REPORT 367

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ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

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MATHEMATICAL MODELS FOR MISSILES

by

W. S. BROWN and D. I. PADDISON

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NORTH ATLANTIC TREATY ORGANISATION

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(5) ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT,

Paris (France).

(6) MATHEMATICAL MODELS FOR MISSILES

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W.S. Brown and D.I. Paddison

This Report is one in the Series 334-374, inclusive, presenting papers, with discussions, given at the AGARD Specialists' Meeting on 'Stability and Control', Training Center for Experimental Aerodynamics, Rhode-Saint-Genèse, Belgium, 10-14 April 1961, sponsored jointly by the AGARD Fluid Dynamics and Flight Mechanics Panels

SUMMARY

This paper describes the use of a large analogue computer, supplemented by digital computations, to study the effects of high-incidence aerodynamic non-linearities and cross-couplings on the performance of a hypothetical cruciform missile. Aerodynamics data, obtained from wind-tunnel tests of a suitable model, were simulated in detail, and linear aerodynamic characteristics were subsequently substituted and simulated for purposes of comparison.

The effects of the aerodynamics on response and on homing performance against targets turning at a constant rate are displayed as graphs, and the importance of accurately simulating the aerodynamics is demonstrated.

Techniques which proved useful in simulation are described, and the accuracy and limitations of the analogue method are indicated by comparisons of the simulator results with others obtained digitally.

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CONTENTS

LIST OF FIGURES

NOTATION

G, XYZ	principal axes in test vehicle (see Fig. 1)
G	centre of gravity
m	mass of test vehicle
S	representative area on test vehicle
l	representative length on test vehicle
A, B, C	principal moments of inertia
X, Y, Z	aerodynamic forces in principal axes
L, M, N	aerodynamic moments about principal axes
V	total linear velocity (relative wind velocity)
u, v, w	component linear velocities
p, q, r	component angular velocities
b, c	component lateral accelerations
θ	total body incidence to relative wind
ϕ	azimuth angle of wind vector, tan ϕ = v/w
α, β	partial incidences
$\bar{\alpha}$	sin α = w/V
$\bar{\beta}$	sin β = v/V
$\bar{\theta}$	sin θ, cos θ = u/V
σ	control panel deflection
ξ, η, ζ	equivalent aileron, elevator and rudder angles
c_X, c_Y, c_Z	aerodynamic force coefficients
c_l, c_m, c_n	aerodynamic moment coefficients
l_p, m_q, n_r	aerodynamic rotary derivatives of L, M, N
M	Mach number

γ	ratio of specific heats of air
p	ambient air static pressure
$G, XY'Z'$	rectangular axes obtained by rotating G, XYZ system about GX through angle ϕ so that plane GXZ' contains wind vector
	prime denotes quantity measured in $G, XY'Z'$ axes
Demand Axes	rectangular axes with centre G and same X-axis as G, XYZ system, but not rotating with test vehicle. Demand always for rotation about Y-axis of demand-axis system
X_{Go}	initial angle of rotation of G, XYZ axes relative to demand axes
R	Range
Suffix D	= demanded
d	= in demand axes
m	= in missile axes
o	= initial value
i	= indicated value

MATHEMATICAL MODELS FOR MISSILES

A Simulator Study of the Effects of Aerodynamic Non-Linearities and Cross-Couplings on the Performance of a Guided Missile

W.S. Brown and D.I. Paddison*

1. INTRODUCTION

As the science of aeronautics advances and aircraft performance steadily improves, the problem of intercepting high-flying assault aircraft by anti-aircraft guided missiles becomes increasingly difficult. Missiles which are not steered by such means as thrust deflection depend for manoeuvre entirely upon the aerodynamic forces and moments which they experience when their fixed or movable surfaces are inclined to the relative wind. A simple approximate calculation shows that, in the stratosphere, the incidence necessary to achieve any desired rate of turn of the flight path varies directly as the density of the missile relative to the air and inversely as the Mach number of flight. Hence, for any particular missile and Mach number, the required incidence increases with altitude approximately as the inverse of the air density. The same law, of course, applies to the aircraft to be intercepted, but the effect on the missile is likely to be more severe because its design is usually determined not solely by aerodynamic considerations but also by those of logistics which do not apply to the aircraft - questions of transport, ease of deployment, suitability for loading on and firing from aircraft or launching platforms in a limited environment. Because of these special requirements, the designer is seldom free to select the optimum aerodynamic configuration for his missile but must, in general, restrict the size of its lifting surfaces. In consequence, the range of incidence required for adequate manoeuvre under all conditions may be much larger in the case of the missile than in that of the aircraft. It is important, therefore, that due attention be paid to the aerodynamic characteristics of winged missiles required to operate at high altitudes.

Whatever its design, it is unlikely that the aerodynamic forces and moments acting on a missile will vary linearly with incidence over more than a limited range. This suggests that, at high altitudes, when adequate manoeuvrability may require large incidences, any aerodynamic non-linearities which may be present may have an important effect on performance. If this is so, the customary assumption of linear aerodynamic characteristics made in design studies, and commonly employed also in more detailed investigations conducted with the aid of simulators or digital computers, may lead to erroneous conclusions. It was such considerations that led us to plan a programme of research combining flight experiments, using a test vehicle, with a detailed simulation which would include any aerodynamic non-linearities which happened to be present in the design. One object of this dual programme was to determine to what extent the observed performance of the test vehicle in flight, in response to selected demands for manoeuvre, was reproducible on the simulator. Given good agreement, we would then use the simulator to extend the investigation by adding some form of homing system to the model and determining whether or not homing

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**United Kingdom*

performance was likely to be affected by aerodynamic characteristics of the type mentioned. It is the latter work which we propose to describe in this paper. We shall give some account of the aerodynamic features of the test vehicle, the mathematical model on which the simulation was based, the techniques which we saw fit to employ in order to represent the aerodynamic characteristics with reasonable accuracy and the limitations of our methods. In discussing some of the results obtained we shall note an instance in which the use of the customary approximate mathematical model would have led to erroneous conclusions.

2. CHOICE OF AERODYNAMIC CONFIGURATION

For a variety of reasons, we selected a fixed wing cruciform design having four movable tail surfaces in line with the wings as most suitable for our work. From the point of view of aerodynamic non-linearities, fixed-wing designs are likely to be more interesting than their moving-wing counterparts because, with the former, large wing incidence is associated with a large body incidence. Furthermore, a Cartesian missile is often regarded as a combination of two 'single plane' missiles at right angles and, from this standpoint, is a simpler proposition to analyse than a missile of the twist and steer variety. This apparent simplicity might be deceptive however if, in an actual missile, aerodynamic cross-coupling between the planes were present in addition to the customary inertial and gyroscopic couplings. In any case, it was of some interest and importance to determine the effect of such features on the ideal performance.

Our choice was finally determined by the fact that the Royal Aircraft Establishment already had a general-purpose test vehicle of this type which had been extensively tested in flight and in the wind tunnels as part of a programme of basic research. However, as our investigation was primarily one of the effects of large incidences, it was necessary to supplement the existing wind-tunnel data by further tests on the model. These were conducted at a variety of supersonic Mach numbers over a range of incidence up to 35° to the relative wind. In the meantime, the flight tests already completed and the wind-tunnel data already available served as a useful starting point and check on the simulator model.

3. AERODYNAMIC CHARACTERISTICS OF TEST VEHICLE

3.1 General

A diagrammatic representation of the test vehicle is shown in Figure 1, which also indicates the system of co-ordinate axes adopted for the mathematical model and the scheme of notation. G, XYZ are principal axes defining the longitudinal axis of the test vehicle and the planes through the two pairs of wings. The centre of gravity is assumed to be located at G ; θ is the angle between the velocity vector GV and the longitudinal axis and, hence, represents the total incidence, and ϕ is the angle between the XZ plane and the plane containing the longitudinal axis and the velocity vector. Hence, $\tan \varphi = v/w$, where v and w are the sideslip velocities in the directions of GY and GZ . We may also here define partial incidences α and β with $\sin \alpha = \bar{\alpha} = w/V$ and $\sin \beta = \bar{\beta} = v/V$, V being the total velocity of the missile, so that $w = \bar{\theta} \cos \phi$ and $v = \bar{\theta} \sin \varphi$, where $\bar{\theta} = \sin \theta$. We shall have occasion to refer to these later. The angular deflections of the control surfaces are denoted by σ with an appropriate suffix, and linear combinations of these quantities are equivalent to the ξ, η, ζ of the standard notation for aileron, elevator and rudder angles.

Since the wind-tunnel model with its control surfaces undeflected had geometrical symmetry in each wing plane, all the aerodynamic quantities measured were periodic functions of ϕ, a feature which was of considerable value in both the wind-tunnel testing and the subsequent simulation. Only effects resulting from deflections of the controls disturbed the symmetry in the four quadrants and, even so, it was possible to reduce the amount of wind-tunnel work by considerations of symmetry. In order to cover fully all contingencies, the effects of deflecting the control surfaces were determined over ranges of incidence of the latter between 35° positive and negative.

