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INITIAL FRAGMENT VELOCITIES FROM HOLLOW WARHEADS (U)

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FOREWORD

Since their publication in 1943, Gurney's formulae for predicting the initial velocities of fragments from high explosive warheads have been widely used in analyzing fragment vulnerability environments. This report extends Gurney's formulae to hollow warheads and, it is hoped, will prove similarly useful to Navy weapon designers.

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ABSTRACT

Formulae are presented to predict the initial fragment velocities from hollow spherical and cylindrical warheads. These formulae are modified from those of R. W. Gurney, which predict the initial fragment velocities from solid spherical and cylindrical warheads. The results of the new formulae are tabulated for a number of ratios of both the explosive charge to the case mass and the inner to the outer radius of the explosive charge. The results obtained with the new formulae agree with the results obtained from use of T. E. Sterne's and L. H. Thomas' theoretical formulae and with test data.

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INTRODUCTION

R. W. Gurney developed theoretical formulae for predicting both the initial velocity of fragments from a spherical case surrounding a solid spherical charge of explosive and the initial velocity of fragments from cylindrical charges of similar construction. L. H. Thomas rigorously developed a formula for predicting the initial velocity of fragments from a simple cylindrical warhead with a geometry such that it may be considered to approach a flat plate. (See Appendix 1 for the geometry of a cylinder which can be considered to approach that of a flat plate.) T. E. Sterne developed a theoretical formula to predict the initial velocity of fragments from a flat slab of metal in contact with a flat slab of explosive, all in free space.

Little work has been published, however, on the prediction of the initial velocities of fragments from cases surrounding hollow explosive charges. (A warhead with this type construction will hereafter be referred to as a hollow warhead.) This report presents working formulae for predicting the initial velocity of fragments from a case surrounding the explosive charge of a hollow warhead.

NOMENCLATURE

a	Maximum radius of a cylindrical or spherical warhead at the moment of fragmentation
A	Surface area of a cylinder
B	Surface area of a sphere
C	Mass of the explosive per unit length or per unit radius

D	Degree of confinement of a warhead
E	Kinetic energy per unit mass
E_o	Useful kinetic energy per unit mass directed away from the central axis in the case of a cylinder and away from the center in the case of a sphere
K	Constant
L	Represents a line
M	Mass of the case per unit length or per unit radius
N	Represents a plane tangent to a sphere
R	Any radius
R_1	Internal radius of the explosive charge
R_2	External radius of the explosive charge
S	Represents a plane tangent to a cylinder
v	Initial velocity of the fragments from the case
v_{HC}	Initial velocity of the fragments from the case surrounding a hollow cylindrical warhead
v_{HS}	Initial velocity of the fragments from the case surrounding a hollow spherical warhead
v_P	Initial velocity of the fragments from the case surrounding the cylindrical or spherical warhead that is approximated by a flat plate
v_{sc}	Initial velocity of the fragments from the case surrounding a solid cylindrical warhead
v_{ss}	Initial velocity of the fragments from the case surrounding a solid spherical warhead

α A parameter that transforms the equation for the initial velocity of fragments from a solid warhead to the equation for the initial velocity of fragments from a hollow warhead, where $\alpha < 1$

ϕ The angle between a spherical surface and a plane tangent to the surface

θ The angle between a cylindrical surface and a plane tangent to the cylinder

HISTORICAL BACKGROUND

R. W. Gurney developed an equation for predicting the initial velocity of the fragments from the case of a solid cylindrical warhead. This work is presented in Ref. 1. The equation he developed is

$$v_{sc} = \sqrt{2E}\sqrt{\frac{\frac{C}{M}}{1 + \frac{C}{2M}}} \qquad (1)$$

In developing this equation Gurney assumed the following:

1. Maximum confinement of explosive was along the axis of the cylinder.

2. The gases moved radially outward from the central axis of the explosive charge.

3. The radial velocity of the gases varied directly with the distance from the central axis to the metal case surrounding the charge, with zero velocity along the central axis and maximum velocity at a distance a, the radius at the moment of fragmentation.

4. The gases moved outward with zero kinetic energy along the central axis and maximum kinetic energy at the outside of the charge at the time of fragmentation.

Using a similar line of reasoning and assuming that the contribution to the kinetic energy made by the detonation of each unit mass of a particular explosive is the same in all types of projectiles, Gurney developed an equation for predicting the initial velocity of fragments from the case of a solid spherical warhead. This equation is

$$(2) \qquad v_{ss} = \sqrt{2E}\sqrt{\frac{\frac{C}{M}}{1 + \frac{3C}{5M}}}$$

L. H. Thomas presented, in Ref. 2, an equation for predicting the initial velocity of fragments from a cylindrical warhead that approaches a flat plate (see Appendix A). The explanation of his final equation is beyond the scope of this report. T. E. Sterne, in the Appendix of Ref. 3, presents a summary of the development of Thomas' equation. To introduce the equation here with an explanation of terms, factors or both, would require a duplication of T. E. Sterne's work in Ref. 3. For the reader who is relatively new to the field of fragmentation but is interested in following this development, it is suggested that he first read Sterne's report for an introduction to the Thomas equation. Sterne's brief explanation will give the reader enough knowledge and understanding to work with the final equation. For the rigorous and somewhat lengthy development of Thomas' equation, the reader, of course, should read Thomas' original report. (The formulae necessary for computing values of $v/\sqrt{2E}$ as a function of C/M for plotting Thomas' equation in Fig. 1 are presented in Appendix C.)

T. E. Sterne, in Ref. 3, extended the equations of Gurney and Thomas to allow for prediction of the initial fragment velocities of a plate in contact with a flat explosive charge. Although Thomas had already done this rigorously, his equation was difficult to handle. Sterne wished to develop a formula that would facilitate computations and, at the same time, give results related to those predicted by the

Thomas formula. The validity of his equation depends on the requirement that the slab of explosive is supposed to be thin in comparison t its surface area, so that the motions are all substantially normal to plane of the slab.

Sterne concluded that the final value of the fragment velocity, approached asymptotically as the expansion of gas progresses, is given by

$$v_p = \sqrt{2E}\sqrt{\frac{\frac{3C}{5M}}{1 + \frac{C}{5M} + \frac{4M}{5C}}} \qquad (3)$$

Sterne computed several values of C/M versus $v/\sqrt{2E}$ using the rigorous equation of Thomas and Eq. 3. The results showed that Eq. 3 is in close agreement with the results obtained from use of Thomas' equation.

The flat plate theory assumes that:

1. The metal casing is thin but heavy, with maximum confinement at the charge-case interface.
2. The kinetic energy is zero at the charge-case interface.
3. The gases move in both directions.
4. The case is pushed out by the gases.

HOLLOW CYLINDRICAL AND SPHERICAL WARHEADS

In the case of a hollow warhead detonated on its charge-case interface, as assumed by T. E. Sterne, part of the energy of the detonation will move toward the center of the warhead, and part of the energy wil move away from the center. T. E. Sterne points out that the presence a cavity permits a loss of energy, thereby causing the fragment veloci to be less than that predicted by the solid warhead formulae set forth by R. W. Gurney. This loss of energy is represented by that part of the kinetic energy moving toward the central cavity after detonation.

