DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION. ALEXANDRIA. VIRGINIA

64-11

ESD-TDR-63-477

TM-3641

A BRIEF SURVEY OF SATELLITE ORBIT COMPUTATION METHODS AND MAJOR PERTURBATIVE EFFECTS

TECHNICAL DOCUMENTARY REPORT NO.   ESD-TDR-63-477

MARCH 1964

R.H. Greene

Prepared for

496L SYSTEM PROGRAM OFFICE

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L.G. Hanscom Field, Bedford, Massachusetts

Project 496L
Prepared by
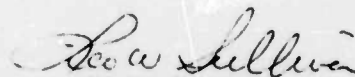
THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF19(628)-2390

APR 7 1964

A BRIEF SURVEY OF SATELLITE ORBIT COMPUTATION
METHODS AND MAJOR PERTURBATIVE EFFECTS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-477

MARCH 1964

R.H. Greene

ABSTRACT
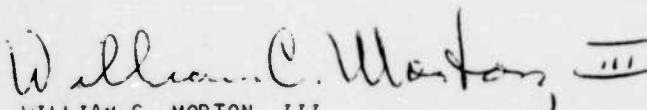
A brief descriptive survey of the various satellite orbit computation methods is presented here in order to give the systems engineer an introduction to the various methods and problems which currently exist.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

LEO W. SULLIVAN
Colonel   USAF
System Program Director
496L System Program Office
Deputy for Systems Management

WILLIAM C. MORTON, III
Acting Technical Director
496L System Program Office
Deputy for Systems Management

A BRIEF SURVEY OF SATELLITE ORBIT COMPUTATION METHODS AND MAJOR PERTURBATIVE EFFECTS

SECTION I

INTRODUCTION

Many systems engineers who have very limited knowledge of celestial mechanics, but who are, nevertheless, occasionally concerned with designing space systems, may find a brief introduction to the practical problems of computing satellite orbits useful for their work. The following exposition makes no attempt at completeness, but merely discusses some of the more common and useful computation methods. The mathematics involved is kept to a bare minimum, since it would be almost impossible to give even a fairly complete listing of the various formulae used in these orbit computation methods without filling several books. However, references and bibliography compiled at the end of this paper will serve to guide those more interested in details to some of the most important sources.

SECTION II

REFERENCE SYSTEMS
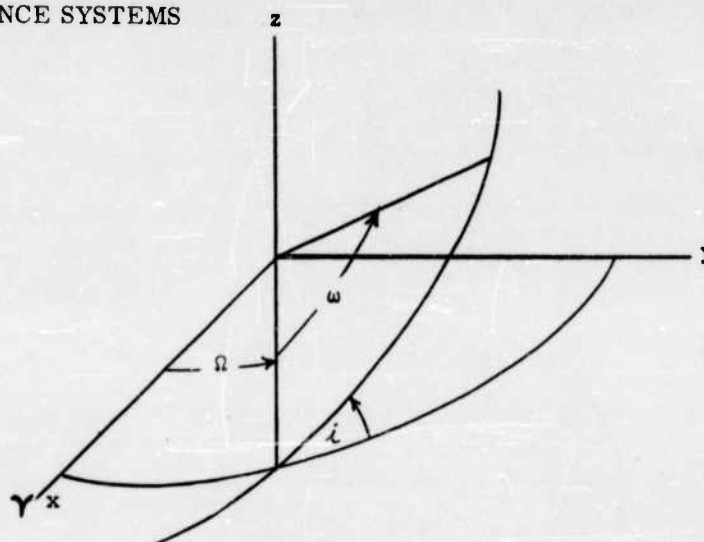
Fig. 1 Geocentric Reference System

The most commonly used reference system for earth satellites is the geocentric equatorial system which is fixed in inertial space (see Fig. 1). The x-axis points toward the vernal equinox γ which is that point in inertial space where the apparent orbit of the sun crosses the equator from south to north, the z-axis points toward the North Pole, and the y-axis forms a right-hand system. The x-y plane, which is the equatorial plane, is the fundamental plane of the system.

The orbit of a satellite is an ellipse with one focus at the center of the earth (see Fig. 2), and is completely described by six independent parameters. Three of these parameters Ω, ω, and i fix the orientation of the orbit in space, and the other three parameters a, e, and T fix the size and shape of the ellipse and the time of the point of closest approach of the satellite. Using Figs. 1 and 2 for illustration, we have:

a = the semi-major axis of the satellite orbit.

e = the eccentricity of the ellipse.

i = the inclination of the orbit plane.

Ω = the longitude of the node.

ω = the argument of perigee.

T = the time of perigee passage.

Fig. 2 Reference System

o = center of earth   B = apogee height
A = perigee height   q = perigee distance

Normally, the element M, the mean anomaly, is used in place of the time of perigee passage.

$$M = k_e a^{-3/2} (t - T)$$

where k_e is the Gaussian constant.

4

SECTION III

THE DETERMINATION OF SATELLITE ORBITS

The determination of a satellite orbit may be outlined as follows:

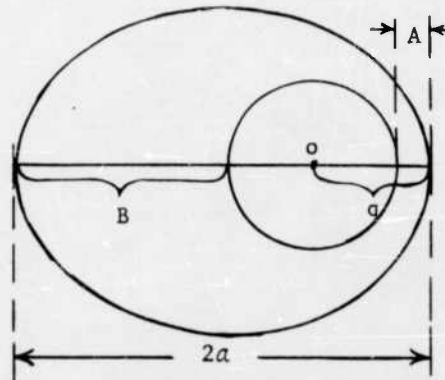(a) The first determination of "intermediate elements" E_i from observations. "Intermediate elements" will be regarded as geocentric position and velocity, or the standard elements a, e, i, Ω, ω, M or some other set of parameters which are normally obtained in first approximation methods using incomplete observations.

(b) The "representation" or calculation of what the observations would be if the basic intermediate elements were correct. This process, which ends in the determination of residuals, may involve additional observations as well as those upon which the intermediate elements are based.

(c) The differential correction. The differential correction utilizes residuals in differential formulae that relate them to corrections of the adopted elements ΔE_i. Finally, we transform the set of intermediate elements into an adopted set of "terminal elements" $E_{Ti} = E_i + \Delta E_i$. Terminal elements may be initial position and velocity, but more often they will be elliptical parameters such as a, e, i, Ω, ω, M or some other parameters especially suited in a particular problem to ephemeris computation and analytical correction programs.

It is in the first determination of the intermediate elements that one finds the greatest number of alternative procedures or methods. This variety is a

5

result of mathematical complexities and of differing observational patterns. With observations scattered in time, kind, and quality, it is not possible to select a single orbit-determination method that will be satisfactory in all circumstances.

An orbit may be determined from a six-dimensional fix, i.e., topocentric position and velocity. The simple coordinate transformation performed in this method makes it possible to determine the orbit without approximations, and hence, no corrections are needed until additional observational data are available.

Two or more complete three-dimensional topocentric position fixes may be available with present radar equipment. Geocentric fixes may be obtained immediately from the relation

$$\underline{\textit{r}} = \underline{\rho} - \underline{R}$$

and used as intermediate elements. The quantities $\underline{\rho}$ and $\underline{\textit{r}}$ are the vectors directed to the satellite from the observer and the dynamical center, respectively, and $\underline{R}$ is the vector from the observer to the center of the earth. If three or more fixes are available, it may be preferable to determine the position and velocity at some central date by the use of the Herrick-Gibbs formula[1] and use these as intermediate elements. If more than three observations are used, a least-squares reduction of the velocity at the central date is obtained. Since the velocity is obtained from series expressions neglecting higher terms, representation is necessary to determine if differential correction is necessary.

A limitation of the Herrick-Gibbs program is its requirement for three radar fixes separated by not more than one radian of arc as seen from the center of the earth. Observations spaced over any arc can be processed by the "Two-Position Program" developed by Aeronutronic; the data may be separated

6

by several revolutions of the satellite in its orbit. Observations spaced at geocentric angles 0 and 180 degrees apart produce singularities in the method, as two-position vectors with either of these particular singularities make it impossible to ascertain the orbit plane.

Aeronutronic has conducted much experimentation with the Herrick-Gibbs initial orbit programs to demonstrate the usefulness of range rate measurements in accurate determination of the orbital elements, particularly in the determination of the period. The observations were those made by the Millstone radar over a period of about one week. The elements were differentially corrected with all the observations so as to obtain a standard against which elements out of the initial orbit program could be compared. The results are good, considering the short radar track of one and two minutes. The use of range rate significantly improved the period determination in all but one test case. The effect on the rest of the elements is not conclusive and, in any event, the error in the period is orders of magnitude more significant than the errors in any of the other elements.

A great deal of literature is available on orbit determination from sets of angles and angular rates. The basic methods are frequently associated with the names of LaPlace, LaGrange, Gauss, and Gibbs, though there have been many modifications by subsequent writers. The relatively poor observations of earth satellites limit the usefulness of some of the classical methods.

In the LaPlacian method, one determines position and velocity at a central date, to be used as "intermediate elements" for correction purposes prior to the determination of the "terminal elements." The first approximation is based upon Taylor's series expansions of the observed angular coordinates or direction cosines; the large angular rates further limit its usefulness for satellites. This approach fails for close earth satellites because the rapid topocentric angular

7

motion gives rise to prohibitive truncation errors. In order to use the method successfully for such satellites, one either processes a large number of angles in a least-squares reduction or employs additional data, e.g., range rate observations.

The methods of LaGrange, Gauss, and Gibbs, in their simplest and most effective forms, all make use of the same first approximation. This approximation is also based on Taylor's series expansions, but in the dynamical coordinates (e.g., geocentric coordinates for geocentric orbits) rather than in the observational ones. Thus, for geocentric orbits, not only is the angular motion greatly reduced, but also it is possible to include some of the higher derivatives in the series with the aid of the dynamical properties of motion. The resulting first approximation, which yields three geocentric position vectors, may be more successful in one of the methods than in the others because of the way in which the position vectors are used in the determination of the terminal elements.

The first approximations of LaPlace and LaGrange-Gauss-Gibbs share an indeterminacy that occurs when the basic observations lie on a great circle arc. The indeterminacy may be overcome with three observations if the plane of the great circle does not pass through the dynamical center. If it does, one must seek a fourth observation from which the indeterminacy is eliminated.

It would be impossible to go into the details of these orbit computations in this short report. Reference 1 provides equations and procedures for use of the Herrick-Gibbs method and for those of LaPlace, LaGrange and Gauss. Moulton[2] provides a thorough discussion of the methods of LaPlace and Gauss. Herget's privately published book[3] provides detailed numerical procedures and many numerical examples for computing orbits using the methods of LaPlace and Gauss. Watson's book,[4] though published in 1885, is still a useful guide.

Many new methods for computing earth satellite orbits using canonical and other element sets have been proposed. The most significant recent works are papers by Brouwer, Kozai, Garfinkel, and Vinti.[5,6,7,8]

Once the intermediate orbital elements are determined from the initial orbit routines, the approximate position can be easily computed and compared to the observed position of the satellite. One thus gets a residual between the observed and computed position

$$\alpha_0 - \alpha_c = \Delta\alpha$$

where α may be range, altitude, or azimuth (or right ascension and declination). Since α is a function of the six orbital elements e_1, e_2, ... e_6. We can expand α into a Taylor series of the parameters e_1, ... e_6. Thus,

$$\Delta\alpha = \alpha_0 - \alpha_c = \frac{\partial\alpha}{\partial e_1}\Delta e_1 + \frac{\partial\alpha}{\partial e_2}\Delta e_2 + ... + \frac{\partial\alpha}{\partial e_6}\Delta e_6 + \text{higher}$$

order terms.

Neglecting the higher order terms, we have the equations for the differential correction. Since the partial derivatives $\partial\alpha/\partial e_i$ are functions of the orbital elements and can easily be computed (actually, the formulas are somewhat long and complicated), we have an expression for the error in the observations due to the errors in the orbital elements. What we wish to find, however, are the errors in e_1, e_2, ... e_6 due to the differences in the observed and computed positions. Therefore, if we have six observed differences $\Delta\alpha_1$, $\Delta\alpha_2$, ... ,$\Delta\alpha_6$, we can invert the matrix of coefficients of the Δe_i's:

9

$$\Delta\alpha_1 = \frac{\partial\alpha_1}{\partial c_1}\,\Delta e_1 + \frac{\partial\alpha_1}{\partial c_2}\,\Delta e_2 + \ldots + \frac{\partial\alpha_1}{\partial e_6}\,\Delta e_6 \quad,$$

$$\Delta\alpha_2 = \frac{\partial\alpha_2}{\partial e_1}\,\Delta e_1 + \frac{\partial\alpha_2}{\partial e_2}\,\Delta e_2 + \ldots + \frac{\partial\alpha_2}{\partial e_2}\,\Delta e_6 \quad,$$

$$.$$
$$.$$
$$.$$

$$\Delta\alpha_6 = \frac{\partial\alpha_6}{\partial e_1}\,\Delta e_1 + \frac{\partial\alpha_6}{\partial c_2}\,\Delta e_2 + \ldots + \frac{\partial\alpha_6}{\partial e_6}\,\Delta e_6 \quad,$$

and solve for the corrections to the elements, the Δe_i .

