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Nonlinear Oscillations and Electroencephalography

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EDMOND M. DEWAN

MICROWAVE PHYSICS LABORATORY PROJECT 5635
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES, OFFICE OF AEROSPACE RESEARCH, UNITED STATES AIR FORCE, L.G. HANSCOM FIELD, MASS.

Abstract

Pilots have noticed that when the sun is viewed through an idling propeller one can be subjected to an unusual form of vertigo. This is apparently due to an interaction of the flickering light and certain rhythmic electrical activity taking place in the brain. Other such phenomena that may occur when a man is placed in an unusual environment suggest the investigation of the nature of brain rhythms, especially in regard to malfunction.

It has been found that almost all of the well-known phenomena of nonlinear oscillations have counterparts in the macroscopic neuroelectric behavior of the human brain. Examples of such correspondences are:

a. soft limit cycles—"spontaneous" oscillatory brain activity

b. hard limit cycles involving "inertial" nonlinearities—initiation of epileptic after-discharges by local electrical stimulation of the cortex

c. nonlinear entrainment—hypersynchrony, photic driving of brain rhythms (mentioned in the first paragraph)

d. oscillation hysteresis—paradoxical duration of certain after discharges

e. variation of the parameter μ in van der Pol's equation

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$$

—evolution of epileptic after discharge from high frequency waves into low frequency relaxation oscillations and a connection between "exhaustion" and "epileptic foci".

f. asynchronous actions—blocking of the "alpha rhythm", "facilitation" and excitation and suppression activity associated with attention.

These correspondences permit the construction of a very suggestive "phenomenological" theory to "explain" electroencephalographic data in terms of nonlinear stability which in turn reveals unsuspected relationships in the data and suggests new experiments.

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1. INTRODUCTION

The great hopes that accompanied the discovery of EEG have not yet been realized. A large mass of data exists, but an understanding of its significance is all but absent. Although some correlations exist between certain waveforms and their spatial distributions on the one hand and certain states of consciousness and abnormal brain activity (for example, epilepsy) on the other hand, very little has been said concerning the mechanism of such brain phenomena. Clearly, from both the clinical and purely scientific point of view, it is very important that we take any possible steps that show promise of helping to solve this great enigma.

A very powerful approach, used successfully throughout the history of physics,* is one that organizes data by means of a mathematical or "phenomenological" model. To call such a model a theory would be incorrect. It merely serves the purpose of (1) displaying the underlying order and uniformities in the data, and (2) guiding further experiment. Only after this is done can one formulate a truly theoretical scheme.[1]

The purpose of this paper is to point out certain analogies between EEG activity and the phenomena of nonlinear oscillations (NLO) and to use these analogies to show how a semi-quantitative phenomenological model could be constructed to explain

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*This approach is of course not limited to physics. For example Freud's "id", "ego", "superego", and so on, were concepts formulated directly from a large amount of observational data.

EEG data. Many mathematical and neurological details will be omitted and attention will be confined to macroscopic observations alone since a much more extensive treatment has been published elsewhere.[1]

2. NONLINEAR ENTRAINMENT

The analogy between entrainment of nonlinear oscillations and certain EEG behavior has already been pointed out in the literature.* Grey Walter, who was the first to notice the possible analogy, was concerned with subharmonic and superharmonic entrainment, while more recently Norbert Wiener discovered the possible presence of harmonic entrainment. While both possibilities require more experimental investigation before they can be considered as definitely established, they are nevertheless extremely suggestive.

Harmonic entrainment consists of locking together two oscillations at the same frequency, while sub- and super-harmonic entrainment consists of locking at a submultiple or integral multiple frequency. Wiener first suspected the presence of harmonic entrainment when he saw a very accurate spectrum of brain waves in the alpha frequency region. He noticed that the 10-cps line was quite narrow and that there was a dip on each side of it (at about ±1/2 cps). This could be explained by hypothesizing a strong pulling together of frequencies. Robinson[2] has subsequently shown that, in general, there are no dips at ±1/2 cps and that the alpha rhythm is not quite as narrow or constant as originally supposed. Still more recently** experimental evidence seems to indicate that Wiener's original idea of entrainment might be correct but that the dips are to be found several cycles away from the main peak. In fact, two power spectra of one subject showed a distribution identical in form to the diagram to be found in Wiener's work.[3] The surprising fact is that this diagram describes the entire resting EEG spectrum rather than a small section of it.

Another possible role of harmonic entrainment might be found in the well-known "hypersynchrony" of epileptic discharges. Although a central pacemaker might be important in some or all cases of epileptic hypersynchrony, entrainment would very likely also be involved (the two are not mutually exclusive).

An interesting phenomenon which may link epilepsy to entrainment is the so-called photic flicker effect. Pilots of propeller-driven aircraft have noticed that occasionally the sunlight shining through propeller blades can cause them to experience a peculiar form of vertigo and, in extreme cases, loss of consciousness.

