

UNCLASSIFIED

AD **431961**

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

64-10

SPRINGFIELD ARMORY

SPRINGFIELD, MASSACHUSETTS

RESEARCH AND ENGINEERING

CATALOGED BY DDC

431961

AS AD No.

431961

Report: SA-TR15-1104

Date: 30 April 1963

CMS Code: 5011.11.83800.08

Report Title: Design Analysis of Belleville Washer Springs

Author H. P. Swieskowski
H. P. SWIESKOWSKI
Mathematician

Approved Stanley C. Skeiber
STANLEY C. SKEIBER
Lt Col, Ord Corps
Chief, Res and Eng Div

MAR 10 1964
DISPOSITIVE
315A A



The findings in this report are not to be construed as an official Department of the Army position.

DDC AVAILABILITY NOTICE. Qualified requesters may obtain copies of this report from DDC.

Report: SA-TR15-1104

Date: 30 April 1963

CMS Code: 5011.11.83800.08

Report Title: Design Analysis of Belleville Washer Springs

Author H. P. Swieskowski
H. P. SWIESKOWSKI
Mathematician

Approved Stanley C. Skeiber
STANLEY C. SKEIBER
Lt Col, Ord Corps
Chief, Res and Eng Div

Preparing Agency: Springfield Armory, Springfield, Mass.

Project Title: Basic Research in Physical Sciences (Engineering Sciences)

This TECHNICAL REPORT, to the extent known, does not contain any patentable material, copyrighted and/or copyrightable material, trade secrets, or trade names.

NONLIMITATION ON REPRODUCTION AND DISTRIBUTION. This is a nonlimited distribution report. Initial distribution of the report has been made in accordance with the attached distribution list. Reproduction and/or distribution by any installation or agency is authorized.

DESTROY. Do not return.

REPORT
SA-TR15-1104

ABSTRACT

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{OD}{TD}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

CONTENTS

	<u>Page</u>
Nomenclature	(ii)
Subject	1
Object	1
Summary of Conclusions	1
Introduction	2
Discussion of Analysis	2
Part I - Basic Assumptions and Derivation of Design Formulas	2
Part II - Comparison of a Single Spring with a Nested Arrangement	7
Part III - Comparison of a One-Parallel Series with a Two-Parallel Series and a Three-Parallel Series	10
General Remarks and Useful Formulas	13
Conclusions and Recommendations	14
Appendix A - Illustrations	16
Appendix B - Reference	23
Appendix C - Distribution	24

REPORT
SA-TR15-1104

NOMENCLATURE

<u>Symbols</u>	<u>Definitions</u>	<u>Dimensions</u>
$D_o = \text{O.D.}$	Outside diameter	in.
I.D.	Inside diameter	in.
h	Disk height	in.
t	Thickness of washer	in.
H_s	Solid height of spring assembly	in.
H_f	Free height of spring assembly	in.
F	Deflection	in.
$F_s = H_f = H_s$	Total stroke	in.
P	Load	lb
E_N	Energy capacity	in.-lb
S	Stress	psi
S_s	Final stress	psi
E	Modulus of elasticity	psi
Q	Poisson's ratio	-
A	Diameter ratio	-
B	Height-thickness ratio	-
Y	Constant	-
C_1	Constant	-
C_2	Constant	-
N	Number of washers in one-parallel series	-

NOMENCLATURE - Continued

<u>Symbols</u>	<u>Definitions</u>	<u>Dimensions</u>
S_1^i	Variation index of final stress	-
E_N^o, S_s^o	Energy capacity and final stress of outer spring in nested design	in.-lb, psi
E_N^i, S_s^i	Energy capacity and final stress of inner spring in nested design	in.-lb, psi
<u>For two-parallel series</u>		
N_2	Number of parallel units each containing two washers	-
B_2	Height-thickness ratio	-
S_{s2}	Final stress	psi
<u>For three-parallel series</u>		
N_3	Number of parallel units each containing three washers	-
B_3	Height-thickness ratio	-
S_{s3}	Final stress	psi

Basic Equations

$$(a) B = h/t = F_s/H_s \quad (b) H_s = Nt \quad (c) A = O.D./I.D.$$

$$(d) Y = \frac{6}{\pi \ln A} \left[\frac{A-1}{A} \right]^2$$

$$(e) C_1 = \left[\frac{A-1}{\ln A} - 1 \right] \frac{6}{\pi \ln A}$$

$$(f) C_2 = \frac{3(A-1)}{\pi \ln A}$$

REPORT
SA-TR15-1104

SUBJECT

An analytical method for the design of Belleville washers for energy storage was developed.

OBJECTIVES

1. To develop an analytical procedure for the design of Belleville washers for energy capacity.
2. To determine the stress reduction that is obtained by replacing a single washer assembly by a nested arrangement.
3. To determine the optimum stacking arrangement of Belleville washers.

SUMMARY OF CONCLUSIONS

Results of this study show that the final working stress is directly proportional to the square root of the energy requirement, and inversely proportional to the outside diameter and the square root of the solid height of the spring assembly. These results also show that the final stress is at a minimum when the diameter ratio $A = \frac{O.D.}{I.D.}$ equals 1.7. However, the range $1.5 \leq A \leq 2.0$ should be considered a favorable design range since the stress increases in this interval, at most, by 3 per cent. In the practical range of the height-thickness ratio (B), the final stress varies directly with B.

A stress reduction of 14 per cent is obtained by the substitution of a 2-spring nest for a single spring. The reduction is independent of the space envelope of the single spring.

Of the three stacking arrangements that were investigated - one-parallel series, two-parallel series, and three-parallel series, it was shown that (in the practical range of B values) the one-parallel series is the most efficient for energy storage.

1. INTRODUCTION

Previous design methods for Belleville washers for energy storage frequently involved the use of a maze of complex nomographs, numerous tables, and a series of tedious calculations. The method outlined in this report is a direct method requiring a minimum amount of calculations. This method shows immediately whether Belleville washers can be designed within safe stress levels to accommodate a given set of energy-space requirements. Also it represents a departure from previous published design methods since the solid height of the spring assembly is introduced as a basic design parameter and the entire spring assembly (rather than the elementary washer) is considered at the outset of design. This method shows the effect of the outside diameter, the energy capacity, and the solid height on the variation of the maximum working stress and indicates the optimum design values for ratios $A = \frac{O.D.}{I.D.}$ and $B = \frac{h}{t}$.

2. DISCUSSION OF ANALYSIS

Part I - Basic Assumptions and Derivation of Design Formulas

For simplification of the analysis, the minimum working height of the spring is considered equal to the solid height and no spring compression is required for assembly, i.e., the stroke, $F_s = H_f - H_s$. Basic dimensions of a single washer and an assembly of washers stacked in one-parallel series arrangement are shown in Figure 1. This type of arrangement is exclusively treated in the first part of the discussion.