If wc have more than six observational residuals and hence, more than six equations of condition, then we can perform a ieast-squares fit. In matrix form, we have

$$A\,X = R \;,$$

where

$$A = \begin{pmatrix} \dfrac{\partial\alpha_1}{\partial e_1} & \dfrac{\partial\alpha_1}{\partial e_2} & \cdots & \dfrac{\partial\alpha_1}{\partial e_n} \\[2ex] \dfrac{\partial\alpha_2}{\partial e_1} & \dfrac{\partial\alpha_2}{\partial e_2} & \cdots & \dfrac{\partial\alpha_2}{\partial e_n} \\[2ex] & \vdots & \\[2ex] \dfrac{\partial\alpha_m}{\partial e_1} & \dfrac{\partial\alpha_m}{\partial e_2} & \cdots & \dfrac{\partial\alpha_m}{\partial e_n} \end{pmatrix} \;,$$

10

$$X = \begin{pmatrix} \Delta e_1 \\ \Delta e_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta e_n \end{pmatrix},$$

$$R = \begin{pmatrix} \Delta \alpha_1 \\ \Delta \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta \alpha_m \end{pmatrix},$$

and $m > n$.

Usually $n = 6$ (the number of orbital elements), but sometimes a seventh "element" or drag parameter is used in the differential correction.

It is appropriate to point out that each equation of condition

$$\Delta \alpha = \frac{\partial \alpha}{\partial e_1} \Delta e_1 + \frac{\partial \alpha}{\partial e_2} \Delta e_2 + \ldots + \frac{\partial \alpha}{\partial e_6} \Delta e_6$$

11

may be weighted by multiplying it by $1/\sigma$ where σ is the root-mean-square (r.m.s.) of the error of the observation. The values of σ are different for different radars and various types of optical observations and are determined independently of the differential correction process. Quite often, however, since the weighting of these equations does not contribute substantially to the accuracy of the final results, and since the values of σ are often difficult to determine, a common practice is to omit the weighting altogether and assign each equation a unit weight of one (e.g., SPADATS).

Since little is known about the values of σ for the various tracking installations, a study of this problem is being conducted at Aeronutronic. NASA, on the other hand, does make use of a weighted least-squares technique in the Project Mercury Program.

In matrix form, we may get the normal equations (usually six) by multiplying by the transpose of A.

$$A^T A X = A^T R.$$

$$B X = C$$

We will then have six equations in six unknowns which can be solved by standard elimination methods such as the Gauss-Jordan reduction. The solution of these equations will give us the correction to the elements $\Delta e_1, \Delta e_2, \ldots \Delta e_6$.

At Space Track, after forming a set of residuals corresponding to a set of observations, the magnitudes of the angle and range residuals are compared with an absolute maximum value of 1000 km, and the range rate residual is compared with an absolute maximum value of 0.5 km/sec. All residuals exceeding these limits are rejected from the current iteration. Upon completion of the first residual rejection test, a second similar residual rejection test is

12

performed with a new maximum value equal to (1.5) (r.m.s. value of the previously accepted residuals). The remaining number of residuals is then used in the differential correction equations. Upon completion of the calculation of the new set of corrected elements, however, the absolute maximum value of the first rejection test is subject to change from 1000 to 75 km if the new r.m.s. value of the residuals is less than 50 km. With the existing system of not too accurate radars, it is necessary to have many observations to get a good satellite orbit. It is necessary to use a large number of observations (usually 20 or more per satellite) in a "least-squares differential correction" every one or two weeks to update a satellite orbit; however, satellite orbits not appreciably affected by drag can often be accurately predicted without running the differential corrections for periods of from 30 to 60 days.

The literature contains three excellent and most complete references on the differential correction methods. [4,9,10] References 4 and 10 contain excellent sections on the least-squares technique and on the weighting of residuals, though the notation is somewhat clumsy since matrix notation is not used.

One should point out that the particular set of orbital parameters e_1, e_2, ... , e_6 one chooses and the particular method of orbit computation depend on the kind of orbit. Since the perigee ω is not well-determined in a nearly circular orbit, it is necessary to use other orbital element sets which can be better determined. For orbits of low eccentricity, the elements a, $e \cos \omega$, $e \sin \omega$, Ω, i, M are useful. Since the node Ω is not well-determined in a nearly equatorial orbit, it is also necessary to use a different orbital element set in this case. For orbits of low inclination, the elements a, e, ω, $\sin i \sin \Omega$, $\sin i \cos \Omega$, M are sometimes used. For orbits of low inclination and low eccentricity, Herrick has found orbital elements that depend on vector quantities such as $\underline{U}_o$, $\underline{V}_o$ most useful, where $\underline{U}_o$ is a unit vector

in the direction of the satellite at time t_o and $\underline{V}_o$ is perpendicular to $\underline{U}_o$ and in the orbital plane of motion of the satellite. A differential correction developed Eckert and Brouwer[11] makes use of infinitesimal rotations and corrects on $\Delta \psi_1$, $\Delta \psi_2$, and $\Delta \psi_3$ where these are the resultants of rotations of the orbit about the x, y, and z axes, respectively. Actually the Eckert-Brouwer method uses combinations of these and other elements to yield several sets of elements, each suited to a particular type of orbit (for example $\Delta M_o +$ $\Delta \psi_3$, $\Delta \psi_1$, $\Delta \psi_2$, $e\Delta \psi_3$, $\Delta a/a$, and Δe are used for low eccentricity orbits). All in all, there are many different sets of orbital elements and computation methods, each with its own advantages and disadvantages.

SECTION IV

PERTURBATIONS OF THE SATELLITE ORBIT

The significant perturbative forces are oblateness of the earth, drag, and solar radiation pressure. Less significant perturbative forces are those due to magnetic effects, lunar and solar gravitational attraction.

Oblateness

The potential function for the earth can be written as

$$
U = \frac{\mu}{r} \left[1 - \frac{J_2}{2r^2} (3 \sin^2 \delta - 1) - \frac{J_3}{2r^3} (5 \sin^3 \delta - 3 \sin \delta) \right.
$$

$$
- \frac{J_4}{8r^4} (35 \sin^4 \delta - 30 \sin^2 \delta + 3)
$$

$$
- \frac{J_5}{8r^5} (63 \sin^5 \delta - 70 \sin^3 \delta + 15 \sin \delta)
$$

$$
\left. - \frac{J_6}{16r^6} (231 \sin^6 \delta - 315 \sin^4 \delta + 105 \sin^2 \delta - 5) \right],
$$

where

δ is the instantaneous latitude of the satellite,

μ is the gravitational constant for the earth 1.407639,

r is the radial distance of the satellite from the center of the earth in earth equatorial radii,

$J_2 = 1.08228 \times 10^{-3} (\pm .00003 \times 10^{-3})$,

$J_3 = -2.3 \times 10^{-6} (\pm .2 \times 10^{-6})$,

$J_4 = -2.12 \times 10^{-6} (\pm .05 \times 10^{-6})$,

15

$$J_5 = -.2 \times 10^{-6} \, (\pm .1 \times 10^{-6}), \quad \text{and}$$

$$J_6 = 1.0 \times 10^{-6} \, (\pm .8 \times 10^{-6}).$$

For most practical estimates of the oblateness effect, the J_2 term is the only one that need be considered. These oblateness effects are quite considerable on close earth satellites and can affect the perigee and nodal points by as much as several degrees per day. One can see by looking at the potential function that as r increases, the effect of the oblateness is diminished.

The oblateness of the earth primarily affects three orbital elements Ω, ω, and M; however, the small effect on the remaining elements can be accurately determined analytically. [5, 6, 7, 8, 12] The equatorial bulge causes the node Ω to precess in a manner similar to that of a spinning top. The formula for this precession is given by

$$\dot{\Omega} = -\frac{J_2' \, n \, \cos i}{a^2 (1 - e^2)^2} \quad ,$$

where

$$J_2' = 1.62345 \times 10^{-3},$$

a = the semi-major axis,

e = the eccentricity,

i = the inclination of the orbit to the equator, and

n = the mean motion (angular velocity) of the satellite
in its orbit.

We should note, however, that at i = 90 degrees, cos i = 0 and hence, $\dot{\Omega}$ = 0; also, $\dot{\Omega}$ can reach a maximum of slightly more than 10 deg/day for certain low inclination orbits. Actually, Ω is referred to as the regression of the node since its sign is minus. Figures 3 and 4 are most instructive.
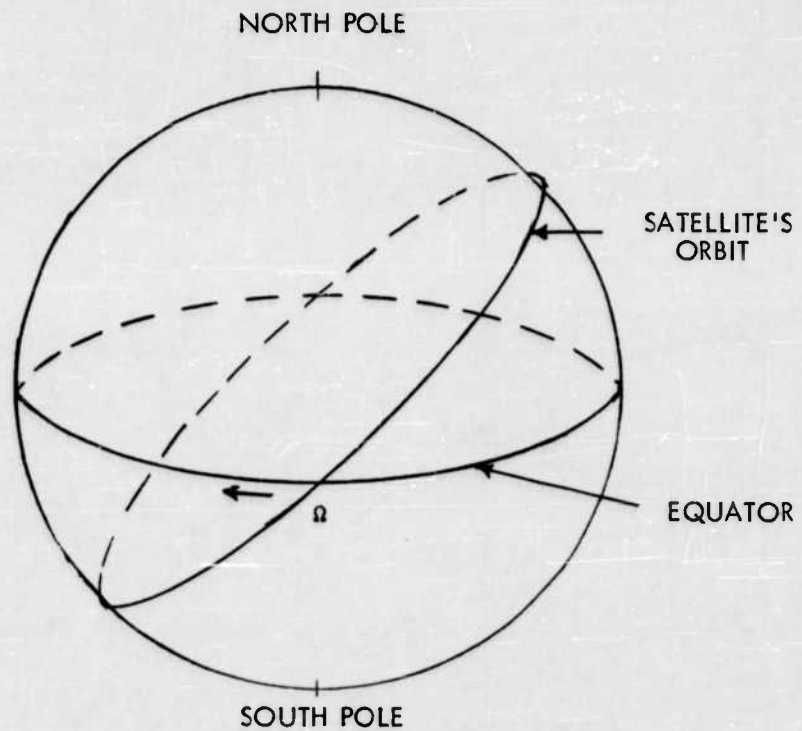
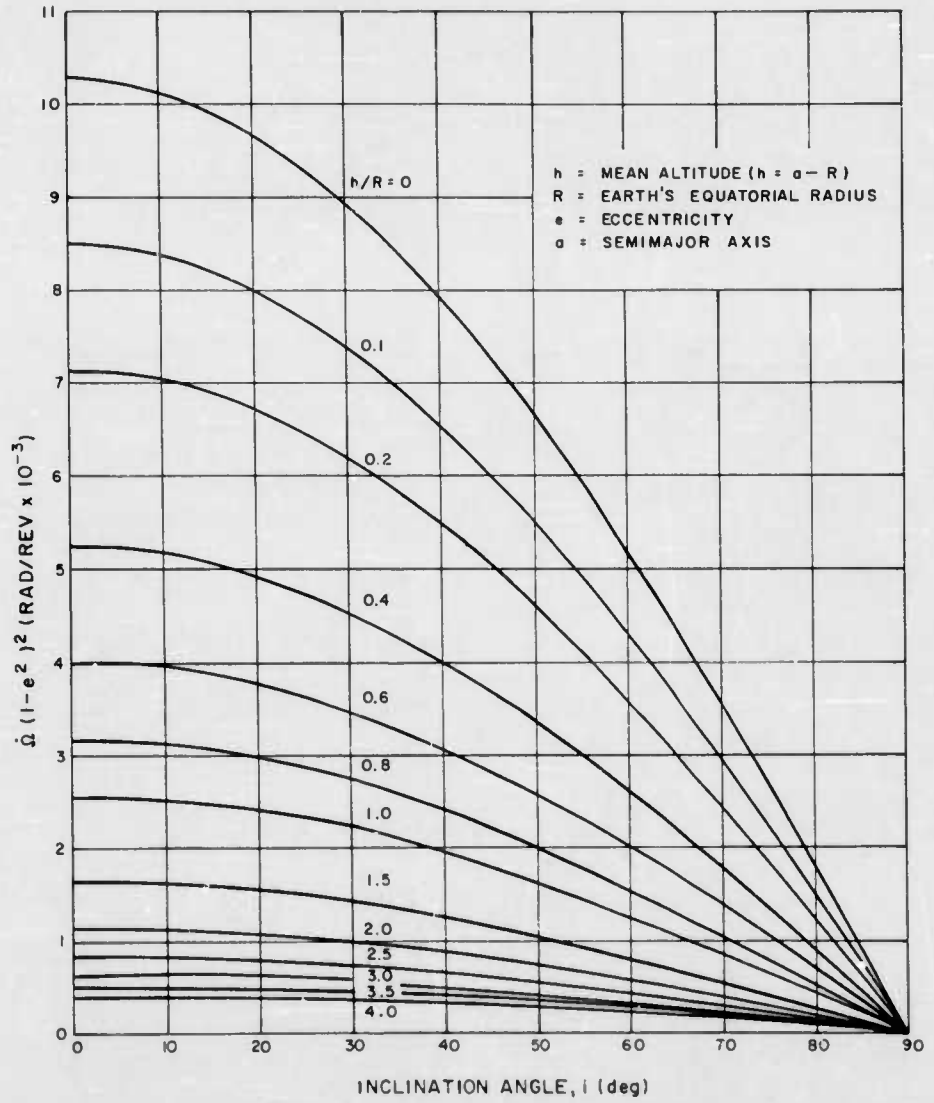16