3.2 Wing-Body Tail-Fixed Characteristics

The general pattern of the results is indicated in Figures 2, 3 and 4, Figures 2 and 3 relating to the model with controls undeflected. In Figure 2, the symbols without primes denote lift and pitching-moment coefficients in the principal plane GXZ, and primed symbols the same quantities measured in the plane containing the wind vector and the X-axis, with the addition of C_Y' as the lateral force at right angles to this plane. Figure 2a shows that the Z force varies almost linearly with incidence, in the XZ plane, and Figure 2b that it varies little with ϕ, the trend at 25° incidence being plotted to indicate this. On the other hand, the pitching moment, as Figure 2c shows, is markedly non-linear with incidence in the XZ plane and, in the wind plane, changes so rapidly with ϕ as to become negative around $\phi = 45^\circ$, as indicated by Figure 2d. The same figure indicates the presence of a yawing moment C_n' at right angles to the pitching moment, which is zero only in planes of symmetry. The non-linearity in the pitching moment is attributable to a longitudinal movement of the centre of pressure with change of incidence, which is different in different wind planes; and the side force and moment are indicative of aerodynamic cross-coupling.

The rolling moment about the X-axis, with the wind plane inclined at 15° to the XZ plane, is shown in Figure 3a as a function of the total incidence, and its variation with ϕ at 25° incidence is indicated in Figure 3b. The rolling moment, therefore, is markedly non-linear with respect to both incidence and angle of roll and, although zero in planes of geometrical symmetry, that is, in either of the wing planes or in the planes through the longitudinal axis of the model equally inclined to the wings - planes referred to later as the 45° planes, is always of such a sign as to rotate the model into the nearest 45° plane, where the roll equilibrium is stable in contrast to that in the wing planes, which is unstable. It may be noted in passing that the rolling moment at any incidence is a maximum for a value of ϕ in the neighbourhood of 22½°.

To sum up, therefore, our tests showed that, in all planes other than those of geometrical symmetry, the resultant force and moment on the model did not lie in the plane of total incidence as defined above, and the pitching, rolling and yawing moments were markedly non-linear with respect to both total incidence and angle of roll. The stability in pitch was greatest in the wing planes and least in the 45°

planes, the opposite being true of the stability in roll. These results are attributable to aero-dynamic non-linearities and cross-couplings. Had such effects been absent, the quantities C'_Y and C'_n would have been small and C'_Z and C'_m almost independent of ϕ. The rolling moment coefficient C_l would have been identically zero. The effects noted varied to some extent with Mach number but their general character was unchanged.

3.3 Control Surface Characteristics

Deflection of the control surfaces revealed a complicated situation. With the body in any selected attitude, the tail moments were non-linear functions of the control-surface deflections relative to the wings; but the shapes of the curves depended also on the model's attitude to the wind, since part of the non-linearity was due to wing-body interference. The method of investigation adopted in the wind-tunnel was first to deflect only one tail fin at a time, and then to move two adjacent ones together by varying amounts. This was done over the full range of attitudes of the model covered in the tests. The results revealed that the mutual interference of the fins was relatively small and that it was possible to find approximate mathematical expressions for this and the other effects observed. Figure 4, which is a plot of elevator trim angles in terms of the partial incidences α and β for a typical Mach number and centre of gravity, is indicative of the overall non-linearities and cross-couplings.

3.4 Effect of Aerodynamic Characteristics

These peculiarities in the aerodynamics revealed by a study of the model might be expected to have important effects on the performance of the test vehicle. In particular, the presence of aerodynamic cross-couplings implied that, if the vehicle were required to manoeuvre under aerodynamic force in any plane other than a principal plane of symmetry, the control surfaces would have to provide not only the requisite force and moment in the desired plane of manoeuvre but also those necessary to resist a couple tending to rotate the vehicle out of the plane. Hence, unless the control system were designed to prevent this, the vehicle would tend to yaw and roll away from the desired plane of manoeuvre. In particular, unless the roll control system provided stabilisation of roll position, the vehicle would, in general, roll under the application of a demand for incidence until it reached a position of stable roll equilibrium; in other words, the incidence would be developed ultimately in one of the 45° planes. This would apply equally to a vehicle without any roll autopilot and to one which was stabilised by control of roll rate. For this reason, it was decided to simulate roll rate stabilisation in the first place, although the test vehicle in its earlier flight tests had been roll position stabilised. In any case, an easy transition could be made to an idealised roll position stabilised system, if desired, by suppressing the roll.

4. DETAILS OF SIMULATION

The simulation was undertaken on TRIDAC, a large analogue computer at the Royal Aircraft Establishment, which has been described elsewhere[1,2].

4.1 Wing-Body Tail-Fixed Aerodynamics

The generation of the complex aerodynamics on the simulator posed problems. At the outset, two methods of approach were considered and tried. The first of these involved the expansion of the aerodynamic forces and moments as polynomials in $\bar{\alpha}$ and $\bar{\beta}$, a method which had been adopted previously. On this basis, the total incidence θ of the model is given by $\bar{\theta}^2 = \bar{\alpha}^2 + \bar{\beta}^2$, where $\bar{\theta} = \sin\theta$. TRIDAC is well suited to such a method, for it is equipped with several sets of high-grade linear potentiometers ganged together and driven by high-performance hydraulic servo-motors. However, although the method of polynomials is extremely effective over a limited range of incidence if the functions to be generated are reasonably linear, it becomes rapidly less satisfactory if marked non-linearities occur at larger incidences; for then it is necessary to include many more terms in the expansion. Invariably, both positive and negative terms occur and the associated constant multipliers often vary greatly in magnitude. A formidable scaling problem therefore arises, added to which the successive multiplications necessarily introduce inaccuracies. In the end, it usually happens that, while the value of the function generated is correct at each of the points selected to define the polynomial, the accuracy of the representation at intermediate points is poor. It was found to be much better, when generating the non-linear wing-body aerodynamics, to take advantage of the aforementioned symmetry of the cruciform design when the tail control surfaces are undeflected. This results in the forces and moments being periodic functions of the angle ϕ, which enables one to express these quantities as Fourier sine or cosine series in ϕ, the coefficients being functions of θ.

The general form adopted to express the wing and body forces and moments with the controls undeflected was, therefore,

$$F'(\bar{\theta},\phi) = \sum_n \left[G_n(\bar{\theta}) \sin 4n\phi + H_n(\bar{\theta}) \cos 4n\phi \right]$$

the fundamental variable in the trigonometrical terms being 4ϕ rather than ϕ, since the symmetry repeats in each of the four quadrants formed by the wings.

TRIDAC is well equipped to generate such expressions also, since it has several highly accurate multiple sine and cosine resolvers driven by hydraulic servo-motors. The general term in $\sin 4n\phi$ or $\cos 4n\phi$ is, of course, easily obtained as a sum of products of powers of $\sin 4\phi$ and $\cos 4\phi$ but it was, in fact, never found necessary to include more than two terms of the series to obtain an adequate fit. The wind-tunnel data had been obtained over series of values of θ and ϕ covering the required ranges and were therefore, in a suitable form from the start. The ordinates of the $G_n(\bar{\theta})$ and $H_n(\bar{\theta})$ functions at each experimental value of θ were determined by applying the method of least squares to the appropriate section of the data at that incidence, i.e. to the set of values of each quantity at all the values of ϕ,

4.2 Function Generators

The G and H functions were simulated by means of diode function-generators of a pattern developed by the staff of TRIDAC. One of these generators is illustrated in Figure 5. The basic unit has six sections and it is mounted on a card measuring approximately 8.5 cm by 19.5 cm, which also carries the silicon diodes and the miniature potentiometers used to fix the start and slope of each link of the chain of tangents to the curve used as the first approximation to it. A printed circuit is employed, and the card also carries input sockets for the signal, bias and a.c. smoothing voltages, and output sockets for the connections to the associated amplifiers. The smoothing voltage, which is the special feature of these units, has a saw-tooth waveform, is adjustable in amplitude, has a frequency of 100 kc/sec, and is superimposed on the input voltage. It modifies the output of the function generator by a process which amounts to sampling the slopes of the tangents in succession. As each junction of tangents is approached, the next slope is sampled to an increasing extent. It may readily be shown that the process rounds off the corners at the junctions, which are commonly referred to as 'break points', and, since the amount of rounding is adjustable by varying the input summing resistor of the smoothing voltage, it results in a close fit to the curve at all points. If higher accuracy is required, two function generators may be used in series, thus doubling the number of sections. In our experience, however, one six-section unit with smoothing is as good as one of twelve-sections without smoothing. Invariably, we have been able to generate the required functions with error everywhere less than one per cent of the maximum value of the function in the range. This is certainly as good as, and probably better than, the accuracy of the wind-tunnel data.

