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Department of Meteorology and Oceanography
Geophysical Sciences Laboratory Report No. 63-11

ESTIMATES OF THE POWER SPECTRA FOR FULLY DEVELOPED SEAS FOR WIND SPEEDS OF 20 TO 40 KNOTS

by

Lionel Moskowitz

Technical Report Prepared for
U. S. Naval Oceanographic Office
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Table of Contents

Abstract

Various criteria pertaining to the synoptic situation are presented in order to determine when a fully developed wind generated sea might be found in the North Atlantic Ocean. 460 wave records, corresponding to various synoptic situations, were digitized and spectrally analyzed as a first step in the preparation of a climatology of ocean wave spectra. The wave records were taken by the Ocean Weather Ships of the United Kingdom by means of a Tucker shipborne wave recorder.

Selected subsets from the available spectra based on these synoptic criteria were averaged in order to produce spectra for various wind speeds. These selected subsets were tested to see if they came from the same population by means of the Kolmogorov-Smirnov test, and the results show that, within the accuracy to be expected because of inaccuracies in the winds, the samples chosen represent fully developed seas. A second subset chosen at random without using these criteria was tested and the results showed that wind speed alone does not properly characterize the sea state.

A nested family of spectra was obtained for wind speeds of 20, 25, 30, 35, and 40 knots in which the frequency of the maximum appeared to be inversely proportional to the wind speed and the significant height was proportional to the square of the wind speed. The spectra and the results deducible from the spectra yield results that appear to be a compromise among the various published theoretical forms for the spectra and the equations for the significant height of a fully developed sea.

Introduction

In recent years, the study of wind generated ocean waves and ocean wave forecasting has become important. Meteorologists and oceanographers are, only now, just beginning to learn how energy is transferred to the waves. The use of wave spectra, both theoretical and observed, has contributed greatly to the developments in the field of ocean waves. The wind effect on the sea surface is not simple, basically due to turbulence in both sea and air. It can be seen that the waves of different heights and periods occur. A sea is then assumed to be an infinite number of sinusoidal waves of different amplitudes and periods traveling in different directions. The summation of all these waves produces the visible sea conditions. Since an infinite number of periods (frequencies) are present, a spectrum is implied.

At present there is disagreement as to the form of the wave spectrum of a fully developed sea for a particular wind speed. It is known that as the wind speed increases, if fetch (the distance over water a wind of constant speed and direction travels) and duration are adequate, the area under the spectral curve (total variance) will increase and shift toward lower frequencies (longer periods). With a constant wind speed, and an adequate fetch and duration, a point will be reached where the spectrum will no longer grow. When this point is reached the sea is called fully developed. The spectrum will change only if additional energy is either added to or taken away from the waves.

In this paper an attempt will be made to determine the wave spectra for fully developed seas for various wind speeds. Various synoptic conditions, for which wave records were known to exist, were chosen. The synoptic situations cover the five year period April 1955 through March 1960.

History

The study of wind generated ocean waves and ocean wave forecasting first became important during World War II. Knowledge of sea, swell, and surf conditions was necessary for many applications. The methods devised by Sverdrup and Munk (1947) were improvements over previous empirical techniques. The empirical techniques prior to those of Sverdrup and Munk produced inconsistent results. The results of Sverdrup and Munk pertained only to the significant height and period and therefore could not give a complete description of the waves.

Many scientists have studied ocean waves using the concepts of a wave spectrum. Early results from these studies pertaining to ocean waves were not very good because the instruments were set in relatively shallow water. This would tend to produce a distorted spectrum because the distribution of spectral energy in shallow water is different from that in the deep ocean. Although inconclusive results were obtained from these studies, they paved the way for future developments. These future developments were along the lines of specifying mathematically, the shape and form of the spectrum.

In 1952, Tucker (1956) developed a shipborne wave recorder for use on board the Ocean Weather Ships of Great Britain. (Several other countries have since installed the Tucker recorder on some of their research vessels.) This instrument produces a continuous record of the height of the waves passing by the ship. The recorder does this by a combination of pressure recorders and accelerometers. These measure the varying pressure of the overlying water at two points on opposite sides of the ship. Coupled accelerometers make allowance for the vertical motion of the measuring points. From a wave record a spectrum may be calculated by either digital (Blackman and Tukey, 1958) or analog techniques.

The Neumann (1953) spectrum represented a great achievement in ocean wave forecasting. Neumann used many visual observations and the results of Longuet-Higgins (1952) to arrive at his results. The results were later incorporated into a wave forecasting manual (Pierson, Neumann, and James (1955), henceforth denoted as PNJ). According to PNJ, in order to forecast the state of the sea, one needs only the co-cumulative spectral curves (CCS curves) and the wind speed with some knowledge of how the wind has changed with time at the forecast site. Additional aids are given which allow a detailed forecast of the sea conditions to be made.