The height-thickness ratio $B = h/t$ determines the slope of the load-deflection curve. The general shapes of load-deflection curves obtained from various B values when the washer or an assembly is compressed from free height to solid height are illustrated in Figure 2.

The conventional load and the stress formulas for Belleville washers are:

$$\text{Load} = P = \frac{4EF}{(1-Q^2)YD_0^2} \left[(h-F)(h-\frac{F}{2})t + t^3 \right] \quad (1)$$

$$\text{Stress} = S = \frac{4EF}{(1-Q^2)YD_0^2} \left[C_1(h-\frac{F}{2}) + C_2t \right] \quad (2)$$

2. DISCUSSION OF ANALYSIS - Continued

Equation (2) gives the value of the compressive stress that is acting on the convex side of the inner diameter and has a maximum value when $F = h$ of

$$S_s = \frac{4EBt^2}{(1-Q^2)YD_o^2} \left[C_1 \frac{B}{2} + C_2 \right] \quad (3)$$

The energy stored in one washer, when compressed from free height to the flat position, is obtained by integration of equation (1)

$$E_N = \int_0^h PdF = \frac{4E}{(1-Q^2)YD_o^2} \left[\frac{th^4}{8} + \frac{t^3 h^2}{2} \right] \quad (4)$$

And for an assembly of N washers in series

$$E_N = \frac{N4E}{(1-Q^2)YD_o^2} \left[\frac{th^4}{8} + \frac{t^3 h^2}{2} \right] \quad (5)$$

With the aid of equations a and b, equation (5) is transformed to

$$E_N = \frac{EH_s t^4 B^2 [B^2 + 4]}{2(1-Q^2)YD_o^2} \quad (6)$$

Equations (3) and (6) combined give the expression

$$S_s = \frac{4}{D_o} \sqrt{\frac{2EE_N}{(1-Q^2)H_s(B^2+4)Y}} \left(C_1 \frac{B}{2} + C_2 \right) \quad (7)$$

and for the usual spring steel materials where $E = 30 \times 10^6$ p.s.i. and $Q = .3$, equation (7) can be written as

$$S_s = \frac{32,480}{D_o} \sqrt{\frac{E_N}{H_s (B^2+4) Y}} \left(C_1 \frac{B}{2} + C_2 \right) \quad (8)$$

For all practical purposes, the stress at solid height (final stress) can be considered the maximum working stress in the spring. This is particularly true for small arms applications where spring space is limited and where the impact type loading is inherent in weapon function. In the design of Belleville washers, one of the main considerations is that of keeping the final stress at a safe and reasonable level. Equation (8) shows that the final stress is inversely proportional to the outside diameter and the square root of the solid height. Therefore,

2. DISCUSSION OF ANALYSIS - Continued

these two values should be made as large as space requirements will allow. Doubling the value of the outside diameter (keeping all other variables constant) will result in a 50 per cent reduction in the final stress. A similar increase in the solid height will produce a 30 per cent reduction. Equation (8) further shows that the final stress is directly proportional to the energy capacity, e.g., a two-fold increase in the energy requirement will effect a 41 per cent stress increase.

The values for the outside diameter, the solid height, and the energy capacity are usually given within narrow limits for a particular application. The outside diameter is determined by the hole diameter into which the spring must fit. The solid height is usually taken as .90 times the minimum operating height; the energy capacity is prescribed by functional considerations.

The question now is, What values of $A = \frac{O.D.}{I.D.}$ and $B = h/t$ should be selected so that the final stress is at a minimum? Visual examination of Equation (8) does not readily show the stress effect of the two ratios. However, a nomogram clearly illustrates their influence. In Figure 3, a family of curves is obtained by plotting the final stress factor S'_s with respect to ratio A with ratio B acting as a parameter. For graphical expediency, a modified expression of equation (8) is plotted in Figure 3.

$$S'_s = \sqrt{\frac{1}{(B^2+4)Y} \left(C_1 \frac{B}{2} + C_2 \right)} \quad (8A)$$

Final stress as shown in Figure 3 is at a minimum when the diameter ratio $A = 1.7$ for all values of B. However, for a given B value, the final stress increases at most by 3 per cent in the range $1.5 \leq A \leq 2.0$. Therefore, the range $1.5 \leq A \leq 2.0$ should be considered as a favorable design range. Furthermore, Figure 3 shows that in the favorable diameter ratio range, the final stress increases with increasing values of B. This condition is true except for the height-thickness ratio of $B = 3$ since in the range $1.5 \leq A \leq 2.0$, the final stress is less than that for smaller B values ($1 \leq B \leq 2$). However, it should be pointed out that Belleville washers have been generally designed for energy capacity with B values less than unity because of the required short working stroke and the necessity to provide a high degree of stability.

REPORT
SA-TR15-1104

2. DISCUSSION OF ANALYSIS - Continued

Design Example

Application of the method is outlined in the following example.

Given space-energy requirements:

Outside diameter, $D_o = .900$ in.

Solid height, $H_s = 2.035$ in.

Stroke, $F_s = .407$ in.

Energy requirement, $E_N = 100$ in.-lb

Material, AISI 6150

Step I - To obtain minimum final stress S_s , select $A = O.D./I.D. = 1.7$, therefore $Y = .61$ $C_1 = 1.15$ $C_2 = 1.26$.

Step II - In the absence of stroke requirement, select B as small as practical. In this case a stroke of .407 is required which means $B = \frac{h}{t} = \frac{F_s}{H_s} = .20$

Step III - Calculate final working stress with equation (8)

$$S_s = \frac{32,480}{D_o} \sqrt{\frac{E_N}{H_s(B^2+4)Y}} (C_1 \frac{B}{2} + C_2)$$
$$= \frac{32,480}{.900} \sqrt{\frac{100}{2.035(4.04)(.61)}} (1.15(.1) + 1.26) = 222,000 \text{ p.s.i.}$$

If the stress value of 222,000 p.s.i. is acceptable, the thickness t now can be calculated from either equation (3) or (6)

Step IV
From Equation (3)

$$t = \sqrt{\frac{S_s (1-Q^2) Y D_o^2}{4EB (C_1 \frac{B}{2} + C_2)}} = \sqrt{\frac{222,000(.91)(.61)(.81)}{3(30 \cdot 10^6)(.2)(1.375)}} = .055 \text{ in.}$$

Step V - Solve for disk height h , $h = Bt = .2(.055) = .011$

2. DISCUSSION OF ANALYSIS - Continued

Step VI - Solve for number of washers - $N = \frac{H_s}{t} = \frac{2.035}{.055} = 37$

Complete Design Data - Spring A

Thickness, .055"

Disk height, .011"

Number of washers, 37

Outside diameter, .900

$A = O.D./I.D. = 1.7$; therefore, $I.D. = .530$, $Y = .61$, $C_1 = 1.15$,
and $C_2 = 1.26$

Height-thickness ratio, .20

Energy capacity, 100 in.-lb

Final stress, 222,000 p.s.i.

