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Group Report

61G-3

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The Aperture Efficiency of Some Horn-Fed Paraboloidal Reflectors

30 December 1963

Prepared under Electronic Systems Division Contract AF 19(628)-500 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

DDC
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

THE APERTURE EFFICIENCY OF SOME HORN-FED PARABOLOIDAL REFLECTORS

A. DION

Group 61

GROUP REPORT 61G-3

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ABSTRACT

The optimum aperture efficiency of a reflector antenna is calculated for the cases where it is fed by one of three horn feeds of common occurrence. The effects, on the aperture efficiency, of varying the radiation characteristics of these feeds are also determined.

This technical documentary report is approved for distribution.

Franklin C. Hudson, Deputy Chief
Air Force Lincoln Laboratory Office

I. INTRODUCTION

The efficiency of a perfect paraboloidal reflector antenna is a function, principally, of the radiation characteristics of its feed. In order to calculate this efficiency, it is necessary to know the feed gain and its radiation pattern (amplitude and phase) over the conical region bounded by the reflector. From these characteristics, the absolute value of the aperture field may be determined by application of the laws of geometric optics. The on-axis far field resulting from this aperture field may then be obtained in absolute value, and hence the gain and efficiency can be calculated.

The determination of the aperture field by application of the laws of geometric optics is well justified for large reflectors and for points in the aperture a few wavelengths inside the boundary. Near the boundary, diffraction effects take place which modify somewhat the distribution in this area, but which can be neglected for apertures large with respect to the wavelength.

The electrical characteristics of simple feed configurations may be obtained analytically. For complex feed structures it may be more desirable to measure these characteristics. However, even for simple horn feeds, the analysis is complex except for horns with dimensions larger than about one wavelength (λ) for which the effect of currents circulating outside the horns may be neglected. The present study will make use of such approximations and therefore the results will be applicable to reflectors fed with horns of side dimensions somewhat larger than one wavelength. Feeds of such dimensions find some applications in paraboloid reflectors with large f/D ratio, but more generally in Cassegrainian feed systems.

Three different types of horn feeds will be considered in the study; the rectangular horn with equal E- and H-plane 10-db beamwidths, the diagonal horn, and a cluster of four square horns commonly used in monopulse feed systems. The aperture efficiency will be derived under the assumption that the reflector is perfect and is in the far field of a horn feed, also assumed perfect, and will not account for aperture block. It is, therefore, the maximum efficiency attainable with such systems.

II. APERTURE EFFICIENCY OF PARABOLOIDAL REFLECTOR ANTENNAS

The amplitude of the electric field at a far-field distance R along the axis of a paraboloid of focal length f and illuminated by a feed at its focus is given by Silver[1] and is after rearrangement of the variables:

$$E = \frac{2f}{\lambda R}\left[\frac{\zeta_o P G_F}{2\pi}\right]^{1/2} \left| \int_0^{\beta_o} \int_0^{2\pi} F(\beta,\xi) \tan(\beta/2)\, d\beta d\xi \right| \qquad (1)$$

where P is the power radiated, ζ_o is the characteristic impedance of free space, G_F is the gain of the feed whose normalized radiation function is $F(\beta,\xi)$ and the other parameters are defined in Fig. 1.

The gain of the reflector giving rise to this field is

$$G = \frac{E^2/2\zeta_o}{P/4\pi R^2}$$

or $$G = 4\, G_F\, (D/\lambda)^2\, (f/D)^2 \left| \int_0^{\beta_o} \int_0^{2\pi} F(\beta,\xi) \tan(\beta/2)\, d\beta d\xi \right|^2 \qquad (2)$$

and its efficiency is

$$\eta = G/(\frac{\pi D}{\lambda})^2$$

or

$$\eta = \frac{4}{\pi^2} G_F (f/D)^2 \left| \int_0^{\beta_o} \int_0^{2\pi} F(\beta, \xi) \tan(\beta/2) \, d\beta d\xi \right|^2 \qquad (3)$$

__Cassegrainian Feed Systems:__– The basic geometry of the Cassegrainian feed system is shown in Fig. 2. Its efficiency may be found by applying Eq. (3) to the virtual image of the feed. Is is therefore necessary to derive the gain function of this apparent feed. Let this gain function be $G(\beta, \xi)$ and let the gain function of the feed be $G(\alpha, \xi)$. These two functions are related by the condition that the total power contained in the incident bundle of rays must be equal to the total power contained in the associated reflected bundle of rays. The infinitesimal solid angle corresponding to the incident bundle of rays is

$$\sin\alpha \, d\alpha \, d\xi$$

while that corresponding to the reflected one is

$$\sin\beta \, d\beta \, d\xi.$$

Application of the principle of conservation of energy then yields:

$$G(\alpha, \xi) \sin\alpha \, d\alpha = G(\beta, \xi) \sin\beta \, d\beta. \qquad (4)$$

