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MT. AUBURN RESEARCH ASSOCIATES, INC.
12 Norfolk Street
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Contract No. AF19(628)-2836

Scientific Report No. 1

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ABSTRACT

The coupling of magnetohydrodynamic waves to electromagnetic and sound waves at a plane interface between a plasma and a neutral gas is considered for arbitrary direction of the incident wave vector and for the constant external magnetic field lying either in the plane of incidence or perpendicular to it. The relations between the angle of incidence, reflection, and transmission, the ratio of the field amplitude transmitted or reflected to the corresponding amplitude incident, and the energy coupling coefficients are calculated.

An incident fast magnetoacoustic mode generates a sound wave whose wave propagation vector makes an angle with the normal to the boundary of the order of the sound speed to Alfvén speed, when this ratio is small. In this case, the energy coupling coefficient of sound to the fast magnetoacoustic mode is also of the order of this ratio. An incident slow magnetoacoustic mode generates a sound wave with energy coupling coefficient of the order of unity, and an electromagnetic wave with energy coupling coefficient of the order of the ratio, speed of sound to speed of light.

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1. INTRODUCTION

Although magnetohydrodynamic wave propagation has been thoroughly treated for the case of a homogeneous infinite medium,[1] in many cases of practical significance the results are not applicable since the medium is inhomogeneous due to spatial changes in the ionization density. The present paper is concerned with wave propagation in a medium with a spatial discontinuity in the ionization density. In particular, this discontinuity will be taken so that the gaseous medium changes from one which is fully ionized to one which is totally un-ionized. The ionized medium can support three distinct magnetohydrodynamic modes (the Alfvén, the fast magnetoacoustic, and the slow magnetoacoustic), whereas the un-ionized medium can support electromagnetic waves and sound waves. The coupling of the magnetohydrodynamic modes to the electromagnetic and acoustic modes at a plane plasma-neutral gas interface will be discussed in the present paper.

This problem is similar to two others which also involve discontinuities in the gaseous medium. All three may be described in terms of the ionization concentration on each side of the interface:

A. the media on both sides of the interface are fully ionized (plasma-plasma problem);

B. the medium from which the waves are approaching the interface is totally un-ionized, whereas the other medium is fully ionized (neutral gas-plasma problem);

C. the medium from which the waves are approaching the interface is fully ionized, whereas the other medium is totally un-ionized (plasma-neutral gas problem).

A. Plasma-Plasma Problem

Any of the three magnetohydrodynamic modes (Alfvén, fast magneto-acoustic, and slow magnetoacoustic) may be incident, and in the most general case all three modes can be reflected or transmitted. Ferraro[2] considers reflection and transmission of Alfvén waves in incompressible media for the magnetic field of the incident wave parallel to the interface. Roberts[3] extends Ferraro's work to arbitrary orientations of the magnetic field of the incident wave. Simon[4], Williams[5], and Raju and Verma[6] consider reflection and transmission of all three modes in compressible media and examine in detail the case where the external magnetic field is normal to the interface. Talwar[7] extends this analysis by assuming the sound velocity is much smaller than the Alfvén velocity. Fejer[8] examines dispersion relations and Snell's laws when the two plasma regions have a relative velocity parallel to the interface. He chooses the external magnetic field parallel to the interface. Frieman and Kulsrud[9] calculate amplitude relations for all three modes when the external magnetic field is parallel to the interface and lies in the incident plane. Pridmore-Brown[10] gives a graphical presentation of Snell's laws to determine the nature and disposition of reflected and refracted waves. Bazer[11] calculates amplitude relations in detail for the external magnetic field in the incident plane.

B. Neutral Gas-Plasma Problem

The two electromagnetic modes and the sound mode may be incident on the interface and reflected from it, while all three magnetohydrodynamic modes can be transmitted. Turcotte and Schubert[12] calculate the coupling of an incident electromagnetic wave with magnetohydrodynamic waves. They consider the cases where the external magnetic field is parallel to the interface, the magnetic field of the incident wave is parallel to the external magnetic field, and the incident wave vector is either normal to or parallel to the interface. Kornhauser[13] considers in greater detail the case where the external magnetic field is parallel to the interface and the incident wave vector is normal to the interface. Kontorovich and Glutsyuk[14] calculate the coupling of an incident sound wave with magnetohydrodynamic waves for the Alfvén velocity much less than the sound velocity.

C. Plasma-Neutral Gas Problem

Any of the three magnetohydrodynamic modes can be incident on and reflected from the interface, while acoustic waves and electromagnetic waves of both polarizations can be transmitted. Kahalas[15] considers the case in which the sound velocity is zero and both the incident wave vector and the external magnetic field are normal to the interface. Ullah and Kahalas[16], and Kahalas[17] discuss the case in which the sound velocity is zero, the incident wave vector has arbitrary orientation, and the external magnetic field lies either in the plane of incidence or perpendicular to it. In the above analyses only the electromagnetic waves are transmitted since with sound velocity zero sound waves do not propagate.

This paper is an extension of the latter work to the case of non-zero sound velocity. The incident wave vector will be allowed to have an arbitrary orientation with respect to the interface, but the constant external magnetic field will be constrained to lie either in the plane of incidence or perpendicular to it.

The mathematical model analyzed in this paper has features in common with the physical situation which describes the propagation of geomagnetic disturbances down through and below the ionosphere. The ionosphere and atmosphere are considered here as two distinct regions, with the ionosphere as a homogeneous plasma, the atmosphere as a homogeneous neutral gas, and the transition between the two as a plane surface.

Within the ionosphere and the atmosphere, the sound velocity is much less than the Alfvén velocity; consequently, this fact will be used to simplify some of the equations.

The physical model discussed here is lacking because the external magnetic field is assumed to be uniform whereas the earth's field is not, collisions between positive ions and neutral molecules in the lower ionosphere have not been taken into account, the ionospheric boundary is not necessarily as abrupt as assumed, and the effect of the conducting earth itself has not been included. Numerical studies have been made by Francis, Karplus, and Dragt[18] on the propagation of magnetohydrodynamic waves through the lower ionosphere incorporating the positive ion-neutral particle collisions, the effect of the earth, and the gradual change of ionospheric parameters. The present analytical approach should be considered as a supplement to their work.

In Section 2, a derivation is given of the boundary conditions relating field quantities across the fluid interface. These boundary conditions are valid for the plasma-plasma, neutral gas-plasma, and plasma-neutral gas problems. Section 3 gives an overall statement of the problem. Section 4 discusses the properties of magnetohydrodynamic waves. In particular, the dispersion relations, relation between field quantities, and energy transport vector are exhibited. Section 5 discusses the same topics for the electromagnetic and acoustic waves propagating in the neutral gas. Section 6 treats the relations between the angles of incidence, reflection, and transmission and the relations between the field quantities on both sides of the interface for the case in which the constant external magnetic field lies normal to the plane of incidence. Section 7 similarly discusses the case in which the constant external magnetic field lies in the plane of incidence.

2. BOUNDARY CONDITIONS

The boundary conditions relating the field quantities on two sides of a discontinuity have been derived by several authors using somewhat different methods.[19,20] The present derivation involves an integration of the conservation equations across the boundary. The boundary conditions which result are equally valid for the plasma-plasma, plasma-neutral gas, and neutral gas-plasma cases.

In order to derive the boundary conditions at a plasma-neutral gas interface, it is important to ensure that the equations used are valid in both the plasma and the neutral gas. This means, for example, that the $\partial \underset{\sim}{E}/\partial t$ term (needed to describe electromagnetic waves) must be included in the $\nabla \times \underset{\sim}{B}$ Maxwell equation, although it is often left out of the magnetohydrodynamic equations. Similarly, Ohm's law is written with a finite conductivity, since the variation of the conductivity (from infinity in the plasma to zero in the neutral gas) must be included. The equations used are[21]

$$\bar{\rho}(\partial \underset{\sim}{\bar{v}}/\partial t) = -\bar{\rho}(\underset{\sim}{\bar{v}}\cdot\nabla)\underset{\sim}{\bar{v}} - \nabla\bar{p} + (\underset{\sim}{\bar{j}}\times\underset{\sim}{\bar{B}})/c + \bar{\rho}_e\underset{\sim}{\bar{E}} \quad , \qquad (2.1)$$

$$(\partial\overline{\rho}/\partial t) + \nabla\cdot(\overline{\rho}\underset{\sim}{\overline{v}}) = 0 \quad , \qquad (2.2)$$

$$\nabla\times\underset{\sim}{\overline{E}} = -(1/c)(\partial\underset{\sim}{\overline{B}}/\partial t) \quad , \qquad (2.3)$$

$$\nabla\times\underset{\sim}{\overline{B}} = (1/c)(\partial\underset{\sim}{\overline{E}}/\partial t) + 4\pi\underset{\sim}{\overline{j}}/c \quad , \qquad (2.4)$$

$$\nabla\cdot\underset{\sim}{\overline{B}} = 0 \quad , \qquad (2.5)$$

$$\nabla\cdot\underset{\sim}{\overline{E}} = 4\pi\overline{\rho}_e \quad , \qquad (2.6)$$

$$\underset{\sim}{\overline{j}} = \sigma(\underset{\sim}{\overline{E}} + \underset{\sim}{\overline{v}}\times\underset{\sim}{\overline{B}}/c) \quad , \qquad (2.7)$$

and

$$\nabla\overline{p} = s^2\,\nabla\overline{\rho} \quad , \qquad (2.8)$$

where $\underset{\sim}{\overline{E}}$, $\underset{\sim}{\overline{B}}$, $\underset{\sim}{\overline{v}}$, $\overline{p}$, $\underset{\sim}{\overline{j}}$, $\overline{\rho}$, $\overline{\rho}_e$, c, s, and σ are total electric field, magnetic field, fluid velocity, pressure, electric current density, mass density, electric charge density, velocity of light, velocity of sound, and electrical conductivity, respectively. A bar over a field quantity denotes a total unlinearized quantity, a subscript zero denotes constant quantities, and field quantities without bars are used to denote quantities of small amplitude.

These equations may be rearranged to express the conservation of momentum[19]

$$(\partial/\partial t)\left[\overline{\rho}\underset{\sim}{\overline{v}} + (1/4\pi c)\,\underset{\sim}{\overline{E}}\times\underset{\sim}{\overline{B}}\right]_i + \sum_j \partial\overline{\pi}_{ij}/\partial x_j = 0 \qquad (2.9)$$

and conservation of energy

$$(\partial/\partial t)\left[(\overline{\rho}\overline{v}^2/2) + (\overline{E}^2 + \overline{B}^2)/8\pi\right] + \nabla\cdot\underset{\sim}{\overline{T}} - \overline{p}(\nabla\cdot\underset{\sim}{\overline{v}}) + \overline{j}^2/\sigma - (\underset{\sim}{\overline{v}}\cdot\underset{\sim}{\overline{E}})(\nabla\cdot\underset{\sim}{\overline{E}})/4\pi = 0 \quad (2.10)$$

where

$$\bar{\pi}_{ij} \equiv \bar{\rho}\bar{v}_i\bar{v}_j + \bar{p}\delta_{ij} + \left[(\bar{E}^2 + \bar{B}^2)/8\pi\right] \delta_{ij} - (1/4\pi)(\bar{E}_i\bar{E}_j + \bar{B}_i\bar{B}_j) \quad , \quad (2.11)$$

and

$$\bar{\underset{\sim}{T}} \equiv \bar{p}\underset{\sim}{\bar{v}} + (c/4\pi)\, \underset{\sim}{\bar{E}}\mathrm{x}\underset{\sim}{\bar{B}} + (\bar{v}^2/2)\, \bar{\rho}\underset{\sim}{\bar{v}} \quad . \quad (2.12)$$

The integration of Eq. (2.3) over a closed line path lying on both sides of the interface yields

$$\underset{\sim}{\hat{n}}\mathrm{x}\underset{\sim}{\bar{E}_1} = \underset{\sim}{\hat{n}}\mathrm{x}\underset{\sim}{\bar{E}_2} \quad . \quad (2.13)$$

Similarly, the integration of Eqs. (2.2), (2.5), and (2.9) over the volume of a Gaussian pillbox extending through the interface gives, respectively,

$$\bar{\rho}_1(\underset{\sim}{\bar{v}_1}\cdot\underset{\sim}{\hat{n}}) = \bar{\rho}_2(\underset{\sim}{\bar{v}_2}\cdot\underset{\sim}{\hat{n}}) \quad , \quad (2.14)$$

$$\underset{\sim}{\bar{B}_1}\cdot\underset{\sim}{\hat{n}} = \underset{\sim}{\bar{B}_2}\cdot\underset{\sim}{\hat{n}} \quad , \quad (2.15)$$

and

$$\underset{\sim}{\bar{\pi}_1}\cdot\underset{\sim}{\hat{n}} = \underset{\sim}{\bar{\pi}_2}\cdot\underset{\sim}{\hat{n}} \quad , \quad (2.16)$$

where subscripts 1 and 2 refer to quantities on the two sides of the boundary and $\underset{\sim}{\hat{n}}$ is the unit normal to the interface. Equations (2.13), (2.14), and (2.16) are the boundary conditions for the problem. Equation (2.15) is obtainable from Eq. (2.13) and thus provides no further restriction on the system.

