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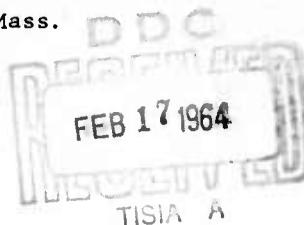
ESD-TDR-63-649

INITIAL ORBIT DETERMINATION FROM LEAST
SQUARES REDUCTION OF GEOCENTRIC POSITION VECTORS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-649
JANUARY 1964

T. L. Johnston
T. P. Smith

496L SYSTEM PROGRAM OFFICE
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Mass.



Prepared under Contract No. AF 19(628)-562 by Aeronutronic,
a Division of Ford Motor Company, Newport Beach, California

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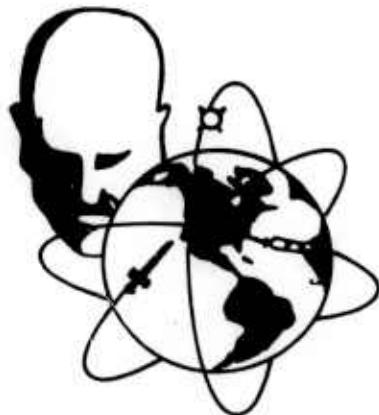
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FOREWORD

Contractor's Report Publication No. U-2442

INITIAL ORBIT DETERMINATION FROM LEAST
SQUARES REDUCTION OF GEOCENTRIC POSITION VECTORS

ABSTRACT

A computer program for initial orbit determination based on a theory employing a linear least squares fit to the geocentric position vectors has been developed. Observations can be spread over a number of days since this procedure does not use the times of observations; however, they are used in the differential correction procedure. This program coded for the Philco 2000, is designed for operational use at the Space Detection and Tracking Center in Colorado Springs, Colorado.

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SECTION 1

INTRODUCTION

The computer program discussed in this report computes and differentially corrects the preliminary orbital elements of a satellite when given an aggregate of observation position fixes (maximum of 150). The Initial Orbit Least Squares Program (IOLSQ) is coded for the Philco 2000 computer and is intended to be a part of the Semi-Automatic Program System (SPS) at the SPADATS Center in Colorado Springs, Colorado. This program operates in conjunction with the SPS executive routines.

IOLSQ is based on a theory employing a least-squares fit to the computed geocentric position vectors. The least squares procedure does not use the times of the observations; time enters into the formulation only as the epoch time, i.e., the time of the first observation. As the least squares method is not iterative, it involves only linear equations. The result is an initial orbit that is not severely affected by specific errors due to a few pieces of data. This is in contrast to an exact orbit based on only two position vectors.

Included in Section Two is the actual formulation used in IOLSQ to provide a detailed guide to the program operation. Section Three describes the input and output requirements while the flow charts are contained in Section Four. A summary of the experimentation that has been conducted with the program to evaluate the theory is included in Section Five.

For a complete discussion of the differential correction portion of IOLSQ, see Section 3 of Reference 2 listed at the end of this document. A discussion of the ephemeris part of differential correction can be found in Reference 1.

SECTION 2

FORMULATION

Computation for the Initial Orbit Least Squares Program involves the solution of a number of linear equations. These equations, solved in a straight-forward manner, yield the classical orbital elements, a , e , i , Ω , and ω . Each of the observations must initially be converted to position vectors, \underline{r}_i , in the inertial system thus requiring complete observational fixes of range and two angles for this procedure. The formulation follows:

- (1) Determination of the Geocentric Position Vector, \underline{r}_i , and the Unit Position Vector \underline{U}_i :

\underline{r}_i and \underline{U}_i are calculated for each observation for
 $i = 1, 2, 3, \dots, N$; where N is the number of
acceptable observations

$$\underline{r}_i = \rho_i \underline{L}_i - \underline{R}_i, \text{ where } \rho_i \text{ is the observed range.}$$

$$\underline{U}_i = \frac{\underline{r}_i}{\rho_i}$$

The relationship between the satellite position, the dynamical center, and the observer is illustrated in Figure 1.

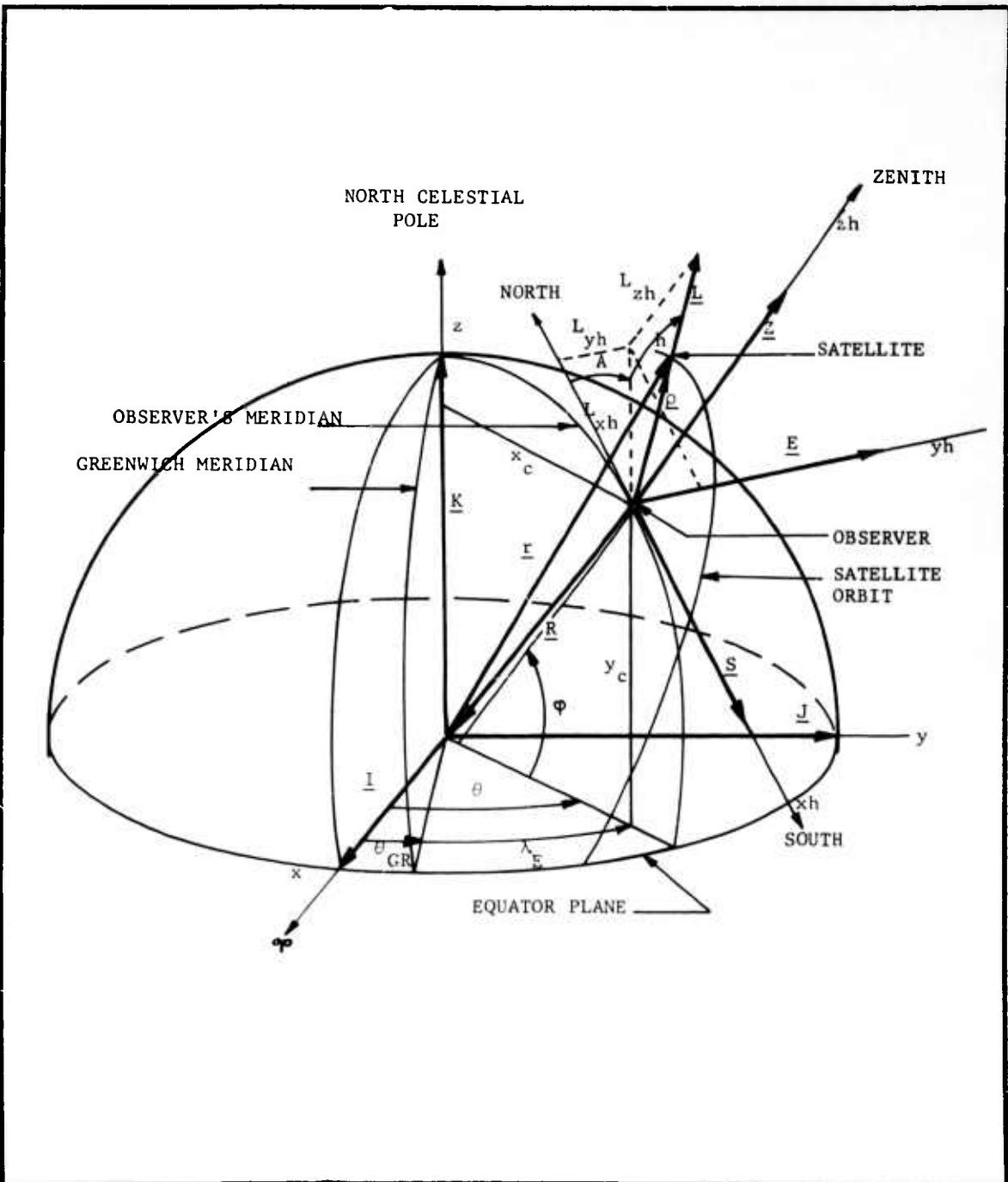


FIGURE 1. OBSERVATIONAL GEOMETRY

The position of the Station, R_i , is calculated as follows:

$$x = \left[\frac{x}{\cos \theta} \right] \cos \theta$$

$$y = \left[\frac{x}{\cos \theta} \right] \sin \theta$$

$$z = z$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

where θ , the local sidereal time, is computed from

$$\theta = \theta_{gr} + \lambda_E$$

$$\theta_{gr} = \theta_{gr_0} + D(0.9856472) + F(360.9856472)$$

θ_{gr_0} = Greenwich sidereal time at the beginning of the year,
and D and F are days and fractions of days, respectively,
from the start of the epoch year to epoch.