Material, AISI 6150 ($E = 30.10^6$ and $Q = .3$)

It is interesting to note that the analysis shows that increased spring travel can be obtained with no corresponding increase in final stress. At point $A = 2.0$ and $S_s = 1.07$ (Figure 3) the two curves, $B = 1$ and $B = 3$, intersect. This, therefore, indicates that two spring designs occupying the same spring space (i.e., H_s and D_o) will have equal energy capacities and equal final stresses. However, their total travel, F_s , will be in the proportion of 3 to 1. For a numerical example, the following spring requirements are considered:

Outside diameter, $D_o = 2.300$ "

Inside diameter, $D_i = 1.150$ "

Solid height, $H_s = 1.650$ "

Total travel, $F_s \leq 3.000$ "

Energy requirement, 342 in.-lb

Material, AISI 6150

REPORT
SA-TR15-1104

2. DISCUSSION ANALYSIS - Continued

A pair of designs that corresponds to the intersection $B = 1$ and $B = 3$ is as follows:

<u>Nomenclature</u>	<u>Spring B</u>	<u>Spring C</u>
Outside diameter, D_o (in.)	2.300	2.300
Inside diameter, D_i (in.)	1.150	1.150
Height, h (in.)	.055	.075
Thickness, t (in.)	.055	.025
Height-thickness ratio, $h/t = B$	1.0	3.0
Diameter ratio, O.D./I.D. = A	2.0	2.0
Number of washers	30	66
Solid height, H_s (in.)	1.65	1.65
Final stress, S_s (p.s.i.)	218,000	218,000
Energy capacity, E_N (in.-lb.)	342	342
Material	AISI 6150	AISI 6150

It is to be observed that the solid height, the outside diameter, the final stress, and the energy capacity of both designs are equal. The load-deflection diagrams for each design are shown in Figure 4. The energy capacity of Design B is represented by the positive inclined lines; that of Design C, by the negative inclined lines. The energy content that is common to both designs is indicated by the double-cross hatched area. The total travel for Design C (4.95 in.) is three times that of Design B (1.65 in.).

Part II - Comparison of a Single Spring with a Nested Arrangement

For simplification of the analysis, it is assumed that there is no diametral clearance between the nested springs. Furthermore, for a meaningful comparison, both assemblies should have the same values for the following characteristics.

2. DISCUSSION OF ANALYSIS - Continued

- a. Energy capacity, $E_N = E_N^0 + E_N^i$
- b. Stroke, F_s
- c. Solid height, H_s
- d. Diameter ratio, $A = 1.7$
- e. Material, AISI 6150
- f. Outside diameter (i.e., outside diameter of nested arrangement equals outside diameter of single spring)

Properties of the single spring are given by Equation (8). Similarly, the nested system (Figure 5) is described.

for outer spring

$$S_s^0 = \frac{32,480}{D_o} \sqrt{\frac{E_N^0}{H_s(B^2+4)Y}} \left(C_1 \frac{B}{2} + C_2 \right) \quad (9)$$

for inner spring

$$S_s^i = \frac{32,480}{\frac{D_o}{1.7}} \sqrt{\frac{E_N^i}{H_s(B^2+4)Y}} \left(C_1 \frac{B}{2} + C_2 \right) \quad (10)$$

For efficient design, the stress in the nested arrangement should be equally distributed, i.e., $S_s^0 = S_s^i$. Therefore, Equations (9) and (10) result in:

$$2.89 E_N^i = E_N^0 \quad (11)$$

From basic assumption $E_N = E_N^0 + E_N^i$ and Equation (11), it follows that

$$E_N = 1.346 E_N^0 \quad (12)$$

The relationship between the final stress of the single spring and that of the nested spring is derived from Equations (8), (9) and (12) as

$$S_s = 1.16 S_s^0 \quad (13)$$

REPORT
SA-TR15-1104

2. DISCUSSION OF ANALYSIS - Continued

Therefore, the percentage reduction in final stress that is gained by the substitution of a nested arrangement for a single spring is

$$AS = \% \text{ reduction} = \left[\frac{S_s - S_s^0}{S_s} \right] 100 = \left[\frac{S_s - 1.16 \frac{S_s}{1.16}}{S_s} \right] 100 = 14\%$$

The stress reduction is constant and applies generally since it is independent of the solid height and outside diameter of the single spring.

Numerical Example

If the final stress (222,000 p.s.i.) of Spring A in Part I is assumed to be excessive, then the use of a nested arrangement will allow this stress to be reduced to 191,000 p.s.i., i.e., a stress reduction of 31,000 p.s.i. A nested arrangement that is equivalent to Spring A in energy and space conditions is as follows:

<u>Nomenclature</u>	<u>Outer Spring</u>	<u>Inner Spring</u>
Thickness (in.)	.051	.030
Disk height (in.)	.0102	.006
Number of washers	40	68
Outside diameter (in.)	.900	.530
Inside diameter (in.)	.530	.312
A = O.D./I.D.	1.7	1.7
Height-thickness ratio	.20	.20
Energy capacity (in.-lb.)	74	26
Final stress (p.s.i.)	191,000	191,000
Stroke (in.)	.408	.408
Solid height (in.)	2.040	2.040
Material	AISI 6150	AISI 6150

2. DISCUSSION OF ANALYSIS - Continued

The solid height and the total stroke of the nested design are not exactly equal to those of the single spring because the number of washers in each spring has to be a whole number. However, the differences are negligible and, for comparison purposes, are considered equal.

Part III - Comparison of a One-Parallel Series with a Two-Parallel Series and a Three-Parallel Series

Again, for a valid comparison, all spring assemblies should have the same values for:

- a. Energy capacity, E_N
- b. Stroke, F_S
- c. Solid height, H_s
- d. Diameter ratio, $A = 1.7$
- e. Material, AISI 6150
- f. Outside diameter, D_o

For two washers in parallel, the load-deflection formula is

$$\text{Load} = P = 2 \left\{ \frac{4EF}{(1-Q^2)YD_o^2} \left[\frac{(h-F)(h-F)t}{2} + t^3 \right] \right\} \quad (1A)$$

The energy stored in the washers upon compression from free to solid height is

$$E_N = \int_0^h P \, dF = \frac{8E}{(1-Q^2)YD_o^2} \left[\frac{th^4}{8} + \frac{t^3h^2}{2} \right] \quad (4A)$$

and for N_2 pairs of washers stacked in a two-parallel series, as shown in Figure 5, the energy is

$$E_N = \frac{N_2 \ 8 \ E}{(1-Q^2)YD_o^2} \left[\frac{th^4}{8} + \frac{t^3h^2}{2} \right] \quad (5A)$$