Consider now only small perturbations about an unperturbed solution of the system. Let $\underset{\sim}{E}$, $\underset{\sim}{B}$, $\underset{\sim}{v}$, p, $\underset{\sim}{j}$, and ρ be the small deviations of electric field, magnetic field, fluid velocity, pressure, electric current density, and mass density from their equilibrium values which are 0, $\underset{\sim}{B_o}$, 0, p_o, 0, and ρ_o. $\underset{\sim}{B_o}$ and ρ_o are the uniform magnetic field and mass density throughout the entire system. Then Eqs. (2.13) – (2.16) may be written to first order in the perturbation as

$$\hat{\mathbf{n}} \times \mathbf{E}_1 = \hat{\mathbf{n}} \times \mathbf{E}_2 \quad , \tag{2.17}$$

$$\mathbf{v}_1 \cdot \hat{\mathbf{n}} = \mathbf{v}_2 \cdot \hat{\mathbf{n}} \quad , \tag{2.18}$$

$$\mathbf{B}_1 \cdot \hat{\mathbf{n}} = \mathbf{B}_2 \cdot \hat{\mathbf{n}} \quad , \tag{2.19}$$

and

$$\left[p_1 + (\mathbf{B}_o \cdot \mathbf{B}_1)/4\pi\right]\hat{\mathbf{n}} - (1/4\pi)\left[\mathbf{B}_1(\mathbf{B}_o \cdot \hat{\mathbf{n}}) + \mathbf{B}_o(\mathbf{B}_1 \cdot \hat{\mathbf{n}})\right]$$

$$= \left[p_2 + (\mathbf{B}_o \cdot \mathbf{B}_2)/4\pi\right]\hat{\mathbf{n}} - (1/4\pi)\left[\mathbf{B}_2(\mathbf{B}_o \cdot \hat{\mathbf{n}}) + \mathbf{B}_o(\mathbf{B}_2 \cdot \hat{\mathbf{n}})\right] \quad . \tag{2.20}$$

With the vector identity $\mathbf{B} = (\mathbf{B} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} - \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{B})$ along with Eq. (2.19), Eq. (2.20) may be separated into its normal and tangential parts, giving

$$p_1 + (\mathbf{B}_o \cdot \mathbf{B}_1)/4\pi = p_2 + (\mathbf{B}_o \cdot \mathbf{B}_2)/4\pi \tag{2.21}$$

and

$$(\mathbf{B}_o \cdot \hat{\mathbf{n}})\,\hat{\mathbf{n}} \times \mathbf{B}_1 = (\mathbf{B}_o \cdot \hat{\mathbf{n}})\,\hat{\mathbf{n}} \times \mathbf{B}_2 \quad . \tag{2.22}$$

There exist two separate cases depending on whether $\mathbf{B}_o \cdot \hat{\mathbf{n}} = 0$ or $\mathbf{B}_o \cdot \hat{\mathbf{n}} \neq 0$.

When $\mathbf{B}_o \cdot \hat{\mathbf{n}} = 0$, the boundary conditions are Eqs. (2.17), (2.18), and (2.21). A discontinuity in $\mathbf{B}$, resulting from a surface current density, $\mathbf{j}^*$, is determined by $\hat{\mathbf{n}} \times (\mathbf{B}_2 - \mathbf{B}_1) = 4\pi \mathbf{j}^*/c$.

When $\mathbf{B}_o \cdot \hat{\mathbf{n}} = 0$, the boundary conditions reduce to Eqs. (2.17) and (2.18) along with

$$\hat{\mathbf{n}} \times \mathbf{B}_1 = \hat{\mathbf{n}} \times \mathbf{B}_2 \quad , \tag{2.23}$$

and

$$p_1 = p_2 \quad . \tag{2.24}$$

It will be shown now that Eq. (2.10) is consistent with the boundary conditions to first and second order in the perturbation. To first order Eq. (2.10) is the scalar product of Eq. (2.3) with $\underset{\sim}{B}_o$. The resulting boundary condition duplicates the scalar product of Eq. (2.17) with $\underset{\sim}{B}_o$. To second order, Eq. (2.10) becomes

$$\partial W/\partial t + \nabla\cdot\underset{\sim}{T} + j^2/\sigma = 0 \quad , \tag{2.25}$$

where

$$\underset{\sim}{T} = p\underset{\sim}{v} + c(\underset{\sim}{E}\times\underset{\sim}{B})/4\pi \quad , \tag{2.26}$$

and

$$W \equiv \rho_o v^2/2 + (E^2 + B^2)/8\pi + s^2\rho^2/2\rho_o \quad .$$

The integration of Eq. (2.25) over a Gaussian pillbox of base area δA and volume δV extending through the surface gives

$$\underset{\sim}{T}_1\cdot\underset{\sim}{\hat{n}} = \underset{\sim}{T}_2\cdot\underset{\sim}{\hat{n}} + \Lambda \tag{2.27}$$

where $\Lambda \equiv \lim_{\delta A\to 0} (1/\delta A)\iiint_{\delta V} dV(j^2/\sigma)$. For the case $\underset{\sim}{B}_o\cdot\underset{\sim}{\hat{n}} \neq 0$ there is no surface current density and $\Lambda = 0$. Then Eq. (2.27) is seen to be consistent with Eqs. (2.17), (2.18), (2.23), and (2.24). For the case $\underset{\sim}{B}_o\cdot\underset{\sim}{\hat{n}} = 0$, there is a surface current density. Λ is calculated directly from $j^2/\sigma = \underset{\sim}{j}\cdot(\underset{\sim}{E} + \underset{\sim}{v}\times\underset{\sim}{B}_o/c)$ evaluated over the surface. The surface current density is by definition tangential to the interface and $(\underset{\sim}{E} + \underset{\sim}{v}\times\underset{\sim}{B}_o/c)_{tangential}$ is continuous across the boundary so that Λ is given by $\Lambda = \underset{\sim}{j}^*\cdot(\underset{\sim}{E}_1 + \underset{\sim}{v}_1\times\underset{\sim}{B}_o/c)$. The term in parentheses vanishes when evaluated in the plasma medium where the conductivity is infinite so that $\Lambda = 0$ and Eq. (2.27) becomes $\underset{\sim}{T}_1\cdot\underset{\sim}{\hat{n}} = \underset{\sim}{T}_2\cdot\underset{\sim}{\hat{n}}$. If $(\underset{\sim}{T}_2 - \underset{\sim}{T}_1)\underset{\sim}{\hat{n}}$ is calculated directly from the definition, Eq. (2.26), using Eqs. (2.17), (2.18) and (2.21), one gets

$$(\underset{\sim}{T}_2 - \underset{\sim}{T}_1)\cdot\hat{\underset{\sim}{n}} = (c/4\pi)(\underset{\sim}{B}_2 - \underset{\sim}{B}_1)\cdot\left[\hat{\underset{\sim}{n}}\times\underset{\sim}{E}_1 - (\underset{\sim}{n}\cdot\underset{\sim}{v}_1/c)\underset{\sim}{B}_o\right].$$

Again, the bracketed term vanishes when the conductivity is infinite so that Eq. (2.10) is consistent with the boundary conditions to first and second order in the perturbation.

3. DESCRIPTION OF PROBLEM

In the ionized medium, the equations governing extremely low frequency wave motion (i.e., frequencies less than the ion cyclotron frequency) are given by Eqs. (2.1) - (2.8) written to first order in the perturbation and are

$$\rho_o(\partial\underset{\sim}{v}/\partial t) = -\nabla p + \underset{\sim}{j}\times\underset{\sim}{B}_o/c \quad , \tag{3.1}$$

$$\partial\rho/\partial t + \rho_o\nabla\cdot\underset{\sim}{v} = 0 \quad , \tag{3.2}$$

$$\nabla\times\underset{\sim}{E} = -(1/c)\partial\underset{\sim}{B}/\partial t \quad , \tag{3.3}$$

$$\nabla\times\underset{\sim}{B} = 4\pi\underset{\sim}{j}/c \quad , \tag{3.4}$$

$$\nabla\cdot\underset{\sim}{B} = 0 \quad , \tag{3.5}$$

$$\nabla\cdot\underset{\sim}{E} = 4\pi\rho_e \quad , \tag{3.6}$$

$$\underset{\sim}{E} = -\underset{\sim}{v}\times\underset{\sim}{B}_o/c \quad , \tag{3.7}$$

and

$$\nabla p = s^2\,\nabla\rho \quad , \tag{3.8}$$

where infinite conductivity has been assumed and the $\partial\underset{\sim}{E}/\partial t$ term in Eq. (2.4) has been neglected.

In the neutral gas the conductivity is assumed to be zero, so that the electric charge and current densities are zero. The electromagnetic equations are then uncoupled from the fluid flow equations, and the system is described by

$$\rho_0(\partial \underset{\sim}{v}/\partial t) = -s^2 \nabla\rho \qquad , \qquad (3.9)$$

$$\partial\rho/\partial t + \rho_0\nabla\cdot\underset{\sim}{v} = 0 \qquad , \qquad (3.10)$$

$$\nabla\times\underset{\sim}{E} = -(1/c)\,\partial\underset{\sim}{B}/\partial t \qquad , \qquad (3.11)$$

$$\nabla\times\underset{\sim}{B} = (1/c)\,\partial\underset{\sim}{E}/\partial t \qquad , \qquad (3.12)$$

$$\nabla\cdot\underset{\sim}{B} = 0 \qquad , \qquad (3.13)$$

and

$$\nabla\cdot\underset{\sim}{E} = 0 \qquad . \qquad (3.14)$$

Since Eqs. (3.1) - (3.14) are linear and homogeneous, the solutions may be obtained by assuming that the field quantities vary as $\exp\left[i(\underset{\sim}{k}\cdot\underset{\sim}{r} - \omega t)\right]$ where ω is the angular frequency and $\underset{\sim}{k}$ is the wave vector. Then Eqs. (3.1) - (3.8) describe magnetohydrodynamic waves of three modes, Eqs. (3.9) and (3.10) describe a sound wave, and Eqs. (3.11) - (3.14) describe electromagnetic waves with two independent polarizations.

The geometry is chosen so that the x-y plane at z = 0 is the plane of the interface, and the normal, $\hat{\underset{\sim}{n}}$, lies along the positive z axis and points from the plasma into the neutral gas. Without loss of generality $\underset{\sim}{k}$ will be taken to lie in the y-z plane which is the plane of incidence. Let θ be the angle made by $\underset{\sim}{k}$ with the z axis. Then $\underset{\sim}{k}$ may be written

$$\underset{\sim}{k} = k(0, \sin\theta, \cos\theta) \tag{3.15}$$

The calculations will be made for two cases: $\underset{\sim}{B}_o$ perpendicular to the plane of incidence and $\underset{\sim}{B}_o$ in this plane. In the first case, only the fast magnetoacoustic mode propagates, and in the second case the Alfvén mode is uncoupled from the magnetoacoustic modes.

The geometry is shown in Fig. 1 for the case in which $\underset{\sim}{B}_o$ lies in the plane of incidence. Superscripts i, r, and t refer to the incident, reflected, and transmitted waves, respectively. There is, in general, more than one mode reflected and transmitted (for a given incident mode) but this is not shown in the figure. The ratios of the transmitted to incident fields and the ratios of energy transmitted in any mode to the incident energy will be calculated. The energy coupling coefficient is defined to be the latter ratio, $\langle \underset{\sim}{T}^t \cdot \hat{\underset{\sim}{n}} \rangle / \langle \underset{\sim}{T}^i \cdot \hat{\underset{\sim}{n}} \rangle$, where the brackets denote a time average.