ϕ is the station latitude (positive to the north)

λ_E is the longitude east

$\frac{x}{\cos \theta}$ is the given distance of the station from the polar axis

and Z is the polar component of the station's geocentric position.
These quantities are normally acquired in referencing the SPS
system for station coordinates.

The unit vector, L_i , is computed from the observed quantities
azimuth (A) and elevation (h):

$$L_x = -\cos h \cos A \sin \phi \cos \theta - \cos h \sin A \sin \theta \\ + \sin h \cos \phi \cos \theta$$

$$L_y = -\cos h \cos A \sin \phi \sin \theta + \cos h \sin A \cos \theta$$

$$+ \sin h \cos \phi \sin \theta$$

$$L_z = \cos h \cos A \cos \phi + \sin h \sin \phi$$

$$\text{such that } L = \sqrt{L_x^2 + L_y^2 + L_z^2}$$

(2) Calculation of S_{xx} , S_{xy} , S_{yy} , S_{xz} , S_{yz} , S_{zz} , the Least Squares Quantities used in the Solution of \underline{w}_o :

$$S_{xx} = \sum_{i=1}^N U_{x_i}^2$$

$$S_{xy} = \sum_{i=1}^N U_{x_i} U_{y_i}$$

$$S_{xz} = \sum_{i=1}^N U_{x_i} U_{z_i}$$

$$S_{yz} = \sum_{i=1}^N U_{y_i} U_{z_i}$$

$$S_{yy} = \sum_{i=1}^N U_{y_i}^2$$

$$S_{zz} = \sum_{i=1}^N U_{z_i}^2$$

(3) Calculation of \underline{w}_o , the Unit Vector Perpendicular to the Orbit
Plane:

$$\underline{w}_o = \frac{\hat{\underline{w}}}{\hat{W}} \quad (\text{See Figure 2.})$$

where $\hat{W} = \sqrt{\hat{w}_x^2 + \hat{w}_y^2 + \hat{w}_z^2}$

and $\hat{w}_x = \frac{s_{yz}s_{xy} - s_{xz}s_{yy}}{s_{xx}s_{yy} - s_{xy}^2}$

$$\hat{w}_y = \frac{s_{xz}s_{xy} - s_{yz}s_{xx}}{s_{xx}s_{yy} - s_{xy}^2}$$

$$\hat{w}_z = 1.0$$

(4) Calculation of \underline{u}_o and \underline{v}_o , where \underline{v}_o is the Unit Vector in the
Orbit Plane Perpendicular to \underline{u}_o :

\underline{u}_o and \underline{v}_o form an orthogonal system with \underline{w}_o

Let $n = \underline{u}_1 \cdot \underline{w}_o$ (subscript 1 denotes the first observation)

Then $\underline{u}_o = \frac{\hat{\underline{u}}_o}{\hat{U}_o}$

where $\hat{\underline{u}}_o = \underline{u}_1 - n\underline{w}_o$

$$\hat{U}_o = \sqrt{\hat{U}_{x_o}^2 + \hat{U}_{y_o}^2 + \hat{U}_{z_o}^2}$$

and

$$\underline{v}_o = \underline{w}_o \times \underline{u}_o$$

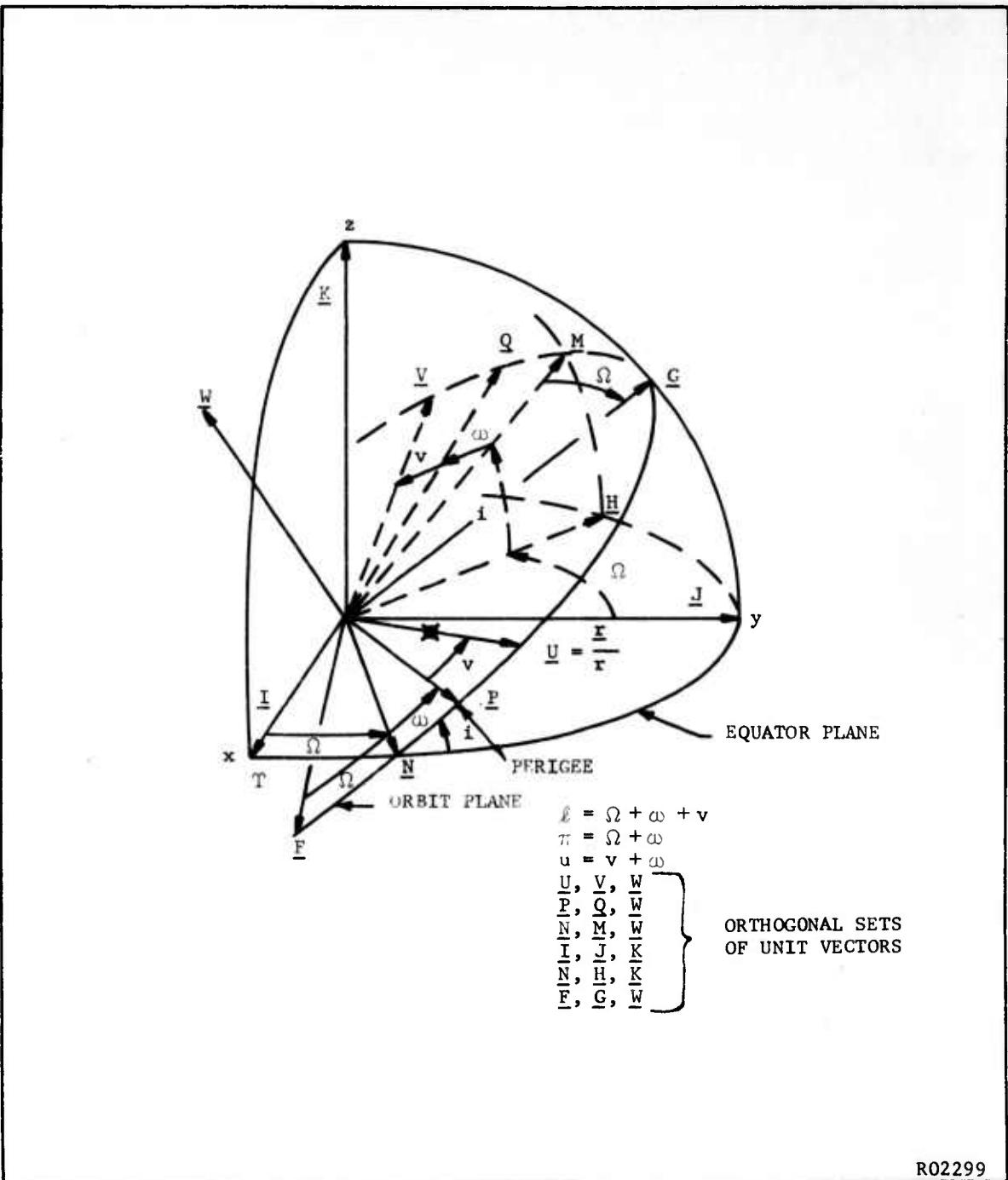


FIGURE 2. PROJECTION OF ORBIT-PLANE ON CELESTIAL SPHERE WITH ORIENTATION
UNIT VECTORS AND ANGLES

Restricting W_{z_o} to a positive value will generate the wrong solution when the true orbit is retrograde and, in this case, the vector \underline{v}_o will be calculated in the wrong direction. To avoid this problem, the direction of motion of the satellite is determined by examining a pair of vectors, \underline{u}_i and \underline{u}_{i+1} , such that $0 < t_{i+1} - t_i \leq 40$ minutes. If W_{z_o} is selected correctly, then

$$\text{sign of } (\underline{u}_i \times \underline{u}_{i+1})_j = \text{sign of } \underline{w}_j$$

where j denotes the vector component with the largest magnitude.

If this sign equality does not occur, W_{z_o} has been chosen in the wrong direction. The signs of the components of \underline{w}_o and \underline{v}_o are then changed, and the calculations continue.

