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Decay Prediction of 1963-21 Using U.S. Naval Space Surveillance System Observations

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ABSTRACT

The multiple launch of 1963-21 suffered a malfunction in the orbital injection motor that left the orbiting bodies in the transfer ellipse rather than the intended circular orbit of 500 miles. Because of the approximately 100-mile-high perigees, the periods of the several objects decayed to reentry in time intervals of some 2 to 8 weeks. Early observation suggested that decay rates were approximately inversely proportional to the ballistic coefficients. From this, it was possible to predict the reentry times of the longer lasting satellites from the behavior of the first one to reenter. Simple curve fitting was tried as a method of refining predictions.

PROBLEM STATUS

This is a final report on one phase of the problem; work on other phases continues.

AUTHORIZATION

DECAY PREDICTION OF 1963-21 USING
U.S. NAVAL SPACE SURVEILLANCE
SYSTEM OBSERVATIONS

INTRODUCTION

Satellite launch 1963-21 was a multiple operation intended to place six payloads in a circular orbit at a nominal height of 500 miles, but the orbit injection motor failed to fire. The six payloads separated on schedule but remained in the transfer orbit with perigee below 100 statute miles and with apogee of approximately 500 miles. When the observed orbital periods were plotted as functions of elapsed time since launch, it was obvious that the lifetimes would be short. Of particular interest to NRL was 1963-21B, Lofti-2. This payload was equipped with a very low frequency receiver and a 10-foot antenna which could be extended to 40 feet by ground command. For this reason an early lifetime estimate of this satellite was required in order to decide the advisability of extending the antenna to the full length with the attendant increase in drag and decrease in lifetime of the experiment.

For this estimate a quick method was needed which did not require a full set of orbital elements. It appeared that the variation in orbital period would provide the needed information.

PROCEDURE

In order to determine period using Space Surveillance System observations, it was necessary to determine the time of passage on successive passes of a satellite and then to correct the elapsed time for the different latitudes of the two observations. Additional corrections for the advance of the perigee and the eccentricity of the orbit could have been made, but these can be ignored in the case of a rapidly decaying orbit. In addition to the 1-revolution measurement, the elapsed time for 15 or 16 revolutions (about 24 hours) could be measured, and the mean period for that interval could be determined.

In preparation for this and other multiple launches, a group of receivers was set up at the Fort Stewart receiving station using the 5600-ft alert antenna, with receivers tuned to the several telemeter frequencies. The satellites passed through the main lobe in approximately 0.15 second. This short observation time permits measuring the time of satellite passage to a precision of approximately 0.1 second. Reflected signals were received so that position as well as time of passage could be measured. By applying the latitude corrections the period was determined to a precision of ±0.1 second.

When the period was plotted as a function of increasing time, a curve was formed whose slope was a measure of the rate of decay of the period with respect to time (Fig. 1). Note that each plot appears to be linear for the first few days except for minor variations about the straight line. During this period, the energy loss was impulsive, coming in the short period of time about the perigee. Each satellite shows a different slope with all the lines converging to the time of launch and separation.

Preliminary estimates of lifetime were found by dividing the difference between the observed period and 88 minutes by the rate of change of the period. The 88-minute figure is based on previous experience with such satellites as Mercury and Vostak. This rate

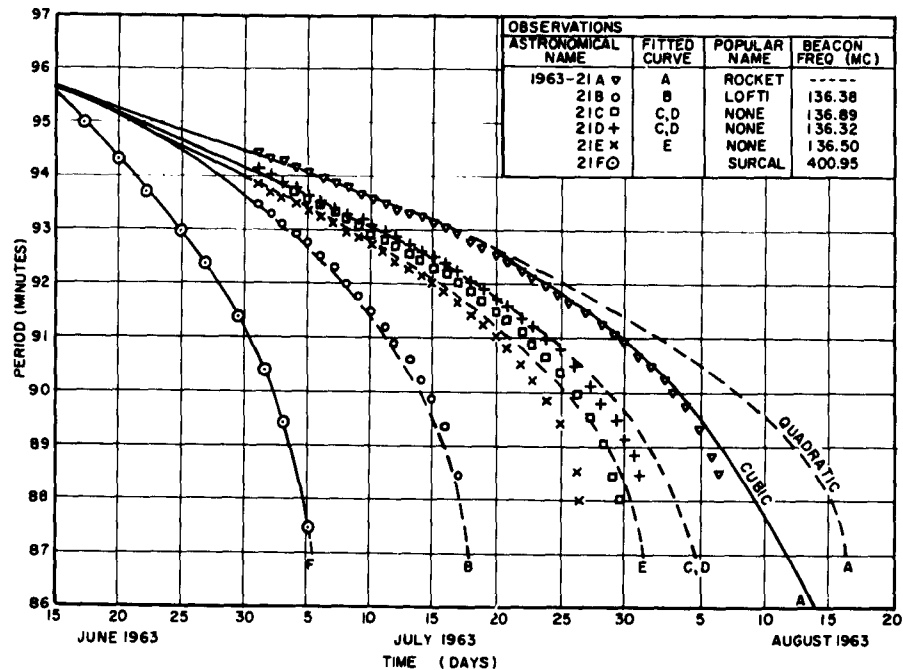| OBSERVATIONS			
ASTRONOMICAL NAME	FITTED CURVE	POPULAR NAME	BEACON FREQ (MC)
1963-21 A ▽	A	ROCKET	-----
21B o	B	LOFTI	136.38
21C □	C,D	NONE	136.89
21D +	C,D	NONE	136.32
21E ×	E	NONE	136.50
21F ⊙		SURCAL	400.95

