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MEASUREMENT OF PHASE CONSTANT
FOR ROCK PROPAGATED SIGNALS

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MEASUREMENT OF PHASE CONSTANT FOR ROCK PROPAGATED SIGNALS

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ABSTRACT

A method is described for determining the phase constant of the rock medium below the surface of the earth. The constant is deduced from the measured phase angle of the mutual impedance between two vertically polarized antennas placed in drill holes extending into the rock.

Experiments were conducted on Cape Cod, Massachusetts, using the method to determine the phase constant of the rock between two 1000-foot drill holes, separated by a distance of 6000 feet, at frequencies between 100 and 4200 cps. Electrically short monopoles were used in the drill holes, with the overburden above the rock serving as a ground plane. Reference phase was derived from Loran C transmissions at both the transmitting and receiving ends of the path.

The measured phase constants at these frequencies indicate a large loss tangent for the medium. The bulk conductivity was thus deduced to be 1.1×10^{-3} mhos/meter, and the measurements did not yield an estimate of the relative dielectric constant.

I. INTRODUCTION

The rock strata below the surface of the earth is a dielectric medium which, although lossy, can support propagation of radio waves.[1] The medium is generally inaccessible except through deep drill holes in which antennas for transmitting and receiving can be inserted. Since the electrical characteristics of the rock strata vary from location to location, the complex phase constant $k = \beta - j\alpha$ of any transmission path between two antennas must be estimated either through surface measurement or by measuring the transmission loss of the path.

The transmission loss or attenuation method has been successfully used to evaluate the attenuation constant α from which the conductivity of the medium can be deduced if the loss tangent is high.[1] An equally important method of evaluating the medium characteristics is to measure the phase constant β, i.e., the transmission delay. For a very lossy medium the transmission delay (or phase shift) measurements yield only the conductivity. When the loss tangent is small, the transmission delay is a measure of the relative dielectric constant of the medium; whereas the transmission loss method does not result in an independent estimate of either conductivity or the relative dielectric constant.

The phase method described in this report is based on the measurement of the phase angle of the mutual impedance between two antennas as described in Section II. A practical, simple, and convenient way of providing reference phase through the use of Loran C transmission is shown in Section III. Section IV shows the experimental results of a test conducted on Cape Cod involving a transmission path length of 6000 feet in the granitic strata below a layer of overburden.

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II. THEORY

In conducting deep strata propagation investigations, access to the rock medium is usually through a limited number of deep vertical drill holes. Measurements of medium characteristics should then be those which use the least number of access holes. For in situ measurement of the phase constant of a propagation medium, two drill holes are required. We shall show that the phase constant of a path in a dissipative medium can be determined from the measurement of the phase angle of the mutual impedance between two antennas placed in drill holes at two ends of a path.

If a linear antenna of effective length h_1 has an input current I_1, an electric field is excited in its surrounding medium. If a receiving antenna is parallel to the transmitting antenna and both antennas have the same equatorial plane, it can be shown that for an effective receiving antenna height of h_2, the mutual impedance Z_m between the antennas is given by

$$Z_m = \frac{h_1 h_2 \omega \mu_o}{2 R^2 k} \left(jkR + 1 + \frac{1}{jkR}\right) e^{-jkR} \tag{1}$$

where

$$
\begin{aligned}
\omega &= \text{radian frequency} \\
\mu_o &= \text{permeability of free space} \\
k &= \beta(1 - j\frac{\alpha}{\beta}) \\
R &= \text{distance between antennas}
\end{aligned}
\tag{2}
$$

and α and β are respectively the attenuation and phase constants of the medium defined as

$$\beta - j\alpha = \omega \sqrt{\mu_o \varepsilon_o} \sqrt{\varepsilon_r} \sqrt{1 - jp} \tag{3}$$

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where

$$\varepsilon_r = \text{relative dielectric constant}$$

$$p = \frac{\sigma}{\omega \varepsilon_o \varepsilon_r} \tag{4}$$

The quantity p is known as the loss tangent and σ is the conductivity. MKS units are used and the medium is assumed nonmagnetic.