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*Perhaps the first analogy between nonlinear phenomena and nerve activity was between the all-or-none character of neuron firing and the hysteresis and relaxation phenomenon of NLO. Grey Walter has also made use of the analogy between relaxation oscillation and cortical spike activity.

**Brodkey and Dewan (to be published). In this paper, power spectra of both "resting alpha rhythms" and EEG during photic "driving" have been obtained. The latter show marked superharmonic resonance effects.

This effect has also been studied in the laboratory by means of stroboscopes,[4] and variations of it have been used to induce epileptic fits in normal subjects.* One of the proposed mechanisms of this effect[5] is entrainment between the flashing light and the brain rhythms. Although more experiments are needed to confirm this,[6] it is significant that Grey Walter has succeeded in locking together certain harmonics of the interictal petit mal EEG by triggering the photic stimulus[7] at certain phase relationships with the harmonics. The result of this entrainment of frequency multiples is the well known "wave-and-spike" waveform of a petit mal fit.

The above observations and the suggestion that the wave-spike pattern "is produced by the synchronization of local cortical physiological rhythms by a large slow discharge originating in the diencephalon",[8] leads quite naturally to the following proposal for the mechanism of petit mal epilepsy:** (1) In the abnormal brain of the petit mal type there is an overly great tendency for subharmonic resonance and entrainment to take place in cortical oscillations. (2) Occasionally, when the subharmonic at about 3 cps is especially effective in stimulating certain subcortical structures, the latter give forth a resonant oscillation which feeds back to the cortex. This in turn drives the various harmonics of the cortical oscillations, and a high voltage-locked wave-and-spike pattern develops.

This model would explain not only the prominence of the 3 cps frequency in wave-and-spike patterns but also the beneficial effects of the drug Tridione and of maturation upon petit mal. (This disease is a childhood ailment which is usually outgrown.) Both Tridione and maturation have the effect of raising[9] the dominant cortical frequencies which, in turn, raise the subharmonic of the cortical waves to a higher frequency than the hypothesized subcortical resonance, and hence destroy the abnormal feedback loop. Clearly this hypothesis suggests several interesting experiments such as finding out if indeed there is a subcortical resonant frequency at about 3 cps and if this frequency actually becomes detuned with respect to cortical subharmonics.

3. LIMIT CYCLES AND NONLINEARITY

Up to this point we have represented the output of the cortex as if it were caused by a system of coupled nonlinear oscillators. This model not only explains the periodicity in the data but also its random aspects as implied by Wiener[3] and treated mathematically more recently by Martin Schnetzen.[10] We now shift our point of view by considering only the periodic component of the EEG and, in addition, regard the output of a single EEG channel (for example, the potential variation from one pair of

*This effect is being given serious consideration in the design of the proposed tunnel between England and France. An unfortunate choice of spacing of the lights might cause a motorist to lose consciousness at certain velocities.

**Several other observations not mentioned here also play a role.

adjacent electrodes on the scalp) as if it were from a single nonlinear oscillator. This oscillator is to be considered as being susceptible to stimulation either by external means (for example, electrical stimulation by electrodes) or by internal means (for example, natural or induced influences from subcortical structures).

3.1 Spontaneous EEG Activity and Induced After-discharges

Many neurologists seem to regard the spontaneous oscillatory electrical activity of the central nervous system (CNS) as a great enigma. Grey Walter called it "the most bewildering of the many unsolved problems in central neurobiology".[11] This bewilderment stems from the assumption that all oscillatory activity is caused by an initiating event and that it in some sense preserves the information that the event occurred. Such an assumption is true for linear systems but does not hold generally for nonlinear systems. The evident nonlinearity of the CNS is sufficient to disqualify any assumption based on linearity and the above enigma disappears when one realizes that there is a large class of nonlinear systems which (a) can behave in a manner entirely independent of initial conditions (that is, "initiating events"), and hence preserve no information, and (b) can be initiated by arbitrarily small perturbations, that is, by the ubiquitous random background fluctuations of Nature. This will be clarified in the discussion of limit cycles. The main point is that "spontaneity" is to be expected of nonlinear systems and hence should no longer seem bewildering.

A second puzzling phenomenon concerns the mechanism by which repetitive electrical stimulation can cause normal neurons to become excited and to exhibit epileptic oscillations. Such "epileptic after-discharges"* are induced by direct electrical cortical stimulation with square-wave pulses. The following parameters of this stimulation have been found to determine its efficacy: pulse width (PW), pulse-repetition frequency (PRF), pulse amplitude (PA), and pulse-time duration (PTD).