4.3 ϕ Servo

By such means, the aerodynamic forces and moments in any wind plane were expressed as functions of $\bar{\theta}$ and ϕ, the variables defining the position of the wind vector, and were then resolved, as required, to form the forces and moments in the principal planes of the model. The servo-resolver which provided sin 4ϕ and cos 4ϕ also generated sin ϕ and cos ϕ for these resolutions, a gear box of ratio 4:1 having been provided. The method of driving this servo may be of interest. In the course of the simulation, the quantities v, w and V, defined earlier, were generated. By feeding the voltages representing v and w to two resolvers driven by the servo, the quantity

$$\epsilon = v \cos\phi - w \sin\phi$$

could be formed. If the shaft angle of the servo were correct, this quantity would be zero. Hence, ϵ could be employed as an error signal to drive the servo as a position servo. The servo was also able to provide

$$V\bar{\theta} = v \sin\phi + w \cos\phi$$

and, therefore the $\bar{\theta}$ necessary to generate the G and H functions. Difficulty arises with such a system, however, when $\bar{\theta}$ is small, because then both v and w are small; hence, ϵ is small even when ϕ is substantially in error. For this reason, it was necessary to provide the servo with a form of automatic gain control to increase the error signal when $\bar{\theta}$ was small. A simple arrangement utilising an auxiliary servo driving a potentiometer to vary the gain of the error signal in proportion to the inverse of $\bar{\theta}$ over a suitable range of the latter was found to be completely effective.

It will have been noted also that, when ϕ is 0 or $\pi/2$, so that v or w is zero, ϵ is zero for any value of the incidence. Hence, if the wind vector happens to lie in either of the wing planes, where there is no rolling moment, there will be no error signal whatever the value of the incidence, large or small, positive or negative. Hence, the incidence could, apparently, change sign without the servo being aware of the fact. We feared this might be a fatal objection to the method but, in practice, it was found that, with the a.g.c. control, the small electrical drifts which were always present sufficed to cause the servo to swing rapidly through 180° in such instances. This also implies that the incidence, as defined in our system, is always positive, the position of the wind vector being determined by the value of the angle ϕ. A similar situation occurs when the wind vector is in either of the 45° planes, in which case the component velocities v and w are numerically equal. Once again, however, no trouble was experienced in practice, ϕ being precisely defined whenever θ exceeded 0.2°.

4.4 Control Surface Aerodynamics

As regards the generation of the contributions of the tail surfaces when deflected, it was found, as one might expect, that these consisted of a major term which was a linear function of the control angle relative to the appropriate wing, and which could, therefore, be expressed as a quantity multiplying ξ, η or ζ, and a part which contained cross products of these with $\bar{\alpha}$ and $\bar{\beta}$ or, alternatively, the equivalent functions of $\bar{\theta}$ and ϕ. This secondary part of the effect of the tail was found to be most easily expressed by polynomials in ξ, η, ζ, $\bar{\alpha}$, $\bar{\beta}$, and the terms were generated by electric servo-multipliers driven by these quantities.

4.5 Rotary Derivatives and Gravity

No attempt was made in the simulation to generate unsteady aerodynamic effects, but provision was made to include derivatives such as l_p, m_q and n_r, values of which were obtained by theoretical calculation. Their effect was found to be small. Gravity forces were not simulated, since their inclusion in the earlier stages of the work would have involved adding an earth-to-missile axis transformation to the simulation. After the latter was installed for other reasons, the gravity terms in the equations were still omitted since their effect seemed likely to be secondary and in the nature of a bias which was regarded as liable to obscure the main issue so far as homing performance was concerned. The effect of gravity was included, however, in digital studies which were undertaken to check the performance of the early flight rounds.

4.6 Mach Number Variation

A further point may be mentioned regarding the generation of the aerodynamics. As already noted, the curves of force and moment varied with Mach number. It was found upon analysis that the overall variations of the G and H functions were considerably less if the coefficients were multiplied by the Mach number. This was therefore done, and the resultant forces and moments were obtained from the resulting expressions by multiplying them by $\frac{1}{2}S\gamma pM$, or $\frac{1}{2}Sl\gamma pM$, in the usual notation, rather than these factors with M^2. The effect of varying Mach number was then studied by interpolating linearly between the outputs of successive pairs of function generators.

4.7 Altitude Variation

In addition to the means described for varying the Mach number, provision was made to alter the altitude. The interest being in large incidences, only altitudes exceeding the height of the tropopause were considered. The simulation of height variation was accomplished, therefore, simply by varying p, the ambient pressure, in the factor multiplying the aerodynamic force and moment coefficients.

4.8 Centre of Gravity Variation

Provision was also made to vary the position of the centre of gravity in accordance with changes which would be produced by the burning of the vehicle's sustainer rocket motor. The effect is expressible as a change in the leverage of the forces which contribute to the moments about the centre of gravity. The variation was assumed to be linear in time and was, therefore, readily accomplished with the aid of a servo-driven potentiometer which varied the gain of the force contributions to the moments.

4.9 Control System

It has already been mentioned that a scheme of roll rate stabilisation was selected for the first phase of the investigation. A system utilising rate feedback was also selected for the pitch and yaw control system, since this had been employed on the test vehicle.

4.10 Dynamical Equations

The simulation of the dynamical equations was straightforward. The equations of motion were formed in moving axes coincident with the principal axes of the vehicle, and were integrated once to produce the linear and angular velocities in the same system. The former were then employed to generate the aerodynamics after derivation of $\bar{\theta}$ and ϕ, as already described. The equation of motion in u, the velocity in the direction of the longitudinal axis of symmetry, was not simulated but was replaced by the equation $V = f(t)$, V being the total velocity and $f(t)$ a function of time which, for much of the work, was assumed constant. The value of u was then obtained from the relation $u = V \cos \theta$, $V \sin \theta$ being available from the ϕ servo, as explained earlier. The cosine was derived from the sine with the aid of a diode function generator utilising the relationship

$$\cos \theta = \sqrt{(1 - \sin^2\theta)}$$

or

$$u = \sqrt{V^2 - (V\theta)^2}$$

A block diagram of the simulation is shown in Figure 6, and the mode of generating the aerodynamics in Figure 7.

missile passes the target on the opposite side and that there is an optimum initial range for which the miss distance is zero. The growth of the maximum lateral acceleration with decrease of initial range is indicated in Figure 12.

8.2 Homing in Three Dimensions

Curve 1 of Figure 13 shows the effect of both non-linearities and cross-couplings when the missile is roll-rate stabilised and, therefore, tends to roll in its progress towards the target, seeking to complete its manoeuvre in a plane of stability in roll. The abscissa here is the initial angle of roll of the missile relative to the plane in which the target is turning aside at constant rate. All the engagements start from the same range, with the missile and target approaching each other in line. It will be seen that the miss distance depends markedly on the initial angle of roll, being a maximum when the angle is near 0 or 90°, and a minimum near 65°, when the demand plane is initially nearer to a stable plane and the missile, therefore, does not roll much throughout its manoeuvre. That this minimum is the least of all may be explained by the fact that, as was noted earlier, the stability in pitch is least in the 45° plane and, consequently, the response is more rapid in this neighbourhood. The asymmetry of the curve is due to the fact that the dish gimbal system was necessarily asymmetric since we assumed an outer and inner gimbal at right angles. The curve is also somewhat idealised, since the end points correspond to single plane manoeuvres in planes of unstable roll equilibrium.

Curve 2 shows the effect of suppressing the roll, i.e. of fitting a perfect system of roll position stabilisation in place of the roll rate system of Curve 1. Naturally, the two curves have the same ordinates at roll angles of 0 and 90° and cross near 45°. The reduction in miss distance at intermediate angles in this second case is very marked.

Curve 3 shows the effect of linearising the aerodynamics in pitch and yaw while retaining the cross-coupling in roll. This, and Curve 4, are somewhat artificial for the reason given above in discussing the single plane case. In particular, the assumption made in linearising the pitching moment resulted in the static stability in the 45° plane, and neighbouring planes, being somewhat greater than it was in fact, and it is probably for this reason that the miss distance recorded is somewhat greater than in the fully non-linear case. It is clear from the curve that the linearising process has had almost as much effect on the miss distance as the suppression of the roll, the additional effect of which is shown by Curve 4 to be relatively small.

This last curve shows the result which would be obtained by assuming the aerodynamics of the missile to be completely linear. This is an assumption which is frequently made in theoretical analyses and in simulations, where investigations are often further restricted to single plane studies. It will be clear from Figure 13 that such assumptions may lead to erroneous conclusions. It should be pointed out, however, that although the curves of Figure 13 differ so much, the r.m.s. values of the miss distance, averaged in each case over the whole range of the roll angle, do not differ greatly. Since the assumed manoeuvre is somewhat artificial, it might be argued from this that aerodynamic non-linearities and cross-couplings are not of great importance and that there is little to choose between the different systems of roll control. It is important to note, however, that the curves of Figure 13 were all obtained for one value of the initial range and it does not follow that the effects must be the same at other ranges. The whole problem is extremely complicated and so many factors are involved that it is almost impossible to reach general conclusions. The type of manoeuvre discussed above is illustrated in Figure 14.