2. DISCUSSION OF ANALYSIS - Continued

With the Equations $H_s = 2 N_2 t$ and $B_2 = \frac{h}{t}$,

Equation (5A) is transformed to

$$E_N = \frac{EH_s t^4 B_2^2 (B_2^2 + 4)}{2(1-Q^2)Y D_0^2} \quad (6A)$$

The solid stress of the two-parallel series is

$$S_{s2} = \frac{4E B_2 t^2}{(1-Q^2)Y D_0^2} \left(C_1 \frac{B_2}{2} + C_2 \right) \quad (3A)$$

Equations (3A) and (6A) combined give the following expression for the two-parallel series:

$$S_{s2} = \frac{4}{D_0} \sqrt{\frac{2E E_N}{(1-Q^2) H_s (B_2^2 + 4) Y}} \left(C_1 \frac{B_2}{2} + C_2 \right) \quad (7A)$$

The relationship between the final stress of the one-parallel series and the final stress of two-parallel series is derived from Equations (7) and (7A) as

$$\frac{S_s}{S_{s2}} = \sqrt{\frac{B_2^2 + 4}{B^2 + 4}} \left[\frac{C_1 \frac{B}{2} + C_2}{C_1 \frac{B_2}{2} + C_2} \right] \quad (14)$$

From the given condition that both assemblies should have equal strokes, it follows that

$$2B = B_2 \quad (15)$$

Therefore, Equation (14) is rewritten as

$$\frac{S_s}{S_{s2}} = \sqrt{\frac{B^2 + 1}{B^2 + 4}} \left[\frac{C_1 B + 2C_2}{C_1 B + C_2} \right] \quad (16)$$

A graph of Equation (16) is shown in Figure 6. It can be seen that in the practical range of B, ($B \leq 1$), the one-parallel series is more efficient than the two-parallel series. This is particularly true for B values between .3 and .6 where an 8 per cent stress reduction can be realized.

2. DISCUSSION OF ANALYSIS - Continued

Similarly, the final stress of the three-parallel series, shown in Figure 5, is

$$S_{s3} = \frac{4}{D_o} \sqrt{\frac{2 E E N}{(1-Q^2)H_s (B_3^2 + 4)Y}} \left(C_1 \frac{B_3}{2} + C_2 \right) \quad (17)$$

Since both assemblies have equal strokes when Equation (17) and Equation (7) are combined, it follows that

$$\frac{S_s}{S_{s3}} = \sqrt{\frac{9B^2 + 4}{B^2 + 4}} \left[\frac{C_1 \frac{B}{2} + C_2}{C_1 \frac{3B}{2} + C_2} \right] \quad (18)$$

Equation (18) is also plotted (Figure 6). It is shown that, in the practical B range, the one-parallel series offers a better utilization of spring space than a three-parallel series. For example, two spring assemblies corresponding to point B = .4 and $\frac{S_s}{S_{s3}} = .87$ - a one-parallel series (B = .4) and a three-parallel series (B₃ = 1.2) - will have equal strokes, energy capacities, and occupy the same space. However, the final stress of the one-parallel series will be 13 per cent less. A pair of assemblies that corresponds to this particular point is tabulated below:

<u>Nomenclature</u>	<u>One-Parallel Series</u>	<u>Three-Parallel Series</u>
N, Number of individual washers	26	48
N ₃ , Number of parallel units, each containing three washers		16
D _o , Outside diameter (in.)	1.87	1.87
D _i , Inside diameter (in.)	1.10	1.10
A = D _o /D _i , Diameter ratio	1.7	1.7
h, Height (in.)	.034	.055
t, Thickness (in.)	.085	.046
B = h/t, Height-thickness ratio	.4	1.2

REPORT
SA TR15-1104

2. DISCUSSION OF ANALYSIS - Continued

<u>Nomenclature</u>	<u>One-Parallel Series</u>	<u>Three-Parallel Series</u>
F_s , Stroke (in.)	.884	.884
H_s , Solid height (in.)	2.21	2.21
E_N , Energy capacity (in.-lb)	600	600
S_s , Final stress (p.s.i.)	266,000	305,000

3. GENERAL REMARKS AND USEFUL FORMULAS

For simplification of the analysis, it was assumed that there was no initial spring compression and also that there was no clearance between the minimum operating height and solid height. However, in actual practice, it is recommended that a small precompression be applied to prevent looseness and that clearance be provided to avoid loading to flat position. The two recommendations can be easily satisfied by designing for a total energy capacity slightly larger than actually required.

Stress values given by Equations (2) and (8) are localized stresses that occur at the inner diameter and not throughout the entire cross section. Therefore, calculated stress values may, at times, exceed the yield point of the spring material and yet be permissible.

Formulas that can be used advantageously for load-deflection calculations are:

$$\text{Rate} = \frac{P}{F} = \frac{4E \tau^3}{(1-Q^2)D_0^2 Y N} \quad (19) \quad \text{and} \quad \text{Rate} = \frac{P}{F} = \frac{4E\tau^4}{(1-Q^2)D_0^2 Y H_s} \quad (20)$$

For the usual spring materials where

$E = 30.10^6$ p.s.i. and $Q = .3$, the above equations are reduced to

$$\text{Rate} = \frac{P}{F} = \frac{132.10^6 \tau^3}{D_0^2 Y N} \quad (19A) \quad \text{and} \quad \text{Rate} = \frac{132.10^6 \tau^4}{D_0^2 Y H_s} \quad (20A)$$

The formulas are more convenient to use than the Equation (1) and are acceptably accurate for B values less than or equal to unity where the rate is essentially linear (Figure 2).

4. CONCLUSIONS AND RECOMMENDATIONS

It was found that the final stress is inversely proportional to the outside diameter and the square root of the solid height. Therefore, these two dimensions should be as large as space requirements will allow. Furthermore, the stress is at a minimum when the diameter ratio $A = 1.7$. However, the range $1.5 \leq A \leq 2.0$ should be considered a favorable design range since the stress increases slightly above the minimum in this interval. Because the final stress increases directly with the height-thickness ratio, it is recommended that B be selected as small as practical, which in effect means that the total stroke be held at a minimum since $B = F_s/H_s$.

In the practical range of the height-thickness ratio, the one-parallel series offers better utilization of spring space and is preferred to the two- and three-parallel series. In overstressed applications, where longitudinal and radial space is limited, consideration should be given to a 2-spring nested arrangement. The substitution of a nest for a single spring will result in a 14 per cent stress reduction.

