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AN ECONOMETRIC MODEL OF METROPOLITAN EMPLOYMENT AND POPULATION GROWTH

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PREFACE

In August 1960 the Ford Foundation made a grant to The RAND Corporation for an exploratory study of urban transportation. An objective of this study was the development of a land-use model which could be employed in forecasting the changing land-use patterns of an idealized metropolitan area. Work has been underway on this phase of the study since 1961, first with the support of the Ford Foundation grant, and subsequently as part of The RAND Corporation's program of self-sponsored research.

The present RAND Memorandum embodies the results of this latest research and gives a full description of the land-use model in its latest and most complete form. The Memorandum then employs the model to forecast employment and population growth in 36 metropolitan areas, and discusses the implications of the resulting developmental patterns on the spatial configuration of urban areas.

Aspects of an earlier version of the model were reported on in two studies by J. H. Niedercorn and J. F. Kain: An Econometric Model of Metropolitan Development (P-2663, December 1962), and Suburbanization of Employment and Population, 1948-1975 (P-2641, January 1963). Land-use data obtained through questionnaires sent to city planning commissions of the largest cities of the United States were analyzed and reported on by J. H. Niedercorn and E. F. R. Hearle in Recent Land-Use Trends in Forty-Eight Large American Cities (RM-3664-FF, June 1963).

SUMMARY

This Memorandum presents an econometric model describing the growth of metropolitan employment and population. The analysis carried out for three geographical areas, the entire Standard Metropolitan Statistical Area (SMSA), the central city, and the metropolitan ring (the part of the SMSA lying outside the central city). A mathematical model was constructed to describe changes taking place in each of the three geographical areas in manufacturing, wholesaling, retailing, and selected service trades; in finance, insurance, and real estate; in transportation, communications, and public utilities; and in government administrative employment, and resident population. The model's parameters were estimated using data taken from a sample of 41 of the largest SMSAs. The model was then used to forecast ten-year changes in employment and population for 36 of the nation's most important metropolitan areas.

Given the limitations of the data and the uncertainties inherent in econometric models of this kind, and assuming that future urban growth is like that in the recent past, it appears that population and employment growth will be rapid in the majority of these metropolitan areas, and especially rapid in the metropolitan rings. Population and all types of employment will increase substantially in the ring. On the average, the forecasts indicate that less than 15 per cent of total metropolitan employment growth and less than 5 per cent of the total population growth will locate in the central city. The main sources of central city growth will be business services, finance, insurance, real estate, and government administration. Wholesaling and retailing will remain fairly steady in the central city but manufacturing should decline substantially.

The model covers approximately 72 per cent of total metropolitan employment. The only omitted categories are construction, professional services, private household services, and central offices of manufacturing firms. As both professional services and central

offices are highly concentrated in core areas it appears likely that increases in these categories should offset possible declines in construction and private household services. If this is true, the model's forecasts of total central city employment change should not be significantly affected.

The model predicts that central city employment will grow considerably faster than the employed resident labor force. If this conclusion is accepted, it follows that many of the new jobs will be filled by people living outside the central city. There would therefore be increased commuter traffic between central city and metropolitan ring (and more rush hour traffic congestion) unless the capacity to transport these persons is correspondingly enlarged.

ACKNOWLEDGMENTS

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I. INTRODUCTION

During recent years social scientists, among others, have paid increasing attention to the phenomenon of urban growth. Because of its complexity many researchers have come to the conclusion that only through the use of mathematical models can the maze of interrelated factors that determine the rates and patterns of growth in population and employment be identified. At present, several studies are underway that include attempts to simulate the growth of a metropolitan community.* All of these models are complex, embodying many equations and fine areal disaggregation.

The construction of a simpler aggregative model explaining growth in the larger metropolitan areas of the United States was not attempted until recently when The RAND Corporation undertook such a study as a part of its Ford Foundation sponsored Urban Transportation Project. A dynamic cross-sectional econometric model of metropolitan growth has been developed that explains changes in population and manufacturing employment in the Standard Metropolitan Statistical Area (SMSA) and then partitions this growth between central city and metropolitan ring (defined as the difference between SMSA and central city). Changes in the following types of employment have also been explained for both central city and metropolitan ring: wholesaling, retailing, and service trades; financial, insurance, and real estate; transportation, communications, and public utilities; and government administration. Comparisons of the projected growth of the labor force living in the central city and the employment available there yield interesting inferences about future commuter traffic between metropolitan ring and central city.

The model is used to forecast changes in population and employment taking place in 36 large metropolitan areas over the ten-year

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*The work of the Penn-Jersey Transportation Study, The Traffic Research Corporation, The Pittsburgh Regional Study, and Ira S. Lowry of The RAND Corporation, all deserve mention.

period 1958-1968. The magnitude of these changes and their implications for the future of the nation's urban areas are then discussed.

II. METROPOLITAN DEVELOPMENT

Although there are many partial theories of urban development, no satisfactory general theory has yet been formulated and tested. The model presented in this Memorandum embodies an attempt to synthesize several partial theories and come up with a unified picture of metropolitan growth. The various theories are formulated mathematically when discussed, and then the entire model is assembled, estimated, and tested. Although it would be presumptuous to call this model a general theory, the equations taken as a group embody a first attempt to specify the interrelationships between population and employment growth, and locational behavior in the automotive age.

The model is based on four assumptions about metropolitan growth. They are:

(1) Manufacturing employment and population growth interact in the SMSA in a mutually reinforcing manner.

(2) Changes in SMSA manufacturing employment and population can be allocated between central city and metropolitan ring if the amount of vacant land in the city and the size of the city are known.

(3) Growth of employment in central city and metropolitan ring is determined by population changes. Employment categories included in this assumption are: wholesaling, retailing, services; finance, insurance, and real estate; transportation, communications, and public utilities; and government administration.

(4) Given changes in employed central city labor force and total employment, the increase or decrease in people traveling to work from ring to central city can be ascertained.

SMSA GROWTH IN MANUFACTURING EMPLOYMENT AND POPULATION

Increases in "basic" employment are usually considered to be the driving force behind metropolitan growth. Basic employment is conceived as export oriented employment, in other words, as employment providing goods and services for nonlocal markets. This category ordinarily includes farmers, miners, federal and state government employees, people providing services for transients, and workers in manufacturing concerns whose sales are chiefly outside the SMSA of origin. Growth of basic employment attracts job seekers to settle in the area, thereby causing population growth over and above the natural increase from any excess of births over deaths.