Darbyshire (1955, 1956, 1959) studied many wave records and drew conclusions that differed greatly from those of Neumann. First, Darbyshire concluded that the significant height, $\overline{H}_{1/3}$ (the average height of the one-third highest waves) was proportional to the square of the wind speed. Neumann concluded that the significant height was proportional to the 2.5 power of the wind speed. Darbyshire also found that the sea would grow faster and reach full development sooner than the sea predicted by PNJ. As an example, assuming a 40-knot wind, Darbyshire concluded that saturation (fully developed state) would be reached after a duration of 12 hours and a fetch of approximately 200 miles. PNJ predicted full development for a 40-knot wind after a duration of 42 hours and a fetch of at least 710 miles.

The large differences between the results of Darbyshire, PNJ, and other methods were the basis for comparisons of the different wave forecasting methods (Rattray and Burt, 1956; Neumann and Pierson, 1957; Darbyshire, 1957; and Walden, 1963). Pierson (1959), DeLeonibus (1962), and Bretschneider, Crutcher, et al (1963) have studied the wave spectra of waves generated by particular storms in the North Atlantic Ocean. A recent review "Ocean Wave Spectra" (Proceedings of a conference, Easton, Md., 1961) shows how widely the various attempts to describe the spectra of fully developed seas differ from one another.

Data

A search of the Daily Series Synoptic Weather Maps (U.S. Weather Bureau) for the period April 1955 through March 1960 was undertaken to find periods of time when wind direction and speed were relatively constant. The particular locations examined were those of the Ocean Weather Ships of Great Britain. The geographical locations of the Ocean Weather Ships are:

Position A - 62°N, 33°W

Position I - 59°N, 19°W

Position J - 52.5°N, 20°W

Position K - 45°N, 16°W

A further study of the data chosen from the Daily Series Synoptic Weather Maps was then made with the aid of the Six-Hourly Synoptic Weather Maps of the North Atlantic Ocean (U.S. Weather Bureau, April 1955-March 1960) and a table (Table I) showing the location of the shipborne wave recorder (Tucker, 1956). This study yielded rules for the selection of wave data that were recorded at times when particular wind speeds, as determined from the synoptic charts, maintained the same direction for as long a duration as possible.

Table I. Date of sailing to station and date of return to port of the Ocean Weather Ships.

OWS WEATHER EXPLORER

Date of sailing	Station	Date of return
3 Apr 1955	I	2 May 1955
21 May 1955	J	16 June 1955
29 June 1955	A	2 Aug 1955
25 Aug 1955	K	27 Sep 1955
12 Oct 1955	I	10 Nov 1955
26 Nov 1955	J	23 Dec 1955
7 Jan 1956	A	11 Feb 1956
5 Mar 1956	K	6 Apr 1956
21 Apr 1956	I	20 May 1956
5 June 1956	J	4 July 1956
26 July 1956	I	23 Aug 1956
9 Sep 1956	J	8 Oct 1956
30 Oct 1956	I	28 Nov 1956
18 Dec 1956	I	15 Jan 1957
3 Feb 1957	I	4 Mar 1956
20 Mar 1957	J	18 Apr 1957
1 May 1957	A	4 June 1957
27 June 1957	I	26 July 1957
14 Aug 1957	I	12 Sep 1957
28 Sep 1957	I	28 Oct 1957
9 Nov 1957	A	13 Dec 1957
5 Jan 1958	I	3 Feb 1958
22 Feb 1958	I	24 Mar 1958
8 Apr 1958	J	9 May 1958

Vessel withdrawn from service.

Table I (continued)

OWS WEATHER REPORTER

Date of sailing	Station	Date of return
20 May 1958	A	23 June 1958
17 July 1958	I	9 Aug 1958
28 Aug 1958	J	24 Sep 1958
14 Oct 1958	I	10 Nov 1958
24 Nov 1958	J	23 Dec 1958
4 Jan 1959	A	6 Feb 1959
3 Mar 1959	K	4 Apr 1959
18 Apr 1959	I	18 May 1959
2 June 1959	J	2 July 1959
15 July 1959	A	18 Aug 1959
11 Sep 1959	K	12 Oct 1959
27 Oct 1959	I	26 Nov 1959
11 Dec 1959	J	9 Jan 1960
23 Jan 1960	A	26 Feb 1960

The particular wind speeds in this study begin at 20 knots and increase in intervals of 5 knots. Cases of wind speeds less than 20 knots were omitted because of the difficulty of finding conditions when duration, and especially direction, were constant for long periods of time. Except for the summer months it is difficult to say that the sea conditions, for light winds, are truly wind generated. Usually the sea conditions are due to a stronger wind system that had previously passed through the observational area.

In order to determine which cases should be selected and in order to say that the state of the sea was due solely to the wind, for these cases, certain synoptic wind criteria were used.

The first step was to examine the synoptic charts at the location of the Ocean Weather Ships for winds of long duration, as previously described. The wind speed differences during the mean wind period were held as small as possible. Wind speeds at the beginning and end of the period had to be less than the mean wind speed. A steadily increasing wind speed would produce a building or developing sea and, it was hoped, would rule out cases involving remnants of previous storms or periods of higher winds prior to the period studied. In addition, effects of previous high winds would not be masked within the observed spectra. The presence of swell is very difficult to determine from the use of the synoptic charts, and therefore could not be eliminated. Spectra with swell were eliminated by inspection after the wave records were analyzed.