REPORT
SA-TR15-1104

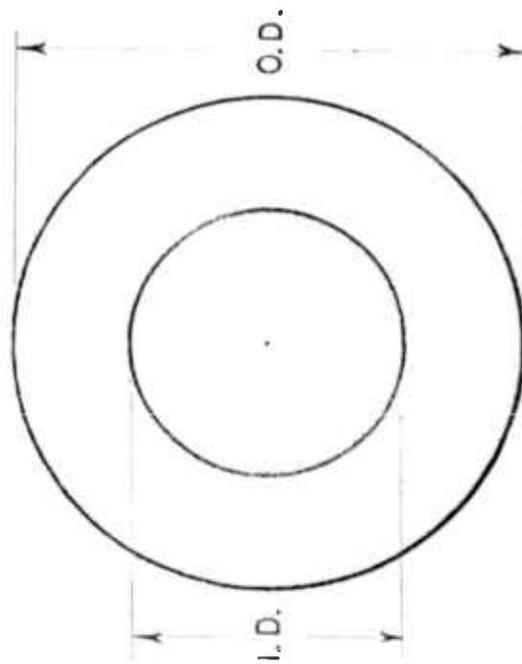
APPENDICES

- A - Illustrations
- B - Reference
- C - Distribution

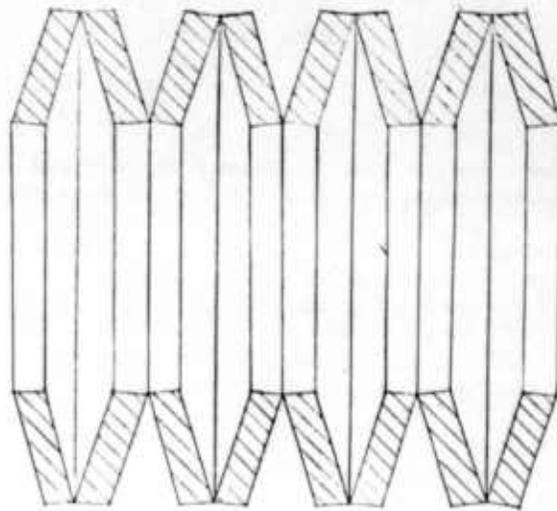
ILLUSTRATIONS

- Figure 1. Drawing - Single Washer and One-Parallel Series Assembly
- Figure 2. Nomograph - General Shapes of Load-Deflection Curves that are Obtained for Various B Values
- Figure 3. Nomograph - Variation Index of Final Stress with Respect to Diameter Ratio with Height-Thickness Ratio B as a Parameter
- Figure 4. Graph - Load-Deflection Curves of Springs B and C Showing Equal Energy Contents
- Figure 5. Drawing - Nested Arrangement, Two-Parallel Series Assembly and Three-Parallel Series Assembly
- Figure 6. Graph - Stress Ratio S_s/S_{s_2} , S_s/S_{s_3} with Respect to Height-Thickness Ratio B