Although this may be sufficient explanation of metropolitan growth for the late 19th century, it is clearly inadequate for the middle of the 20th century. The continuing industrialization and mechanization of the American economy have made manufacturing increasingly market oriented rather than natural resource oriented. Value added by labor and capital in the process of fabrication is increasing much more rapidly than total inputs of raw materials.* Consequently, raw material costs are becoming a smaller fraction of total costs, and the expense of transporting raw materials is no longer so burdensome. Since the different stages of fabrication are often undertaken in different plants, businessmen have an incentive to locate near other plants rather than raw material sources.**

As a consequence, manufacturers can often minimize total transportation and marketing costs by locating in the large population centers, near other manufacturers and near large consumer markets. Markets thus become the strongest determinant of industrial location. Of course, the supply of skilled labor available in the popula[illegible]on

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*For a discussion of this hypothesis see Alexander Gerschenkron, A Dollar Index of Soviet Machinery Output, 1927-28 to 1937, The RAND Corporation, R-197, April 6, 1951, pp. 47-48.

**See Benjamin Chinitz and Raymond Vernon, "Changing Forces in Industrial Location," Harvard Business Review, Vol. XXXVIII (January-February 1960).

centers is another important factor in their favor. Large SMSAs are therefore likely to become highly diversified manufacturing centers, and the ratio of employment in this activity to population in the future will be distributed more evenly across the major regions of the nation. These ideas might be formulated in the following manner. If an SMSA has less than the equilibrium amount of manufacturing employment in relation to its population, it will gain in the manufacturing sector; if it has more it will decline because markets outside the region and the local SMSA are gradually lost as other areas develop a diversified manufacturing capability.

The growth of SMSA population has two sources: natural increase (the excess of births over deaths) and net migration. Metropolitan areas have been net recipients of migration from foreign countries as well as small towns and the countryside for well over a hundred years. There is also a substantial interchange among metropolitan areas. A large part of the net migration is attributable to increases in basic employment, and the resulting high levels of wages and salaries. The remainder, consisting in large part of retired people, depends upon amenities such as a pleasant climate. In the equations that follow, manufacturing employment has been used as a proxy for basic employment. This has been done because data corresponding exactly to the concept of basic employment are not available. The first equation states that the equilibrium amount of manufacturing employment at any point of time, M^s_{eq}, is a constant fraction α_{11} of total SMSA population P^s. This assumption will later be modified to allow α_{11} to vary over time since the ratio of manufacturing employment to population is falling in the United States.

Equation (2) says that the annual percentage change in SMSA manufacturing employment depends on the difference between the equilibrium and actual levels of manufacturing employment, and itself lagged one year. The latter term is included because deviations from the equilibrium growth path have a strong effect in the short run; the size of the coefficient α_{22} measures the strength of this effect.

Equation (3) shows that the annual percentage change in SMSA population depends on the percentage change in SMSA manufacturing employment and a constant term α_{32} which measures the percentage rate of growth common to the larger SMSAs. The constant term may be interpreted as the sum of two components, one being the rate of natural increase and the other the average rate of migration from non-SMSA areas.

$$M^s_{eq} = \alpha_{11} P^s , \tag{1}$$

$$\Delta M^s/M^s = \alpha_{21} (M^s_{eq} - M^s)/M^s + \alpha_{22} (\Delta M^s/M^s)_{t-1}, \tag{2}$$

$$\Delta P^s/P^s = \alpha_{31} (\Delta M^s/M^s) + \alpha_{32} . \tag{3}$$

These three equations constitute the SMSA growth model. They form a system of simultaneous equations that is nonlinear with respect to the variables, although linear with respect to the parameters. The latter fact permits estimation of the parameters by the usual statistical techniques.

ALLOCATION OF CHANGES IN SMSA VARIABLES TO CENTRAL CITY AND RING

It is generally assumed that manufacturing firms choose central city locations because of proximity to complementary plants, labor force, and commodity transport terminals. This drawing power of the city is counteracted by the lack of usable vacant land. Since the most efficient plant layout often requires assembly line production, single story plants are desirable but sites large enough to accommodate them are difficult to find in the city. Parking lot requirements further increase the need for land. Therefore, manufacturing employment in the central city is likely to increase only if significant amounts of vacant land exist or can be created by redevelopment at a reasonable cost.

Since most of the larger cities contain very little usable vacant land, by far the greater part of new industrial development is likely to take place in the suburbs. In addition, there will probably be some net movement from central city to suburbs as firms outgrow their old facilities and want to build newer and larger plants. Although many new firms are born in the city, these are not likely to grow fast enough to fully replace those that leave because skilled labor is also moving to the suburbs.

Population can be assumed to behave in a manner similar to manufacturing. The city offers many amenities such as a short journey to work, a wide choice of consumer's goods and services, and proximity to educational and cultural institutions. Again, the absorptive power of the city is limited by the amount of usable vacant land. If the city has considerable amounts of such land, most of the new residential development will take place within the city limits. On the other hand, if little or no vacant land exists, the increase in SMSA population must settle largely in the suburbs.

A further powerful factor that influences residential location is space preference.* Many people prefer the roomier houses, privacy, and lawns of the suburbs to the apartments and row houses of the city. The private automobile has given these people the opportunity to enjoy suburban living and still retain close contact with their jobs and places of recreation in the city. One might expect that the denser the city the greater will be the incentive to move to the suburbs because of the overcrowding associated with high densities.

Equations (4) and (5) explain annual changes in central city population and manufacturing employment.

$$\Delta M^c = \alpha_{41} (V/L) (M^c/M^s) \Delta M_g^s + \alpha_{42} (M^c/M^s) \Delta M_d^s + \alpha_{43} P^a + \alpha_{44} , \tag{4}$$

$$\text{when } \Delta M_g^s > 0, \ \Delta M_d^s = 0$$

$$\text{when } \Delta M_d^s < 0, \ \Delta M_g^s = 0$$

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*For a discussion of this concept see John F. Kain, A Multiple Equation Model of Household Locational and Tripmaking Behavior, The RAND Corporation, RM-3086-FF, April 1962.

$$\Delta P^c = \alpha_{51} (V/L) (P^c/P^s) \Delta P^s + \alpha_{52} (P^c/L) P^c + \alpha_{53} P^a + \alpha_{54} \quad . \qquad (5)$$

The first term on the right side of Equation (4) is used when SMSA manufacturing employment is increasing, and the second term then equals zero. If SMSA manufacturing is decreasing, the second term is used and the first equals zero. In the former case, growth of central city manufacturing employment is limited by the amount of vacant land, but this factor need not be taken into account if SMSA manufacturing employment is decreasing because employment losses do not require additional land. In both cases, however, the change in SMSA manufacturing employment is weighted by the ratio of central city to SMSA employment. This indicates that increases or decreases in the SMSA will be allocated between central city and metropolitan ring roughly in proportion to the existing relative employment levels of these areas. Of course, in the case of increases the vacant land factor modifies this allocative mechanism.