The wind direction was also examined to see that it did not deviate more than ± 45° from the mean wind direction. The purpose of maintaining a predominantly constant wind direction would tend to eliminate effects of cross or opposing seas. However, in some cases involving high wind speeds, the wind deviated by slightly more than $\pm 45°$ from the mean wind direction.

After the wind data had been compiled the wave records for the selected periods were obtained from the National Institute of Oceanography. All the wave records studied were taken by either the OWS Weather Explorer or the OWS Weather Reporter and the logs of these ships were used to determine the wind speeds as given to the nearest knot.

The wave records were examined to find the speed of the ship which is also entered on every wave record. If it was found that the ship was moving at a speed greater than 2 knots, the record was not used in the final analyses. At observation time (usually every three hours) the ship should be in a stopped position, but sea conditions may force the ship to speed up. It was believed that at a speed less than 2 knots there would be little or no frequency distortion of the spectra (Pierson, 1959). [The spectra obtained when the ship is moving at a speed greater than 2 knots compared with the spectra obtained when the ship is relatively stationary under the same conditions, would show a shift in the frequency of the spectral maximum. This shift would be towards higher frequencies if the ship were heading into the waves, and toward the lower frequencies if the ship were heading away from the waves.]

Approximately 1000 wave records were obtained from the National Institute of Oceanography and 420 were selected with 40 more records available from another study (Bretschneider, Crutcher, et al, 1962), bringing the total to 460. Every wave record was approximately fifteen minutes long although several were as short as seven minutes. Bounds were then set on each record just above the highest crest and below the lowest trough and the records were read to an accuracy of one part in a thousand at $1\frac{1}{2}$ second intervals. Each wave record was thus represented as a time series of approximately 600 points. The time series was then spectrally analyzed on a CDC 1604 computer so as to estimate the energy spectrum of the waves at 60 points over the frequency range 0 to 0.333 cycles per second by means of the procedures of Blackman and Tukey (1958).

The spectral estimates still had to be corrected for noise from both the original wave records and the digitization procedures. Also, at the high frequency end a smoothing operator was applied to comply with results from other investigators (Bretschneider, Crutcher, et al, 1962) who used this type of analysis. The noise level was determined by averaging the last ten values of the smoothed spectrum and then this average was subtracted from each of the spectral estimates of the smoothed spectrum. The resulting spectrum was then multiplied by the calibration curve of the recorder for the particular ship that took the record to determine the final spectrum. The original values that were computed represented the spectral estimates in terms of the resolution of the variance of the wave record into frequency intervals.

The results of these computations should yield fairly reliable spectral estimates for frequencies ranging from 0 to 0.25 cycles per second. Since the Tucker recorder is not accurate at higher frequencies than 0.25 cycles per second, the values at the high frequencies should not be used to decide on any of the features of this end of the spectrum.

<u>The synoptically chosen subsets</u>

In order to evaluate a mean spectrum for each of the wind speed subsets, the records chosen for evaluation had to conform to the synoptic criteria as closely as possible. By the time all the criteria were applied, not many cases at exactly the chosen wind speeds (20, 25, 30, etc.) were available. It was decided then to allow the wind speed at observation time, as determined from the logs of the Ocean Weather Ships, to vary within ± 2 knots of the mean wind speed. In very few cases the wind speed prior to observation time was slightly higher [lower] than two knots above [below] the mean wind speed. All the synoptically chosen spectra were examined to be sure that there was no contamination from the presence of swell. Darbyshire (1959) also used spectra free of swell and accepted cases of higher wind speeds prior to observation time. Some of the records analyzed were the same as those used by Darbyshire (1959). It is of interest to note that different significant wave heights were obtained using the same data. The results of Darbyshire were lower because he used an earlier calibration of the Tucker recorder. This calibration has been improved on the basis of recent comparisons with the buoy described by Longuet-Higgins, Cartwright, and Smith (1963).

The spectra chosen according to the synoptic criteria for each subset are given in Table 2a along with additional synoptic data. The spectra as well as all the processed wave records obtained from the National Institute of Oceanography are given in graphical and tabular form in several technical reports (Moskowitz, Pierson, and Mehr, 1962, 1963).

Although little has been said of fetch, estimates of the fetches for the synoptically chosen spectra are given in Table 2a. As pointed out by Walden (1963) fetch requirements for the occurrence of fully developed seas for lighter winds (up to about 30 knots) are frequent in most areas. Wind speeds greater than 30 knots rarely obtain fetches great enough to

Table 2a. Synoptically chosen subsets .