9. EFFECT OF ANGULAR NOISE ON MISS DISTANCE

The homing system which produced the results of Figures 11-13 was not afflicted by noise, radome aberration, or any other of the troubles which beset real missiles. It would have taken too long to investigate all these effects on the simulator, but we did consider it worthwhile to obtain an estimate of the probable effect of angular noise on the results. The conclusions are indicated in Figure 15. These results were obtained digitally. It is fairly clear that the general trend of the curve without noise is unaltered.

10. PROBLEMS OF A LARGE SIMULATION

10.1 Simulator Errors

By the time the simulation had reached the above stage, it had grown to an assembly of some 500 amplifier units and had absorbed most of the available capacity and special facilities of TRIDAC. The problem of keeping the computer 'on the rails' had by then grown formidable, and a considerable portion of the working time had to be spent in tracing and eradicating intermittent faults in the equipment. This was a difficult matter on account of the many loops in the system, and here the numerous complete and partial digital solutions mentioned earlier proved most useful. There were certain shortcomings in the equipment, however, which we could not overcome, notably the limited resolving power of the sine and cosine potentiometers in the axis transformations, and this feature was most serious when it came to generating the sight line error angles in dish axes. Although the computer behaved remarkably well on the whole, we were driven to rely more and more on digital methods where high accuracy was essential, and, indeed, although TRIDAC confirmed the general trends of all the curves, the results shown in Figure 11 and 13 were, in fact, obtained digitally.

10.2 Comparison of Analogue and Digital Results in Single Plane

Figure 16 compares the digital and TRIDAC solutions in the case of single plane homing. The initial range was varied from 20,000 ft downwards. It will be seen that the simulator confirms the general trend of the digital results, though there is some scatter in the experimental points. Four configurations were investigated, namely, target turning upwards or downwards in the vertical plane or to port or starboard in the horizontal plane. Hence, four values of miss distance were obtained for each value of the initial range. At values of the latter exceeding 10,000 ft, all four results were valid, but at lesser ranges, some amplifiers in the system were overloaded. The trouble could have been overcome by rescaling, but this was not considered worthwhile. Since the overload limit points of the amplifiers are higher for positive than for negative output voltages, it was possible to obtain only two valid measurements for initial ranges less than 10,000 ft.

10.3 Comparison of Analogue and Digital Results in Three Dimensions

Figure 17 compares the digital and analogue results in the case of three-dimensional homing from a constant initial range. The digital result was given previously, on a different scale, in Figure 13, Curve 1. There are four results at each value of initial roll angle, namely, the two obtained by simulating the manoeuvre and its mirror image in the yaw plane, and the two obtained by repeating this process in the pitch plane. The discrepancies between the points so obtained are probably due to slight asymmetries and inaccuracies in the axis transformations which could not be removed. It is interesting to note, however, that over much of the range of roll angle, the mean of the four solutions at each angle is close to the digital curve.

CONCLUSIONS

Aerodynamic non-linearities and cross-couplings may have important effects on the response of a guided missile, but these effects may be greatly modified if the missile is provided with a suitable autopilot.

A cruciform missile, which is roll-rate stabilised, has preferred planes of manoeuvre and will, in general, attempt to manoeuvre in these planes when homing. The rolling motion may affect miss distance adversely.

Within the limited field explored, roll-position stabilisation appeared to be preferable to roll-rate stabilisation, although the r.m.s. miss distances, averaged over all angles of roll, may not differ widely in the two cases.

Incorrect and misleading results may be obtained from incomplete simulations, for example, those which represent the aerodynamics by linear approximations, or which are capable of simulating only single plane manoeuvres.

On the other hand, a large and complex simulation is fraught with many difficulties. Digital check solutions are indispensable in such cases.

REFERENCES

1.	Spearman et alii	*TRIDAC, A Large Analogue Computing Machine.* Proc. Inst. Elect. Engrs., Vol.103, Pt. 13, 1956, p.375
2.	Gait, J.J., Nutter, J.C.	*TRIDAC. A Research Flight Simulator.* Elect. Eng. Vol.28, 1956, pp.368 and 430.

