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Generating a Variable from the Tail of the Normal Distribution

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GENERATING A VARIABLE FROM THE TAIL OF THE NORMAL DISTRIBUTION

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Very fast procedures for generating normal variables may be based on representing the density function as a mixture, with the dominant terms chosen so that they lead to fast procedures in the computer. See [1] - [3]. There is always the problem of handling the tail of the normal distribution. Variables from the tail are needed so infrequently that convenience is a more important consideration than speed in searching for methods. The following method is very convenient - easy to understand and easy to program. At the same time, it is reasonably fast, requiring a logarithm and a square root.

The idea is to transform the tail of the normal distribution to the unit interval and then use the rejection technique. It goes as follows:

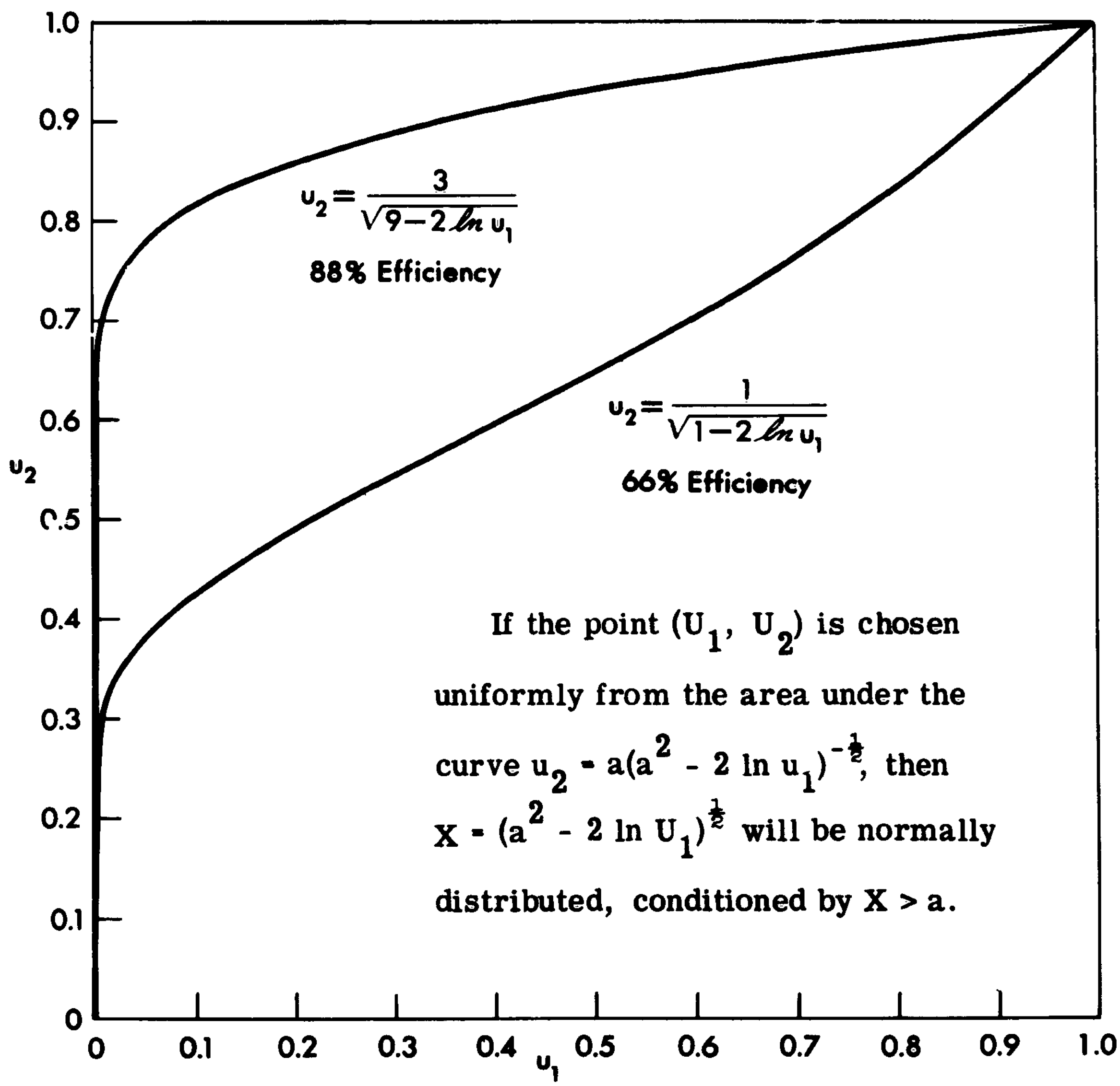
To generate a standard normal variable X, _conditioned by_ $X > a$, i.e., _with density_ $ce^{-.5x^2}$, $x > a$, _generate pairs of uniform over_ (0,1) _random variables_ U_1, U_2 _until_

$$(1) \qquad U_2 < a(a^2 - 2 \ln U_1)^{-\frac{1}{2}}$$

then put $X = (a^2 - 2 \ln U_1)^{\frac{1}{2}}$.

The density of U_1, given condition (1), is a multiple of $(a^2 - 2 \ln u_1)^{-\frac{1}{2}}$, $0 < u_1 < 1$, hence the density of $X = (a^2 - 2 \ln U_1)^{\frac{1}{2}}$ is a multiple of $e^{-.5x^2}$, $a < x < \infty$.

Graphs of $u_2 = a(a^2-2 \ln u_1)^{-\frac{1}{2}}$ for $a = 1$ and 3 are plotted in this figure. When $a = 3$, the probability of event (1) is .88, so that the efficiency of the rejection technique is satisfactorily high.



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[1] G. Marsaglia, "Expressing a Random Variable in Terms of Uniform Random Variables", Annals Math. Stat. 32, (1961), pp. 894-898.

[2] G. Marsaglia "Random Variables and Computers", Transactions of the Third Prague Conference, to appear.

[3] G. Marsaglia, M. D. MacLaren, T. A. Bray, "A Fast Procedure for Generating Normal Variables", Comm. of the ACM, to appear.