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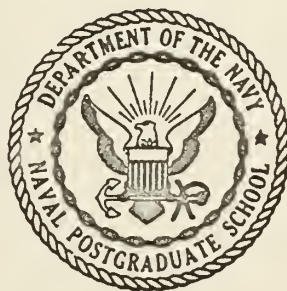
Willard Evan Bleick

ORBITAL TRANSFER WITH MINIMUM FUEL.

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by

W. E. Bleick and F. D. Faulkner

Professors of Mathematics and Mechanics

RESEARCH PAPER NO. 40

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W. E. BLEICK and F. D. FAULKNER*

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A note in this Journal, Ref. 1, discussed the problem of scheduling the direction p of constant momentum thrust of a rocket, which loses mass at a constant rate, so that it transfers to a known earth satellite orbit in minimum time T after launching. A numerical solution was obtained, using rectangular coordinates, for the case of fixed launching conditions. The method of Ref. 1 is extended here to solve the problem of orbital transfer of such a rocket with minimum fuel consumption. All of the symbols, units, and end conditions of Ref. 1 are used here without redefinition.

Statement of the Problem

The time of flight T in minimum fuel transfer must be longer than in the minimum time transfer of Ref. 1, unless these two trajectories turn out to be identical. This implies at least one interruption in rocket thrust during minimum fuel transfer. The problem solved here assumes exactly one such interruption, i.e. launch at $t=0$, thrust interruption at $t=t_1$, thrust resumption at $t=t_2$, and final thrust termination at transfer $t=T$. The problem of minimum fuel transfer is equivalent to the Lagrange calculus of variations problem of requiring the integral

$$J = \int_0^T (f + \lambda\varphi_1 + \mu\varphi_2 + \pi\varphi_3 + \rho\varphi_4) dt \quad (1)$$

to be stationary, where f is the fuel consumption rate, λ, μ, π, ρ are continua of Lagrangian multipliers, and $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0$ are the first order equations of rocket motion of Ref. 1. The f function and the

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rocket thrust per unit remaining mass function a are defined as follows: For $0 < t < t_1$, $f=1$ and $a=c\dot{M}/(1-\dot{M}t)g$. For $t_1 < t < t_2$, $f=0$ and $a=0$. For $t_2 < t < T$, $f=1$ and $a=c\dot{M}/[1-\dot{M}(t+t_1-t_2)]g$. Note that for $t_2 < t < T$ $\partial a/\partial t_1 = -\partial a/\partial t_2 = ga^2/c$. The varied time subinterval end points in Eq.(1) are taken as $t_1+\Delta t_1$, $t_2+\Delta t_2$ and $T+\Delta T$. The vanishing first variation δJ and its partial integration are computed as in Ref. 1. The coefficients of $\delta u, \delta v, \delta x, \delta y, \delta p$ in $\delta J=0$ give the Euler Eqs.(2) and (3), consisting of the adjoint equations

$$\begin{aligned} \dot{\lambda} + \pi &= 0, & \dot{\mu} + \rho &= 0, \\ \dot{\pi} + g_{1x}\lambda + g_{2x}\mu &= 0, & \dot{\rho} + g_{1y}\lambda + g_{2y}\mu &= 0, \end{aligned} \quad (2)$$

and the control equation

$$\tan p = \mu/\lambda. \quad (3)$$

The coefficient of ΔT in $\delta J=0$ gives, with the aid of Eq.(3), the transversality condition

$$(a \cdot \wedge)_T = (a \wedge)_T = 1 \quad (4)$$

where the adjoint vector $\wedge = i\lambda + j\mu$, $|\wedge| = (\lambda^2 + \mu^2)^{1/2}$, and $a = a(i \cos p + j \sin p)$. The coefficients of Δt_1 and Δt_2 in $\delta J=0$ give, with the aid of Eq.(4), the corner conditions

$$H(t_1) = [a \wedge]_{t_1}^T - \frac{g}{c} \int_{t_1}^T a^2 \wedge dt = 0, \quad H(t_2) = 0. \quad (5)$$

Eqs.(5) are equivalent, by the definition of a , to

$$\wedge(t_1) = \wedge(t_2) \quad (6)$$

and, by partial integration, to

$$\int_{t_2}^T a \wedge dt = 0. \quad (7)$$

Numerical Solution

Let $\lambda_i, \mu_i, \pi_i, \rho_i$, $i=1,2,3,4$, be four linearly independent solutions of the adjoint Eqs.(2) corresponding to the columns of the matrix $E(t)$ of Ref. 1. The control angle p of Eq.(3) is defined by

$$\tan p = (\mu_1 + 1\mu_2 + m\mu_3 + n\mu_4) / (\lambda_1 + 1\lambda_2 + m\lambda_3 + n\lambda_4) \quad (8)$$

