AD NUMBER

AD441598

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BRL

REPORT NO. 1244
MARCH 1964

EQUATIONS OF MOTION OF A RIGID PROJECTILE

Robert F. Lieske
Robert L. McCoy

RDT & E Project No. 1M523801A287

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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BALLISTIC RESEARCH LABORATORIES

REPORT NO. 1244

MARCH 1964

EQUATIONS OF MOTION OF A RIGID PROJECTILE

Robert F. Lieske
Robert L. McCoy

Computing Laboratory

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ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 1244

RFLieske/RLMcCoy/jag
Aberdeen Proving Ground, Md.
March 1964

EQUATIONS OF MOTION OF A RIGID PROJECTILE

ABSTRACT

The basic vector equations of motion of a rigid body are described, and an aerodynamic force-moment system is developed. The vector forces and moments due to gravity and rotation of the earth are added to the system. Forces and moments produced by a rocket motor are included, and the resulting vector differential equations of motion of a symmetric rigid projectile are derived.

An appendix is added, which describes a set of initial conditions imposed on pertinent parameters for rigid body trajectory simulation.

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Table of Symbols

Term	Definition	Units
$A(t)$	Axial moment of inertia at time t	$lb\text{-}ft^2$
$\dot{A}(t)$	Rate of change of A at time t	$\frac{lb\text{-}ft^2}{sec}$
A_e	Area of jet exit	ft^2
A_Z	Azimuth of line of fire (clockwise from north)	deg
$B(t)$	Transverse moment of inertia at time t	$lb\text{-}ft^2$
$\dot{B}(t)$	Rate of change of B at time t	$\frac{lb\text{-}ft^2}{sec}$
CG	Distance from nose of rocket to the center of mass	ft
d	Reference diameter of missile	ft
$\vec{E}$	Position of missile with respect to spherical earth surface	ft
ϵ	Fin cant angle	rad
f	Deceleration due to friction	ft/sec^2
$\vec{g}$	Acceleration due to gravity	ft/sec^2
g_o	Acceleration due to gravity (surface)	ft/sec^2
g_c	Constant used in the conversion of thrust (lbf) to force ($lb\text{-}ft/sec^2$)	ft/sec^2
$\vec{H}$	Total angular momentum	$lb\text{-}ft^2\text{-}rad/sec$
$\vec{h}$	Angular momentum divided by B	rad/sec
$\dot{\vec{h}}$	Rate of change of $\vec{h}$	rad/sec

Term	Definition	Units
h_{2_1}	h_2 at end of launch (t_L)	rad/sec
h_{3_1}	h_3 at end of launch (t_L)	rad/sec
$\underrightarrow{h}(t_L)$	$\underrightarrow{h}$ at end of launch (t_L)	rad/sec
I_{SP}	Specific impulse per unit of fuel mass	$\frac{\text{lbf-sec}}{\text{lbm}}$
I_{ST}	Total impulse at standard conditions	lbf-sec
K_A	Spin damping moment coefficient	———
K_{D_o}	Drag coefficient	———
K_{D_α}	Yaw drag coefficient	$1/\text{rad}^2$
K_E	Fin cant coefficient	———
K_F	Magnus force coefficient	———
K_H	Damping moment coefficient	———
K_L	Lift force coefficient	———
K_M	Overturning coefficient	———
K_S	Pitching force coefficient	———
K_T	Magnus moment coefficient	———
K_{XF}	Magnus cross force coefficient	———
K_{XT}	Magnus cross moment coefficient	———
l_e	Length of missile	ft
L	Latitude of launch point	deg
M	Mach number	———

Term	Definition	Units
m_f (t)	Mass of fuel at time t	lb
m (t)	Mass of missile at time t	lb
$\dot{m}$ (t)	Rate of change of mass at time t	lb/sec
N	Axial spin	rad/sec
P_a	Static atmospheric pressure	lb/ft^2
P_e	Jet pressure at nozzle exit	lb/ft^2
r	Distance between center of earth and body	ft
r_1	Distance from CG (t_B) to CG of fuel	ft
r_2	Radius of gyration of fuel mass	ft
r_e	Distance from CG of missile to nozzle exit	ft
r_F	Distance from tail to CG of fuel	ft
r_s	Radius of spin rocket action	ft
r_t	Distance from CG of missile to nozzle throat	ft
r_3 (t)	Distance of CG shift due to fuel mass change at time t	ft
t	Time	sec
T*	Effective thrust	lb
t_B	Motor burnout time	sec
t_{BM}	Time from ignition until first motion	sec
t_{BMST}	Time from ignition until first motion at standard condition	sec
t_{BST}	Motor burnout time at standard condition	sec
T_F	Thrust factor	—

Term	Definition	Units
t_L	Time at end of launch	sec
T_R (t)	Thrust produced by motor at time t	lbf
T_s	Spin rocket thrust	lbf
T	Thrust	lbf
$\vec{u}$	Velocity of missile with respect to ground	ft/sec
$\dot{\vec{u}}$	Acceleration of missile with respect to ground	ft/sec^2
$\vec{v}$	Velocity of missile with respect to air	ft/sec^2
X_{CG} (t)	Distance from nose of missile to center of mass at time t	ft
X_{CP}	Normal force center of pressure from nose	ft
$\vec{X}$	Position of missile with respect to ground	ft
$\vec{x}$	Unit vector along longitudinal axis of missile	——
$\dot{\vec{x}}$	Rate of change of $\vec{x}$	rad/sec
$\vec{x}(t_L)$	$\vec{x}$ at end of launch (t_L)	
$\vec{y}$	Unit vector perpendicular to $\vec{x}$	——
$\dot{\vec{y}}$	Rate of change of $\vec{y}$	rad/sec
$\vec{y}(t_L)$	$\vec{y}$ at end of launch (t_L)	
$\vec{z}$	Unit vector perpendicular to both $\vec{x}$ and $\vec{y}$, $\vec{z} = (\vec{x} \times \vec{y})$	——
$\dot{\vec{z}}$	Rate of change of $\vec{z}$	
$\vec{z}(t_L)$	$\vec{z}$ at end of launch (t_L)	rad/sec

Term	Definition	Units
α	Yaw of missile	deg
γ	Angle between the $\underset{\rightarrow}{x}\ \underset{\rightarrow}{y}$ plane and the angular thrust malalignment	deg
Γ	Angle between the $\underset{\rightarrow}{x}\ \underset{\rightarrow}{y}$ plane and the linear thrust malalignment	deg
Δ	Linear thrust malalignment	ft
δ	Angular thrust malalignment	deg
$\underset{\rightarrow}{\Lambda}$	Acceleration due to rotation of earth	$\mathrm{ft/sec^2}$
ρ	Air density (varies with altitude)	$\mathrm{lb/ft^3}$
ϕ	Angle of elevation of launch	deg
ψ	Orientation of yaw	deg

Introduction

The purpose of this report is to present a general and flexible mathematical model for rigid body trajectory simulation.

The total vector force-moment system presented is based on a collection of the ideas of several authors. In a basic paper published in 1920 [Ref. 1], Fowler, Gallop, Lock, and Richmond developed the basic aerodynamic theory and the concise vector notation of the present report. (See also Reference 5, and unpublished work by A. S. Galbraith). Fowler's aerodynamic hypothesis contained a logical contradiction, as was pointed out by Nielsen and Synge in 1946 [Ref. 2]. Kelley, McShane, and Reno in their text on exterior ballistics [Ref. 4] extended the work of Nielsen and Synge and developed the basic aerodynamic hypothesis used in the present report.

The force-moment equations produced by a rocket motor were obtained by collecting the basic principles expounded in two texts [Ref. 3, 6].

The resulting equations were brought into logical consistency and expressed in the vector notation of the present report.

The usefulness of any mathematical model is determined by its ability to closely match the results of physical experiments over a spectrum of test conditions. The model presented in this report has successfully simulated free flight rocket trajectories over a wide spectrum of test conditions. A limited amount of simulation has been performed on conventional spin stabilized projectiles, and the results are quite encouraging.

BASIC LAWS OF MOTION OF A RIGID BODY

Consider a ground-fixed, right-handed coordinate system, whose origin is located at the center of mass of a rigid body, as shown in Fig. 1 below.

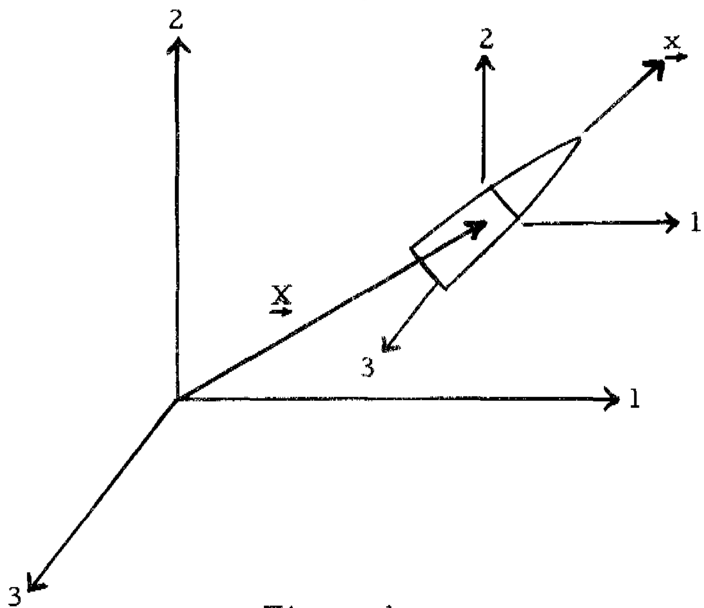


Figure 1.

Assume that the body can be considered a solid of revolution, and assign to the axis of rotational symmetry a unit vector $\vec{x}$ in the chosen coordinate system. Since the rigid body is to represent a rocket or shell, the direction of $\vec{x}$ from tail to nose is defined as positive.

The total angular momentum of the body can now be expressed as the sum of two vectors in the ground-fixed coordinate system:

(a) The angular momentum about $\vec{x}$.

(b) The total angular momentum about an axis perpendicular to $\vec{x}$.

The angular momentum about $\underset{\rightarrow}{x}$ has the magnitude AN, where A is the moment of inertia of the body about $\underset{\rightarrow}{x}$, and N is the axial spin or angular velocity about $\underset{\rightarrow}{x}$ [1], in radians per second, hence, the total angular momentum about $\underset{\rightarrow}{x}$ can be represented by the vector AN $\underset{\rightarrow}{x}$.