(5) Computation of $\cos(v_i - v_o)$ and $\sin(v_i - v_o)$:

$$\cos(v_i - v_o) = \underline{u}_i \cdot \underline{u}_o$$

$$\sin(v_i - v_o) = \underline{u}_i \cdot \underline{v}_o$$

(6) Formulation of $s_1, s_{11}, s_{12}, s_{13}, s_2, s_{22}, s_{23}, s_3, s_{33}$, the Least Squares Quantities used in the Solution of $p, e \sin v_o$ and $e \cos v_o$:

$$s_1 = \sum_{i=1}^N \sin(v_i - v_o)$$

$$s_{11} = \sum_{i=1}^N \sin^2(v_i - v_o)$$

$$s_{12} = \sum_{i=1}^N \sin(v_i - v_o) \cos(v_i - v_o)$$

$$s_{13} = \sum_{i=1}^N \frac{1}{r_i} \sin(v_i - v_o)$$

$$s_2 = \sum_{i=1}^N \cos(v_i - v_o)$$

$$s_{22} = \sum_{l=i}^N \cos^2(v_i - v_o)$$

$$s_{23} = \sum_{i=1}^N \frac{1}{r_i} \cos(v_i - v_o)$$

$$s_3 = \sum_{i=1}^N \frac{1}{r_i}$$

$$s_{33} = \sum_{i=1}^N \left(\frac{1}{r_i} \right)^2$$

(7) Calculation of p, e sin v_o, e cos v_o:

The following least squares equations are solved to obtain p, e sin v_o, e cos v_o.

$$S_1 = (e \sin v_o) S_{11} - (e \cos v_o) S_{12} + p S_{13}$$

$$S_2 = (e \sin v_o) S_{12} - (e \cos v_o) S_{22} + p S_{23}$$

$$S_3 = (e \sin v_o) S_{13} - (e \cos v_o) S_{23} + p S_{33}$$

(8) Determination of e, a, e cos E_o, e sin E_o:

$$e = \sqrt{(e \cos v_o)^2 + (e \sin v_o)^2}$$

$$a = \frac{p}{1-e^2}$$

$$e \cos E_o = \frac{e(e + \cos v_o)}{1 + e \cos v_o}$$

$$e \sin E_o = \frac{\sqrt{1-e^2} (e \sin v_o)}{1 + e \cos v_o}$$

$$E_o = \tan^{-1} \left[\frac{\sqrt{1-e^2} \sin v_o}{e + \cos v_o} \right] \quad 0 \leq E_o < 2\pi$$

(9) Calculation of \underline{h} , the Angular Momentum Vector Directed Along \underline{w}_o :

$$\underline{h} = \sqrt{p} \underline{w}_o$$

(10) Calculation of Ω and i , the Longitude of the Ascending Node and the Inclination of the Orbit Plane:

Since the three equatorial component equations of \underline{w}_o are

$$w_{x_o} = \sin \Omega \sin i$$

$$w_{y_o} = -\cos \Omega \sin i$$

$$w_{z_o} = \cos i$$

$$\text{then } \Omega = \tan^{-1} \left[\frac{w_{x_o}}{-w_{y_o}} \right] \quad 0 \leq \Omega < 2\pi$$

$$\text{and } i = \tan^{-1} \left[\frac{1-w_{z_o}^2}{w_{z_o}} \right] \quad 0 \leq i < \frac{\pi}{2}$$

(11) Calculation of \underline{a} , the Vector Directed to the Perifocus, and Its Components, a_{xN} and a_{yN} :

a_{xN} is the component of \underline{a} along \underline{N} , the unit vector directed to ascending node.

a_{yN} is the component of \underline{a} along \underline{M} , the unit vector in the orbit plane perpendicular to \underline{N} , in the direction of motion.

$$\underline{a} = \underline{U}_o e \cos v_o - \underline{V}_o e \sin v_o$$

$$a_{xN} = \frac{a_y w_x - a_x w_y}{\sqrt{1 - w_z^2}}$$

$$a_{yN} = \frac{-w_z (a_x w_x + a_y w_y)}{\sqrt{1 - w_z^2}} + a_z \sqrt{1 - w_z^2}$$

(12) Calculation of ω , the Argument of Perigee:

$$\omega = \tan^{-1} \left(\frac{a_{yN}}{a_{xN}} \right) \quad 0 \leq \omega < 2\pi$$

(13) Calculation of $\dot{\Omega}$, $\dot{\omega}$, P_a , and H_q :

$\dot{\Omega}$ is the rate of change of the longitude of ascending node.

$\dot{\omega}$ is the rate of change of the argument of perigee.

P_a is the anomalistic period.

H_q is the altitude of perigee in statute miles.

$$\dot{\Omega} = \frac{k_e w_{z_0}}{a^{3/2} p^2} \quad (1.62329 \times 10^{-3})$$

$$\dot{\omega} = \frac{k_e}{2 a^{3/2} p^2} \quad (1.62329 \times 10^{-3}) (5 w_{z_0}^2 - 1)$$

$$P_a = \frac{2\pi}{k_e} a^{3/2}$$

$$H_q = [a(1-e) - 1] (3963.199)$$

(14) Calculation of the Mean Longitude at epoch, L_o :

$$L_o = \pm \Omega + \omega + E_o - e \sin E_o \quad \text{if } i \leq 90^\circ, \text{ use } + \Omega$$

if $i > 90^\circ$, use $- \Omega$

SECTION 3

PROGRAM OPERATIONS

This program is executed in conjunction with the B-2 Semi-Automatic Program System (SPS) job schedule mode of operation. The normal operating procedures required for this mode of program execution are described in Reference 3.

Observations are read into memory from cards. Sensor coordinates are stored in core from cards or from the system SEAI file. The position vectors of the satellite are then calculated and stored in memory.

3.1 INPUT

IOLSQ has two input options:

0 : Observation Cards.

1 : Observation and Sensor Cards.

Required input data consists of observation types 2 or 3 and/or the Sensor Cards.* These data cards must be used in conjunction with the usual SPS schedule tape mode control cards.

*Observation Types: 2 - contains elevation, azimuth and range
3 - contains elevation, azimuth, range and range-rate.

The tape setup required for input is arranged as follows:

<u>TAPE</u>	<u>DESCRIPTION</u>
1	SPS Master Tape
2	Schedule Tape
4	SEAI Tape
11	Output Tape

The Schedule Tape used for input is generated from the following punched cards:

ID Card (70 SC TP)
JOB Card
REM Card
SPSJOB Card (IOLSQ)
Sensor Cards - optional
Observation Cards
END Card (END OF JOB)
END Card (ENDSCHED)
Blank Card.

The items shown in parentheses above are the code names to be used in the respective cards. These cards, along with the JOB, REM, Sensor and Observation Cards are described in Section 5.4 of Reference 2.

3.2 OUTPUT

All printed or punched output may be obtained from logical tape unit 11, using data select 1 for printed output and data select 2 for punched card output.

The printed output of IOLSQ begins with a heading and the date. This is followed by the N, M Elements and the Classical Elements at the time of the first observation (epoch time). The differential correction output follows, showing the observation residuals and correction data at each iteration followed by the final correction summary. Printout for the differential correction portion is exactly like that of all SPS differential correction packages. (See Reference 2.). In this standard differential correction output, the corrected elements are displayed, plus the corrected elements updated to the final revolution. For this program, the results of the two sets of new elements will be the same. See Figure 3, page 19.

The punched card output consists of two standard SPS 7-Card Element sets. System routines were used to obtain the punched output so that the two sets are duplicates. This is because the corrected set and the **updated set** are the same in IOLSQ.

3.3 PROGRAM NOTES

Comments may be printed indicating the operation of the IOLSQ Program. Any of the comments from the following list may be produced during the operation of the initial orbit portion of IOLSQ:

(1) "EXPONENT OVERFLOW AT THE ^{LEFT} HALF OF LOC. XXXXX"
^{RIGHT}

(2) "SUBROUTINE ERROR AT THE ^{LEFT} HALF OF LOC. XXXXX"
^{RIGHT}

(3) "NO COORDS SPECIFIED FOR SENSOR NO. XXXX"

This comment indicates that the OBSGET subroutine could not find from either the SEAI Tape or from the Sensor Cards the coordinates corresponding to the sensor number on one of the Observation Cards.

(4) "THERE ARE XXX OBSERVATIONS WHICH HAVE NO RANGE"

Only observations types 2 and 3 can be used by the initial orbit part of IOLSQ. This comment indicates the number of input observations that were rejected during the initial orbit determination.

(5) "NUMBER OF GOOD OBS. IS LESS THAN 5"

The least-squares theory in IOLSQ requires that there be a minimum of 5 acceptable observations in order that the calculations can proceed.

(6) "OBSERVATION BUFFER FULL, ONLY THE FIRST 150 TYPE 2 OR 3 INPUT OBS. WILL BE USED"

This comment indicates that the maximum of 150 acceptable observations was exceeded. Therefore, calculations will proceed using only the first 150 type 2 and 3 observations.