and its variation δp is obtained in terms of $\delta l, \delta m, \delta n$ by total differentiation as in Ref. 1. The Bliss fundamental formulas are obtained by assuming that a solution of the rocket motion equations $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0$ has been found, corresponding to Eq.(8), which does not necessarily satisfy the terminal conditions at $t=T$ or the corner condition Eqs.(5). Using this solution and holding T fixed, but allowing t_1 and t_2 to vary, find the variation of the vanishing matrix integral

$$\int_0^T [\varphi_1, \varphi_2, \varphi_3, \varphi_4] E(t) dt = 0 \quad (9)$$

with the terminal constraints at $t=T$ removed. Since the columns of $E(t)$ satisfy the adjoint Eqs.(2), one obtains the system of Bliss formulas in the 1×4 matrix equation

$$[\delta u, \delta v, \delta x, \delta y]_T E(T) + [G(t_1) - (apF)_T] \Delta t_1 - [G(t_2) - (apF)_T] \Delta t_2 = [0, \delta l, \delta m, \delta n] A \quad (10)$$

where the matrix A has been defined in Ref. 1, and where the matrix

$$G(t) = (apF)_t^T - \frac{g}{c} \int_t^T a^2 p F dt \quad (11)$$

where the 2×4 matrix $F(t)$ is the first two rows of $E(t)$, and where the matrix $p = [\cos p, \sin p]$. Substitution of

$$[\delta u, \delta v, \delta x, \delta y]_T = [U-u, V-v, X-x, Y-y]_T + [\dot{U}-\dot{u}, \dot{V}-\dot{v}, \dot{X}-\dot{x}, \dot{Y}-\dot{y}]_T \Delta T \quad (12)$$

into Eq.(10) gives four of the required six Newton-Raphson equations for the determination of $\Delta T, \Delta t_1, \Delta t_2, \delta l, \delta m, \delta n$ on the varied trajectory.

The remaining two equations attempt to satisfy the corner condition Eqs.(5) on the varied trajectory. Involved here are the differentials

$$\begin{aligned} da &= \delta a + \dot{a} dt \\ &= (\partial a / \partial t_1) \Delta t_1 + (\partial a / \partial t_2) \Delta t_2 + (ga^2/c) dt \end{aligned} \quad (13)$$

$$\begin{aligned} \text{and } d\Lambda &= \delta \Lambda + \dot{\Lambda} dt \\ &= [0, \delta l, \delta m, \delta n] F' p' + (\dot{\Lambda} \cos p + \dot{\mu} \sin p) dt \end{aligned} \quad (14)$$

where the primes on F and p indicate matrix transposition. Use of Eqs.(6) and (14) yields the Newton-Raphson equation

$$\dot{\Lambda}(t_1) \Delta t_1 - \dot{\Lambda}(t_2) \Delta t_2 - [0, \delta l, \delta m, \delta n] [F' p']_{t_1}^{\Delta t_2} = \Lambda(t_2) - \Lambda(t_1) \quad (15)$$

Use of Eqs.(13) and (14), and the first of Eqs.(5), yields the Newton-Raphson equation

$$(\dot{a}^{\wedge})_T \Delta T + (K - a^{\wedge})_{t_1} \Delta t_1 - K(t_2) \Delta t_2 + [0, \delta l, \delta m, \delta n] G'(t_1) = -H(t_1) \quad (16)$$

where

$$K(t) = \frac{g}{c} [a^{2\wedge}]_t^T - 2 \left(\frac{g}{c}\right)^2 \int_t^T a^{3\wedge} dt \quad (17)$$

The iteration to successive varied trajectories, using Eqs.(10), (12), (15) and (16), may be carried out as in Ref. 1. Two devices were used to stabilize the course of the iteration. The first was to adjust the m and n values of the new T, t₁, t₂, i, m, n sextuple, found by solving the Newton-Raphson equations, to satisfy the corner condition Eqs.(5) before proceeding with the next iteration. The second device was to modify the $[\dot{U}, \dot{V}, U, V, X, Y]_T$ terms in the Newton-Raphson equations, before solving these equations, so as to minimize the sum of the squares of the elements of $[U-u, V-v, X-x, Y-y]_T$.

The numerical example of minimum fuel transfer given here involves the same launching conditions, mass loss parameters and circular orbit used in the minimum time transfer of Ref. 1. The results for minimum fuel transfer are T=0.353977, t₁=0.210293, t₂=0.275349, l=-0.820196, m=-0.708727, n=-1.181390, and the transfer sector angle B=0.189345 rad. Since the minimum time trajectory of Ref. 1 gave T=0.289869, the net fuel saving in minimum fuel transfer over minimum time transfer is measured by $0.289869 - 0.353977 + t_2 - t_1 = 0.000948$, or an unspectacular one-third per cent. Figure 1 shows the trajectories and thrust directions for minimum time and minimum fuel transfer.