If we write

$$Z_m = \left| Z_m \right| e^{j\theta_m}$$

the phase angle θ_m of the mutual impedance may be obtained by substituting equation (2) in equation (1), whence

$$\theta_m = -\beta R + 2 \tan^{-1} \frac{\alpha}{\beta} + \tan^{-1} \frac{\beta R - \frac{1}{\beta R} - \frac{\alpha}{\beta} - (\beta R) \left(\frac{\alpha}{\beta}\right)^2}{1 + 2 \beta R \left(\frac{\alpha}{\beta}\right)} \tag{5}$$

θ_m is a function of ω, σ, and ε_r. We observe in equations (3) and (4) that p is a positive real quantity so that the ratio $\frac{\alpha}{\beta}$ is limited to values between 1 and 0, that is

$$0 \leq \frac{\alpha}{\beta} \leq 1$$

The ratio $\frac{\alpha}{\beta}$ will assume a value close to 1 or to 0 for only a very small range of frequency. We thus may simplify equation (5) by considering βR as an independent variable. Figure 1 shows the relation between θ_m and βR with $\frac{\alpha}{\beta}$ as a parameter.

It is seen in Figure 1 that for βR greater than 3 radians, θ_m is essentially independent of $\frac{\alpha}{\beta}$. Therefore, if θ_m is a measured quantity less than -100°, the phase constant of a path may be determined with very little error by assuming $\frac{\alpha}{\beta} = 1$. For lower values of βR, the actual ratio $\frac{\alpha}{\beta}$ will generally be a

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function of frequency. Furthermore, since the slopes of the θ_m vs βR curves are very small when θ_m is small, it will be quite difficult to determine βR with reasonable accuracy. Thus, the transmission delay method is primarily useful when the electrical path length is sufficiently large so that the receiving antenna is in the far field zone of the transmitting antenna.

It should be noted that in a lossy medium, when the frequency is sufficiently low, $\frac{\alpha}{\beta}$ will be nearly unity, so that θ_m approaches zero as βR approaches zero. This is an important feature because it can be used to resolve ambiguity of an integral number of cycles, which may not be otherwise apparent in measuring θ_m.

If loss tangent p is small, say $p \leq 0.6$, then

$$\beta \cong \omega\sqrt{\mu_o \varepsilon_o \varepsilon_r} \tag{6}$$

$$\alpha \cong \frac{\sigma}{2}\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_r}} \tag{7}$$

The attenuation constant α is a function of both σ and ε_r and is independent of frequency while β is only a function of ε_r and frequency. Thus, the dielectric constant of the medium can be determined only if both attenuation and phase methods are used. On the other hand, when p is large, say $p \geq 10$

$$\beta \cong \alpha \cong \sqrt{\frac{\omega\mu_o \sigma}{2}} \tag{8}$$

The phase and attenuation constants are very nearly equal and proportional to the square root of σ and frequency, but independent of ε_r. These remarks apply to a dissipative medium in which σ and ε_r are constant with frequency.

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III. INSTRUMENTATION

Common reference phase must be provided at receiving and transmitting ends of a path if the phase constant of the medium is to be determined. In coastal regions of the United States, this reference is readily available in the form of Loran C transmission. Loran C is a radio system primarily intended as a navigational aid.[2,3]

Briefly, Loran C signal is a pulse-modulated 100-kc carrier-frequency signal. The basic repetition rate is 20 cps. Modulation and carrier frequencies are phase coherent. The transmission mode of Loran C for short ranges is based on ground wave propagation. The rise time of the pulse envelope is 30 microseconds, so that by observing the RF cycles, the waveform of a pulse may be used as an accurate time marker. Being a ground wave signal, the received waveform of Loran C is delayed from the transmitted waveform by an amount proportional to the known distance between the receiving site and the Loran C station.

In our deep strata experiment, two receiving sites are involved. The time base at each site is derived from the Loran C signal. There is a finite discrepancy between the time bases corresponding to the difference in distances from these sites to the Loran C station. This discrepancy can be easily accounted for if one so wishes. The time base generated by the Loran C signal is relative, in the form of time markers at a rate of 20 pulses per second. The block diagram in Figure 2 shows the instrumentation required. A 100-kc receiver is used to amplify the Loran C signal. The output of the receiver is displayed on a monitor scope. The purpose of this scope is to generate a more convenient waveform. The scope is internally triggered, and the trigger circuit is adjusted so that the sweep of the scope starts at the desired point on the waveform of the Loran C pulses. The gate output of the scope is a 20-cps rectangular wave with its leading edge coincident in time of arrival with the phase of the Loran C pulse. If this same instrumentation is used at both test sites, there will then be a common reference phase at 20 cps.