The total angular velocity of the body about an axis perpendicular to $\underset{\rightarrow}{x}$ is given by the vector $(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$, where the superscript dot refers to differentiation with respect to time. Since the body possesses rotational symmetry, every axis through the center of mass and perpendicular to $\underset{\rightarrow}{x}$ is a principal axis of inertia. If the moment of inertia of the body about any transverse axis is B, the total angular momentum about an axis perpendicular to $\underset{\rightarrow}{x}$ has the vector representation B $(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$.

Let $\underset{\rightarrow}{H}$ denote the total vector angular momentum of the body. The vector representation of $\underset{\rightarrow}{H}$ is:

(1.1) $\underset{\rightarrow}{H} = AN\underset{\rightarrow}{x} + B\ (\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$

Let $\underset{\rightarrow}{h} = \underset{\rightarrow}{H}/B$, and divide both sides of Equation (1.1) by B.

(1.2) $\underset{\rightarrow}{h} = \frac{AN}{B}\underset{\rightarrow}{x} + (\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$

Let $\Sigma\underset{\rightarrow}{M}$ be the sum of the vector applied moments, and set $\Sigma\underset{\rightarrow}{M}$ equal to the vector rate of change of angular momentum.

(1.3) $\dot{\underset{\rightarrow}{h}} = \frac{A\dot{N}}{B}\underset{\rightarrow}{x} + \frac{AN}{B}\dot{\underset{\rightarrow}{x}} + (\underset{\rightarrow}{x} \times \ddot{\underset{\rightarrow}{x}}) = \Sigma\ (\underset{\rightarrow}{M}/B)$

[1] In this report a positive N is defined as rotation which would cause a right-hand screw to advance in the direction of $\underset{\rightarrow}{x}$.

In addition to these basic equations, two additional expressions are needed for use in the force-moment system.

(1.4) $(\vec{h} \times \vec{x}) = \dot{\vec{x}}$

(1.5) $(\vec{h} \cdot \vec{x}) = \frac{AN}{B}$

Equation (1.3) is the basic vector differential equation of angular motion in a fixed coordinate system. The basic equation of motion of the center of mass is:

(1.6) $\dot{\vec{u}} = \Sigma\,(\vec{F}/m)$,

where $\Sigma\vec{F}$ denotes the sum of the vector applied forces, m is the mass of the body, and $\dot{\vec{u}}$ is the vector acceleration of the center of mass in the fixed coordinate system.

It is now necessary to determine the forces and moments which comprise $\Sigma\vec{F}$ and $\Sigma\vec{M}$. In the next section the basic aerodynamic forces and moments are considered; those of non-aerodynamic origin are deferred until a later section.

AERODYNAMIC FORCES AND MOMENTS

A force or moment is defined to be aerodynamic in origin if it is produced by interaction of a rigid body and its atmospheric medium. The aerodynamic forces and moments presented in this report are those considered necessary to form a logically consistent mathematical model for a rigid body possessing rotational symmetry.

DRAG

Drag is historically defined as resistance to forward motion of a projectile. The magnitude of drag force is represented in classical exterior ballistics as $\rho d^2 K_{D_o} v^2$, where ρ is the density of the atmospheric medium, d is the reference diameter of the projectile, v is the forward velocity of the center of mass with respect to the air, and K_{D_o} is a dimensionless number called the drag coefficient. If the forward velocity with respect to air is represented by the vector $\vec{v}$, then the direction of drag is $-\vec{v}$.

Figure 2 illustrates the general case of a projectile with the unit vector $\vec{x}$ pointing along the axis of symmetry, and the forward velocity represented by $\vec{v}$. The angle between $\vec{v}$ and $\vec{x}$, denoted by α, is traditionally called the angle of yaw.

Yaw increases drag on a projectile by presenting to the air stream an enlarged cross-sectional area. The effect of yaw on drag is accounted for by allowing K_{D_o} to increase with yaw squared. The yaw drag coefficient is denoted by K_{D_α}, and the increase in the drag coefficient due to yaw is then given by $K_{D_\alpha} \alpha^2$. The effective drag coefficient is $(K_{D_o} + K_{D_\alpha} \alpha^2)$. Based on the above definitions, drag force is represented by the vector equation:

$$(2.1) \quad \text{DRAG FORCE} = -\rho d^2 (K_{D_o} + K_{D_\alpha} \alpha^2)\, v\, \vec{v}$$

SPIN DAMPING MOMENT

The spin damping moment is an aerodynamic moment produced by viscous friction of the air on ths surface of a spinning shell, and is a couple tending to destroy axial spin. (See Figure 3.)

The magnitude of the spin damping moment is given by $\rho d^4 K_A Nv$, where K_A is a dimensionless number called the spin damping moment coefficient. Since the spin damping moment opposes axial spin, its direction is given by $-\vec{x}$. The vector representation of the spin damping moment is given by:

(2.2) SPIN DAMPING MOMENT $= -\rho d^4 K_A N v \vec{x}$

Replacing N with its equivalent from Equation (1.5):

(2.3) SPIN DAMPING MOMENT $= -\frac{\rho d^4 B}{A} K_A v (\vec{h} \cdot \vec{x}) \vec{x}$

FIN CANT MOMENT

If a projectile is symmetrically finned, and the fins form equal non-zero angles with the axis of symmetry, a pure spin generating couple will be produced during flight. The fin cant angle, ϵ, is defined as positive if it generates a positive spin. Figure 4 illustrates the fin cant moment for a positive fin cant angle.

The magnitude of the fin cant moment is given by $\rho d^3 K_E \epsilon v^2$, where K_E is the dimensionless fin cant moment coefficient, and ϵ is the fin cant angle. The direction of the moment is $\vec{x}$ for positive values of ϵ. Accordingly, the vector representation of the fin cant moment is:

(2.4) FIN CANT MOMENT $= \rho d^3 K_E \epsilon v^2 \vec{x}$

LIFT FORCE

Aerodynamic lift is created by an asymmetric air flow over a yawed body. The magnitude of the lift force is given by $\rho d^2 K_L v^2 \sin \alpha$, where K_L is the dimensionless lift force coefficient. The lift force acts in the plane of yaw and is perpendicular to the direction of motion of the projectile.

Figure 5 illustrates the vector representation of the lift force. Consider the vector $[\underset{\rightarrow}{v} \times (\underset{\rightarrow}{x} \times \underset{\rightarrow}{v})]$. This vector has the magnitude $v^2 \sin \alpha$, and is perpendicular to $\underset{\rightarrow}{v}$ in the plane containing $\underset{\rightarrow}{v}$ and $\underset{\rightarrow}{x}$. The vector representation of lift force is then given by:

(2.5) $\text{LIFT FORCE} = \rho d^2 K_L [\underset{\rightarrow}{v} \times (\underset{\rightarrow}{x} \times \underset{\rightarrow}{v})]$

Expanding the triple vector product, and rewriting:

(2.6) $\text{LIFT FORCE} = \rho d^2 K_L [v^2 \underset{\rightarrow}{x} - (\underset{\rightarrow}{v} \cdot \underset{\rightarrow}{x}) \underset{\rightarrow}{v}]$

The vector sum of the lift and drag forces can be expressed as a total resistance vector. If the components of vector resistance are resolved parallel and perpendicular to $\underset{\rightarrow}{x}$ instead of $\underset{\rightarrow}{v}$, the axial drag force and normal force result. The normal force is the component of the vector resistance that produces the overturning moment.

OVERTURNING MOMENT

If the line of action of the aerodynamic normal force does not pass through the center of mass of the projectile, an overturning moment will be produced. The magnitude of the overturning moment due to normal force is given by $\rho d^3 K_M v^2 \sin \alpha$, where K_M is the dimensionless overturning moment coefficient. The overturning moment is perpendicular to the plane of yaw, or to both $\underset{\rightarrow}{v}$ and $\underset{\rightarrow}{x}$, as is illustrated in Figure 6. The vector $(\underset{\rightarrow}{v} \times \underset{\rightarrow}{x})$ has the proper direction, and in magnitude is equal to $v \sin \alpha$. The overturning moment vector is then given by:

(2.7) $\text{OVERTURNING MOMENT} = \rho d^3 K_M v (\underset{\rightarrow}{v} \times \underset{\rightarrow}{x})$

If the line of action of normal force intersects the axis of symmetry at a point behind the center of mass, the overturning moment acts as a restoring moment. This situation is met by allowing K_M to be negative. The addition of fins at the rear of a body of revolution moves the center of pressure of the normal force to the rear, and usually results in a negative K_M, or restoring moment.

MAGNUS FORCE

The Magnus force arises from the interaction of the air stream and the boundary layer of a yawed spinning body. The magnitude of the Magnus force is given by $\rho d^3 K_F N v \sin \alpha$, where K_F is the dimensionless Magnus force coefficient. The direction of the Magnus force is perpendicular to the plane of yaw and is represented by the vector $(\underline{x} \times \underline{v})$ for positive values of N, as illustrated in Figure 7. The Magnus force is represented by the vector:

(2. 8) $\text{MAGNUS FORCE} = \rho d^3 K_F N (\underline{x} \times \underline{v})$

Replacing N with its equivalent from Equation (1. 5):

(2. 9) $\text{MAGNUS FORCE} = \frac{\rho d^3 B}{A} K_F (\underline{h} \cdot \underline{x}) (\underline{x} \times \underline{v})$

MAGNUS MOMENT

If the line of action of the Magnus force does not pass through the center of mass of the projectile, a Magnus moment will be produced. The magnitude of the Magnus moment is given by $\rho d^4 K_T N v \sin \alpha$, where K_T is the dimensionless Magnus moment coefficient. The Magnus moment lies in the plane of yaw, and is perpendicular to $\underline{x}$. The vector $[\underline{x} \times (\underline{x} \times \underline{v})]$ has the magnitude $v \sin \alpha$ and has the proper direction, as illustrated in Figure 8. The Magnus moment is represented by the vector:

(2. 10) $\text{MAGNUS MOMENT} = \rho d^4 K_T N [\underline{x} \times (\underline{x} \times \underline{v})]$