Record no.	Date	Time	Position	Wind speed	$\overline{H}_{1/3}$	Upper $\overline{H}_{1/3}$	Lower $\overline{H}_{1/3}$	$T_A - T_s$	Duration	Fetch NM	n
		20 kts									12
JHC 015	19/9/55	3	K	21	8.1	8.3	7.2	-0.5°F	6	250	
JHC 016	19/9/55	9	K	21	8.0	8.4	7.5	-0.2°	12	250	
JHC 063	19/6/56	14	J	20	7.1	7.3	6.2	0.7°	6	175	
JHC 064	19/6/56	18	J	19	7.9	8.5	7.3	0.7°	9	200	
JHC 065	20/6/56	0	J	20	7.0	7.5	6.4	0.2°	15	250	
JHC 066	20/6/56	3	J	18	6.7	7.2	6.2	-0.7°	18	250	
JHC 067	20/6/56	6	J	19	7.1	7.3	6.3	-0.3°	21	250	
JHC 068	20/6/56	9	J	20	7.2	7.8	6.7	0.1°	24	200	
JHC 069	20/6/56	12	J	19	7.3	7.5	6.4	-0.5°	27	100	
DL 45	16/1/59	15	A	22	7.5	8.2	6.9	-3.6°	6	250	
DL 88	2/10/59	9	K	21	8.1	8.8	7.5	-1.2°	6	125	
DL 97	25/7/59	0	A	20	6.8	7.1	6.1	-0.9°	18	125	
		25 kts									8
JHC 029	7/6/55	18	J	24	13.3	13.9	11.8	0.3°	6	150	
JHC 030	7/6/55	21	J	25	14.3	15.1	12.8	0.4°	9	175	
JHC 032	8/6/55	12	J	26	13.5	14.2	12.2	-0.1°	21	300	
JHC 033	8/6/55	15	J	25	13.1	13.6	11.5	-0.8°	24	450	
JHC 116	23/5/57	3	A	27	13.8	14.8	12.5	0.3°	9	100	
JHC 117	23/5/57	6	A	25	13.4	14.6	12.4	-1.6°	12	125	
JHC 127	18/4/58	6	J	24	11.5	11.9	10.3	-0.3°	12	700	
JHC 128	18/4/58	12	J	25	12.0	12.5	10.6	1.1°	18	350	

Table 2a (continued)

Record no	Date	Time	Position	Wind speed	$H_{1/3}$	Upper $H_{1/3}$	Lower $H_{1/3}$	T_A-T_s	Duration	Fetch NM	n
		30 kts									12
JH 36	23/1/59	15	A	32	19.2	20.8	17.7	-4.8°F	6	150	
JH 37	24/1/59	18	A	32	21.0	23.1	18.9	-3.0°	6	200	
DL 8	20/1/58	21	I	30	20.1	21.6	18.7	-17.9°	6	200	
DL 39	29/10/59	9	I	29	19.9	21.9	18.0	-7.0°	6	125	
DL 40	29/10/59	18	I	30	20.3	22.1	18.6	-6.8°	15	125	
JHC 002	23/11/56	15	I	32	19.7	21.0	18.4	-2.2°	6	150	
JHC 084	20/3/56	18	K	29	20.6	22.0	18.7	-3.4°	12	125	
JHC 086	21/3/56	0	K	29	21.2	21.9	18.0	-1.9°	18	125	
JHC 098	4/11/55	0	I	28	19.0	19.6	16.9	-1.9°	9	175	
JHC 105	5/11/55	6	I	32	22.5	23.1	19.9	0.4°	6	125	
JHC 106	5/11/55	15	I	30	21.8	22.2	18.5	0.8°	15	100	
JHC 132	19/4/58	9	J	30	20.5	20.9	17.7	-2.1°	18	225	
		35 kts									8
JHC 050	6/11/56	18	I	34	22.8	23.8	20.2	0.2°	12	450	
JHC 104	5/11/55	3	I	35	24.4	22.8	19.4	-0.1°	6	125	
JHC 147	11/12/58	18	J	35	22.2	23.7	19.9	-5.7°	6	100	
DL 7	20/1/58	15	I	35	23.0	24.8	21.4	-18.4°	6	225	
DL 18	24/4/58	15	J	35	24.6	26.7	22.7	-3.9°	18	400	
DL 19	24/4/58	18	J	35	24.0	26.6	22.6	-3.3°	21	400	
DL 20	24/4/58	21	J	35	25.2	27.4	23.1	-4.0°	24	425	
DL 27	25/4/58	18	J	36	24.0	26.0	22.2	1.1°	3	300	

Table 2a (continued)

Record no.	Date	Time	Posi-tion	Wind speed	$\bar{H}_{1/3}$	Upper $\bar{H}_{1/3}$	Lower $\bar{H}_{1/3}$	T_A-T_s	Dura-tion	Fetch NM	n
		40 kts									14
JHC 012	25/11/56	3	I	42	31.6	34.3	29.1	-1.0°F	6	350	
JHC 013	25/11/56	9	I	38	34.5	35.6	29.3	-2.4°	12	300	
JHC 076	11/5/56	9	I	40	33.3	35.5	30.0	-2.2°	15	150	
JHC 077	11/5/56	12	I	42	34.4	35.6	30.7	0.6°	18	175	
JHC 078	11/5/56	14	I	41	33.5	35.0	30.0	-0.2°	20	175	
JHC 079	11/5/56	15	I	40	31.1	32.8	28.0	-0.2°	21	175	
JHC 081	11/.5/56	21	I	38	31.1	33.5	28.3	-1.1°	27	150	
JHC 092	21/3/56	21	K	39	32.4	32.5	28.1	-1.5°	15	150	
JHC 138	10/12/58	9	J	40	31.0	34.1	28.2	-8.3°	6	175	
JHC 151	12/12/58	6	J	40	29.4	32.5	26.2	-11.8°	9	700	
JHC 153	12/12/58	12	J	40	29.2	31.8	26.2	-10.6°	15	450	
JHC 215	21/11/56	3	I	42	33.1	36.1	30.3	-2.3°	15	100	
JHA 16	17/12/59	21	J	41	31.5	34.5	28.7	---	21	425	
JHA 17	18/12/59	0	J	40	35.3	39.1	31.8	---	24	250	

Table 2b. Randomly chosen subsets.