The semilogarithmic plots of Fig. 2 show the different behavior of Δ versus time in the two problems. For some reason there is much more difference than we expect. The curve increases monotonically for minimum time transfer. The curve for minimum fuel shows a rather char-

acteristic shape. It is large initially and decreasing; then it increases, and then decreases. If the final decreasing interval does not occur, larger values of T lead to lower values of fuel consumption, as may be partially inferred from Eq. (7).

Reference

¹ Bleick, W. E., "Orbital transfer in minimum time," AIAA Journal 1, 1229-1231 (1963).

Figures

Fig. 1 Trajectories and thrust directions

Fig. 2 Λ versus time

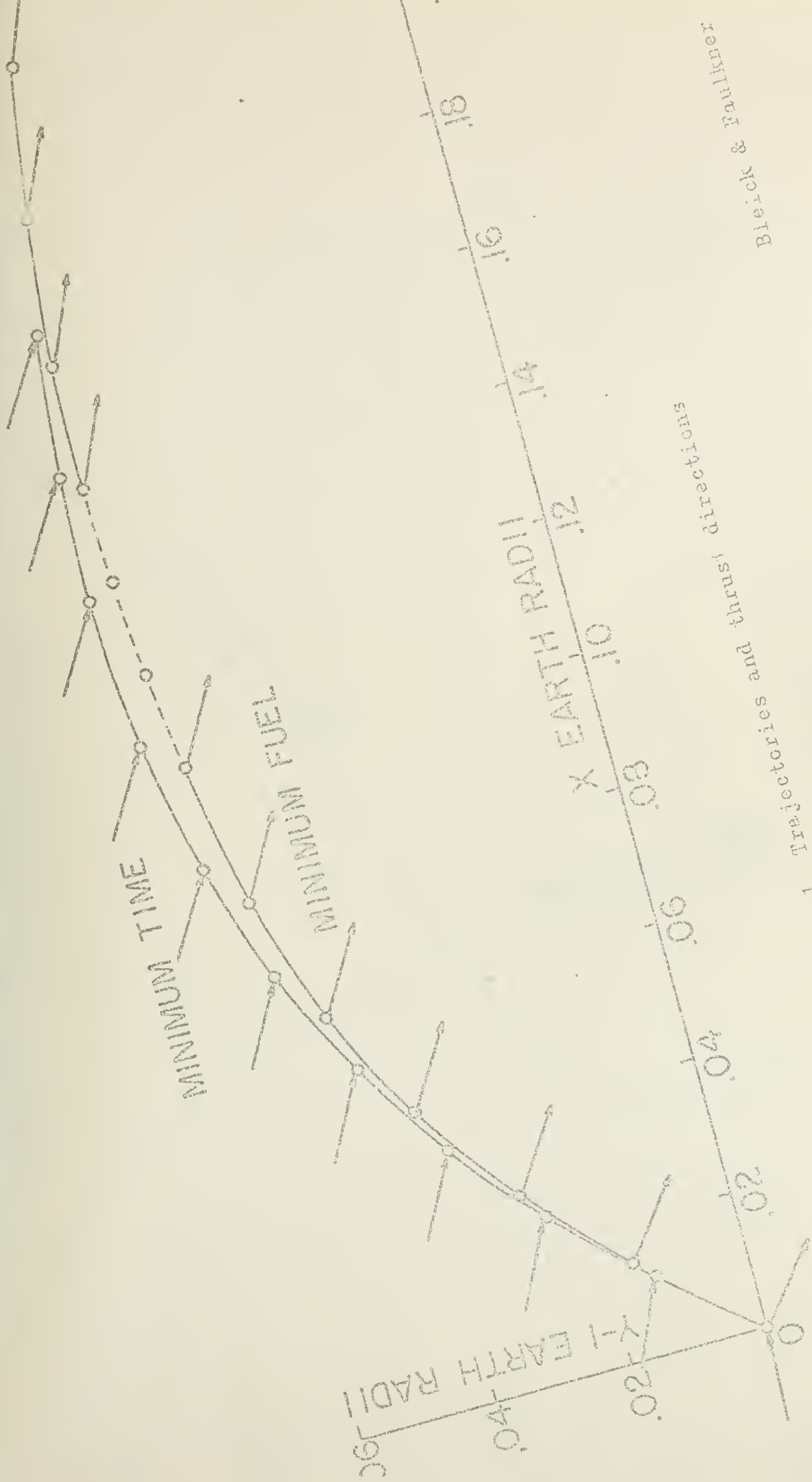
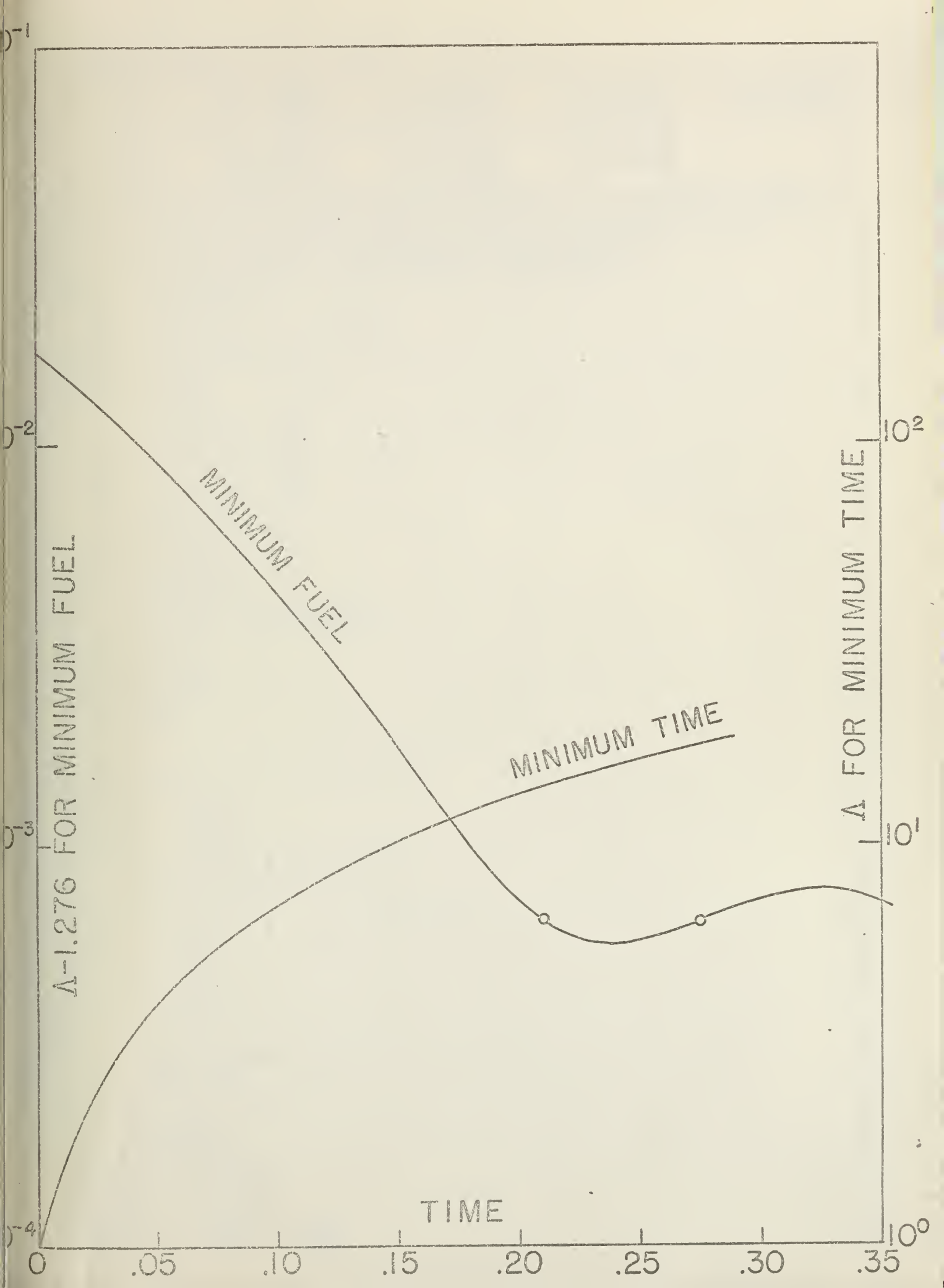