Fig. 3 Regression of the Node

Fig. 4 Nadal Regression Rate as Functions of Mean Altitude and
Orbital Inclination Angle (fram Reference 13)

The second important effect of the bulge at the earth's equator is that the orbital ellipse rotates steadily in its own plane (see Fig. 5). The rotation rate is given by the formula

$$\dot{\omega} = \frac{J_2' n (5 \cos^2 i - 1)}{2a^2 (1 - e^2)^2} \qquad .$$

The rotation is forward, in the same direction as the satellite motion, for $i < 63.4$ degrees, zero when $i = 63.4$ degrees, and backward for $i > 63.4$ degrees. We should note from Fig. 6 that this precession of perigee (often called apsidal precession) can reach 20 degrees in certain near equatorial orbits.
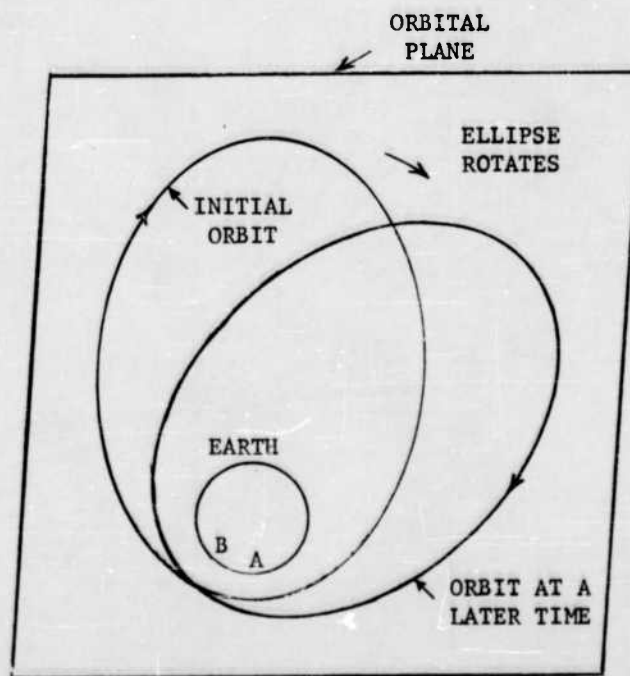
Fig. 5 Diagram Showing Rotation of the Orbital Ellipse in Its Own Plane. (Initially the orbit is nearest to the earth at A, and later, at B.)
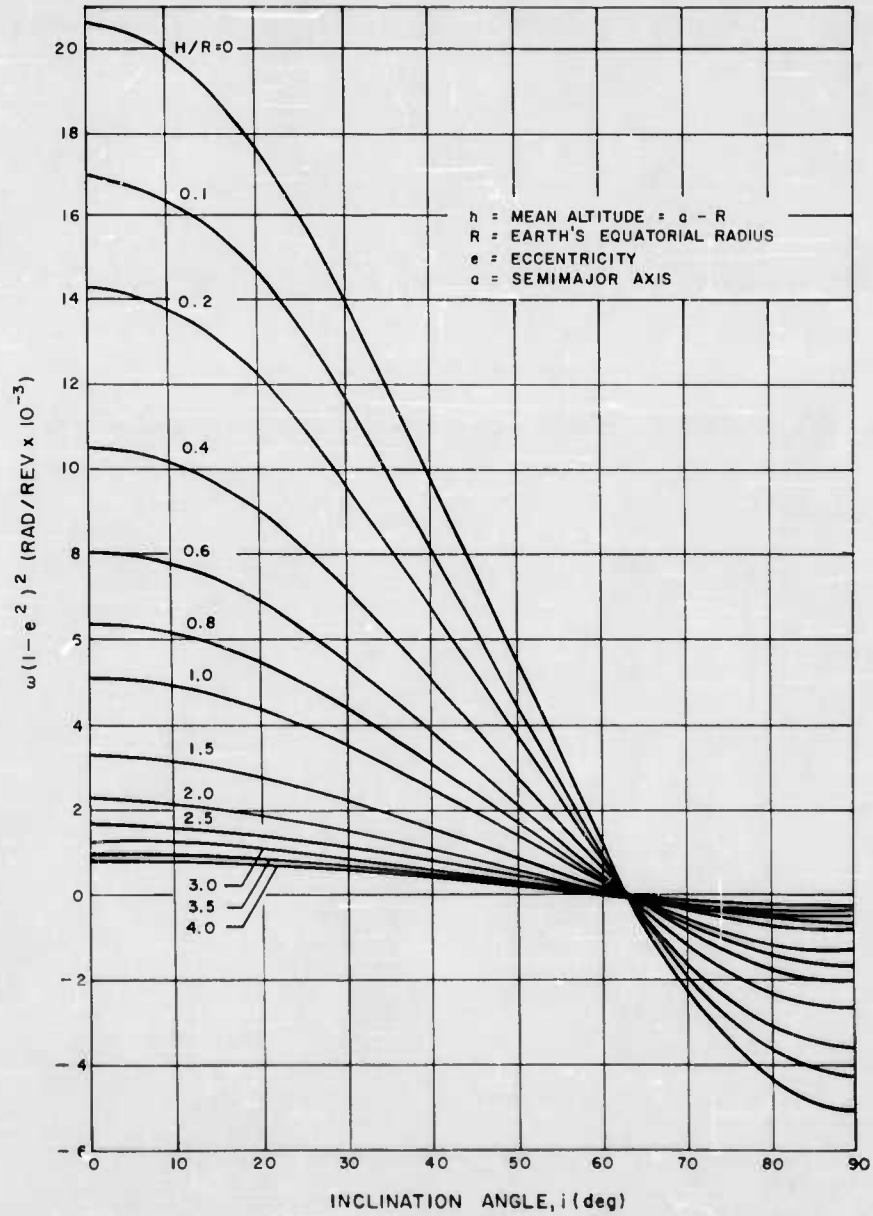
Fig. 6 Apsidal Precession Rate as a Function of Mean Altitude
and Orbital Inclination Angle (from Reference 13)

20

The mean anomaly M of the orbit is also severely perturbed by the earth's oblateness. Thus,

$$\dot{M} = \frac{J_2' n (3 \cos^2 i - 1)}{2a^2 (1 - e^2)^{\frac{3}{2}}} \quad .$$

The oblateness has a small effect on the remaining orbital elements, but they are generally not too important for short time periods. Complete investigations of the effect of oblateness on all the orbital elements may be obtained from the papers of Brouwer, Kozai, and Garfinkel.[5, 6, 7]

Drag

The perturbative effects of air drag are significant at altitudes below about 300 miles. It is usual to express the aerodynamic drag D of a satellite in terms of a drag coefficient C_D, based on the maximum frontal area A_c of the satellite:

$$D = \frac{1}{2} C_D A_c \rho V^2 \quad ,$$

where

ρ is the local air density and

V is the magnitude of the satellite velocity.

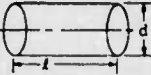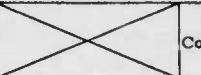In most formulations, however, the drag deceleration is found to be the basic quantity

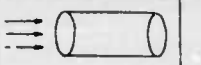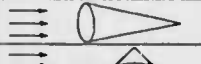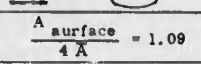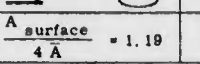$$\frac{D}{m} = \frac{C_D A_e}{2m} \rho V^2 = B \rho V^2 \quad .$$

21

Considerable simplification may be achieved by the use of the ballistic coefficient B, since the problem may be solved parametrically for a series of values of B which eliminates the necessity for variables C_D, A_c, and m.

For a given vehicle mass m is obviously known, but for more complicated shapes each particular case will necessarily determine its own coefficient of drag and reference area. The drag coefficient for upper atmospheres is a function of the flow regime and the body shape wherever the reference area is a function of shape and body dynamics such as orientation tumbling, etc. It can be shown that at about 75 statute miles the satellite is out of the slip flow region, and free molecular flow conditions apply for any velocity and any satellite size. For free molecular flow, the fraction of diffusely reflected particles is 0.9; thus, the curve for diffuse reflection should be used for design consideration. It is noted in Reference 13 that for high molecular speed ratios, $C_D \approx 2$ is reached for both spherical and cylindrical satellites. This value is a valid first approximation. The reference area for steady flight is the cross-sectional area perpendicular to the air stream, i.e., for a sphere $A_c = \frac{\pi}{4} D^2$. For tumbling satellites it is assumed that each orientation is equally probable, and the average reference area is given by the total surface area divided by four. Numerical results from Reference 13 are compared in Fig. 7 for two satellites. It can be seen that for a cylinder with a diameter-to-length ratio of 0.078, the relative drag for random tumbling is 11.9 times higher than for an orientation with the main axis parallel to the air stream, and 28 percent lower than for an orientation with the axis perpendicular to the air stream.

In addition to the satellite drag based strictly on the air densities of an atmospheric model, such as ARDC 1959, certain special effects exist which should be considered.

Assumed Surface Interaction: Diffuse Re-emission $V_i \rightarrow \infty$
Accommodation Coefficient = 0.50

Satellite	1958 82 Sputnik III				1958 ε Explorer IV			
Shape	Cone				Cylinder			
Dimensions	d = 5.67 ft ℓ = 11.75 ft $\frac{d}{\ell}$ = 0.48				d = 0.52 ℓ = 6.67 $\frac{d}{\ell}$ = 0.078			
Flight Attitude		Drag Coefficient C_D	Relative* Reference Area A'	Relative Drag $C_D A'$		Drag Coefficient C_D'	Relative† Reference Area A'	Relative Drag $C_D A'$
"Forward"		2.22	1.00	2.22		2.94	1.00	2.94
"Backward"		2.94	1.00	2.94	Same as "Forward"			
"Sideways"		2.75$^\Delta$	1.32	3.63		2.75	16.3	44.9
Random Tumbling	$\frac{A_{surface}}{4\bar{A}}$ = 1.09	2.88	1.21	3.48	$\frac{A_{surface}}{4\bar{A}}$ = 1.19	3.13	11.2	35.0

*Normalized on the base area $\frac{\pi}{4}d^2$ = 25.2 ft.

Δ Approximated by drag coefficient of cylinder with axis normal to stream.

†Normalized on the base area $\frac{\pi}{4}d^2$ = 0.212 ft.

Fig. 7 Effect of Attitude and Tumbling on Drag of
Representotive Sottellites (from Reference 13).

The earth's oblateness basically adds a latitude effect into the atmospheric tables, which is presently missing in all the existing air density models. Good discussions of the oblateness effects on the air drag are given in the Literature, References 14, 15 and 16.

Rotation of the atmosphere causes the orbital lifetimes of satellites launched in retrograde orbits to be different from those of satellites in corresponding direct orbits. This problem has received some attention.[17,18]

23

Diurnal atmospheric bulge exists above 125 statute miles and is believed to result from thermal and gravitational air expansion effeets. Also, the atmosphere is very unstable between 50 and 70 statute miles due to large waves and tidal motions in the air. At 125 statute miles, the solar flares may cause ehanges of 40 pereent in air density, beeoming more important above 175 statute miles. There seems to be large variations related to the seven-year sun spot eyele, whieh would imply that the satellite lifetime is also a funetion of the year in whieh the orbit is established.

Sinee it is well-known that drag is the major influenee on satellite life-time, it is appropriate to mention that a fairly aeurate estimate of a satellite's lifetime ean be obtained at the beginning of its eareer if the initial eecentricity e_o is known and is not larger than about 0.2, and if the period of revolution has been measured for several days. The total lifetime of the satellite, t_L, is given by

$$t_L = \frac{3}{4} \frac{e_o T_o}{x} \; ,$$

where

T_o is the initial period of revolution, and

x is the daily deerease in the period of revolution.

If, for example, $e_o = 0.1$, $T_o = 100$ minutes and $x = 0.05$ minutes/day, we find $t_L = 150$ days, and this estimate of lifetime should not be in error by more than about 15 days.