Record no.	Date	Time	Position	Wind speed	$\bar{H}_{1/3}$	Upper $\bar{H}_{1/3}$	Lower $\bar{H}_{1/3}$	n
		20 kts						12
JHA 19	18/12/59	6	J	20	20.6	22.5	18.9	
JHB 15	24/12/59	12	J	20	17.4	19.1	15.9	
JHB 16	24/12/59	18	J	20	14.8	16.4	13.3	
JHB 18	25/12/59	6	J	20	14.6	15.9	13.3	
JHB 26	26/12/59	12	J	20	26.9	29.9	24.2	
JHC 028	19/6/55	14	J	20	11.3	12.2	10.5	
JHC 063	19/6/55	14	J	20	6.7	7.3	6.2	
JHC 068	20/6/55	9	J	20	7.2	7.8	6.7	
JHC 144	11/12/58	3	J	20	18.8	20.7	17.1	
JHC 120	16/4/58	12	J	20	6.3	6.8	5.9	
JHC 174	12/12/55	15	J	20	12.2	13.1	11.3	
JHC 205	6/3/68	18	I	20	18.4	19.8	17.1	
		25 kts						8
JHA 30	19/12/59	15	J	25	13.4	14.8	12.2	
JHB 20	25/12/59	15	J	25	21.2	23.4	10.2	
JHB 24	26/12/59	6	J	25	29.9	33.4	26.8	
JHB 31	27/12/59	15	J	25	16.1	17.6	14.7	
JHC 020	20/9/55	6	K	25	11.4	12.3	10.6	
JHC 115	22/5/57	0	A	25	11.4	12.3	10.5	
DL 43	17/1/59	6	A	25	15.0	16.9	13.3	
DL 99	25/7/59	6	A	25	8.7	9.4	8.0	

Table 2b (continued)

Record no.	Date	Time	Position	Wind speed	$\bar{H}_{1/3}$	Upper $\bar{H}_{1/3}$	Lower $\bar{H}_{1/3}$	n
		30 kts						12
JHA 25	19/12/59	0	J	30	16.4	17.9	15.1	
JHB 14	24/12/59	6	J	30	25.5	27.7	23.4	
JHC 083	12/5/56	3	I	30	23.2	25.5	21.1	
JHC 143	11/12/58	0	J	30	24.3	26.8	22.1	
JHC 154	12/12/58	15	J	30	30.7	34.1	27.6	
DL 3	19/1/58	12	I	30	16.4	17.8	15.1	
DL 8	20/1/58	21	I	30	20.1	21.6	18.7	
DL 29	26/4/58	0	J	30	17.7	19.3	16.3	
DL 30	26/4/58	3	J	30	16.1	17.4	14.9	
DL 108	6/4/57	15	J	30	16.0	17.2	14.9	
DL 114	7/4/57	18	J	30	16.5	18.1	15.1	
DL 115	8/4/57	3	J	30	17.2	18.8	15.8	
		35 kts						
JH 3	17/1/59	15	A	35	14.5	15.8	13.4	
JHC 010	24/11/56	18	I	35	31.4	34.0	29.0	
JHC 014	25/11/56	12	I	35	29.4	32.0	27.1	
JHC 087	21/3/56	3	K	35	23.7	25.8	21.8	
JHC 107	5/11/55	21	I	35	19.2	20.9	17.7	
JHC 217	21/11/56	12	I	35	30.4	33.2	27.8	
DL 7	20/1/58	15	I	35	23.0	24.8	21.4	
DL 54	8/11/59	21	I	35	25.0	27.5	22.8	

Table 2b (continued)

Record no.	Date	Time	Position	Wind speed	$\bar{H}_{1/3}$	Upper $\bar{H}_{1/3}$	Lower $\bar{H}_{1/3}$	n
		40 kts						14
JH 28	29/1/59	0	A	40	28.2	30.9	25.7	
JH 43	19/12/58	0	J	40	27.1	29.7	24.7	
JHA 17	18/12/59	0	J	40	35.2	39.1	31.8	
JHB 4	22/12/59	15	J	40	38.6	42.2	35.2	
JHB 12	23/12/59	21	J	40	27.6	30.0	25.3	
JHC 011	25/11/56	0	I	40	26.0	28.2	23.9	
JHC 149	12/12/58	0	J	40	26.8	29.7	24.2	
JHC 156	6/6/59	0	J	40	18.9	20.6	17.4	
JHC 200	5/3/58	15	I	40	20.5	22.2	18.9	
JHC 213	20/11/56	18	I	40	26.8	29.2	24.7	
DL 46	7/11/59	21	I	40	25.6	28.1	23.4	
DL 48	7/11/59	15	I	40	23.7	25.8	21.7	
DL 56	9/11/59	6	I	40	28.3	30.9	25.9	
DL 82	31/1/60	12	A	40	22.1	24.2	20.2	

produce fully developed seas. The minimum fetch requirements necessary to produce fully developed seas for various wind speeds as given by PNJ (1955) and Darbyshire (1959) are:

v	PNJ	Darbyshire
20	.75 miles	100-200 miles
25	155 "	"
30	280 "	"
35	460 "	"
40	710 "	"