Again replacing N with its vector equivalent, and expanding the vector triple product:

$$(2.11)\quad \text{MAGNUS MOMENT} = \frac{\rho d^4 B}{A} K_T (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x}) [(\underset{\rightarrow}{v} \cdot \underset{\rightarrow}{x}) \underset{\rightarrow}{x} - \underset{\rightarrow}{v}]$$

PITCHING FORCE

The pitching force is a force opposing any change in the direction of the longitudinal axis of a projectile. If the unit vector $\underset{\rightarrow}{x}$ is in motion, its rate of change, $\dot{\underset{\rightarrow}{x}}$, is in the plane of motion, and is perpendicular to $\underset{\rightarrow}{x}$. If $|\dot{\underset{\rightarrow}{x}}|$ represents the scalar magnitude of $\dot{\underset{\rightarrow}{x}}$, the magnitude of the pitching force is $\rho d^3 K_S v |\dot{\underset{\rightarrow}{x}}|$, where K_S is the dimensionless pitching force coefficient. The direction of the pitching force is parallel to $\dot{\underset{\rightarrow}{x}}$ and oppositely sensed, as is illustrated in Figure 9. The vector representation of the pitching force follows:

$$(2.12)\quad \text{PITCHING FORCE} = -\rho d^3 K_S v \dot{\underset{\rightarrow}{x}}$$

Replacing $\dot{\underset{\rightarrow}{x}}$ with its equivalent from Equation (1.4):

$$(2.13)\quad \text{PITCHING FORCE} = -\rho d^3 K_S v (\underset{\rightarrow}{h} \times \underset{\rightarrow}{x})$$

DAMPING MOMENT

The damping moment is a moment opposing angular velocity of the longitudinal axis of a projectile, and is the aerodynamic moment associated with pitching force. If the unit vector $\underset{\rightarrow}{x}$ is in motion, its vector rate of change is $\dot{\underset{\rightarrow}{x}}$, and the transverse angular velocity vector is $(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$. The magnitude of the damping moment is $\rho d^4 K_H v |\dot{\underset{\rightarrow}{x}}|$, where K_H is the dimensionless damping moment coefficient. The direction of the damping moment is parallel to $(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$ and oppositely sensed, as is illustrated in Figure 10. The damping moment is represented by the vector:

$$(2.14)\quad \text{DAMPING MOMENT} = -\rho d^4 K_H v (\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$$

Replacing $\underset{\rightarrow}{\dot{x}}$ with its vector equivalent, and expanding the resulting triple vector product:

$$(2.15)\quad \text{DAMPING MOMENT} = -\rho d^4 K_H v\, [\underset{\rightarrow}{h} - (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\, \underset{\rightarrow}{x}]$$

MAGNUS CROSS FORCE

The Magnus cross force is a magnus force arising from a transverse angular velocity of the shell axis. The magnitude of the Magnus cross force is $\rho d^4 K_{XF} N |\underset{\rightarrow}{\dot{x}}|$, where K_{XF} is the dimensionless Magnus cross force coefficient. The direction of the Magnus cross force is parallel to the vector $(\underset{\rightarrow}{x} \times \underset{\rightarrow}{\dot{x}})$, and has the same sense for positive values of N, as shown in Figure 11. The Magnus cross force is represented by the vector:

$$(2.16)\quad \text{MAGNUS CROSS FORCE} = \rho d^4 K_{XF} N (\underset{\rightarrow}{x} \times \underset{\rightarrow}{\dot{x}})$$

Replacing N and $\underset{\rightarrow}{\dot{x}}$ with their respective vector equivalents, and expanding the resulting triple vector product:

$$(2.17)\quad \text{MAGNUS CROSS FORCE} = \frac{\rho d^4 B}{A} K_{XF} (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\, [\underset{\rightarrow}{h} - (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\, \underset{\rightarrow}{x}]$$

MAGNUS CROSS MOMENT

The Magnus cross moment is the aerodynamic moment associated with the Magnus cross force. The magnitude of the Magnus cross moment is $\rho d^5 K_{XT} N |\underset{\rightarrow}{\dot{x}}|$, where K_{XT} is the dimensionless Magnus cross moment coefficient. Since the direction of the Magnus cross moment is parallel to $\underset{\rightarrow}{\dot{x}}$ and oppositely sensed, as is shown in Figure 12, the Magnus cross moment is represented by the vector:

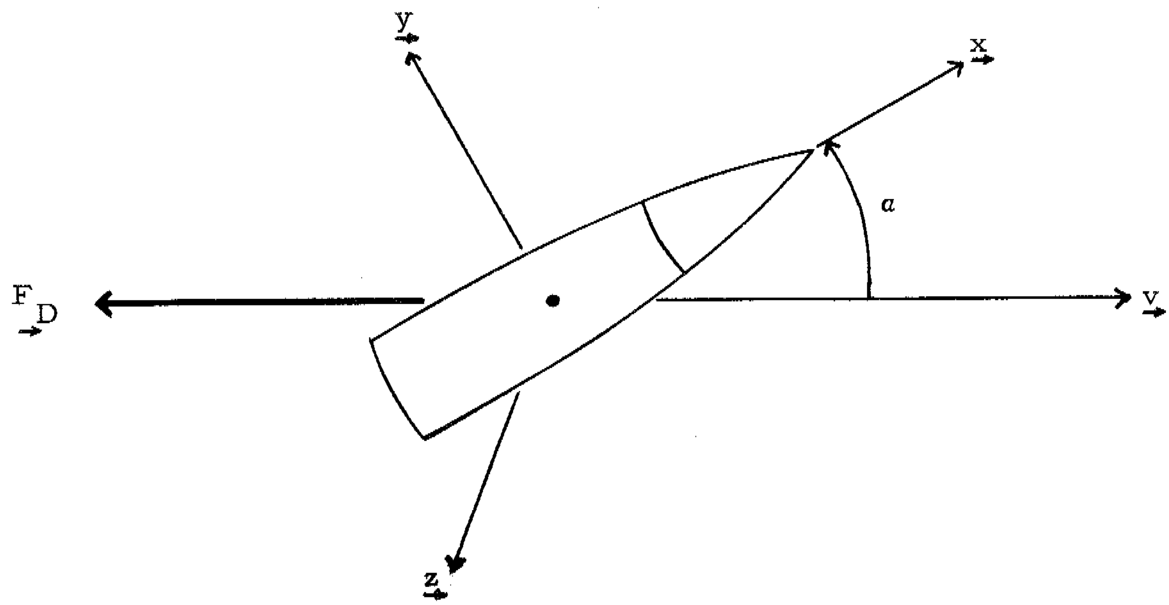
$$(2.18)\quad \text{MAGNUS CROSS MOMENT} = -\rho d^5 K_{XT} N \underset{\rightarrow}{\dot{x}}$$

Replacing N and $\dot{\underset{\rightarrow}{x}}$ with their respective vector equivalents:

$$(2.19) \quad \text{MAGNUS CROSS MOMENT} = -\frac{\rho d^5 B}{A} K_{XT} (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x}) (\underset{\rightarrow}{h} \times \underset{\rightarrow}{x})$$

The dimensionless aerodynamic coefficients are functions of many dimensionless power products, including the dimensionless shape parameters, Reynolds number and Mach number. Aerodynamic coefficients are defined with reference to a specific set of shape parameters, and may be expressed as functions of Mach number. Additional aerodynamic coefficients may be included to account for the variation of aerodynamic forces and moments with yaw.

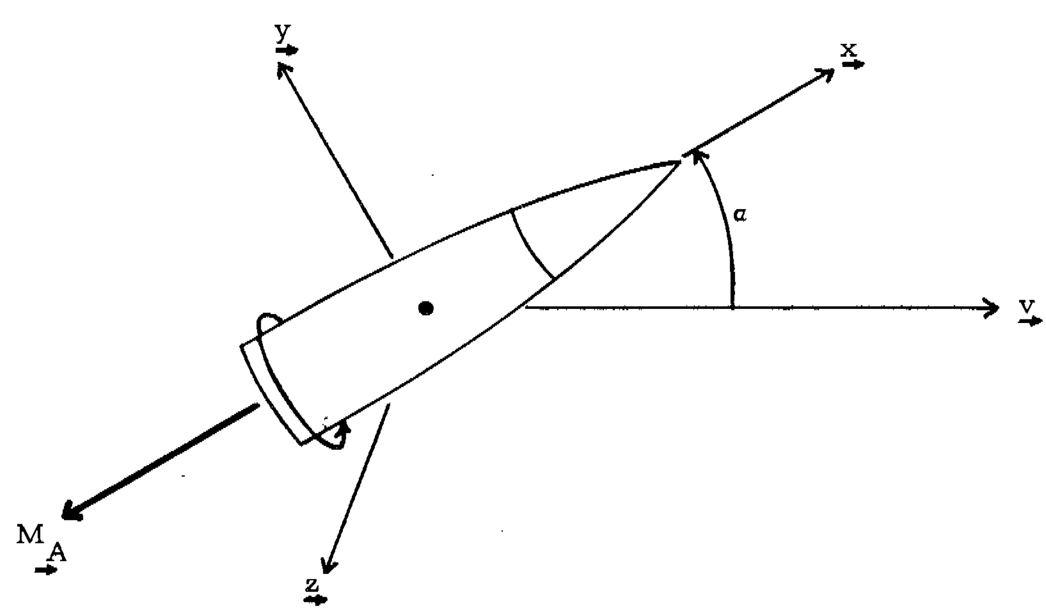
DRAG FORCE



$$\text{DRAG FORCE} = \underset{\rightarrow}{F_D} = -\rho d^2 (K_{D_o} + K_{D_a} a^2)\, v\, \underset{\rightarrow}{v}$$

Figure 2.