The most important faets to remember about air drag are that it varies widely and it is difficult to prediet these short period fluetuations. Figures 8, 9 and 10, dealing with air density and drag, should prove useful and informative.
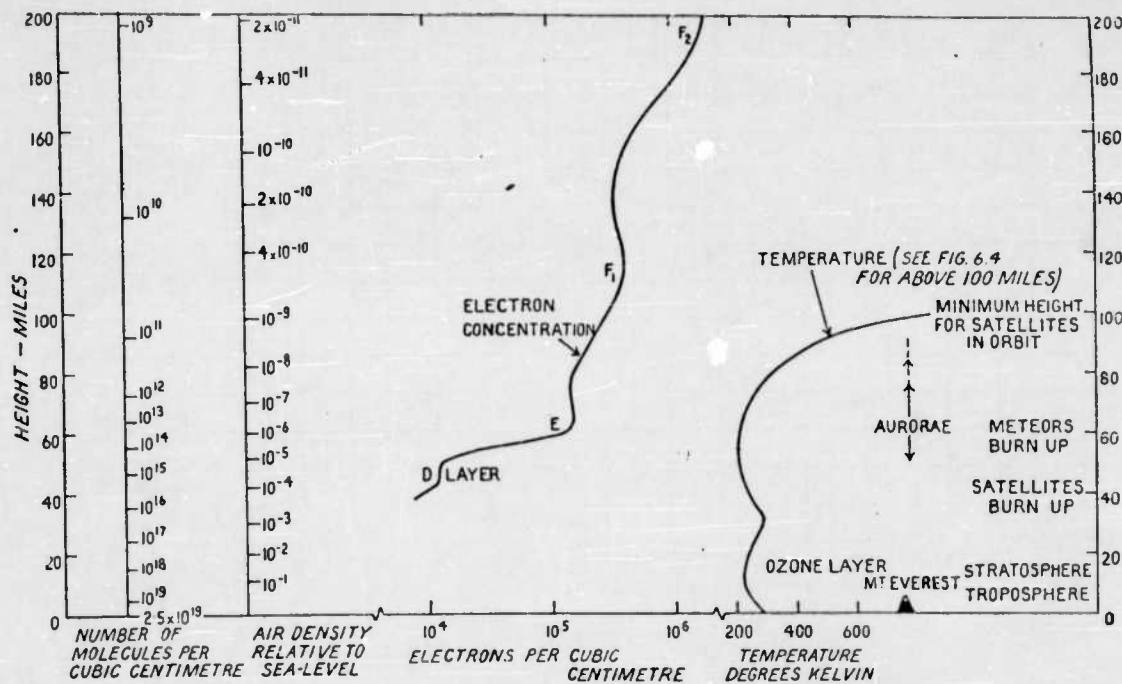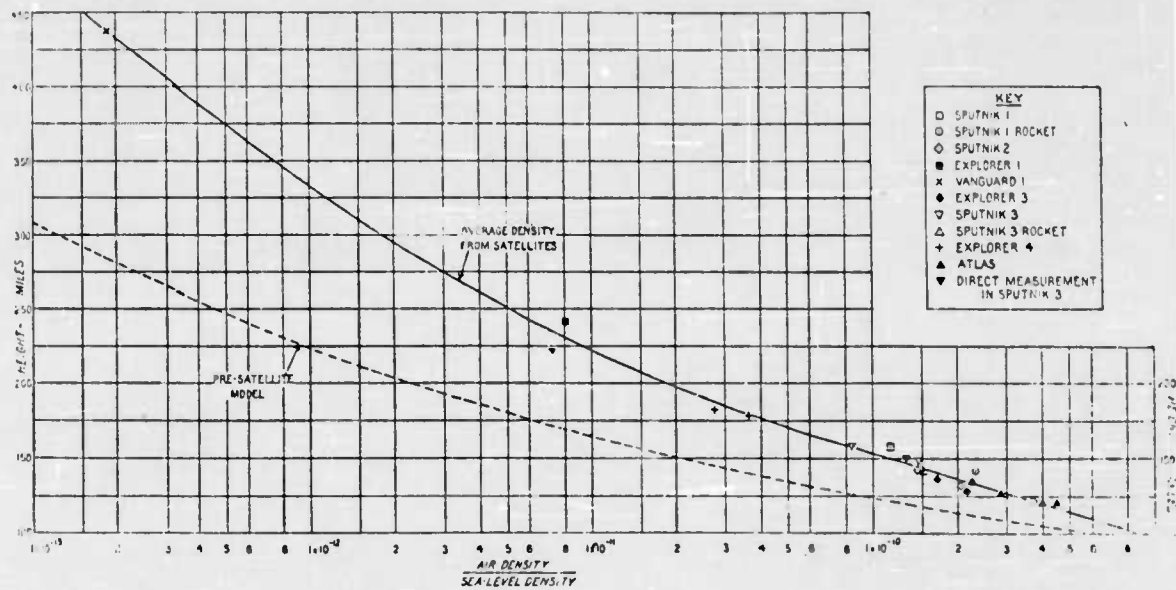
24

Fig. 8   Chart of the Atmosphere Below 200 Miles Altitude, Showing Air
Density, Temperature, and Concentrations of Molecules and of
Free Electrons (from Reference 19)

The points plotted are the values of air density obtained from the orbits of each of the satellites, with the full line drawn through them representing the average density. The broken line shows the density according to the pre-satellite ARDC model, which is too low by a factor of between 3 and 11 at heights between 120 and 250 miles. Sea-level density is 0.0765 lb/cu. ft., or 0.00123 grams/cm^3.

Fig. 9 The Variation of Air Density with Height (from Reference 19)
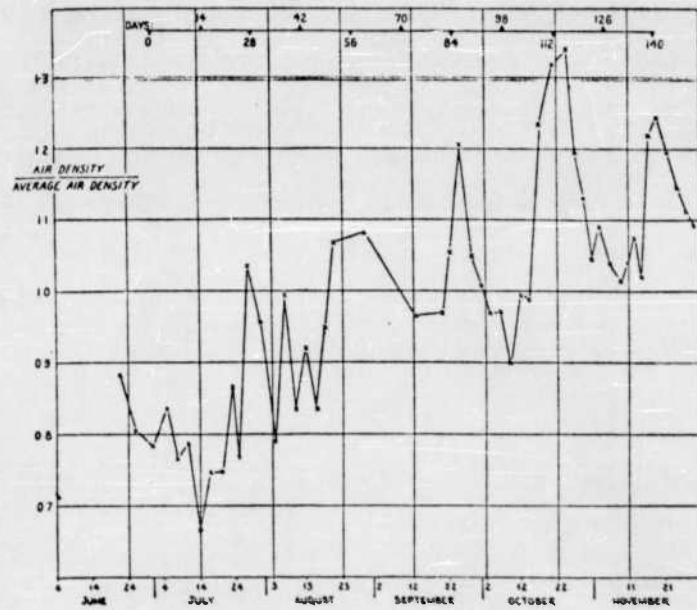
The Figure shows that density departs from its average value by up to 35 percent, and that the maximum values of density occur at intervals of about 28 days.

Fig. 10   The Variation of Air Density during 1958, at Heights between 100 and 150 Miles, as Given by Sputnik 3 Rocket (from Reference 19)

Aerodynamic drag primarily affects the elements M, a, and e. In a circular orbit, the air drag slows down the speed of a satellite slightly, but as its speed drops below the orbital speed proper to its height, it begins to descend at a small angle. As soon as this happens, gravity accelerates it on its down-hill path. So the satellite begins to go faster, and the speed increases beyond its original value until it reaches the slightly higher speed, which is the proper orbital speed at this new and slightly lower height. Then drag begins to slow it, and the whole cycle begins again. The net result is that the satellite slowly descends at a steadily increasing speed in a nearly circular spiral. In this case, the semi-major axis a is continually decreasing and since the period

$$P = \left(\frac{2\pi}{k_e}\right) a^{3/2} \quad ,$$

the period also decreases.

If the initial orbit, instead of being circular, is appreciably elliptic, the effect of aerodynamic drag is quite different at first. Since air density falls off rapidly as height increases, a satellite in an elliptic orbit will get a much greater wallop from drag at or near perigee than at or near apogee. In fact, the drag can be largely ignored except in the region near perigee. Over this short section of the orbit, drag causes a small loss in speed of the satellite. The minimum height of the satellite is reduced only to a very negligible degree, but the maximum height at apogee is reduced to a much greater extent. The effect of aerodynamic drag on an elliptic orbit, therefore, is to make the orbit more nearly circular by steadily reducing the maximum height and scarcely reducing the minimum height. As in the circular case, the semi-major axis a and the period P are also decreasing. In eccentric orbits, however, the eccentricity is also decreased since the orbit continually grows more nearly circular.

28

Very gross errors in satellite position and the orbital elements would develop in a few orbits if the major perturbations such as oblateness and drag were ignored. One can fairly easily account for the major changes in earth satellite orbits caused by oblateness by using the formulas for the secular variations in Ω, ω, and M as given in the section on oblateness. The effect on the mean anomaly M by drag is a very considerable one. If one writes the mean anomaly as a function of time

$$M = M_o + c_1 (t - T) + c_2 (t - T)^2 + c_3 (t - T)^3 + \text{periodic terms} \quad ,$$

then the c_1 constant is just the Kepler two-body change in the orbit plus the secular perturbations due to oblateness. The c_2 and c_3 constants which are the drag terms are usually determined empirically by a least-squares fit, i.e., the differential corrections corrects on the seven elements a, e, i, Ω, ω, M, and c_2 and c_3 are only used in the case of very high drag satellites; Smithsonian and SPASUR have found that the use of the c_3 term with low drag satellites often decreases rather than increases the accuracy of prediction. Drag corrections based on this empirical c_2 can also be made to the elements a and e in a straightforward manner:

$$a = a_o - \frac{4}{3} a_o c_2 (t - T) \quad ,$$

and

$$e = 1 - \frac{a_o (1 - e_o)}{a} \quad .$$

All existing operational systems in NASA, Space Track, NAVSPASUR, JPL, Smithsonian, etc., make use of these simple corrections for drag and oblateness; however, the inclusion of more terms (i.e., short and long periodic

perturbations[5] and more sophisticated techniques is quite prevalent in the above tracking and computational centers.   Figures 11 through 17 serve to illustrate and clarify this discussion.
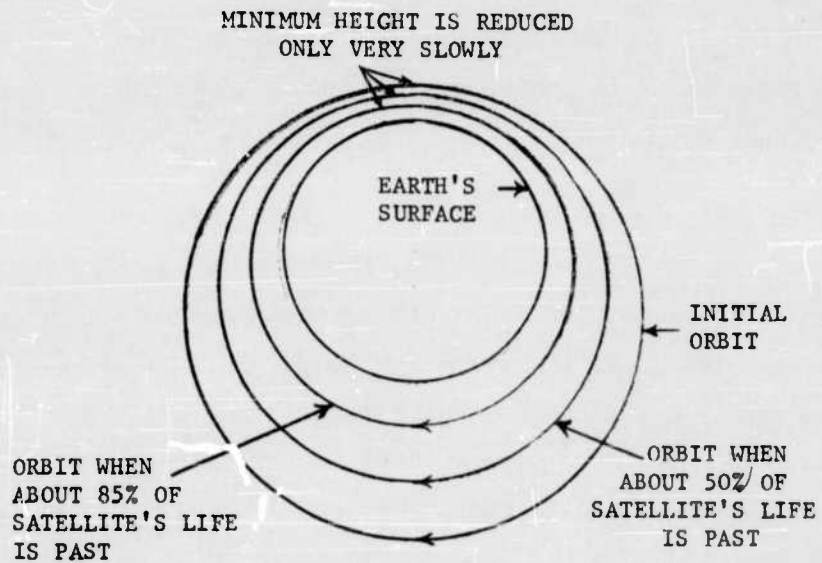

Fig. 11   Shrinking of Satellite Orbit Under the Action of Air
Drag — Not Exactly to Scale (from Reference 19)

Solar Radiation Pressure

Solar radiation pressure produces significant perturbations only on large light satellites such as Echo. More specifically, this perturbation is significant for satellites with area-to-mass ratios of approximately 25 cm^2/gm or greater,[13] although solar radiation pressure also produces measurable effects on small, dense satellites at high altitudes. Solar radiation pressure is estimated to be equal to the air drag at a 500-statute mile altitude. Its most important effect is displacing the geocentric center of the orbit. For certain resonance conditions, the

30
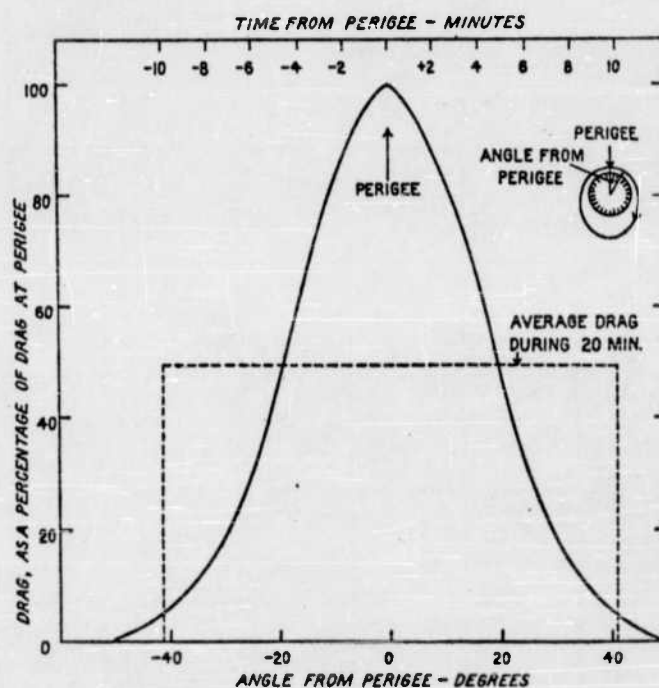
The average drag during the 20 minutes when drag is appreciable is just under 50 percent of the maximum drag, the area af the rectangle shown being equal to the area under the curve. The graph applies to a satellite with arbital eccentricity near 0.1.