The ray angles α and β may be shown to be related by the expression

$$\tan(\beta/2) = M \tan(\alpha/2) \qquad (5)$$

where M is the magnification factor of the hyperboloid, and consequently

$$\sec^2 (\beta/2) \, d\beta = M \sec^2 (\alpha/2) \, d\alpha. \tag{6}$$

Substitution of (6) in (4) and manipulation with the help of (5) finally yields:

$$G(\beta,\xi) = \frac{G(\alpha,\xi)}{M^2} \frac{\sec^4 (\beta/2)}{\sec^4 (\alpha/2)} \tag{7}$$

The gain of the virtual feed G_V is therefore given by

$$G_V = G_F/M^2 \tag{8}$$

and the normalized radiation function $F(\beta,\xi)$ of this feed is related to that of the real feed, $F(\alpha,\xi)$, by the expression

$$F(\beta,\xi) = F(\alpha,\ ) \frac{\sec^2 (\beta/2)}{\sec^2 (\alpha/2)}. \tag{9}$$

Substitution of (8), (9) and (6) in (3) yields the efficiency of the Cassegrainian antenna in terms of the feed characteristics,

$$\eta = \frac{4}{\pi^2} G_F M^2 (f/D)^2 \left| \int_0^{\alpha_0} \int_0^{2\pi} F(\alpha,\xi) \tan(\alpha/2) \, d\alpha d\xi \right|^2 \tag{10}$$

A comparison of the efficiency expressions of the feed-at-focus system (Eq.3) and of the Cassegrainian feed system (Eq. 10) confirms the equivalence between this latter and a feed-at-focus system having a focal length M times that of the Cassegrainian system.

As $4f/D = \cot(\beta_0/2) = \cot(\alpha_0/2)/M$ where β_0 is the angular aperture of the reflector and α_0 that of the subreflector, Eq. (10) may be written in the more appropriate form:

$$\eta = (1/4\pi^2)\, G_F \cot^2(\alpha_0/2) \left| \int_0^{\alpha_0} \int_0^{2\pi} F(\alpha,\xi) \tan(\alpha/2)\, d\alpha d\xi \right|^2 \qquad (11)$$

III. APPLICATION TO SOME PRACTICAL HORN FEEDS

The aperture efficiency of paraboloidal reflector antennas was evaluated for three different types of horn feeds commonly used in practice; a rectangular horn with equal E- and H-plane 10-db beamwidths, a diagonal horn, and a 4-horn monopulse feed (Fig. 3). The gain functions of these feeds, obtained analytically, do not incorporate the effect of currents flowing on the external walls of the horns and for this reason are accurate only for horns with dimensions somewhat larger than 1λ.

The feed horns considered were assumed to be ideal and therefore devoid of phase errors. When these horns are excited with a TE_{10} mode only and are polarized as shown in Fig. 3, their gains $\left[0.81\ (4\pi\ A/\lambda^2)\right]$ and radiation functions are:

(a) For the rectangular horn with an H-plane dimension equal to 1.37 times the E-plane dimension:

$$G_T = 4.44\ \pi(d/\lambda)^2$$

$$F(\alpha,\xi) = \left(\frac{1 + \cos\alpha}{2}\right) \left[\frac{\sin(u \cos\xi)}{u \cos\xi}\right] \left[\frac{\cos(1.37\ u \sin\xi)}{1 - \frac{7.52}{\pi^2} u^2 \sin\xi}\right].$$

(b) For the diagonal horn:

$$G_T = 3.24\ \pi(d/\lambda)^2$$

$$F(\alpha,\xi) \simeq \left(\frac{1+\cos\alpha}{2}\right)\left[\frac{\sin\left[u\sin(\xi-\pi/4)\right]}{u\sin(\xi-\pi/4)}\ \frac{\cos\left[u\cos(\xi-\pi/4)\right]}{\left[1-\frac{4u^2}{\pi^2}\cos^2(\xi-\pi/4)\right]}\right.$$

$$\left.+\frac{\sin\left[u\cos(\xi-\pi/4)\right]}{u\cos(\xi-\pi/4)}\ \frac{\cos\left[u\sin(\xi-\pi/4)\right]}{\left[1-\frac{4u^2}{\pi^2}\sin^2(\xi-\pi/4)\right]}\right]$$

(This approximation for $F(\alpha,\xi)$ holds good up to moderately large off-axis angles.)