The frequency for the propagation test is derived from the 20-cps gate output of the monitor scope. The waveform of the gate output is rich in harmonics. A suitable instrumentation at the transmitter end of the deep strata propagation path is shown in Figure 3. A wave analyzer selects the desired harmonic frequency of the 20-cps gate output waveform. The selected harmonic is available at the output of the wave analyzer (Hewlett-Packard Type 302A) for driving the transmitter. A current transformer in series with the antenna converts the current waveform into a voltage waveform which is applied to the vertical input of a measuring scope. The measuring scope is used to determine the phase of the antenna current relative to the 20-cps reference phase. For this measurement, it is necessary to use only the gate output from the monitor scope to trigger a delayed sweep on the measuring scope. The calibrated delay adjustment on the scope is used to determine the phase of the current on the transmitting antenna. This phase is denoted θ_I.

At the receiving end of the path a measuring scope is also used, as shown in Figure 4. Here, the vertical input to this scope corresponds to the voltage induced on the receiving antenna. In this case, the wave analyzer is used as a narrowband receiver. The phase of the antenna voltage is denoted by θ_V and if the phase shift of the receiver is negligible, the measuring scope will measure θ_V directly. Since the angles θ_V and θ_I measured by the scopes are in terms of phase delay, the phase angle of the mutual impedance is then

$$\theta_m = -(\theta_V - \theta_I) \tag{9}$$

This represents, of course, an idealized situation. In practice, corrections may be required to account for phase shifts in transmission lines connecting the antennas and for phase shift in the receiver. The correction can be readily measured or calculated if the impedances of the antennas and transmission lines are known.

IV. CAPE COD EXPERIMENT

The instrumentation described above for phase measurement was employed recently in tests on Cape Cod, Massachusetts. The drill holes are located on the Town Dump, Brewster and Tubman Road, Brewster. These are referred to as Brewster and Tubman drill holes, respectively. The drill holes are 6000 feet apart and are both 1000 feet deep. The granitic rock in that area is separated from the surface of the earth by a layer of overburden about 400 feet thick.

Antennas were inserted into the drill hole just below the overburden, which was used as a ground plane for the monopole antennas. The antennas used were the insulated type with short-circuit termination and low impedances.[1] The test frequencies ranged from 100 cps to 4220 cps. No correction for phase shift in the antenna systems was found necessary. The phase shift in the receiver amounted to 90° at the lowest frequency and reduced to negligible amount when frequency was increased.

The measured phase angle θ_m (in degrees), determined from equation (9), is plotted as a function of frequency in Figure 5. It is observed that the phase angle θ_m can be extrapolated to the origin ($\theta_m = 0$, $\beta R = 0$), indicating that there is no ambiguity of multiples of 360° involved. The measured phase angles, θ_m, are used to determine βR from the curve in Figure 1 for $\frac{\alpha}{\beta} = 1$. The resultant values of βR, in radians, are plotted in Figure 6 as a function of frequency. A straight line drawn through these points on log-log paper has a slope of 0.5, indicating that β is proportional to the square root of frequency as in equation (6) and that the loss tangent is large.

One may then deduce from these measurements that the average conductivity of the path between the Tubman and Brewster drill holes was 1.1×10^{-3} mho/meter. The value of σ deduced from the attenuation constant[1] was very close to this value. Such values indicate a poor medium for rock propagation.

V. DISCUSSION

A method of determining the real phase constant β of a conducting medium has been devised. The phase method is useful primarily in the far field when the phase angle of the mutual impedance between two antennas is essentially independent of the loss tangent of the medium. If loss tangent is large, the phase method has no greater utility than the simpler attenuation constant method. The attenuation method, however, must be implemented by the phase method to yield the conductivity and relative dielectric constant of the medium, if the loss tangent is small.

The application of the phase method is particularly simple in areas where reception of Loran C is good. Otherwise, more cumbersome means must be employed to provide common reference phase or time base.

One interesting aspect of the phase method is that the transmission is phase coherent with the reference, so that arbitrarily narrow bandwidths may be used in detection to enhance the signal-to-noise ratio. This not only increases the usable frequency for phase measurement, it also improves the reception for attenuation measurement.