The relation between the minimum effective PA and PRF with constant PW and PTD is shown in Figure 1.** As can be seen, an increase PRF lowers the threshhold of PA, and, as the log-log plot shows, the relation

$$(PRF)\,(PA)^2 = \text{constant}$$

holds over the range 10 cps to 100 cps. As Walter has pointed out, this implies a certain energy (or more correctly, power) requirement. We shall see that almost

*Grey Walter pointed out to the author in a private communication that the word "epileptic" is so ambiguous that it could easily lead to misunderstanding as used here. Nevertheless Hsiang-Tung Chang used the term "epileptiform after-discharge" in his article "The Evoked Potentials", vol. 1, sec. 1, p. 299 et seq. Handbook of Physiology: Neurophysiology. American Physiological Society, 1960. Perhaps a more suitable terminology would be "electric after-discharge". In any case, if the theoretical model proposed here proves to be "correct", then "epileptic after-discharge" would be an appropriate term to use since the model implies that normal cells should theoretically be capable of producing epileptic waveforms.

**Figure 1 is from Grey Walter, op. cit. p. 229, Figure 7-1B.

all properties of after-discharges in addition to the phenomenon of spontaneity can be explained simultaneously in a unified way in terms of limit cycles with inertial nonlinearities.

3.2 Limit Cycles*

Limit cycles can be classified as stable, unstable, and neutral. A stable limit cycle is a mode of oscillation that will return to its original state after being perturbed,

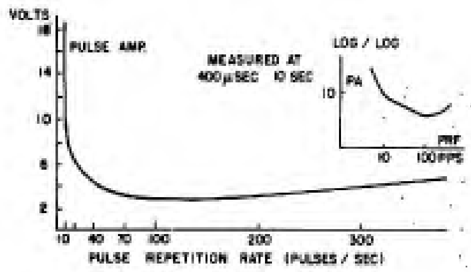

Figure 1. Effective Pulse Amplitude vs Pulse Repetition Rate

whereas an unstable limit cycle never returns to its original mode even after an arbitrarily small perturbation. The neutral limit cycle, on the other hand, depends only upon initial conditions and any perturbation will alter it in proportion to the magnitude of the perturbation. In fact, perturbations can be considered as new initial conditions and, indeed, the case considered by Grey Walter of the frictionless pendulum that "preserves information" about its past is here exemplified.

Stable limit cycles can be classified as "hard" or "soft". Oscillations having an unstable "off" configuration which causes them to build up spontaneously are called "soft" whereas those that must be "jolted" to become initiated are called "hard". In both cases the final steady-state oscillations reflect essentially nothing of the initial conditions and hence do not preserve information. On the contrary they are in fact to be regarded as self-sustained.

Multiply stable systems also exist in which there is a combination of limit cycles. Perhaps the most interesting of these, for our purposes, is one with both a soft and a hard mode. In this case, there would be a spontaneous oscillation which, when jolted enough by an external influence, could go into a second and entirely different stable mode. An example of an equation with such properties is:

$$\ddot{X} + (-a + bX^2 - cX^6 + dX^{10})\dot{X} + X = 0,$$

with all constants positive. The form of the "damping coefficient" (that is, the coefficient of $\dot{X}$) is shown in Figure 2. We see that there are two regions of X with negative damping separated by a region of positive damping. It is the region with

*Details of limit cycle phenomena will be found in N. Minorsky, Nonlinear Oscillations, van Nostrand Co., Inc., 1962. More elementary treatments are given in F. Clauser, J. Aeronaut. Sci., 23:411, 1956 and also in AFCRL-63-149 (I).

larger X which is responsible for the hard mode of oscillation. (Figure 3 shows the phase plot for this system.)* Clearly there is an analogy between the behavior of

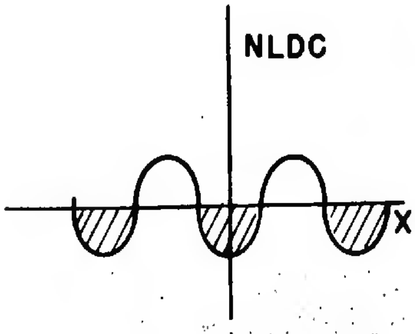

Figure 2. Nonlinear Damping Coefficient for Double-Mode Oscillation

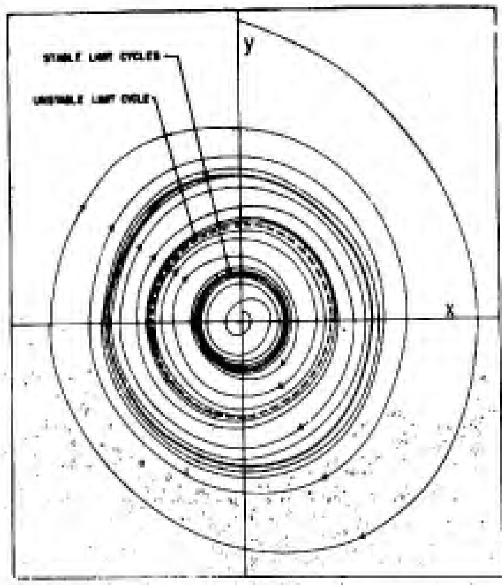

Figure 3. Phase Plane of an Oscillator That Can Perform Either of Two Independent Oscillations

this system and the behavior of the cortex since both have activity which can be induced by external stimulation in addition to spontaneous activity. In order to account for the "parameters of efficacy" mentioned earlier, a further refinement is necessary: namely, the introduction of inertial nonlinearity into the model.