(7) "THERE IS NO PAIR OF OBSERVATIONS WHICH ARE SEPARATED IN TIME BY MORE THAN 1 MIN. AND LESS THAN 40 MIN."

To determine the direction of motion of the satellite, the program attempts to examine two observations which are greater than 1 minute but less than 40 minutes apart. The above comment indicates that no such pair was found and the job is terminated.

The error comments that may result from the differential correction portion of the program are fully described in Section 3, pages 3-84 and 3-85, of Reference 2.

FIGURE 3: IOLSO OUTPUT FORMAT

SECTION 4

FLOW DIAGRAMS

The following illustrations indicate the computational procedures of IOISQ in diagrammatic form. Diamonds indicate decisions; arrows indicate the direction of logical flow; ovals indicate subroutines, and rectangles indicate mathematical or bookkeeping operations. The small circles indicate starts and exits and also serve as connectors from one page to the next. The oval at the entrance to a subroutine contains the name of that particular subroutine. A rectangle connected to a subroutine oval contains a brief description of that particular subroutine. Dotted lines from a subroutine oval indicate possible exits.

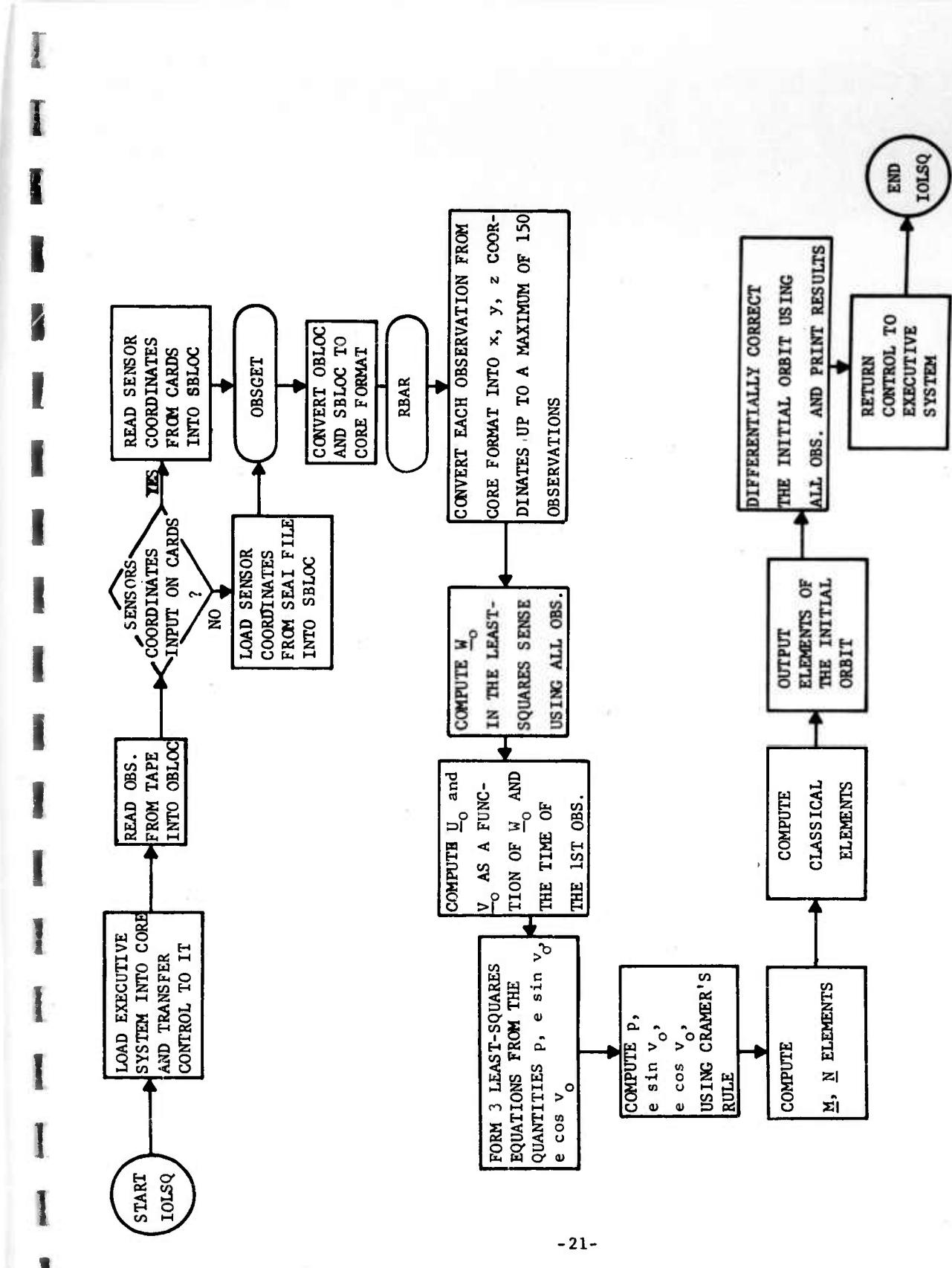


FIGURE 4. IOISQ GENERAL FLOW DIAGRAM

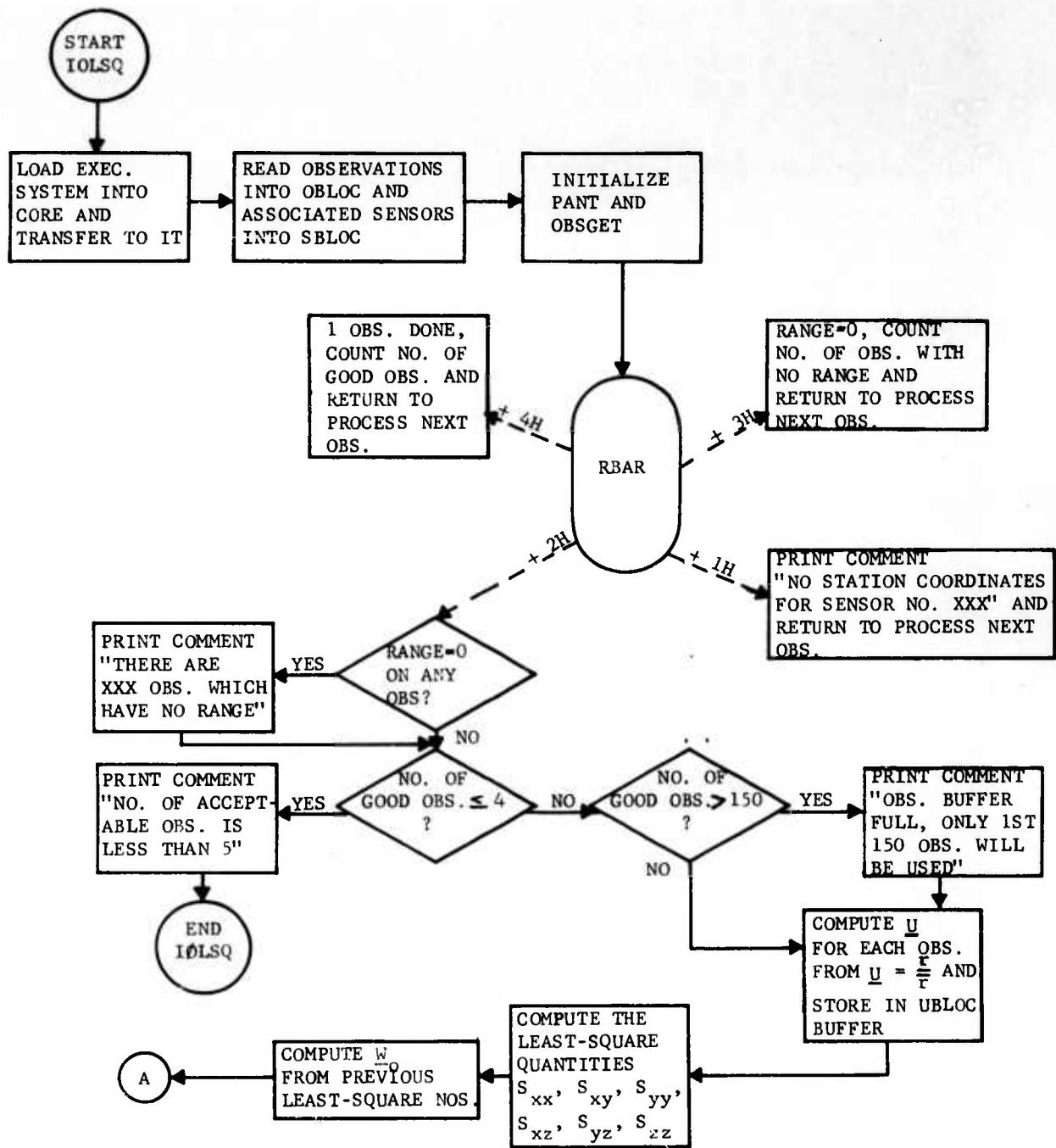


FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (1 OF 3)

