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PACIFIC MISSILE RANGE

POINT MUGU, CALIFORNIA

Technical Memorandum No. PMR-TM-63-15

ERROR PROPAGATION IN SHORT-BASE-LINE, THREE-DIMENSIONAL UNDERWATER TRACKING SYSTEM

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SUMMARY

This technical memorandum reports the results of a theoretical analysis of errors in a short-base-line, underwater tracking system. The analysis considers only the random errors of the computed positions (as described in cartesian coordinates) which result from errors existing in the measurements of the slant distances from which these positions are determined. It is assumed that the original errors are normally distributed with some degree of statistical correlation. The precision of the system is shown to be a very strong function of this statistical correlation.

INTRODUCTION

The two special cases considered in this technical memorandum were included in a number of similar, unpublished, in-house studies made at the Pacific Missile Range as background for a proposal for an underwater tracking range in the PMR for Fleet training. These in-house studies indicated that almost any system of tracking that uses long base lines would be superior to a short-base-line system, such as that installed at Dabob Bay, unless the slant-range measurements made by the short-base-line system showed almost perfect, positive statistical correlations among the errors of measurement. It was assumed that such correlation could not be expected for a system operating in the open sea, and therefore, a long-base-line system with hydrophones placed in a rectangular pattern was chosen for the proposal. This proposal is presented in PMR Technical Memorandum No. PMR-TM-61-11 of 3 July 1961, entitled, "Proposed Underwater Tracking Range for Fleet Training Exercises in the PMR."

Subsequently to the publishing of this proposal, a rigid, short-base-line array was activated at San Clemente Island for the Naval Ordnance Test Station, Pasadena, and a similar array was installed and tested at the Atlantic Undersea Test and Evaluation Center (AUTEC). It is usually easier and cheaper to settle scientific questions by analyzing pertinent data, if such data exist, than it is to make new experiments specifically designed for the problem. It is very likely that the data taken to evaluate the two installations mentioned above can be analyzed to estimate the extent of statistical correlations of the errors in the distances measured during the evaluations. This memorandum may prove useful in such a study.

TRACKING SYSTEM

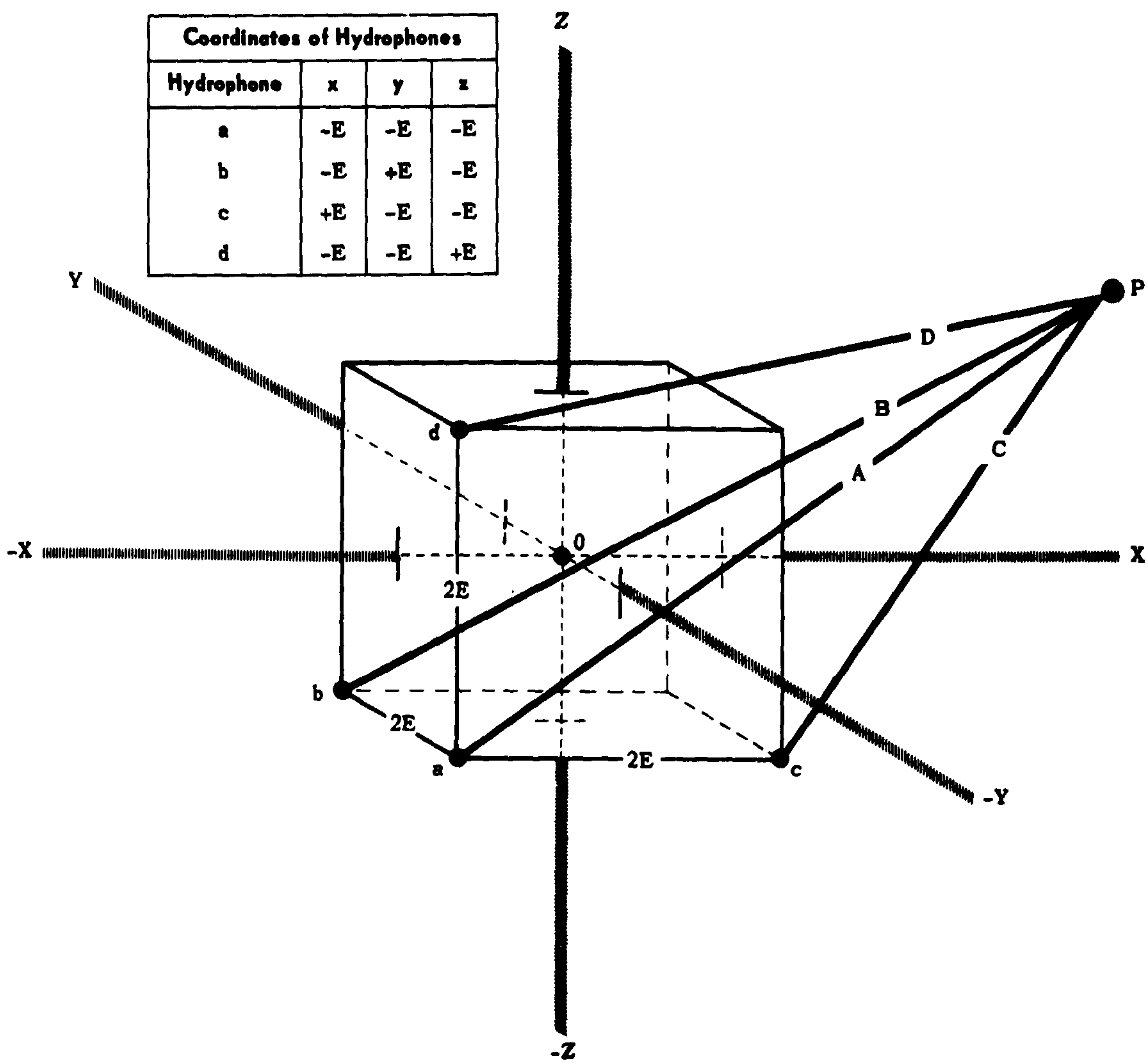
The tracking system under consideration is known as a "spherical system" because it is instrumented to measure, by sonic means, the distances from the target of tracking to several receivers (hydrophones). Each measured distance defines a spherical surface on which the target is located, and the point-position of the target is determined from the intersections of these surfaces.