(c) For a 4-horn monopulse feed (sum channels):

$$G_T = 3.24\ \pi(d/\lambda)^2$$

$$F(\alpha,\xi) = \left(\frac{1+\cos\alpha}{2}\right)\left[\frac{\sin(u\cos\xi)}{u\cos\xi}\right]\left[\frac{\cos^2\ (u/2)\sin\xi}{1-\frac{u^2}{\pi^2}\sin^2\xi}\right]$$

where for all three cases $u = \pi(d/\lambda)\sin\alpha$, and d is the dimension of one side of the horn feeds as shown in Fig. 3. Substitution in (11) of the feeds respective gains and radiation expressions and numerical integrations of various values of d/λ yielded the results illustrated in Figs. 4, 5 and 6. These results were obtained by setting the angular aperture (α_o) equal to 5^o. The calculations were repeated with the angular aperture set at 20^o but no appreciable differences were observed in the results. This could be expected because under the conditions where the gains and radiation functions expressed above apply the same total amount of power is intercepted by the reflector.

The E-plane edge illumination corresponding to the calculated efficiencies is also plotted in Figs. 4, 5 and 6. It is observed that a maximum efficiency results with an edge illumination of about -11.4 db for the rectangular and 4-horn feed and of about -10 db for the diagonal horn feed. With a rectangular horn feed, a maximum aperture efficiency of 75.4 per cent is reached. With the diagonal horn feed, this maximum is 72.4 per cent while with the 4-horn monopulse feed it is 58.4 per cent. Even though the diagonal horn, with its low side-lobe level, gives rise to a smaller amount of spillover than does the rectangular horn, this latter still yields a larger aperture efficiency. This is due to the appreciable amount of cross-polarized energy radiated by the diagonal horn.

The smaller value of aperture efficiency observed with the 4-horn feed as compared to the rectangular, or diagonal, horn feeds is due to the increased spillover resulting, principally, from two relatively high side lobes (-7.9 db) in the radiation field of the 4-horn cluster. The level of these side lobes may be reduced by the introduction of metal septa in the horn apertures such as to produce a more uniform taper of the energy in the aperture of each horn. This method has been investigated and the results are reported in a separate report.

The variation of aperture efficiency as a function of frequency may be obtained from the efficiency curves of Figs. 4, 5 and 6 under the condition that no spurious modes are generated in the horn apertures.

The calculated efficiencies are plotted in Fig. 7 as a function of edge illumination in the E-plane. The diagonal horn efficiency curve is seen to decrease faster, as a function of illumination, than the other two curves. The efficiency of a reflector fed by a diagonal horn is, therefore, more frequency sensitive than that of a reflector fed by a rectangular horn or a 4-horn cluster.

This behavior is due to the existence in the 45° plane of the diagonal horn of a side lobe which, for small values of edge illumination, is directed toward the reflector.[2]

IV. CONCLUSION

The calculated aperture efficiencies justify the commonly used rule that the efficiency of a horn-fed reflector is optimized when the illumination at the edge is between 8 to 18 db less than the illumination on axis. These calculations also provide maximum efficiencies attainable with feeds of practical occurrence. In practice, the aperture efficiency of a reflector antenna will be appreciably less than optimum due to reflector surface deviations, to aperture block, and to phase errors in the aperture of the feed horn. The reduction of efficiency resulting from reflector surface deviations may be obtained from Ruze[3] while that due to aperture block is proportional to the percentage of block. Phase errors in the horn aperture do not, in general, modify the relative illumination on the paraboloid and consequently lead to a reduction of efficiency corresponding only to the gain loss of the feed horn.

ACKNOWLEDGMENT

The numerical evaluation of the aperture efficiency integrals was programmed on a high-speed general-purpose computer by R. J. Pieculewicz.

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3. J. Ruze, "The Effect of Aperture Errors on the Antenna Radiation Pattern," Supplemento del Nuovo Cimento 9 No. 3, (1952), p. 364.

FIGURE CAPTIONS

Fig. 1. Geometry of paraboloidal reflector.

Fig. 2. Geometry of hyperboloidal reflector.

Fig. 3. Feed horns.

Fig. 4. Aperture efficiency (solid curve) and edge illumination (dashed curve) of a Cassegrainian antenna fed by a rectangular horn.

Fig. 5. Aperture efficiency (solid) and edge illumination of a Cassegrainian antenna fed by a diagonal horn.

Fig. 6. Aperture efficiency (solid curve) and edge illumination of a Cassegrainian antenna fed by a 4-horn monopulse feed.

Fig. 7. Aperture efficiency vs. edge illumination.