The term P^a denotes population annexed by the central city from the metropolitan ring, and has been included in Equation (4) to account for central city employment growth attributable to annexations. Manufacturing employment in annexed areas is assumed to be roughly proportional to population residing in these areas. This correction was necessary because the Bureau of the Census reports employment data on a legal area rather than a constant area basis.

The first term on the right side of Equation (5) is analogous to the corresponding one in Equation (4). Since all SMSAs are increasing in population, only one term of this general type is needed. The term $(P^c/L) P^c$ denotes gross population density times central city population. This term is included because the exodus from the city depends on density, and the latter must be weighted by population size. Again, the annexed population should be included as an independent variable.

Once changes in central city manufacturing and population are determined, changes in the metropolitan ring can be found by subtracting central city changes from SMSA changes.

$$\Delta M^r = \Delta M^s - \Delta M^c \quad , \tag{6}$$

$$\Delta P^r = \Delta P^s - \Delta P^c \quad . \tag{7}$$

DEMAND OR POPULATION ORIENTED EMPLOYMENT

Locations assumed to be demand or population oriented are: wholesaling, retailing, service; financial, insurance, and real estate; transportation, communications, and public utilities; and government administrative employment.* In other words, these kinds of employment are assumed to follow population growth. The model is quite simple. Changes in both central city and metropolitan ring populations are included as independent variables when explaining central city employment changes because people who live in the ring often travel to the city to obtain highly specialized kinds of goods and services. This will be true of governmental services as well as those supplied by private business. Consumers' demands for things such as food, drugs, and soft goods can usually be satisfied locally. For this reason, metropolitan ring employment, which is less specialized than central city employment, is assumed to depend only on changes in ring population.

$$\Delta E_i^c = \alpha_{61} \Delta P_k^c + \alpha_{62} \Delta P^r + \alpha_{63} P^a + \alpha_{64} \quad , \tag{8}$$

$$\Delta E_i^r = \alpha_{71} \Delta P_k^r + \alpha_{72} P^a + \alpha_{73} \quad . \tag{9}$$

Again, the term P^a has been included in both equations to account for annexations. These two equations illustrate the behavior

*The locational behavior of construction, professional services, private household services, and central offices of manufacturing firms has not been discussed because accurate data for them were not available. Of course, only a portion of state and federal government employment can be regarded as population oriented. However, since there is no way of separating population oriented and basic components, the entire category is treated as population oriented.

of changes in demand oriented employment, denoted by ΔE_1.* The subscript k has been added to ΔP^c and ΔP^r in two places to denote changes in constant-area population.** Variables that refer to legal areas have no subscripts.

The equation for changes in central city transportation, communications, and public utilities employment also includes as an independent variable the central city level of this employment. It was included because transportation is a declining industry and part of its annual employment loss can reasonably be assumed proportional to the existing level of service.

CHANGE IN COMMUTER TRAVEL

Once the changes in central city population and employment are known, the change in the number of commuters traveling to work from metropolitan ring to central city can be determined. The total number of commuters depends on the difference between the number of jobs located in the central city and the number of employed workers living there. Therefore, the change in commutation should depend on the change in central city jobs minus the change in central city working population. This can be represented by Equation (10) where ΔJ is the change in number of commuters traveling to work and α_{81} is the labor force participation ratio of the population settling in or emigrating from the central city.

$$\Delta J = \Sigma \Delta E_i^c + \Delta M^c - \alpha_{81} \Delta P^c \quad . \tag{10}$$

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*One exception should be noted here. Changes in wholesaling employment have been assumed to depend on changes in retailing, and thus indirectly, rather than directly, on changes in population.

**Equations (8) and (9) are simplified versions of more complicated equations derived in an earlier study. They originally made allowances for changes in productivity, but have been simplified because changes in productivity do not seem to have a significant effect in highly aggregated sectors. For derivation, see John H. Niedercorn and John F. Kain, Changes in the Location of Food and General Merchandise Store Employment Within Metropolitan Areas, 1948-1958, The RAND Corporation, P-2614, August 1962.

Of course, this equation cannot be expected to give an accurate picture of changes in demand for urban transportation in the SMSA, but it does give a rough idea of what will happen to the total number of people commuting to work between central city and metropolitan ring.* Equation (10) completes the basic structure of the model.

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*More specifically, it neglects the effects of reverse commuting. However, because reverse commuting is thought to be increasing (owing to the rising proportion of Negroes in the central city population), Equation (10) will underestimate the increase in the number of workers traveling from suburbs to central city in the morning rather than overstate it when central city jobs are growing faster than the central city labor force.

III. THE DATA AND PARAMETER ESTIMATION

DATA

The data used in estimating the parameters of the model are taken from a sample of 41 SMSAs.* Several different sources were used in obtaining these data. Changes in manufacturing, wholesaling, retailing, and selected service trades were calculated from Census of Manufactures and Census of Business data for the years 1954 and 1958. Changes in population are estimates for the period 1954-1958 constructed by interpolating between the census years 1950 and 1960. Population annexation data are estimates calculated from figures appearing in various issues of the Municipal Year Book. The data for transportation, communications, and public utilities, finance, insurance, and real estate, and government administration were obtained from the 1950 and 1960 Censuses of Population. Changes over the ten-year period were then calculated.