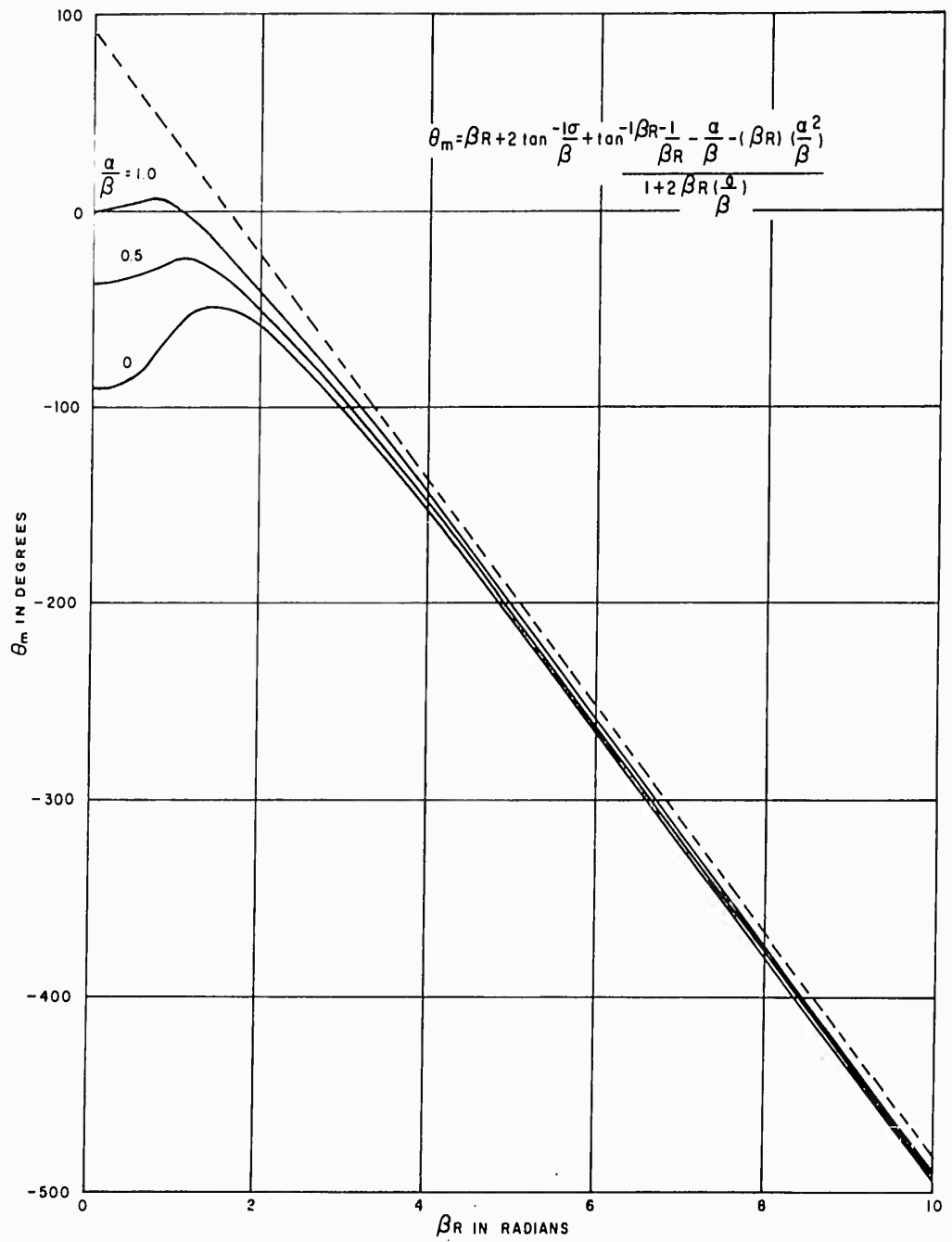## VI. ACKNOWLEDGMENT

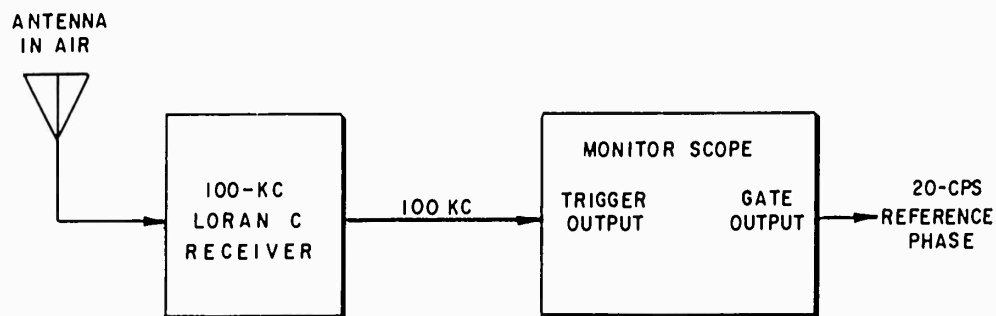Figure 1.   Phase Angle of Mutual Impedance