Let R_1/R_2 be the ratio of the internal radius of the charge to the external radius of the charge. R_2 is also the distance from the center to the charge-case interface. Consider the physical aspects of R_1/R_2 as it varies from zero to one, but is never equal to one. $(0 \leq R_1/R_2 < 1)$

Consider a cylindrical warhead. When $R_1/R_2 = 0$, then the special case of a solid cylindrical warhead is considered and Gurney's formula applies. Figure 1 shows a curve of C/M versus $v/\sqrt{2E}$ using Gurney's formula for a cylinder.

When R_2 becomes very large, so that the surface of a cylinder approaches a plane tangent to the surface, then the special case of a thin plate can be considered, Appendix A. If the charge is very thin so that R_1 approaches R_2, then the ratio R_1/R_2 approaches unity. L. H. Thomas' formula predicts the initial fragment velocities rigorously for $R_1/R_2 = 1$ and gives a good approximation for a small range of R_1/R_2 slightly less than one. As stated previously, Sterne's formula is in close agreement with Thomas' formula. Figure 1 shows a curve of C/M versus $v/\sqrt{2E}$ using Thomas' formula for a thin plate. Figure 2 shows the relationship between Sterne's simple formula and Thomas' rigorous formula. Since the graph of Sterne's equation follows that of Thomas' very closely, it was not plotted on Fig. 1, for purposes of clarity.

The curves of the two special cases mentioned above, Fig. 1, meet at $C/M \simeq 5.45$, $v/\sqrt{2E} = 1.21$. If it is true that a solid cylindrical warhead should predict greater fragment velocities than either a hollow cylindrical warhead or a flat plate, then the curve representing $v/\sqrt{2E}$ versus C/M for a flat plate should not meet the curve representing the same function for a solid cylindrical warhead. For the curves to meet would contradict the physical characteristics of the problem in question when considering practical types of cylindrical warheads.

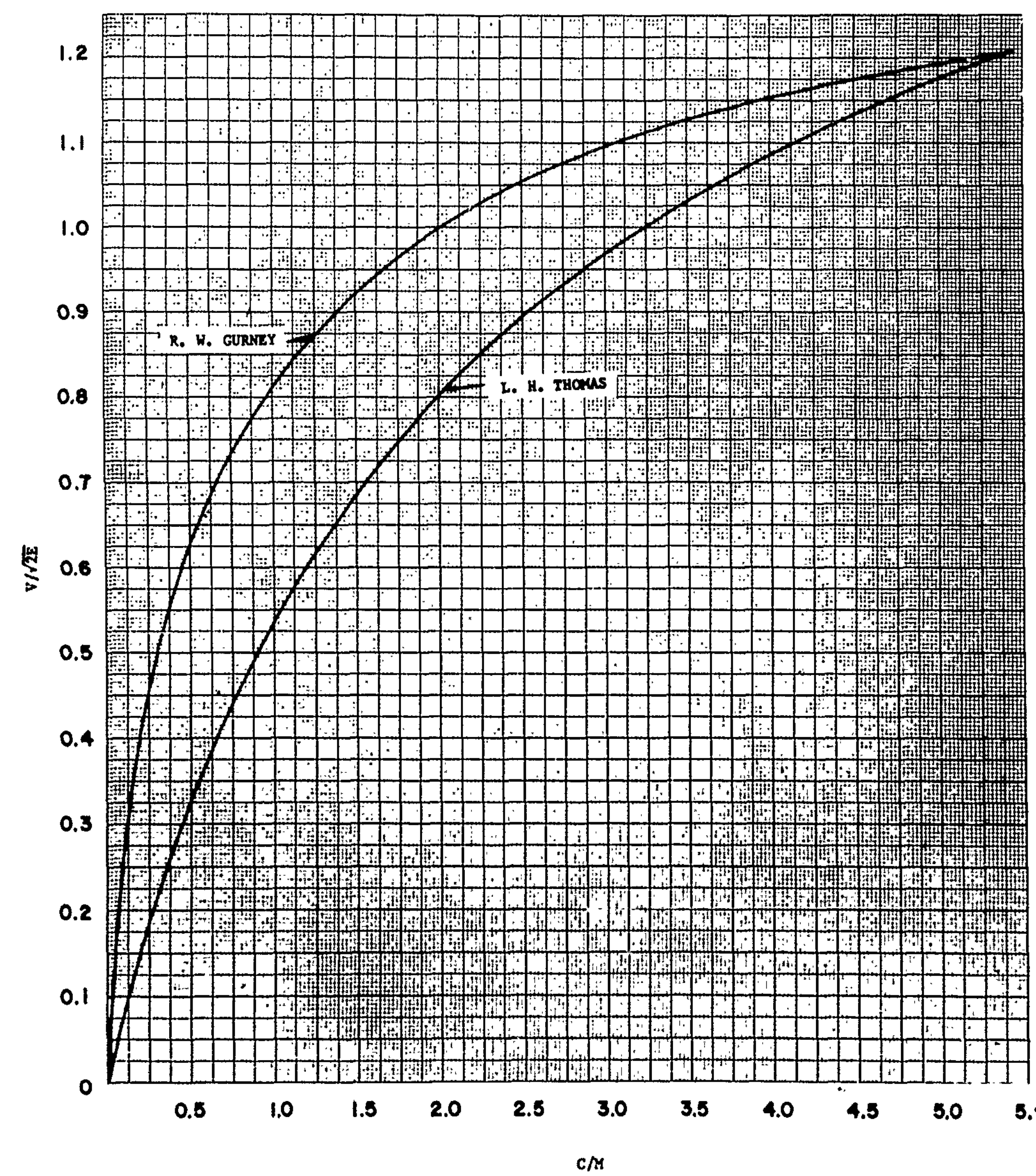

FIG. 1. C/M Versus $v/\sqrt{2E}$ Predicted by the Gurney Formula for a Solid Cylinder and by the Thomas Formula for a Thin Plate.