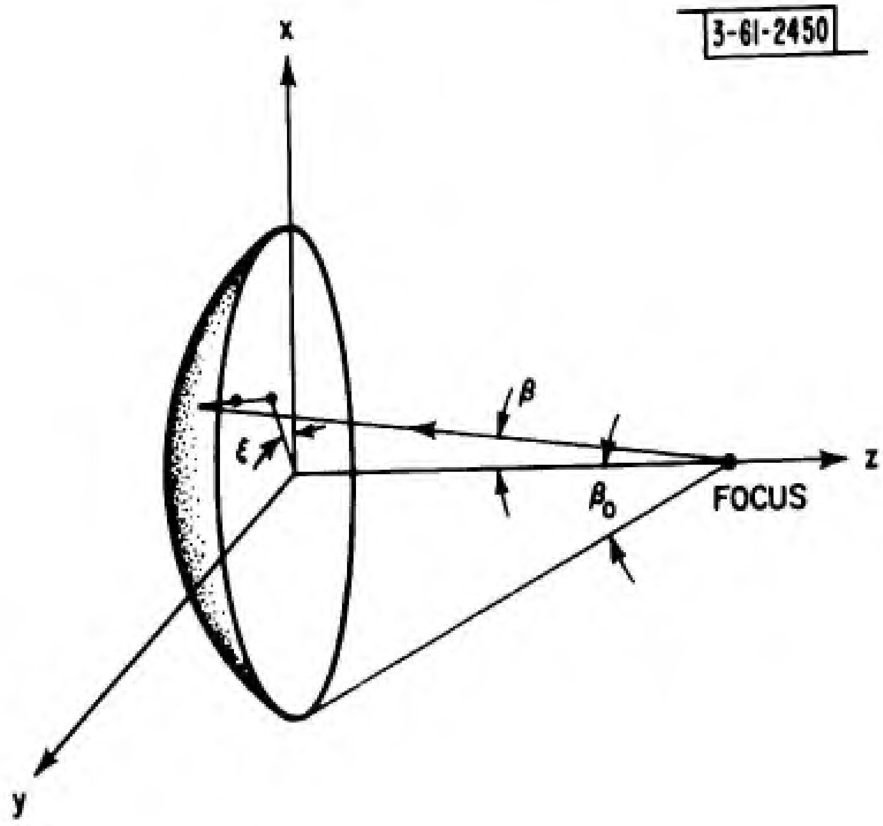

Fig. 1 Geometry of paraboloidal reflector.

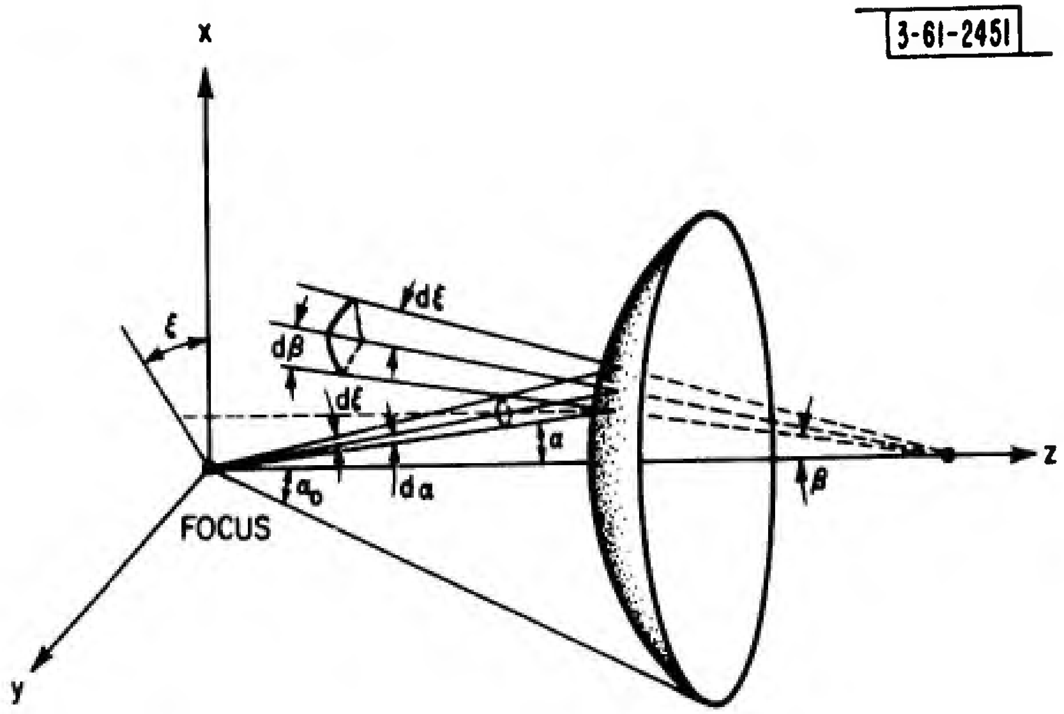

Fig. 2 Geometry of hyperboloidal reflector.