Fig. 1 Trajectories and thrust directions

Bleick & Faulkner



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1 PROGRAM MINFUEL
CYVARS (1) = XU YVARS(5) = XLM1 YVRS(9) = XLM2 YVRS(13) = XLM3 YVRS(17) = XLM4
(2) = XV (6) = XMU1 (10) = XMU2 (14) = XMU3 (18) = XMU4
(3) = XX (7) = XPI1 (11) = XPI2 (15) = XPI3 (19) = XPI4
(4) = XY (8) = XRO1 (12) = XRO2 (16) = XRO3 (20) = XRC4
CYVARS (21) = AA12 YVRS(25) = AA23 YVRS(29) = AA44 YVRS(33) = C2ASQ
(22) = AA13 (26) = AA24 (30) = A2LAM (34) = C3ASQ
(23) = AA14 (27) = AA33 (31) = A3LAM (35) = C4ASQ
(24) = AA22 (28) = AA34 (32) = C1ASQ

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2 DIMENSION YVARS(35), AK(4,35), DY(35), YC(35), C(4), XU(500), XV(500),
+R(6), XX(500), XY(500), TAU(500), CAPLAM(500), CGALAM(500), A2LAM(500),
+P(500), CC(4), A(6,6), AI(6,6), DEL(6), CAPV(4), CAPVD(4), CIT1(4),
+CIT2(4), GCIA2T(4), AAA(4), AA(4,4), TVAR(6), GCIA2T(4)
3 EQUIVALENCE (T, TVAR(1)), (T1, TVAR(2)), (T2, TVAR(3)),
(EL, TVAR(4)), (EM, TVAR(5)), (EN, TVAR(6))