The latter data caused some difficulty because the 1950 employment figures are listed by place of residence rather than place of

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*These include Akron, Baltimore, Boston, Buffalo, Chicago, Cincinnati, Cleveland, Columbus (Ohio), Dallas, Dayton, Denver, Detroit, Ft. Worth, Houston, Kansas City, Los Angeles-Long Beach, Louisville, Memphis, Miami, Milwaukee, Minneapolis-St. Paul, New Orleans, Newark, Norfolk-Portsmouth, Oklahoma City, Philadelphia, Phoenix, Pittsburgh, Portland (Oregon), Rochester, St. Louis, San Antonio, San Diego, San Francisco-Oakland, Seattle, Albany-Schenectady-Troy, Birmingham, Hartford, Providence-Pawtucket, Syracuse, and Youngstown-Warren. Where more than one central city exists in an SMSA they are added together to form a consolidated central city for the statistical analysis. The Boston SMSA has been redefined to include all of Essex, Middlesex, Norfolk, and Suffolk Counties, and the Providence-Pawtucket SMSA has also been redefined as the sum of Bristol, Kent, and Providence Counties. The Hartford SMSA is redefined as all of Hartford County. The New York SMSA has been excluded from this study because of its size and other special characteristics. The Washington, D.C. area has also been omitted owing to its atypical pattern of employment. Where possible, data for all SMSAs have been made to correspond to 1958 areal definitions. This was not always possible in the cases of finance, insurance and real estate, transportation, communications, and public utilities, and government administration.

work. A plausible assumption was then required to reduce the 1950 data to a place of work basis. This was done by assuming that for each type of employment, the ratio of employment to resident population in the ring increased or decreased by the same percentage as the ratio changed in the economy as a whole over the ten-year period 1950-1960.* This assumption did not prove tenable for government administrative employment so the data were recalculated under the assumption that the ratio of employment to resident population in the ring remained constant.**

Figures for total land area of the central city and vacant land were obtained from a questionnaire sent by The RAND Corporation to various city planning commissions in the spring of 1962.***

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*The ratio increased 24.2 per cent for finance, insurance, and real estate, 19.1 per cent for government administration, and decreased 5.0 per cent for transportation, communications, and public utilities.

**Even this latter assumption gave odd results. It attributed decreases in government administrative employment to 11 central cities including Albany-Schenectady-Troy, Boston, Buffalo, Chicago, Dayton, Louisville, Newark, Phoenix, San Diego, San Francisco-Oakland, and Youngstown-Warren. These decreases seemed to contradict data collected by Senator Byrd's Joint Committee on Nonessential Federal Expenditures that show the number of Federal Government employees working in central counties (counties containing central cities) generally increased substantially between 1950 and 1960. Combined with the well-known effects of Parkinson's Law and fragmentary evidence of government office building construction in central cities, Byrd's data led to the tentative conclusion that employment in government administration probably did not decline in central cities during the period 1950-1960. Consequently, the above mentioned 11 SMSAs were dropped from the sample used in estimating the government employment equations. Government administration includes only employees involved in administrative tasks. Neither public school personnel nor workers in government-owned public utilities are included. See U.S. Congress, Federal Civilian Employment 1950 (With Tables Showing Employment by States and Localities), and Federal Civilian Employment by County (1960), Reports of the Joint Congressional Committee on Reduction of Nonessential Federal Expenditures, Government Printing Office, Washington, D.C., 1951 and 1961.

***The data from these questionnaires are now available in published form. See John H. Niedercorn and Edward F. R. Hearle, Recent Land Use Trends in Forty-Eight Large American Cities, The RAND Corporation, RM-3664-FF, June 1963.

ESTIMATION

The entire model is recursive in Wold's terminology (rather than simultaneous) because each dependent variable is a function only of predetermined variables and dependent variables determined by earlier equations.* In other words, causality within each equation is unidirectional throughout the model. This considerably simplifies the estimation problem.

Ideally, it would be appropriate to estimate the parameters for the entire model by one of the latest econometric techniques -- limited information maximum likelihood, or two stage least squares.** However, this is not possible for two reasons. First, the non-linearities in Equations (2), (3), (4), and (5) render the simultaneous estimation of the entire model impossible using standard available techniques. Second, the differences in time periods over which the variables are measured necessitates separate estimation of the equations for (1) transportation, communications, and public utilities, (2) government administration, and (3) finance, insurance, and real estate.

As a consequence of these problems, the model has been estimated in three separate blocks. The SMSA equations form a small system that is independent of the equations that follow it. The parameter α_{11} was set equal to the mean value of the 1954 quotient of M^s and P^s for the entire sample of 41 SMSAs. The other parameters were estimated by the limited information maximum likelihood method. The central city equations for population, manufacturing, wholesaling, retailing, and selected service trades, and the metropolitan ring equations for wholesaling, retailing, and selected services have also

*See Herman Wold in association with Lars Jureen, Demand Analysis, John Wiley and Sons, Inc., New York, 1953.

**For a discussion of these methods see Henri Theil, Economic Forecasts and Policy, North-Holland Publishing Co., Amsterdam, Netherlands, 1958.

been estimated as a separate system by the method of limited information. Finally, the equations for transportation, communication, and public utilities, government administration, and finance, insurance, and real estate for both central city and ring have been estimated by the classical method of single equation least squares, and the labor force participation ratio, α_{81}, has been set equal to its 1960 mean value for the central cities of the sample. This parameter refers to employed labor force only. The unemployed part of the labor force is not taken into account.

IV. THE ESTIMATED MODEL AND ITS INTERPRETATION

Employment change variables not heretofore introduced are denoted as follows: retailing ΔR, wholesaling ΔW, selected services ΔS, transportation, communications, and public utilities, ΔT, government administration ΔG, and finance, insurance, and real estate ΔF. A superscript defines the area to which the variable refers and the subscript tells whether the variable refers to paid employees, e , or proprietors, p. The absence of a subscript means that employment was not broken down into paid employees and proprietors. The variable ΔJ denotes the change in the number of commuters traveling to work between metropolitan ring and central city that can be expected as a result of population and employment shifts.

All population and employment variables and the commutation variable have been measured in thousands. Land areas have been measured in square miles. Where necessary, the parameters of the equations that were estimated over intervals of four to ten years have been reduced to an annual basis by dividing by four or ten to facilitate comparability. In the cases of retailing, wholesaling, and selected service trades, separate equations have been estimated for proprietors and paid employment. Data for the 1947-1954 period were used for the lagged values of $\Delta M^s/M^s$. The estimated model follows:

$$M^s_{eq} = .155\ P^s \qquad (11)$$

$$\Delta M^s/M^s = \underset{(.005)}{.018}\ (M^s_{eq} - M^s)/M^s + \underset{(.121)}{.869}\ (\Delta M^s/M^s)_{t-1} \qquad (12)$$

$$\Delta P^s/P^s = \underset{(.045)}{.386}\ (\Delta M^s/M^s) + \underset{(.007)}{.025} \qquad (13)$$

$$\Delta M^c = \underset{(.496)}{1.881}\ (V/L)(M^c/M^s)\ \Delta M^s_g + \underset{(.110)}{1.332}\ (M^c/M^s)\ \Delta M^s_d$$

$$+ \underset{(.041)}{.050}\ P^a - \underset{(.443)}{.666} \qquad (14)$$

$$\Delta P^c = \underset{(.089)}{1.277}\,(V/L)(P^c/P^s)\,\Delta P^s - \underset{(0.00006)}{.00033}\,(P^c/L)(P^c) + \underset{(.080)}{.882}\,P^a - \underset{(.895)}{2.065} \qquad (15)$$