Fig. 1 - Decay of 1963-21

of change of the period was found by observing the successive differences between periods. When the first differences were constant (second difference equal to zero), the decay was linear. When the second difference increased from zero, the period decayed at an increasing rate. A second estimate of lifetime was made by assuming a third difference of zero. Toward the end of the life of each satellite the third difference increased from zero as the satellite spent a greater portion of each revolution in the denser atmosphere near the perigee altitude.

As shown in Fig. 1 the periods of the various elements of 1963-21 decayed at different rates. After observing the varying slopes for several days it appeared as if the ratios of the several slopes were constant. Thus by fitting a curve to the record of the fastest decaying satellite the rate of decay of the other elements could be predicted.

After 10 days of observation, it was possible to fit a parabola to the decaying orbit of 1963-21F (Surcal II) that allowed us to predict its demise within a half day. Extrapolation, using the ratio of slopes, gave predicted reentries of the other bodies. In general, toward the end of the lifetimes of the satellites the respective periods decayed faster than predicted by the parabolic model. This error indicated the need of a polynomial of higher order to obtain improved accuracy.

When the period decays of the longer lived satellites were plotted, it was seen that during the first two weeks the smooth decrease in period was masked by a roughly sinusoidal variation (see Fig. A1 in Appendix A). This variation in turn showed a period of roughly 9 days. On July 15, 1963, the curves (Fig. 1) showed a small but definite bend to a new slope, so that the satellite 21A reentered some 10 days ahead of the prediction based on the reentry of the shortest period satellite.

DISCUSSION

In spite of the poor launch, calculations of the rates of decay of the satellite periods during the first 3 days after launch indicated that the payloads would remain in orbit for several weeks. After 8 days of observation, it was decided to command the extension of the antenna on satellite 1963-21B (Lofti-2) to the 40-foot length. After the antenna was extended, the satellite period began to decay more rapidly, as shown in Fig. 1, curve B.

With the practical matter of Lofti disposed of, it seemed valuable to determine how well lifetimes could be predicted using only anomalistic periods as determined by Space Surveillance System observations. Accordingly, the periods were plotted as functions of days after launch (Fig. 1).

Several ways of interpreting the data were tried — both graphical and analytical. The slopes were measured and compared. It appeared that the ratios between the slopes remained constant as the slopes changed. This is a property of a family of parabolas and suggested a quick way to predict the reentry times of the various bodies after the one with the greatest decay rate had reentered. A parabola was fitted to the data on 21F, which indicated reentry 22 days after launch. Similar fitting to the data of 21B, taken after the antenna was extended, gave a reentry date of 34 days after launch. In both cases reentry occurred on the day predicted.

Parabolic fits to the other data were made to predict the reentries of 21A, C, D, and E. As shown in Fig. 1 these bodies reentered 5 to 10 days ahead of prediction. Study of the data curves showed that on the 31st day after launch, all plots showed a marked increase in slope, as though the drag had increased suddenly. Curve (parabolic) fitting to data taken after that break gave reentry predictions within 1 to 2 days of actual reentry.

A cubic curve was fitted to the data of 21A and showed a better fit. Obviously, the discontinuity on the 31st day after launch made it impractical to try to fit data on both sides of the discontinuity with any low order polynomial. Appendix A shows the least squares fit of the data to a straight line and the sinusoidal appearance of the deviations from the straight line. Appendix B gives examples of the curve fitting and shows the approximate relationship between ballistic coefficients and lifetimes.