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4 REARTH = 20.925 OCC.
5 GACCEL = 32.086
6 TUNIT = SQRTF(REARTH/GACCEL)
7 CCC = 70.000.
8 COVERG = CCC/(GACCEL * TUNIT)
9 FMDOT = 0.0036
10 OMEGA = FMDOT * TUNIT
11 VSTART = 0.585
12 THETA = 0.928
13 T = C.353 966 649
14 T1 = C.210 274 520
15 T2 = C.275 321 127
16 EL = -C.820 214 924
17 EM = -C.708 762 302
18 EN = -1.181 456 519
19 R = 1.075 698 925
20 V = SQRTF(1.0/R)
21 VSQDR = V*V/R
22 DB = C.189 335 935
23 TFIN = C.28972 53036
24 XSTEP = TFIN/176.
25 XU(1) = VSTART * COSF(THETA)
26 XV(1) = VSTART * SINF(THETA)
27 XX(1) = C.0
28 XY(1) = 1.0
29 TAU(1) = 0.0
31 A2LAM(1) = 0.0
32 C(1) = C.0
33 C(2) = C.5
34 C(3) = C.5
35 C(4) = 1.0

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36 KK = C
37 DO 271 L=1,3
38 XVAR = 0.0
39 YVARS(1) = XU(1)
40 YVARS(2) = XV(1)
41 YVARS(3) = XX(1)
42 YVARS(4) = XY(1)
43 CAPLAM(1) = SQRTF(1.0 + EL*EL)
44 P(1) = 57.2957 * ATANF(EL)
45 XA = COVERG * OMEGA
46 CGALAM(1) = COVERG * XA * CAPLAM(1)
47 DO 46 I=6,35
48 YVARS(I) = 0.0
49 DO 48 I=5,20,5
50 YVARS(I) = 1.0
51 N1 = T1/XSTEP + 1.0
52 XN1 = N1
53 STEP1 = T1/XN1
54 N2 = (T2-T1)/XSTEP + 1.0
55 XN2 = N2
56 STEP2 = (T2-T1)/XN2
57 N2 = N1 + N2
58 N3 = (T-T2)/XSTEP + 1.0
59 XN3 = N3
60 STEP3 = (T-T2)/XN3
61 N3 = N2 + N3
62 SINP = SINF(BB)
63 COSB = COSF(BB)
64 CAPV(1) = V * COSB
65 CAPV(2) = -V * SINB
66 CAPV(3) = R * SINB
67 CAPV(4) = R * COSB
68 CAPVD(1) = -VSQDR * SINB
69 CAPVD(2) = -VSQDR * COSB