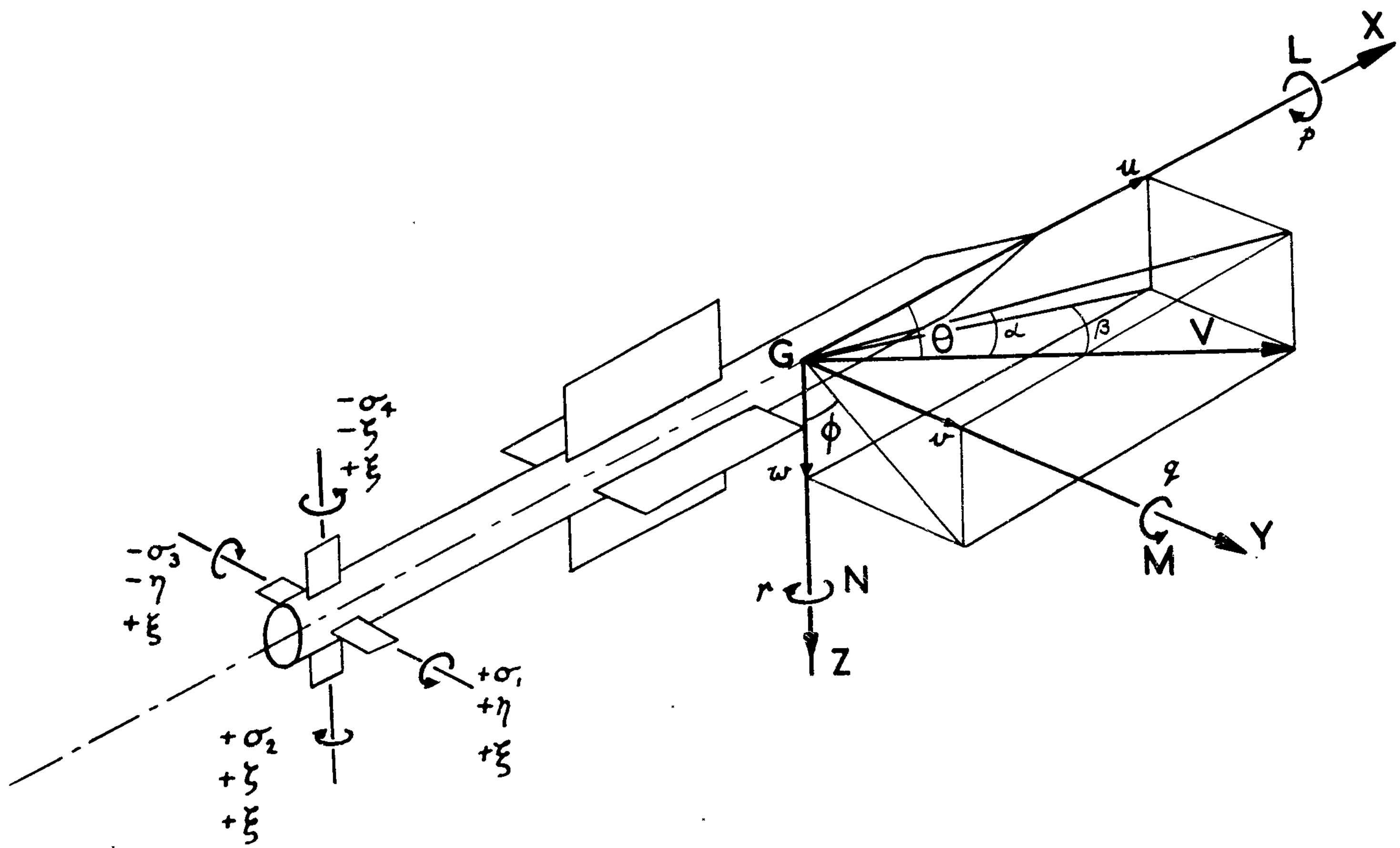

Fig.1 Diagrammatic representation of test vehicle

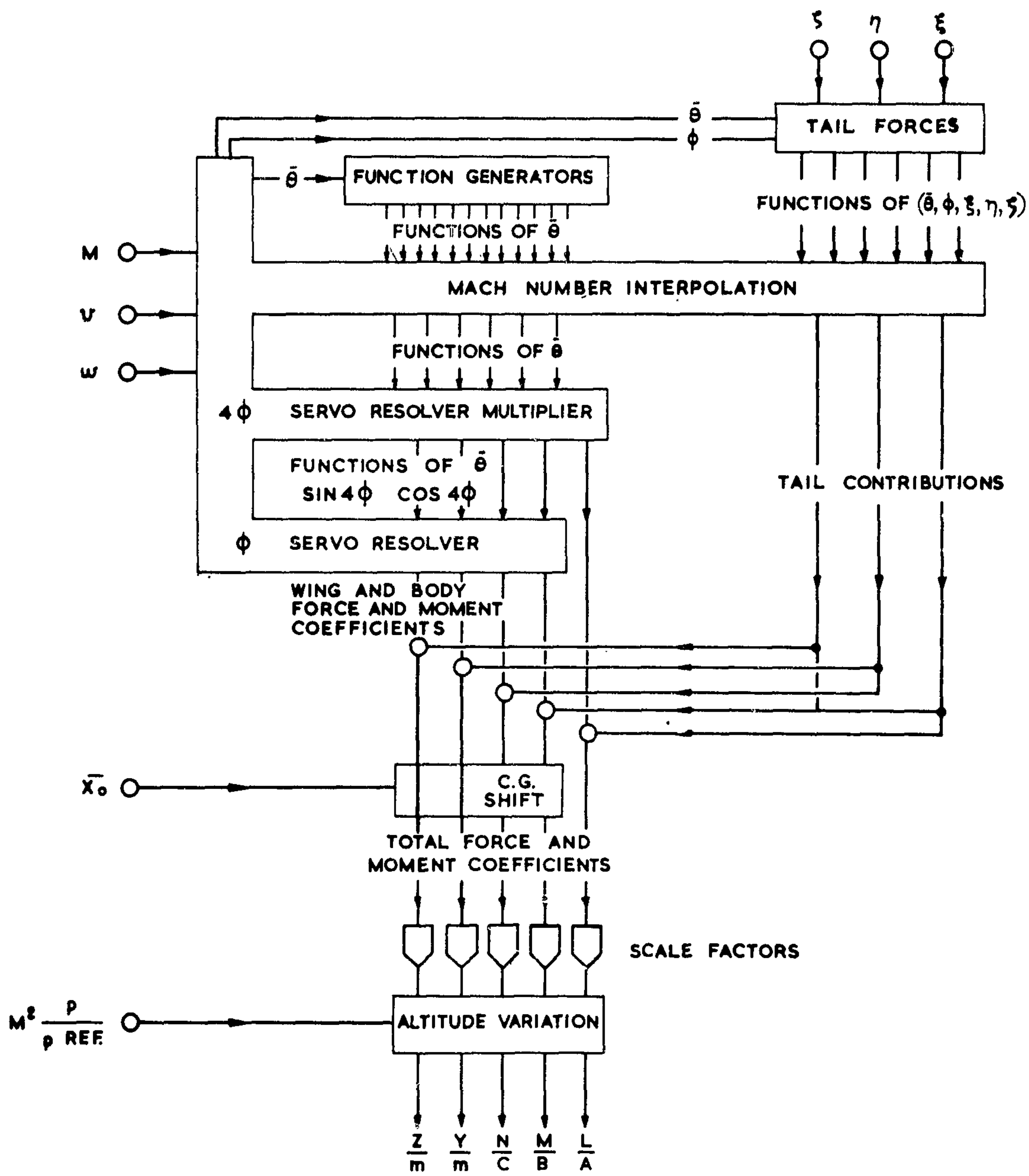

Fig. 7 Generation of aerodynamic forces and moments

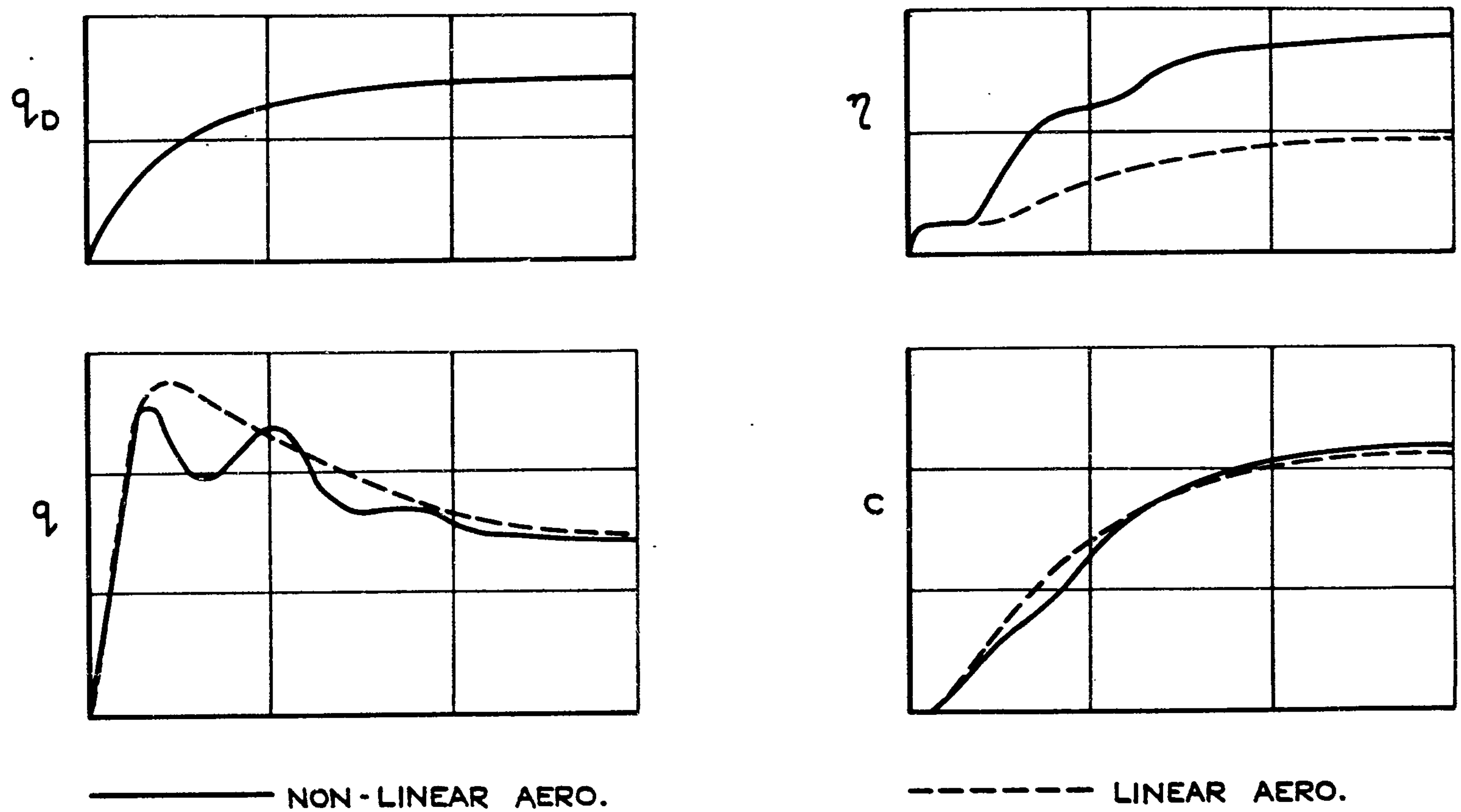

Fig. 8 Responses in single plane with linear and non-linear aerodynamics