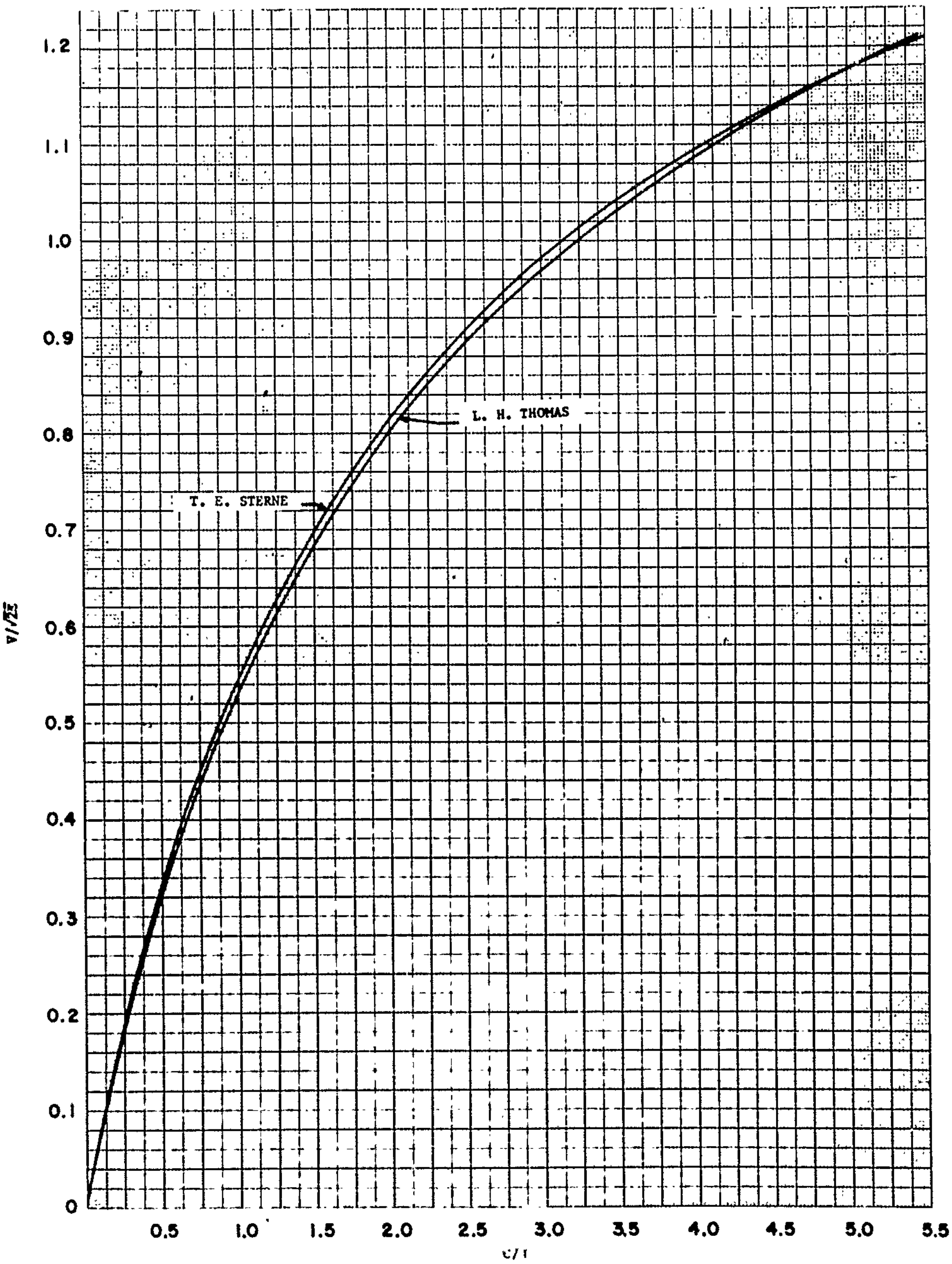

FIG. 2. Comparison of C/M Versus $v/\sqrt{2E}$ Predicted by Thomas' Rigorous Formula and Sternes' Simple Formula for a Thin Plate.

Sterne remarks that Thomas' flat plate formula would clearly be inapplicable for the prediction of the velocity of fragments from a hollow metal cylinder. He adds, however, that when the metal and high explosive cylinders are thin relative to the air cavity of a hollow cylindrical warhead, the flat plate theory should be approximately applicable.

It should be noted that both Gurney and Sterne assumed that all of the kinetic energy of the gases was transferred into the kinetic energy of the metal case resulting in the initial velocity of the fragments. Other energies resulting from the detonation and the subsequent shock wave in a particular explosive were not considered. These energies are available to do useful work. On the other hand, the energy required to expand the metal casing and break the metal casing into fragments was also not taken into account by Gurney or Sterne. The processes of expansion and fragmentation would result in a loss of kinetic energy. Although some of the energy available to do work and other energy lost in doing work were not considered, Gurney's equation for a solid cylindrical warhead agrees with experimental data over a range of $.07 \leq C/M \leq 5.6$.

Since Gurney has been successful using the above assumption and Sterne follows a similar line of reasoning, the same assumption will be used in developing a working formula for a hollow cylindrical warhead. Another assumption of Gurney that will be considered in this report is that considering a given explosive "the contribution to the kinetic energy made by the detonation of each unit mass of this explosive is the same in all types of projectiles".

Sterne relates in his report several experiments performed by Robert Fleming and Harold Breidenbach at the Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland. Their experiments used steel cylinders. One was solidly filled with Composition C3 explosive, the second had a metal core, and the third contained an air cavity. The same explosive was used in all experiments. Table 1 shows given data and initial fragment velocities determined from instantaneous X-ray photographs.

TABLE 1. Experimental Data for Three Types of Warheads with Same C/M Ratio

	External Radius of Metal Case, inches	Internal Radius of Metal Case Equals External Radius of Charge, inches	Internal Radius of Charge = External Radius of Air Cavity or Metal Core	Ratio C/M	Fragmentation Velocity from Experiment, fps	Gurney's Velocity for Equivalent Solid Warhead, fps
Solid Cylinder	0.75	0.615	0	0.43	5,144 ± 177	5,235
Cylinder with Metal Core	1.79	1.67	1.375	0.43	4,465 ± 358	5,235
Cylinder with Air Cavity	1.79	1.67	1.375	0.43	3,107 ± 213	5,235

It should be noted that in Gurney's formula, $\sqrt{2E}$ is a constant dependent on the type of explosive used ($\sqrt{2E}$ = 8,000 fps for TNT and 8,800 fps for Comp. C3); therefore, v is a function of C/M. Gurney's results compare very well with experimental data for a solid warhead. A metal-core cylinder shows a decrease in initial fragment velocity for the same C/M ratio; and the hollow cylinder with an air cavity has a greater decrease in initial velocity of fragments. The velocity of the fragments from the cylinder with the metal core is approximately 85 percent of that predicted by Gurney's formula; the velocity of the fragments from the cylinder with the air cavity is approximately 59 percent of that predicted by Gurney's formula.

Since the experimental results show that there is a decrease in the initial velocity of fragments for cylindrical warheads not solidly packed with explosives, there must be less effective kinetic energy of the gases available that can be transferred to the kinetic energy of the metal case resulting in a lower initial velocity for the fragments.

It is stated in Ref. 4 that "the rate of detonation of a given explosive, provided that a sufficient initiator or booster explosive is used, is determined by its degree of confinement or loading density". The degree of confinement of an explosive is a function of the material encasing the explosive, the diameter of the explosive, and the loading density. For example, a heavy steel tube surrounding a cylindrical charge affords a greater degree of confinement than a glass tube or no casing at all. The degree of confinement increases as the diameter of a cylindrical charge increases. If a charge is solidly packed so that its density is almost equal to its maximum density, the degree of confinement is increased. If the charge is in the shape of a cylinder or sphere of sufficient diameter, the material near the center of mass may be regarded as completely confined. In general, the rates of detonation of explosives are reflected by their relative brisance (shattering or fragmentation) values. The higher the rate of detonation, the greater the brisance.

Since the rate of detonation of gases varies directly with the degree of confinement of the charge and since the initial velocity of the fragments varies directly with the rate of detonation, then the initial fragment velocity varies directly with the degree of confinement.

