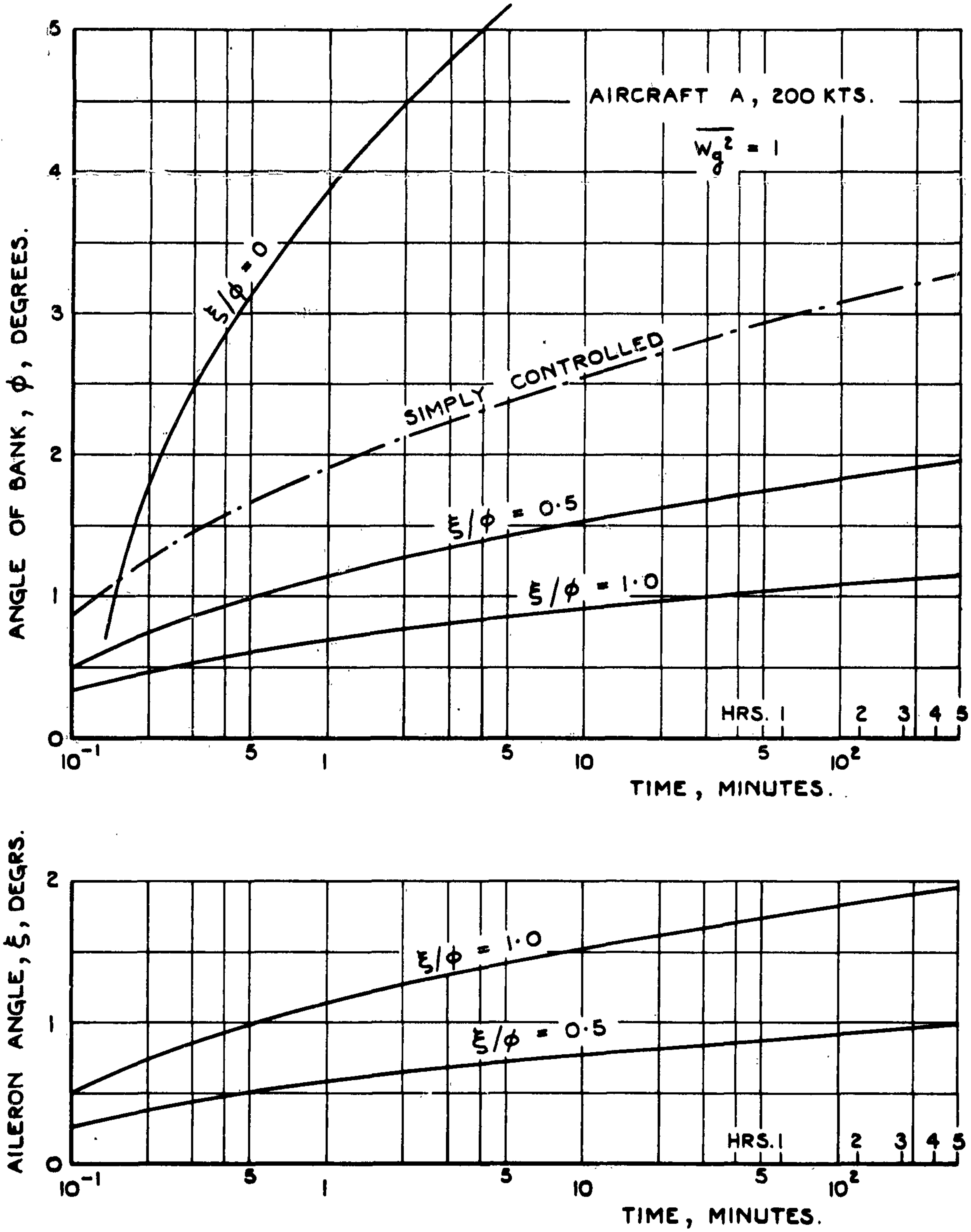
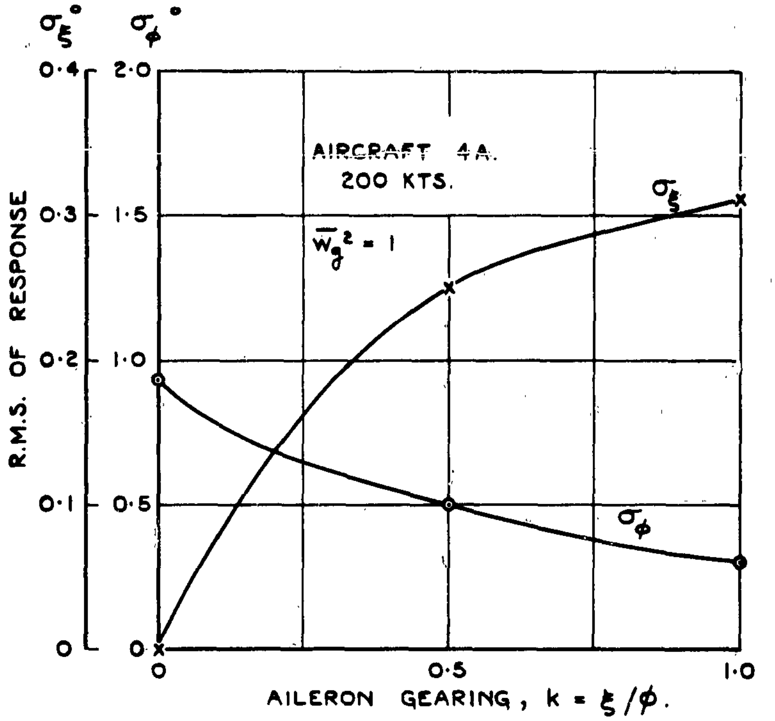
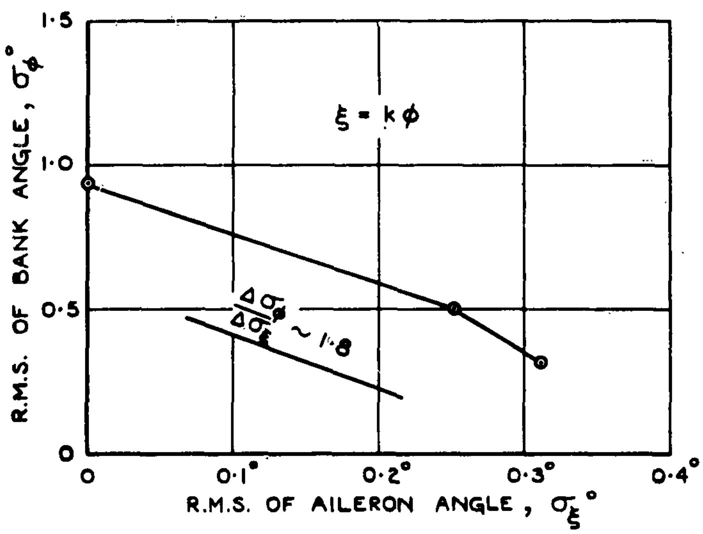