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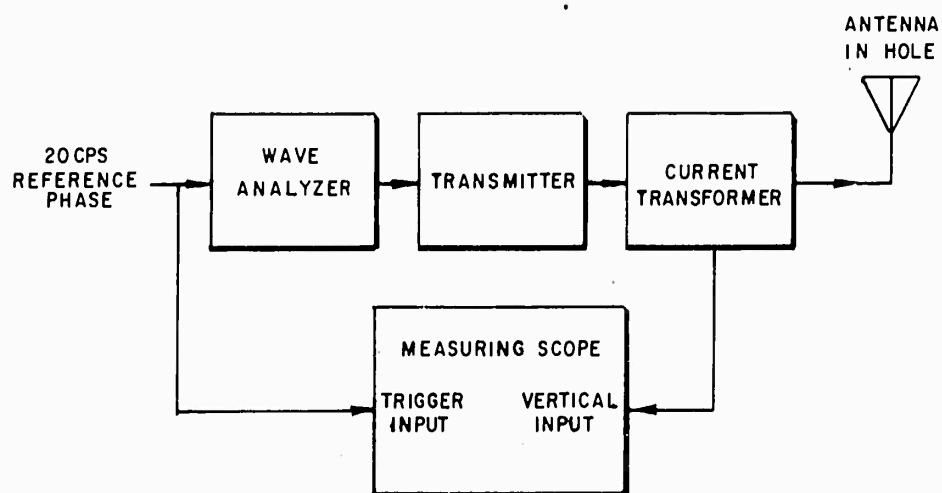Figure 2.   Generation of 20-CPS Reference Phase

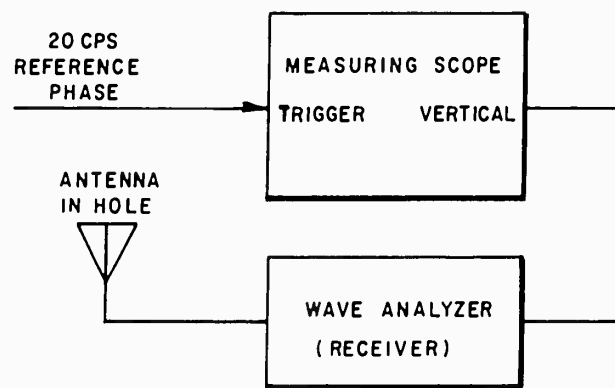Figure 3.   Block Diagram for Phase Measurement at Transmitting Site