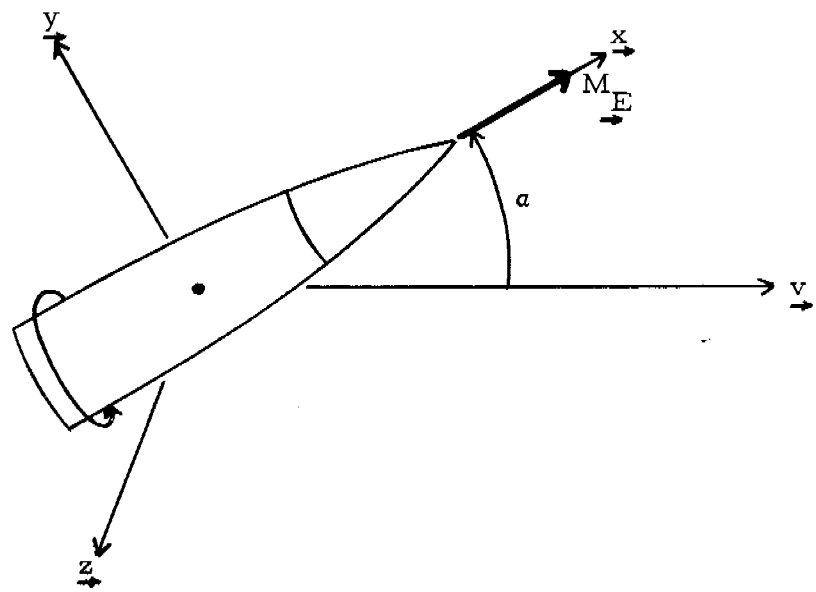
SPIN DAMPING MOMENT



$$\text{SPIN DAMPING MOMENT} = \underset{\rightarrow}{M_A} = -\rho d^4 K_A N v \underset{\rightarrow}{x}$$

Figure 3.

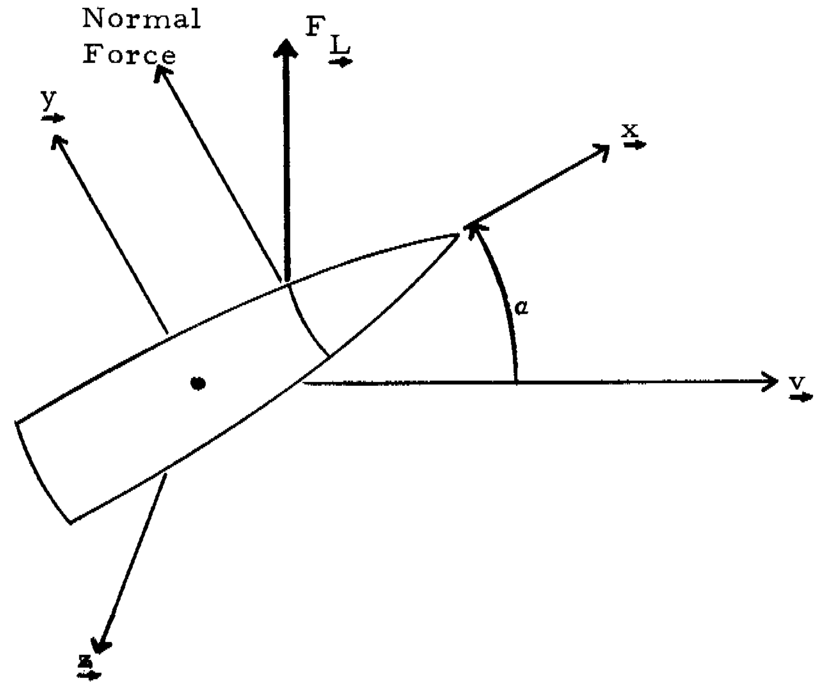
FIN CANT MOMENT



$$\text{FIN CANT MOMENT} = \vec{M}_E = \rho d^3 K_E \epsilon v^2 \vec{x}$$

Figure 4.

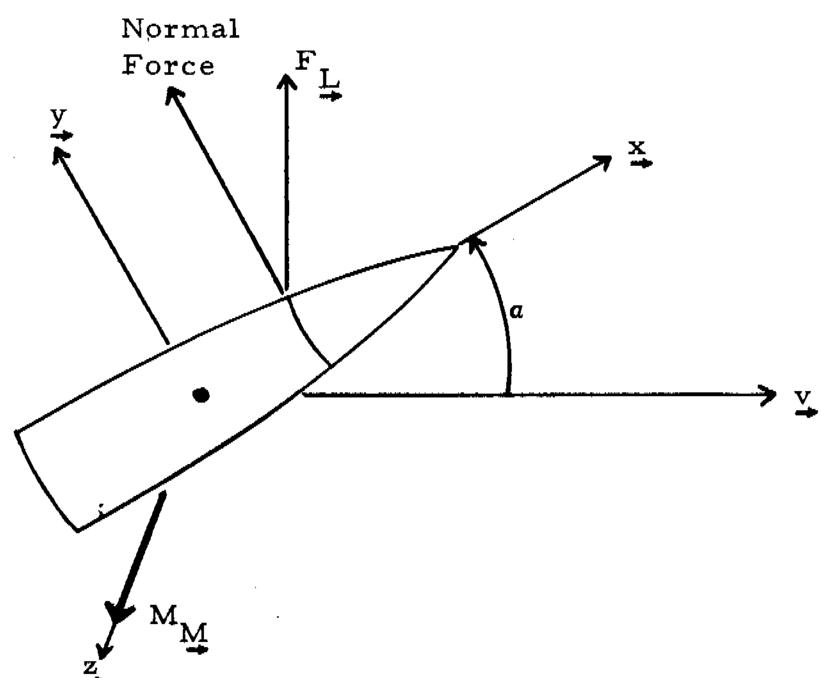
LIFT FORCE



$$\text{LIFT FORCE} = \vec{F}_L = \rho d^2 K_L [\vec{v} \times (\vec{x} \times \vec{v})]$$

Figure 5.

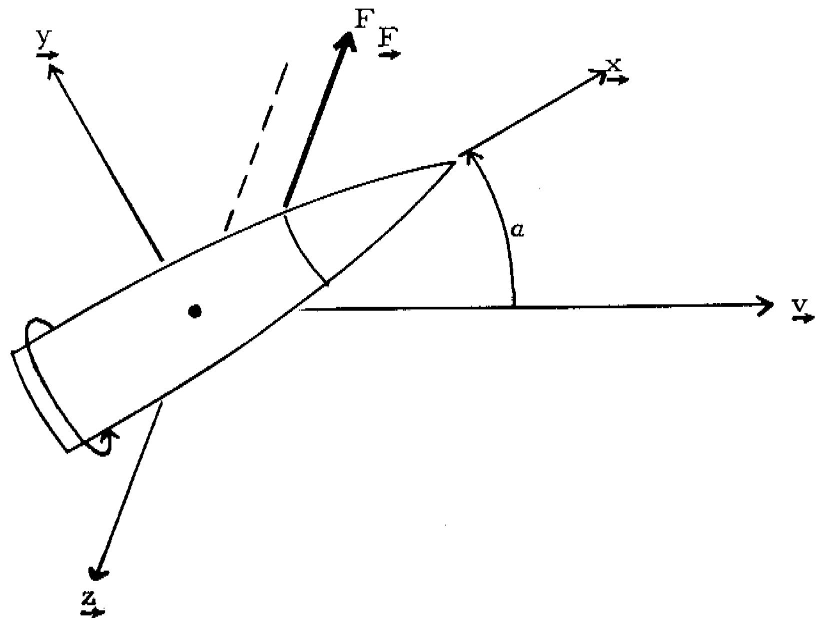
OVERTURNING MOMENT



$$\text{OVERTURNING MOMENT} = \vec{M}_M = \rho d^3 K_M v (\vec{v} \times \vec{x})$$

Figure 6.

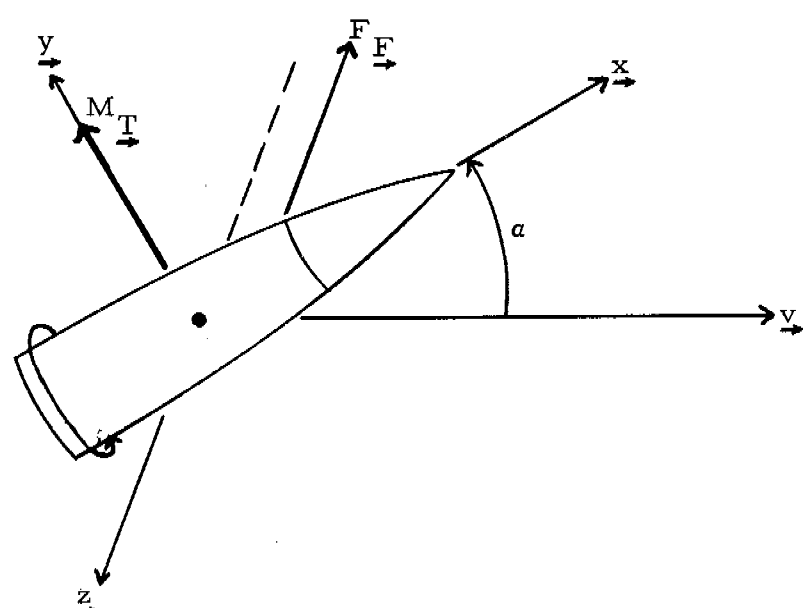
MAGNUS FORCE



$$\text{MAGNUS FORCE} = \underset{\to}{F_F} = \rho d^3 K_F N (\underset{\to}{x} \times \underset{\to}{v})$$

Figure 7.

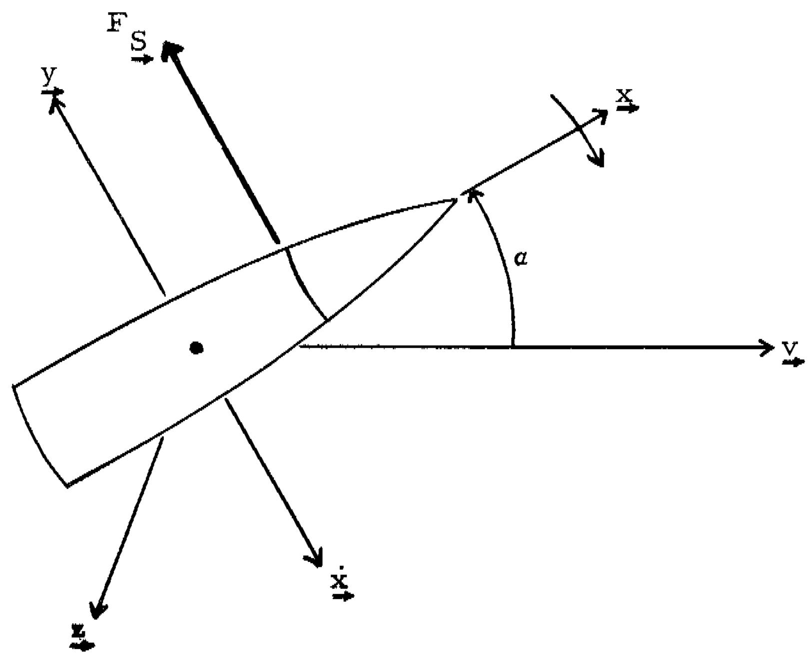
MAGNUS MOMENT



$$\text{MAGNUS MOMENT} = \underset{\to}{M_T} = \rho d^4 K_T N [\underset{\to}{x} \times (\underset{\to}{x} \times \underset{\to}{v})]$$

Figure 8.