Fig.16 Angle of bank and aileron angle expected within a given time for three aileron gearings; aircraft A, 200 Kts., S.L. continuous turbulence, $\overline{w_g^2} = 1 \text{ (f.p.s.)}^2$



(a) RESPONSE PARAMETERS AS A FUNCTION OF AILERON GEARING.



(b) EXCHANGE RATE BETWEEN R.M.S. OF AILERON ANGLE σ_ξ AND R.M.S. OF BANK ANGLE, σ_ϕ.

Fig.17 Effect of aileron gearing $\xi = k\phi$ on the response to continuous turbulence of a large aircraft; aircraft 4A, b = 140 ft

FIG. 18.

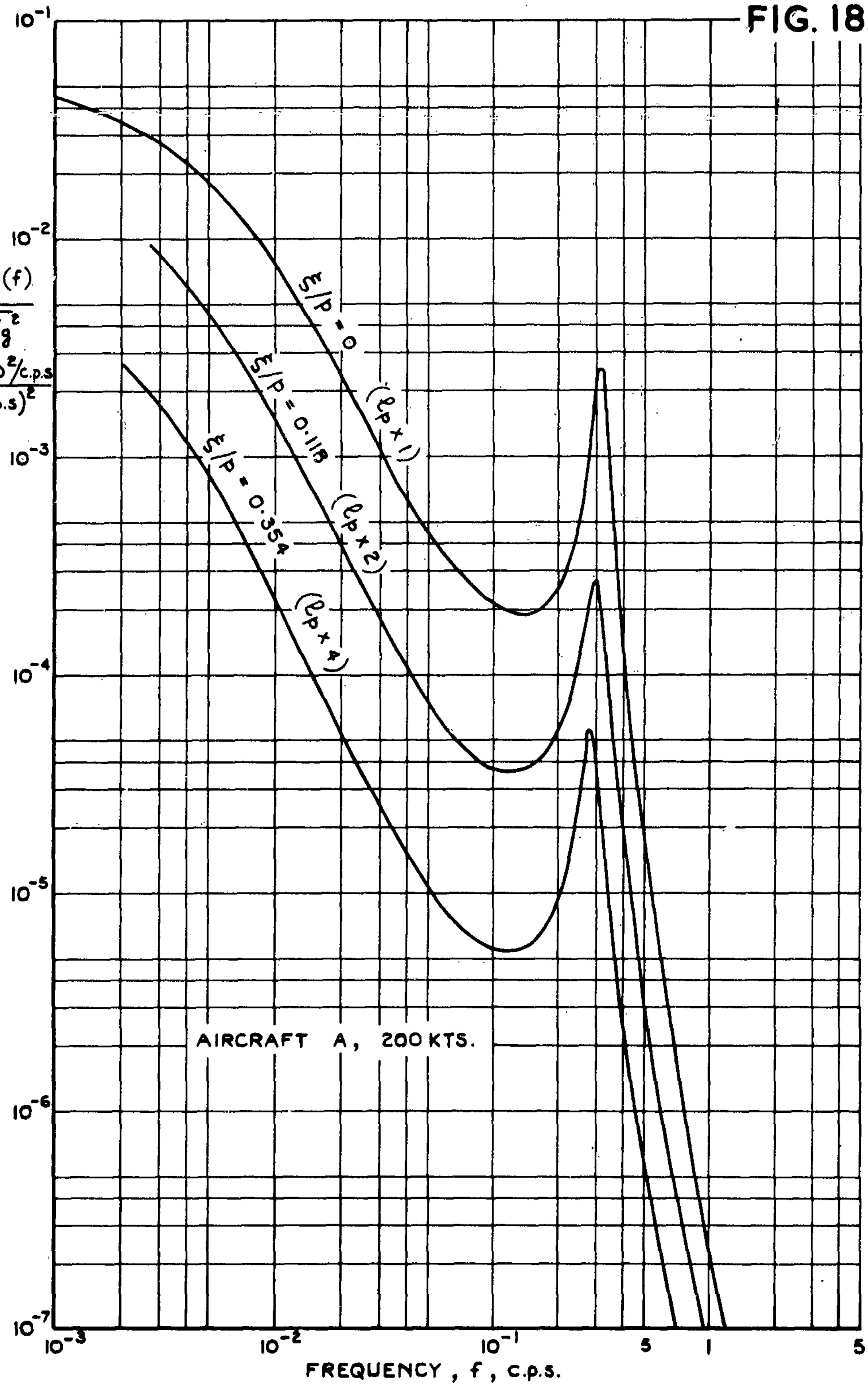
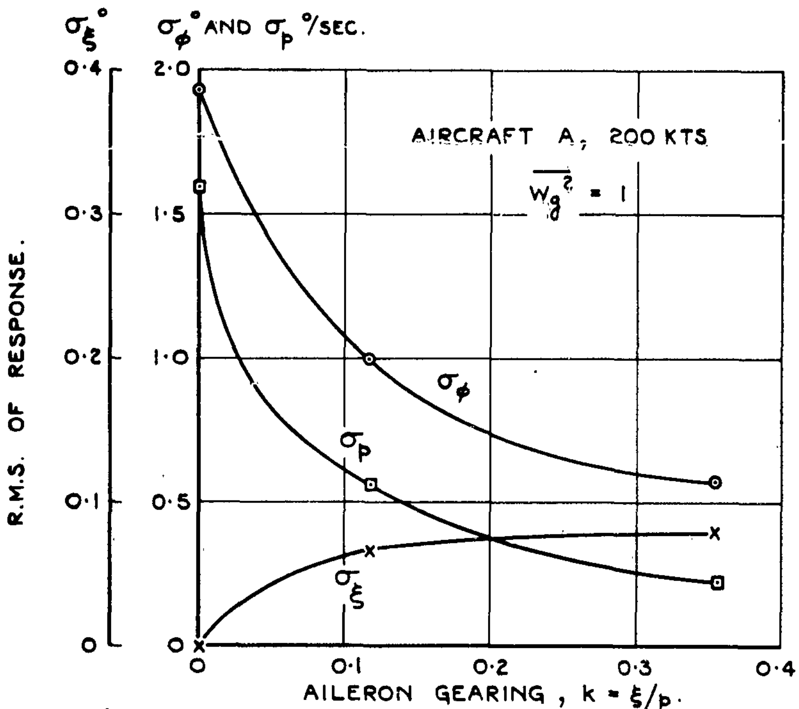
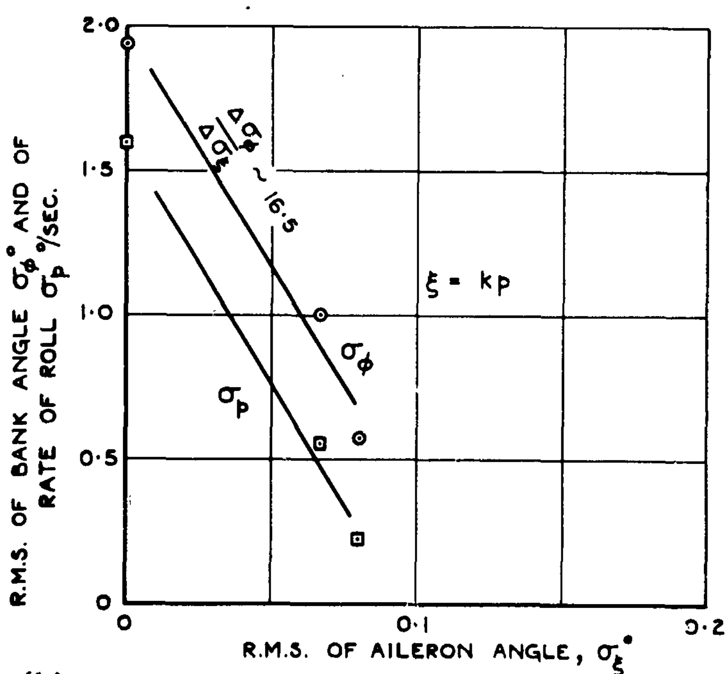