3.3 Inertial Nonlinearities

An inertial or "delay"** nonlinearity is one which depends upon the integral of the dependent variable over a finite time. An example of a circuit element with such nonlinearity is the thermistor since its resistance depends upon a finite history of the current flow through it. If such a device is incorporated in a van der Pol oscillator, one can obtain a system described by the following equation:

$$\ddot{X} - \mu\left[A - (X^2 + b_1\rho_o - b_3\rho_o^3 + b_5\rho_o^5)\right]\dot{X} + X = 0 \qquad (1)$$

where b_1, b_3, and b_5 are positive constants and ρ_o is the integrated value of X^2 over a certain interval T,

$$\rho_o \equiv \int_{t-T}^{t} X^2(t)\,dt\,.$$

*Clauser op. cit. and AFCRL-63-149(I) elaborate further on this.

**The term "delay nonlinearity" is used in a simplified treatment by A.A. Kharkevich, Nonlinear and Parametric Phenomena in Radio Engineering, Rider Pub. Co., New York, 1962.

Such a system has a soft limit cycle, but, if it is stimulated with sufficient amplitude, pulse width, frequency, and duration, the value of ρ_o will increase in such a way that when it reaches a certain value, a new form of stable oscillation will result. Notice that as ρ_o is increased the amplitude of stable oscillation is decreased since the negative damping region is diminished by the $b_1 \rho_o$ term. If ρ_o is sufficiently increased, negative damping is removed altogether and replaced by increasing positive damping until ρ_o reaches a value such that the $b_3 \rho_o^3$ overrides the $b_1 \rho_o$ term; at this point, the damping starts to return toward the negative region for small X (assuming the b's have appropriate magnitudes). Finally, a point is reached where self-sustained oscillations return and ρ_o can continue to grow until it reaches a stable value.

Two things result from this model:

1. In the case of an integration over a series of square waves of equal amplitude, we get*

$$\rho_o = (PA)^2 (PRF)(PW)(T) .$$

If the requirement for setting up the second stable limit cycle is $\rho_o \geq$ constant, then we obtain Grey Walter's previously mentioned "energy" requirement.

2. Before the second limit cycle is reached, there is a range of ρ_o where the amplitude X_{max} of the self-sustained oscillation is cut down until the negative resistance is replaced by positive resistance. In view of this, it is extremely interesting to note that the response of the cortex to electrical stimulation must first decrease to zero before an afterdischarge occurs. (This is a necessary but not sufficient condition.) In other words our model accounts for more of the observed neurological phenomena than has been "fitted" into the model.

The value of T, as can be seen from Figure 1, is about 7 seconds since beyond this time there is no increase in effectiveness. The details of much of the mathematics needed for a more quantitative understanding (for example, the "stroboscopic method") in addition to further refinements describing some of certain other interesting aspects of after-discharges (for example, alternation, latency, and demultiplication during initiation) have been omitted here since they have been treated somewhat extensively elsewhere.[1]

4. THE TRANSITION TO RELAXATION OSCILLATION

4.1 Waveform and Damping Magnitude

The van der Pol equation is usually written in the form[12]

$$\ddot{X} + \mu (X^2 - 1)\dot{X} + X = 0 .$$

*PW has a maximum value beyond which it effectively cannot be increased. This feature is discussed in AFCRL-63-149(I), p. 29.

As the magnitude of the damping term increases by making μ larger, the solution becomes less "simple harmonic" in form ($\mu \to 0$, of course, corresponds to pure sinusoidal motion). Figure 4 shows the waveform for various values of μ and it is clear that for large μ one has relaxation oscillations.[13] This alteration with increasing μ is qualitatively very analogous to that found in after-discharges as shown in Figure 5. In the case of the latter, there is a change from sinusoid waves (tonic stage) to relaxation oscillation (clonic stage). Indeed, "clonic spikes" are very analogous to the current output of a gas discharge tube relaxation oscillator, as has already been pointed out by Grey Walter.* It thus appears that the so-called "exhaustion" due to continued high voltage activity[14] could be represented mathematically by an increase in the magnitude (μ) of the nonlinear damping coefficient (NLDC).