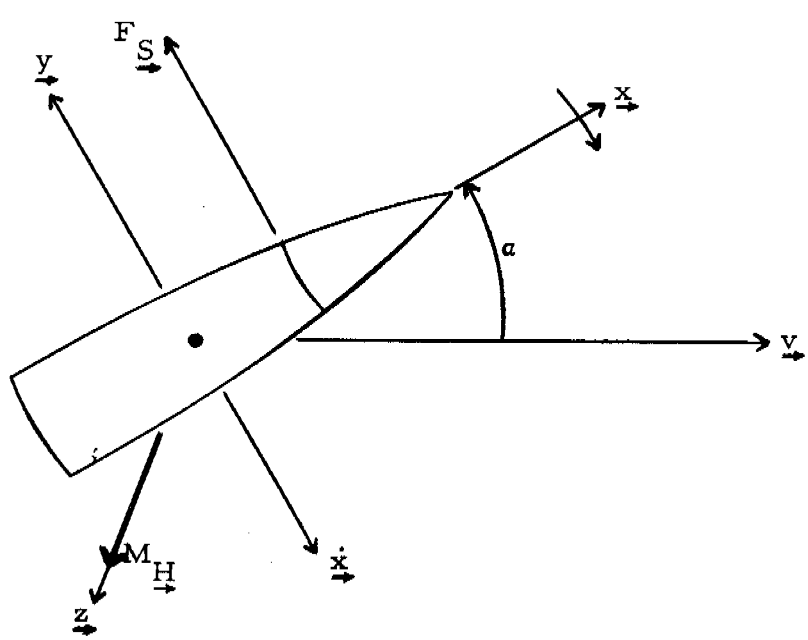
PITCHING FORCE



PITCHING FORCE $= \underset{\rightarrow}{F_S} = - \rho d^3 K_S v \underset{\rightarrow}{\dot{x}}$

Figure 9.

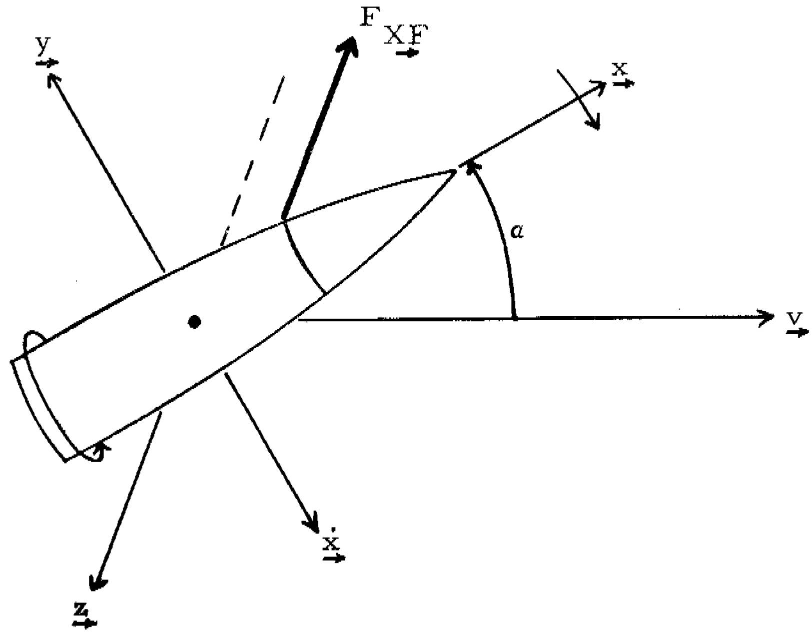
DAMPING MOMENT



DAMPING MOMENT $= \underset{\rightarrow}{M_H} = - \rho d^4 K_H v (\underset{\rightarrow}{x} \times \underset{\rightarrow}{\dot{x}})$

Figure 10.

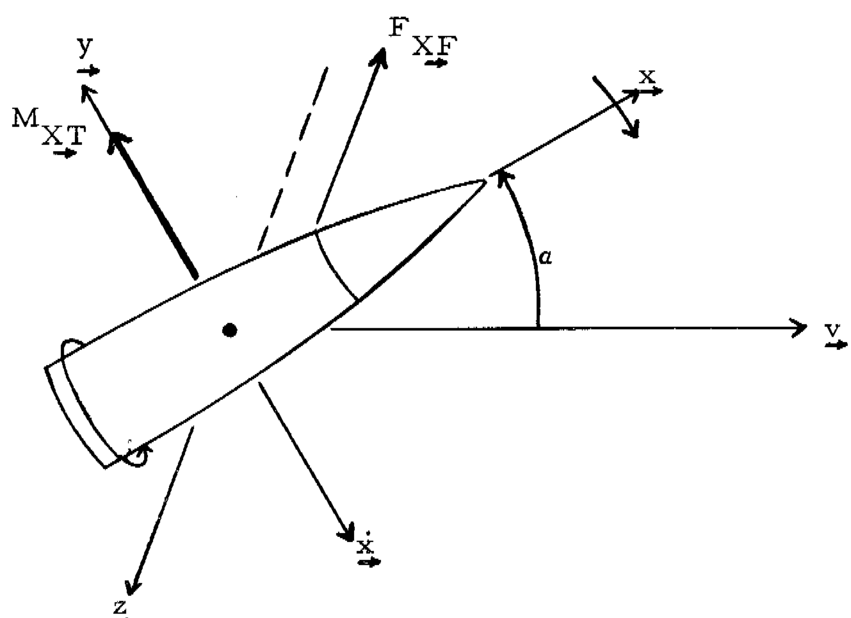
MAGNUS CROSS FORCE



MAGNUS CROSS FORCE $= \underset{\rightarrow}{F_{XF}} = \rho d^4 K_{XF} N (\underset{\rightarrow}{x} \times \underset{\rightarrow}{\dot{x}})$

Figure 11.

MAGNUS CROSS MOMENT



MAGNUS CROSS MOMENT $= \underset{\rightarrow}{M_{XT}} = - \rho d^5 K_{XT} N \underset{\rightarrow}{\dot{x}}$

Figure 12.

GRAVITY AND ROTATIONAL ACCELERATIONS

The acceleration due to gravity is caused by the force of gravitational attraction between a body and the earth. The magnitude of gravitational force is proportional to the mass of the body, and inversely proportional to the squared distance between the centers of mass of the body and of the earth. The line of action is directed from the mass center of the body to the center of mass of the earth.

It is assumed that a spherical earth is a sufficiently good approximation for the purpose of trajectory simulation. The position of the missile center of mass is specified by the vector $\underset{\rightarrow}{X}$ in the ground fixed coordinate system. The force of gravity is approximated by the vector:

$$(3.1) \qquad \underset{\rightarrow}{g} = -g_o \frac{R^2}{r^3} \begin{bmatrix} X_1 \\ (X_2 + R) \\ X_3 \end{bmatrix}$$

$$r = [X_1^{\,2} + (X_2 + R)^2 + X_3^{\,2}]^{1/2}$$

where g_o = value of gravitional acceleration at point of launch

R = radius of the earth at point of launch

r = distance between center of earth and body

X_1, X_2, X_3 = components of $\underset{\rightarrow}{X}$ in the ground fixed coordinate system.

In the derivation of the force-moment system, all vectors have been referenced to the ground fixed coordinate system. Since the earth is spinning about an axis passing through its center of mass, the acceleration produced by rotation of the earth must be added to the force equation. The acceleration due to rotation of the earth is represented by the vector:

$$(3.2) \qquad \underset{\rightarrow}{\Lambda} = \begin{bmatrix} -\lambda_1 u_2 - \lambda_2 u_3 \\ \lambda_1 u_1 + \lambda_3 u_3 \\ \lambda_2 u_1 - \lambda_3 u_2 \end{bmatrix}$$

The λ's are defined by the following equations: *

$$(3.3) \qquad \begin{aligned} \lambda_1 &= 2\,\Omega \cos L \sin A_z \\ \lambda_2 &= 2\,\Omega \sin L \\ \lambda_3 &= 2\,\Omega \cos L \cos A_z \end{aligned}$$

where Ω = Angular velocity of the earth (radians/sec)

L = Latitude of launch point

A_z = Azimuth of fire measured clockwise from North

u_1, u_2, u_3 = Components of $\underset{\rightarrow}{u}$, the vector velocity of the missile with respect to ground

* The λ's given are for the northern hemisphere. For the southern hemisphere replace L by (-L).

JET FORCES AND MOMENTS

The simulation of free flight rocket trajectories necessitates the inclusion in the simulator of the forces and moments produced by the rocket motor. The rocket is assumed to be a rigid body, and all jet forces are assumed to originate at the nozzle.

THRUST

The thrust of the rocket motor is a vector assumed rigidly attached to the rocket, and is allowed to vary in magnitude as a function of time.

Consider two thrust-time functions as illustrated in Fig. 13 below.