The values tabulated in Table 2a for fetch and duration represent the distance upwind over which the wind was substantially the value chosen at the time of observation and the duration at the ship of that wind speed. Before the number of hours indicated, the wind would not be zero, but perhaps five or ten knots lower (and before that even lower) than the tabulated value, and upwind of the distance indicated the winds would be different from the value tabulated (usually less), but not zero. The durations and fetches do not therefore satisfy the theoretical requirement that they describe the time required and the distance needed to generate a fully developed sea starting from zero wave conditions. For the higher winds in particular, the tabulated values often represent the time required and the distances involved for a sea raised to a given height by a wind of lesser velocity to grow to full development at the higher velocity. Further study will be needed to decide these questions.

No subsets were found for wind speeds greater than 40 knots. All but two of the wave records obtained when the winds were greater than 40 knots showed waves that were no higher than the waves for 40 knots.

One spectrum for a wind speed of 45 knots, JHC 41, and one spectrum for a wind speed of 50 knots, JHC 141, appear to reach the fully developed state. From all the available data it was possible to find several wave records at the lower wind speeds that, according to PNJ, represented saturated conditions. It was not possible to find any records for higher winds that approached the fully developed state as given by PNJ.

It is evident from Table 2a that only 54 spectra were selected for further evaluation from a total of 460 spectra. Of the 460 spectra, 101 were not chosen from the application of the criteria to the synoptic conditions. These were obtained for use in other studies. The 101 spectra are comprised of a set of 72 from the period 15-28 December 1959 and a set of 29 from a hurricane in September 1961. Only two spectra were chosen from the set of 72 spectra, and no spectra were used from the set of 29 because wind speed estimates could be determined only in terms of the Beaufort number. When the higher wind speed subsets (see above), the spectra pertaining to the beginning and end of the mean wind speed periods, and those spectra containing extraneous swell were removed, the remaining spectra (plus the two spectra from the set of 72 already discussed) are those that represent the spectra for fully developed seas for the various mean wind speeds as given in Table 2a according to the synoptic criteria that were used.

The synoptically chosen spectra were then analyzed to determine a mean spectrum for each subset. These mean spectra are tabulated in Table 3 and graphed in figure 1. The frequency is given by dividing the number in the first column by 180. The values tabulated are in units of $(FT)^2$ and represent an estimate of the integral of the spectral density