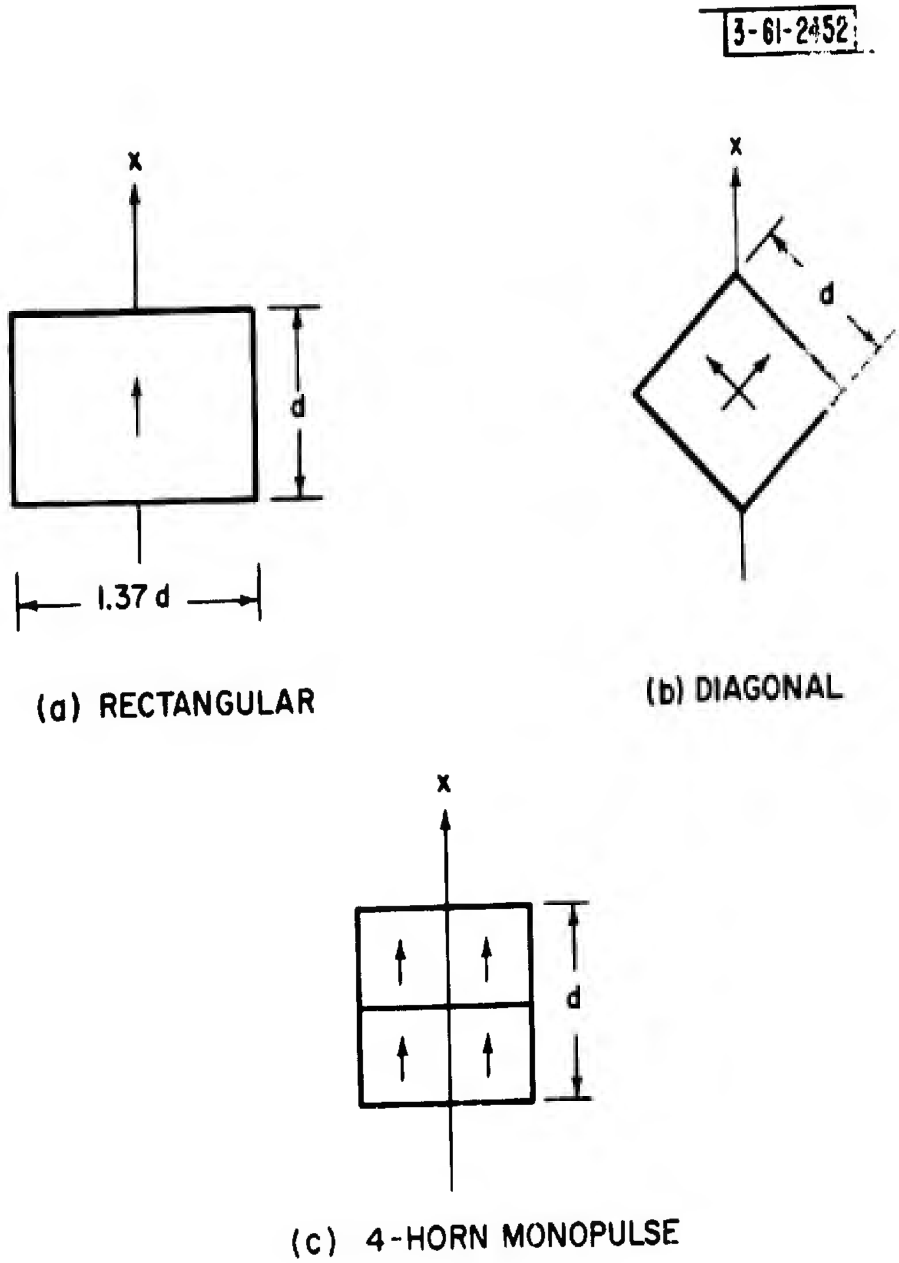

Fig. 3 Feed horns.