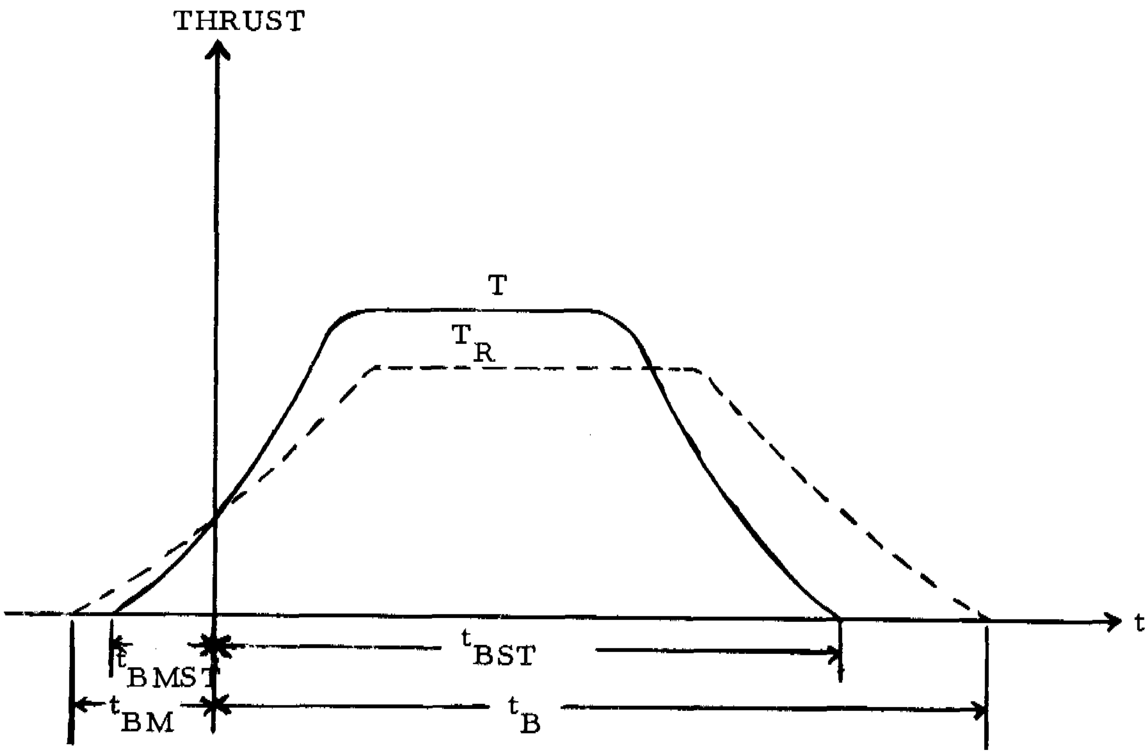


Figure 13.

The relevant symbols are defined below:

A_e = Area of jet exit

F[ ] indicates "a function of"

I_{SP} = Specific impulse per unit of fuel mass

I_{ST} = Total impulse at standard conditions

m_f = Mass of fuel

P_a = Static atmospheric pressure

P_e = Jet pressure at nozzle exit

T* = Effective thrust

t_B = Motor burnout time

t_{BM} = Time of ignition

t_{BMST} = Time of ignition at standard conditions

t_{BST} = Motor burnout time at standard conditions

T_F = Thrust factor

$T_R(t)$ = Thrust produced by motor at time t

T = Thrust

Let:

$$(4.1) \quad T = F\left[(t - t_{BM})\left(\frac{t_{BST} - t_{BMST}}{t_B - t_{BM}}\right) + t_{BMST}\right]$$

The thrust of the rocket motor as a function of time may be linearly transformed, maintaining the same total impulse, by the following equation:

$$(4.2) \quad T_R = T_F m_f \left(\frac{I_{SP}}{I_{ST}}\right)\left(\frac{t_{BST} - t_{BMST}}{t_B - t_{BM}}\right) \quad T$$

The force due to jet pressure at the rocket nozzle is given by $P_e A_e$. The static atmospheric pressure produces a force equal to $-P_a A_e$. The total thrust contributed by pressure is then given by $(P_e - P_a) A_e$.

The effective thrust of the rocket motor is the algebraic sum of the components considered, and is given by:

$$(4.3) \qquad T^* = T_F m_f \left(\frac{I_{SP}}{I_{ST}} \right) \left(\frac{t_{BST} - t_{BMST}}{t_B - t_{BM}} \right) T \quad + \quad (P_e - P_a) A_e$$

The velocity-time history of a rocket during the burning period is reproduced in the simulator by linearly transforming the standard thrust curve in time, and by iteratively determining the value of T_F required to match the observed burnout velocity.

Figure 14 is a diagram of the thrust vector in the missile fixed coordinate system.

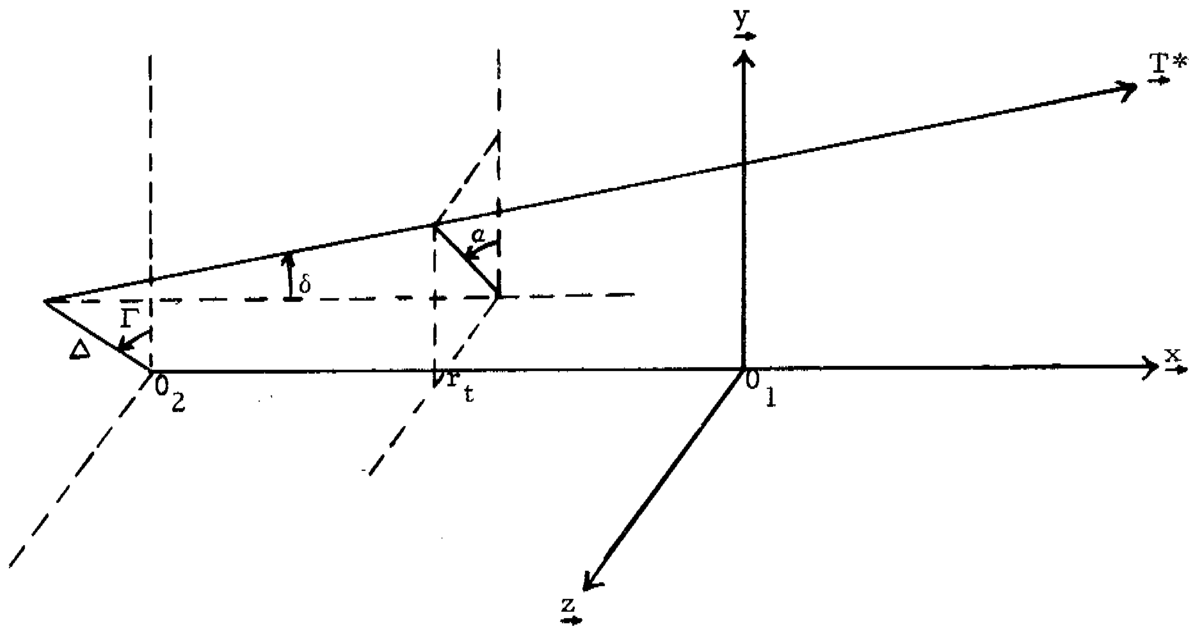


Figure 14.

The point 0_1 represents the position of the center of mass, and 0_2 represents the position of the throat of the rocket nozzle. The distance from 0_1 to 0_2 is r_t, and is defined as a positive number.

Two types of rocket thrust malalignments are considered in this report. The first is historically defined as linear malalignment, in which the nozzle is displaced from the axis of symmetry of the rocket. The amount of displacement is denoted by Δ, and the direction by an angle Γ, measured clockwise from the plane containing $\vec{x}$ and $\vec{y}$.

The second type is defined as angular malalignment, in which the direction of effective thrust forms a fixed angle with the axis of symmetry. This direction in the missile fixed coordinate system is specified by two angles; the magnitude of angular malalignment, denoted by δ, and the direction angle γ, measured clockwise from the plane containing $\underset{\rightarrow}{x}$ and $\underset{\rightarrow}{y}$.

The vector thrust force can be resolved into components along the missile fixed coordinate system, and the following expression results:

(4. 4) $\text{THRUST FORCE} = g_c\ T^* (\cos\delta\ \underset{\rightarrow}{x} + \cos\gamma\ \sin\delta\ \underset{\rightarrow}{y} + \sin\gamma\ \sin\delta\ \underset{\rightarrow}{z})$

Consider a vector normal to the thrust vector, passing through 0_1, and directed from 0_1 toward the vector thrust. For small values of the angle δ, this vector is approximated by:

$$[(\Delta \cos\Gamma + r_t \cos\gamma \sin\delta)\ \underset{\rightarrow}{y} + (\Delta \sin\Gamma + r_t \sin\gamma \sin\delta)\ \underset{\rightarrow}{z}]$$

The vector thrust moment is the cross product of this vector onto the vector thrust force. If the indicated expansion is performed, the following equation results:

(4. 5) $\text{THRUST MOMENT} = g_c\ T^*\ \{[(\Delta\cos\Gamma + r_t\cos\gamma\sin\delta)\sin\gamma$
$- (\Delta\sin\Gamma + r_t\sin\gamma\sin\delta)\cos\gamma]\ \sin\delta\ \underset{\rightarrow}{x} + (\Delta\sin\Gamma + r_t\sin\gamma\sin\delta)\cos\delta\ \underset{\rightarrow}{y}$
$- (\Delta\cos\Gamma + r_t\cos\gamma\sin\delta)\cos\delta\ \underset{\rightarrow}{z}\}$

Equations (4. 3), (4. 4) and (4. 5) represent the total vector force and moment due to the thrust of the rocket motor.

JET DAMPING

Consider a burning rocket which has acquired a transverse angular velocity. In addition to a loss in mass, the rocket is also losing transverse angular momentum. It is assumed that the jet stream passes from subsonic to supersonic velocity at a point midway between the throat and the exit of the nozzle. The nozzle is then removing transverse angular momentum from the system at the rate $\dot{m}\ r_e\ r_t\ |\dot{\underset{\rightarrow}{x}}|$, where r_e is the distance from the rocket center of mass to the nozzle exit. The effect is analogous to a damping force and moment, and the terms are therefore called jet damping force and moment.

The jet damping force has the magnitude $(\frac{\dot{B}}{r_t} - \dot{m}\ r_e)\ |\dot{\underset{\rightarrow}{x}}|$, and the direction is parallel to $\dot{\underset{\rightarrow}{x}}$. The jet damping force is represented in Figure 15.

(4.6) $\text{JET DAMPING FORCE} = (\frac{\dot{B}}{r_t} - \dot{m}\ r_e)\ \dot{\underset{\rightarrow}{x}}$

Replacing $\dot{\underset{\rightarrow}{x}}$ with its equivalent from Equation (1.4):

(4.7) $\text{JET DAMPING FORCE} = (\frac{\dot{B}}{r_t} - \dot{m}\ r_e)\ (\underset{\rightarrow}{h} \times \underset{\rightarrow}{x})$

Figure 16 illustrates the jet damping moment associated with the jet damping force. The magnitude of the jet damping force is given by $(\dot{B} - \dot{m}\ r_e\ r_t)\ |\dot{\underset{\rightarrow}{x}}|$. It is parallel to the vector $(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$, and oppositely sensed.