FIGURE 1

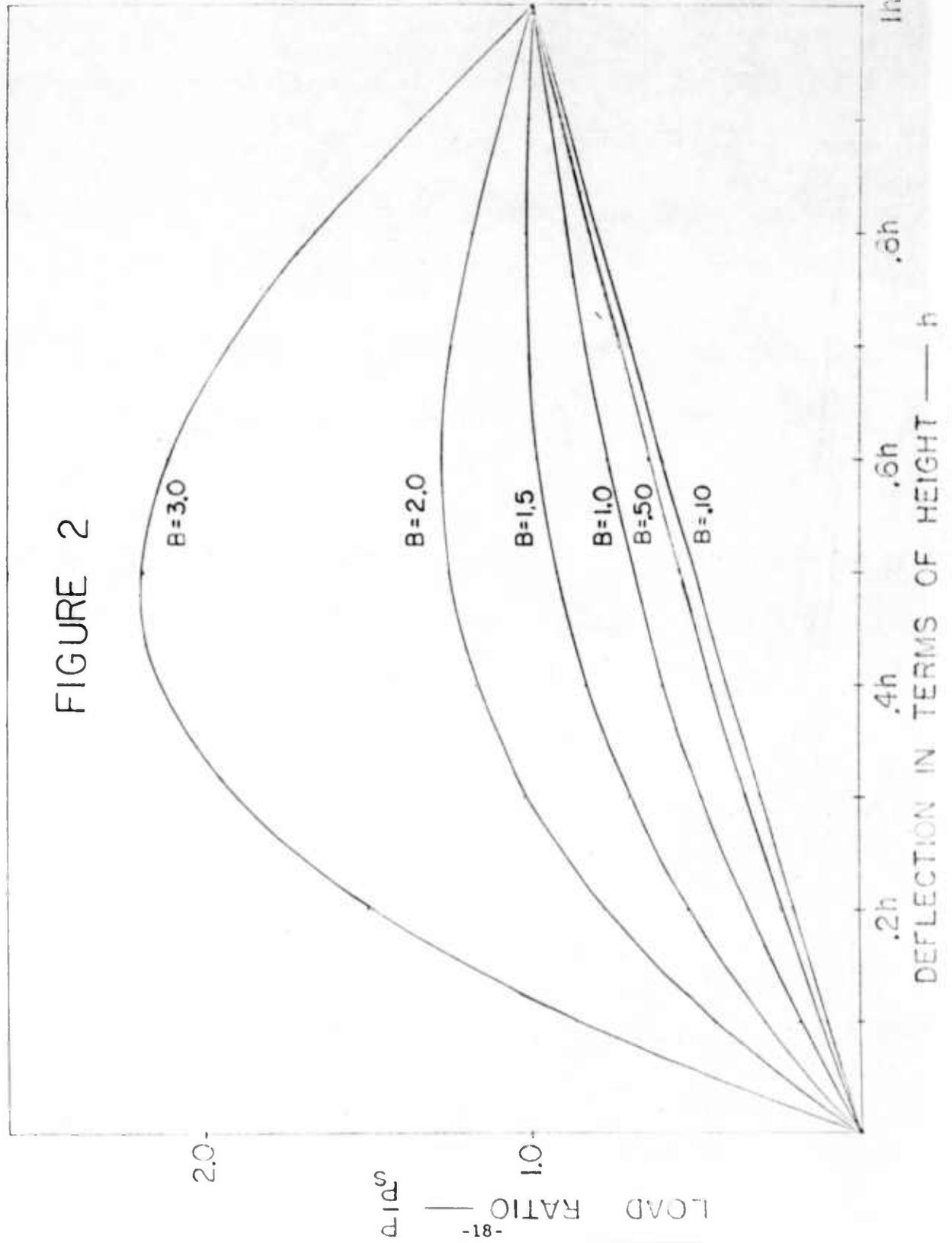


BELLEVILLE WASHER



ONE - PARALLEL
SERIES

FIGURE 2



LOAD RATIO —
-18-

DEFLECTION IN TERMS OF HEIGHT — h

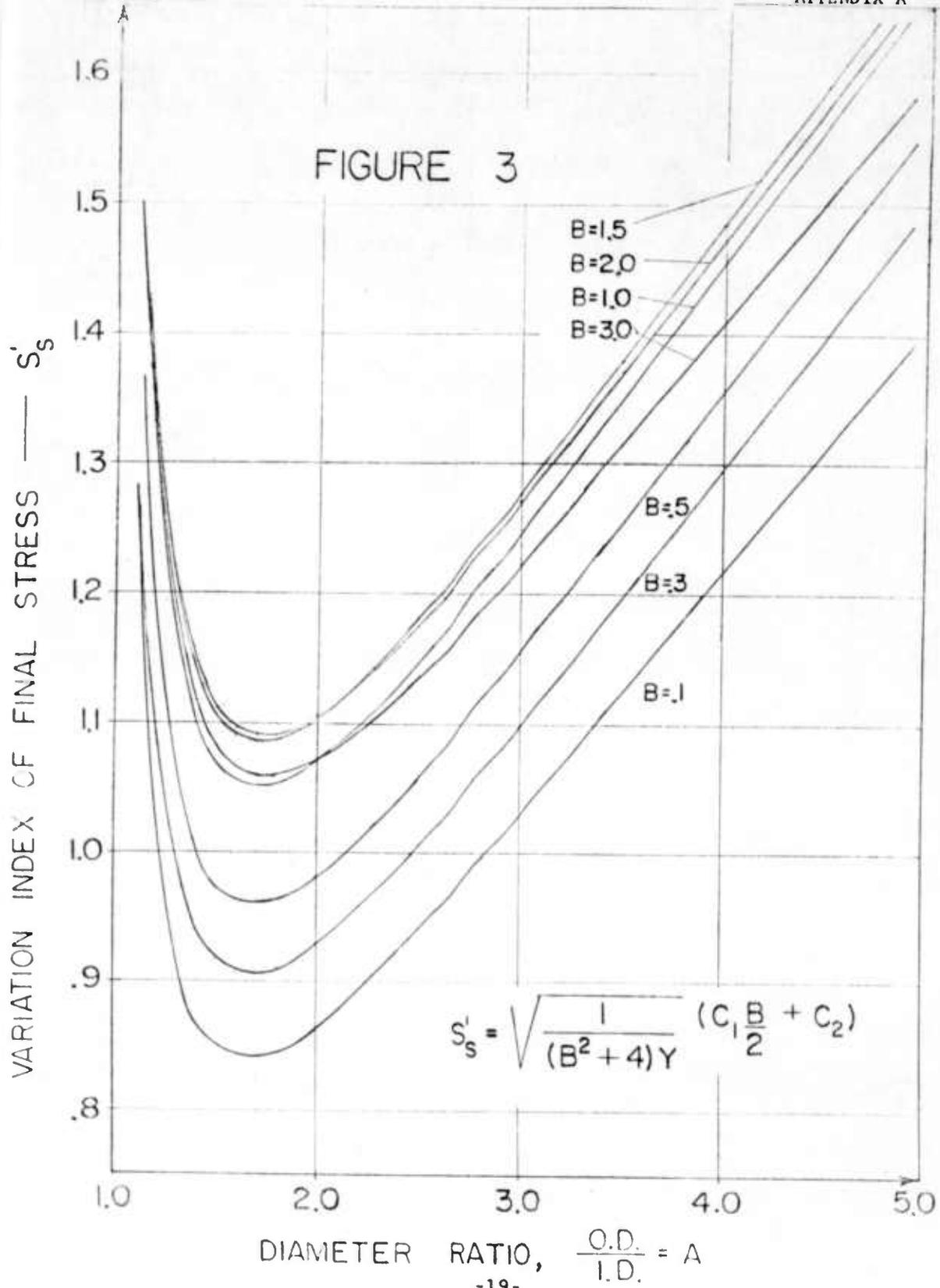


FIGURE 4

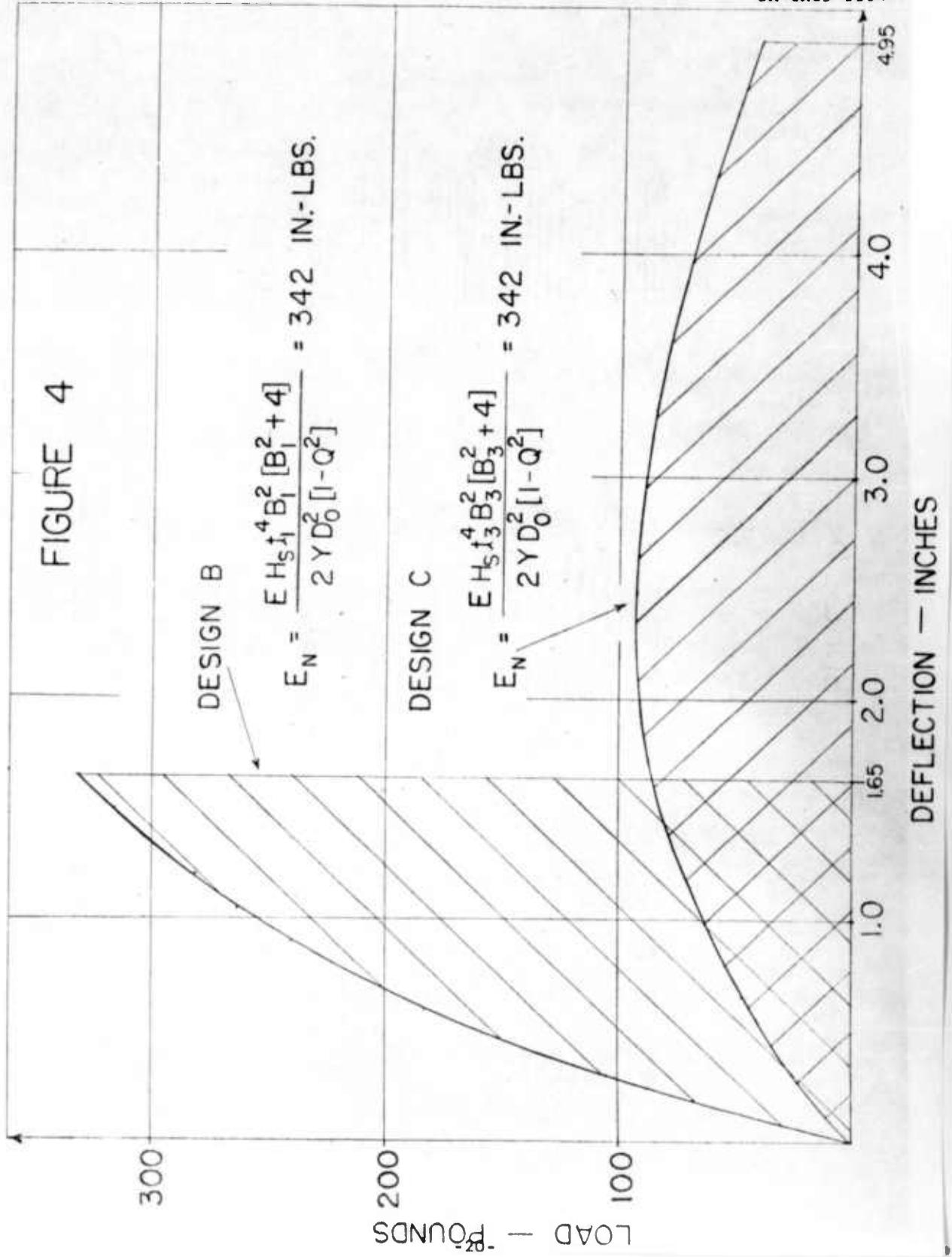
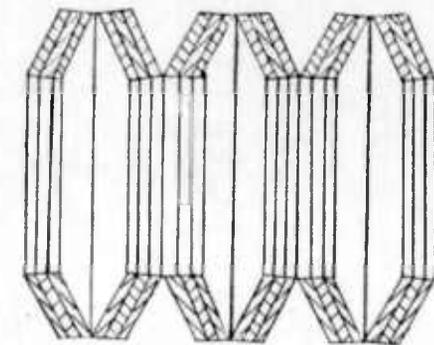
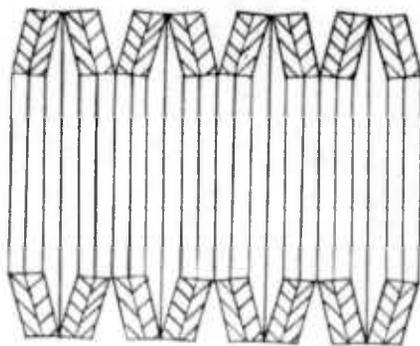