Fig.18 Spectral densities of bank angle for three aileron gearings (aileron angle proportional to the rate of roll) aircraft A, 200 Kts)



(a) RESPONSE PARAMETERS AS A FUNCTION OF AILERON GEARING.



(b) EXCHANGE RATE BETWEEN R.M.S. OF AILERON ANGLE σ_ξ AND R.M.S.'s OF BANK ANGLE σ_ϕ, AND RATE OF ROLL σ_p

Fig.19 Effect of aileron gearing, $\xi = kp$, on the response to continuous turbulence of aircraft A

DISCUSSION

G.H. Lee (U.K.): It is stated in the paper that increasing the size of the fighter aeroplane four times (linear scale) only slightly reduced the response in bank due to side gusts. Does the same apply to the slender-wing layout (aircraft B)?

Because of the large rolling moment due to sideslip of aircraft B, could this aircraft be controlled better (in bank) by means of rudder, rather than ailerons? It is suggested that rudder angle might be controlled by angle of bank or, perhaps, rate of roll (i.e. an artificial n_ϕ or n_p term).

Reply by H.H.B.M. Thomas: No calculations were made on the effect of size for an aircraft of the layout B. It is expected on general grounds that the effect of scale for aircraft B would be similar to its effect for aircraft A.

Reply by author: The angular responses of an aircraft (p, q and r) decrease with increase in size of aircraft (increasing exactly in proportion to the ratio of aircraft size to turbulence scale). The control power, however, decreases with increasing size of aircraft. Roughly speaking the value of $pb/2V$ is independent of aircraft size, thus the available p (rate of roll) is decreasing with increasing span b. The net answer is that the aileron angle required to correct gusts increases with increasing size of aircraft.