It may happen that for certain directions of $\underset{\sim}{k}^i$, some $\underset{\sim}{k}^r$ and $\underset{\sim}{k}^t$ will have imaginary z components. This corresponds to the physical fact that these reflected and transmitted waves are exponentially attenuated in the z direction. The attenuation is of the form $\exp(-|z|/d)$ where

$$d \equiv 1/|k_z| \quad . \tag{3.16}$$

Now $k_z = k(1 - \sin^2\theta)^{\frac{1}{2}}$. Whenever $\sin\theta \gg 1$, then

$$d \approx (k|\sin\theta|)^{-1} = |k_y|^{-1} \quad . \tag{3.17}$$

4. PROPERTIES OF MAGNETOHYDRODYNAMIC WAVES

Equations (3.1) - (3.8) give

$$\partial^2 \underset{\sim}{v}/\partial t^2 = s^2\nabla(\nabla\cdot\underset{\sim}{v}) - a^2\hat{\underset{\sim}{B}}_o x \left\{\nabla x\left[\nabla x(\underset{\sim}{v} x \hat{\underset{\sim}{B}}_o)\right]\right\} \tag{4.1}$$

where the caret denotes a unit vector, s is the sound velocity, and a is the Alfvén velocity, $B_o/(4\pi\rho_o)^{\frac{1}{2}}$. Equation (4.1) can be written

$$\sum_j M_{ij} v_j = 0 \quad , \tag{4.2}$$

where

$$M_{ij} \equiv \left[\omega^2 - a^2(\hat{\underset{\sim}{B}}_o\cdot\underset{\sim}{k})^2\right]\delta_{ij} - (s^2 + a^2)k_i k_j + a^2(\hat{\underset{\sim}{B}}_o\cdot\underset{\sim}{k})\left[k_i \hat{B}_{o_j} + \hat{B}_{o_i} k_j\right]. \tag{4.3}$$

The condition for a non-trivial solution of Eq. (4.2) is that det M vanish; this gives the dispersion relations for three modes

$$\omega = a\left|\underset{\sim}{k}_\alpha\cdot\hat{\underset{\sim}{B}}_o\right| \quad , \tag{4.4}$$

where subscript α is used to denote the Alfvén mode,

$$\omega = k_+ a\left[1 + (s^2/2a^2)(\hat{\underset{\sim}{k}}_+ x \hat{\underset{\sim}{B}}_o)^2 + O(s/a)^4\right] \quad , \tag{4.5}$$

where subscript + is used to denote the fast wave of the magnetoacoustic mode, and

$$\omega = s\left|\underset{\sim}{k}_-\cdot\hat{\underset{\sim}{B}}_o\right| \left[1 - (s^2/2a^2)(\hat{\underset{\sim}{k}}_- x \hat{\underset{\sim}{B}}_o)^2 + O(s/a)^4\right] \tag{4.6}$$

where subscript - is used to denote the slow wave of the magnetoacoustic mode. The dispersion relations are exhibited as a power series in $(s/a)^2$ for application to the geophysical problem where $s/a \ll 1$.

The eigenvectors associated with each mode are determined from Eq. (4.2) with the aid of the dispersion relations. The results are that $\underset{\sim}{v}_\alpha$ is perpendicular to both $\underset{\sim}{k}_\alpha$ and $\underset{\sim}{B}_o$, and $\underset{\sim}{v}_+$ and $\underset{\sim}{v}_-$ lie in the plane containing $\underset{\sim}{B}_o$ and the corresponding wave vector.

When $\underset{\sim}{B}_o$ is normal to the plane of incidence, i.e., $\underset{\sim}{B}_o = B_o(1, 0, 0)$, only the fast magnetoacoustic mode propagates. Its dispersion relation is exactly $\omega = k_+(s^2 + a^2)^{\frac{1}{2}}$ and $\underset{\sim}{v}_+$ is parallel to $\underset{\sim}{k}_+$,

$$\underset{\sim}{v}_+ = v_+(0, \sin\theta_+, \cos\theta_+) \quad . \tag{4.7}$$

When $\underset{\sim}{B}_o$ lies in the plane of incidence, $\underset{\sim}{v}_+$ and $\underset{\sim}{v}_-$ also lie in this plane, while $\underset{\sim}{v}_\alpha$ is perpendicular to it. Let φ and β be the angles made by $\underset{\sim}{B}_o$ and $\underset{\sim}{v}$, respectively, with the z axis.

For

$$\underset{\sim}{B}_o = B_o(0, \sin\varphi, \cos\varphi) \quad , \tag{4.8}$$

then

$$\underset{\sim}{v}_\alpha = v_\alpha(1, 0, 0) \quad , \tag{4.9}$$

and

$$\underset{\sim}{v}_\pm = v_\pm(0, \sin\beta_\pm, \cos\beta_\pm) \quad , \tag{4.10}$$

where

$$\beta_+ = \varphi + \pi/2 - (s^2/2a^2)\sin\left[2(\theta_+ - \varphi)\right] + O(s/a)^4 \quad , \tag{4.11}$$

and

$$\beta_- = \varphi - (s^2/2a^2)\sin\left[2(\theta_- - \varphi)\right] + O(s/a)^4 \quad . \tag{4.12}$$

Thus, to zeroth order in s/a, $\hat{\underset{\sim}{v}}_+$ is perpendicular to $\hat{\underset{\sim}{B}}_o$ and $\hat{\underset{\sim}{v}}_-$ is parallel to $\hat{\underset{\sim}{B}}_o$.

The energy transport vector is given by Eq. (2.26) which, when written here in terms of the fluid velocity, $\underset{\sim}{v}$, is

$$\underset{\sim}{T} = (a^2\rho_o/\omega)\left\{\underset{\sim}{k}(\underset{\sim}{v}\times\hat{\underset{\sim}{B}}_o)^2 - (\underset{\sim}{v}\times\hat{\underset{\sim}{B}}_o)\left[\underset{\sim}{k}\cdot(\underset{\sim}{v}\times\hat{\underset{\sim}{B}}_o)\right] + (s^2/a^2)\,\underset{\sim}{v}(\underset{\sim}{k}\cdot\underset{\sim}{v})\right\} \quad . \tag{4.13}$$

For the different modes, this reduces to

$$\underset{\sim}{T}_\alpha = \hat{\underset{\sim}{B}}_o\rho_o a(v_\alpha)^2\sigma(\underset{\sim}{k}_\alpha\cdot\underset{\sim}{B}_o) \tag{4.14}$$

and

$$\underset{\sim}{T}_\pm = (a^2\rho_o/\omega)\left[\underset{\sim}{k}_\pm(\underset{\sim}{v}_\pm\times\hat{\underset{\sim}{B}}_o)^2 + (s^2/a^2)\,\underset{\sim}{v}_\pm(\underset{\sim}{k}_\pm\cdot\underset{\sim}{v}_\pm)\right] \tag{4.15}$$

where $\sigma(f) = f/|f|$. Equation (4.14) shows that the energy in the Alfvén mode propagates in the direction of the constant magnetic field.

When $\underset{\sim}{B}_o$ is normal to the plane of incidence, the relevant field quantities, written in terms of v_+, are

$$\underset{\sim}{E}_+ = (B_o/c)\,v_+(0,\ -\cos\theta_+,\ \sin\theta_+) \quad , \tag{4.16}$$

and

$$\underset{\sim}{B}_+ = (B_o k_+ v_+/\omega)\,(1,\ 0,\ 0) \quad . \tag{4.17}$$

From Eq. (4.15)

$$\underset{\sim}{T}_+ = (\rho_o a^2 \underset{\sim}{k}_+/\omega)\,(v_+)^2\left[1 + (s^2/a^2)\right] \quad , \tag{4.18}$$

which shows that the energy transport is entirely in the $\underset{\sim}{k}_+$ direction.

When $\underset{\sim}{B}_o$ lies in the plane of incidence, the field quantities, written in terms of v, are

$$\underset{\sim}{E}_\alpha = (B_o/c)v_\alpha(0,\ \cos\varphi,\ -\sin\varphi) \quad , \tag{4.19}$$

$$\underset{\sim}{E}_\pm = -(B_o/c)v_\pm\,\sin(\beta_\pm - \varphi)\,(1,\ 0,\ 0) \quad , \tag{4.20}$$

$$\underset{\sim}{B}_\alpha = -(B_o/\omega)k_\alpha v_\alpha\,\cos(\theta_\alpha - \varphi)\,(1,\ 0,\ 0) \quad , \tag{4.21}$$

and

$$\underset{\sim}{B}_{\pm} = k_{\pm}(B_o v_{\pm}/\omega)\ \sin(\beta_{\pm} - \varphi)\ (0,\ -\cos\theta_{\pm},\ \sin\theta_{\pm}) \quad . \tag{4.22}$$

From Eqs. (4.14) and (4.15)

$$\underset{\sim}{T}_{\alpha} = \hat{\underset{\sim}{B}}_o \rho_o a (v_{\alpha})^2 \sigma \left[\cos(\theta_{\alpha} - \varphi)\right] \quad , \tag{4.23}$$

$$\underset{\sim}{T}_{+} = (a^2\rho_o/\omega)\ k_{+}(v_{+})^2 \left[\hat{\underset{\sim}{k}}_{+} + \hat{\underset{\sim}{v}}_{+}\ O(s^2/a^2)\right] \quad , \tag{4.24}$$

$$\underset{\sim}{T}_{-} = (s^2\rho_o k_{-}/\omega)\ (v_{-})^2 \cos(\theta_{-} - \varphi) \left[\hat{\underset{\sim}{B}}_o + \hat{\underset{\sim}{k}}_{-}\ O(s^2/a^2)\right] \quad . \tag{4.25}$$

The energy in the fast wave of the magnetoacoustic mode propagates mainly in the $\underset{\sim}{k}_{+}$ direction with a component $O(s^2/a^2)$ less in the $\underset{\sim}{v}_{+}$ direction, while the energy in the slow wave of the magnetoacoustic mode propagates mainly in the $\underset{\sim}{B}_o$ direction with a component $O(s^2/a^2)$ less in the $\underset{\sim}{k}_{-}$ direction.

5. PROPERTIES OF SOUND AND ELECTROMAGNETIC WAVES

Equations (3.9) and (3.10) give

$$\partial^2 \underset{\sim}{v}/\partial t^2 = s^2 \nabla(\nabla \cdot \underset{\sim}{v}) \tag{5.1}$$

which is the equation of a longitudinal wave, i.e., $\hat{v}_s = \hat{k}_s$, where the subscript s denotes sound waves. The dispersion relation is $\omega = k_s s$, and the energy transport vector is

$$\underset{\sim}{T}_s = p\underset{\sim}{v}_s = (s^2\rho_o/\omega)\ \underset{\sim}{k}_s (v_s)^2 \tag{5.2}$$

Equations (3.11) - (3.14) describe electromagnetic waves. The dispersion relation is $\omega = k_e c$, where the subscript e denotes electromagnetic waves. With $k_e = (0,\ \sin\theta_e,\ \cos\theta_e)$, the electromagnetic modes may be described completely by specifying E_x and E_y. Then $E_z = -E_y \tan\theta_e$, $B_x = -E_y \sec\theta_e$, $B_y = E_x \cos\theta_e$, $B_z = -E_x \sin\theta_e$, and

$$\underset{\sim}{T}_e = (c/4\pi)\,\hat{\underset{\sim}{k}}_e \left[E_x^2 + E_y^2 \sec^2\theta_e\right] \tag{5.3}$$

6. CALCULATIONS FOR $\underset{\sim}{B}_o$ NORMAL TO THE PLANE OF INCIDENCE

When $\underset{\sim}{B}_o$ is normal to the plane of incidence, only the fast magneto-acoustic mode propagates. The requirement that the boundary conditions, Eqs. (2.17), (2.18), and (2.21), be valid for all times over the entire interface, in the usual manner, leads to the conditions that ω and $\hat{\underset{\sim}{n}} x \underset{\sim}{k}$ are constant. These, along with the dispersion relations, are sufficient to calculate $\underset{\sim}{k}^r$ and $\underset{\sim}{k}^t$ in terms of $\underset{\sim}{k}^i$. One finds, then, that the angle of incidence equals the angle of reflection.

$$\theta_+^r = \pi - \theta_+^i \quad , \tag{6.1}$$

$$k_+^r = k_+^i \quad , \tag{6.2}$$

$$k_e^t = k_+^i (s^2 + a^2)^{\frac{1}{2}}/c \quad , \tag{6.3}$$

$$\sin\theta_e^t = c(s^2 + a^2)^{-\frac{1}{2}} \sin\theta_+^i \quad , \tag{6.4}$$

$$k_s^t = k_+^i (s^2 + a^2)^{\frac{1}{2}}/s \quad , \tag{6.5}$$

$$\sin\theta_s^t = s(s^2 + a^2)^{-\frac{1}{2}} \sin\theta_+^i \quad . \tag{6.6}$$

Equation (6.4) shows that whenever θ_+^i is greater than $(s^2 + a^2)^{\frac{1}{2}}/c$, $\sin\theta_e^t$ is greater than unity so that $\cos\theta_e^t$ is imaginary. In this case, the electromagnetic wave is exponentially damped in the z direction. Thus, the incident fast magnetoacoustic mode must have its wave vector lie within a small cone of half angle $(s^2 + a^2)^{\frac{1}{2}}/c$ with cone axis along the normal to the boundary in order for the electromagnetic wave to propagate unattenuated. When the incident wave vector lies outside this cone, the transmitted electromagnetic wave is exponentially damped, proportional to exp(-z/d) where, from Eq.(3.17), $d \cong (s^2 + a^2)^{\frac{1}{2}}/|\omega \sin \theta_+{}^i|$. Equation (6.6) shows that when $s/a \ll 1$, the fast magnetoacoustic mode incident on the boundary gives rise to a sound wave lying within a cone of half angle (s/a) with cone axis along the boundary normal. That is, a fast magnetoacoustic wave, in this approximation, gives rise to a sound wave which propagates at a small angle to the normal regardless of the angle of incidence.