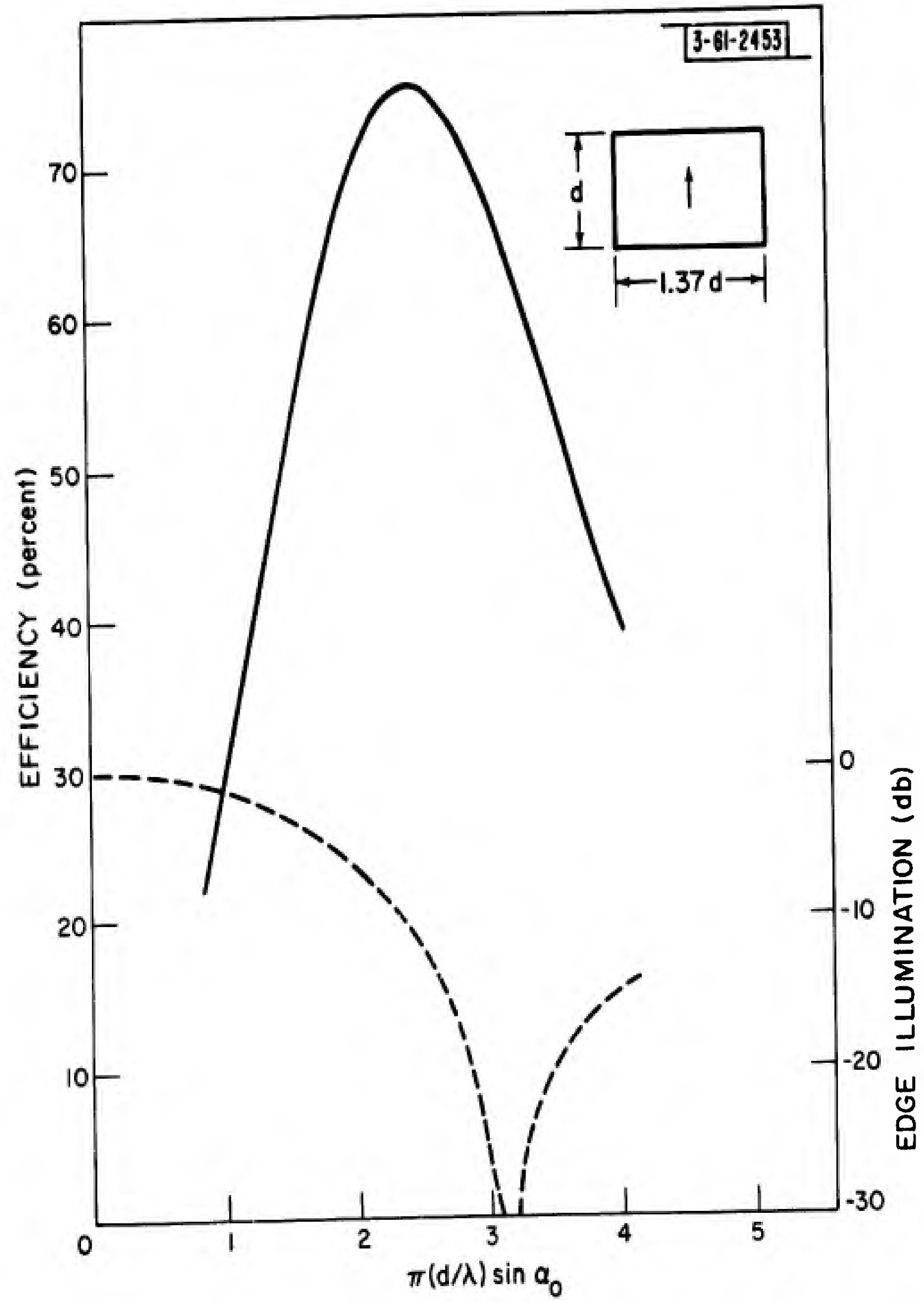

Fig. 4 Aperture efficiency (solid curve) and edge illumination (dashed curve) of a Cassegrainian antenna fed by a rectangular horn.