Fig. 12   The Variation in the Drag Acting on a Satellite Near Perigee (fram Reference 19)
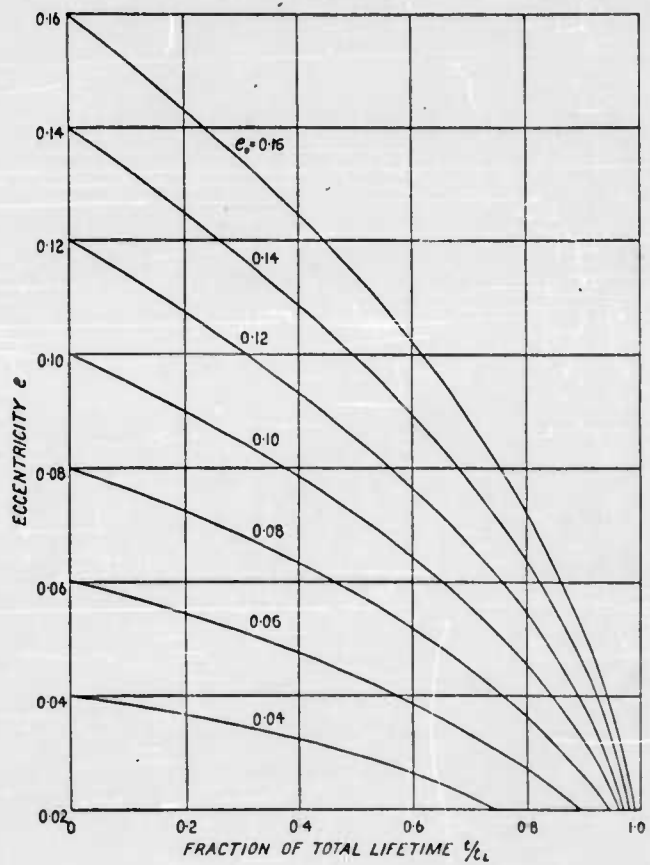
Numbers on the curves indicate the initial eccentricity e_a.

Fig. 13   Variation of the Eccentricity e of the Orbit during a
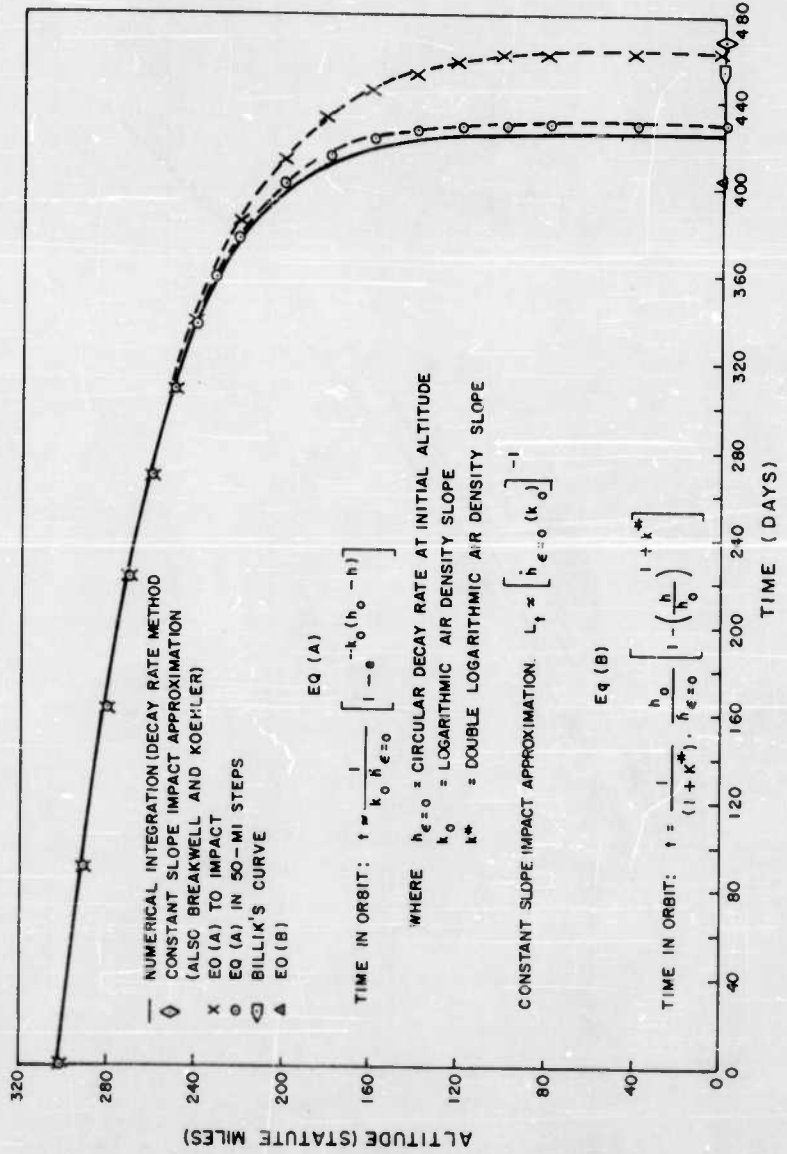Satellite's Lifetime   (from Reference 19)

32

Fig. 14 Altitude Decay History for a Circular Orbit (from Reference 13)

— NUMERICAL INTEGRATION (DECAY RATE METHOD
◇ CONSTANT SLOPE IMPACT APPROXIMATION
    (ALSO BREAKWELL AND KOEHLER)
× EQ (A) TO IMPACT
○ ⌀ EQ (A) IN 50-MI STEPS
▽ BILLIK'S CURVE
▲ EQ (B)

EQ (A)

TIME IN ORBIT: $t = \dfrac{1}{k_0 \, \dot{h}_{\epsilon=0}} \left[1 - e^{-k_0 (h_0 - h)} \right]$

WHERE $\dot{h}_{\epsilon=0}$ = CIRCULAR DECAY RATE AT INITIAL ALTITUDE
k_0 = LOGARITHMIC AIR DENSITY SLOPE
k^{**} = DOUBLE LOGARITHMIC AIR DENSITY SLOPE

CONSTANT SLOPE IMPACT APPROXIMATION: $L_t = \left[\dot{h}_{\epsilon=0} (k_0) \right]^{-1}$

EQ (B)

TIME IN ORBIT: $t = \dfrac{1}{(1+k^{**})} \cdot \dfrac{h_0}{\dot{h}_{\epsilon=0}} \left[1 - \left(\dfrac{h}{h_0} \right)^{1+k^{**}} \right]$

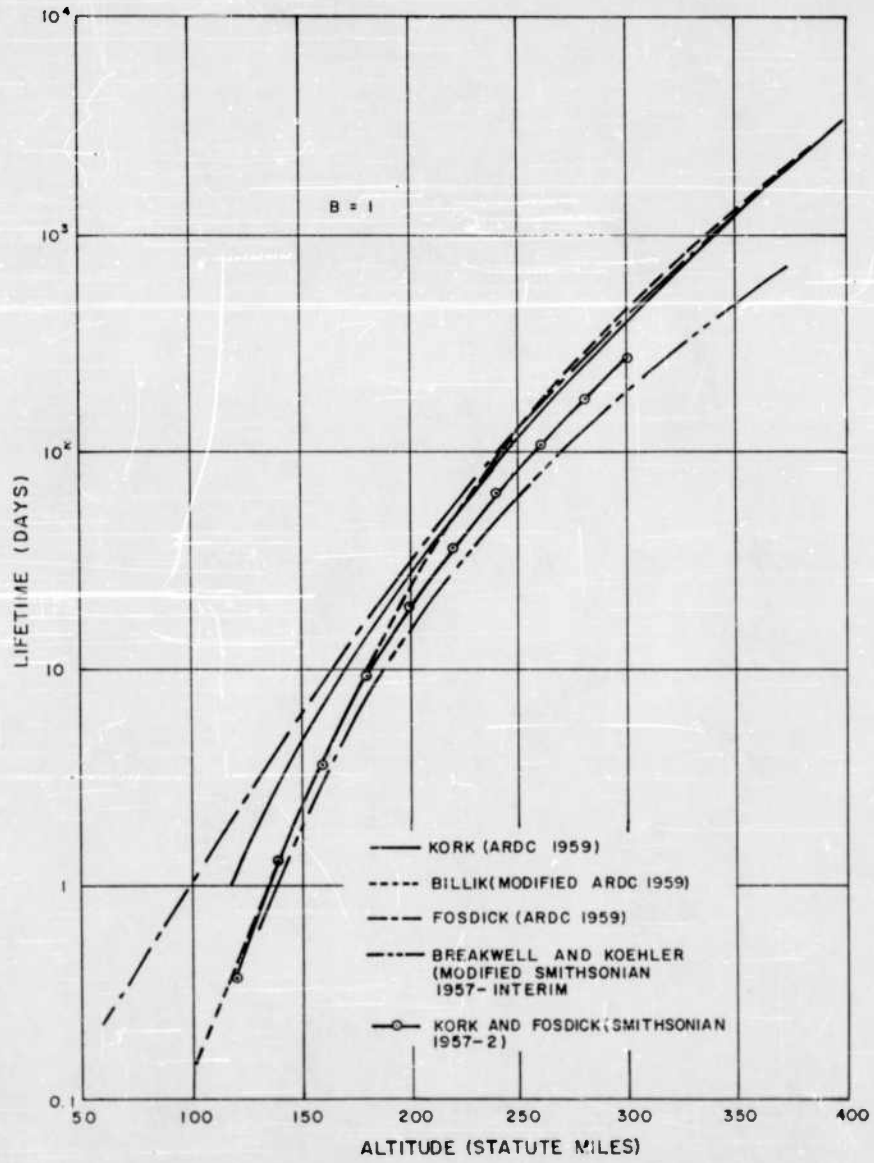ALTITUDE (STATUTE MILES)

TIME (DAYS)