By applying the boundary conditions, Eqs. (2.17), (2.18), (2.21), and (4.16) - (4.18), one finds

$$-(B_o/c)(v_+{}^i \cos \theta_+{}^i + v_+{}^r \cos \theta_+{}^r) = E_y{}^t \quad , \tag{6.7}$$

$$v_+{}^i \cos \theta_+{}^i + v_+{}^r \cos \theta_+{}^r \doteq v_s{}^t \cos \theta_s{}^t \tag{6.8}$$

$$(s^2\rho_o/\omega)(k_+{}^i v_+{}^i + k_+{}^r v_+{}^r) + (a^2\rho_o/\omega)(k_+{}^i v_+{}^i + k_+{}^r v_+{}^r)$$
$$= (s^2\rho_o/\omega)(k_s{}^t v_s{}^t) - (B_o/4\pi)(E_y{}^t/\cos \theta_e{}^t) \tag{6.9}$$

From the latter equations, with the aid of Eqs. (6.1) - (6.6), one can show that

$$E_y{}^t / E_{+,y}{}^i = (v_+{}^i - v_+{}^r)/v_+{}^i$$
$$= 2c(s^2 + a^2)^{\frac{1}{2}} \Big/ \left[c(s^2 + a^2)^{\frac{1}{2}} + a^2 \cos \theta_+{}^i/\cos \theta_e{}^t + sc \cos \theta_+{}^i/\cos \theta_s{}^t\right]$$
$$= 2 \qquad (s \ll a \ll c) \quad , \tag{6.10}$$

$$E_z^{\,t}/E_{+,z}^{\,i} = (\tan\theta_e^{\,t}/\tan\theta_+^{\,i})(E_y^{\,t}/E_{+,y}^{\,i})$$

$$= 2c\cos\theta_+^{\,i}/a\cos\theta_e^{\,t} \qquad (s \ll a \ll c) \quad , \tag{6.11}$$

$$B_x^{\,t}/B_{+,x}^{\,i} = \left[(s^2 + a^2)^{\frac{1}{2}}/c\right]\left[\cos\theta_+^{\,i}/\cos\theta_e^{\,t}\right]\left[(v_+^{\,i} - v_+^{\,r})/v_+^{\,i}\right]$$

$$= 2a\cos\theta_+^{\,i}/c\cos\theta_e^{\,t} \qquad (s \ll a \ll c) \quad , \tag{6.12}$$

$$\frac{\langle \hat{\underset{\sim}{n}}\cdot \underset{\sim}{T}_e^{\,t}\rangle}{\langle \hat{\underset{\sim}{n}}\cdot \underset{\sim}{T}_+^{\,i}\rangle} = \frac{4a^2c(\cos\theta_+^{\,i}/\cos\theta_e^{\,t})(s^2 + a^2)^{\frac{1}{2}}}{\left[c(s^2 + a^2)^{\frac{1}{2}} + a^2\cos\theta_+^{\,i}/\cos\theta_e^{\,t} + sc\cos\theta_+^{\,i}/\cos\theta_s^{\,t}\right]^2}$$

$$= (4a/c)(\cos\theta_+^{\,i}/\cos\theta_e^{\,t}) \qquad (s \ll a \ll c) \quad , \tag{6.13}$$

$$\frac{\langle \hat{\underset{\sim}{n}}\cdot \underset{\sim}{T}_s^{\,t}\rangle}{\langle \hat{\underset{\sim}{n}}\cdot \underset{\sim}{T}_+^{\,i}\rangle} = \frac{4sc^2(\cos\theta_+^{\,i}/\cos\theta_s^{\,t})(s^2 + a^2)^{\frac{1}{2}}}{\left[c(s^2 + a^2)^{\frac{1}{2}} + a^2\cos\theta_+^{\,i}/\cos\theta_e^{\,t} + sc\cos\theta_+^{\,i}/\cos\theta_s^{\,t}\right]^2}$$

$$= (4s/a)(\cos\theta_+^{\,i}/\cos\theta_s^{\,t}) \qquad (s \ll a \ll c) \quad , \tag{6.14}$$

Equation (6.13) gives the energy coupling coefficient for electromagnetic to magnetohydrodynamic waves, and Eq. (6.14) gives the energy coupling coefficient for acoustic to magnetohydrodynamic waves. In the limit of $s = 0$, Eqs. (6.10) - (6.12) reproduce Eqs. (41) - (43) of Ullah and Kahalas[16].

7. CALCULATIONS FOR $\underset{\sim}{B}_o$ IN THE PLANE OF INCIDENCE

In contrast to the case of $\underset{\sim}{B}_o$ perpendicular to the plane of incidence where only the fast magnetoacoustic mode can propagate, in general now all three magnetohydrodynamic modes can propagate. It will be shown in this section that the two magnetoacoustic modes are coupled, but propagate independently from the Alfvén mode. Before discussing the coupling of wave amplitudes across the boundary, we shall consider the relations between the incident wave vector and the reflected and transmitted ones.

A. Relations between Wave Vectors

The calculation of the reflected and transmitted wave vectors in terms of the incident wave vector involves the requirement that the boundary conditions, Eqs. (2.17), (2.18), (2.23), and (2.24), are satisfied over the entire boundary plane for all times. This leads to the statement that ω and $\hat{n} \times k$ are constant. In addition, one must impose the requirement that an incident wave propagating in the plasma transports energy towards the boundary and a reflected wave propagating in the plasma transports energy away from the boundary. These requirements must be kept in mind because it is quite possible for the Alfvén and slow magnetoacoustic modes to transport energy in such a manner that $T \cdot \hat{n}$, the projection of the energy transport vector in the z direction, has opposite sign to $k \cdot \hat{n}$. This situation will be discussed in more detail below.

For B_0 lying in the plane of incidence, there are several different situations that can arise. These are specified in terms of the wave mode incident.

1) Alfvén Mode Incident

Consider now the relation between wave vectors of the reflected Alfvén mode and the incident Alfvén mode. Since ω is constant,

$$k_\alpha^i \left|\cos(\theta_\alpha^i - \varphi)\right| = k_\alpha^r \left|\cos(\theta_\alpha^r - \varphi)\right| \quad . \tag{7.1}$$

Since $\hat{n} \times k$ is constant,

$$k_\alpha^i \sin \theta_\alpha^i = k_\alpha^r \sin \theta_\alpha^r \quad . \tag{7.2}$$

From these two equations one finds two solutions

$$(\cot \theta_\alpha^i + \tan \varphi) = \pm (\cot \theta_\alpha^r + \tan \varphi) \quad . \tag{7.3}$$

From Eq. (4.23) $< \hat{n} \cdot T_\alpha > = \rho_0 a < (v_\alpha)^2 > |\cos \varphi \cos(\theta_\alpha - \varphi)| \sigma(\cos \varphi) \sigma\left[\cos(\theta_\alpha - \varphi)\right]$. In order for the incident wave energy to impinge on the boundary plane, $\sigma(\cos \varphi)\sigma\left[\cos(\theta_\alpha^i - \varphi)\right] > 0$, and for the reflected wave energy to be propagated into the plasma away from the boundary, $\sigma(\cos \varphi)\sigma\left[\cos(\theta_\alpha^r - \varphi)\right] < 0$. Thus it is required that

$$\sigma(\cos \varphi) = \sigma\left[\cos(\theta_\alpha^i - \varphi)\right] = -\sigma\left[\cos(\theta_\alpha^r - \varphi)\right], \tag{7.4}$$

Equation (7.4) is consistent with Eq. (7.3) only when the minus sign is chosen. This leads to

$$\cot \theta_\alpha^r = -\cot \theta_\alpha^i - 2 \tan \varphi, \tag{7.5}$$

Figure 2 shows a plot of Eq. (7.5) for various values of φ in the first and fourth quadrants. θ^i is restricted without loss of generality to lie in the range $0 < \theta^i < \pi$. Only for $\varphi = 0$ or for very small angles of incidence does the angle of incidence equal the angle of reflection (i.e., $\theta^i = \pi - \theta^r$).[22]

A graphical construction, Fig. 3, is valuable in seeing the relationship between the reflected k_α^r and the incident k_α^i. The dot-dashed line is the locus of all vectors k for which $\hat{n} x k = \hat{n} x k_\alpha^i$. The dashed lines are the locus of all vectors k for which $|k \cdot \hat{B}_0| = |k_\alpha^i \cdot \hat{B}_0|$. In Fig. 3a, the intersections of the dashed lines with the dot-dashed line gives both the incident wave vector and the single possible reflected wave vector at A. In Fig. 3b, B_0 lies along the y axis, and the two equations, (7.1) and (7.2), provide the same information. In this case, there is no unique solution. In Fig. 3c, both intersections lie in the upper half plane. Note that $< \underset{\sim}{T}_{\alpha}^{r} \cdot \hat{\underset{\sim}{n}} > < 0$, as required, whereas $\underset{\sim}{k}_{\alpha}^{r} \cdot \hat{\underset{\sim}{n}} > 0$.

The relationship between $\underset{\sim}{k}_e^t$ and $\underset{\sim}{k}_{\alpha}^i$ (obtained from ω and $\hat{\underset{\sim}{n}} \times \underset{\sim}{k}$ constant) is given by

$$\sin \theta_e^t = (c/a)\left[\sin \theta_{\alpha}^i \Big/ \left|\cos(\theta_{\alpha}^i - \varphi)\right|\right] \tag{7.6}$$

and

$$k_e^t = k_{\alpha}^i a \left|\cos(\theta_{\alpha}^i - \varphi)\right| / c \quad . \tag{7.7}$$

For the electromagnetic wave to propagate unattenuated, $\sin \theta_e^t$ must be less than or equal to unity. Equation (7.6) shows that the incident wave vector must lie within a cone (the cone for radiation) whose axis lies along the normal to the boundary and with half angle for $a/c \ll 1$ given by $a|\cos \varphi|/c$. When the electromagnetic wave is damped, proportional to $\exp(-z/d)$, d is given by Eq. (3.17)

$$d \simeq a \left|\cos(\theta_{\alpha}^i - \varphi)\right| / \omega \left|\sin \theta_{\alpha}^i\right| \quad . \tag{7.8}$$

2) Fast Mode Incident

Consider first the relation between wave vectors of the reflected and the incident fast magnetoacoustic mode. To lowest order in s/a, the dispersion relation of the fast magnetoacoustic mode is independent of the orientation of $\underset{\sim}{k}_+$. Since ω is constant

$$k_+^i = k_+^r \quad , \tag{7.9}$$

and since $\hat{\underset{\sim}{n}} \times \underset{\sim}{k}$ is constant,

$$k_+^i \sin \theta_+^i = k_+^r \sin \theta_+^r \tag{7.10}$$

Thus, the angle of incidence equals the angle of reflection,

$$\theta_+{}^r = \pi - \theta_+{}^i \quad . \qquad (7.11)$$

From the requirement that $< \underset{\sim}{T}_+{}^i \cdot \hat{\underset{\sim}{n}} > \ > 0$ and $< \underset{\sim}{T}_+{}^r \cdot \hat{\underset{\sim}{n}} > \ < 0$ and from the fact that $\underset{\sim}{T}_+$ is in the direction of $\underset{\sim}{k}_+$ to lowest order in s/a, $0 \leqslant \theta_+{}^i \leqslant \pi/2 \leqslant \theta_+{}^r \leqslant \pi$.

Next the relation between wave vectors of the reflected slow magnetoacoustic mode to the incident fast magnetoacoustic mode is considered. With ω constant

$$k_+{}^i\, a = k_-{}^r\, s\, \left|\cos(\theta_-{}^r - \varphi)\right| \quad , \qquad (7.12)$$

and $\hat{\underset{\sim}{n}} \times \underset{\sim}{k}$ constant

$$k_+{}^i \sin\theta_+{}^i = k_-{}^r \sin\theta_-{}^r \quad . \qquad (7.13)$$

From these two equations one finds two solutions

$$a/\left[s \sin\theta_+{}^i\, \left|\cos\varphi\right|\right] = \pm\,(\cot\theta_-{}^r + \tan\varphi) \quad . \qquad (7.14)$$

Without loss of generality θ is taken so that $0 \leqslant \theta \leqslant \pi$ for all modes. From Eq. (4.25) $< \hat{\underset{\sim}{n}} \cdot \underset{\sim}{T}_- > = (\rho_o s^2 k_-/\omega) < (v_-)^2 > \left|\cos\varphi \cos(\theta_- - \varphi)\right| \sigma(\cos\varphi)\sigma\left[\cos(\theta_- - \varphi)\right]$. The requirement that $< \hat{\underset{\sim}{n}} \cdot \underset{\sim}{T}_- > \ < 0$ leads to

$$\sigma\,(\cos\varphi) = -\sigma\left[\cos(\theta_-{}^r - \varphi)\right] \quad . \qquad (7.15)$$

Since $\sigma(\sin\theta_-{}^r) = +1$, there results $\sigma\left[\cos(\theta_-{}^r - \varphi)\right] = \sigma(\cos\varphi)\sigma(\cot\theta_-{}^r + \tan\varphi)$. This combined with Eq. (7.15) leads to the condition that $\sigma(\cot\theta_-{}^r + \tan\varphi) = -1$. Thus Eqs. (7.14) and (7.15) are consistent only when the minus sign is chosen, which gives

$$\cot \theta_-^{\,r} = -\left[a/(s \sin \theta_+^{\,i} \,|\cos \varphi|)\right] - \tan \varphi \tag{7.16}$$

Equation (7.16) shows that for any angle of incidence the reflected wave lies within a cone of half angle $s|\cos \varphi|/a$ whose axis lies along the normal to the boundary.