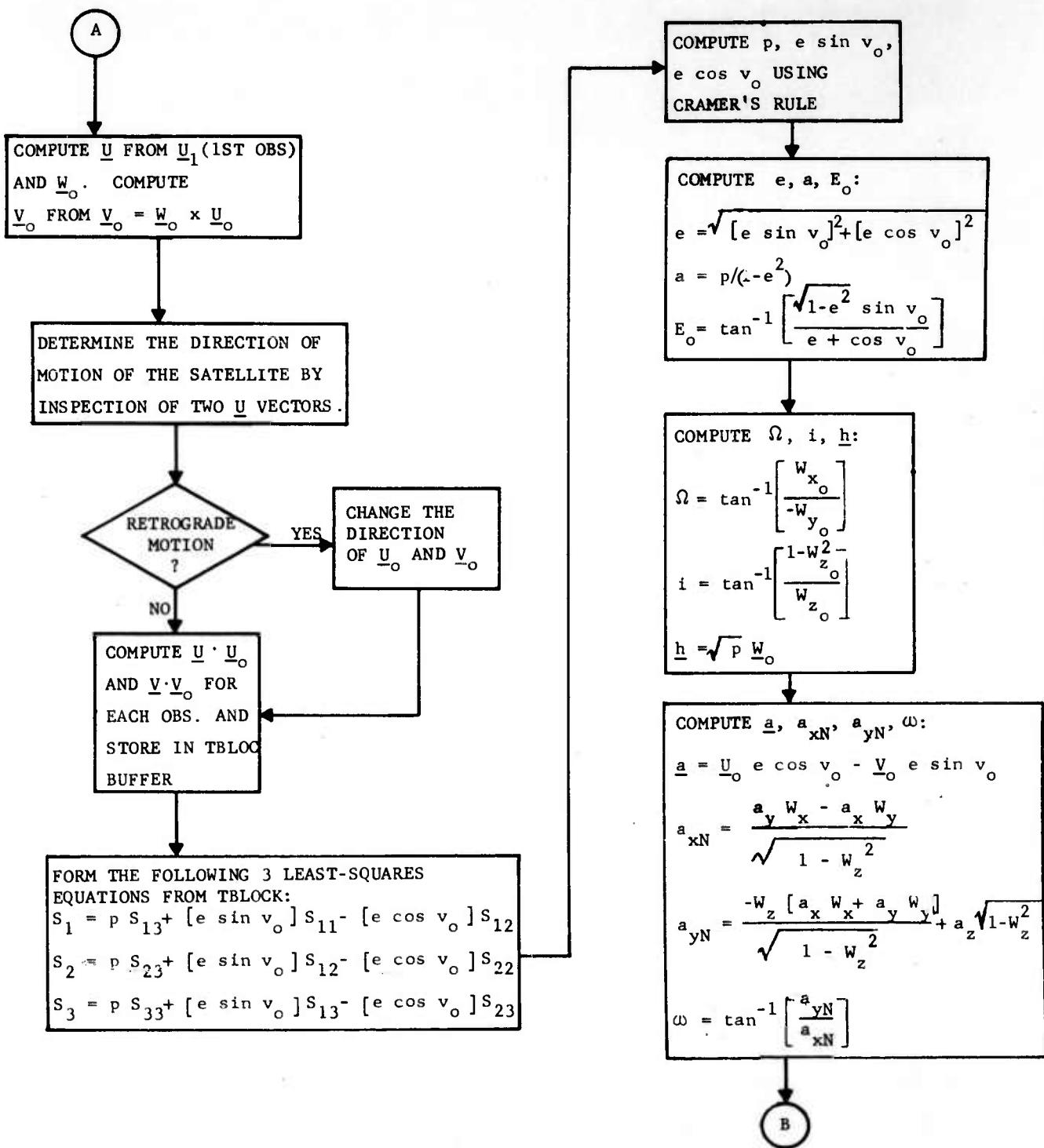


FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (2 OF 3)

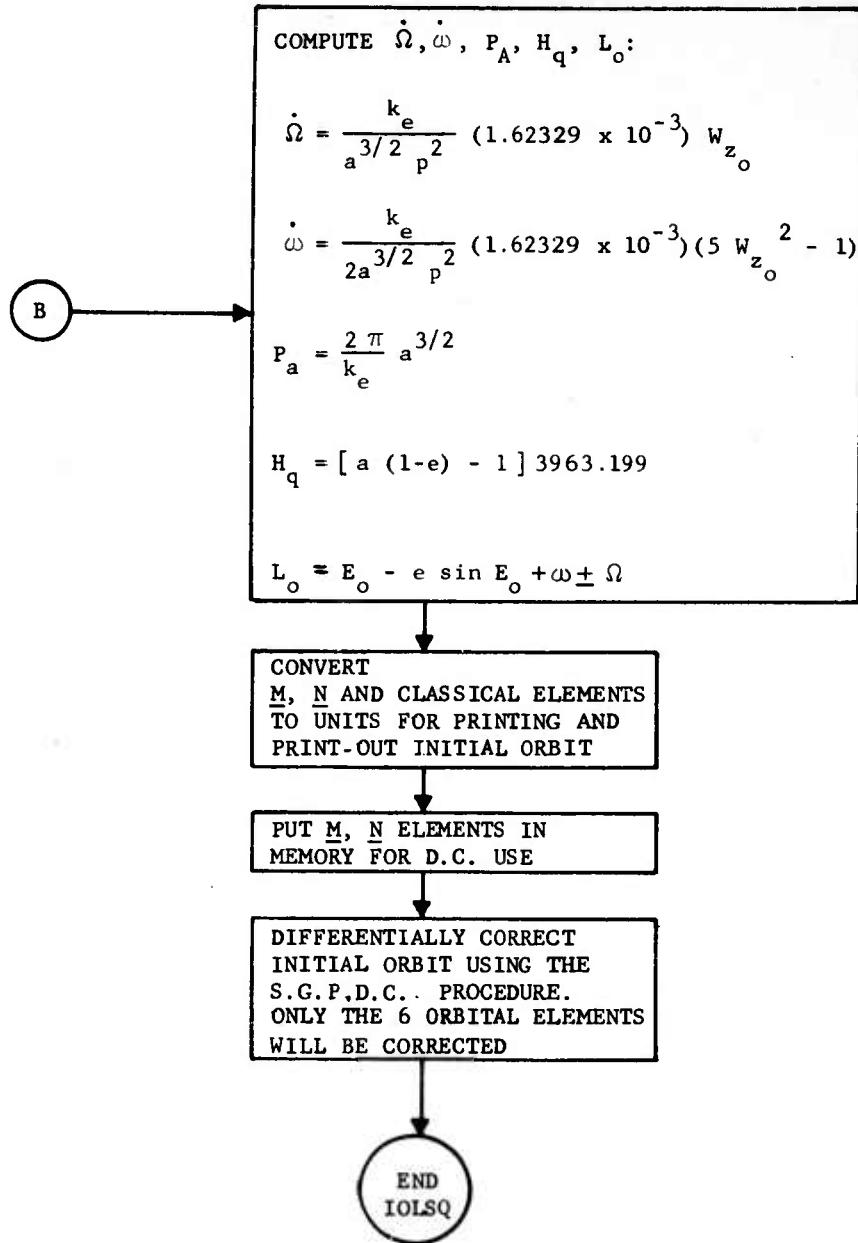


FIGURE 5. IOLSQ DETAILED FLOW DIAGRAM (3 OF 3)

SECTION 5

PROGRAM EXPERIMENTATION

5.1 SUMMARY OF TESTING

In this section a summary of twenty cases that were run with the IOLSQ Program is presented to illustrate the capabilities of the initial orbit determination.

Observations were chosen from four different satellites, each satellite representing a different type of orbit. The following table summarizes the different orbits used.