(4.8) $\text{JET DAMPING MOMENT} = -(\dot{B} - \dot{m}\ r_e\ r_t)\ (\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$

Replacing $\dot{\underset{\rightarrow}{x}}$ with its vector equivalent, and expanding the resulting triple vector product:

(4.9) $\text{JET DAMPING MOMENT} = -(\dot{B} - \dot{m}\ r_e\ r_t)\ [\underset{\rightarrow}{h} - (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\ \underset{\rightarrow}{x}]$

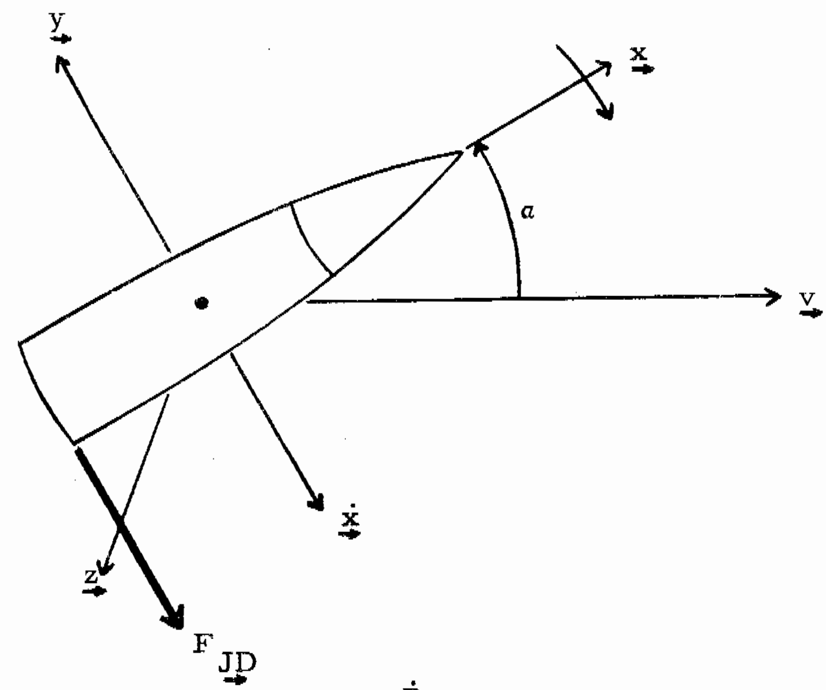
SPIN ROCKET MOMENT

Spin rockets are utilized for imparting spin to a ballistic rocket after it has left the launcher. If a symmetric bank of spin rockets is mounted perpendicular to the longitudinal axis of the rocket, the result is a pure spin generating couple. The total thrust of the spin rockets is denoted by T_s, and the radius of spin rocket action by r_s. The magnitude of the spin rocket moment is given by $g_c\, r_s\, T_s$, where g_c is a constant used in the conversion of thrust (lbf) to force (lb-ft/sec^2). The direction of the spin rocket moment is $\underrightarrow{x}$ for a positive spin (Fig. 17). The vector representation of the spin rocket moment is:

(4.10) SPIN ROCKET MOMENT = $g_c\, r_s\, T_s\, \underrightarrow{x}$

The thrust of the spin rockets is linearly transformed in time by the same method previously described for the rocket motor.

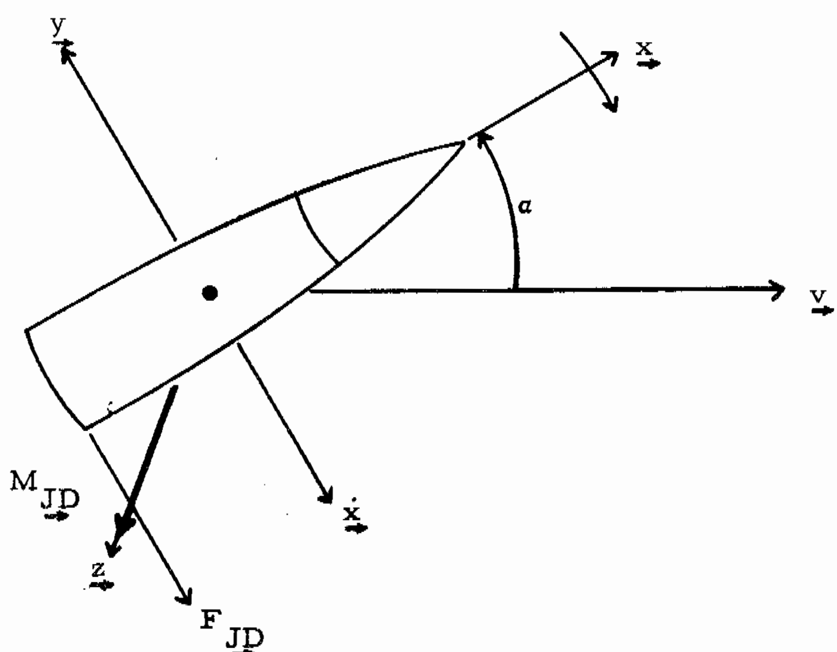
JET DAMPING FORCE



JET DAMPING FORCE = $\underset{\rightarrow}{F}_{JD} = \left(\frac{\dot{B}}{r_t} - \dot{m}\, r_e\right) \dot{\underset{\rightarrow}{x}}$

Figure 15.

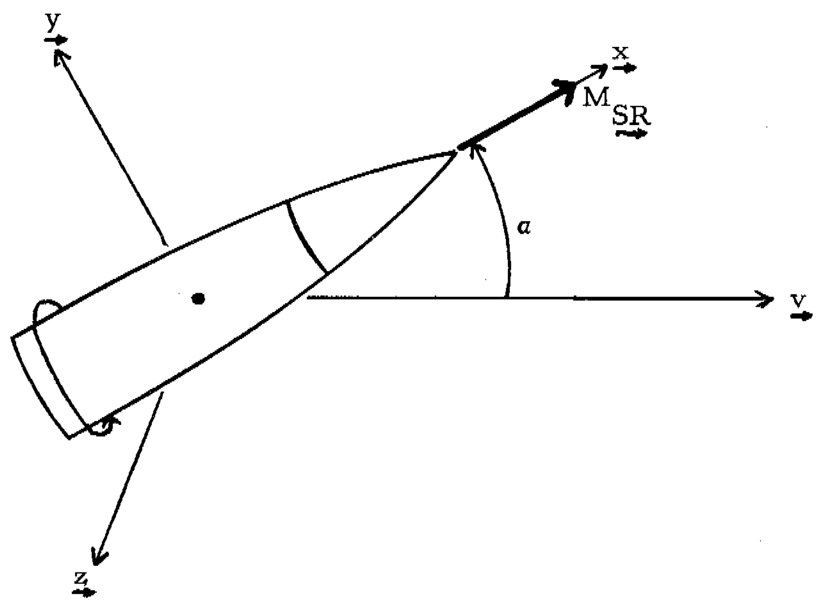
JET DAMPING MOMENT



JET DAMPING MOMENT = $\underset{\rightarrow}{M}_{JD} = -(\dot{B} - \dot{m}\, r_e\, r_t)\,(\underset{\rightarrow}{x} \times \dot{\underset{\rightarrow}{x}})$

Figure 16.

SPIN ROCKET MOMENT



SPIN ROCKET MOMENT = $M_{\underset{\rightarrow}{SR}} = g_c \, r_s \, T_s \, \underset{\rightarrow}{x}$

Figure 17.

EQUATIONS OF MOTION

In the previous four sections, the forces and moments acting on a shell or rocket have been derived. If these forces and moments are summed and substituted in equations (1. 6) and (1. 3), the following vector differential equations of motion result:

$$(5.1) \quad \dot{\underset{\rightarrow}{u}} = -\frac{\rho d^2}{m} (K_{D_o} + K_{D_\alpha} \alpha^2) v \underset{\rightarrow}{v} + \frac{\rho d^2}{m} K_L [v^2 \underset{\rightarrow}{x} - (\underset{\rightarrow}{v} \cdot \underset{\rightarrow}{x}) \underset{\rightarrow}{v}]$$

$$- \frac{\rho d^3}{m} K_s v (\underset{\rightarrow}{h} \times \underset{\rightarrow}{x}) + \frac{\rho d^3}{m} \frac{B}{A} K_F (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x}) (\underset{\rightarrow}{x} \times \underset{\rightarrow}{v})$$

$$+ \frac{\rho d^4}{m} \frac{B}{A} K_{XF} (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x}) [\underset{\rightarrow}{h} - (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x}) \underset{\rightarrow}{x}] + \underset{\rightarrow}{g}$$

$$+ \underset{\rightarrow}{\Lambda} + \frac{g_c T^*}{m} (\cos \delta \underset{\rightarrow}{x} + \cos \gamma \sin \delta \underset{\rightarrow}{y} + \sin \gamma \sin \delta \underset{\rightarrow}{z})$$

$$+ \frac{1}{m} (\frac{\dot{B}}{r_t} - \dot{m} r_e) (\underset{\rightarrow}{h} \times \underset{\rightarrow}{x})$$

$$(5.2)\qquad \dot{\underline{h}} = \frac{\rho d^3}{B} K_M v (\underline{v} \times \underline{x}) - \frac{\rho d^4}{B} K_H v [\underline{h} - (\underline{h} \cdot \underline{x}) \underline{x}]$$

$$- \frac{\rho d^4}{A} K_A v (\underline{h} \cdot \underline{x}) \underline{x} + \frac{\rho d^3}{B} K_E \epsilon v^2 \underline{x}$$

$$+ \frac{\rho d^4}{A} K_T (\underline{h} \cdot \underline{x}) [(\underline{v} \cdot \underline{x}) \underline{x} - \underline{v}]$$

$$- \frac{\rho d^5}{A} K_{XT} (\underline{h} \cdot \underline{x}) (\underline{h} \times \underline{x}) + \frac{g_c r_s T_s}{B} \underline{x}$$

$$+ \frac{g_c T^*}{B} \{[(\Delta \cos \Gamma + r_t \cos \gamma \sin \delta) \sin \gamma$$

$$- (\Delta \sin \Gamma + r_t \sin \gamma \sin \delta) \cos \gamma] \sin \delta \underline{x}$$

$$+ (\Delta \sin \Gamma + r_t \sin \gamma \sin \delta) \cos \delta \underline{y} - (\Delta \cos \Gamma$$

$$+ r_t \cos \gamma \sin \delta) \cos \delta \underline{z}\}$$

$$- \left(\frac{\dot{B} - \dot{m} r_e r_t}{B}\right) [\underline{h} - (\underline{h} \cdot \underline{x}) \underline{x}]$$