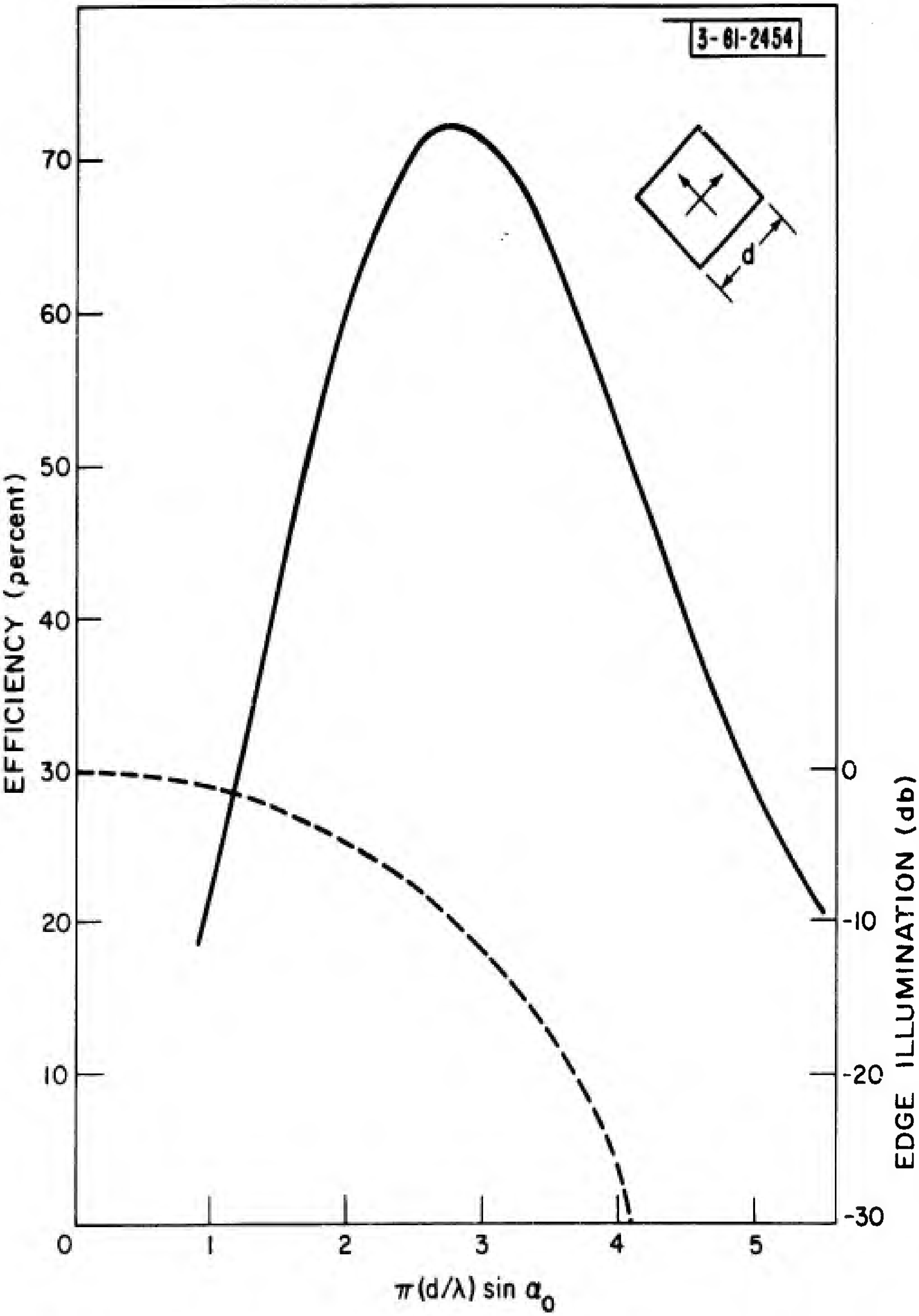

Fig. 5 Aperture efficiency (solid) and edge illumination of a Cassegrainian antenna fed by a diagonal horn.