As the geometry of the charge is changed from a solid cylinder to a hollow cylinder by moving the mass of charge radially outward from the center, leaving an internal air cavity, the experimental data in Table 1 show that, for a constant ratio of C/M, there is a decrease in initial velocity of fragments. Although experimental data are limited, it can be assumed by the foregoing that the degree of confinement of a cylindrical warhead decreases as the air cavity increases, for the same C/M ratio.

Further study of Fig. 1 shows that the greatest deviation of $v/\sqrt{2E}$ between Gurney and Thomas is at C/M = 0.5. For C/M = 0.5, $v/\sqrt{2E}$ from Gurney's formula is 0.632 and $v/\sqrt{2E}$ from Thomas' formula is 0.326. For values of C/M greater than 0.5, the value $\Delta v/\sqrt{2E}$ decreases until C/M is approximately 5.45, at which point $\Delta v/\sqrt{2E} = 0$.

For values greater than C/M = 5.45, Thomas' equation predicts greater initial velocity of fragments from a flat plate than Gurney's equation for a solid cylindrical warhead. For example, at C/M = 18.23, $v/\sqrt{2E}$ by Gurney's equation equals 1.34 and $v/\sqrt{2E}$ by Thomas' equation equals 1.60. Bearing in mind Sterne's remark that Thomas' equation should be applicable to a hollow warhead if the metal and high explosive cylinders are thin <u>relative</u> to the air cavity, then to meet these conditions for a high value of C/M, the air cavity must be extremely large, or beyond the limits of a practical warhead. This report is considering only the practical type of hollow warhead.

Apparently there is a limiting value of C/M for the Thomas formula to be applicable to the condition set forth above. This limiting value is not known by the writers at this time. Since Gurney's formula for a solid cylindrical warhead has been shown to fit experimental data very well over a wide range of C/M, and since Thomas' equation for a flat plate is approximately true for a hollow warhead when C/M is small, and using the assumption that the degree of confinement of a cylindrical warhead decreases as the air cavity increases for the same C/M ratio, the graph for a hollow warhead must lie between the graphs of Gurney and Thomas. Since the air cavity of a hollow warhead is a variable, then there must exist a family of lines for varying degrees of confinement between the lower limit of a flat plate and the upper limit of a solid warhead. As the air cavity increases, the internal radius of the charge increases. It follows from the above assumption that degree of confinement is a function of the ratio of the internal radius of charge, R_1, to the external radius of charge, R_2, or

(4) $$D = f \left[\frac{1}{R_1/R_2} \right]$$

Since

(5) $$v_H \propto D$$

and

(6) $$E_o \propto v_H$$

then

(7) $$E_o = f \left[\frac{1}{R_1/R_2} \right]$$

or the useful kinetic energy of the gases assumed to give an initial velocity to the fragments at the instant of fragmentation decreases as the internal radius of the charge increases, for the same ratio of C/M.

As R_1/R_2 approaches zero, E_o approaches the upper limit given by the Gurney formula.

As R_1/R_2 approaches 1, E_o approaches the lower limit given by the Thomas formula.

Thus, it can be deduced that there exists a family of lines for a range of ratios R_1/R_2 between zero and one. Since Gurney's assumptions concerning the kinetic energy of the gases are being considered, let the equation for a hollow cylindrical warhead be a function of Gurney's equation. The final values of v_H will be less than Gurney's values for v_s; hence, Gurney's equation should be multiplied by a factor that is less than 1.

(8) $$v_H = \alpha v_s$$

where α is less than one. Since

(8a) $$v_H = f\left[\frac{1}{R_1/R_2}\right]$$

let

(9) $$\alpha = f\left[\frac{C}{M}, \frac{1}{R_1/R_2}\right]$$

A rigorous derivation of a formula as a function of R_1/R_2 has been attempted, but without success. Continued efforts are being made to rigorously develop a formula for a hollow warhead. In the meantime, a working formula has been developed by attempting to fit an equation to a family of lines for $0 \leq R_1/R_2 < 1$. Two such equations have been developed: one for a hollow cylindrical warhead and one for a hollow spherical warhead. They are working formulae that were deduced from theoretical mathematical data and not from experimental data, since the latter were too limited.

The factor α provides a good fit for a hollow cylindrical warhead when

(9a) $$\alpha_c = \sqrt{\frac{\frac{C}{M}}{\frac{C}{M} + \frac{R_1}{R_2}}}$$

Then

$$v_{HC} = v_{ss}\sqrt{\frac{\frac{C}{M}}{\frac{C}{M} + \frac{R_1}{R_2}}} \tag{10}$$

Dividing through by C/M and substituting Eq. 1 for v_{sc} for a cylinder yields

$$v_{HC} = \sqrt{2E}\sqrt{\frac{\frac{C}{M}}{1 + \frac{C}{2M}}}\sqrt{\frac{1}{1 + \frac{R_1}{R_2}\frac{M}{C}}} \tag{10a}$$

for a hollow cylindrical warhead.

It was shown in Table 1 that experimental results for a hollow cylindrical warhead yield a fragmentation velocity of 3107 fps $\pm$ 213 fps standard deviation, determined by flash radiographs. Equation 10a for a hollow cylinder with C/M = 0.43, R_1/R_2 = 0.8234 gives a fragmentation velocity,

$$v_{HC} = 3066 \text{ fps}$$

which is well within the range of experimental error of the data.

Following the same line of reasoning but considering the geometry of a sphere, a good fit to a family of lines is obtained when the factor α is related to C/M as follows

$$\alpha_s = \sqrt{\frac{\frac{C}{M}}{\frac{C}{M} + \left(\frac{R_1}{R_2}\right)^2}} \tag{11}$$

The initial velocity of fragments for a hollow spherical warhead is

$$(12) \qquad v_{HS} = v_{ss} \sqrt{\frac{\frac{C}{M}}{\frac{C}{M} + \left(\frac{R_1}{R_2}\right)^2}}$$

Dividing through by C/M and substituting Eq. 2 for v_{ss} for a sphere yields

$$(12a) \qquad v_{HS} = \sqrt{2E} \sqrt{\frac{\frac{C}{M}}{1 + \frac{3C}{5M}}} \sqrt{\frac{1}{1 + \left(\frac{R_1}{R_2}\right)^2 \frac{M}{C}}}$$

for a hollow spherical warhead. The term $2/\sqrt{2E}$ is a constant dependent on the type of explosive used.

COMPARISON WITH FORMULAE OF GURNEY, THOMAS, AND STERNE

Consider Eq. 10a and 12a. R_1 has been defined as the inner radius of the charge and R_2 as the outer radius of charge. R_2 is also the distance from the center of a sphere or central axis of a cylinder to the charge-case interface. As R_1 approaches zero, the cylinder or sphere approaches the conditions of a solid cylindrical or spherical warhead. When $R_1 = 0$, Eq. 10a becomes

$$(13) \qquad v_{HC} = \sqrt{2E} \sqrt{\frac{\frac{C}{M}}{1 + \frac{C}{2M}}}$$

or

$$v_{HC} = v_{sc} \tag{13a}$$

and Eq. 12a reduces to

$$v_{HS} = \sqrt{2E}\sqrt{\frac{\frac{C}{M}}{1 + \frac{3C}{5M}}} \tag{14}$$

or

$$v_{HS} = v_{ss} \tag{14a}$$

both of which are identical to Gurney's formulae for a cylinder and a sphere, respectively.