Figure 4 shows the construction by which $\underset{\sim}{k}_-^{\,r}$ is found given $\underset{\sim}{k}_+^{\,i}$. Again, the dot-dashed line is the locus of all vectors $\underset{\sim}{k}$ for which $\hat{\underset{\sim}{n}} \times \underset{\sim}{k} = \hat{\underset{\sim}{n}} \times \underset{\sim}{k}_+^{\,i}$. The circle has a radius $ak_+^{\,i}/s$, and the dashed lines are the locus of all vectors $\underset{\sim}{k}$ for which $|\underset{\sim}{k} \cdot \hat{\underset{\sim}{B}}_o| = ak_+^{\,i}/s$. In Fig. 4a, the intersection of these loci occurs at A and A'. By considering all possible orientations of $\underset{\sim}{B}_o$, one can show that $\underset{\sim}{k}_-^{\,r}$ is given by the intersection at A in the lower half plane and that the $\underset{\sim}{k}_-^{\,r}$ always lies in the lower half plane. Figure 4b shows the case when $\varphi = \pm\, \pi/2$. In this case, there is no solution.

The relations between the transmitted wave vectors and the wave vector of the incident magnetoacoustic mode (given by ω and $\hat{\underset{\sim}{n}} \times \underset{\sim}{k}$ constant) are

$$k_e^{\,t} = k_+^{\,i}\, a/c \tag{7.17}$$

and

$$\sin \theta_e^{\,t} = c \sin \theta_+^{\,i}/a \tag{7.18}$$

for the electromagnetic wave and

$$k_s^{\,t} = k_+^{\,i}\, a/s \tag{7.19}$$

and

$$\sin \theta_s^{\,t} = s \sin \theta_+^{\,i}/a \tag{7.20}$$

for the sound wave. For the electromagnetic wave to propagate unattenuated

$\sin\theta_e^{\,t}$ must be less than or equal to unity. Eqution (7.18) shows thst the incident wave vector must lie within a cone whose axis lies along the normal to the boundary and with half angle $\sin^{-1}(a/c)$. The angle at which the sound wave is transmitted is, from Eq. (7.20), close to the normal for any $\theta_+^{\,i}$ when $s/a \ll 1$. When the electromsgnetic wave is exponentially dsmped, proportional to $\exp(-z/d)$, d is given by Eq. (3.17)

$$d \simeq a/\ \omega|\sin\theta_+^{\,i}| \tag{7.21}$$

3) Slow Mode Incident

Consider first the relation between wave vectors of the reflected fast magnetoacoustic mode to the incident slow magnetoscoustic mode. Since ω is constant,

$$k_+^{\,r}a = k_-^{\,i}s\ |\cos(\theta_-^{\,i} - \varphi)| \quad , \tag{7.22}$$

and since $\hat{n}\times k$ is constant,

$$k_+^{\,r}\sin\theta_+^{\,r} = k_-^{\,i}\sin\theta_-^{\,i} \quad . \tag{7.23}$$

These two equations have two solutions

$$a/(s\sin\theta_+^{\,r}\ |\cos\varphi|\ ) = \pm\ (\cot\theta_-^{\,i} + \tan\varphi) \quad . \tag{7.24}$$

Without loss of generslity θ is taken so thst $0 \leqslant \theta_-^{\,i} \leqslant \pi$. From Eq. (4.25)
$\langle \hat{n}\cdot T_- \rangle = (\rho_o s^2 k_-/\omega)\langle (v_-)^2 \rangle\ |\cos\varphi\,\cos(\theta_- - \varphi)|\ \sigma(\cos\varphi)\,\sigma\left[\cos(\theta_- - \varphi)\right]$.
The requirement that $\langle \hat{n}\cdot T_-^{\,i} \rangle > 0$ lesds to

$$\sigma(\cos\varphi) = \sigma\left[\cos(\theta_-^{\,i} - \varphi)\right] \tag{7.25}$$

With θ_-^i in the first and second quadrants, $\sigma\left[\cos(\theta_-^i - \varphi)\right] = \sigma(\cos\varphi)$ $\sigma(\cot\theta_-^i + \tan\varphi)$. The latter equation combined with Eq. (7.25) leads to the condition that $\sigma(\cot\theta_-^i + \tan\varphi) = +1$. Thus Eqs. (7.24) and (7.25) are consistent only when the plus sign is chosen which gives

$$\sin\theta_+^r = (a/s)\left[|\cos\varphi|(\cot\theta_-^i + \tan\varphi)\right]^{-1} \tag{7.26}$$

For the reflected fast magnetoacoustic mode to propagate unattenuated, $\cot\theta_-^i \geq (a/s\,|\cos\varphi|) - \tan\varphi$, which may be approximated by $\theta_-^i \leqslant (s/a)\,|\cos\varphi|$. That is, the incident slow magnetoacoustic mode must be within a cone of half angle $(s/a)\,|\cos\varphi|$. When the fast magnetoacoustic mode is attenuated, proportional to $\exp(-|z|/d)$, d is given by Eq. (3.17)

$$d \simeq s\,|\cos(\theta_-^i - \varphi)|\,/\,\omega|\sin\theta_-^i| \tag{7.27}$$

Figure 5 shows the construction by which $\underset{\sim}{k}_+^r$ is found, given $\underset{\sim}{k}_-^i$. Again, the dot-dashed line is the locus of all vectors $\underset{\sim}{k}$ for which $\hat{\underset{\sim}{n}}\times\underset{\sim}{k} = \hat{\underset{\sim}{n}}\times\underset{\sim}{k}_-^i$. The circle has a radius $(s/a)\,|\underset{\sim}{k}_-^i\cdot\hat{\underset{\sim}{B}}_o| = \omega/a$. Then $\underset{\sim}{k}_+^r$ is determined by the intersection of the circle and the dot-dashed line. There can be two intersections, and the one in the lower half plane is chosen, according to the requirement $\langle \underset{\sim}{T}_+^r\cdot\hat{\underset{\sim}{n}}\rangle < 0$, since the fast magnetoacoustic mode transports energy in the direction of $\underset{\sim}{k}_+^r$. In Fig. 5a the intersections occur at A and A', and the one in the lower half plane is chosen. Figure 5b shows a construction in which no solution exists. In this case $\sin\theta_+^r > 1$, and Eq. (7.26) leads to $\cot\theta_-^i < (a/s\,|\cos\varphi|) - \tan\varphi$. Figure 5c gives the construction for $\varphi = \pm\pi/2$ for $s/a < 1$. In this case no solution exists.

Next, the relation between wave vectors of the reflected slow magnetoacoustic mode to the incident slow magnetoacoustic mode is considered. With ω constant,

$$k_-^{\,i}\,|\cos(\theta_-^{\,i} - \varphi)| = k_-^{\,r}\,|\cos(\theta_-^{\,r} - \varphi)| \quad , \tag{7.28}$$

and with $\hat{n} \times k$ constant,

$$k_-^{\,i} \sin\theta_-^{\,i} = k_-^{\,r} \sin\theta_-^{\,r} \quad . \tag{7.29}$$

These two equations are the same as the ones for Alfvén mode incident and reflected. The discussion is the same as for that case, with the resulting equation

$$\cot\theta_-^{\,r} = -\cot\theta_-^{\,i} - 2\tan\varphi \quad . \tag{7.30}$$

Figure 2 shows the relation between the angles of incidence and reflection, and Fig. 3 shows the construction of $k_-^{\,r}$ from $k_-^{\,i}$.

The relations between the transmitted wave vectors and the wave vector of the incident slow magnetoacoustic mode (given by ω and $\hat{n} \times k$ constant) are

$$k_e^{\,t} = k_-^{\,i}\,|\cos(\theta_-^{\,i} - \varphi)|\, s/c \tag{7.31}$$

and

$$\sin\theta_e^{\,t} = (c/s)\left[\sin\theta_-^{\,i} \Big/ |\cos(\theta_-^{\,i} - \varphi)|\right] \tag{7.32}$$

for the electromagnetic wave, and

$$k_s^{\,t} = k_-^{\,i}\,|\cos(\theta_-^{\,i} - \varphi)| \tag{7.33}$$

and

$$\sin \theta_s{}^t = \sin \theta_-{}^i / \left|\cos(\theta_-{}^i - \varphi)\right| \tag{7.34}$$

for the sound wave. Equation (7.34) is shown graphically in Fig. 6. Equation (7.32) shows that for the electromagnetic wave to propagate unattenuated, the incident wave vector of the slow magnetoacoustic mode must lie within a cone whose axis lies along the normal to the boundary and with half angle given by $s\left|\cos\varphi\right|/c$. When the electromagnetic wave is exponentially damped, proportional to $\exp(-z/d)$, d is given by Eq. (3.17)

$$d \simeq s\left|\cos(\theta_-{}^i - \varphi)\right| / \omega\left|\sin\theta_-{}^i\right| \quad . \tag{7.35}$$

Equation (7.34) shows that the sound wave is damped unless

$$\cot \theta_-{}^i \geqslant (1/\left|\cos\varphi\right|) - \tan\varphi \quad . \tag{7.36}$$

Whenever $-\pi/2 \leqslant \varphi \leqslant \pi/2$, Eq. (7.36) is equivalent to $\theta_-{}^i \leqslant \varphi/2 + \pi/4$. Whenever $\pi/2 \leqslant \varphi \leqslant 3\pi/2$, Eq. (7.36) is equivalent to $\theta_-{}^i \leqslant \varphi/2 - \pi/4$. When the sound wave is exponentially damped, proportional to $\exp(-z/d)$, d is given by Eq. (3.17)

$$d \simeq \left[s\left|\cos(\theta_-{}^i - \varphi)\right|\right] \Big/ \omega\left[\left|\cos\varphi\cos(2\theta_-{}^i - \varphi)\right|\right]^{\frac{1}{2}} \tag{7.37}$$

B. Relations between Wave Amplitudes

The boundary conditions, Eqs. (2.17), (2.18), (2.23), and (2.24), are now applied, along with the equations of Sections 4 and 5 in which the field vectors are expressed in terms of specific amplitudes and angles. The resulting equations separate into two sets. The first set contains only $v_\alpha{}^i$,

$v_\alpha^{\,r}$, and $E_y^{\,t}$; the second contains only $v_\pm^{\,i}$, $v_\pm^{\,r}$, $E_x^{\,t}$, and $v_s^{\,t}$. Thus, an incident Alfvén mode gives rise only to a reflected Alfvén mode and a transmitted electromagnetic wave. An incident magnetoacoustic mode gives rise only to reflected magnetoacoustic modes and transmitted electromagnetic and sound modes.

1) Alfvén Mode Incident

The equations relating amplitudes on the two sides of the boundary are

$$B_o \cos\varphi(v_\alpha^{\,i} + v_\alpha^{\,r})/c = E_y^{\,t} \tag{7.38}$$

and

$$(B_o/\omega)\left[k_\alpha^{\,i} v_\alpha^{\,i} \cos(\theta_\alpha^{\,i} - \varphi) + k_\alpha^{\,r} v_\alpha^{\,r} \cos(\theta_\alpha^{\,r} - \varphi)\right] = E_y^{\,t}/\cos\theta_e^{\,t} \tag{7.39}$$

These equations determine $v_\alpha^{\,r}$ and $E_y^{\,t}$ in terms of $v_\alpha^{\,i}$. From Eqs. (7.38) and (7.39), the ratios of field quantities are

$$\begin{aligned} E_y^{\,t}/E_y^{\,i} &= (v_\alpha^{\,i} + v_\alpha^{\,r})/v_\alpha^{\,i} \\ &= 2\cos\theta_e^{\,t}\Big/\left[\cos\theta_e^{\,t} + (a/c)\,|\cos\varphi|\right] \\ &= 2 \qquad (a/c \ll 1) \quad , \end{aligned} \tag{7.40}$$

$$\begin{aligned} E_z^{\,t}/E_z^{\,i} &= (\tan\theta_e^{\,t}/\tan\varphi)(E_y^{\,t}/E_y^{\,i}) \\ &= 2\sin\theta_e^{\,t}\cot\varphi\Big/\left[\cos\theta_e^{\,t} + (a/c)\,|\cos\varphi|\right] \\ &= 2\tan\theta_e^{\,t}\cot\varphi \qquad (a/c \ll 1) \quad , \end{aligned} \tag{7.41}$$

$$\begin{aligned} B_x^{\,t}/B_x^{\,i} &= (2a/c)\,|\cos\varphi|\Big/\left[\cos\theta_e^{\,t} + (a/c)\,|\cos\varphi|\right] \\ &= (2a/c)\,|\cos\varphi|/\cos\theta_e^{\,t} \qquad (a/c \ll 1) \quad . \end{aligned} \tag{7.42}$$