CONCLUSIONS

1. Some worthwhile predictions of the lifetime of rapidly decaying orbits can be made from measured anomalistic periods of satellites using Space Surveillance System data.

2. When the reentry time of one member of a multiple launch or of another satellite in a similar orbit can be measured, the reentry times of the remaining elements can be estimated quite accurately (1 to 5 day errors in lifetimes of 8 to 10 weeks).

3. If the ballistic coefficient of one or more objects is known, that of the others can be derived from the ratio of decay curve slopes.

RECOMMENDATION

It is recommended that a lightweight, high-drag body of adequate area be included in future multiple launches to provide data for quick lifetime predictions. This is particularly true for launches that use a restartable motor for apogee injection.

Appendix A

VARIATION OF THE ORBITAL PERIOD FROM LINEAR DECAY

Visual inspection of the decay curves showed that for the first few weeks the period oscillated about a straight line. The period and magnitude of this oscillation were studied more intensively by using the method of least squares.

The method of least squares obtains the best curve through a given set of points by minimizing the squares of the differences between the observed values y_i and the values y obtained from the assumed functional relationship, $y = f(x)$. For this case $y = f(x) = ax + b$, where a and b are to be determined. Minimizing

$$\sum_{i=1}^{n} (y_i - y)^2 = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

$$= \sum_{i=1}^{n} (y_i^2 + a^2 x_i^2 + b^2 - 2ax_i y_i - 2y_i b + 2x_i ab).$$

Let (*) represent the last line of the above equation. To determine the best values for a and b in terms of x_i and y_i, take the partials of (*) with respect to a and b, set the results equal to zero, and solve for a and b. Doing this yields:

$$0 = \frac{\partial(*)}{\partial a} = \sum_{i=1}^{n} (2ax_i^2 - 2x_i y_i + 2bx_i)$$

$$= a \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i + b \sum_{i=1}^{n} x_i,$$

and

$$0 = \frac{\partial(*)}{\partial b} = \sum_{i=1}^{n} (2b - 2y_i + 2ax_i)$$

$$= nb - \sum_{i=1}^{n} y_i + a \sum_{i=1}^{n} x_i.$$

Cramer's Rule gives

$$a = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} (x_i)^2 - \left[\sum_{i=1}^{n} x_i \right]^2}$$

$$b = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n} (x_i)^2 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{n \sum_{i=1}^{n} (x_i)^2 - \left[\sum_{i=1}^{n} x_i\right]^2}.$$

The actual values for y_i were reduced by 5600 seconds for ease in computation. With this consideration in mind the best straight line relationship for 21C (the solar radiation satellite transmitting at 136.89 Mc.) for the period from June 16 to July 5 was

$$y = -6.21977 x + 138.479 \text{ min}.$$

The observed periods oscillated about this line with a period of 9 days and an amplitude of 1.4 seconds. The best linear relationship for 21A (the rocket body) for the same period was

$$y = -4.90608 x + 139.503 \text{ min}.$$

The variations from this line had a maximum amplitude of about 1 second and a period of 9 days. In fact, to a remarkable degree, the plots of the variation for 21A and 21C were in phase with very similar shapes (see Fig. A1).
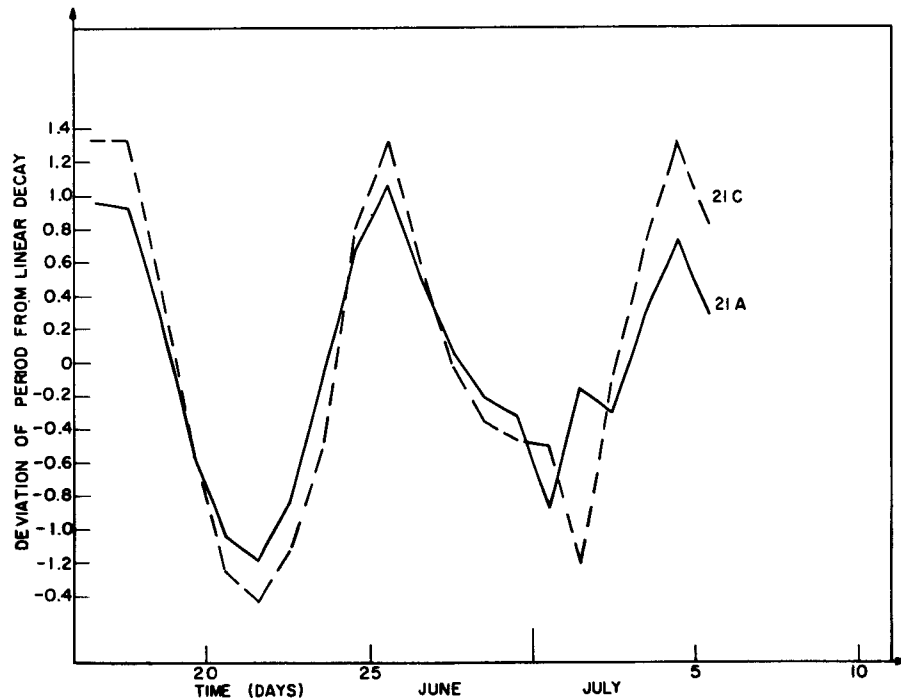
Fig. A1 - Departure of the period from linear decay