The particular arrangement of hydrophones in the system under study is exemplified by the unit hydrophone cluster of the Dabob Bay Range. In this arrangement, four hydrophones are rigidly mounted at the ends of three adjacent edges of an imaginary cube, with three hydrophones in a horizontal plane and the fourth vertically above this plane (see figure 1). In this study, no account is taken of the biases resulting from deviations of the hydrophone array from an exactly vertical and horizontal setting, it being assumed that these biases will yield to careful calibration. Motions of the hydrophone array resulting from bottom currents are also neglected.

The root-mean-square errors in measuring the slant distances are assumed to be small in comparison with the slant distances measured. The magnitude of each of these errors is a function of the design of the elements of the tracking system, the background noise of the environment, the slant distance measured, and many other factors. No *a priori* estimate of these errors can be expected to have much validity. Such estimates are useful in the selecting and designing of a system, but they will have to be superseded, finally, by actual test results which can properly be stated in meaningful, statistical terms. But whatever these basic errors may turn out to be, they are inexorably subject to the geometric distortions discussed below.

ANALYSIS

If the origin of a set of cartesian coordinates is placed at the center of the hydrophone-array cube and the axes are made parallel to the edges of the cube, the slant distances from a point, P, in space to the four hydrophones of the array are transformed into rectangular coordinates by the following set of symmetrical equations (see figure 1):



Coordinates of Hydrophones			
Hydrophone	x	y	z
a	-E	-E	-E
b	-E	+E	-E
c	+E	-E	-E
d	-E	-E	+E

Figure 1. Geometry of the Problem.

$$x = \frac{A^2 - C^2}{4E} \tag{1}$$

$$y = \frac{A^2 - B^2}{4E} \tag{2}$$

$$z = \frac{A^2 - D^2}{4E} \tag{3}$$

In these equations, A, B, C, and D are the slant distances or "ranges" from point P to hydrophones a, b, c, and d, respectively, and E is a distance equal to one-half the edge of the hydrophone-array cube. Each slant range is determined, in practice, by measuring the time interval between the emission of a sonic pulse at point P, and its reception at a hydrophone. These time intervals are multiplied by the mean velocity of sound to produce the slant ranges appearing in the equations.

There will be errors in the slant ranges so measured, and these errors will, in turn, result in errors in the positional coordinates as calculated from the equations above. It can be anticipated that the root-mean-square (rms) errors (standard deviations) in measuring the slant ranges A, B, C, and D will not be always the same, but there is no reason to expect them to be markedly different. By assuming them to be equal in every case, one can write a set of simple equations for any point P which will relate the errors in A, B, C, and D to the resulting errors in x, y, and z. If the rms error in the measurement of any slant range R is denoted by σ_R, the rms error in any one of the coordinates can be expressed as σ_R multiplied by a factor which will hereafter be called the geometric error multiplier (GEM).

Assume, first, as an extreme case, that the rms errors in measurement of A, B, C, and D are all equal to σ_R and that they are entirely uncorrelated statistically.