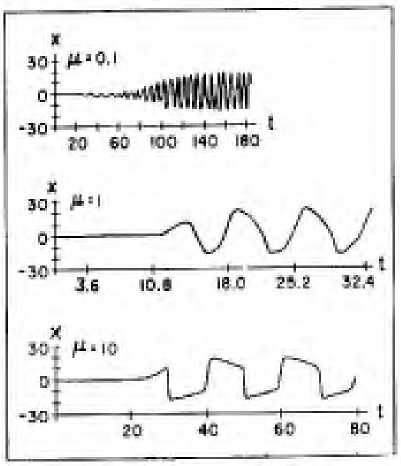

Figure 4. Displacement-time Curves for Van der Pol Oscillators

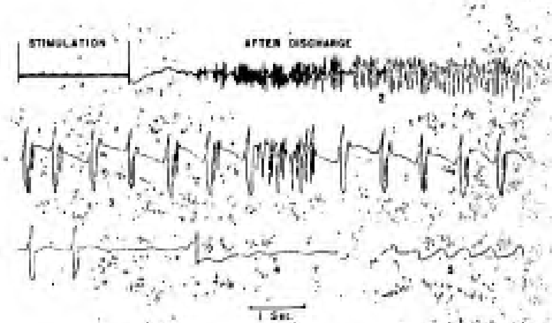

Figure 5. Electric After-Discharge

It is noteworthy that the increase of μ not only causes the transition to relaxation oscillations but also lowers the frequency.[15] Both of these effects occur as an after-discharge approaches a stage of "exhaustion". Indeed, many of the spike discharges found in EEG (in particular, those found in spike foci) are associated with low frequencies. This suggests the possibility that such "noninduced" spike activity might be due to "permanently exhausted" brain oscillators where μ remains large. It is interesting, therefore, that clinical neurologists regard a predominant amount of EEG activity at low frequency to be indicative of pathological conditions; the lower this frequency, the more ominous the diagnosis.

These considerations are very suggestive. Perhaps large μ and consequent spike activity** are always associated with "exhaustion". Physiologically, the

*GREY WALTER, Chap. VIII, Hill and Parr, op. cit., p. 262. The voltage output of such a circuit is a sawtooth wave while the current through the gas tube has a spike pulse waveform.

**In the case of the van der Pol equation the relaxation oscillations are square waves. Perhaps it is the differentiated solution of the equation which is more relevant to EEG, since the latter differs markedly from the actual solution only when ψ is large.

exhaustion during after-discharges is in many cases* accompanied by a decrease of oxygen availability, no doubt due to an abnormally high metabolism rate. In other words, an insufficiency of oxygen and most likely other materials necessary for metabolism might be very intimately related to a large μ in our model. Such an hypothesis is consistent with the fact that tumors which cause epileptic foci could also cause a deficiency in blood supply which in turn could impede the flow of metabolic materials. It is also consistent with the observation that the removal of cortical material can aid in the control of diffuse cerebral abnormality involving spike activity. It appears that such an operation increases the blood supply to the remaining parts of the brain and this, in turn, decreases the spike activity.

4.2 Mathematical Model for After-discharge Evolution

It is possible to generalize the mathematical model given previously for the initiation of after-discharges to include the evolution and eventual self-termination of these discharges as follows:

Let

$$f(\rho, X) \equiv \left[A - X^2 - b_1\rho + b_3\rho^3 - b_5\rho^5\right]$$

as in the X coefficient in Eq. (1) and ρ is (as before) defined as

$$\rho = \int_{t-T}^{t} X^2 \, dX \, .$$

Further, let there be a monotonically increasing function $\psi(\rho')$ where

$$\rho' \equiv \int_{t-\tau}^{t} X^2 \, dX$$

and τ is much larger than T.** Then the generalized equation we shall consider is

$$\ddot{X} + \psi(\rho')\, f(\rho)\, \dot{X} + X = 0 \, .$$

If we set T = 10 sec and τ = 10 min, then the behavior of this equation during and after stimulation is described as follows:

*There are exceptions to this. See Grey Walter, Grenell, op. cit., p. 240. In such cases projection and asynchronous quenching might also play an important role.

**It should be emphasized that the experimental significances of τ and T are a bit complex. They are the maximum times for effects to occur assuming that the parameters of stimulation involved are at their minimum value for effectiveness. For example, Figure 1 shows the minimum PA which can induce an AD for a given PRF. Such a PA would take T seconds to initiate an AD, however a larger PA would take less time. In other words T = PTD for minimum effective parameters. Similarly τ is the maximum time for minimum effective stimulation (or self stimulation) to terminate an AD or to produce sleep.

1. If $\psi(\rho')$ is chosen so that it is $<< 1$ in the neighborhood of $\rho' = 0$, then it will be possible to replace it by small μ during the after-discharge initiation phase (that is, during forced oscillations which increase ρ so that the new stable limit cycle becomes possible) and simply keep our previous description for after-discharge initiation.