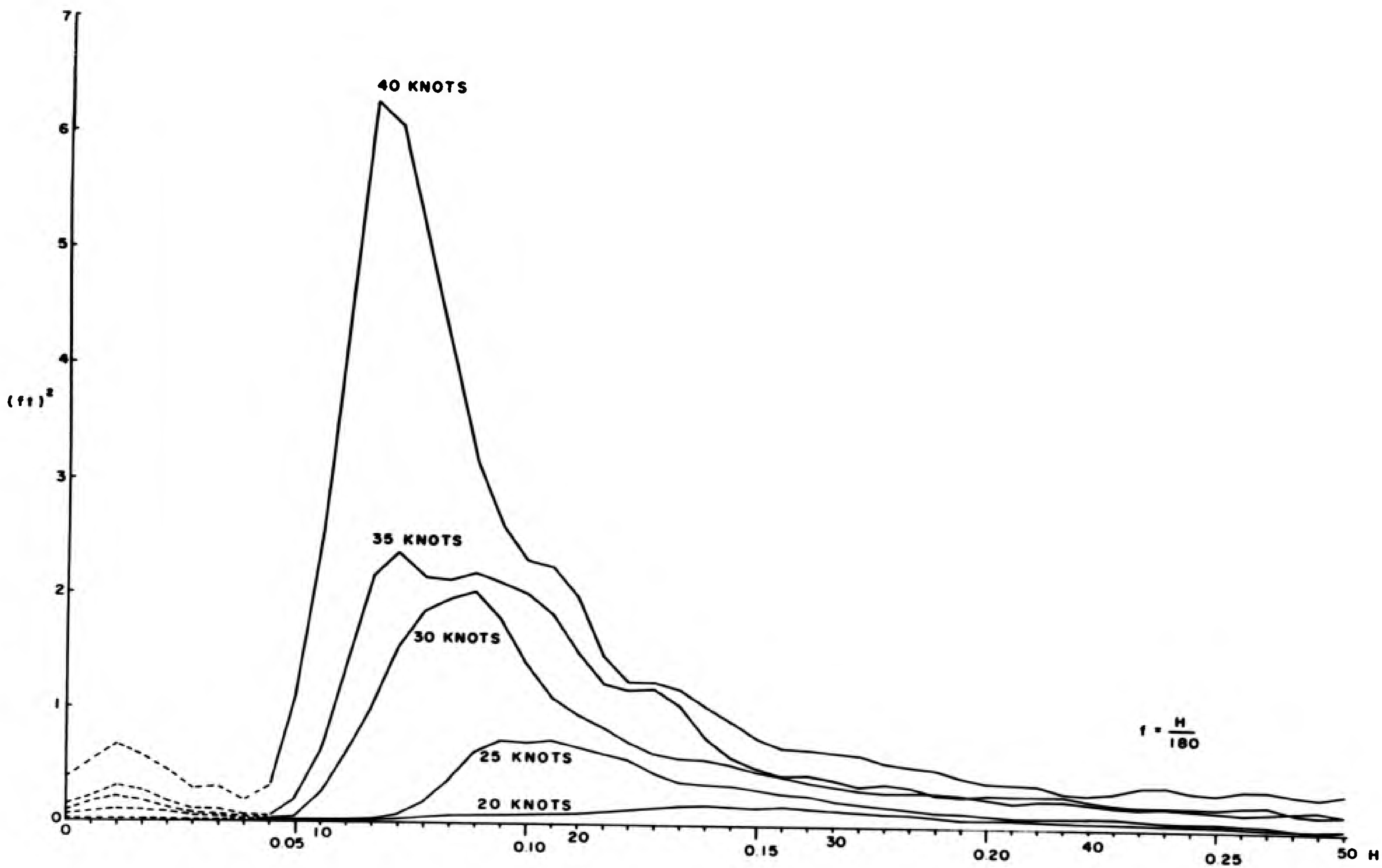

FIG. I GRAPHS OF THE SYNOPTICALLY CHOSEN SPECTRA

TABLE 3a. SYNOPTICALLY CHOSEN SPECTRA

	n = 12, v = 20 (19.5) kts				n = 8, v = 25 (25.8) kts				n = 12, v = 30 (31.6) kts				n = 8, v = 35 (33.6) kts				n = 14, v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P. F.
0	.0195	.0009	.7490	F	.0601	.0030	.6143	F	.0933	.0055	.8182	F	.1415	.0070	.8889	F	.3797	.1282	.8355	F
1	.0188	.0006	.6319	F	.0841	.0078	.6074	F	.1618	.0087	.3313	P	.2167	.0138	.4384	F	.5309	.1658	.3526	F
2	.0154	.0002	.5264	F	.1024	.0151	.5287	F	.2081	.0119	.3280	P	.3063	.0283	.4288	F	.6671	.1797	.3612	F
3	.0101	.0001	.5208	F	.0983	.0097	.4966	F	.1753	.0098	.3930	F	.2670	.0160	.3384	P	.5790	.1509	.3531	F
4	.0061	.0000	.3479	F	.0744	.0039	.4790	F	.1160	.0049	.3103	P	.1701	.0026	.2658	P	.4484	.0599	.2481	P
5	.0034	.0000	.2696	P	.0475	.0024	.5698	F	.0683	.0019	.3864	F	.1038	.0006	.2759	P	.2841	.0149	.2391	P
6	.0032	.0000	.4162	F	.0430	.0037	.6683	F	.0634	.0014	.3865	F	.1001	.0015	.3053	P	.3091	.0255	.3301	F
7	.0025	.0000	.4167	F	.0226	.0009	.6758	F	.0391	.0005	.2989	P	.0568	.0006	.3621	P	.1843	.0137	.3249	F
8	.0025	.0000	.5864	F	.0133	.0003	.6863	F	.0290	.0006	.4061	P	.0528	.0011	.4777	F	.3172	.1591	.5180	F
9	.0030	.0001	.8081	F	.0086	.0001	.7744	F	.0491	.0020	.5400	P	.1909	.0412	.6250	F	1.1197	3.3802	.7480	F
10	.0075	.0004	.9000	F	.0079	.0001	.7605	F	.2413	.1288	.6802	P	.6181	.5519	.6625	F	2.4644	7.3835	.5758	F
11	.0121	.0010	.9056	F	.0101	.0001	.4865	F	.6043	.9813	.7373	P	1.3742	2.1744	.7289	F	4.3553	4.4986	.2800	P
12	.0134	.0006	.7792	F	.0228	.0003	.4259	F	1.0174	1.2829	.5225	P	2.1524	2.6901	.4978	F	6.2600	7.6949	.1581	P
13	.0229	.0011	.7585	F	.0694	.0026	.5447	F	1.5269	.6834	.2500	P	2.3646	2.0511	.4563	F	6.0404	8.4801	.2385	P
14	.0447	.0058	.8202	F	.1780	.0207	.6430	F	1.8469	.4581	.2242	P	2.1503	.9754	.3536	P	4.9884	2.8723	.2378	P
15	.0634	.0151	.8204	F	.3787	.0831	.5240	F	1.9556	.1722	.2271	P	2.1250	.5166	.2845	P	4.1104	2.2609	.2348	P
16	.0632	.0171	.8117	F	.6244	.1204	.5225	F	2.0255	.5126	.2825	P	2.1917	.5310	.3072	P	3.1776	2.1790	.2670	P
17	.0673	.0150	.7697	F	.7270	.0968	.3177	P	1.7871	.4118	.2751	P	2.1138	.3631	.2271	P	2.6325	1.5151	.2147	P
18	.0827	.0189	.6770	F	.7091	.1191	.3653	P	1.3800	.1575	.2220	P	2.0063	.4546	.3486	P	2.3042	1.5944	.2216	P
19	.0881	.0106	.5589	F	.7223	.1500	.4436	F	1.0981	.0896	.1966	P	1.8438	.3531	.3051	P	2.2431	.9250	.2262	P
20	.0964	.0038	.3008	P	.6708	.0487	.3065	P	.9477	.0659	.1741	P	1.4942	.2238	.3408	P	1.9756	.3660	.1709	P
21	.1105	.0025	.2790	P	.6049	.0347	.3205	P	.8418	.0357	.1862	P	1.2218	.1023	.2080	P	1.4772	.1309	.1865	P
22	.1352	.0017	.2072	P	.5643	.0922	.4110	F	.7135	.0349	.1603	P	1.1738	.0619	.2716	P	1.2430	.1370	.2186	P
23	.1567	.0030	.2153	P	.4464	.0346	.3809	P	.6192	.0417	.2019	P	1.1850	.1202	.3475	P	1.2368	.2634	.2590	P
24	.1690	.0036	.1878	P	.3636	.0096	.2325	P	.5711	.0289	.1918	P	1.0451	.1591	.3809	P	1.1732	.4425	.3359	F
25	.1779	.0053	.2830	P	.3571	.0223	.2491	P	.5715	.0251	.2819	P	.7595	.0714	.2641	P	1.0278	.2538	.2973	P
26	.1530	.0037	.2124	P	.3430	.0194	.2968	P	.5245	.0259	.1932	P	.5834	.0385	.2427	P	.8955	.1277	.2272	P
27	.1506	.0046	.2427	P	.3015	.0085	.2326	P	.4624	.0542	.3475	F	.4928	.0313	.3071	P	.7560	.0790	.2206	P
28	.1673	.0062	.2597	P	.2775	.0040	.3397	P	.4226	.0685	.4676	F	.4305	.0363	.3734	P	.6723	.0834	.2247	P
29	.1478	.0047	.2613	P	.2649	.0071	.3506	P	.3831	.0339	.4164	F	.4490	.0446	.4504	F	.6590	.0714	.2162	P
30	.1320	.0027	.2192	P	.2196	.0041	.1993	P	.3473	.0170	.2035	P	.4052	.0409	.4227	F	.6391	.0669	.2816	P