In addition to these two basic differential equations, the equations relating the $(\underline{x}, \underline{y}, \underline{z})$ coordinate system and the ground fixed coordinate system are needed. Equation (1.4) gives $\dot{\underline{x}} = (\underline{h} \times \underline{x})$. The rates of change of $\underline{y}$ and $\underline{z}$ in the ground fixed system are given by:

$$(5.3)\qquad \dot{\underset{\rightarrow}{y}} = (\underset{\rightarrow}{h} \times \underset{\rightarrow}{y}) + \left(\frac{B - A}{A}\right) (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\, \underset{\rightarrow}{z}$$

$$(5.4)\qquad \dot{\underset{\rightarrow}{z}} = (\underset{\rightarrow}{h} \times \underset{\rightarrow}{z}) - \left(\frac{B - A}{A}\right) (\underset{\rightarrow}{h} \cdot \underset{\rightarrow}{x})\, \underset{\rightarrow}{y}$$

The position of the missile center of mass in the ground fixed coordinate system is represented by the vector $\underset{\rightarrow}{X}$, and is given by the following integral equation:

$$(5.5)\qquad \underset{\rightarrow}{X} = \int_o^t \underset{\rightarrow}{u}\, dt$$

The missile center of mass position with respect to the spherical earth surface is denoted by $\underset{\rightarrow}{E}$. The vector $\underset{\rightarrow}{E}$ is related to the radius of the earth, R, and to $\underset{\rightarrow}{X}$ by the following:

$$(5.6)\qquad \underset{\rightarrow}{E} = \begin{bmatrix} R \left[\operatorname{Arctan} \left(\dfrac{\sqrt{X_1^{\,2} + X_3^{\,2}}}{R + X_2} \right) \right] \dfrac{X_1}{\sqrt{X_1^{\,2} + X_3^{\,2}}} \\ \hline \sqrt{(R + X_2)^2 + (X_1^{\,2} + X_3^{\,2})} - R \\ \hline R \left[\operatorname{Arctan} \left(\dfrac{\sqrt{X_1^{\,2} + X_3^{\,2}}}{R + X_2} \right) \right] \dfrac{X_3}{\sqrt{X_1^{\,2} + X_3^{\,2}}} \end{bmatrix}$$

The physical properties consisting of mass, center of mass, and moments of inertia must be considered functions of time for rocket trajectory simulation. The rate of change of mass with respect to time is given at any instant by the ratio of motor thrust to motor specific impulse. The change in center of mass and axial moment of inertia are represented by linear functions of time, and the transverse moment of inertia is found by applying the parallel axis theorem to the changing mass. The following equations and definitions summarize the method used to account for time variation of the rocket's physical properties.

$$\dot{m}(t) = -\frac{T_R(t)}{I_{SP}} \tag{5.7}$$

$$\dot{A}(t) = \dot{m}(t) \left[\frac{A(t_{BM}) - A(t_B)}{m(t_{BM}) - m(t_B)}\right] \tag{5.8}$$

$$B(t) = B_{(t_B)} + m_f(t)\, r_2^{\,2} + m_{(t_B)}\, r_1^{\,2} - m(t)\,[r_1 - r_3(t)]^2 \tag{5.9}$$

$$\dot{B}(t) = \dot{m}(t)\ \{r_2^{\,2} - [r_1 - r_3(t)]^2\} + 2\, m(t)\,[r_1 - r_3(t)]\ \dot{r}_3(t) \tag{5.10}$$

$$r_3(t) = \left[\frac{X_{CG}(t_{BM}) - X_{CG}(t_B)}{m(t_{BM}) - m(t_B)}\right] [m(t) - m(t_B)] \tag{5.11}$$

$$\dot{r}_3(t) = \left[\frac{X_{CG}(t_{BM}) - X_{CG}(t_B)}{m(t_{BM}) - m(t_B)}\right] \dot{m}(t) \tag{5.12}$$

$$m_f(t)\, r_1 = m(t)\, r_3(t) \tag{5.13}$$

Where r_1 = Distance from CG (t_B) to CG of fuel

r_2 = Radius of gyration of fuel mass

$r_3(t)$ = Distance of CG shift due to fuel mass change at time t

$X_{CG}(t)$ = Distance from nose of missile to center of mass at time t

YAW AND ORIENTATION OF YAW

The yaw of a projectile, denoted by α, is the included angle between the vectors $\vec{x}$ and $\vec{v}$, and is always defined as a positive quantity. Since the dot product of two vectors equals the product of their magnitudes multiplied by the cosine of their included angle, the magnitude of yaw is given by:

$$\alpha = \mathrm{Cos}^{-1}\left[\frac{(\vec{v} \cdot \vec{x})}{|\vec{v}|}\right] \tag{5.14}$$

The orientation of yaw is the angle between the plane containing both $\vec{v}$ and $\vec{x}$ and a vertical plane containing $\vec{v}$, and is measured clockwise from the vertical plane. Yaw orientation is denoted by ψ, and is given by the following expression:

$$\psi = \mathrm{Sin}^{-1}\left[\frac{x_3\,(v^2 - v_3^{\,2})^{1/2} - v_3\,(1 - x_3^{\,2})^{1/2}}{v \sin \alpha}\right] \tag{5.15}$$

Φ

ψ becomes indeterminate when α is zero in Equation (5.15). If ψ is defined as zero when α equals zero, Equation (5.15) becomes continuous for all values of α.

Robert F. Lieske

Robert F. Lieske

Robert L. McCoy

Robert L. McCoy

References

1. R. H. Fowler, E. G. Gallop, C. N. H. Lock, H. W. Richmond, The Aerodynamics of a Spinning Shell, Phil. Trans. Roy. Soc. London (A) 221, 295-387 (1920).

2. K. L. Nielsen, J. L. Synge, On the Motion of a Spinning Shell, Q. A. M. Vol. IV, No. 3 (1946).

3. J. B. Rosser, R. R. Newton, G. L. Gross, Mathematical Theory of Rocket Flight, New York and London, McGraw-Hill Book Company, Inc., 1947.

4. E. J. McShane, J. L. Kelley, and F. Reno, Exterior Ballistics, University of Denver Press, 1953.

5. R. H. Kent, Notes on a Theory of Spinning Shell, Ballistic Research Laboratories Report 898, 1954.

6. L. Davis, J. W. Follin, L. Blitzer, Exterior Ballistics of Rockets, Princeton, New Jersey, D. Van Nostrand Company, Inc., 1958.

APPENDIX

INITIAL CONDITIONS.

The launching phase of a burning rocket is characterized by a period of constrained motion prior to launching into free flight. It is assumed that both the rocket and the launcher are rigid structures. It is further assumed that the rocket can acquire no angular momentum while on the launcher, and that the center of mass is constrained to move in a straight line on the launching rail.

The forces permitted to act on the rocket during launching are thrust, drag, gravity, and rocket-launcher friction. If ϕ is the angle of elevation of the launcher, and F is the deceleration due to rocket-launcher friction, the following equations describe the motion of the rocket center of mass during the launching phase:

$$\text{(A. 1)} \qquad \dot{\underset{\to}{u}} = \begin{bmatrix} \frac{g_c T^*}{m} \cos\phi - \frac{\rho d^2}{m} K_D v^2 \cos\phi \\ - g_o \sin\phi \cos\phi - F \cos^2\phi \\ \hline \frac{g_c T^*}{m} \sin\phi - \frac{\rho d^2}{m} K_D v^2 \sin\phi \\ - g_o \sin\phi - F \cos\phi \sin\phi \\ \hline 0 \end{bmatrix}.$$

If the projectile is launched from a gun tube, the launching phase may be omitted, and an initial velocity is assumed at end of launch.

At time t_L, the initial conditions of axial spin and transverse angular momentum are assigned. The direction cosines of the missile axis of symmetry are given by the components of $\vec{x}$. The unit vector $\vec{y}$ is defined perpendicular to $\vec{x}$, and lies initially in the vertical plane containing $\vec{x}$. The unit vector $\vec{z}$ is then given by $(\vec{x} \times \vec{y})$. The initial values of $\vec{x}$, $\vec{y}$, and $\vec{z}$ are given by the following equations:

(A. 2) $$\vec{x}(t_L) = \begin{bmatrix} \cos\phi \\ \sin\phi \\ 0 \end{bmatrix}$$

(A. 3) $$\vec{y}(t_L) = \begin{bmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{bmatrix}$$

(A. 4) $$\vec{z}(t_L) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$h_{2_1} + h_{3_1}$ are the end of launch components of $\vec{h}$ with respect to the shell fixed axis system. The initial $\vec{h}$ used for simulation must include the effects of initial angle of elevation of the trajectory and initial axial spin. The component of $\vec{h}$ due to axial spin is $\frac{AN}{B}\ \vec{x}$. The initial value of $\vec{h}$ at end of launch is given by:

(A. 5) $$\vec{h}(t_L) = \begin{bmatrix} -\frac{x_2}{x_1}\ h_{2_1} + \frac{AN}{B}\ x_1 \\ h_{2_1} + \frac{AN}{B}\ x_2 \\ h_{3_1} \end{bmatrix}$$

Values of $\underset{\rightarrow}{h}_{(t_L)}$ are chosen so as to allow the simulator to match the observed initial angular momentum of the projectile.

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EQUATIONS OF MOTION OF A RIGID PROJECTILE
Robert F. Lieske, Robert L. McCoy

BRL Report 1244 March 1964

RDT & E Project No. 1M523801A287
UNCLASSIFIED Report

Exterior ballistics-Equations of motion
Bodies of revolution-Equations of motion
Rigid body trajectory simulation.

The basic vector equations of motion of a rigid body are described, and an aerodynamic force-moment system is developed. The vector forces and moments due to gravity and rotation of the earth are added to the system. Forces and moments produced by a rocket motor are included, and the resulting vector differential equations of motion of a symmetric rigid projectile are derived.

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