2. Once the after-discharge is initiated, however, the amplitude is self-sustained at a large value. This causes ρ' to build up and hence $\psi(\rho')$ increases. As we have seen, this eventually would have the tendency to lower the frequency of the oscillations and to cause them to become more of the relaxation type (which can include the spike-and-wave property).

3. As $\psi(\rho')$ continues to grow, the frequency of the oscillations will eventually become so low that the value of ρ will decrease (because of finite integration time) until finally self-sustained after-discharge oscillation will cease. After a period of time the soft (noninertial) mode of oscillation will again become operative, but initially, since ρ' decreases much more gradually then ρ, the waveform will be abnormal (that is, low frequency with a tendency toward a relaxation waveform). Finally, the oscillations will become normal.

4. The following numerical predictions are made by this model:

   a. Initiation would take a time = T seconds with minimum effective parameter settings.

   b. Duration of the after-discharge would have a maximum of something on the order of τ minutes.

   c. The post-discharge "silent period" or "latent time" should be about T seconds.

   d. Normal rhythms would resume at a time τ after the after-discharge.

The writer does not have confirmation of these numerical predictions at this time; but if indeed they are confirmed, the model should be on relatively solid ground.

4.3 Connection Between After-discharge Model and Sleep

The introduction of the function $\psi(\rho')$ above to account for after-discharge phenomena may also account for some of the effects of sleep upon EEG waveforms. One of the most predominant characteristics of sleep is low (delta) frequency waves. If we assume that the oscillator in our model is operating in its normal (soft) mode with ψ larger than normal, then we would expect such a lowered frequency without too much "relaxation characteristic" in the waveform. But this would imply that ρ' would be larger than normal. This in turn could be explained, however, by the hypothesis that the oscillator is being externally stimulated at low frequency (not to build up ρ too much but to predominantly affect ρ' because of the latter's longer period of integration); indeed, experiments have been performed in which low-frequency stimulation to certain subcortical structures (which connect to the cortex) under certain conditions gives rise to sleep both in terms of behavior and of EEG. (The recent

Russian "sleep machine" also gives indications in this direction.)*

The so-called 14-cps spindle waveforms superposed on the delta waves during the transition from wakefulness to sleep might involve "combination oscillations", a common nonlinear oscillatory phenomenon. They are, however, at this time, in some sense outside the mechanism of our model, since the manner in which the cortex is stimulated by subcortical structures during the transition to sleep is not yet clear.

The fact that $\psi(\rho')$ helps to explain sleep implies that there is a connection between sleep and epilepsy (since after-discharges have an epileptic quality). This is experimentally true in several senses:

a. At the end of an epileptic fit there is a short period of complete silence followed by an EEG record and behavior corresponding to deep sleep,

b. Sleep has been known to induce fits in susceptible people, and

c. $\psi(\rho')$ is, as we have seen, related to "exhaustion" which in turn can be due to deprivation of oxygen. Lack of oxygen can also cause sleep both in terms of EEG and behavior. Theoretically, one might say that sleep discharges and epileptic discharges have a common increase of $\psi(\rho')$.

Quantitatively, in terms of the model, the following correspondences which depend on the ρ' and ρ inertial nonlinearities, respectively are predicted:

τ = time for evolution of an after-discharge
= approximately the time needed to "fall asleep"
= time needed to fully recover from sleep
= the time for cells to recover from after-discharge
≈ range of approximately 4 min to 15 min; that is, all these times should be on the order of a few minutes

T = time needed to electrically induce after-discharge with min. effective parameters
= time duration of isoelectric activity at termination of after-discharge
= time to regain consciousness from sleep (but without full recovery
≈ 10 sec or the order of a few seconds

If a coefficient $B(\rho')$ (which, like $\psi(\rho')$, depends on an integral of X^2 over a time τ) is introduced in the following way, then the power of the model is greatly expanded. Let the damping coefficient be

$$-\psi(\rho')\left[A - (B(\rho')X^2 + b_1X - b_3\rho^3 + b_5\rho^5)\right] .$$

$B(\rho')$ is a monotonically decreasing function of ρ'. The following consequences are the result of such a modification:

a. The EEG during sleep would have a higher voltage than during normal wakefulness.

b. Slow and fast stimulation (apparently most effectively applied through the caudate nucleus and reticular formation respectively) would be antagonistic in the sense that the slow stimulation would induce sleep waves and the fast would cause EEG appropriate for wakefulness. Increasing the voltage of either could make one override the other.