Reply by H.H.B.M. Thomas: On Mr. Lee's second point it is important to note that the nature of the Dutch roll mode for this aircraft, which is the result of its large inertia in yaw compared with roll and the large $-l_v$, argues against use of rudder as means of controlling. Further, the use of rudder controlled by bank or rate of roll has only an indirect effect on the aircraft damping.

Reply by author: Since a common misconception is involved it may be worth expanding the above somewhat. The explanation why artificial n_ϕ and n_p do not contribute much to minimization of the gust response is straightforward. From the equations of motion it can be seen that derivatives in ϕ and p, etc., such as n_ϕ, l_ϕ, n_p, l_p, etc. do not affect the excitation by the gust, thus by changing these derivatives we minimize the response to the turbulence, by stabilizing the aircraft, i.e. by modifying the characteristics equation only (improving the stability of different modes of aircraft motion). Further, any cross-derivatives like n_p, n_ϕ, l_r cannot affect the sum of dampings of different modes, they can only redistribute the energy among the modes. On that basis, one cannot expect any basic improvement in the lateral stability by introducing n_ϕ or n_p derivatives, and thus one cannot expect any substantial improvement in gust response. Of course, some improvement could be obtained through the change of frequencies, but this effect would be only of secondary importance.

To illustrate the point, the effect of rudder gearings $\xi = k\phi$ and $\xi = kp$ on the stability roots for aircraft B, $C_L = 0.2$, is shown in the accompanying sketch (overleaf).

H. Runyan (U.S.A.): I would like to mention some flight test work that is being pursued in the U.S.A. An instrumented subsonic airplane has been flown through a series

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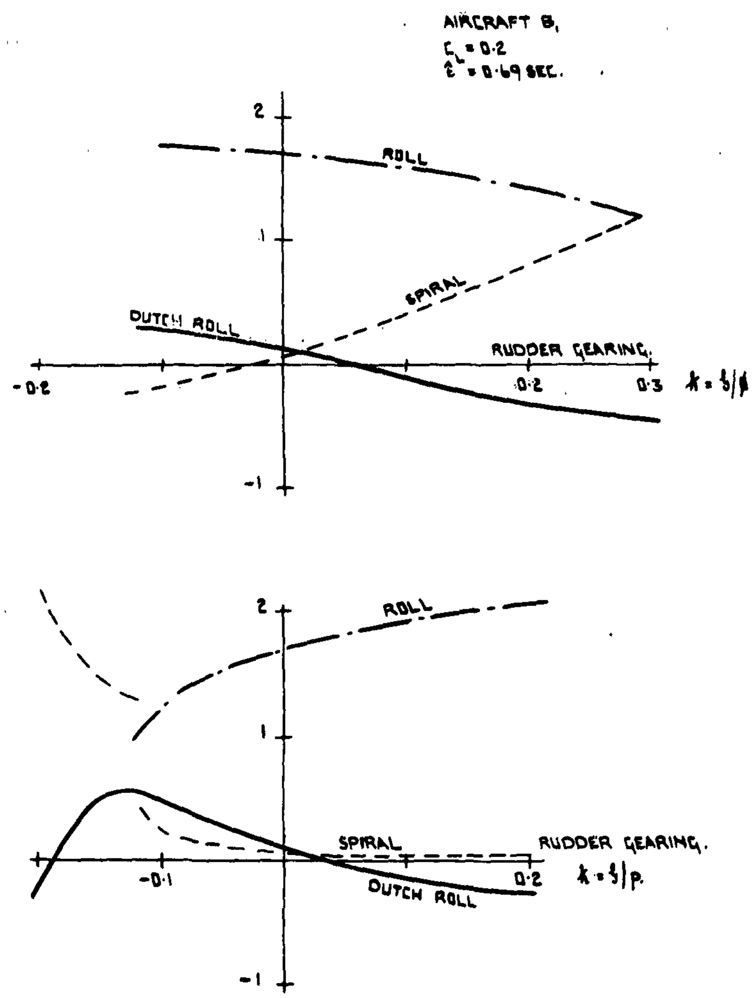
APP.