$$\Delta R_p^c = \underset{(.003)}{.009}\,\Delta P_k^c + \underset{(.003)}{.010}\,P^a - \underset{(.029)}{.115} \qquad (16)$$

$$\Delta R_e^c = \underset{(.013)}{.079}\,\Delta P_k^c + \underset{(.015)}{.007}\,P^a - \underset{(.149)}{.097} \qquad (17)$$

$$\text{[illegible]}^c = \underset{(.006)}{.029}\,\Delta R_e^c - \underset{(.0005)}{.00006}\,P^a + \underset{(.005)}{.016} \qquad (18)$$

$$\Delta W_e^c = \underset{(.071)}{.306}\,\Delta R_e^c + \underset{(.007)}{.023}\,P^a + \underset{(.072)}{.097} \qquad (19)$$

$$\Delta S_p^c = \underset{(.001)}{.009}\,\Delta P_k^c + \underset{(.0004)}{.005}\,\Delta P^r + \underset{(.001)}{.009}\,P^a + \underset{(.019)}{.007} \qquad (20)$$

$$\Delta S_e^c = \underset{(.010)}{.019}\,\Delta P_k^c + \underset{(.003)}{.026}\,\Delta P^r + \underset{(.011)}{.028}\,P^a + \underset{(.141)}{.086} \qquad (21)$$

$$\Delta T^c = \underset{(.003)}{-.017}\,T^c + \underset{(.009)}{.046}\,\Delta P_k^c + \underset{(.017)}{.016}\,P^a + \underset{(.334)}{.132} \qquad (22)$$

$$\Delta G^c = \underset{(.0052)}{.0020}\,\Delta P_k^c + \underset{(.0016)}{.0032}\,\Delta P^r + \underset{(.0080)}{.0134}\,P^a + \underset{(.387)}{.011} \qquad (23)$$

$$\Delta F^c = \underset{(.002)}{.017}\,\Delta P_k^c + \underset{(.001)}{.006}\,\Delta P^r + \underset{(.004)}{.018}\,P^a + \underset{(.440)}{.066} \qquad (24)$$

$$\Delta R_p^r = \underset{(.001)}{.005}\,\Delta P_k^r - \underset{(.002)}{.003}\,P^a + \underset{(.032)}{.007} \qquad (25)$$

$$\Delta R_e^r = \underset{(.00\text{[illegible]})}{.055}\,\Delta P_k^r - \underset{(.012)}{.045}\,P^a + \underset{(.157)}{.064} \qquad (26)$$

$$\Delta W_p^r = \underset{(.001)}{.012} \Delta R_e^r + \underset{(.0002)}{.00003} P^a - \underset{(.004)}{.003} \tag{27}$$

$$\Delta W_e^r = \underset{(.028)}{.387} \Delta R_e^r - \underset{(.006)}{.001} P^a - \underset{(.076)}{.146} \tag{28}$$

$$\Delta S_p^r = \underset{(.001)}{.009} \Delta P_k^r - \underset{(.002)}{.006} P^a - \underset{(.460)}{.020} \tag{29}$$

$$\Delta S_e^r = \underset{(.002)}{.027} \Delta P_k^r - \underset{(.008)}{.014} P^a - \underset{(.870)}{.217} \tag{30}$$

$$\Delta T^r = \underset{(.001)}{.012} \Delta P_k^r - \underset{(.004)}{.009} P^a - \underset{(.548)}{.050} \tag{31}$$

$$\Delta G^r = \underset{(.001)}{.010} \Delta P_k^r - \underset{(.003)}{.009} P^a - \underset{(.615)}{.005} \tag{32}$$

$$\Delta F^r = \underset{(.001)}{.015} \Delta P_k^r - \underset{(.004)}{.011} P^a - \underset{(.615)}{.102} \tag{33}$$

$$\Delta J = \sum_{i=1}^{9} \Delta E_i^c + \Delta M^c - .380 \Delta P^c . \tag{34}$$

Several changes must be made in the model before it can be considered complete and ready to use for forecasting. Because the ratio of manufacturing employment to population is declining in the United States, Equation (11) should be replaced by the following:

$$M_{eq}^s = .109 e^{-.015t} P^s , \tag{35}$$

where .109 is the average 1958 value of M^s/P^s (adjusted for the business cycle) and .015 is the rate at which this ratio declined in the economy as a whole over the period 1954-1962. The figure .109 was obtained by interpolating between 1954 and 1962 values.

Also, the feedback between population and employment changes and vacant land absorption must be taken into account. The function showing the behavior of the vacant land ratio over time has been defined as follows:

$$V/L = (V_{t-1} - .142 \, \Delta P^{c}_{t-1})/L \text{ if } \Delta P^{c}_{t-1} > 0 \, , \qquad (36)$$

$$V/L = .97 \, (V_{t-1}/L) \qquad \text{if } \Delta P^{c}_{t-1} < 0 \, . \qquad (37)$$

Equation (36) states that the amount of central city vacant land absorbed into urban use depends on the increase in central city population. The coefficient .142 was estimated by the method of least squares from a sample of 22 cities.* In the older cities where population is decreasing vacant land is also decreasing. Equation (37) shows that this rate of decrease has been arbitrarily set at the annual rate of 3 per cent. This decline of vacant land largely reflects continued expansion of residential uses, but any resultant population increase is more than offset by falling net densities in the rest of the city.**

A rough idea of the model's validity can be obtained by matching its predictive power against that of a naive model. This has been done by comparing the coefficients of determination of predicted and actual 1954-1958 changes with those obtained by correlating actual 1948-1954 and 1954-1958 changes. These data appear in Table 1. In addition, the least squares coefficients of multiple determination have been included for each equation of the model to give some idea of the accuracy of each component. Unfortunately, the model could not be used to test the predictive power of the equations for finance, insurance, and real estate, transportation, communications, and public

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*This figure includes the effects on land absorption of employment increases that run parallel to the population increases. It was too complicated to account for the employment changes separately. For a more complete discussion of this problem, see Niedercorn and Hearle, *op. cit.*, pp. 14-15.