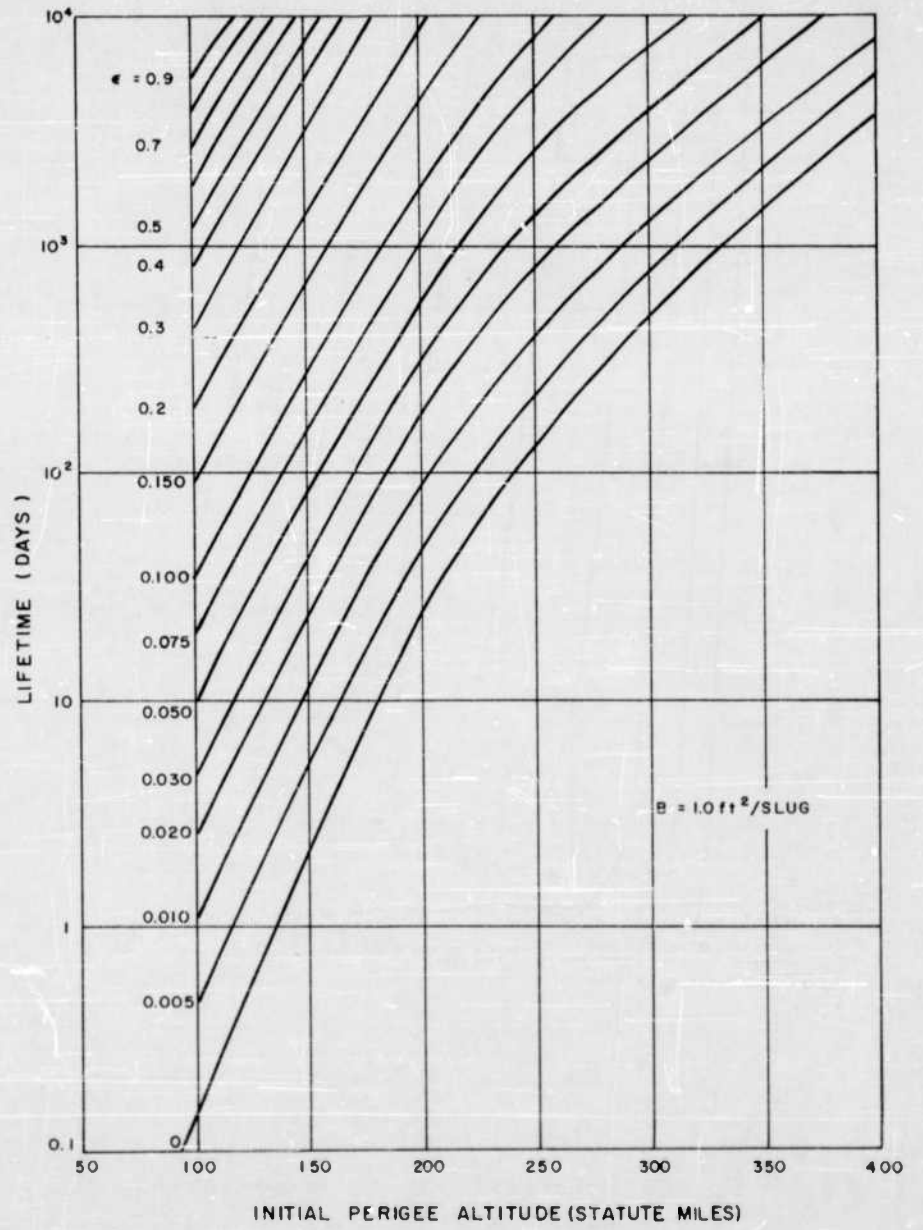Fig. 15 Lifetimes for Circular Orbits (from Reference 13)
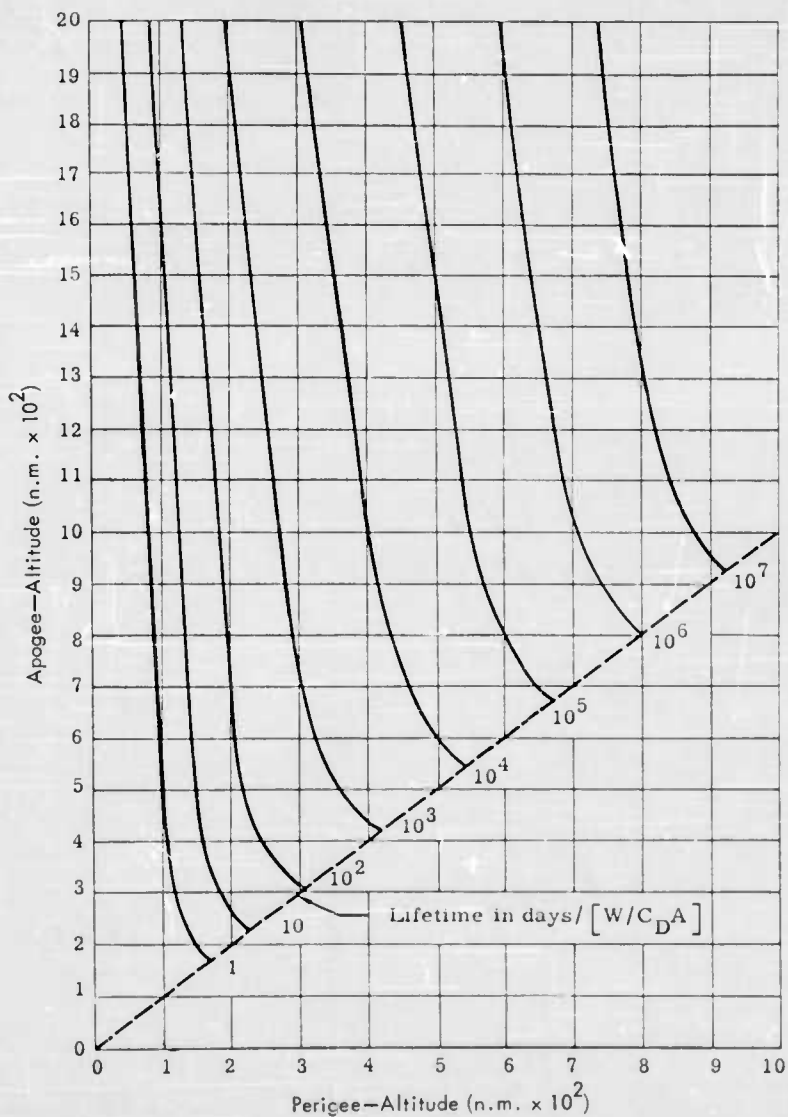
Fig. 16 Satellite Lifetimes (from Reference 13)

Fig. 17   Satellite Lifetimes as Functions of Apogee and Perigee
Altitudes (from Reference 13)

orbital lifetime is changed by significant amounts. For the Beacon satellite (area-to-mass ratio of 23.2 cm^2/gm) at a mean altitude of 950 miles, the lifetime can vary by a factor of 10, depending on the hour of launch.

In general, during a complete orbital period, solar radiation pressure causes a first-order perturbation of all six orbital parameters. However, the most conspicuous effect for a nearly circular orbit is a displacement of its geometric center. This displacement is perpendicular to the earth-sun line in the orbit plane and in a direction such as to decrease the altitude of that part of the orbit in which the satellite moves away from the sun. Calculations show that at a mean altitude of 1000 miles, radiation pressure can displace the orbit of the 100-foot Echo balloon at rates up to 3.7 miles/day, the orbit of the 12-foot Beacon satellite at 0.7 miles/day, and even the orbit of Vanguard I at a much slower rate of about one mile/year.

For certain resonance conditions, these perturbations due to solar radiation pressure accumulate and drastically affect the satellite's lifetime. For the Beacon satellite at a mean altitude of 950 miles, an initial eccentricity of 0.106 and an inclination of 40 degrees, the lifetime can vary by a factor of 10, depending on the hour of launch. For an inclination of 48 degrees of the Beacon satellite, these conditions are no longer resonant, and the variation in lifetime is reduced to a factor of 2.

For the Echo balloon placed in a 1000-mile altitude circular orbit and an inclination of 35 degrees, we find that the lifetime is 240 days. For an initially circular equatorial orbit, the resonance altitude of 4000 miles leads to a 1.3-year lifetime, while the same orbit at 1000 miles altitude has an extremely long lifetime.

For an excellent report on the effects of solar radiation pressure on satellite orbits, one is urged to read papers of I. I. Shapiro. [20, 21] They are

37

brief, lucid and give very valuable information. For the present, we offer Figures 18 through 23, which depict the radiation effects on eccentricity, perigee height, and the argument of perigee, ω.
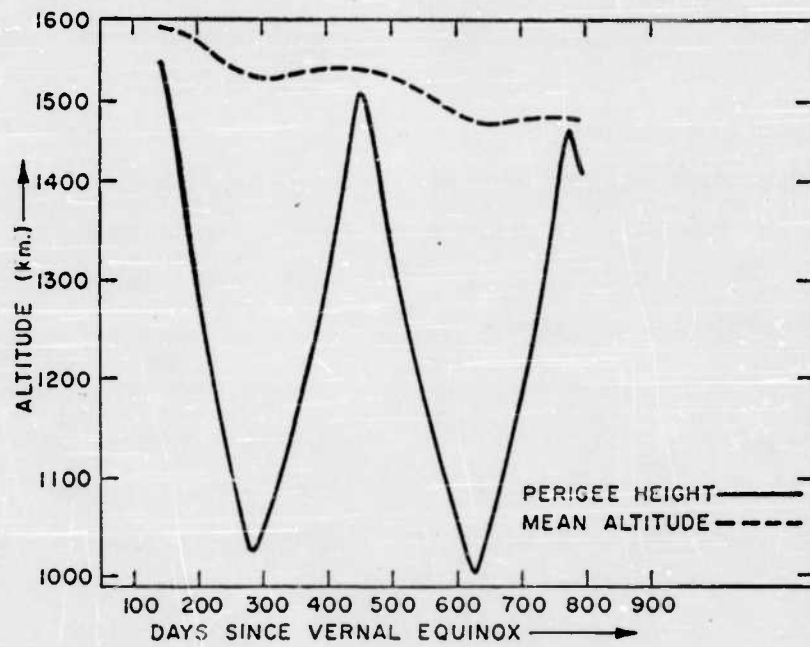

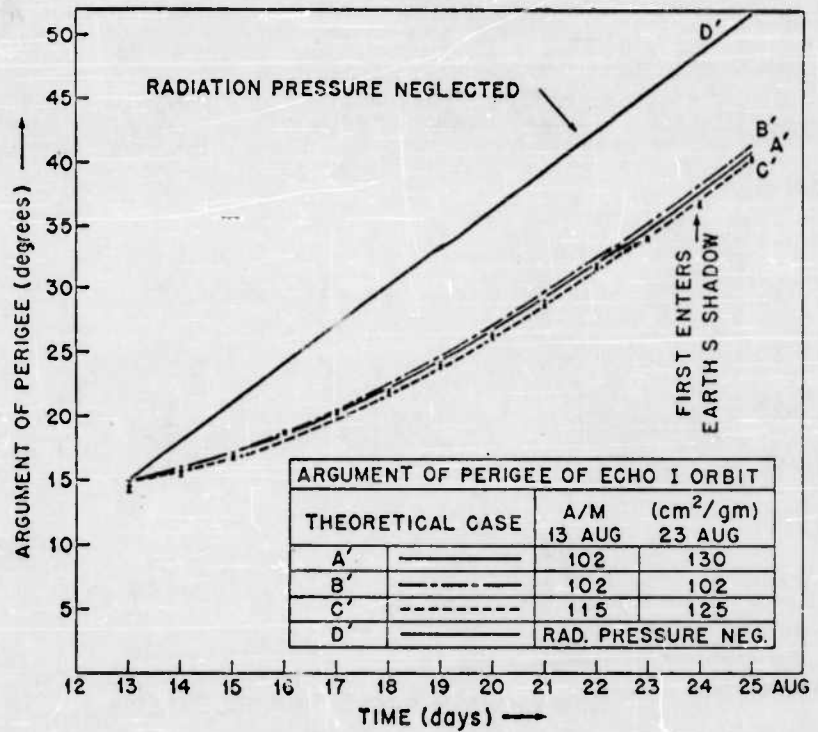Fig. 18 Time Variation of Perigee Height and Mean Altitude for Echo I (from Reference 21)

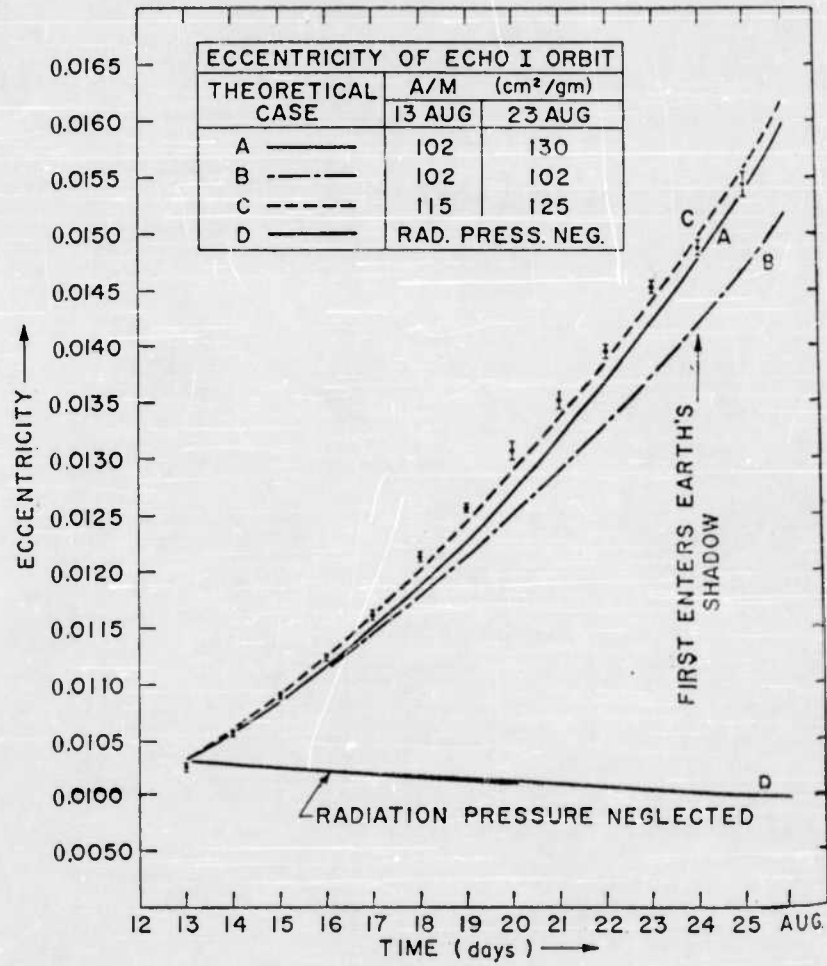Fig. 19 Time Variation of Argument of Perigee
(from Reference 21)