Appendix B

CURVE FITTING TO THE PERIOD VS TIME PLOT

Take three points on the plot of period vs time for 21F. Let $x = 0$ on June 15. We have the following:

Date	x (days)	y (min)	y' (min)	x' (days)
June 20	5	94.3	4.3	1
25	10	92.95	2.95	2
30	15	91.15	1.15	3

To simplify the arithmetic subtract 90 from each value of y and divide each value of x by 5. To fit the quadratic equation $ax^2 + bx + c = y$ simply evaluate the equation at the three points.

$$a + b + c = 4.3$$

$$4a + 2b + c = 2.95$$

$$9a + 3b + c = 1.15$$

Solving the equations for a, b, and c allows us to write the equation

$$y = 5.16 - 0.615 x - 0.245 x^2 .$$

Thus for the period of 88 minutes

$$y = -2 .$$

Solving for x gives

$$x = 4.003 .$$

Converting back to dates, the fitted curve predicted that the satellite would decay to 88 minutes on July 5. The observed value was 87.7 minutes at noon on July 5, and the satellite was not seen after that pass. The dotted curve for 21A was formed by operating on the x values according to the ratio of the slopes during the early, nearly linear decay period.

A cubic polynomial fitted to the plot of 21A for the dates July 1, 10, 20, and 30 was plotted. Note that it fits the observed data more closely than the scaled up quadratic. Only satellite 21B followed the scaled up prediction closely. The other satellites all showed rather abrupt changes in slope on July 15. Object 21B had nearly decayed by that date so that the unknown force had negligible effect. To date no explanation has been advanced for the change in slope on July 15 or the roughly sinusoidal variation during the first three weeks. The diurnal bulge in the atmosphere was suggested as a possible disturbing influence. However, during the period of interest, the satellite orbit plane was well removed from the right ascension of the diurnal bulge. Figure B1 shows a plot of the orbital planes and the diurnal bulge during the period of interest.
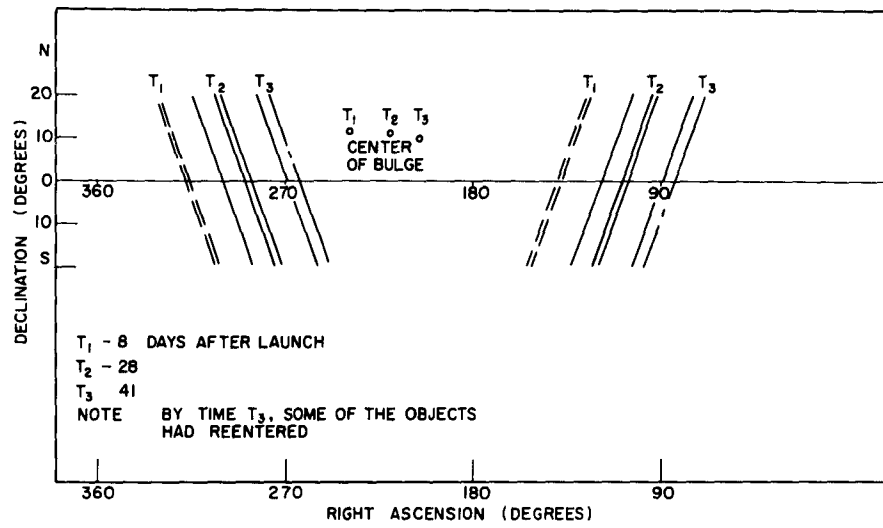
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Fig. B1 - Relative position of satellite tracks
and atmospheric diurnal bulge

Naval Research Laboratory. Report 6049. DECAY PREDICTION OF 1963-21 USING U.S. NAVAL SPACE SURVEILLANCE SYSTEM OBSERVATIONS, by R. Brescia, D. Ballou, and R. Zirm. 7 pp. and figs.

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