From equation (1), $\frac{\partial x}{\partial A} = \frac{A}{2E}$ and $\frac{\partial x}{\partial C} = \frac{-C}{2E}$. If σ_R is small as compared with the range to which it applies,

$$\sigma_x = \sigma_R \sqrt{\left(\frac{A}{2E}\right)^2 + \left(\frac{-C}{2E}\right)^2},$$

$$\sigma_x = \sigma_R \sqrt{\frac{A^2 + C^2}{4E^2}}.$$

Similarly,

$$\sigma_y = \sigma_R \sqrt{\frac{A^2 + B^2}{4E^2}},$$

and

$$\sigma_z = \sigma_R \sqrt{\frac{A^2 + D^2}{4E^2}}$$

The factors expressed by the radicals in these equations are the GEM factors for the three coordinates. "Since the dimensions of the hydrophone array are small compared to the slant ranges, these GEM factors for the three coordinates are not much different from each other except when point P is in the immediate vicinity of the hydrophone array." There is a volume of uncertainty about point P. To characterize this volume completely under the assumption of a trivariate normal density of error, it is necessary to specify its six parameters, σ_x, σ_y, σ_z, σ_{xy}, σ_{xz}, and σ_{yz}. A much simpler measure of imprecision, accurate enough for this discussion, is the rms value of σ_x, σ_y, and σ_z. This measure of imprecision is defined as

$$\sigma_P = \sqrt{\frac{\sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2}{3}}.$$

This measure of imprecision has the desirable property of being invariant with the orientation of the axes. Substituting the values for σ_x, σ_y, and σ_z into this definition of σ_P gives:

$$\sigma_P = \frac{\sigma_R}{2E}\sqrt{A^2 + \frac{B^2 + C^2 + D^2}{3}}\,. \tag{4}$$

Since E is relatively small, each of A, B, C, and D is approximately equal to the slant range R. Replacing each of A, B, C, and D in equation (4) by its approximate value, R, reduces this equation to

$$\sigma_P \approx \frac{\sigma_R\, R}{\sqrt{2}\, E}\,. \tag{5}$$

The surfaces of equal accuracy in the region served by the hydrophone array are, from this approximate equation, concentric spherical shells, and the accuracy decreases directly in proportion to the slant range. For short base lines and long slant ranges, σ_P will be excessive if the range measurements are uncorrelated, and accurate tracking cannot be realized unless σ_R can be made extremely small.

Assume, as a second extreme case, that the range-measurement errors are *perfectly* correlated. In this extreme case,

$$\sigma_x = \sigma_R\left(\frac{A - C}{2E}\right),\ \sigma_y = \sigma_R\left(\frac{A - B}{2E}\right),\ \text{and}\ \sigma_z = \sigma_R\left(\frac{A - D}{2E}\right).$$

$$\sigma_P = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{3}}$$

now becomes, upon substitution,

$$\sigma_P = \frac{\sigma_R}{2E}\sqrt{\frac{(A - B)^2 + (A - C)^2 + (A - D)^2}{3}}\,. \tag{6}$$

From figure 1, it can be seen that in equation (6), the maximum value of any one of the three terms in the numerator under the radical is $(2E)^2$ so that the value for σ_P will everywhere be less than σ_R.

The short-base-line system clearly depends for its accuracy largely on the extent to which the errors in the basic range measurements are correlated.

Neither of these extreme assumptions will ever be realized in actual practice. There will always be some degree of statistical correlation between the errors of measurements of the slant ranges. From the great difference between the results stemming from the two extreme assumptions considered above, it is quite obvious that no valid conclusion regarding the possible accuracy of the short-base-line system can be made (1) without an analysis of the intermediate case, in which the statistical correlations of the errors are considered, and (2) without a sample of experimental data adequate for determining the level of correlation to be expected in practice under the oceanographic conditions anticipated.

If the errors of measurement of the pairs of ranges AB, AC, and AD are assumed to be correlated to some intermediate degree,

$$\sigma_x^2 = \left(\frac{\partial x}{\partial A}\,\sigma_A\right)^2 + \left(\frac{\partial x}{\partial C}\,\sigma_C\right)^2 + 2\,\frac{\partial x}{\partial A}\,\frac{\partial x}{\partial C}\,\sigma_{AC}\ .$$

As before, let

$$\sigma_A = \sigma_C = \sigma_R$$

and let

$$\rho_{AC} = \frac{\sigma_{AC}}{\sigma_A\,\sigma_C},$$

ρ_{AC} being the correlation coefficient for the errors of measurement of A and C, for example. Let

$$\sigma_A = \sigma_B = \sigma_C = \sigma_D = \sigma_R,$$

as before, and assume

$$\rho_{AB} = \rho_{AC} = \rho_{AD} = \rho.$$

Then,

$$\sigma_x^2 = \left(\frac{A}{2E}\,\sigma_R\right)^2 + \left(\frac{-C}{2E}\,\sigma_R\right)^2 + 2\left(\frac{A}{2E}\right)\left(\frac{-C}{2E}\right)\rho\,\sigma_R^2.$$

