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Research Report

Analysis of Transients and Stability in an Idealized Two-Level Laser System

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Analysis of Transients and Stability in an Idealized Two-Level Laser System

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There have been some recent discussions of the stability and transient behavior (spiking, etc.) in various lasers. The analysis of three-or four-level laser systems involves several coupled differential equations which do not yield analytical solutions.

It is the purpose of this note to point out a relatively simple analytical solution to an idealized two-level laser system. Such a two-level system bears a resemblance to the diode laser system.

The equations describing the idealized two-level system shown in Fig. 1 may be written as follows

$$dn_2/dt = -An_2 - B\rho n + R + B\rho n_1, \qquad (1)$$

$$d\rho/dt = -B\rho n_1 + B\rho n_2 - \alpha\rho + An_2, \qquad (2)$$

where n_1 and n_2 are the populations of the lower and upper states (electrons per unit volume).

$\rho(t)$ is the radiation field intensity (photons per unit volume). [Since there is only one frequency, the frequency dependence has been neglected. Also the energy density would be $h\,\nu\rho\,(t)$.]

R is the rate at which electrons are pumped from state 1 to state 2 (electron per unit volume per sec). The pump rate R is constant.

α is the attenuation constant and $\alpha\rho$ is the energy loss rate of the system (photons per unit volume per sec). [For a unit volume laser with the radiation in one plane, $\alpha = 3 \times 10^{10}/\text{sec}^{-1}(1 - r)$, where r is the reflectance of the walls.]

$n_1 + n_2 = c$ (constant).

A is the coefficient of spontaneous emission.

B is the coefficient of stimulated emission. [The frequency dependence of B, like $\rho\,(t)$, is neglected.]

The addition of (1) to (2) leads to

$$\frac{d\rho}{dt} + \frac{dn_2}{dt} = R - \alpha\rho. \qquad (3)$$

If a steady-state condition of the laser system does exist, then both dn_2/dt and $d\rho/dt$ must be zero leading to

$$R = \alpha\rho. \qquad (4)$$

This indicates that if a steady state exists (to be shown), then the pump rate (R) must equal the loss rate ($\alpha\rho$).

Use Eq. (1) and $n_1 = c - n_2$ and rearrange

$$dn_2/dt = R + B\rho c - An_2 - 2B\rho n_2. \qquad (5)$$

Use Eq. (2) and $n_1 = c - n_2$ and rearrange

$$d\rho/dt = -\alpha\rho - B\rho c + An_2 + 2B\rho n_2. \qquad (6)$$

(n_2 shall be simply n from here on.)

The last term in Eqs. (5) and (6) being a cross term prevents a general solution in closed form, so as a subterfuge is in order.

Define ρ_0 and n_0 as follows:

$$\rho_0 = R/\alpha, \qquad (7)$$

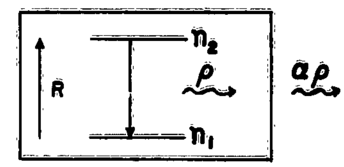

Fig. 1. Idealized two-level laser with populations n_1 and n_2, radiation density ρ, loss rate $\alpha\rho$, and pump rate R.

$$n_0 = \frac{\alpha R + BRc}{A\alpha + 2BR}. \qquad (8)$$

These values are chosen so that in a steady-state condition $n = n_0$ and $\rho = \rho_0$.

Now

$$\rho = \rho_0 + (\rho - \rho_0)$$

and

$$n = n_0 + (n - n_0),$$

$$\rho n = -\rho_0 n_0 + n_0\rho + \rho_0 n + (\rho - \rho_0)(n - n_0). \qquad (9)$$

When $\rho \sim \rho_0$ and $n \sim n_0$ (i.e., near the steady-state position), the last term in (9) is small and may be neglected. This approximation will not hold at points far from the steady state but will permit the determination of the stability of the system near the point of steady-state operation. If the laser is stable about this point, the subsequent solutions will describe its behavior.

Inserting the approximation in Eqs. (5) and (6), we obtain:

$$dn/dt = R + 2B\rho_0 n_0 + (Bc - 2Bn_0)\rho - (A + 2B\rho_0)n, \qquad (10)$$

$$d\rho/dt = -2B\rho_0 - (\alpha + Bc - 2Bn_0)\rho + (A + 2B\rho_0)n. \qquad (11)$$

Rewrite Eq. (10) and differentiate

$$\rho = \frac{1}{(Bc - 2Bn_0)}\left(\frac{dn}{dt} + (A + 2B\rho_0)\,n - (R + 2B\rho_0 n_0)\right), \qquad (10a)$$

$$\frac{d\rho}{dt} = \frac{1}{(Bc - 2Bn_0)}\left(\frac{d^2n}{dt^2} + (A + 2B\rho_0)\frac{dn}{dt}\right). \qquad (10b)$$

Use (10a) and (10b) to remove ρ dependence in (11),

$$\frac{d^2n}{dt^2} + (A + 2B\rho_0 + \alpha + Bc - 2Bn_0)\frac{dn}{dt} + \alpha\,(A + 2B\rho_0 n) - [(\alpha + Bc - 2Bn_0)R + 2\alpha B\rho_0 n_0] = 0 \qquad (12)$$

$$\frac{d^2n}{dt^2} + D\frac{dn}{dt} + En + F = 0. \qquad (12a)$$

Solving this in the usual manner:

$$\left(\frac{d}{dt} + \frac{D + \sqrt{D^2 - 4E}}{2}\right)\left(\frac{d}{dt} + \frac{D - \sqrt{D^2 - 4E}}{2}\right)n + F = 0. \qquad (13)$$

The general solution is then given by

$$n = c_1 \exp \frac{-D + \sqrt{D^2 - 4E}}{2} t + c_2 \exp \frac{-D - \sqrt{D^2 - 4E}}{2} t + \frac{F}{E}, \quad (14)$$

where

$$D = A + 2B\rho_0 + \alpha + Bc - 2Bn_0,$$
$$E = \alpha (A + 2B\rho_0),$$
$$F = (\alpha + Bc - 2Bn_0)R + 2\alpha B\rho_0 n_0,$$

and c_1 and c_2 are integration constants to be determined by the boundary conditions.

The expression $\alpha + Bc - 2Bn_0$ appears in both D and F. Insertion of the value for n_0 [Eq. (8)] leads to

$$\alpha + Bc - 2Bn_0 = \frac{A\alpha^2 + AB\alpha c}{A\alpha + 2BR} \quad (15)$$

Equation (15) may be used to remove the only negative term in D and F; therefore D, E, and F are all positive constants.

Equation (14) may now be inspected to determine the nature of the solution in the vicinity of the steady-state position. The steady-state position for (14) is just $n = F/E$ and the rate at which any deviations from the steady state will be damped out is just $\exp(-Dt/2)$. If $4E > D^2$, then the second part of the exponential $[\sqrt{(D^2 - 4E)}/2t]$ is of the form $\exp(i\omega t)$ and the approach to steady-state operation will be a damped oscillation with the frequency given by $\omega = \sqrt{(4E - D^2)}/2$. If $D^2 > 4E$, then (14) is critically damped and shows no oscillation whatsoever. Since D and E are both positive constants, the exponentials in (14) cannot become positive, therefore the laser cannot become unstable.

It may be concluded, then, that this two-level laser would achieve a steady-state operation with constant pump rate. After a momentary disturbance of the steady state by some means, such as injection of a light pulse or addition of a pulse to the pump, the laser would return to its steady-state operation with a damped oscillation transient. The same transient is expected when the laser pump is first turned on.