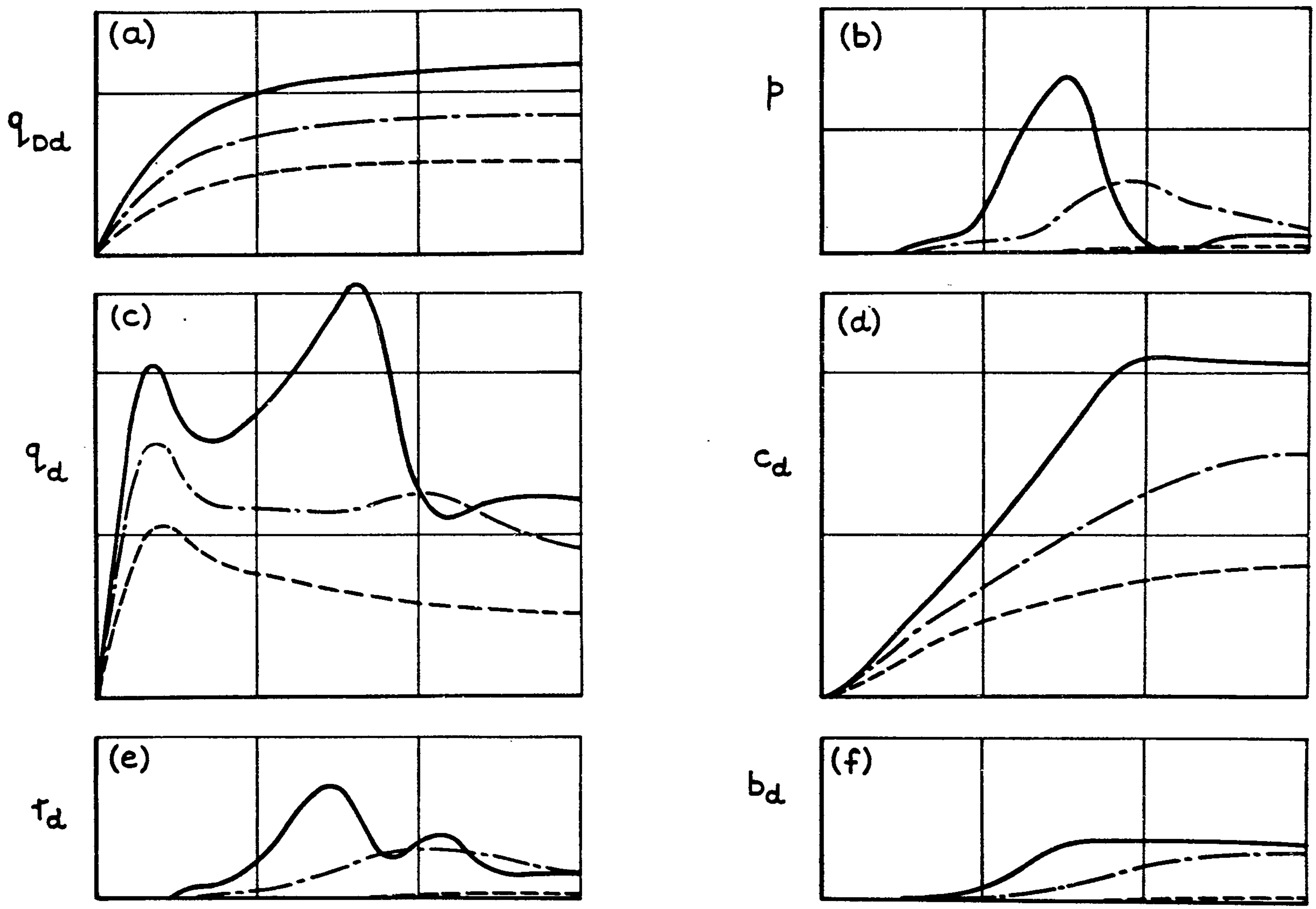

Fig.9 Response in three dimensions to three demands; $X_{Go} = 5^o$

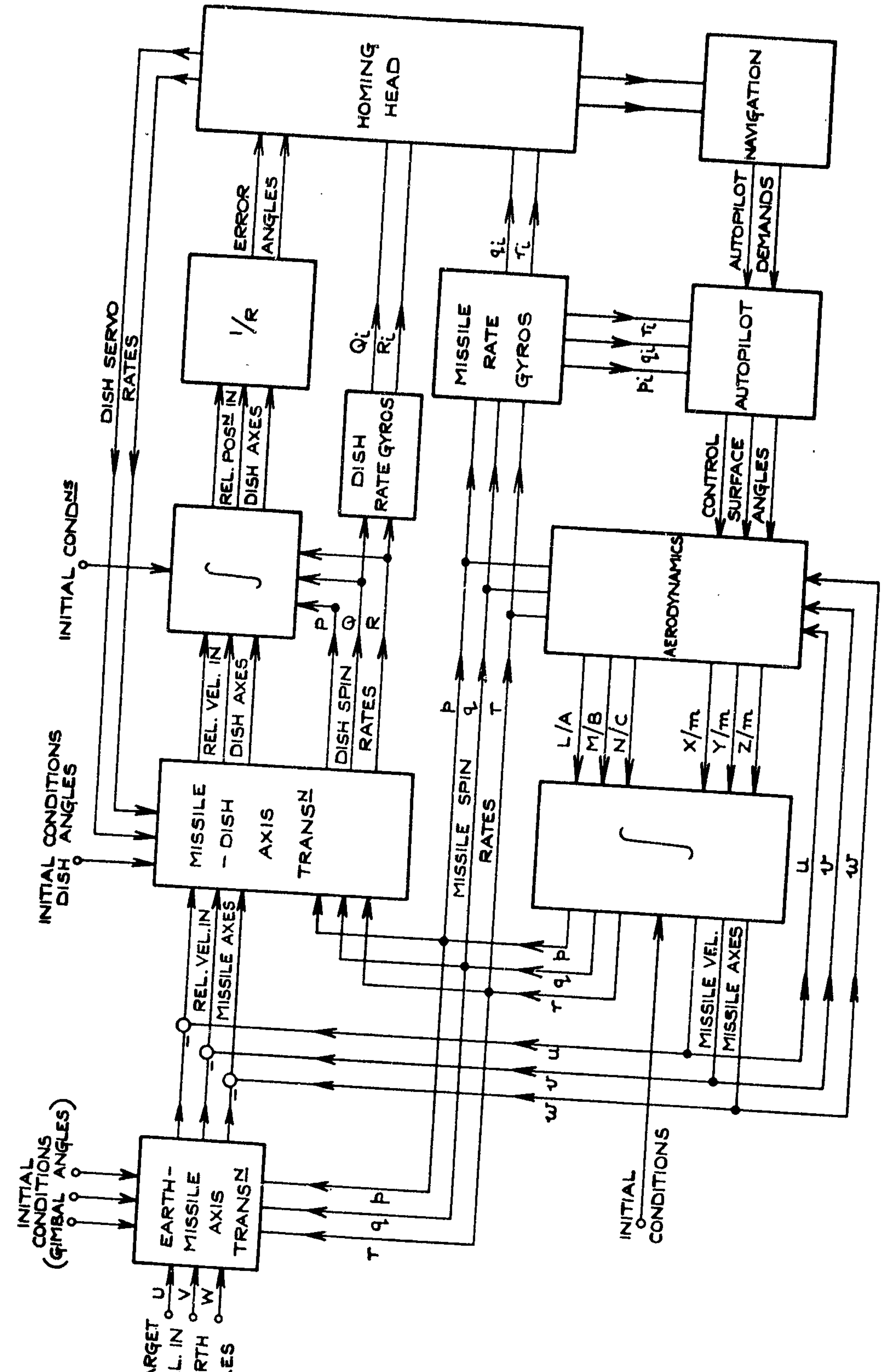

Fig. 10 Block diagram of homing missile simulation

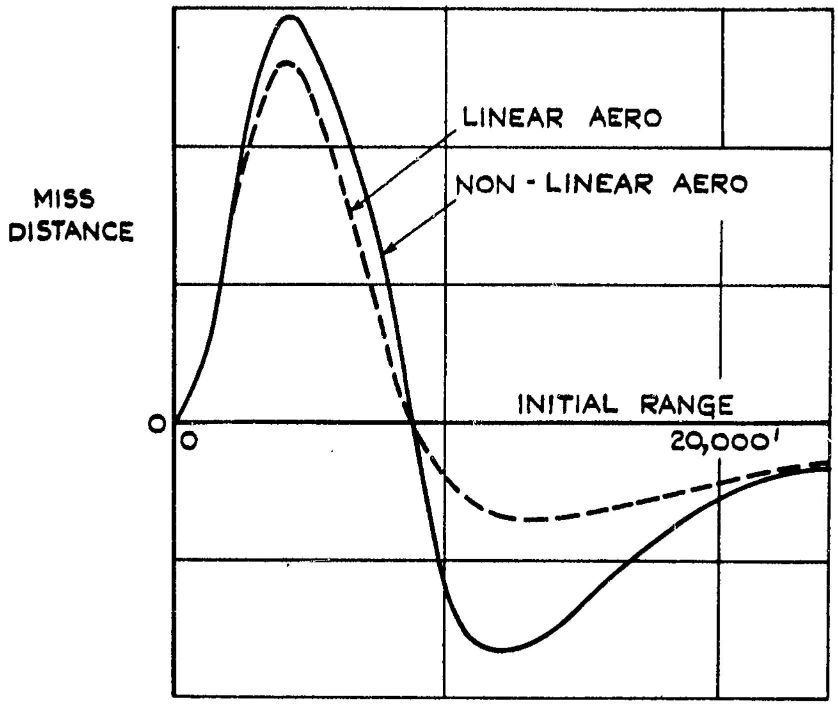