TABLE I
SUMMARY OF TEST ORBITS

<u>Satellite Number</u>	<u>Drag</u>	<u>Eccentricity</u>
011	Low	Moderate
070	Moderate	Low
116	Low	Low
213	High	Low

Five runs were made for each satellite with each successive run having more observations than the previous run and covering a longer time span. The time span and the number of observations used are included in the following tables along with the initial and corrected elements. The test cases are presented here to illustrate the accuracy of the initial orbit portion of IOLSQ rather than the differential correction portion. Each initial orbit is compared with a reference orbit which

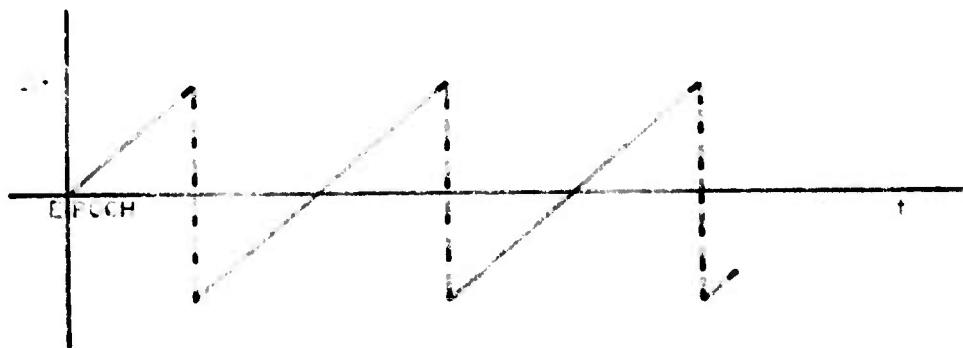
was obtained from the SGPDC Program. (See Reference 2.) The reference orbit was determined using file elements and the same observations as the initial orbit case with the largest number of observations.

5.2 RESULTS

Experimentation indicates that IOLSQ is quite adequate for determining an initial orbit when a group of associated observations is given. The initial orbital parameters are entered as input to the differential correction scheme, and the program appears to yield sufficiently accurate elements for this purpose.

In the early phase of the experimentation, the initial elements were not always sufficiently accurate when corrected over a long time span of observations. This led to divergence in the correction process. To mitigate this problem, a special period correction was included in IOLSQ to improve the semi-major axis. This correction is executed by fitting a straight line in the least-squares sense to the time residuals. The correction to the semi-major axis is then computed from the slope of this line.

When the time span covered by the observations is long, the time residuals may exhibit a "saw-tooth" effect. The time residual is similar to the angle in the orbit plane between the computed position and the observed position. The time residual is thus computed from the angle between two vectors and necessarily lies between $-\pi$ and $+\pi$. Therefore, when the time residuals exceed one-half the period, they will change sign and exhibit the saw-tooth effect. The sketch below illustrates Δt as a function of time:



To fit, in a least-squares sense, a line to the time residuals, it becomes necessary to eliminate the "saw-tooth", i.e., Δt must be made a linear function of time. This is accomplished by adding or subtracting the value of the period when two successive time residuals differ by more than one-half the period. This special period correction is invalid in the case where two successive time residuals are far enough apart that there exists more than one change of sign between them.

IOLSQ differs from other initial orbit programs in that the semi-major axis is determined from geometrical rather than dynamical considerations. Thus, it is to be expected that there will be an error in the semi-major axis when the points in space, as computed from the observations, do not uniquely define an ellipse. The experimentation has shown that the semi-major axis is, indeed, inadequately determined when the observations are concentrated at one point in the orbit but becomes increasingly better as more observations are added. This effect is illustrated in the 3-hour correction of Satellite 011 (see Table II).

The initial orbit formulation assumes that the longitude of the ascending node, Ω , is constant. It can, therefore, be expected that due to the precession of the node, Ω will be poorly determined when the span of the observations is large. This limitation of the program is illustrated by Satellites 070 and 213 (see Tables III and V) where Ω becomes increasingly worse and is in error by as much as 16° when the observation span is 16 days and 15 hours.

Another limitation of the program is illustrated by Satellites 070, 116 (see Table IV), and 213. The argument of perigee, ω , is poorly defined when the eccentricity is low. The large errors in ω are due to the low eccentricity and should be corrected by the differential correction. For low eccentricities, however, a large error in ω does not necessarily indicate an inaccurate orbit.

The eccentricity and the argument of perigee are poorly defined when the observations represent a single point in space. In this case, the program cannot uniquely determine an ellipse to fit the observations. This situation is illustrated by Satellite 011, Case 5. The observations are concentrated at a single point in space, and the differential correction could not adequately determine an orbit except when the correction of the eccentricity and argument of perigee was inhibited.

The effect of high drag is illustrated by Satellite 213. The computation of drag is not included in IOLSQ, and consequently the differential correction cannot always converge to an orbit that closely fits the observations. Cases 3, 4 and 5 could converge to an

orbit only when the correction of the eccentricity was inhibited. When the eccentricity was corrected, the differential correction began to diverge steadily. These cases represent the best results that can be obtained using the usual "no-drag" assumption.

The effect of bad observations is illustrated by the cases run on Satellite 070. Cases 3, 4 and 5 contain a group of observations that represent a single point in space. This point is not compatible with the rest of the orbit, and the differential correction could not determine an orbit to fit all of the observations except when the eccentricity was not corrected. Cases 4 and 5 show that the effect of these bad observations is minimized when observations are added that more clearly define the orbit.

As a result of the testing (and in view of the foregoing comments), it is concluded that IOLSQ is best suited for orbit determinations using groups of observations that cover a time span on the order of four or five days. Enough observations must be available for input to enable the program to geometrically determine the semi-major axis. The time span must be small enough so that the precession of Ω is not a dominating factor. The advantage of IOLSQ is that its accuracy is not severely affected by errors in a few observations as would occur for other initial orbit programs* based on one- or two-position fixes.

5.3 EXAMPLE OF PROGRAM OPERATION

Portions of an IOLSQ test case are presented on the following pages to illustrate the combined operation of the initial orbit determination and the differential correction procedure. The test case is reproduced in the actual computer output format for Satellite 116, Case 3. (See Figure 7.) Case 5 printed at the end of the output indicates the fifth differential correction iteration for Case 3. Due to space limitations, the test case output is abbreviated to show the initial elements, the last pass residuals, and the final element summary. All input data for this case are shown in the reproduced 80-80 listing of the input deck (see Figure 6).

*Such as Initial Orbit Radar Fix (IORF) Program, which computes the initial orbit from 2 or 3 radar fixes, and Initial Orbit Radar Track (IOHG) Program, which calculates the initial orbit from a radar track.

TABLE II
TOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 011

SGP REFERENCE ORBIT		<u>a</u> (E.R.)	<u>e</u>	<u>i</u> (deg)	<u>Ω</u> (deg)	<u>ω</u> (deg)	<u>P_A</u> (min)
		1.302053	0.16459	32.788	110.728	78.091	125.460
CASE 1		INITIAL	1.230400	0.12455	32.598	111.784	64.866
3 Hours		CORRECTED	1.2256795*	0.11639	32.634	109.895	72.190
26 Observations							114.685
CASE 2		INITIAL	1.284587	0.15418	32.704	107.941	82.609
1 Day + 2 Hours		CORRECTED	1.3013319	0.16464	32.647	111.062	78.523
48 Observations							123.013
CASE 3		INITIAL	1.295298	0.16194	32.564	108.238	82.192
2 Days + 1 Hour		CORRECTED	1.3013288	0.16260	32.696	110.988	81.797
70 Observations							124.555
CASE 4		INITIAL	1.292486	0.15988	32.698	10; 144	83.307
3 Days + 2 Hours		CORRECTED	1.3013571	0.16278	32.703	111.107	82.167
96 Observations							124.150
CASE 5		INITIAL	1.324761	0.17994	32.808	104.595	87.558
5 Days + 2 Hours		CORRECTED	1.3013054	0.17994**	32.867	109.051	87.469**
108 Observations							125.457

*All observations except one within 3 min. of one another, hence the poor semi-major axis.