Similarly,

$$\sigma_y^2 = \left(\frac{A}{2E}\,\sigma_R\right)^2 + \left(\frac{-B}{2E}\,\sigma_R\right)^2 + 2\left(\frac{A}{2E}\right)\left(\frac{-B}{2E}\right)\rho\,\sigma_R^2$$

and

$$\sigma_z^2 = \left(\frac{A}{2E}\,\sigma_R\right)^2 + \left(\frac{-D}{2E}\,\sigma_R\right)^2 + 2\left(\frac{A}{2E}\right)\left(\frac{-D}{2E}\right)\rho\,\sigma_R^2.$$

Defining σ_P, as before, as $\sqrt{\dfrac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{3}}$,

$$\sigma_P = \frac{\sigma_R}{2E}\sqrt{A^2 + \frac{B^2 + C^2 + D^2}{3} - \frac{2\rho}{3}\,A\,(B + C + D)}\ . \qquad (7)$$

In the system under consideration, A ≈ B ≈ C ≈ D, and equation (7) can be redúced to the approximate equation

$$\sigma_P \approx \frac{\sigma_R}{E}\sqrt{\frac{x^2 + y^2 + z^2}{2}}\ (1 - \rho),$$

which is:

$$\sigma_P \approx \frac{\sigma_R}{\sqrt{2}}\ \frac{R}{E}\ \sqrt{1 - \rho}. \qquad (8)$$

Equation (8) is the same expression as that given above for the approximate value of σ_P without correlation [equation (5)], multiplied by the factor $\sqrt{1 - \rho}$. If ρ is made unity, equation (7) can be reduced to equation (6). If ρ is made unity, equation (8) reduces to $\sigma_P = 0$, which could be a correct value only if $\sigma_R = 0$, also. It is only near the value +1 for ρ that the approximate equation is seriously in error.

Table 1, by a numerical example, shows the sensitivity of the system under discussion to the degree of correlation of the measurement errors. This table shows the expected errors (in yards) in the positions of a tracked target in the three-dimensional underwater tracking range near San Clemente Island, in which E = 5 yards, at the position x = 1,200 yards and y = 400 yards and z = 300 yards, resulting from an assumed rms error of 0.1 yard in the measurement of all slant ranges and for various values of ρ, the coefficient of correlation for the slant-range measurement errors. The values shown in the table were computed from equation (7). The table shows also, for comparison, these values as computed from equation (8), the approximate equation.

Table 1. Calculated Values for Positioning Errors, in Yards, in San Clemente Underwater Tracking Range

ρ =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
σ_P (Exact)	18.5	17.6	16.5	15.4	14.3	13.1	11.7	10.1	8.2	1.76	0.25
σ_P (Approximate)	18.4	17.5	16.4	15.3	14.2	13.0	11.6	10.0	8.2	0.58	0.0

The assumption of an rms error of 0.1 yard in the range measurements at a slant range of 1,300 yards is considered reasonable or slightly optimistic. This degree of precision can probably be achieved only when background noise conditions permit a high signal-to-noise ratio. The values in the table apply equally well to the Dabob Bay Range if all values are read as feet instead of as yards. The corresponding precision of slant-range measurement assumed for the Dabob Bay Range would be 0.1 foot or 1.2 inches, rms.

There are several reasons for expecting a relatively high degree of correlation between and among the ranging errors of the short-base-line system. Errors due to lack of synchronism of the clocks will obviously be correlated, as will errors resulting from inaccuracies in the values used as the mean velocity of sound. On the other hand, the fact that the ocean consists of blobs of water of varying temperature and salinity in a random pattern of sizes and distributions, which at present are poorly understood, will result in multipath transmission, interference phenomena, velocity fluctuations, and other effects tending to degrade error correlations in the measured slant ranges.

It should be noted also that the transformation to rectangular coordinates of the measured ranges, in the short-base-line system, is greatly facilitated by an arrangement which makes it possible to determine each of the three rectangular coordinates from only two of the slant range coordinates. Therefore, there will be some measure of coordination among the three rectangular coordinates as found by the system.

In this system, four measurements are available for determining three coordinates. It would be possible, by a more sophisticated analysis, to arrive at a data-reduction procedure leading to greater precision than will the procedure which is indicated by the analysis above.

CONCLUSION

It is concluded from this study that knowledge of the magnitudes of the rms errors of slant-range measurements is not sufficient for the prediction of accuracies to be expected in a short-base-line, underwater tracking system. The levels of statistical correlation among the errors in the four simultaneous slant-range measurements must also be known, because the accuracy of the system is strongly dependent upon this correlation. If there is a high degree of positive correlation, the tracking accuracy of the system is excellent, but this accuracy is seriously degraded as the correlation factor decreases. It would not be advisable to install a short-base-line system in an oceanic area for which this correlation factor has not been determined.

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