For the case of a flat plate as considered by Thomas, small sections of the surfaces of the sphere or cylinder are approximately plane surfaces; hence, the radii of curvature approach infinity, and R_1 approaches R_2. Actually, R_1 can never equal R_2, since if $R_1 = R_2$, then $C = 0$; that is, no charge would be present. However, when R_2 becomes very large and R_1 approaches R_2, then R_2 minus R_1 becomes negligible, and the ratio R_1/R_2 approaches unity.

If in Eq. 10a and 12a R_1 is allowed to approach R_2 and $R_1/R_2 \approx 1$, then the resultant equations should produce results equivalent to the flat-plate equation set forth by Thomas or the simpler version of Thomas' equation as set forth by Sterne. For $R_1/R_2 \approx 1$, Eq. 10a for a cylinder becomes

$$(15) \qquad v_{HC} = \sqrt{2E}\sqrt{\frac{1}{1+\frac{M}{C}}}\sqrt{\frac{\frac{C}{M}}{1+\frac{C}{2M}}}$$

or

$$(15a) \qquad v_{HC} = \sqrt{2E}\sqrt{\frac{\frac{2C}{3M}}{1+\frac{C}{3M}+\frac{2M}{3C}}}$$

and Eq. 12a for a sphere becomes

$$(16) \qquad v_{HS} = \sqrt{2E}\sqrt{\frac{1}{1+\frac{M}{C}}}\sqrt{\frac{\frac{C}{M}}{1+\frac{3C}{5M}}}$$

or

$$(16a) \qquad v_{HS} = \sqrt{2E}\sqrt{\frac{\frac{5C}{8M}}{1+\frac{3C}{8M}+\frac{5M}{8C}}}$$

Table 2 shows the comparison of Eq. 15a and 16a with the formulae set forth by Thomas, Appendix B, and Sterne, Eq. 3. The tabulated values

TABLE 2. $v/\sqrt{2E}$, Calculated by Different Equations, for Different C/M Ratios

C/M	Cylinder[1,3] Equation (15a)	Sphere[2,4] Equation (16a)	Thomas'[1,2,5] Equation	Sterne's[3,4] Equation (3)
.0	.0000	.0000	.0000	.0000
.1	.0930	.0926	.0797	.0816
.2	.1741	.1725	.1508	.1543
.3	.2454	.2422	.2147	.2198
.4	.3086	.3036	.2727	.2792
.5	.3651	.3581	.3256	.3333
.6	.4160	.4067	.3742	.3831
.7	.4621	.4505	.4191	.4289
.8	.5040	.4901	.4606	.4714
.9	.5422	.5261	.4993	.5109
1.0	.5774	.5590	.5355	.5477
1.1	.6097	.5892	.5694	.5822
1.2	.6396	.6169	.6012	.6145
1.3	.6673	.6425	.6312	.6449
1.4	.6931	.6662	.6595	.6736
1.5	.7171	.6882	.6864	.7007
1.6	.7396	.7088	.7118	.7263
1.7	.7606	.7279	.7360	.7506
1.8	.7804	.7459	.7590	.7736
1.9	.7990	.7627	.7809	.7956
2.0	.8165	.7785	.8019	.8165
2.1	.8330	.7934	.8219	.8364
2.2	.8487	.8074	.8411	.8555
2.3	.8635	.8207	.8595	.8737
2.4	.8775	.8333	.8772	.8911
2.5	.8909	.8452	.8941	.9078
2.6	.9036	.8565	.9105	.9239
2.7	.9156	.8672	.9262	.9393
2.8	.9272	.8774	.9413	.9541
2.9	.9382	.8871	.9559	.9683
3.0	.9487	.8964	.9699	.9820
3.1	.9587	.9053	.9836	.9952
3.2	.9684	.9138	.9968	1.0079
3.3	.9776	.9219	1.0096	1.0202
3.4	.9864	.9296	1.0219	1.0321
3.5	.9949	.9371	1.0339	1.0435
3.6	1.0031	.9442	1.0455	1.0546
3.7	1.0110	.9511	1.0567	1.0653
3.8	1.0185	.9577	1.0676	1.0757
3.9	1.0258	.9640	1.0783	1.0857
4.0	1.0328	.9701	1.0886	1.0955

(As R_2 becomes very large and R_1 approaches R_2, $R_1/R_2 = 1$, but never = 1.)

1 Correlation coefficient for equations (15a), and that of Thomas = .99710.

2 Correlation coefficient for equations (16a) and that of Thomas = .99546.

3 Correlation coefficient for equations (15a) and (3) = .99781.

4 Correlation coefficient for equations (16a) and (3) = .99635.

5 See Appendix B.

are $v/\sqrt{2E}$ for various values of C/M in the limiting case where the radii of curvature, R_1 and R_2, become very large. The results are substantially in agreement.

RESULTS

Tables 3 and 4, which contain data calculated on a CDC 1604 computer, show various values of $v/\sqrt{2E}$ versus C/M when R_1/R_2 is held constant. Table 3 is for a cylindrical warhead using Eq. 10a.

TABLE 3. Ratio of Initial Fragment Velocity to the Square Root of 2E as Calculated from Various C/M and R_1/R_2 Ratios for a Hollow Cylindrical Warhead

R_1/R_2 \ C/M	.2	.4	.6	.8	1.0	1.5	2.0	2.5	3.0	4.0
.0	.426	.577	.679	.756	.816	.926	1.000	1.054	1.095	1.155
.1	.348	.516	.629	.713	.778	.896	.976	1.034	1.078	1.141
.2	.302	.471	.588	.676	.745	.870	.953	1.014	1.061	1.127
.3	.270	.436	.555	.645	.716	.845	.933	.996	1.044	1.114
.4	.246	.408	.526	.617	.690	.823	.913	.979	1.029	1.101
.5	.228	.385	.502	.593	.667	.802	.894	.962	1.014	1.089
.6	.213	.365	.480	.571	.645	.782	.877	.947	1.000	1.077
.7	.201	.348	.462	.552	.626	.764	.861	.932	.986	1.065
.8	.191	.333	.445	.535	.609	.748	.845	.917	.973	1.054
.9	.182	.320	.430	.519	.592	.732	.830	.904	.961	1.043
*1.0	.174	.309	.416	.504	.577	.717	.816	.891	.949	1.033

* The values tabulated for $R_1/R_2 = 1$ are for mathematical analysis only. These values are for a cylinder and sphere whose surfaces approach a flat plate and the radii of curvature become very large. Hence $R_2 - R_1$ is negligible if R_2 becomes very large and R_1 approaches R_2. $R_1/R_2 \sim 1$, but never equals 1.

Table 4 is for a spherical warhead using Eq. 12a. Figures 3 and 4 are curves representing Tables 3 and 4, respectively.