**e and ω not corrected in the differential correction process.

TABLE III
IOLSO INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 070

	<u>a (E.R.)</u>	<u>e</u>	<u>i (deg)</u>	<u>Ω (deg)</u>	<u>ω (deg)</u>	<u>P_A (min)</u>
SGP REFERENCE ORBIT	1.0797636	0.00584	97.347	324.999	223.992	94.860
CASE 1						
5 Days + 5 Hours						
27 OBSERVATIONS	INITIAL	1.074933	0.01140	99.454	328.325	245.179
	CORRECTED	1.0804762	0.00587	97.297	324.978	233.644
CASE 2						
8 Days + 21 Hours						
56 OBSERVATIONS	INITIAL	1.075284	0.01091	102.152	332.855	244.967
	CORRECTED	1.0804714	0.00584	97.446	324.997	230.667
CASE 3						
13 Days + 11 Hours						
93 OBSERVATIONS	INITIAL	1.073103	0.02721	110.136	338.605	287.341
	CORRECTED	1.0804414	0.02721*	97.055	324.892	287.361*
CASE 4						
15 Days + 15 Hours						
106 OBSERVATIONS	INITIAL	1.075303	0.02107	109.939	339.379	286.639
	CORRECTED	1.0804734	0.00393	97.402	325.003	239.468
CASE 5						
16 Days + 15 Hours						
125 OBSERVATIONS	INITIAL	1.077235	0.01601	109.598	340.7799	289.017
	CORRECTED	1.0806248	0.00404	97.467	325.023	145.955
						94.880

*e and ω not corrected in the differential correction process.

TABLE IV

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 116

		<u>a (E.R.)</u>	<u>e</u>	<u>i (deg)</u>	<u>Ω (deg)</u>	<u>ω (deg)</u>	<u>P_A (min)</u>
SGP REFERENCE ORBIT		1.146960	0.00768	66.799	93.437	137.544	103.815
CASE 1							
17 Hours	INITIAL	1.151539	0.01356	67.098	92.232	110.150	104.406
20 OBSERVATIONS	CORRECTED	1.1473379	0.00762	66.799	93.403	142.030	103.817
CASE 2							
2 Days + 9 Hours	INITIAL	1.51883	0.01326	66.834	89.937	119.695	104.453
71 OBSERVATIONS	CORRECTED	1.1473398	0.00721	66.804	93.431	139.730	103.818
CASE 3							
31 Days + 7 Hours	INITIAL	1.150085	0.01114	67.273	87.723	123.966	104.208
108 OBSERVATIONS	CORRECTED	1.147339	0.00746	66.802	93.429	142.088	103.818
CASE 4							
5 Days + 2 Hours	INITIAL	1.128985	0.01348	66.758	86.177	285.040	101.354
150 OBSERVATIONS	CORRECTED	1.1473421	0.00633	66.749	93.541	127.749	103.818
CASE 5							
6 Days + 1 Hour	INITIAL	1.131450	0.01064	66.444	85.453	283.166	101.686
150 OBSERVATIONS	CORRECTED	1.147341	0.00706	66.799	93.437	140.046	103.818

TABLE V

IOLSQ INITIAL AND CORRECTED ELEMENT SETS FOR SATELLITE 213

		<u>a (E.R.)</u>	<u>e</u>	<u>i (deg)</u>	<u>Ω (deg)</u>	<u>ω (deg)</u>	<u>P_A (min)</u>
SGP REFERENCE ORBIT		1.046857	0.01190	81.206	28.578	304.681	90.560
CASE 1							
INITIAL		1.060330	0.00332	80.907	27.667	82.906	92.251
2 Days + 4 Hours 18 OBSERVATIONS	CORRECTED	1.0472239	0.01185	81.166	28.150	291.194	90.514
CASE 2							
INITIAL		1.060313	0.00378	81.094	26.515	85.832	92.248
5 Days + 4 Hours 40 OBSERVATIONS	CORRECTED	1.0469900	0.01357	81.191	27.996	285.700	90.483
CASE 3							
INITIAL		1.062472	0.00478	78.588	22.222	84.297	92.530
9 Days + 4 Hours 75 OBSERVATIONS	CORRECTED	1.0464638	0.00478*	81.259	29.484	84.332*	90.415
CASE 4							
INITIAL		1.060639	0.00342	78.246	18.317	106.177	92.291
11 Days + 4 Hours 94 OBSERVATIONS	CORRECTED	1.0463125	0.00342*	81.250	29.699	106.209*	90.396
CASE 5							
INITIAL		1.059583	0.00318	78.363	16.989	121.483	92.153
13 Days + 15 Hours 110 OBSERVATIONS	CORRECTED	1.0460965	0.00318*	81.567	28.786	121.513*	90.367

*e and ω not corrected in the differential correction process.

TO SCHTPR

JOB	IOLSQ SAT. 116 3	
REM	INITIAL ORBIT LEAST SQUARE	
REM	WITH DIFFERENTIAL CORRECTION	
SPSJOB IOLSQ 00		
11625 21633071223101369004700	00960000305045-107117850	0000 U
11622003903071403051200014100008750000226129		U
11621 03903071304352000088700	12740000088956	0000 U
11621 03903071317500900055000	14860000115643	0000 U
11621 03903071317522500053200	06860000117867	0000 U
11625 03903071404483252345080	28331000116343-232953530	0000 U
11625 0390307140484452348290	27780000114549-194961900	0000 U
11625 03903071404485652350580	27098000112346-145665450	0000 U
11625 03903071404505652342710	20953000123955 314218540	0000 U
11625 03903071404510252341630	20799000126671 328414430	0000 U
11625 0390307140451205238670	20260000132544 376024430	0000 U
11625 0390307140451325236710	19940000137502 405213110	0000 U
11625 03903071404514452333900	19685000142593 426840540	0000 U
11625 03903071404515052333640	19522000144993 438979330	0000 U
11625 03903071404520253931700	19351000150347 459717200	0000 U
11625 03903071404521453930470	19114000155942 478601810	0000 U
11625 03903071404522653928450	18964000162347 495021610	0000 U
11625 03903071404523854625810	18870000168129 507809030	0000 U
11625 03903071404525054623020	18839000170979 519632770	0000 U
11625 03903071404525654622850	18749000176628 524228830	0000 U
11625003903071418033252346560021755000141386M26745900		U
11622003903071418091700025500001710000181594		U
11625 03903071503142652321880	01654000186378-399375380	0000 U
11625 03903071503143252321970	01757000184059-391406400	0000 U
116220039030715050022000024700027720000172236		U
11625 03903071505063252318520	21353000196841 387588710	0000 U
11625 03903071505064452319200	20926000199817 406788380	0000 U
11625 039030715050650502318190	20966000201902 414108710	0000 U
11625 03903071505070252317460	20928000209132 428953250	0000 U
11625 03903071505070852317350	20801000209882 432011110	0000 U
11625 03903071716591452333640	18322000156333-500933480	0000 U
11625 03903071716592052334690	18217000152337-49335150	0000 U
11625 03903071716593252337150	18048000146394-476470580	0000 U
11625 03903071716593852338050	18101000143455-467760300	0000 U
11625 03903071716595052340340	17879000138662-449227800	0000 U
11625 03903071717012051561760	15489000107607-206266730	0000 U
11625 03903071717013252364310	14209000105291-161974050	0000 U
11625 03903071717013852365430	13656000104974-137140500	0000 U
11625 03903071717015052366770	12770000101936-088770680	0000 U
11625 03903071717015652367030	12106000101389-079319100	0000 U
11625 21633071717135204110150	01454000260317 549840740	0000 U
11625 21633071717102241329040	34679000163186 332176530	0000 U
11625 2163307171527471607420	04269000284459 576027170	0000 U
11625 21633071715260719315680	04959000228193 524486280	0000 U
116220216330717034503172	507002550400 294091 6934556	U 08488 1
11625 21633071702041795811400	15902000244006 609793380	0000 U
11625 21633071701561211321460	00999000148547-491278040	0000 U
11625 2163307170181956303620	11063000309912 544410720	0000 U
11625 21633071700122021924520	05598000172847 083340650	0000 U
11625 21633071622262786601690	05203000326238 268257940	0000 U
11625 21633071622292797303360	04489000314055 207230420	0000 U
11625 2163307160333055809440	23132000258717 275059370	0000 U
11625 21633071603272116008650	28768000264952-304878160	0000 U
11625 21633071603240173613750	23710000227451 203116200	0000 U
11625 21633071603182233708190	29643000267785-399523640	0000 U

FIGURE 6. SAMPLE INPUT DATA - SATELLITE 116, CASE 3 (1 OF 2)