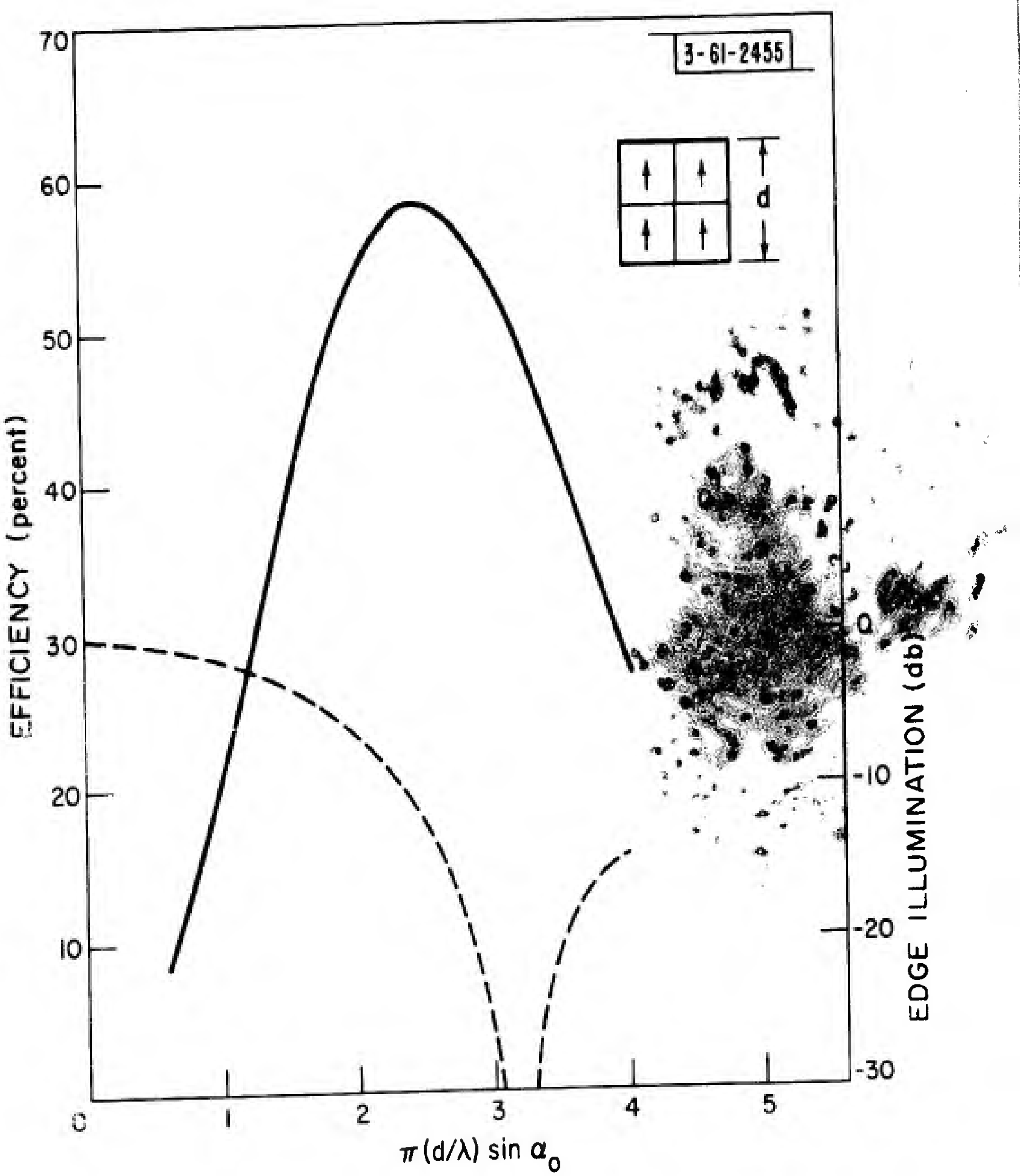

Fig. 6 Aperture efficiency (solid curve) and edge illumination of a Cassegrainian antenna fed by a 4-horn monopulse feed.

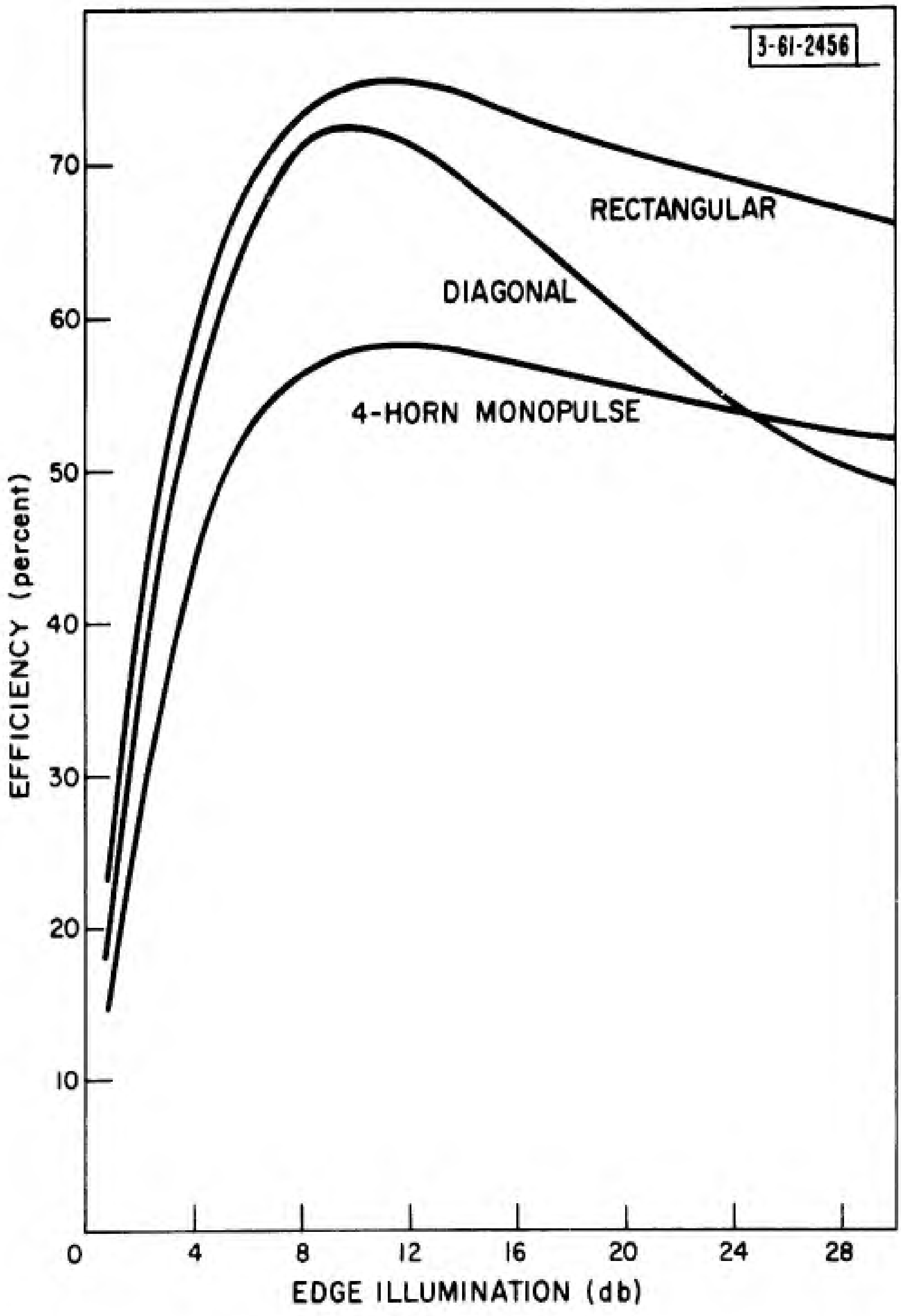

Fig. 7 Aperture efficiency vs. edge illumination.

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