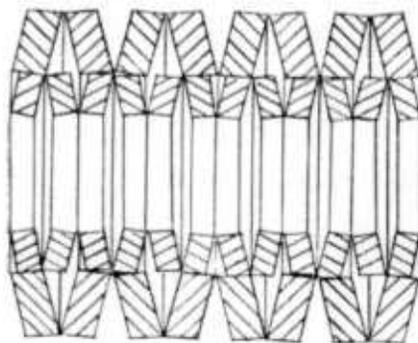
FIGURE 5



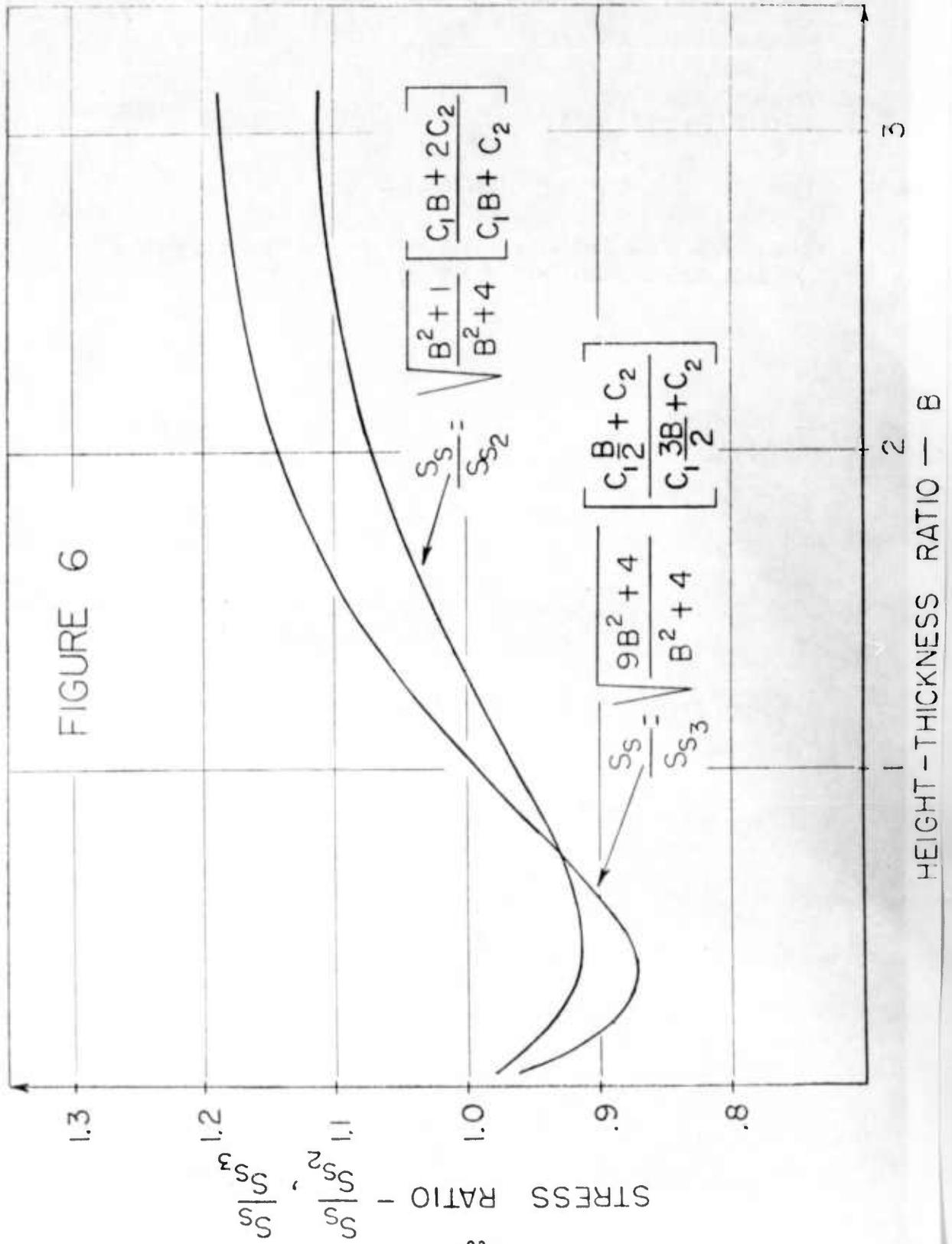
THREE - PARALLEL
SERIES



TWO - PARALLEL
SERIES



NESTED
ARRANGEMENT



REPORT
SA-TR15-1104

APPENDIX B

REFERENCE

Almen, J. O., and Laszlo, A. "The Uniform-Section Disk Spring,"
ASME Transactions, Vol. 58 (May 1936).