***Ibid.*

Table 1

THE MODEL'S PREDICTIVE POWER COMPARED WITH A NAIVE MODEL, PERCENTAGES OF VARIANCES OF 1954-1958 CHANGES EXPLAINED

	Coefficients of Determination		
Equation Number	Least Squares Individual Equation	Entire Model	Naive Model
12	.72	.72	.64
13	.64	.61	.99
14	.85	.47	.50
15	.93	.66	.92
16	.47	.26	.43
17	.53	.38	.04
18	.37	.67	.00
19	.54	.44	.18
20	.93	.86	.53
21	.78	.85	.80
22	.65	-	-
23	.27	-	-
24	.89	-	-
25	.55	.46	.26
26	.88	.88	.84
27	.69	.60	.35
28	.84	.78	.51
29	.88	.89	.72
30	.82	.92	.13
31	.89	-	-
32	.92	-	-
33	.92	-	-

Note:

- means not applicable.

utilities, and government administration because the predicted changes would have to be for the 1950-1960 period rather than for 1954-1958, and a complete set of 1950 data was not available.

The model's forecasts of employment changes are clearly superior to those obtained from the naive model. In 12 of 14 cases the coefficients of determination obtained from the former model are higher than those obtained from the latter. The only exceptions to this pattern are central city manufacturing employment and retail proprietors in the same area. The failure of the model to explain more than 47 per cent of the variance in central city manufacturing employment can be attributed partially to the crudity of the SMSA equations. If changes in SMSA manufacturing employment are taken as given, Equation (14) explains 85 per cent of the total variation in central city manufacturing. The business cycle trough which took place in 1958 helps explain why the model generally predicted an excessively high level of manufacturing employment for that year. This is a long run model and was not designed to account for business cycle variation. The model consistently gives better results for the metropolitan ring than the naive model.

However, the naive model forecasts population changes more accurately. There are two reasons for this. First, the raw population data are only estimates, obtained by interpolation between 1950 and 1960, and extrapolation backward from 1950 to 1948. Consequently, 1948-1954 changes necessarily correlate very strongly with the 1954-1958 changes. Second, population growth is usually more steady over time than employment growth.

Before using the model to forecast future trends in the nation's urban areas, its estimated equations should be discussed more thoroughly to see what the data imply about current locational behavior of both firms and households. Equation (35) is the basic forcing function in the model. It stipulates that the equilibrium ratio of manufacturing employment to population in the nation's metropolitan areas will decrease over time at the rate of 1.5 per cent per year, a rate only slightly smaller than the rate of population increase in the nation as a whole (1.8 per cent). These two figures imply relatively slow growth of manufacturing employment. Equation (12) shows that there is a long run trend toward an evening out of the M^s/P^s ratio across the larger SMSAs. The $(M^s_{eq} - M^s)/M^s$ variable explains roughly one-half of the total variance of the dependent variable when the lagged variable is omitted from the equation. Thus, the ratio of metropolitan manufacturing employment to population will become more uniform in the future; manufacturing will be less concentrated in the East and will develop rapidly in the newer population centers of the West. This is a long run process; its half life is about 38.5 years, assuming a constant value of M^s_{eq}. If $M^s_{eq} > M^s$ and is increasing, the half life is longer than 38.5 years; if $M^s_{eq} < M^s$, but is still increasing, the half life is shorter than 38.5. Thus, the length of the adjustment to equilibrium differs depending on the characteristics of the local area.

Equation (13) indicates that the populations of the larger metropolitan areas are growing at the average rate of 2.5 per cent annually. In addition, a 1 per cent change in manufacturing employment generates a .38 per cent change in population.

Equation (14) demonstrates that the quantity of vacant land and the size of the city play a significant role in determining how much of the increase in SMSA manufacturing employment can be absorbed by the central city. The vacant land variable is especially powerful when it nears zero. At that point none of the increase can locate in the central city. The coefficient of 1.332 of the next term indicates that when a metropolitan area loses manufacturing employment, the city's loss is greater than would be expected on the basis of its share of total employment.

The central city population equation shows that, as in the case of manufacturing, the amount of vacant land and the size of the city limit the absorptive power of the city. However, in the case of resident population a density factor also plays an important role.

The second term on the right side of the central city population equation measures this density factor. Its significance is difficult to fathom in present form. Suppose all other terms on the right side of the equation are set equal to zero. The following differential equation can then be written:

$$dP^c/dt = -.00033\ (P^c)^2/L \quad . \tag{38}$$

This equation yields the solution

$$P^c = (P^c_o\ L)/(.00033\ P^c_o\ t + L) \tag{39}$$

where P^c_o is the value of P^c in an arbitrarily chosen base period. Taking the first derivative with respect to time,

$$dP^c/dt = -\ [.00033\ (P^c_o)^2\ L/(.00033\ P^c_o\ t + L)^2] \tag{40}$$

Central city population declines continuously over time, but at a decreasing rate. There is probably some minimum density below which P^c will not decrease but it has not been ascertained in this work.

The annexation term in Equation (15) has a highly significant positive coefficient as was expected. As a matter of logic it should equal one, but it does not because the population data are estimates rather than actual counts, and contain some error. The constant term in the same equation is significantly negative, indicating that cities are losing population for unspecified reasons other than the adverse effects of high densities.

The meaning of the central city wholesaling and retailing equations should be clear. Employment changes are induced by changes in central city population. The effects of changes in metropolitan ring populations are not statistically significant and as a result this variable has been omitted from Equations (16) through (19). All coefficients except that for population annexations in the wholesale proprietors' equation have the signs expected *a priori* and reasonable magnitudes. On the other hand, services are strongly influenced by population growth in the ring. Apparently the central city acts as a center for specialized services such as business services that cannot locate in the ring because of insufficiently concentrated demand. These services tend to locate near the point of maximum access to the entire metropolitan area.

Central city employment changes in transportation, communications, and public utilities correlate negatively with the central city job level in these categories. This probably reflects continued declines in public transit systems as more riders switch to the automobile. The equation for central city government administrative employment is probably the poorest of the lot, having a coefficient of determination of only .27. It shows some influence of central city population changes although the regression coefficient is not statistically significantly different from zero. The influence of changes in metropolitan ring population is barely significant at the 5 per cent level. Finance, insurance, and real estate behave much like services with changes in ring as well as central city populations, inducing employment changes in the central city.

The metropolitan ring equations are all quite simple. Employment changes depend on ring population changes and annexations in all cases except wholesaling, where retailing employment changes are substituted for population change as the independent variable. Again, the annexation term has the theoretically correct sign for all equations except wholesaling proprietors.

Equation (34) shows that the number of commuters traveling to work between ring and central city will increase, and consequently, rush hour traffic will grow heavier if: (1) central city employment grows faster than the employed labor force residing in the central city, or (2) the employed labor force residing in the central city declines faster than central city employment. The model can be used to predict whether either of these outcomes is likely for many American cities.