TABLE 4. Ratio of Initial Fragment Velocity to the Square Root of 2E as Calculated from Various C/M and R_1/R_2 Ratios for a Hollow Spherical Warhead

R_1/R_2 \ C/M	.2	.4	.6	.8	1.0	1.5	2.0	2.5	3.0	4.0
.0	.423	.568	.664	.735	.791	.889	.953	1.000	1.035	1.085
.1	.412	.561	.659	.731	.787	.886	.951	.998	1.033	1.083
.2	.386	.542	.643	.717	.775	.877	.944	.992	1.028	1.079
.3	.351	.513	.619	.697	.757	.863	.933	.982	1.020	1.073
.4	.315	.480	.590	.671	.734	.845	.917	.969	1.009	1.064
.5	.282	.446	.558	.642	.707	.823	.899	.953	.994	1.052
.6	.253	.412	.525	.611	.678	.798	.878	.935	.978	1.039
.7	.228	.381	.493	.579	.648	.771	.855	.914	.960	1.024
.8	.206	.352	.462	.548	.617	.744	.830	.892	.940	1.007
.9	.188	.327	.433	.518	.588	.716	.804	.869	.919	.989
*1.0	.173	.304	.407	.490	.559	.688	.778	.845	.896	.970

* The values tabulated for $R_1/R_2 = 1$ are for mathematical analysis only. These values are for a cylinder and sphere whose surfaces approach a flat plate and the radii of curvature become very large. Hence, R_2-R_1 is negligible if R_2 becomes very large and R_1 approaches R_2. $R_1/R_2 \sim 1$ but never equals 1.

CONCLUSION

Working formulae for predicting the initial fragment velocities from hollow spherical and cylindrical warheads have been presented. The predictions of the new formulae have been shown to agree substantially with the predictions of Gurney's formulae at one extreme (where the central cavity is quite small relative to the charge) and with the predictions

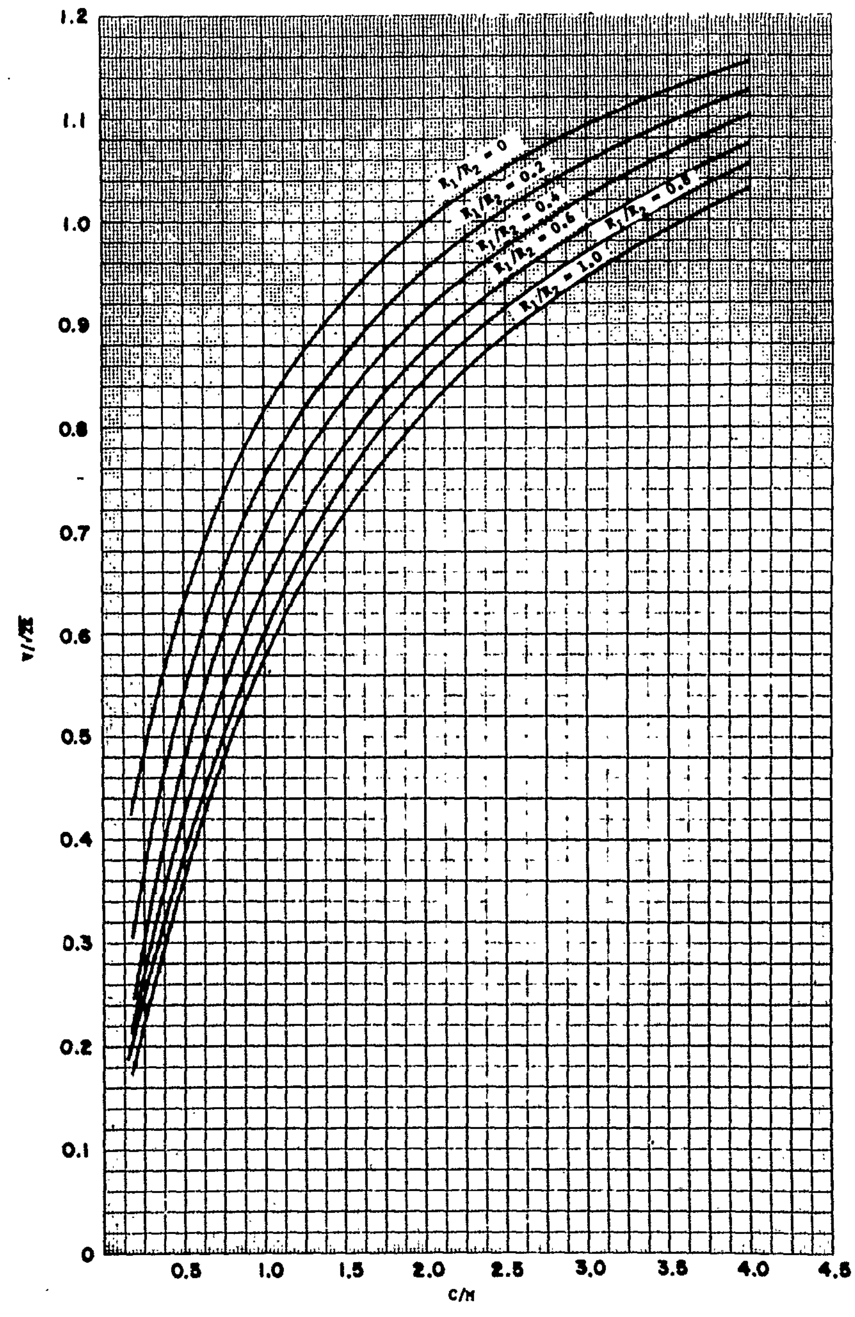

FIG. 3. C/M Versus $v/\sqrt{2E}$ with Parameter $0 \leq R_1/R_2 \leq 1$ for a Hollow Cylindrical Warhead.