APPENDIX C

REPORT
SA-TR15-1104DISTRIBUTION

	<u>Copies</u>
Commanding General U.S. Army Materiel Command Bldg. T-7 (Room 817) Department of the Army Washington 25, D. C. 20310	1
Commanding Officer U.S. Army Coating and Chemical Laboratory Aberdeen Proving Ground, Maryland 21005	2
Headquarters U.S. Army Mobility Command Warren, Michigan 48090	1
Commanding General Aberdeen Proving Ground ATTN: Technical Library Aberdeen, Maryland 21005	1
Commanding General U.S. Army Weapons Command ATTN: AMSWE-RD (1) AMSWE-RDF-3 (1) Rock Island, Illinois 61202	2
Commanding General U.S. Army Ballistic Missile Agency ATTN: Doc & Tec Info Branch Redstone Arsenal, Alabama	2
Commanding General U.S. Army Rocket & Guided Missile Agency ATTN: ORDXR-RGS (1) ORDXR-IQI (1) Redstone Arsenal, Alabama	2
Commanding Officer Frankford Arsenal ATTN: ORDBA-1330 (1) Library-0270 (1) Philadelphia 37, Pennsylvania	2

REPORT
SA-TR15-1104

APPENDIX C

DISTRIBUTION - Continued

	<u>Copies</u>
Commanding Officer Army Materials Research Agency Watertown Arsenal ATTN: RPD Watertown 72. Mass.	1
Commanding Officer Picatinny Arsenal ATTN: Plastics & Packaging Lab Dover, N. J.	1
Commanding Officer Rock Island Arsenal ATTN: Materials Section, Laboratory Rock Island, Illinois 61202	1
Commanding Officer Watertown Arsenal ATTN: ORDSD-LX Watertown 72, Mass.	1
Commanding Officer Watervliet Arsenal ATTN: ORDBF-RR Watervliet, New York	1
Commanding Officer Harry Diamond Laboratories ATTN: AMXDO-TIB Connecticut Avenue and Van Ness St., N.W. Washington 25, D. C. 20438	1
Commanding Officer Engineer R&D Laboratory ATTN: Materials Branch Ft. Belvoir, Virginia	1
Headquarters U.S. Army Signal R&D Laboratory ATTN: Materials Branch Fort Monmouth, N. J.	1

APPENDIX C

REPORT
SA-TR15-1104DISTRIBUTION - Continued

	<u>Copies</u>
Commander Army Research Office Arlington Hall Station Arlington 12, Virginia	1
Commanding Officer U.S. Army Research Office (Durham) Box CM Duke Station Durham, North Carolina	1
Commanding General White Sands Proving Ground Las Cruces, New Mexico	1
Commanding General U.S. Army Test and Evaluation Command ATTN: Technical Library, Bldg. 313 (2) Director of Infantry Materiel Testing (2) Aberdeen Proving Ground, Maryland 21005	4
Commanding Officer U.S. Army Ordnance Training Command ATTN: ORDHB-CR-C Aberdeen Proving Ground, Maryland 21005	1
Commandant U.S. Army Ordnance School ATTN: USAOS Technical Library (1) Non-Resident Training Division (1) Aberdeen Proving Ground, Maryland 21005	2
Department of the Navy Office of Naval Research ATTN: Code 423 Washington 25, D. C.	1
Wright Air Development Division ATTN: ASRCEE-1 (1) WWRCO (1) Materials Central (1) Wright-Patterson Air Force Base, Ohio	3

REPORT
SA-TR15-1104

APPENDIX C

DISTRIBUTION - Continued

	<u>Copies</u>
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	10
Jet Propulsion Laboratory California Institute of Technology ATTN: Dr. L. Jaffe 4800 Oak Grove Drive Pasadena California	1
Director, National Bureau of Standards Department of Commerce Washington 25, D. C.	2
Associated Spring Corporation Research and Development Division ATTN: Mr. H. Perry Smith Bristol, Connecticut	1
Eaton Manufacturing Company Spring Division ATTN: Mr. J. C. Crabtree 9771 French Road Detroit 13, Michigan	1
Chrysler Corporation P.O. Box 1118 ATTN: Mr. Bernard Sterne Detroit 31, Michigan	1

ABSTRACT

AD _____ Accession _____
 UNCLASSIFIED
 Springfield Army, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swieckowski,
 Technical Report SA-TR13-1104, 30 April 1963, 27 pp incl illus.
 OCS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.
 UNLIMITED DISTRIBUTION.

A theoretical study was made to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D_o}{D_i}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

ABSTRACT

AD _____ Accession _____
 UNCLASSIFIED
 Springfield Army, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swieckowski,
 Technical Report SA-TR13-1104, 30 April 1963, 27 pp incl illus.
 OCS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.
 UNLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D_o}{D_i}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

ABSTRACT

AD _____ Accession _____
 UNCLASSIFIED
 Springfield Army, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swieckowski,
 Technical Report SA-TR13-1104, 30 April 1963, 27 pp incl illus.
 OCS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.
 UNLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D_o}{D_i}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

ABSTRACT

AD _____ Accession _____
 UNCLASSIFIED
 Springfield Army, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swieckowski,
 Technical Report SA-TR13-1104, 30 April 1963, 27 pp incl illus.
 OCS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.
 UNLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D_o}{D_i}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

UNCLASSIFIED

UNCLASSIFIED

ABSTRACT

UNCLASSIFIED

Accession

AD

Springfield Armory, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swietkowski,
 Technical Report SA-TR15-1104, 30 April 1963, 27 pp. Incl illus.
 OAS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.

NONLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D}{d}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

ABSTRACT

UNCLASSIFIED

Accession

AD

Springfield Armory, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swietkowski,
 Technical Report SA-TR15-1104, 30 April 1963, 27 pp. Incl illus.
 OAS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.

NONLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D}{d}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

ABSTRACT

UNCLASSIFIED

Accession

AD

Springfield Armory, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swietkowski,
 Technical Report SA-TR15-1104, 30 April 1963, 27 pp. Incl illus.
 OAS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.

NONLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D}{d}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED

ABSTRACT

UNCLASSIFIED

Accession

AD

Springfield Armory, Springfield, Mass.
 DESIGN ANALYSIS OF BELLEVILLE WASHER SPRINGS, by H. P. Swietkowski,
 Technical Report SA-TR15-1104, 30 April 1963, 27 pp. Incl illus.
 OAS Code 5011.11-83800.08
 UNCLASSIFIED REPORT.

NONLIMITED DISTRIBUTION.

A theoretical study was made to obtain data to establish an analytical method for the design of Belleville washers for energy storage and to modify the conventional formulas to replace the dependent variables with the independent or known values. These modified formulas were subsequently used to establish the stress reduction of a nested spring system and to determine an optimum stacking arrangement. A simplified and direct method for the design of washers for energy capacity was established. Final working stress is proportional to the square root of the energy requirement, and is inversely proportional to the outside diameter and the square root of the solid height. The study further shows that the final stress is at a minimum when the diameter ratio $A = \frac{D}{d}$ equals 1.7. The one-parallel series of the stacking arrangements considered is the most efficient for energy storage. Detailed derivations are shown and results discussed.

1. Belleville washer
2. Analytical spring design
3. Applied mathematics
4. Nested springs

UNCLASSIFIED