THE EFFECT OF RUDDER GEARINGS $\zeta = k\phi$ AND $\zeta = kp$ ON THE REAL PART OF STABILITY ROOTS FOR AIRCRAFT B AT $C_L = 0.2$

of thunderstorms for the purpose of measuring the power spectra of turbulence. The project is continuing with a supersonic airplane for the purpose of measuring the low-frequency portion of the spectrum applicable to the supersonic transport.

B. Etkin (Canada): I am pleased to note that Mr. Zbrozek has continued with the study of roll response of airplanes in gusty air, and that he confirms a conclusion I arrived at in an earlier RAE Report (Aero TN 2611) - that is, that the rolling of a slender airplane can be reduced to acceptable levels by the use of roll rate feed back to the ailerons. I wonder, however, about the soundness of his rather arbitrary way of dealing with the pilot. I think it is probably better to put in a simple mathematical model of the pilot, for example as was suggested yesterday morning by M. Grémont*. As to the relation between response to discrete gusts and to random turbulence, we can now say with some confidence just what this is. Whenever the process is one-dimensional (in the sense that we have a function of a single variable only (e.g. time), the following theorem holds: 'The integral of the square of the response to the discrete gust is exactly equal to the mean square response to random turbulence provided that

$$\Phi(\omega) = |F(\omega)|^2$$

where $F(\omega)$ is the Fourier transform of the transient gust input. In order for $F(\omega)$ to represent the spectrum of atmospheric turbulence closely, the gust should be a step input, followed by an exponential decay, i.e. gust velocity = $Ae^{-\gamma t}$. Even the simple step function is useful in this connection. The details of this theorem and its application are given in UTIA Tech. Note No. 32.

Reply by H.H.B.M. Thomas: In reply to Prof. Etkin's first point I cannot agree with the doubts he expressed on the soundness of the approach in dealing with the control by the pilot. All that can be questioned is the frequency value at which the 'cut-off' is applied. As explained in the talk and rather more fully in the written paper, frequencies below 0.02 c.p.s. present no difficulty in control by the pilot. At frequencies between this and something of the order of ten times this the pilot is able to control, although it requires a more conscious effort on his part.

As to the arbitrariness of the approach the results shown in slide 9 (Fig.13 of the paper) are relevant. In the upper diagram (Fig.13(a)) we see that for the aircraft known to have satisfactory dynamic characteristics there is good agreement of the results for 'discrete' and continuous turbulence in spite of the different assumptions regarding the control of the motion by the pilot. This illustrates an insensitivity to exact value of frequency assumed at the 'cut-off'. Matters are quite different for aircraft B, which is dynamically much less satisfactory than aircraft A.

The use of a transfer function for the pilot is an alternative and attractive method of dealing with the controlled case. It does, however, pose the question as to what should be assumed as the input to the pilot.

Reply by author: The mathematical model of the pilot is an attractive concept which works well in some applications but more often than not is misused by people who know more about mathematics than physics.

Reply by H.H.B.M. Thomas: As to Prof. Etkin's remarks on his work on the nature of the equivalence of a discrete gust and the power spectrum approach, I can only say that I have seen his paper and accept his argument. However, the results refer only to mean-square response, which may not be all that is interesting in some applications. Furthermore, there is a difference between discrete gust in this sense and as used in Zbrozek's paper as the latter involves frequency of occurrence of discrete gusts of given magnitude.

Reply by author: It is interesting to learn about the 'theorem', but I am not sure about the practical application of it. The fundamental difficulty, as yet unresolved, is the problem of frequency. The discrete gusts lead to the frequency of occurrence as defined by the gusts per mile measured by, say, counting-accelerometers. The actual frequency of occurrence is the property of the responding system in conjunction with input spectrum; thus there is apparent incompatibility between spectral and discrete gust approaches.

ADDENDUM

AGARD SPECIALISTS' MEETING

on

STABILITY AND CONTROL

Complete List of Papers Presented

Following is a list of the titles and authors of the 41 papers presented at the Stability and Control Meeting held in Brussels in April, 1960, together with the AGARD Report number covering the publication of each paper.