Fig. 11 Miss distance vs initial range in single plane homing

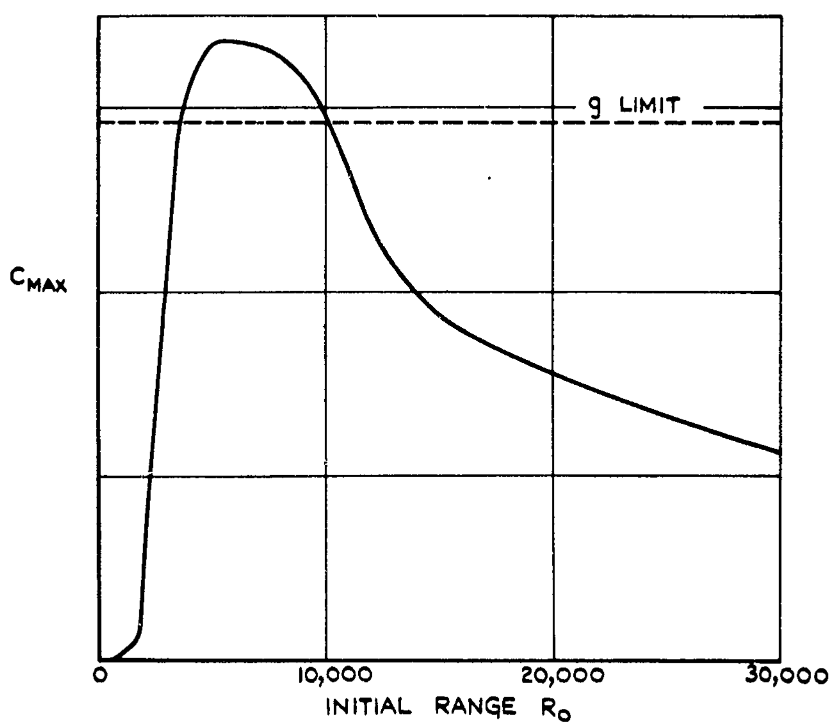

Fig. 12 Maximum lateral acceleration vs initial range in single plane homing

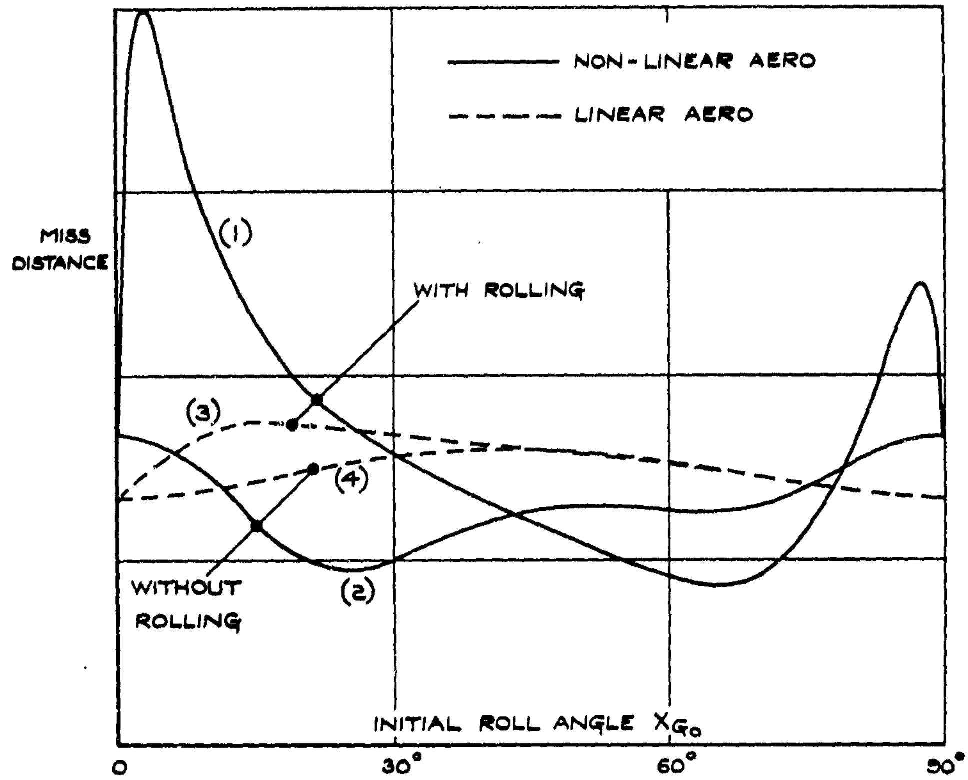

Fig. 13 Miss distance vs initial angle of roll in 3-dimensional homing; initial range fixed