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68	CAPVD(3) = CAPV(1)	79
69	CAPVD(4) = CAPV(2)	80
70	N4 = N3 + 1	81
71	DO 206 K=2,N4	82
72	IF (N1+1-K) 75,73,73	83
73	STEP = STEP1	84
74	GO TO 79	85
75	IF (N2+1-K) 78,76,76	86
76	STEP = STEP2	87
77	GO TO 79	88
78	STEP = STEP3	89
79	DO 124 I=1,4	90
80	XC = XVAR + C(I) * STEP	91
81	DO 82 J=1,35	92
82	YC(J) = YVARS(J) + C(I) * AK(I-1,J)	93
83	XLAM = YC(5) + EL*YC(9) + EM*YC(13) + EN*YC(17)	94
84	XMU = YC(6) + EL*YC(10) + EM*YC(14) + EN*YC(18)	95
85	CLAM = SQRTF(XLAM**2 + XMU**2)	96
86	COSP = XLAM/CLAM	97
87	SINP = XMU/CLAM	98
88	IF (N1+1-K) 91,89,89	99
89	XA = COVERG * OMEGA / (1.0 - OMEGA * XC)	100
90	GO TO 95	101
91	IF (N2+1-K) 94,92,92	102
92	XA = C.0	103
93	GO TO 95	104
94	XA = COVERG*OMEGA / (1.0-OMEGA*(XC-T2+T1))	105
95	XR = SQRTF(YC(3)**2 + YC(4)**2)	106
96	DY(1) = -YC(3)/XR**3 + XA*COSP	107
97	DY(2) = -YC(4)/XR**3 + XA*SINP	108
98	DY(3) = YC(1)	109
99	DY(4) = YC(2)	110
100	G1X = (2.*YC(3)**2 - YC(4)**2)/XR**5	111
101	G1Y = 3.*YC(3)*YC(4)/XR**5	112
102	G2X = G1Y	113
103	G2Y = (2.*YC(4)**2 - YC(3)**2)/XR**5	114
104	DO 108 M=5,17,4	115
105	DY(M) = -YC(M+2)	116
106	DY(M+1) = -YC(M+3)	117
107	DY(M+2) = -G1X*YC(M) - G2X*YC(M+1)	118
108	DY(M+3) = -G1Y*YC(M) - G2Y*YC(M+1)	119
109	DO 110 M=1,4	120
110	AAA(M) = COSP*YC(4*M+2) - SINP*YC(4*M+1)	121
111	DO 112 M=1,3	122
112	DY(20+M) = XA*AAA(1)*AAA(M+1)/CLAM	123
113	DO 114 M=1,3	124
114	DY(23+M) = XA*AAA(2)*AAA(M+1)/CLAM	125
115	DY(27) = XA*AAA(3)*AAA(3)/CLAM	126
116	DY(28) = XA*AAA(3)*AAA(4)/CLAM	127
117	DY(29) = XA*AAA(4)*AAA(4)/CLAM	128
118	DY(30) = XA*XA*CLAM	129
119	DY(31) = XA*DY(30)	130
120	DO 122 Y=1,4	131
121	CC(M) = YC(4*M+1)*COSP + YC(4*M+2)*SINP	132
122	DY(31+M) = CC(M)*XA*XA	133
123	DO 124 J=1,35	134
124	AK(I,J) = STEP*DY(J)	135
125	DO 126 J=1,35	136
126	YVARS(J) = YVARS(J) + (AK(1,J)+2.*AK(2,J)+2.*AK(3,J)+AK(4,J))/6.	137
127	XVAR = XVAR + STEP	138
128	TAU(K) = TAU(K-1) + STEP	139
129	XU(K) = YVARS(1)	140
130	XV(K) = YVARS(2)	141
131	XX(K) = YVARS(3)	142
132	XY(K) = YVARS(4)	143
133	XLAM = YVARS(5) + EL*YVARS(9) + EM*YVARS(13) + EN*YVARS(17)	144
134	XMU = YVARS(6) + EL*YVARS(10) + EM*YVARS(14) + EN*YVARS(18)	145
135	CLAM = SQRTF(XLAM**2 + XMU**2)	146
136	CAPLAM(K) = CLAM	147
137	COSP = XLAM/CLAM	148
138	SINP = XMU/CLAM	149
139	DO 140 M=1,4	150
140	CC(M) = YVARS(4*M+1)*COSP + YVARS(4*M+2)*SINP	151
146	DO 148 M=5,17,4	157
147	DY(M) = -YVARS(M+2)	158
148	DY(M+1) = -YVARS(M+3)	159
149	CGALAM(K) = COVERG*XA*CLAM	160
150	A2LAM(K) = YVARS(30)	161
151	IF (XLAM) 159,152,159	162
152	IF (XMU) 157,155,153	163
153	P(K) = 90.0	164