*See Appendix. If news releases are to be taken seriously, low frequencies are primarily used in sleep induction by this "sleep machine".

c. An appropriate choice of parameters would cause an AD to build up in voltage as it progresses (this is "optional" and is not a necessary consequence of the model).

d. The threshold for after-discharge would be lowered in the sleeping brain.

e. Finally, and this point needs further investigation, there is a possibility that a certain form of $B(\rho')$ might explain "status" epileptic discharges. If this speculation were correct, low frequency stimulation might cause a discharge to be "status" whereas a high frequency stimulation might quench a status discharge if the former were applied for a time = τ . By status we mean that the discharge would last τ minutes but would resume T seconds after termination instead of going into a "sleep-like" state so far as EEG is concerned.

5. ASYNCHRONOUS ACTIONS

Two types of "virtual modification" of the stability of limit cycles can be caused by externally applied asynchronous (that is, of unrelated frequency) oscillations: asynchronous quenching and asynchronous excitation.[16] In the case of a tube oscillator, the external oscillation is applied to the control grid. The mechanism of these effects is most easily visualized as follows. The average value of the damping coefficient determines the stable amplitudes. A high frequency forcing term which is not harmonically related to the spontaneous frequency of a soft oscillator (so as to avoid resonance and entrainment effects) could cause this average to become definitely positive, and hence quench the self-sustained oscillation of a soft oscillator. In the case of a hard oscillator, the forcing term would, for a certain range of amplitudes, cause the average nonlinear damping term to become negative for $X = 0$ and the oscillator would become "soft". If the forcing term were too large, then the average would again become definitely positive. All these facts become clear when one visualizes the physical meaning of the form of the damping term.

5.1 Asynchronous Actions and EEG

Many phenomena of EEG strongly suggest virtual modification of stability effects of the brain oscillations, perhaps due to asynchronous actions. The following are examples of these.

1. Blocking of the alpha rhythm.

The mechanism for the blocking of the alpha rhythm of the wakeful brain due to visual attention undoubtedly involves the reticular formation.[17] The low-voltage high-frequency ($\approx$ 25 cps) "β" activity which is seen during the period of blocking might be a "quenching signal" from this subcortical structure, and this hypothesis could be tested by tracing the origin of the fast activity or by seeing if the application of such frequencies (or higher ones) through the reticular network can indeed quench α rhythms.

On the other hand it is interesting to recall in this connection that there is a frequency region above 100 cps in addition to the one below 10 cps in which the law $(PA)^2(PRF) = const.$ is not obeyed but where it becomes more difficult to induce paroxysmal activity. These regions may both act as "windows" for normal activity. The lower region, where $(PRF) < 10$ cps, covers most of the range of normal EEG activity. The other region $(PRF) > 100$ cps might well be a window for quenching frequencies projected to the cortex from subcortical structures.

Jouvet[18] has found that attention causes not only the rapid low voltage EEG activity but also an augmentation of response to the stimulation attended to at the subcortical level. He thus distinguishes between the neural mechanisms for "EEG arousal" and for attention. In both cases it would seem that the virtual modification of stability plays an important role not only in the suppression of the α rhythm but perhaps also in subcortical interaction and suppression observed when alternative sense modes compete for the field of attention.*

2. Epileptic effects.

EEG observations show that certain types[19] of epileptic seizures give rise to "suppression" in certain parts of the cortex and that these might be correlated in time with "excitation" effects in other parts. Such correlated effects could perhaps be due to asynchronous actions, and it is possible that a single type of high frequency activity quenches or excites cortical oscillators in a way that depends on the nature of their nonlinear stability characteristics. Such phenomena may play a role in other situations in epilepsy where "depression", "suppression" or "excitation" are involved.

Asynchronous actions may also play a role in desynchronization processes. Since entrainment of oscillations depends on the relative influence of the oscillations, anything which tends to decrease voltage output will also decrease entrainment. This might help to explain why drugs like caffeine are effective in stopping petit mal seizures. If, as we assume from the point of view of our model, the attention response of the cortex is due to quenching by an asynchronous "high" frequency, and, if caffeine's ability to increase attentiveness is attributable to an increase in the effectiveness of a quenching signal, then it is possible that this signal is also responsible for breaking up the wave-spike pattern by decreasing entrainment. Such a speculation is very tentative especially in view of the intrinsic involvement of subcortical structures in petit mal; however, an important experiment would be to see if appropriately distributed high frequency stimulation could stop a petit mal attack. A positive result in this direction might lead to the possibility of controlling certain types of seizures by electrical means rather than by means of dangerous drugs.

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*It should also be mentioned that the flicker-fusion rate of cortical neurons can be raised considerably by thalmic or reticular stimulation.[17] This can be considered as evidence of stability modifications at the unit level.