INTRODUCTORY PAPERS

The Aeroplane Designer's Approach to Stability and Control, by G.H.Lee (United Kingdom) .. .. Report 334

The Missile Designer's Approach to Stability and Control Problems, by M.W.Hunter and J.W.Hindes (United States) .. .. Report 335

DESIGN REQUIREMENTS

Flying Qualities Requirements for United States Navy and Air Force Aircraft, by W.Koven and R.Wasicko (United States) .. .. .. Report 336

Design Aims for Stability and Control of Piloted Aircraft, by H.J.Allwright (United Kingdom) .. .. Report 337

Design Criteria for Missiles, by L.G.Evans (United Kingdom) .. Report 338

AERODYNAMIC DERIVATIVES

State of the Art of Estimation of Derivatives, by H.H.B.M.Thomas (United Kingdom) .. .. Report 339

The Estimation of Oscillatory Wing and Control Derivatives, by W.E.A.Acum and H.C.Garner (United Kingdom) .. .. Report 340

Current Progress in the Estimation of Stability Derivatives, by L.V.Malthan and D E.Hoak (United States) .. .. Report 341

Calculation of Non-Linear Aerodynamic Stability Derivatives of Aeroplanes, by K.Gersten (Germany) .. .. Report 342

Estimation of Rotary Stability Derivatives at Subsonic and Transonic Speeds, by M.Tobak and H.C.Lessing (United States) .. .. .. Report 343

Calcul par Analogie Rhéoélectrique des Dérivées Aérodynamiques d'une Aile d'Envergure Finie, by M.Enselme and M.O.Aguesse (France) .. Report 344

A Method of Accurately Measuring Dynamic Stability Derivatives in Transonic and Supersonic Wind Tunnels, by H.G.Wiley and A.L.Braslow (United States) .. .. Report 345

Mesure des Dérivées Aérodynamiques en Soufflerie et en Vol, by M.Scherer and P.Mathe (France) .. .. Report 346

Static and Dynamic Stability of Blunt Bodies, by H.C.DuBose (United States) .. .. Report 347

AEROELASTIC EFFECTS

Effects of Aeroelasticity on the Stability and Control Characteristics of Airplanes, by H.L.Runyan, K.G.Pratt and F.V.Bennett (United States) Report 348

The Influence of Structural Elasticity on the Stability of Airplanes and Multistage Missiles, by L.T.Prince (United States) .. .. Report 349

Discussion de deux Méthodes d'Etude d'un Mouvement d'un Missile Flexible, by M.Bismut and C.Beatrix (France) .. .. Report 350

The Influence of Aeroelasticity on the Longitudinal Stability of a Swept-Wing Subsonic Transport, by C.M.Kalkman (Netherlands) .. .. Report 351

Some Static Aeroelastic Considerations of Slender Aircraft, by G.J.Hancock (United Kingdom) .. .. Report 352

COUPLING PHENOMENA

Pitch-Yaw-Roll Coupling, by L.L.Cronvich and B.E.Amsler (United States) Report 353

Application du Calculateur Analogique à l'Etude du Couplage des Mouvements Longitudineaux et Transversaux d'un Avion, by F.C.Haus (Belgium) .. .. Report 354

Influence of Deflection of the Control Surfaces on the Free-Flight Behaviour of an Aeroplane: A Contribution to Non-Linear Stability Theory, by X.Hafer (Germany) .. .. Report 355

STABILITY AND CONTROL AT HIGH LIFT

Low-Speed Stalling Characteristics, by J.C.Wimpenny (United Kingdom) Report 356

Some Low-Speed Problems of High-Speed Aircraft, by A.Spence and D.Lean (United Kingdom) .. .. Report 357

Factors Limiting the Landing Approach Speed of an Airplane from the Viewpoint of a Pilot, by R.C.Innis (United States) .. .. Report 358

Post-Stall Gyrations and Their Study on a Digital Computer, by S.H.Scher (United States) .. .. Report 359

THE APPLICATION OF SERVO-MECHANISMS

The Place of Servo-Mechanisms in the Design of Aircraft with Good Flight Characteristics, by K.H.Doetsch (United Kingdom) .. .. Report 360

Effects of Servo-Mechanism Characteristics on Aircraft Stability and Control, by F.A.Gaynor (United States) .. .. Report 361

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