The expressions for the coupling coefficients are

$$\frac{<\underset{\sim}{\hat{n}}\cdot\underset{\sim}{T}_e{}^t>}{<\underset{\sim}{\hat{n}}\cdot\underset{\sim}{T}_\alpha{}^i>} = \frac{(4a/c)|\cos\varphi|\cos\theta_e{}^t}{(\cos\theta_e{}^t + a|\cos\varphi|/c)^2}$$

$$= (4a/c)|\cos\varphi|/\cos\theta_e{}^t \qquad (a \ll c) \quad , \qquad (7.43)$$

$$\frac{<\underset{\sim}{\hat{n}}\cdot\underset{\sim}{T}_\alpha{}^r>}{<\underset{\sim}{\hat{n}}\cdot\underset{\sim}{T}_\alpha{}^i>} = -\left[\frac{\cos\theta_e{}^t - a|\cos\varphi|/c}{\cos\theta_e{}^t + a|\cos\varphi|/c}\right]^2$$

$$= -1 \qquad (a \ll c) \quad . \qquad (7.44)$$

2) Fast or Slow Mode Incident

The equations relating the amplitudes on the two sides of the boundary are

$$(B_o/c)\Big[v_+{}^i\sin(\beta_+{}^i - \varphi) + v_-{}^i\sin(\beta_-{}^i - \varphi) + v_+{}^r\sin(\beta_+{}^r - \varphi)$$

$$+ v_-{}^r\sin(\beta_-{}^r - \varphi)\Big] = -E_x{}^t \quad , \qquad (7.45)$$

$$(B_o/\omega)\Big[k_+{}^i v_+{}^i\sin(\beta_+{}^i - \varphi)\cos\theta_+{}^i + k_-{}^i v_-{}^i\sin(\beta_-{}^i - \varphi)\cos\theta_-{}^i + k_+{}^r v_+{}^r\cdot$$

$$\cdot\sin(\beta_+{}^r - \varphi)\cos\theta_+{}^r + k_-{}^r v_-{}^r\sin(\beta_-{}^r - \varphi)\cos\theta_-{}^r\Big] = -\cos\theta_e{}^t E_x{}^t \quad , \qquad (7.46)$$

$$k_+{}^i v_+{}^i\cos(\theta_+{}^i - \beta_+{}^i) + k_-{}^i v_-{}^i\cos(\theta_-{}^i - \beta_-{}^i) + k_+{}^r v_+{}^r\cos(\theta_+{}^r - \beta_+{}^r)$$

$$+ k_-{}^r v_-{}^r\cos(\theta_-{}^r - \beta_-{}^r) = k_s{}^t v_s{}^t \quad , \qquad (7.47)$$

and

$$v_+{}^i\cos\beta_+{}^i + v_-{}^i\cos\beta_-{}^i + v_+{}^r\cos\beta_+{}^r + v_-{}^r\cos\beta_-{}^r = v_s{}^t\cos\theta_s{}^t \quad . \qquad (7.48)$$

For the fast magnetoacoustic mode incident $v_-^i = 0$ and ω is given by Eq. (4.5); for the slow magnetoacoustic mode incident $v_+^i = 0$ and ω is given by Eq. (4.6).

The solution of Eqs. (7.45) - (7.48) for the fast wave incident gives for the ratio of field amplitudes

$$
\begin{aligned}
E_x^t/E_x^i &= (v_+^i + v_+^r)/v_+^i \\
&= 2\cos\theta_+^i \Big/ \left[\cos\theta_+^i + (a/c)\cos\theta_e^t\right] \\
&= 2 \qquad (a \ll c) \ , \qquad (7.49)
\end{aligned}
$$

$$
\begin{aligned}
B_z^t/B_z^i &= E_x^t/E_x^i \\
&= 2\cos\theta_+^i \Big/ \left[\cos\theta_+^i + (a/c)\cos\theta_e^t\right] \\
&= 2 \qquad (a \ll c) \ , \qquad (7.50)
\end{aligned}
$$

and

$$
\begin{aligned}
B_y^t/B_y^i &= (v_+^i - v_+^r)/v_+^i \\
&= (2a/c)\cos\theta_e^t \Big/ \left[\cos\theta_+^i + (a/c)\cos\theta_e^t\right] \\
&= (2a/c)(\cos\theta_e^t/\cos\theta_+^i) \qquad (a \ll c) \ . \qquad (7.51)
\end{aligned}
$$

The expressions for the coupling coefficients are

$$
\begin{aligned}
\frac{\langle \hat{\underset{\sim}{n}}\cdot\underset{\sim}{T}_e^t \rangle}{\langle \hat{\underset{\sim}{n}}\cdot\underset{\sim}{T}_+^i \rangle} &= \frac{(4a/c)\cos\theta_+^i\cos\theta_e^t}{\left[\cos\theta_+^i + (a/c)\cos\theta_e^t\right]^2} \\
&= (4a/c)(\cos\theta_e^t/\cos\theta_+^i) \qquad (a \ll c) \ , \qquad (7.52)
\end{aligned}
$$

$$\frac{\langle \hat{n}\cdot T_s{}^t \rangle}{\langle \hat{n}\cdot T_+{}^i \rangle} = \frac{(4s/a)\sin^2\varphi \cos\theta_+{}^i \cos\theta_s{}^t}{\left[\cos\theta_+{}^i + (a/c)\cos\theta_e{}^t\right]^2 \left[|\cos\varphi| + \cos\theta_s{}^t\right]^2} \tag{7.53}$$

In the above only the lowest order in s/a has been kept. Equation (7.53) gives the coupling coefficient of sound waves to magnetohydrodynamic waves. It is of the order of s/a which has been assumed to be small in this paper. In the lower ionosphere where collisions between heavy ions and neutral particles become important (lower E region), the phase velocity of magnetohydrodynamic wave propagation can be reduced to a small fraction of the Alfvén velocity and, in fact, can become comparable with the sound velocity. In this case there may be a large coupling between magnetohydrodynamic and acoustic waves, although the present model is not adequate to treat that situation because only a fully ionized plasma has been discussed here.

For $s/a \ll 1$, $\sin\theta_s{}^t$ is small and $\cos\theta_s{}^t \simeq 1$. For $\sin\theta_+{}^i <$ a/c, the electromagnetic wave is undamped and Eq. (7.53) can be written

$$\begin{aligned} \frac{\langle \hat{n}\cdot T_s{}^t \rangle}{\langle \hat{n}\cdot T_+{}^i \rangle} &= (4s/a)\tan^2(\varphi/2) \qquad (-\pi/2 \leqslant \varphi \leqslant \pi/2) \\ &= (4s/a)\cot^2(\varphi/2) \qquad (\pi/2 \leqslant \varphi \leqslant 3\pi/2) \quad . \end{aligned} \tag{7.54}$$

The solution of Eqs. (7.45) - (7.48) for the slow wave incident gives for the ratio of field amplitudes

$$E_x{}^t/E_x{}^i = -(a/s)\left[\cos\theta_-{}^i\,\delta^- \Big/ |\cos(\theta_-{}^i - \varphi)|\right] \tag{7.55}$$

where

$$\delta^- \equiv \left\{1 + \frac{\sin(\theta_-^r - \varphi)}{\sin(\theta_-^i - \varphi)} \frac{\cos\theta_-^r}{\cos\theta_-^i} \left[\frac{|\cos\varphi| - \cos\theta_s^t}{|\cos\varphi| + \cos\theta_s^t}\right]\right\} \left[\cos\theta_+^r - (a/c)\cos\theta_e^t\right]^{-1}$$

When the transmitted electromagnetic wave is undamped, $\theta_-^i \simeq 0$, $\theta_s^t \simeq 0$, $\theta_-^r \simeq \pi$, and $\theta_+^r \simeq \pi$. Then Eq. (7.55) becomes

$$E_x^t/E_x^i \simeq (2a/s)/(1 + |\cos\varphi|) \quad . \tag{7.56}$$

Also

$$B_y^t/B_y^i = -(a/c)\cos\theta_e^t\,\delta^- \quad , \tag{7.57}$$

and when the transmitted electromagnetic wave is undamped,

$$B_y^t/B_y^i = (2a/c)\cos\theta_e^t|\cos\varphi|/(1 + |\cos\varphi|) \quad . \tag{7.58}$$

Also

$$B_z^t/B_z^i = -(a/c)\sin\theta_e^t\cot\theta_-^i\,\delta^- \tag{7.59}$$

which becomes, when the electromagnetic wave is undamped

$$B_z^t/B_z^i = (2a/c)\sin\theta_e^t\cot\theta_-^i|\cos\varphi|/(1 + |\cos\varphi|) \, , \tag{7.60}$$

which is of the order of a/s because $\cot\theta_-^i$ is of the order of c/s when θ_-^i lies within the cone for radiation in this case.

The expressions for the coupling coefficients are

$$\frac{\langle \hat{\underset{\sim}{n}}\cdot\underset{\sim}{T}_e^t \rangle}{\langle \hat{\underset{\sim}{n}}\cdot\underset{\sim}{T}_-^i \rangle} = \frac{(4s/c)\cos\theta_e^t\sin^2\varphi\,|\cos\varphi|}{\left[1 + |\cos\varphi|\right]^2} \tag{7.61}$$

and

$$\frac{\langle \underset{\sim}{\hat{n}} \cdot \underset{\sim}{T}_s{}^t \rangle}{\langle \underset{\sim}{\hat{n}} \cdot \underset{\sim}{T}_-{}^i \rangle} = \frac{4|\cos \varphi| \cos \theta_s{}^t}{\left[|\cos \varphi| + \cos \theta_s{}^t\right]^2} \tag{7.62}$$

Figure 7 is a plot of Eq. (7.62). As one might expect, the slow wave of the magnetoacoustic mode couples most strongly to the sound wave and much more weakly (of the order of s/c) to the electromagnetic wave.

APPENDIX

In References 16 and 17, "Coupling of Magnetohydrodynamic to Electromagnetic Waves at a Plasma Discontinuity. I and II", (hereafter called I and II), there is an error made which seriously affects some of the results of those papers. This appendix is devoted to a discussion of the error and its correction. The work of Section 7 is directly related to this discussion, since the problem is the same when $s = 0$.

The error occurs for the case where $\underset{\sim}{B}_o$ lies in the plane of incidence and the Alfvén mode is incident. (In I and II, this is called Case 1b, and the Alfvén mode is called the extraordinary mode.) The error is that the angle of reflection was assumed to be equal to the angle of incidence. This assumption is correct for the fast magnetoacoustic mode incident (called the ordinary wave in I and II), so that all the results of Cases 1a and 2a which refer to the ordinary mode incident are correct. For the Alfvén mode, the relation between the angle of incidence and reflection is given in Eq. (7.5) and is shown graphically in Fig. 2. It is seen that only for $\varphi = 0$ or for θ small does the angle of incidence equal the angle of reflection.

Before proceeding to the discussion of the revision of the equations, we wish to point out that, in spite of the error made, the results of I are correct even for Case 1b. This may be seen from the following reasoning: In I discussion was restricted to the case when electromagnetic waves in the vacuum region propagated unattenuated. This restriction implies that the wave vector of the incident Alfvén mode lies within the cone for radiation, i.e., that $\sin\theta \leqslant V_a \cos\varphi/c$ (Eq. (31) of I in the notation of that paper). When the angle of incidence is very small, it is seen from Eq. (7.5) that for all φ, $\theta_\alpha{}^r = \pi - \theta_\alpha{}^i$; i.e., the angle of incidence equals the angle of reflection if the angle of incidence is very small. Thus the assumption made is correct, snd sll the results of I are correct.

Instead of Eq. (25) of I (in the notstion of that paper), the correct equation is

$$\left|\frac{E_y{}^t}{E_y{}^i}\right| = \frac{\left|k \cos \theta'\right| \left|\sin \theta(\gamma^r - \gamma^i) + \cos \theta(1 - \tan \theta \cot \theta_r)\right|}{\left|k \cos \theta'(\gamma^r \sin \theta - \sin \theta \cot \theta_r) + k'\right|} \qquad (A.1)$$

where θ_r is the angle of reflection (as defined in Fig. 1). The expression for γ^i (Eq. (20) of II) is given correctly in that psper by

$$\gamma^i = \frac{\cos^2(\theta - \varphi)\sin^2\varphi + x^2\left[\cos^2\theta - \cos^2(\theta - \varphi) - \sin^2\varphi\right] + x^4\sin^2\theta}{-\cos^2(\theta - \varphi)\sin \varphi \cos \varphi + x^2(\sin \varphi \cos \varphi + \sin \theta \cos \theta) - x^4\sin \theta \cos \theta} . \qquad (A.2)$$

However, the correct expression for γ^r is

$$\gamma^r = \frac{\cos^2(\theta - \varphi)\sin^2\varphi + x^2\left[\sin^2\theta \cot^2\theta_r - \cos^2(\theta - \varphi) - \sin^2\varphi\right] + x^4(1 - \sin^2\theta \cot^2\theta_r)}{-\cos^2(\theta - \varphi)\sin \varphi \cos \varphi + x^2\left[\sin^2\theta \cot \theta_r + \sin \varphi \cos \varphi\right] - x^4\sin^2\theta \cot \theta_r} \qquad (A.3)$$

In addition Eq. (7.5) is written in slightly different notstion

$$\cot \theta_r = -\cot \theta - 2 \tan \varphi \qquad (A.4)$$

These equations are adequate for discussing the behavior of $E_y{}^t$. First, if there were a trsnsmission resonance in Csse 1b (i.e., Alfvén mode incident) the brscketed psrt of the first term in the denominator of Eq. (A.1) must vanish. This occurs when $\gamma^r = \cot \theta_r$. However, γ^r is spproximated by $\gamma^r \cong - \tan \varphi$ which lesds to the solution, using Eq. (A.4), $\theta - \varphi = \pm \pi/2$. In this case, the incident Alfvén mode does not propagate since $\underset{\sim}{k}$ is perpendicular to $\underset{\sim}{B}_o$. Thus no transmission resonance is possible in Case 1b, contrary to the results of II.