154	GO TO 165	165
155	P(K) = 0.0	166
156	GO TO 165	167
157	P(K) = -90.0	168
158	GO TO 165	169
159	P(K) = 57.2957 * ATANF(XMU/XLAM)	170
160	IF (XLAM) 161,165,165	171
161	IF (XMU) 164,162,162	172
162	P(K) = P(K) + 180.0	173
163	GO TO 165	174
164	P(K) = P(K) - 180.0	175
165	XLAMDCT = DY(5) + EL*DY(9) + EM*DY(13) + EN*DY(17)	176
166	XMUDOT = DY(6) + EL*DY(10) + EM*DY(14) + EN*DY(18)	177
167	IF (N1+1-K) 183,168,183	178
168	DO 169 M=1,4	179
169	CIT1(M) = CC(M)	180
170	A2LAMT1 = XA*XA*CLAM	181
171	CLAMDT1 = COSP*XLAMDCT + SINP*XMUDOT	182
172	CGALDT1 = COVERG*XA*CLAMDT1	183
173	CLAMT1 = CLAM	184
174	C2ACGT1 = COVERG*XA*CC(2)	185
175	C3ACGT1 = COVERG*XA*CC(3)	186
176	C4ACGT1 = COVERG*XA*CC(4)	187
177	AT1 = XA	188
178	CGALMT1 = COVERG*XA*CLAM	189
179	QA2LMT2 = YVARS(3C)	190
180	CA3LMT2 = 2.0*YVARS(31)/COVERG	191
181	DO 182 M=1,4	192
182	QCIA2T2(M) = YVARS(31+M)	193
183	IF (N2+1-K) 190,184,190	194
184	DO 185 M=1,4	195
185	CIT2(M) = CC(M)	196
186	A2LAMT2 = AT1*AT1*CLAM	197
187	CGALAM(K) = COVERG*AT1*CLAM	198
188	CLAMDT2 = COSP*XLAMDCT + SINP*XMUDOT	199
189	CLAMT2 = CLAM	200
190	IF (N4-K) 206,191,206	201
191	CLAMDT = COSP*XLAMDCT + SINP*XMUDOT	202
192	CGALDT = COVERG*XA*CLAMDT	203
193	C2ACGT = COVERG*XA*CC(2)	204
194	C3ACGT = COVERG*XA*CC(3)	205
195	C4ACGT = COVERG*XA*CC(4)	206
196	CGALMT = COVERG*XA*CLAM	207
197	QA2LMT = YVARS(30)	208
198	CA3LMT = 2.0*YVARS(31)/COVERG	209
199	DO 200 M=1,4	210
200	QCIA2T(M) = YVARS(31+M)	211
201	XR = SQRT(YVARS(3)**2 + YVARS(4)**2)	212
202	DY(1) = -YVARS(3)/XR**3 + XA*COSP	213
203	DY(2) = -YVARS(4)/XR**3 + XA*SINP	214
204	DY(3) = YVARS(1)	215
205	DY(4) = YVARS(2)	216
206	CONTINUE	217
207	PRINT 208	218
208	FORMAT(1H06X2HEL13X2HEM13X2HEN13X2HBB13X2HT113X2HT213X1HT)	219
209	PRINT 210, EL, EM, EN, BB, T1, T2, T	220
210	FORMAT(7F15.9)	221
211	PRINT 212	222
212	FORMAT(1H06X2HN113X2HN213X2HN313X1HU14X1HV14X1HX14X1HY)	223
213	PRINT 214, N1, N2, N3, XU(N4), XV(N4), XX(N4), XY(N4)	224
214	FORMAT(3I15,4F15.7)	225
215	PRINT 216	226
216	FORMAT(1H05X4HCAPU11X4HCAPV11X4HCAPX11X4HCAPY)	227
217	PRINT 218, (CAPV(M), M=1,4)	228
218	FORMAT(4F15.7)	229
219	DO 224 I=1,4	230
220	A(I,1) = 0.0	231
221	B(I) = C.0	232
222	DO 224 J=1,4	233
223	A(I,1) = A(I,1) + YVARS(4*I+J)*(DY(J)-CAPVD(J))	234
224	B(I) = B(I) + YVARS(4*I+J)*(CAPV(J)-YVARS(J))	235
225	DO 227 I=1,4	236
226	A(I,2) = AT1*CIT1(I) + (QCIA2T(I)-QCIA2T2(I))/COVERG	237
227	A(I,3) = -AT1*CIT2(I) - (QCIA2T(I)-QCIA2T2(I))/COVERG	238
228	DO 230 J=2,4	239
229	AA(1,J) = YVARS(19+J)	240
230	AA(2,J) = YVARS(22+J)	241
231	AA(3,3) = YVARS(27)	242
232	AA(3,4) = YVARS(28)	243
233	AA(4,4) = YVARS(29)	244
234	DO 236 I=1,4	245