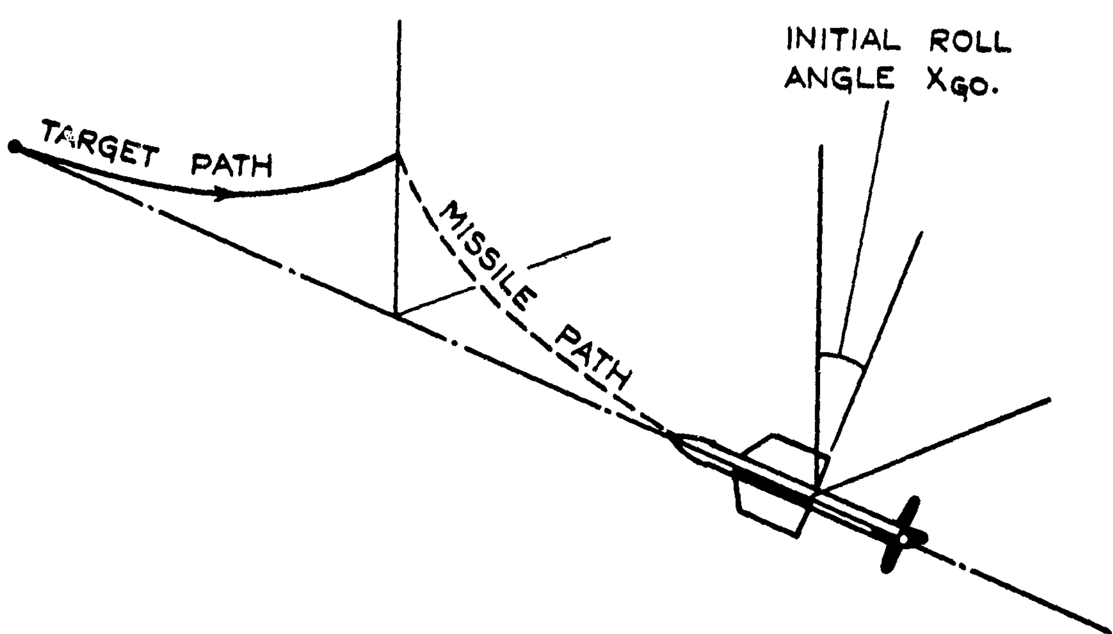

Fig. 14 Diagrammatic representation of homing in three dimensions

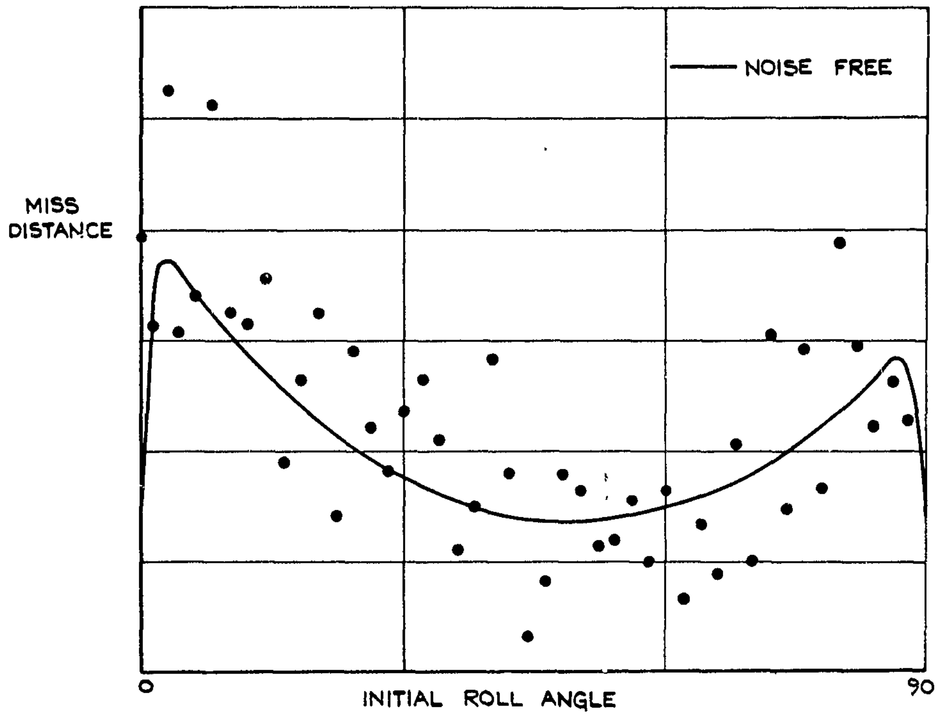

Fig. 15 Effect of angular noise on miss distance in 3-dimensional homing

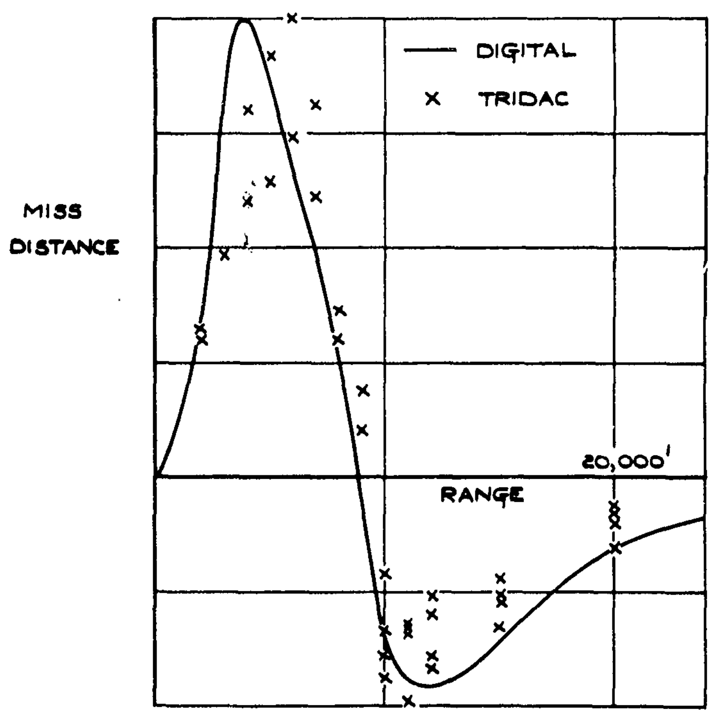

Fig. 16 Comparison of analogue and digital results in single plane homing

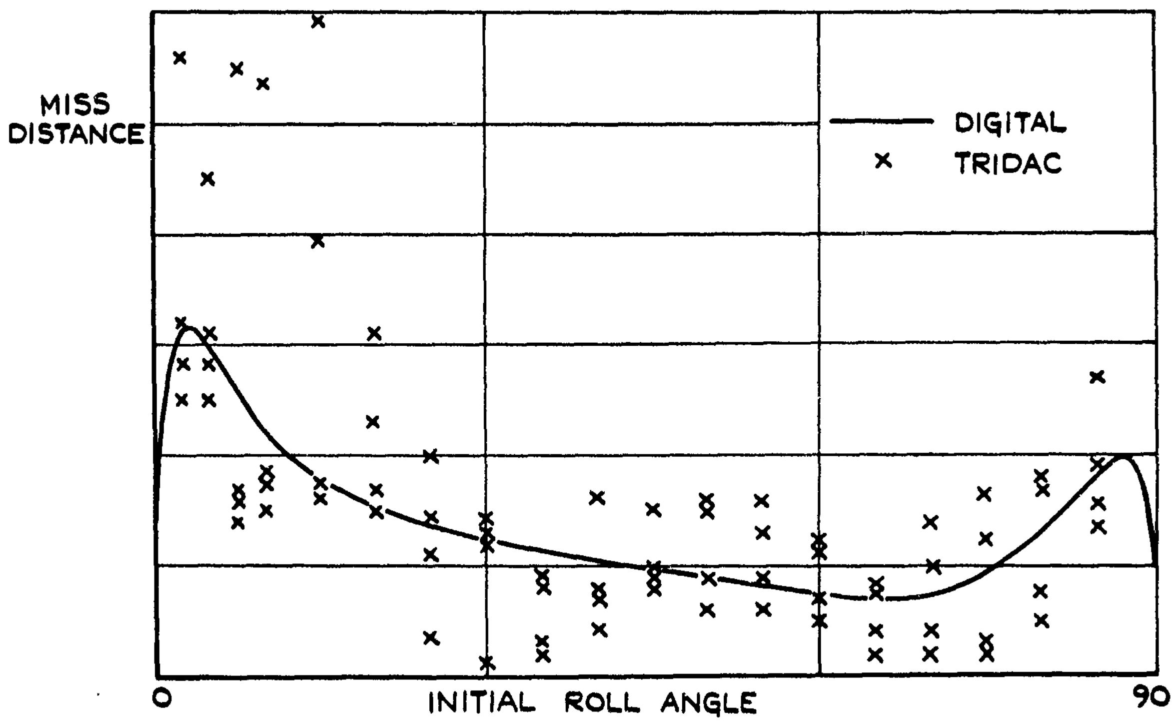

Fig. 17 Comparison of analogue and digital results in three dimensions

DISCUSSION

B.E. Amsler (U.S.A.): I wish to agree strongly with Mr. Brown's conclusion that aerodynamic non-linearities must be considered in the control design. Too often these terms are ignored only in the interest of mathematical siimplicity even though they are critically important. However, I do not feel a large 'all up' dynamic simulation is required to predict flight performance, at least in the case of cruciform configurations. Specifically, if control parameters are selected with a view to overpowering or operating satisfactorily in conjunction with the complete range of non-linear aerodynamic parameters and if interchannel coupling effects are considered and, where necessary, parameters are chosen to ensure no cross-channel resonances (e.g. path spiraling), then a single plane simulation has been found to give a very good prediction of flight performance.

Let me however point out that fundamental non-linear effects (e.g. variations in slope of C_N vs α with increasing α) which cannnot be compensated out by special control parameter selection must be included in performance studies. The significant point is that interchannel coupling has not in this case been found to be important and variations between manoeuver planes are made insignificant.

Reply by W.S. Brown: I would agree with Mr. Amsler to the extent of saying that a large dynamic simulation is not desirable if one can avoid it; for the time required to construct a complex simulation and keep it running smoothly may easily become prohibitive. In missile design, where an approximate answer is required quickly, as for example, when seeking to optimise the control system, a single plane simulation is probably adequate as a means of predicting performance.

However, in the work described, we were not attempting to design a missile. We merely adopted an existing arrangement and sought to reproduce its features as accurately as possible in order to find out whether aerodynamic non-linearities and cross-couplings, which were known to be present, affected performance significantly. Our conclusions, as outlined in the paper, were that the effects could be important in certain circumstances, for example, in a cruciform missile free to roll, which a single plane simulation would not have revealed.

L.T. Prince (U.S.A.): It has been our experience at Honeywell that stability problem areas resulting from non-linear aerodynamic and kinematic factors can be predicted with linear uncoupled analyses of airplanes. These simpler techniques are of great value in control system design studies and are highly recommended. They involve an examination of control system performance at the worst expected values of stability derivatives that may be encountered during maneuvering flight using fixed values of the derivatives.

A. Lightbody (U.K.): I would like to say that we at A.W.A. have carried out a three dimensional analogue simulation of a Seaslug missile which is about the same size as that of Mr. Brown's on TRIDAC. Ours uses electronic multipliers and runs on a 1 : 1 time scale.

We have found many of Mr. Brown's conclusions borne out on our simulator; for example, we also have found that for comprehensive simulation of non-linear aerodynamics the polynominal method becomes too expensive and complex in multiplier. We also use diode function generators.

However, the main point I have to make is that complex three-dimensional simulations such as have been described have two main functions: 1) as a tool for the study of the overall guided missiles system; 2) as a facility for performing pre and post flight simulations. Flight trials can be planned better using such a facility, and the causes of flight failures can be thoroughly investigated after the trial.

I would like to ask Mr. Brown whether he has used TRIDAC in this capacity, i.e., for 1 : 1 flight trial simulation. This would seem to me to be the logical outcome of the comprehensive work carried out on TRIDAC.

Reply by W.S. Brown: Our investigation was intended to be a piece of fundamental research into aerodynamic effects, and was originally planned to include a programme of flight trials. For various reasons, the flight work was abandoned. We did, however, simulate a few of the earlier flights of the test vehicle, and obtained general although not precise, agreement with the telemetered data. Certain discrepancies, however, led us to doubt whether a detailed simulation, such as we had constructed, would be justified in missile design and development work.

L.G. Evans (U.K.): I would like to support Mr. Lightbody's point that the pay-off which justifies these rather extensive computing facilities is the saving in flight trials that they allow, due to the improved ability to understand the meaning of flight results. It is therefore necessary to tie the results of the model back to trial results, and here I would support Mr. Amsler's point.

However, in my experience the designer may well be faced with a severe disagreement between the model and trial results, and in this case something more than a simple overlay system is necessary to help the designer to track down the source of errors in the model.

S.H. Scher (U.S.): I was glad to hear the comments of Mr. Amsler. It indicates an awakening of people to a problem which does exist.

There are times when you can get away with two-dimensional studies using linearized aerodynamics and simplified equations of motion. But, as I mentioned previously in my comments on post-stall gyrations, there are other times when a study of the motion requires that you have enough inputs to completely study the motion.

As Mr. Brown said, when one gets specific and attempts to deal with motions which are completely beyond the realm of normal linear inputs and non-cross coupled inertia factors, one encounters much difficulty.

The inputs needed depend on the motion you are trying to study and, as I see it, you either try to predict what will happen or what you get in attempting to explain what did happen. In either case, you need a complete set of inputs.

ADDENDUM

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