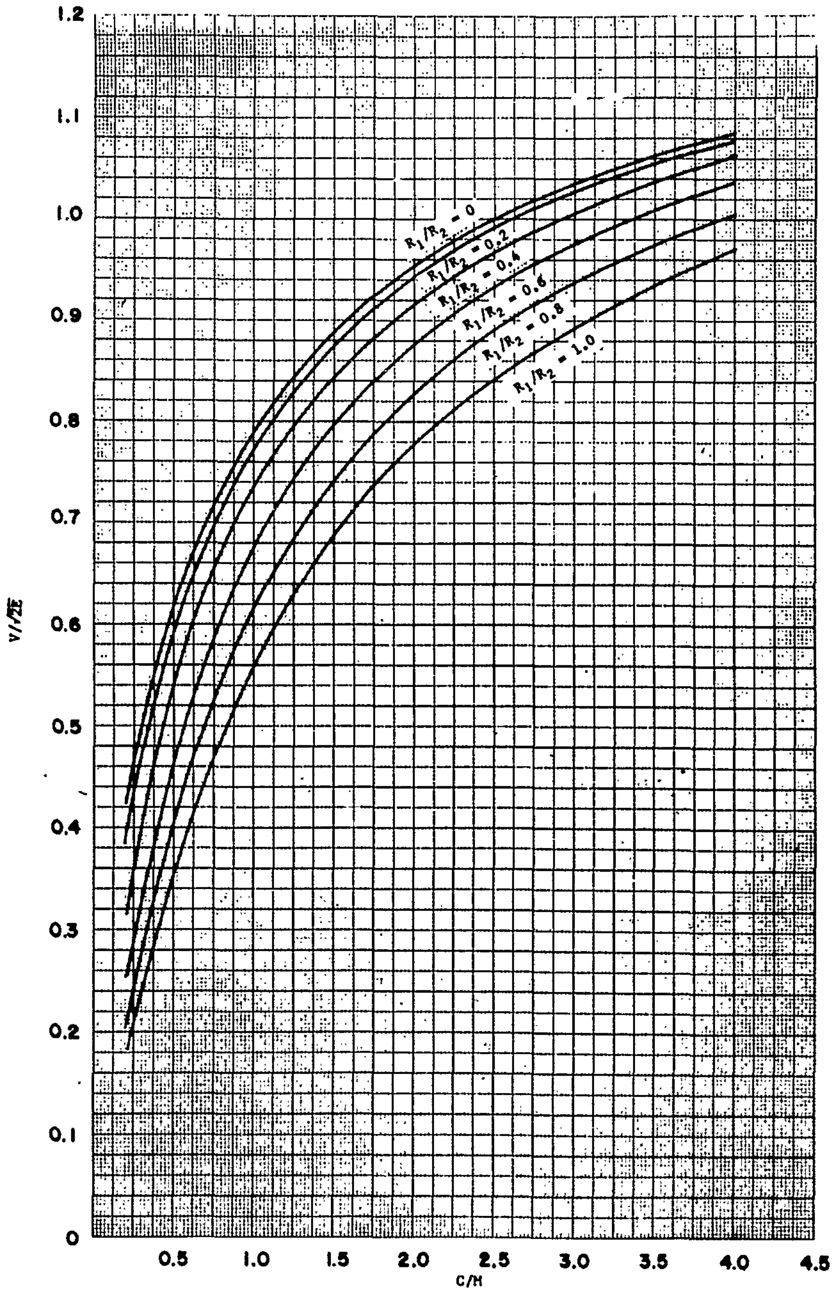

FIG. 4. C/M Versus $v/\sqrt{2E}$ with Parameter $0 \leq R_1/R_2 \leq 1$ for a Hollow Spherical Warhead.

of Thomas' and Sterne's formulae at the other (where the central cavity is very large). The new formulae also agree with the limited experimental data available to the NWEF.

It should be emphasized that the formulae presented in this report are working formulae based on previously derived formulae rather than on experimental data. Only one set of experimental data was available for comparison in this report. This experiment considered a low ratio of C/M (C/M = 0.43). The formulae presented may work very well for low values of C/M, but one cannot be certain about their applicability to high values of C/M, since no comparison was made with experimental data. Unpublished data in the files of military laboratories might not agree with these formulae. Additional data either confirming or denying these formulae will be welcomed. It is hoped that this report will stimulate experimental and theoretical effort toward the analysis of fragmentation of hollow warheads.

These formulae, because of their simplicity, should prove useful in studies of the fragmentation effectiveness of Naval weapons.

Appendix A

DEFINITION OF A CYLINDRICAL WARHEAD APPROACHING A FLAT PLATE

Consider a segment of surface area of a cylinder, A., with plane, S, tangent to the surface of the cylinder along line, L, Fig. 5. Let the angle between plane, S, and cylindrical surface, A, be θ. The distance

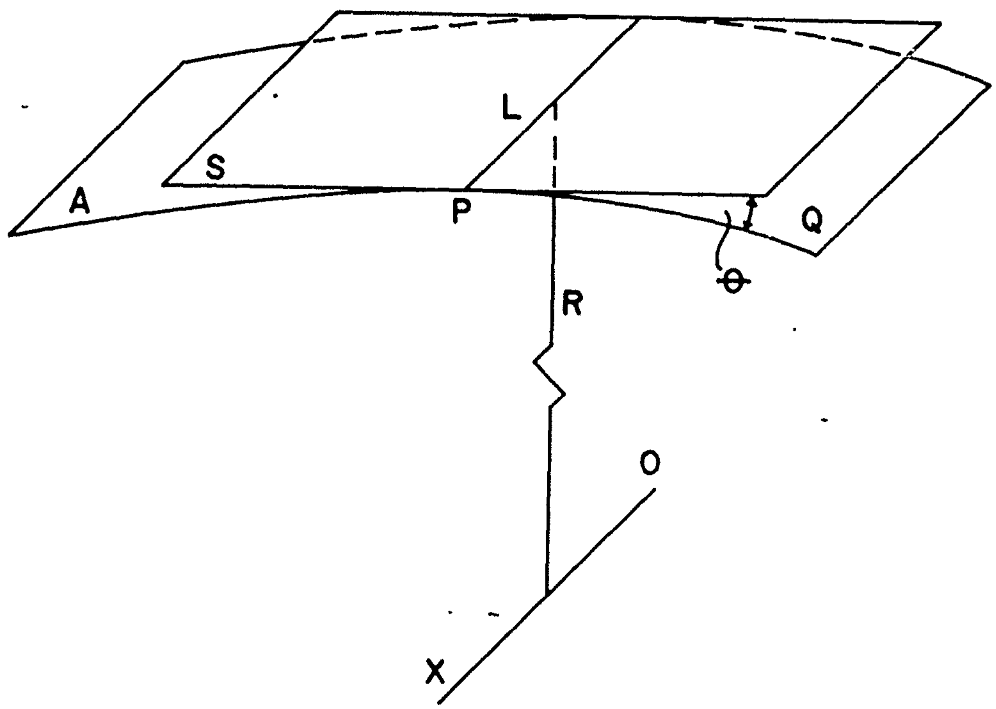

FIG. 5. A Cylindrical Warhead Approaching a Flat Plate.

from the central axis of the cylinder, OX, and line L in the cylindrical surface is R, the radius of the cylinder. As the angle θ approaches zero, the surface, A, approaches the plane S, and the radius approaches infinity.

As $\theta \rightarrow 0$, $A \rightarrow S$, and $R \rightarrow \infty$. If R is held constant and $\overline{PQ}$ decreases and becomes small enough so that θ (radians) equals the sine of θ, then the cylindrical surface, A, approaches the plane S.

If $R = K$, as $\overline{PQ} \rightarrow 0$, $\theta \rightarrow 0$, and $A \rightarrow S$.