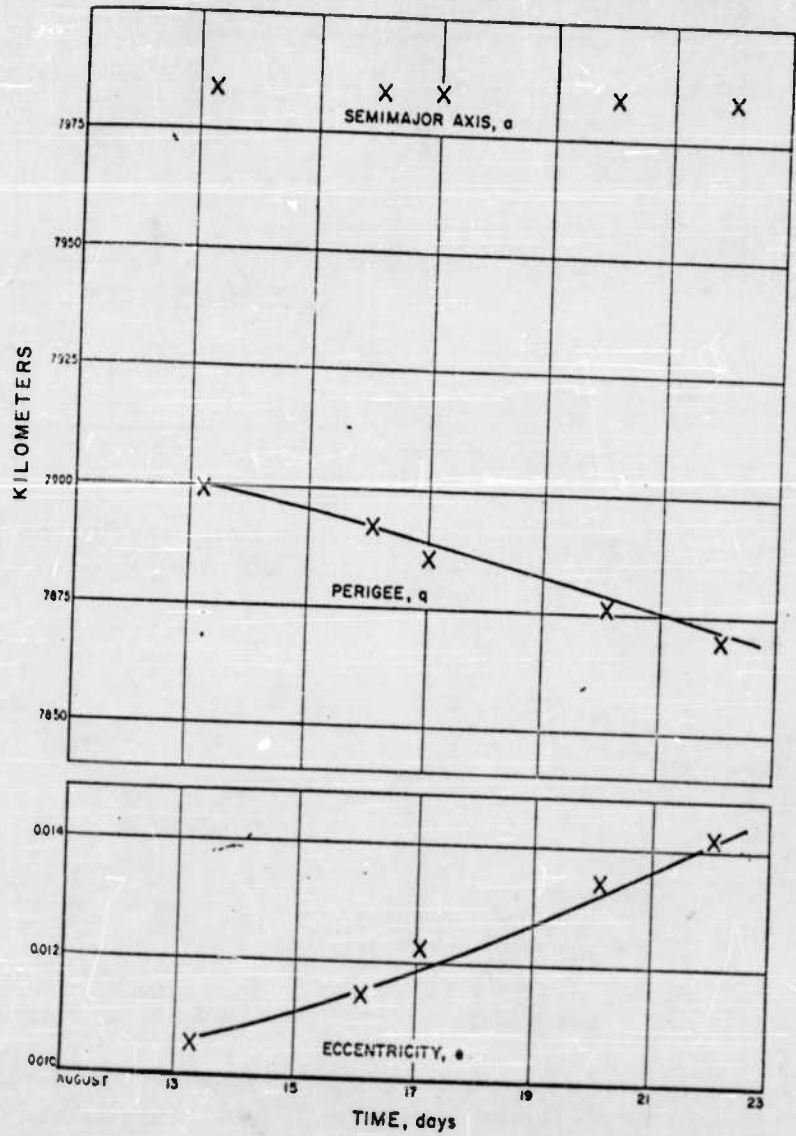Fig. 20   Time Variation of Eccentricity (from Reference 21)

Fig. 21   Effect of Solar Radiation Pressure on Orbital
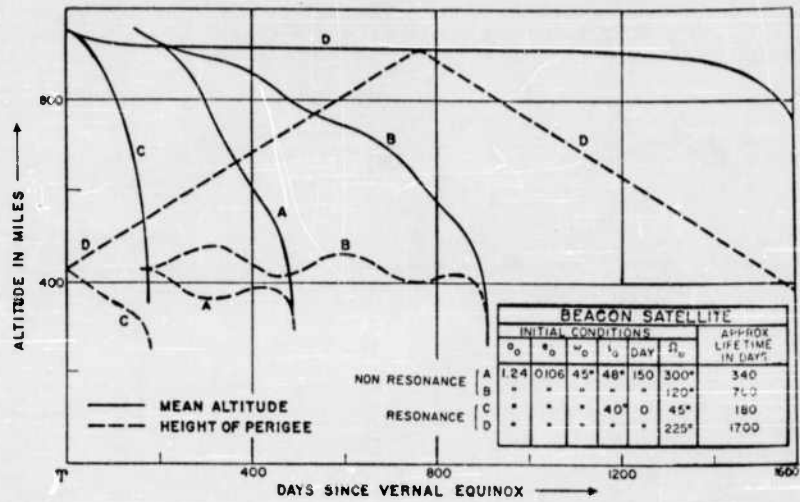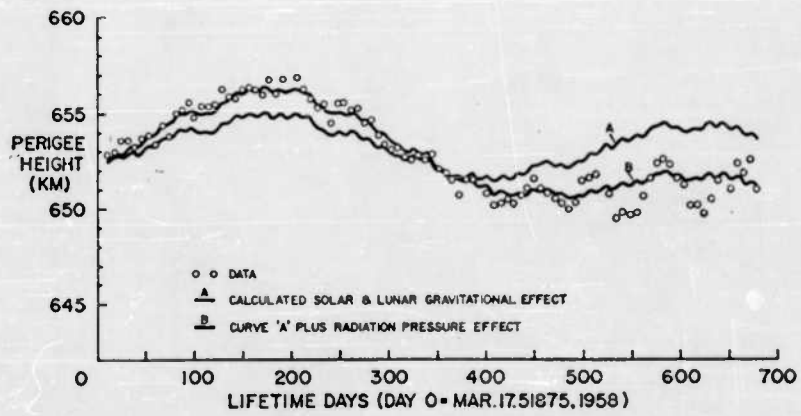Parameters of Echo 1 (from Reference 22)

Fig. 22   Time Variation of Perigee Height and Mean Altitude
far the Beacon Satellite (from Reference 20)

The data are published orbital elements with effect
of the third harmonic removed.

Fig. 23   Perturbations on Perigee Height of Vanguard I
(from Reference 23)

42

Magnetic Effects

If the outer shell of the satellite becomes electrically charged, its movement in a plasma creates three kinds of electromagnetic drag effects: coulomb drag, induction drag, and wave drag. It has been estimated[13] that for a negative potential of 10 volts and a particle density of 10^6 electrons/cm^3, the ratio of electrical-to-aerodynamic drag may be about 0.4 for certain cases, while inductive and wave drag appear to be negligible. At altitudes above about 750 statute miles, the magnetic drag may exceed neutral drag for large balloon satellites. Generally, this source of drag, however, has a negligible effect.

Lunar and Solar Perturbations

Lunar and solar attractions are the major sources of perturbations above the 24-hour orbit altitude of about 22,300 statute miles, but their effects are almost negligible for close earth satellites. The principal effect is a regression of the satellite orbit plane about the normal to the orbit plane of the perturbing body. This regression is given by

$$\dot{\Omega}_d = -\frac{3}{8} \frac{\mu_d}{r_d^3} \sqrt{\frac{a^3}{\mu}} \frac{\cos i_d}{(1-e_d^2)^{\frac{3}{2}}} \cos i \frac{2+3e^2}{\sqrt{1-e^2}} \quad ,$$

where the subscript d indicates a parameter of the disturbing body. This effect is plotted in Figs. 24 and 25 for the sun and the moon. There is also a radial or tidal perturbation, the maximum value being about

$$\Delta r_{max} = \frac{\mu_d}{\mu} \frac{r_c^4}{r_d^3}$$

for circular orbits of radius r_e. This value is plotted in Fig. 26. A summary graph of the major perturbations is given in Fig. 27.
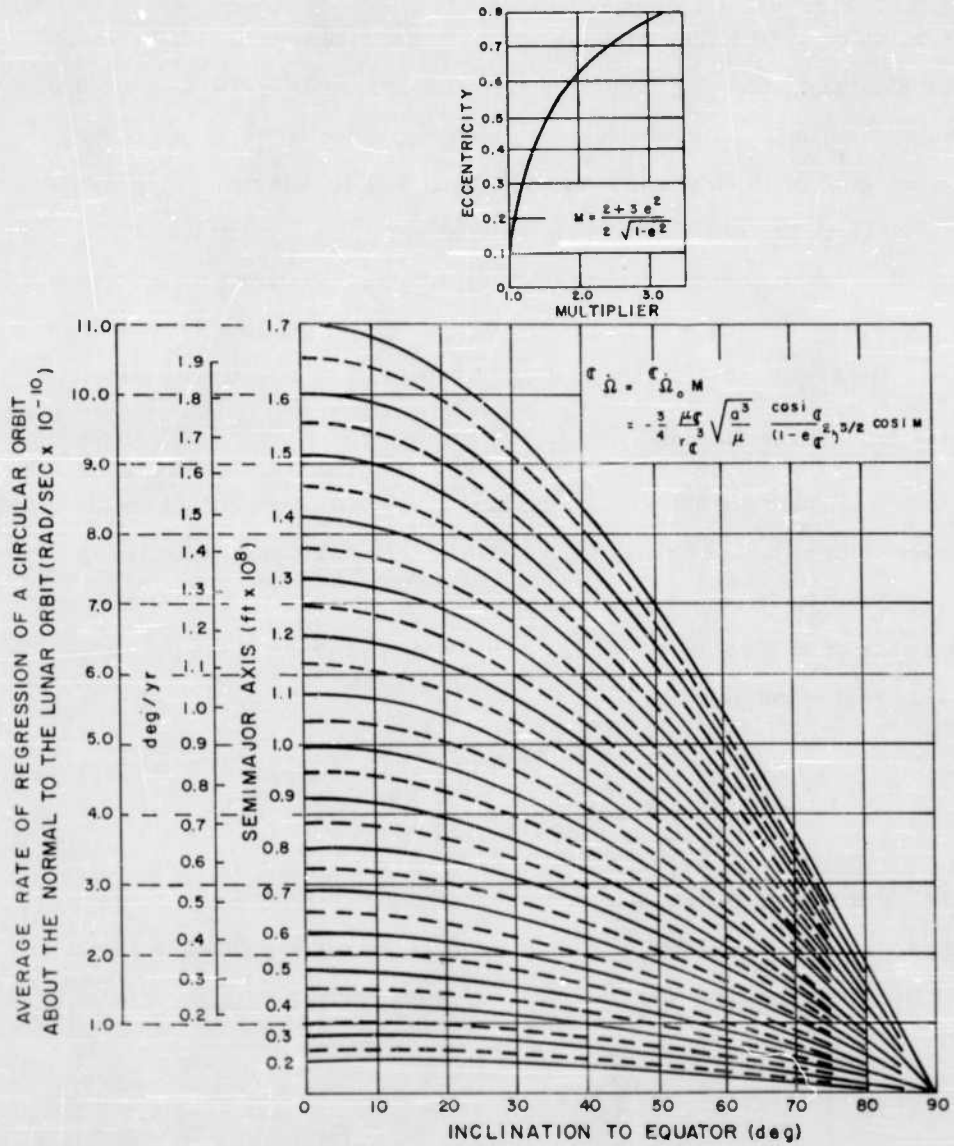
R. H. Greene

43

Fig. 24 Average Regression Rate about the Normal to the Lunar Orbit
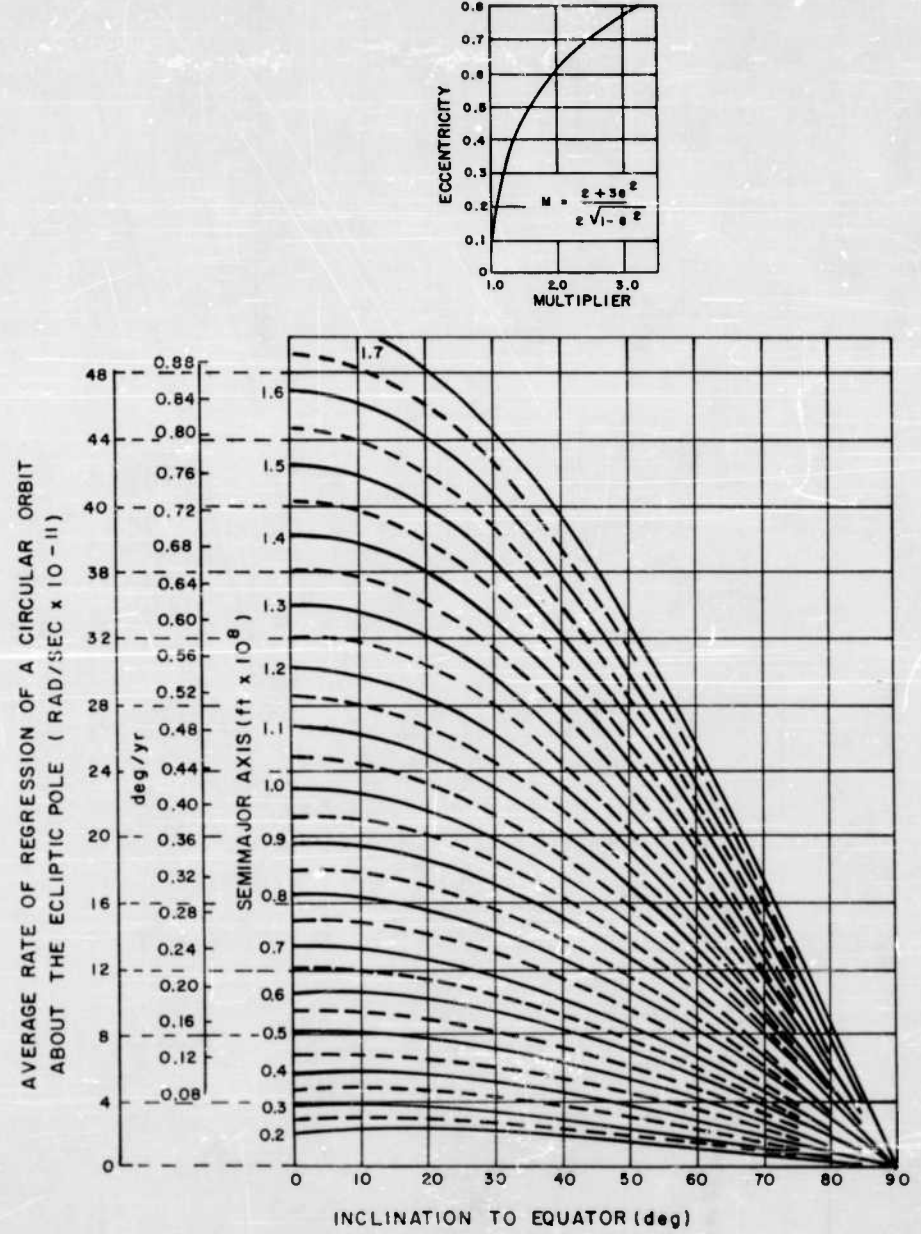Due to Lunar Attraction (from Reference 13)