235	DO 236	J=1,I	246
236	AA(I,J)	= AA(J,I)	247
237	CO 239	I=1,4	248
238	DO 239	J=1,3	249
239	A(I,J+3)	= AA(I,J+1)	250
240	A(5,1)	= -CGALDT	251
241	A(5,2)	= A2LAMT1 + CGALDT1 + CA3LMT - CA3LMT2 - XA*XA*CLAM	252
242	A(5,3)	= -A2LAMT2 - (CA3LMT - CA3LMT2) + XA*XA*CLAM	253
243	A(5,4)	= QCIA2T(2) - QCIA2T2(2) + C2ACGT1 - C2ACGT	254
244	A(5,5)	= QCIA2T(3) - QCIA2T2(3) + C3ACGT1 - C3ACGT	255
245	A(5,6)	= QCIA2T(4) - QCIA2T2(4) + C4ACGT1 - C4ACGT	256
246	B(5)	= CGALMT - CGALMT1 - QA2LMT + QA2LMT2	257
247	A(6,1)	= 0.0	258
248	A(6,2)	= CLAMDT1	259
249	A(6,3)	= -CLAMDT2	260
250	DO 251	J=2,4	261
251	A(6,J+2)	= CIT1(J) - CIT2(J)	262
252	B(6)	= CLAMT2 - CLAMT1	263
272	DO 274	I=1,N4	283
273	CGALAM(I)	= CGALAM(N4) - CGALAM(I)	284
274	A2LAM(I)	= A2LAM(N4) - A2LAM(I)	285
	IF (KK)	320,320,300	263.1
300	KK = KK-1		263.2
	DET = A(5,5)*A(6,6)-A(5,6)*A(6,5)		263.3
	EM = EM + (B(5)*A(6,6)-B(6)*A(5,6))/DET		263.4
	EN = EN + (B(6)*A(5,5)-B(5)*A(6,5))/DET		263.5
	PRINT 301		263.6
301	FORMAT (1H04X3HTAU8X6HCAPLAM7X6HCGALAM8X5HA2LAM/)		263.7
	PRINT 302, (TAU(I),CAPLAM(I),CGALAM(I),A2LAM(I),	I=1,N4)	263.90
302	FORMAT (4F13.9)		263.92
	GO TO 37		263.95
320	DB1 = (XU(N4)-CAPV(1))/CAPV(2)		
	DB2 = -(XV(N4)-CAPV(2))/CAPV(1)		
	DB3 = (XX(N4)-CAPV(3))/CAPV(4)		
	DB4 = -(XY(N4)-CAPV(4))/CAPV(3)		
	BB = BB + (DB1+DB2+DB3+DB4)/4.		
	SINB = SIN(BB)		
	CCSB = COS(BB)		
	CAPV(1) = V*COSB		
	CAPV(2) = -V*SINB		
	CAPV(3) = R*SINB		
	CAPV(4) = R*CCSB		
	CAPVD(1) = -VSQDR*SINB		
	CAPVD(2) = -VSQDR*CCSB		
	CAPVD(3) = CAPV(1)		
	CAPVD(4) = CAPV(2)		
	PRINT 208		263.99
	PRINT 210, EL,EM,EN,BB,T1,T2,T		263991
321	PRINT 216		
322	PRINT 218, (CAPV(M), M=1,4)		
323	DO 328	I=1,4	
324	A(I,1)	= 0.0	
325	B(I)	= 0.0	
326	DO 328	J=1,4	
327	A(I,1)	= A(I,1) + YVARS(4*I+J)*(DY(J)-CAPVD(J))	
328	B(I)	= B(I) + YVARS(4*I+J)*(CAPV(J)-YVARS(J))	
253	CALL GAUSS3 (6, 0.1E-09, A, AI, KER)		264
254	PRINT 255, KER		265
255	FORMAT (5HOKER=I1)		266
256	IF (KER-2)	258,257,258	267
257	STOP 257		268
258	DO 261	I=1,6	269
259	DEL(I)	= 0.0	270
260	DO 261	J=1,6	271
261	DEL(I)	= DEL(I) + AI(I,J)*B(J)	272
262	DO 263	I=1,6	273
263	TVAR(I)	= TVAR(I) + DEL(I)	274
264	BB = BB + V*DEL(1)/R		275
265	PRINT 208		276
266	PRINT 210, EL,EM,EN,BB,T1,T2,T		277
267	IF (T2-T)	303,303,307	278
303	IF (T1-T)	304,304,307	278.1
304	IF (T2)	307,305,305	279
305	IF (T1)	307,306,306	279.1
306	IF (T1-T2)	271,271,307	280
307	GO TO 275		280.1
271	CONTINUE		282
275	PRINT 276		286
276	FORMAT(1H04X3HTAU10X2HXU11X2HXV11X2HXX11X2HXY9X6HCAPLAM7X6HCGALAM8		287
	1X5HA2LAM9X1HP/)		288
278	PRINT 279, (TAU(I),XU(I),XV(I),XX(I),XY(I),CAPLAM(I),CGALAM(I),		289

1	A2LAM(I),P(I), I=1,N4)	290
79	FORMAT (8F13.9, F13.2)	291
80	STOP 280	292
81	END	293
	SUBROUTINE GAUSS3 (N,EP,A,X,KER)	294
	DIMENSION A(6,6), X(6,6)	295
	DO 1 I=1,N	00030
	DO 1 J=1,N	00040
1	X(I,J)=C.0	00050
	DO 2 K=1,N	00060
2	X(K,K)=1.0	00070
10	DO 34 L=1,N	00080
	KP=0	00090
	Z=0.0	00100
	DO 12 K=L,N	00110
	IF(Z-ARSF(A(K,L)))11,12,12	00120
11	Z=ARSF(A(K,L))	00130
	KP=K	00140
12	CONTINUE	00150
	IF(L-KP)13,20,20	00160
13	DO 14 J=L,N	00170
	Z=A(L,J)	00180
	A(L,J)=A(KP,J)	00190
14	A(KP,J)=Z	00200
	DO 15 J=1,N	00210
	Z=X(L,J)	00220
	X(L,J)=X(KP,J)	00230
15	X(KP,J)=Z	00240
20	IF(ABSF(A(L,L))-EP)50,50,30	00250
30	IF(L-N)31,34,34	00260
31	LP1=L+1	00270
	DO 36 K=LP1,N	00280
	IF(A(K,L))32,36,32	00290
32	RATIO=A(K,L)/A(L,L)	00300
	DO 33 J=LP1,N	00310
33	A(K,J)=A(K,J)-RATIO*A(L,J)	00320
	DO 35 J=1,N	00330
35	X(K,J)=X(K,J)-RATIO*X(L,J)	00340
36	CONTINUE	00350
34	CONTINUE	00360
40	DO 43 I=1,N	00370
	II=N+1-I	00380
	DO 43 J=1,N	00390
	S=0.0	00400
	IF(II-N)41,43,43	00410
41	IIP1=II+1	00420
	DO 42 K=IIP1,N	00430
42	S=S+A(II,K)*X(K,J)	00440
43	X(II,J)=(X(II,J)-S)/A(II,II)	00450
	KER=1	00460
	RETURN	00470
50	KER=2	00480
	END	00490
	END	

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