6. SUMMARY

The following table summarizes the correspondences drawn here between NLO and EEG phenomena. The resulting partial semi-quantitative model is very suggestive and promises to be quite helpful in the guidance of experiments. A more comprehensive treatment of the material in this paper and its application to experiment has been published elsewhere.[20]

Nonlinear Oscillation Phenomena	EEG Phenomena
1. Entrainment (a) Harmonic entrainment (b) Sub- and super-harmonic entrainment	1. Synchrony and hypersynchrony (a) Dips in α spectrum (b) Wavespike and locked petit mal rhythms (c) Photic flicker effect (d) Superharmonics induced by photic effect
2. Limit cycles*	2. Spontaneous and after-discharge oscillations
3. Inertial nonlinearity	3. Initiation of after-discharges (a) Constant energy requirement (b) Cortical inertia (c) Latent period following after-discharge
4. Variation of magnitude of damping coefficient as an inertial nonlinearity	4. (a) Evolution of after-discharge to low frequency, relaxation oscillation, and eventual termination in "exhaustion" (b) Explanation of spike foci (c) Possible connection with sleep phenomena such as low frequency (d) Relationships between epilepsy and sleep and quantitative relations between certain time constants in both phenomena
5. Asynchronous actions	5. (a) Alpha block and attention phenomena (b) Epileptic suppression and excitation (c) Drug effects on petit mal

*The continued self-sustained character of after-discharges which persists in spite of changes of parameters (in the model) is an example of "oscillation hysteresis" which is discussed in Appleton and van der Pol, Phil. Mag., 43:177, 1922.

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APPENDIX

MEDICAL ELECTRONICS

It is indicated in an article by N. Teterin (Electro-sleep (Elektroson), Tekhnika Molodezhi, 1957, Nr 4, pp. 28-29) that "the mechanism of the influence of current pulses upon the human brain is being studied with great care". Candidate of Medical Sciences Ye. V. Gurova works at the Scientific-Research Institute of Experimental Surgical Equipment and Tools (Nauchno-Issledovatel' skiy Institut Eksperimental'noy Khirurgicheskoy Apparatury i Instrumentov, NIIEKhAiI) on problems of stimulating sleep by electrical pulses. She has found that "electrical, rectangular-shaped current pulses of the optimum repetition rate and intensity, which are adjusted experimentally, act upon the nerve ends (skin receptors) which receive and then transform them into nerve impulses which are conducted to the central nerve system and cause, in the cortex of the large hemispheres, a distributed relaxation similar to that established during normal healthy sleep".

This same institute, in 1954, developed, under the supervision of Engineer Yu. B. Khudyy, the first prototype model of a seven-channel (for seven persons) "Electrosleep" apparatus. A modified version of this model with five channels was approved for mass-production at the Moscow Plant of Electromedical Instruments (Moskovskiy Zavod Elektromeditsinskikh Instrumentov, EMA). In this apparatus, the pulse shape is controlled visually by a built-in oscilloscope, mechanical and electronic units disconnect the set within two or three thousandths of a second in the case of malfunctioning. Repetition rate of the pulses can be varied continuously from 1 to 130 per second. Pulse duration is variable from 0.4 to 1 msec. It was found that the operation is more effective when a dc component is superimposed upon the pulse series. Amplitude value of pulses is 8 to 12 volts; dc component is adjusted to have one-fifth of the operational pulse amplitude. The operational current is 0.2 to 0.8 milliamperes. Since the resistance of patients' heads varies from 3 to 20 kiloohms and decreases by 8 to 10 percent during sleep, it was necessary to introduce a cathode follower between the power output and the output leads to the head to assure stable output-pulse voltage and provide for the patient's safety.

The latest development by the Institute is a small, portable, two-channel set operating from a power line or from batteries. Several surgical institutions have already accepted this equipment for use in combination with other means of local anesthesia. The main objective of the current work on "Electrosleep" equipment is to eliminate the need for narcotics.

Translated by A. Iwanovsky, The Development of Electronics in the USSR April-June 1957, pp. 243-245.

AF Cambridge Research Laboratories, Bedford, Mass., Rpt No. AFCRL-63-364. NONLINEAR OSCILLATIONS AND ELECTROENCEPHALOGRAPHY, Research Report, August 63, 18 p. incl. illus. and 18 refs. Unclassified Report

Pilots have noticed that when the sun is viewed through an idling propeller one can be subjected to an unusual form of vertigo. This is apparently due to an interaction of the flickering light and certain rhythmic electrical activity taking place in the brain. Other such phenomena that may occur when a man is placed in an unusual environment suggest the investigation of the nature of brain rhythm, especially in regard to malfunction. It has been found that almost all of the well-known phenomena of nonlinear oscillations have counterparts in the macroscopic neuroelectric behavior of the human brain. These correspondences permit the construction of a very suggestive "phenomenological" theory to "explain" electroencephalographic data in terms of nonlinear stability which in turn reveals unsuspected relationships in the data and suggests new experiments.

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2. Nonlinear Oscillations
3. Neurology
4. Nonlinear Circuits

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