In order to examine the behavior of selected services and retailing more closely both were broken down by two-digit Standard Industrial Classification codes and equations similar to Equation (8) were fitted. The results show strong ring influence on central city growth of business services and automobile repair services, and significant but weaker influence on personal services. The automobile services are no doubt related to the fact that commuters from ring to central city are increasingly using the private automobile for their journeys to work. In the retailing category, eating and drinking places, furniture stores, and gasoline service stations also showed ring influence on central city employment. Stores showing a tendency for central location are, as in the case of service activities, the highly specialized ones oriented toward the commuter who works in the downtown offices. Furniture stores are the only exception to this pattern.

V. FORECASTING METROPOLITAN GROWTH

Although a careful examination of the model yields considerable insight into the structure of metropolitan growth, only forecasts can reveal clear-cut quantitative information about it. The model can easily be used to generate such a set of forecasts. The predicted employment and population trends can then be examined and analyzed to determine which will dominate in the future.

These forecasts will be made for each city assuming fixed legal boundaries (no annexations). Consequently, the population annexation term P^a will be set equal to zero in all equations where it appears. The forecasting model includes Equations (6), (7), and (12) through (37).

In order to make a forecast, initial conditions for

$$M^s,\ P^s,\ M^c,\ P^c,\ T^c,\ V,\ L, \text{ and } (\Delta M^s/M^s)_{t-1}$$

must be specified.* This was done for 36 metropolitan areas, using 1958 as the base year; forecasts were made for 1968.** These forecasts are summarized in Table 2 which shows the numbers of areas for which gains in population and each type of employment were predicted.

If the predictions are correct, about half of the 36 metropolitan areas will gain manufacturing employment and half will lose; all will gain population.

Although virtually all metropolitan rings will grow in both population and employment, the future of the central city is much

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*If good forecasts for SMSA population and manufacturing employment can be obtained from outside sources, such as regional economic studies, these forecasts can be inserted directly into Equations (14) and (15), rendering Equations (12) and (13) unnecessary. This would probably improve the accuracy of central city and ring forecasts because some of the error in the SMSA forecasts would be eliminated.

**These areas include 36 of the 41 on which the model's parameters were estimated. Akron, Kansas City, Norfolk-Portsmouth, Albany-Schenectady-Troy, and Youngstown-Warren were omitted owing to poor data on vacant land. The data for the term $(\Delta M^s/M^s)_{t-1}$ were calculated by dividing the 1954-1958 percentage changes by four.

Table 2

NUMBER OF AREAS WITH PREDICTED GAINS IN POPULATION AND VARIOUS TYPES OF EMPLOYMENT, 1958-1968

Population and Employment	Metropolitan Area	Central City	Metropolitan Ring
Population	36	16	36
Manufacturing	19	9	29
Retailing, proprietors	23	6	36
Retailing, paid employees	36	12	36
Wholesaling, proprietors	36	35	36
Wholesaling, paid employees	36	28	36
Services, proprietors	36	36	36
Services, paid employees	36	36	36
Transportation, communications, public utilities	22	8	36
Government administration	36	36	36
Finance, insurance, real estate	36	36	36
Total employment	30	22	36
Commuter trips to work		25	25

less clear-cut. Slightly less than half of the central cities will gain population and only a fourth will gain manufacturing employment. Although a large majority will gain wholesaling employment, the increases are quite small, as can be seen from Table 3. As might be expected from the structure of the model, the central city makes its strongest gains in services, government administration, and finance, insurance, and real estate. Along with central office employment and possibly professional services (neither one included in this model) these sectors are the only areas in which substantial future growth can be expected. L. E. Schwartz has predicted that central office employment in American central cities will increase by about one million over the period 1960-1973.* A substantial part of this increase can be expected to locate in the central business districts of the larger cities. Transportation, communications, and public utilities employment shows losses in about three-fourths of the central cities.

Total employment increases in 22 of 36 central cities and the number of commuters from the ring to the central city increases in 25 cities. Three cases in which decreased commuting is predicted are rapidly growing cities of the Southwest which have made large annexations in recent years.** Since the newer residential areas are generally nearer the periphery than the employment locations, increased rush hour traffic can be expected in these cities even though the traffic does not cross city boundaries. If probable changes in central office employment are taken into account, a few more central cities of the sample might show increases in total employment and in commuters from the metropolitan ring.

*See Lawrence E. Schwartz, From Core to Ring: An Econometric Study of the Relocation of the Central Office and Its Workers (unpublished Ph.D. dissertation, Department of Economics, Harvard University, 1963).

**They are Ft. Worth, Houston, and Oklahoma City.

Table 3

MEAN PREDICTED VALUES OF TEN-YEAR EMPLOYMENT
AND POPULATION CHANGES, 1958-1968
(thousands)

Variable	Value	Variable	Value	Variable	Value
ΔP^s	528.2	ΔP^c	22.1	ΔP^r	506.1
ΔM^s	10.8	ΔM^c	-8.4	ΔM^r	19.2
ΔR_p^s	1.6	ΔR_p^c	-0.9	ΔR_p^r	2.5
ΔR_e^s	29.3	ΔR_e^c	0.8	ΔR_e^r	28.5
ΔW_p^s	0.5	ΔW_p^c	0.2	ΔW_p^r	0.3
ΔW_e^s	10.8	ΔW_e^c	1.2	ΔW_e^r	9.6
ΔS_p^s	[illegible]	ΔS_p^c	3.0	ΔS_p^r	4.4
ΔS_e^s	[illegible]	ΔS_e^c	14.6	ΔS_e^r	11.5
ΔT^s	[illegible]	ΔT^c	-2.7	ΔT^r	5.6
ΔF^s	[illegible]	ΔF^c	4.1	ΔF^r	6.6
ΔG^s	6.8	ΔG^c	1.8	ΔG^r	5.0
$\sum_{i=1}^{9} \Delta E_i^s + \Delta M^s$	106.7	$\sum_{i=1}^{9} \Delta E_i^c + \Delta M^c$	13.6	$\sum_{i=1}^{9} \Delta E_i^r \Delta M^r$	93.1
		$.380 \Delta P^c$	8.4		
		ΔJ	5.2		

A more precise idea of the average magnitudes of the various predicted population and employment trends can be obtained from Table 3.