Equation (A.1) will now be evaluated. The left hand term of the denominator of Eq. (A.1) vanishes only when the angle of incidence lies on the cone for radiation ($\cos\theta' = 0$) or when $\theta - \varphi = \pm\ \pi/2$, as discussed above. Excepting these cases, the right hand term in the denominator is $O(a/c)$ with respect to the left hand term and so is ignored. Thus Eq. (A.1) becomes

$$\left|\frac{E_y{}^t}{E_y{}^i}\right| = \frac{\left|\sin\theta(\gamma^r - \gamma^i) + \cos\theta(1 - \tan\theta\cot\theta_r)\right|}{\left|(\gamma^r - \cot\theta_r)\sin\theta\right|} \tag{A.5}$$

On using $\gamma^i = \gamma^r = -\tan\varphi$ together with Eq. (A.4), one gets that

$$\left|E_y{}^t/E_y{}^i\right| = 2 \quad , \tag{A.6}$$

which is the correct answer for all φ. The only remaining point is to see when γ^i and γ^r may be approximated by $-\tan\varphi$. From Eq. (A.2) and (A.3), one sees that the approximation is permissible when the terms independent of x do not vanish. When these terms do vanish $\varphi = 0,\ \pm\ \pi/2$, and $\theta - \varphi = \pm\ \pi/2$. The latter condition corresponds to the case of no propagation, as mentioned above; $\varphi = \pm\ \pi/2$ corresponds to an indeterminate situation because the conditions ω constant and $\hat{\underset{\sim}{n}}\times\underset{\sim}{k}$ constant are the same to lowest order in x. If one allows finite x, then the angle of incidence again equals the angle of reflection, and the discussion of II is valid (for $\varphi = \pm\ \pi/2$). For $\varphi = 0$ also, the discussion of II is correct.

Equation (21) of II is not changed, since it has no dependence on reflected quantities. Thus, for all values of φ, (in the notation of II)

$$\left|\frac{E_y^{\,t}}{E_{yo}^{\,i}}\right| = \exp\left\{-z'\left[\frac{\sin^2\theta}{\cos^2(\theta - \varphi) - x^2} - \frac{V_a^{\,2}}{c^2}\right]^{\frac{1}{2}}\right\} \tag{A.7}$$

ACKNOWLEDGEMENT

The research reported in this paper was sponsored by the Air Force Cambridge Research Laboratories, Office of Aerospace Research.