Appendix B

DEFINITION OF A SPHERICAL WARHEAD APPROACHING A FLAT PLATE

Consider a segment of surface area of a sphere B with plane N tangent to the surface of the sphere at P (x, y, z), Fig. 6. Let the angle between plane N and spherical surface B equal θ. The radius of the

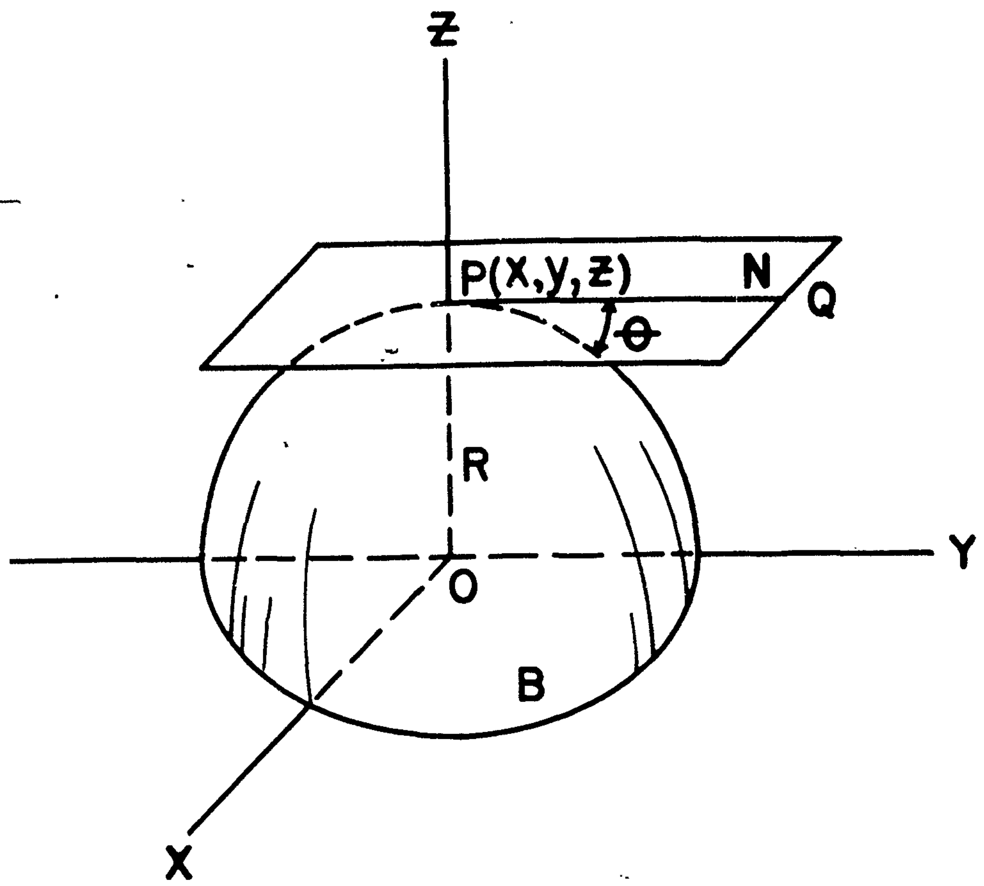

FIG. 6. A Spherical Warhead Approaching a Flat Plate.

sphere is $R = \overline{OP}$. As θ approaches zero, the surface B approaches plane N, and R approaches infinity.

As $\theta \to 0$, $B \to N$, $R \to \infty$. Analogous to the cylinder, if R is held constant, it can be shown if $R = K$, as $\overline{PQ} \to 0$, $\theta \to 0$, and $B \to N$.

Appendix C

THE VELOCITY OF FRAGMENTS FROM A PLANE PLATE
ACCORDING TO L. H. THOMAS' FORMULA

The following are the equations needed to solve L. H. Thomas' formula for $v/\sqrt{2E}$ versus C/M for a flat slab of metal in contact with a flat slab of high explosive. As mentioned in the text of the report, the development of the formula is beyond the scope of this report.

Thomas' final equation for the initial velocity of fragments shows v as a function of the energy E, the mass ratio C/M, and a quantity not discussed heretofore called x_o. The equation is

$$(17) \qquad v^2 = 2E\,\frac{(3\gamma - 1)}{(\gamma - 1)}\left[\frac{\frac{C}{M}}{1 + \frac{C}{Mx_o^2}}\right]$$

where γ is assigned the value of 2.75 by Thomas.

Further, C/M is a function of x_o, that is,

$$(18) \qquad \frac{C}{M} = -\frac{2\gamma}{(\gamma - 1)}\,x_o\left[\frac{1}{(1 - x_o^2)^{\gamma/(\gamma - 1)}}\right]$$

Let the integrand be called J:

$$(19) \qquad J = \int_{x_o}^{1} (1 - x^2)^{\frac{1}{\gamma - 1}} dx$$

then

$$(20) \qquad \frac{C}{M} = -\frac{2\gamma}{(\gamma - 1)} x_o \left[\frac{1}{(1 - x_o^2)^{\gamma/(\gamma - 1)}}\right] \int_{x_o}^{1} (1 - x^2)^{\frac{1}{\gamma - 1}} dx$$

Table 5 shows values of J from $x_o = 0$ to $x_o = -1.00$. Knowing x_o and J, one can solve Eq. 20 for C/M. Then using Eq. 17 one can solve for $v/\sqrt{2E}$ for various values of x_o and the related values of C/M.

TABLE 5. Various Values of J from $x_o = 0$ to $x_o = -1.00$

x_0	J	x_0	J
.00	.764568	-.50	1.239919
-.01	.774568	-.51	1.248370
-.02	.784566	-.52	1.256756
-.03	.794563	-.53	1.265073
-.04	.804556	-.54	1.273320
-.05	.814544	-.55	1.281496
-.06	.824527	-.56	1.289598
-.07	.834502	-.57	1.297626
-.08	.844470	-.58	1.305576
-.09	.854429	-.59	1.313447
-.10	.864377	-.60	1.321237
-.11	.874314	-.61	1.328944
-.12	.884238	-.62	1.336567
-.13	.894148	-.63	1.344101
-.14	.904044	-.64	1.351547
-.15	.913923	-.65	1.358900
-.16	.923785	-.66	1.366160
-.17	.933628	-.67	1.373323
-.18	.943452	-.68	1.380388
-.19	.953255	-.69	1.387351
-.20	.963036	-.70	1.394210
-.21	.972794	-.71	1.400962
-.22	.982527	-.72	1.407605
-.23	.992234	-.73	1.414135
-.24	1.001915	-.74	1.420549
-.25	1.011567	-.75	1.426845
-.26	1.021190	-.76	1.433018
-.27	1.030783	-.77	1.439066
-.28	1.040343	-.78	1.444985
-.29	1.049870	-.79	1.450770
-.30	1.059363	-.80	1.456418
-.31	1.068821	-.81	1.461924
-.32	1.078241	-.82	1.467284
-.33	1.087623	-.83	1.472492
-.34	1.096965	-.84	1.477544
-.35	1.106267	-.85	1.482434
-.36	1.115526	-.86	1.487156
-.37	1.124741	-.87	1.491703
-.38	1.133912	-.88	1.496067
-.39	1.143036	-.89	1.500241
-.40	1.152112	-.90	1.504215
-.41	1.161139	-.91	1.507980
-.42	1.170115	-.92	1.511523
-.43	1.179039	-.93	1.514832
-.44	1.187909	-.94	1.517889
-.45	1.196724	-.95	1.520676
-.46	1.205483	-.96	1.523168
-.47	1.214183	-.97	1.525332
-.48	1.222824	-.98	1.527122
-.49	1.231403	-.99	1.528458
		-1.00	1.529135

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ABSTRACT CARD

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