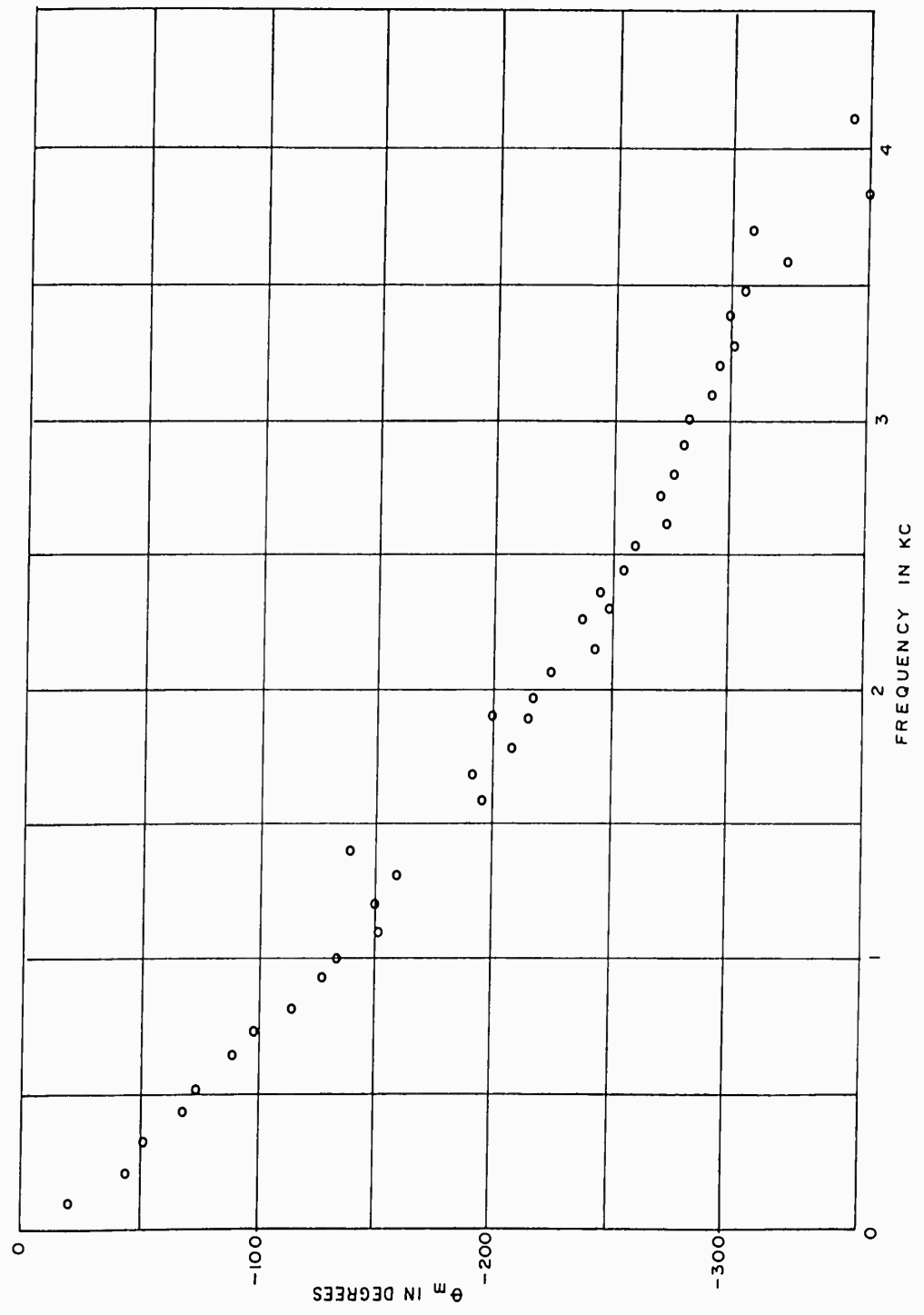Figure 4.   Block Diagram for Phase Measurement at Receiving Site

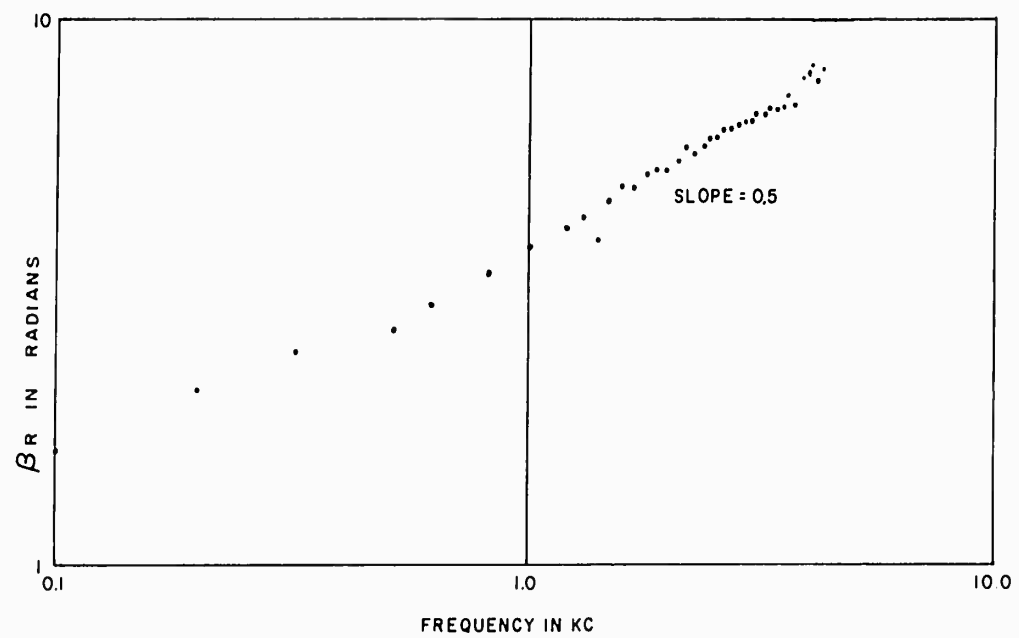Figure 5.   Measured Phase Angle of Mutual Impedance, Tubman-Brewster Path

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Figure 6.   Path Length in Radians as a Function of Frequency,
Tubman-Brewster Path

VII. REFERENCES

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