11625	21633071601505964407330	15172000276264	631772930	0000 U
11625	21633071600020538805770	09068000289513	448097320	0000 U
11625	21633071523590570614080	06615000225692	240588920	0000 U
11625	21633071523523669408280	35709000270465-408623090	0000 U	
11625	21633071522101588001560	03715000333487	201948650	0000 U
11625	21633071522072322203330	01533000315336	002094170	0000 U
11625	21633071503225215502480	19371000321218	523487530	0000 U
11625	2163307150316032132280	25740000181935-036119840	0000 U	
11625	21633071501374347201970	13807000325355	615260470	0000 U
11625	21633071501322403633770	11623000141831	445910480	0000 U
11625	21633071423411455011290	02759000246912-075575540	0000 U	
11625	21633071423385479708040	00591000272296-278765870	0000 U	
11625	21633071423375490105950	35811000290918-343852010	0000 U	
11625	21633071420013410005860	34094000296870	034266590	0000 U
11625	21633071418133046324080	33561000181828	212252720	0000 U
11625	21633071418072751113880	25949000247642-476841230	0000 U	
11625	2163307141854768706560	25055000296763-527138430	0000 U	
11625	21633071414375891005600	10951000305638-125798610	0000 U	
11625	21633071414362906903970	12084000322084-235696340	0000 U	
11625	21633071403032362023800	22340000141403	232212230	0000 U
11625	21633071402593118024700	29710000171611-428100750	0000 U	
11625	21633071411161228033700	06640000142144	129356850	0000 U
11625	21633071401122551021600	00160000186292-439590900	0000 U	
11625	21633071323272740007500	03350000276320	048184500	0000 U
11625	2163307132325354007100	01640000279841-115642800	0000 U	
11625	21633071317593354029900	35030000160862	361198430	0000 U
11625	2163307131755574035700	27680000145295-250374080	0000 U	
11625	2163307131617034002600	04290000295668-570430350	0000 U	
11625	21633071516125380026600	07070000173390	345445800	0000 U
11625	21633071304350461805710	24730000289795	147166580	0000 U
11625	21633071304343467106080	25154000286108	103633740	0000 U
11625	21633071304333477706480	26033000282610	011564280	0000 U
11625	21633071302495895439500	18239000128023	438108300	0000 U
11625	21633071302424971113550	52337000229981-509608060	0000 U	
11625	21633071301053024809580	09861000257567	473338580	0000 U
11625	21633071300591091720690	02291000189782-224539770	0000 U	
11625	21633071223104251004800	01360000302636-071350130	0000 U	
11625	21633071223115921004600	02030000302880-002038580	0000 U	
116250328	30717134507608	320681080591	326555-22621643	U 07792 1
116250328	30717135026981	32488 661678	319982 20198542	U 07806 1
116250328	30717153507430	68472 750573	280539 30189224	U 08012 2
116250329	30716235002989	324882403108	324747-18289677	U 04975 1
116250329	30717013802746	664722725396	291689-18057828	U 05167 1
116250329	30717032409866	324882926802	325859-18479371	U 05460 1
116250329	30717032425686	324882927876	322062-18379255	U 05463 1
116250329	30717032518235	684722929904	286967-18258061	U 05466 1
116250329	30717051110296	324883035026	318636-17968251	U 05776 1
116250329	30717051203422	684723018345	205392-17757479	U 05779 1
116250329	30717065916295	684722977989	285300-13352360	U 06038 1

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FIGURE 6. SAMPLE INPUT DATA - SATELLITE 116, CASE 3 (2 OF 2)

REGTN SCRFI - TAPS
 101 SA SAT. 196 7
 INTRAL OPIR LFACT SQUARE
 WITH DIFFERENTIAL CORRECTION
 SPS.03 TNS 50

START TNS
 19610001-17.9

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TNS AT APPROXIMATE EPOCH
 DATE : NOVEMBER 40 - 1967

N.M ELEMENTS AT TIME OF FIRST OBSERVATION:

67011223113.6 192.45355 -0.1624688 .01921029 .09948560 -.13020417 .4143008

CLASSICAL ELEMENTS AT TIME OF FIRST OBSERVATION:

	A-F-R.	E	I-NG	OMEGA-DEG	ORBITAL DAY	PERIOD	PER. ALT.
TIME-DAYS	67011223113.6	.11114114	.000270504	121.06298	-0.774571	87.72335	544.0
						104.26920	544.0
						104.26920	544.0

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (1 OF 5)

TIME OF EPOCH 103.949974
ELEMNT SET NO. n

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (3 OF 5)

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STANDARD GENERAL PERTURBATION DIFFERENTIAL CORRECTION

ELEMENT #0. SATELLITE NAME: ELEMENT SFT No.: TIME OF EPNCH 193.9449674

ELAB SFTA NO.	ELAB SFTB NO.	TIME	P. R. RANGE	P. R. ISCPN	DECI.	ALTITUDE	ELEV.	RD. RFS:	VECTOR	DELTA	U	ELAB
			MM. SS. SS	A. J. DSS.	MM.	RFS. KM.	RFS. KM.	MAG. MM. / SEC.	MAG. MM. T MIN.	DEG.	DEG.	
145	329	7 17 16 27 51	2.97	1 0 -1.1797+0	-3170+1	1935+1	1438+0	203+2	0.3	51	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	6496+1	1357+3	4788+0	158+2	162	65	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	4114+1	1705+2	4446+0	566+2	0.9	74	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	3120+4	1924+2	4451+1	490+2	0.8	74	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	4820+1	1158+2	4372+1	190+2	0.3	77	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	5308+1	3409+1	4374+1	207+2	0.2	64	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	5770+1	7215+1	4316+1	303+2	0.4	58	0	
145	329	3 07 17 1 58	2.71	1 0 -1.341+4	1561+0	2415+1	3248+1	169+2	0.3	66	0	

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DFCFCMFR 19; 1963

STANDARD GENERAL PERTURBATION DIFFERENTIAL CORRECTION

ELEMENT #0. SATELLITE NAME: ELEMENT SFT No.: TIME OF EPNCH 193.9449674

CASE	LMS NO.	RMS/ KM/SEC	DELTA AVN	DELTA AVN	DELTA AVN	DELTA NOSE	DELTA T	DELTA CO
4	.PR3382+1	.21547+1	.27176577+4	.045442+4	.717124+5	.10550+3	.5nn71+2	.075666+4

CORRECTED ELEMENTS

REV. CASE NO.	LA DEGREES	TO Days	FACT1 DANIT	FACT2 DANIT	E DEG.	I DEG.	NODE DEG.	AMFGA DEG.	CO DA/REV+2	PER ALT ST. MIN.	PA MINUTES
5	93.97215	103.94496	1.1477394	.00748	44.870	93.429	142.0+2	.0000000	549.9	103.818	

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (4 OF 5)

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DECEMBER 19, 1963

STANDARDISATION DIFFERENTIAL CORRECTION

SATELLITE NAME: SATELLITE 10.

TIME OF EPICU 103.9449663

REF. NO.	OLD RMS	NEW RMS	DIFF.	REF. NO.	OLD RMS	NEW RMS	DIFF.
KW.	KW.	KW/SEC	KW/SEC	A	A	A	A
4.943	883.1	3.254	2.151	4.94496	1.473.94	4.94496	1.473.94
NO. OF RESTRAINTS	354	354	0.0	NO. OF PERTURBATIONAL & RF JUNCTIONS	54	54	0.0

REF. NO.	SFT	LEES	TO	E	I	NODE	MEGA	CO	PFR ALT
OLD ELEMENTS	No.		DAY	CAPTION	deg.	deg.	DEG.	DEG.	ST. MI.
NEW ELEMENTS	No.								MINUTES
103.94446	103.94446	103.94446	0.0000000	0.0000000	47.272	47.272	123.922	123.922	544.0
103.97215	103.94496	103.94496	0.0000000	0.0000000	46.802	46.802	142.088	142.088	549.9
103.97215	103.94496	103.94496	0.0000000	0.0000000	46.802	46.802	142.088	142.088	549.9
NO. OF NOT = -7.025 MEG/DAV. MEGA NOT = -.561 DEG/DAV									

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ALL ELEMENTS CORRECTED TO FIRST ATTEMPT

ELEMENTS NOT RECALC. VARIOUSLY

FIND TNS
12-10-11-11-16.2

FIGURE 7. SAMPLE OUTPUT DATA - SATELLITE 116, CASE 3 (5 OF 5)

REFERENCES

1. Hilton, C. G., "The SPADATS Mathematical Model," Aeronutronic Publication No. U-2202, 5 August 1963.
2. "Space Detection and Tracking System (SPADATS) Programming Document" (Revised), Aeronutronic Publication No. U-1691, 1 October 1962.
3. "Spacetrack Operators Manual," System Development Corporation Publication TM-LX-37, 14 February 1963.