Several items in this table stand out as significant. Metropolitan population growth will be considerable, and by far the largest part of it will settle in the metropolitan ring. Roughly 95 per cent of total population growth will take place in the ring. The growth of manufacturing employment in the SMSA will be relatively slow and will actually be negative in the central city. The increases that take place in the metropolitan ring will be a combination of new jobs and the transfer of old jobs from the central city. Total employment growth in the ring will average about 93,100, while it will be only 13,600 in the central city. The strongest gains in the central city will be registered by services, finance, insurance, and real estate, and government administration. Roughly two-thirds of the increase in services will probably be in business services.*

Only about 15 per cent of the total average metropolitan employment increase will locate in the central city. Although this growth is relatively small, the central city's gain in employed labor force will be even smaller, an increase of roughly 8,400. This leaves an average difference of 5,200 additional workers who will commute to the city from the ring or beyond.

These conclusions are based on the assumption that the locational patterns of employment categories not included in the model do not change radically. The validity of this assumption can be clarified by examining more carefully the nonincluded kinds of employment. Table 4 gives national totals for types of employment covered by the model as well as those not included.

Construction, professional services, manufacturing central offices, and private household services have not been included in the model,

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*During the 1954-1958 period, two-thirds of the mean central city increase in services was accounted for by business services. This pattern will probably continue in the future.

Table 4

CIVILIAN EMPLOYMENT IN NONEXTRACTIVE INDUSTRIES
IN THE UNITED STATES, 1960
(thousands)

Type of Employment	Total	Included in Model	Not Included In Model
Construction	3,816		3,816
Manufacturing	17,513	16,762	751[a]
Transportation, communications, and public utilities	4,458	4,458	
Wholesale and retail trade	11,793	11,793	
Finance, insurance, and real estate	2,695	2,695	
Business and repair service	1,611	1,611	
Personal services	3,858	1,941	1,917[b]
Entertainment and recreation services	503	503	
Professional and related services	7,578		7,578
Public administration	3,203	3,203	
Not reported	2,608		2,608
Total	59,616	42,966	16,670

Notes:

[a] Central office employment.

[b] Private household services.

which covers 72 per cent of 1960 employment. The neglected types of employment do not seem likely to significantly change the model's forecasts of total employment. Construction and private household services may decrease slightly in the central city, but this will probably be offset by increases in central office employment and professional services. As a considerable portion of the latter category is highly specialized, one might expect it to behave more like business services than like retail trade.

VI. CONCLUSIONS

Several important conclusions about the structure of metropolitan growth can be inferred from the statistical testing of the model. First, the ratio of manufacturing employment to population is indeed becoming more uniform across the country's metropolitan areas. The old manufacturing centers are losing employment and the newer areas that hitherto have had less manufacturing are gaining. This is a long run process and will probably continue for many years. Second, available vacant land and city size limit the increases in both population and manufacturing employment that a city can absorb. In addition, there is a trend toward lower residential densities. High central city densities cause population declines, but the rate of population decrease diminishes over time. Third, other kinds of employment can be classified into two types: (1) those serving the entire metropolitan area, and (2) those serving only local areas. Certain types of services, especially business services, government administration, and finance, insurance, and real estate fall into the first category and consequently tend to seek central locations. Retailing, wholesaling, and most personal services fall into the second and consequently locate throughout the entire metropolitan area.

The forecasts made with the model show rapid population and employment growth in the largest metropolitan areas, and especially in the metropolitan rings, but show only slow growth in the central cities. All types of employment increase in the ring. However, the central city can expect increases only in services (mainly business services), government administration, finance, insurance, and real estate, and manufacturing central offices. Central city employment in wholesaling, retailing, and most services will remain fairly stable, but manufacturing will decline substantially. White collar employment will continue to increase in the central city, but blue collar jobs will suburbanize rapidly. Because the growth of central city jobs will be greater than the increase in employed central city labor force, there will be increases in the number of people commuting to work in the central city. This will cause increased traffic congestion unless existing highway systems are enlarged to move the extra commuters or they can be diverted to some form of mass transit. Reverse commuting is also likely to increase as minority groups who are constrained by racial segregation to live in the central city find that the construction and manufacturing jobs which they traditionally hold will henceforth be concentrated more in the metropolitan ring.

Appendix

LIST OF VARIABLES

P^s	SMSA population.
P^c	Legal area central city population.
P_k^c	Constant area central city population.
P^r	Legal area metropolitan ring population.
P_k^r	Constant area metropolitan ring population.
P^a	Population annexed by the central city.
M^s	SMSA manufacturing employment.
M_{eq}^s	Equilibrium level of SMSA manufacturing employment.
M_g^s	SMSA manufacturing employment in SMSAs of increasing manufacturing employment, otherwise equal to zero.
M_d^s	SMSA manufacturing employment in SMSAs of decreasing manufacturing employment, otherwise equal to zero.
M^c	Central city manufacturing employment
M^r	Metropolitan ring manufacturing employment.
V	Central city vacant land area.
L	Central city total land area.
t	Time measured in years.
E_i^s	SMSA demand oriented employment.
E_i^c	Central city demand oriented employment.
E_i^r	Metropolitan ring demand oriented employment.
R_p^s	SMSA proprietors in retailing.
R_p^c	Central city proprietors in retailing.
R_p^r	Metropolitan ring proprietors in retailing.
R_e^s	SMSA paid employees in retailing.
R_e^c	Central city paid employees in retailing.
R_e^r	Metropolitan ring paid employees in retailing.
W_p^s	SMSA proprietors in wholesaling.

W_p^c Central city proprietors in wholesaling.

W_p^r Metropolitan ring proprietors in wholesaling.

W_e^s SMSA paid employees in wholesaling.

W_e^c Central city paid employees in wholesaling.

W_e^r Metropolitan ring paid employees in wholesaling.

S_p^s SMSA proprietors in selected service trades.

S_p^c Central city proprietors in selected service trades.

S_p^r Metropolitan ring proprietors in selected service trades.

S_e^s SMSA paid employees in selected service trades.

S_e^c Central city paid employees in selected service trades.

S_e^r Metropolitan ring paid employees in selected service trades.

T^s SMSA employment in transportation, communications, and public utilities.

T^c Central city employment in transportation, communications, and public utilities.

T^r Metropolitan ring employment in transportation, communications, and public utilities.

G^s SMSA employment in government administration.

G^c Central city employment in government administration.

G^r Metropolitan ring employment in government administration.

F^s SMSA employment in finance, insurance, and real estate.

F^c Central city employment in finance, insurance, and real estate.

F^r Metropolitan ring employment in finance, insurance, and real estate.

J Number of commuters traveling to work between ring and central city.

P_o^c Central city population in an arbitrarily chosen base year.