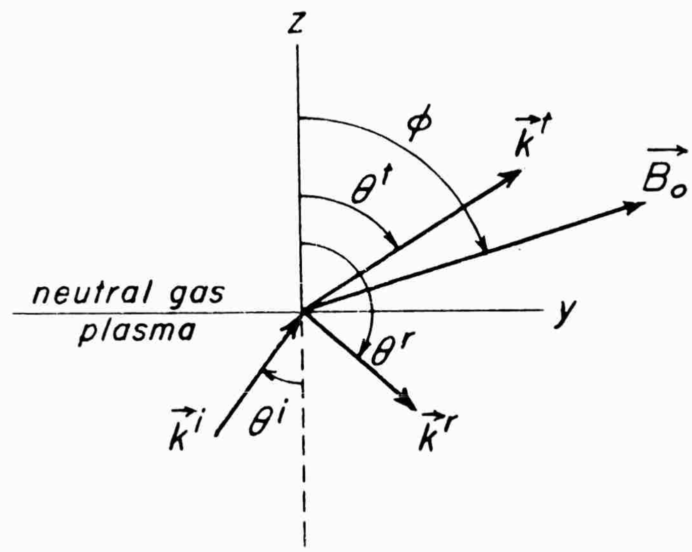

Fig. 1. GEOMETRY OF THE PROBLEM

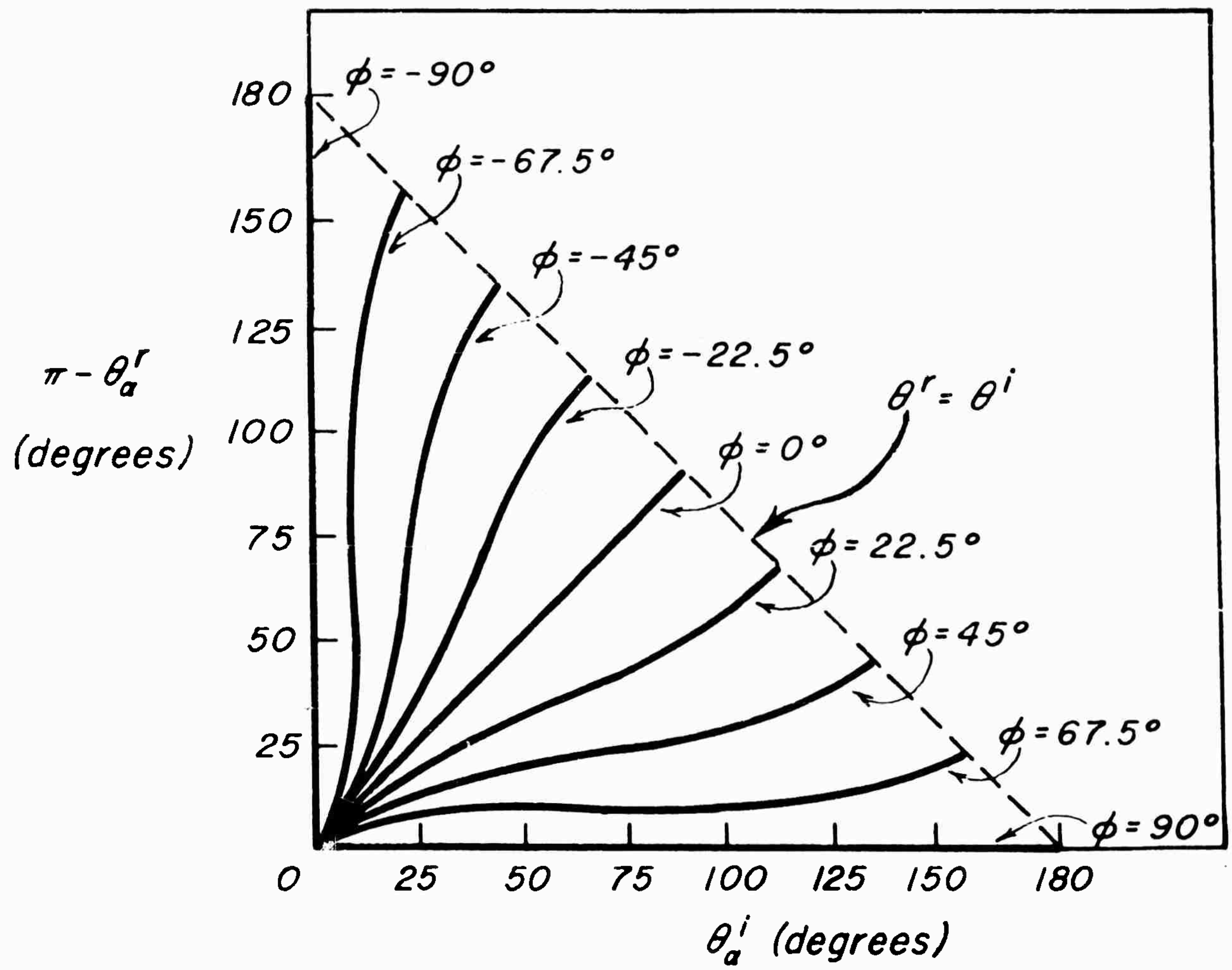

Fig. 2. ANGLE OF REFLECTION VS ANGLE OF INCIDENCE FOR VARIOUS ORIENTATIONS OF THE EXTERNAL MAGNETIC FIELD, ALFVÉN MODE

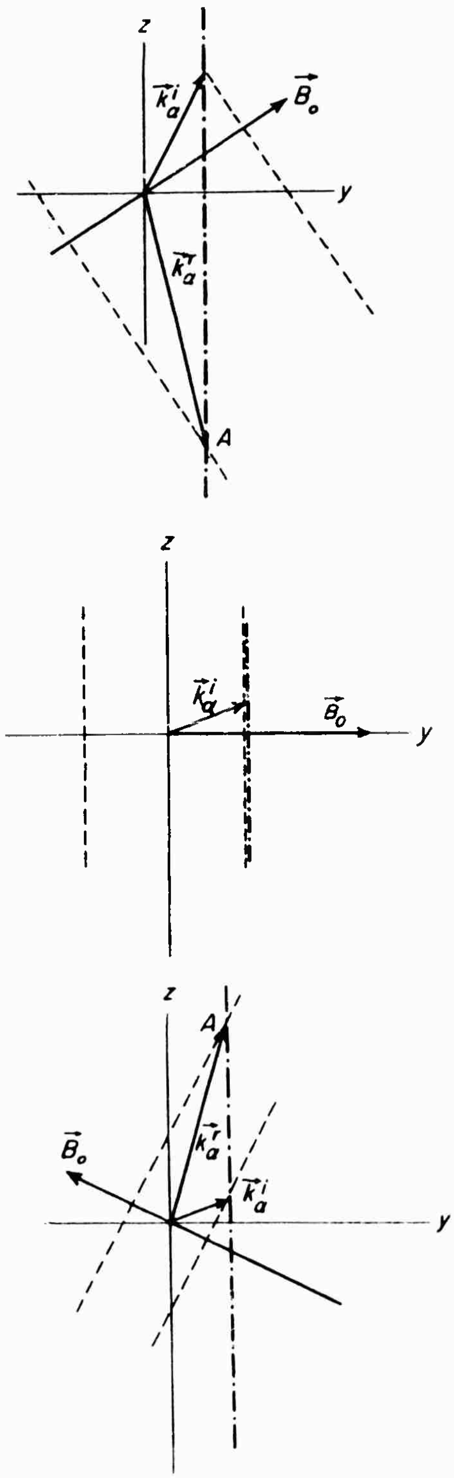

Fig. 3. RELATION BETWEEN INCIDENT AND REFLECTED WAVE VECTORS, ALFVÉN MODE

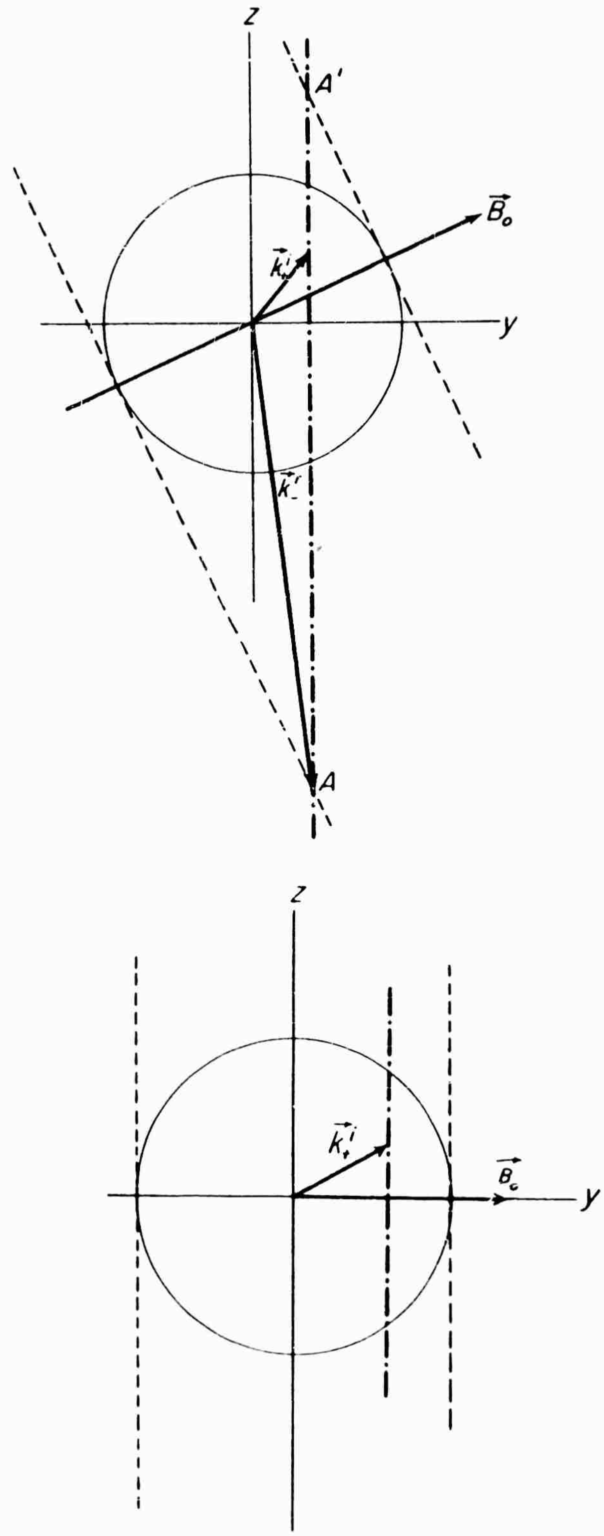

Fig. 4. RELATION BETWEEN REFLECTED SLOW WAVE VECTOR AND INCIDENT FAST WAVE VECTOR

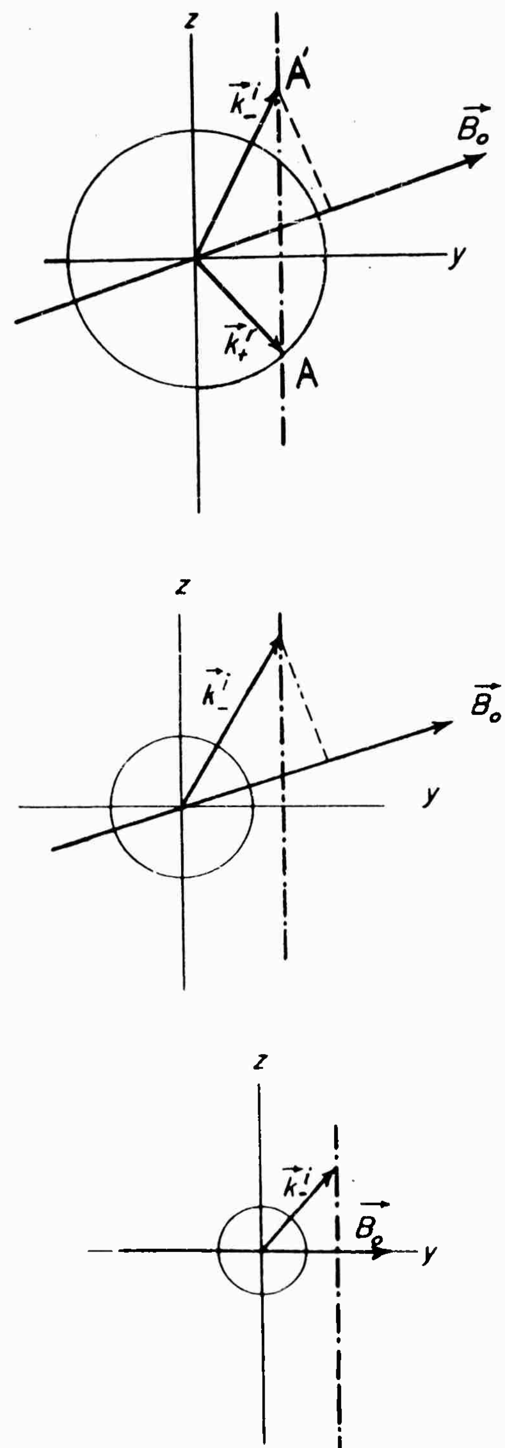

Fig. 5. RELATION BETWEEN FAST REFLECTED WAVE VECTOR AND INCIDENT SLOW WAVE VECTOR

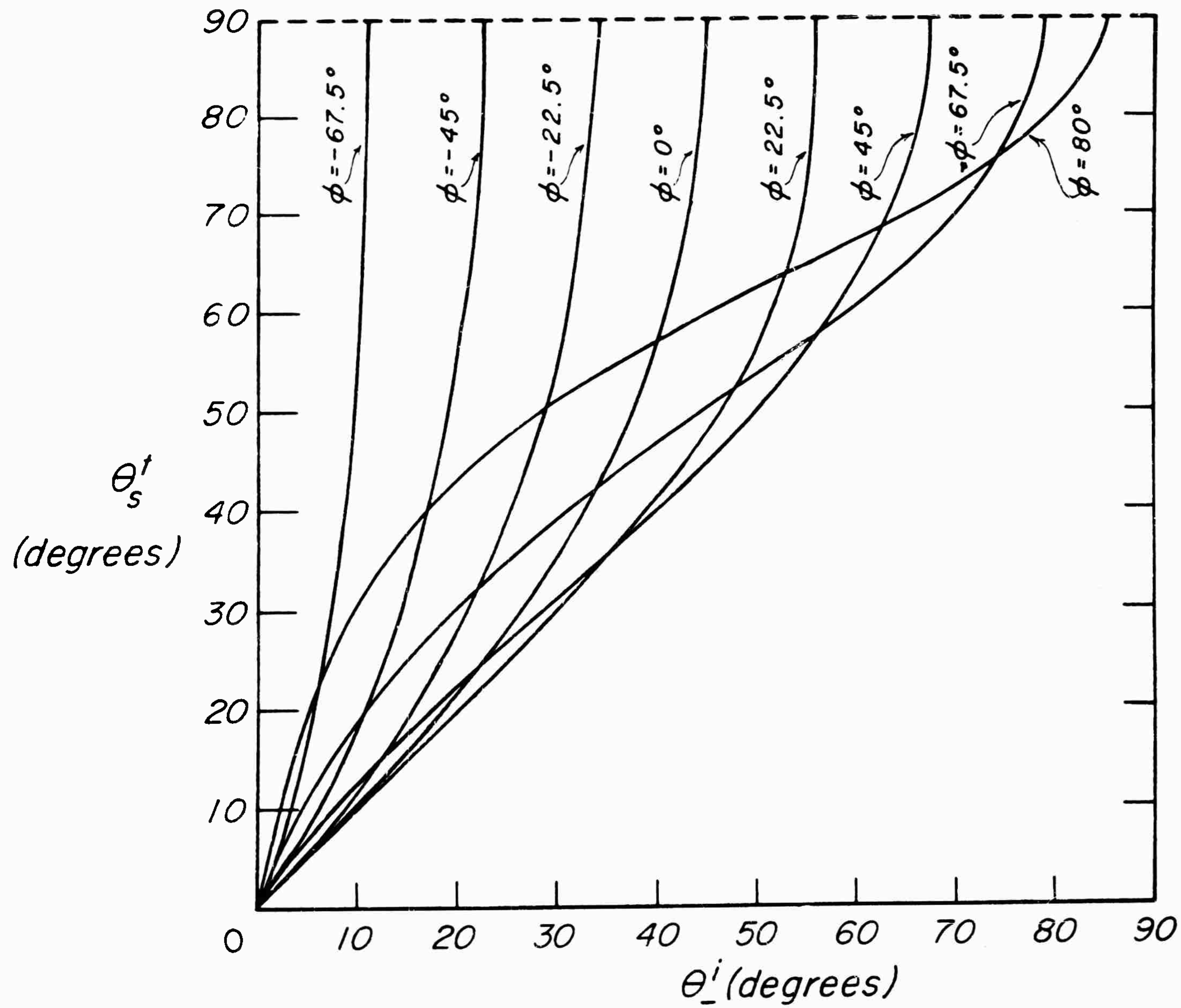

Fig. 6. RELATION BETWEEN THE ANGLE OF TRANSMISSION OF SOUND WAVES AND THE ANGLE OF INCIDENCE OF THE SLOW WAVE FOR VARIOUS ORIENTATIONS OF THE CONSTANT MAGNETIC FIELD

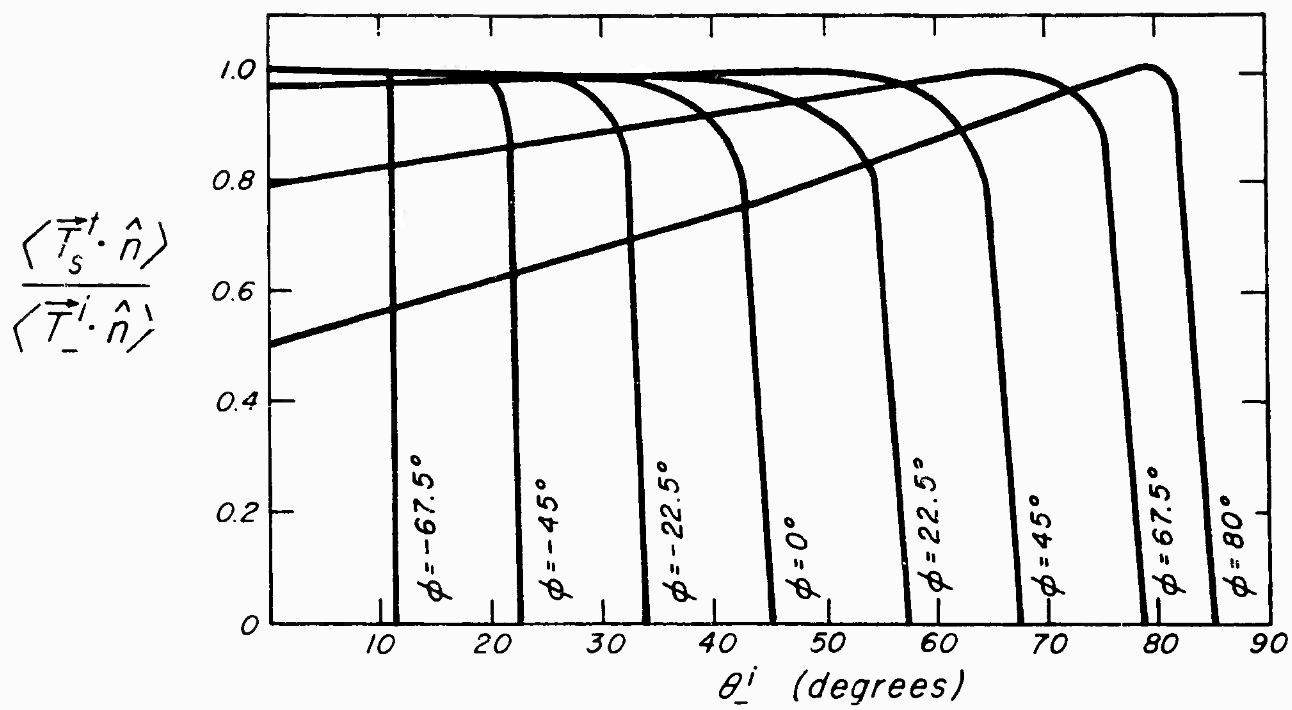

Fig. 7. COUPLING COEFFICIENT OF TRANSMITTED SOUND WAVES FOR THE SLOW MAGNETO-ACOUSTIC MODE INCIDENT

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22. In Ref. 17 there is an error for the case of the Alfvén mode incident. Namely, it was assumed that the angles of incidence and reflection were equal. As shown by Eq. (7.5) and Fig. 2, this is not in general correct. See the appendix to this paper for a discussion.

Air Force Cambridge Research Laboratories,
Bedford, Massachusetts.
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Scientific Report No. 1, 31 October 1963, 40p., figs., 22 refs.
Unclassified Report

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An incident fast magnetoacoustic mode generates a sound wave whose wave propagation vector makes an angle with the normal to the boundary of the order of the sound speed to Alfvén speed, when this ratio is small. In this case, the energy coupling coefficient of sound to the fast magnetoacoustic mode is also of the order of this ratio. An incident slow magnetoacoustic mode generates a sound wave with energy coupling coefficient of the order of unity, and an electromagnetic wave with energy coupling coefficient of the order of the ratio, speed of sound to speed of light.

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An incident fast magnetoacoustic mode generates a sound wave whose wave propagation vector makes an angle with the normal to the boundary of the order of the sound speed to Alfvén speed, when this ratio is small. In this case, the energy coupling coefficient of sound to the fast magnetoacoustic mode is also of the order of this ratio. An incident slow magnetoacoustic mode generates a sound wave with energy coupling coefficient of the order of unity, and an electromagnetic wave with energy coupling coefficient of the order of the ratio, speed of sound to speed of light.

1. Magnetohydrodynamics
2. ELF Wave Motions
3. Ionospheric Propagation
4. Ionospheric Disturbances Detection
5. Acoustic Detectors

I Contract AF19(628)-2836
II MT. AUBURN RESEARCH ASSOCIATES, INC. 12 Norfolk Street, Cambridge, Mass.
IV Kahalas, S. L.
McNeill, D. A.
V Scientific Report No. 1
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Air Force Cambridge Research Laboratories,
Bedford, Massachusetts.
AFCRL-63-944 COUPLING OF MAGNETOHYDRODYNAMIC TO ELECTROMAGNETIC AND ACOUSTIC WAVES AT A PLASMA-NEUTRAL GAS INTERFACE.
Scientific Report No. 1, 31 October 1963, 40p., figs., 22 refs.
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