44

Fig. 25   Average Ecliptic Nodal Regression Rate Due to Solar
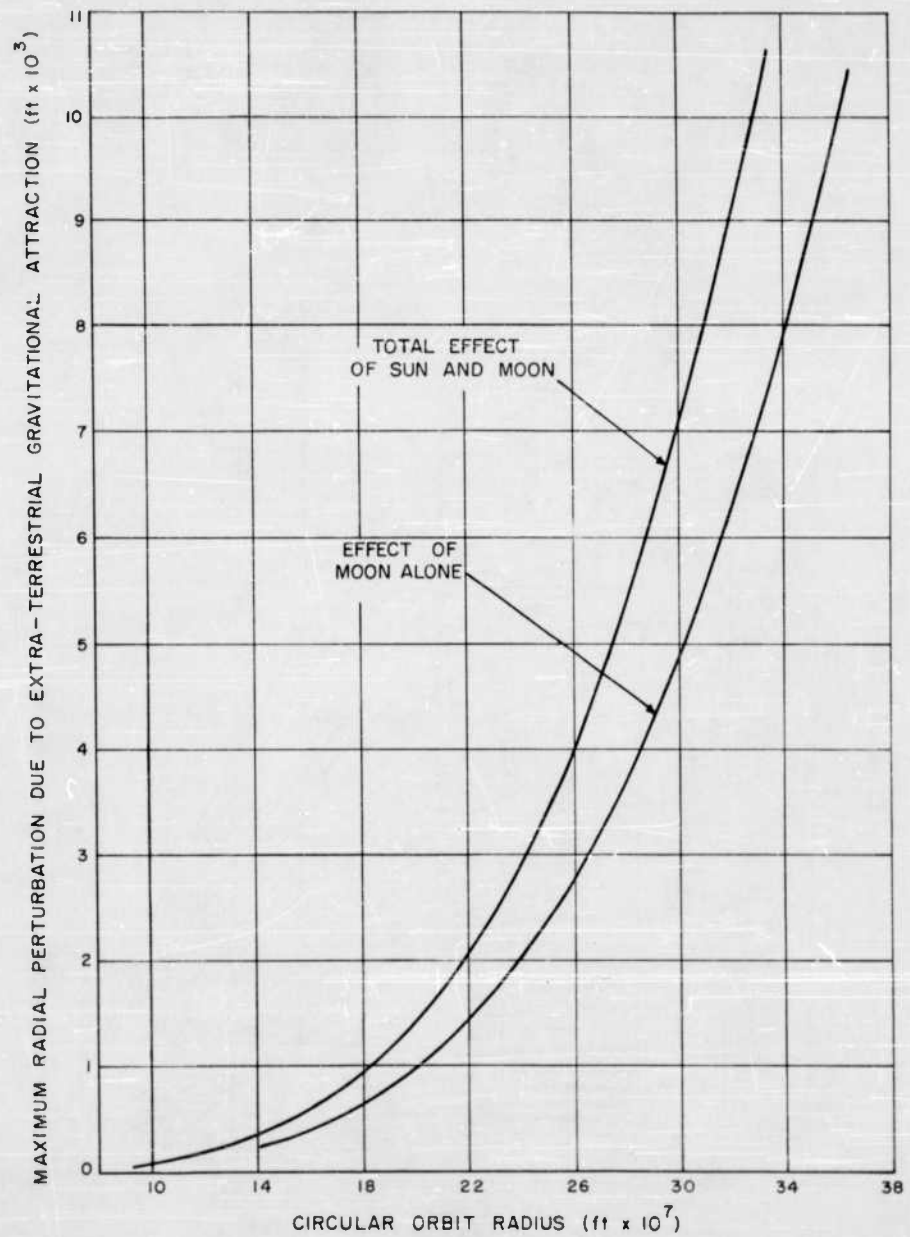Attraction (from Reference 13)

45

Fig. 26   Maximum Radial Perturbation Due to Attraction of the
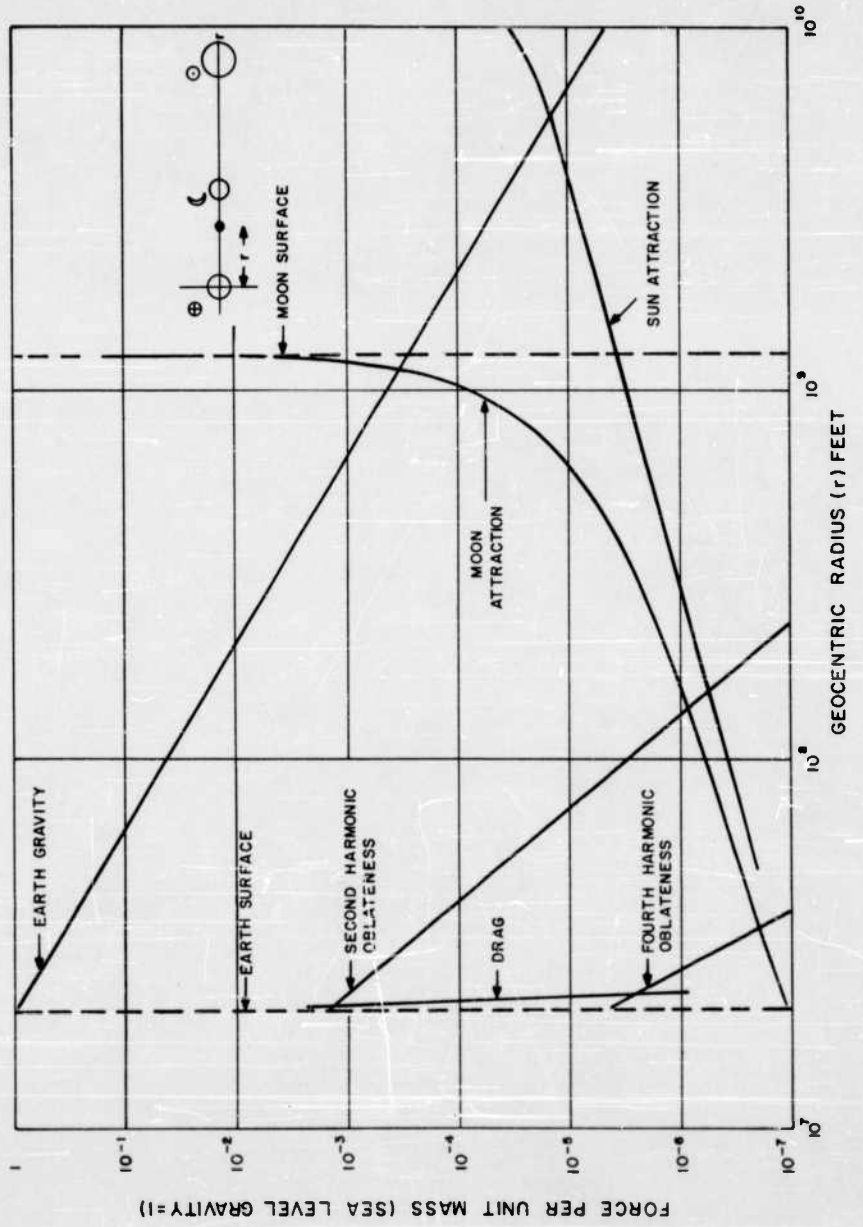Sun and Moon (from Reference 13)

Fig. 27 Comparison of Perturbation Magnitudes (from Reference 13)

REFERENCES

1. Contribution to Astrodynamics, Aeronutronic Publication U-880, Vol. I, June 1, 1960.

2. Moulton, F. R. An Introduction to Celestial Mechanics, New York, The MacMillan Company, 1914.

3. Herget, P. The Computation of Orbits, Cincinnati, Private Publication, 1948.

4. Watson, J. C. Theoretical Astronomy, London, J. B. Lippencott and Company, 1885.

5. Brouwer, D. "Solution of the Problem of Artificial Satellite Theory without Drag," Astronomical J., Vol. 64, No. 9, November 1959.

6. Kozai, Y. "The Motion of a Close Earth Satellite," Astronomical J., Vol. 64, No. 9, November 1959.

7. Garfinkel, B. "The Orbit of a Satellite of an Oblate Planet," Astronomical J., Vol. 66, No. 6, June 1961.

8. Vinti, J. P. "Theory of an Accurate Intermediary Orbit for Satellite Astronomy," J. Research NBS, Vol. 65B, No. 3, July—September 1961.

9. Contribution to Astrodynamics, Aeronutronic Publication U-880, Vol. II, June 1, 1960.

10. Brouwer, D. and Clemence, C. M. Methods of Celestial Mechanics, New York and London, Academic Press, 1961.

11. Eckert, W. J. and Brouwer, D. "The Use of Rectangular Coordinates in the Differential Correction of Orbits," Astronomical J., Vol. 46, No. 13, August 16, 1937.

12. King-Hele, D. G. and Gilmore, D. M. C. "The Effect of the Earth's Oblateness on the Orbit of a Near Satellite," Technical Note No. G. W. 475, Farnborough, England, Royal Aircraft Establishment, October, 1957.

13. Jensen, J., Townsend, G., Kork, J. and Kraft, D. Design Guide to Orbital Flight, New York, McGraw-Hill Book Company, Inc., 1962.

14. Groves, G.V., "Effect of the Earth's Equatorial Bulge on the Life-Time of Artificial Satellites and Its Use in Determining Atmospheric Scale Heights," Nature, April 12, 1958.

15. Taratynova, G.P., "The Motion of an Artificial Earth Satellite in the Eccentric Gravitational Field of the Earth when Atmospheric Resistance is Taken into Account," The Russian Literature of Satellites, New York, International Physical Index, Incorporated, 1958.

16. Parkyn, D.G., "Satellite Orbits in an Oblate Atmosphere," J. of Geophysical Res., Vol. 65, No. 1, January 1960, pp. 9-17.

17. Sterne, T.E., "Effect of the Rotation of a Planetary Atmosphere upon the Orbit of a Close Satellite," ARS J., October 1959.

18. Petersen, N.V., and Swanson, R.S., "Rendezvous in Space — Effects of Launch Conditions," Aerospace Eng., Vol. 19, No. 5, May 1960, pp. 72-73, 106-107.

19. King-Hele, K. Satellites and Scientific Research, London, Routledge and Kegan Paul, Ltd., 1960.

20. Shapiro, I.I., Jones, H.M., and Parkinson, R.W., "Effects of Solar Radiation Pressure on Earth Satellite Orbits," Science, Vol. 131, No. 3404, March 25, 1960.

21. Shapiro, I.I. and Jones, H.M., "Perturbations of the Orbit of the Echo Balloon," Science, Vol. 132, November 18, 1960.

22. Muhleman, D.O., Hudson, R.H., Holdridge, D.B., Carpenter, R.L., and Oslund, K.C., "Observed Solar Pressure Perturbations of Echo I," Science, Vol. 132, November 18, 1960.

23. Musen, P., Bryant, R. and Bailie, A., "Perturbations in Perigee Height of Vanguard I," Science, Vol. 131, March 25, 1960.

BIBLIOGRAPHY

1. Aeronutronic Progress Reports for 496-L, August 1961 to September 1962

2. Baker and Makemson, <u>An Introduction to Astrodynamics</u>, Academic Press, New York and London, 1960

3. Blanco, V. M., and McCuskey, S. W., <u>Basic Physics of the Solar System</u>, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1961

4. Brouwer, D. and Hori, G., "Theoretical Evaluation of Atmospheric Drag Effects in the Motion of an Artificial Satellite," The Astronomical Journal, Volume 66, Number 5, June, 1961

5. Project Mercury, Goddard Processing Programs prepared for NASA Contract Number NAS 1-430 by IBM, August 1, 1961

6. Smart, W. M., <u>Celestial Mechanics</u>, John Wiley & Sons, Inc., New York, 1961

7. <u>Space Technology</u> edited by H. S. Seifert, John Wiley and Sons, Inc., New York, 1959