TABLE 3a (continued)

	n = 12				n = 8				n = 12				n = 8				n = 14			
	v = 20 (19.5) kts				v = 25 (25.8) kts				v = 30 (31.6) kts				v = 35 (33.6) kts				v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
31	.1243	.0021	.2235	P	.1878	.0028	.3099	P	.3034	.0180	.2621	P	.3531	.0366	.5070	F	.6146	.0657	.2612	P
32	.1146	.0027	.2459	P	.1734	.0027	.2480	P	.2992	.0228	.3736	F	.3786	.0178	.3138	P	.5468	.0571	.3029	P
33	.1041	.0027	.2979	P	.1461	.0033	.2775	P	.3011	.0134	.2507	P	.3456	.0119	.2475	P	.5235	.0711	.3141	F
34	.0842	.0014	.2794	P	.1307	.0031	.4025	P	.2921	.0118	.3271	P	.2843	.0172	.3454	P	.4973	.0678	.2971	P
35	.0697	.0012	.2499	P	.1240	.0028	.4143	F	.2832	.0057	.1901	P	.2676	.0066	.3909	P	.4339	.0485	.3674	F
36	.0759	.0019	.3204	P	.0990	.0020	.3760	P	.2422	.0032	.1840	P	.2807	.0042	.3096	P	.3826	.0249	.2219	P
37	.0779	.0016	.2700	P	.0782	.0008	.2803	P	.2160	.0069	.1957	P	.2850	.0064	.2390	P	.3790	.0223	.1428	P
38	.0638	.0009	.3121	P	.0806	.0010	.3355	P	.2351	.0171	.3390	F	.2864	.0161	.2516	P	.3636	.0250	.2573	P
39	.0556	.0021	.4395	F	.0947	.0037	.3890	P	.2380	.0181	.2781	P	.2833	.0092	.2340	P	.3106	.0173	.2382	P
40	.0538	.0012	.3970	F	.0998	.0044	.5557	F	.2090	.0122	.3599	F	.2454	.0093	.4162	F	.2920	.0108	.2733	P
41	.0496	.0007	.2664	P	.0942	.0032	.4763	F	.1808	.0103	.3683	F	.2092	.0128	.3832	P	.3181	.0150	.1933	P
42	.0445	.0006	.2571	P	.0792	.0015	.3825	P	.1721	.0111	.2974	P	.1942	.0095	.4322	F	.3607	.0280	.2605	P
43	.0422	.0008	.2458	P	.0668	.0014	.3707	P	.1804	.0093	.3647	F	.1913	.0041	.2393	P	.3693	.0390	.3257	F
44	.0439	.0016	.3668	F	.0646	.0020	.3750	P	.1732	.0137	.4103	F	.1816	.0051	.2905	P	.3244	.0622	.4323	F
45	.0431	.0017	.4410	F	.0624	.0024	.4849	F	.1484	.0154	.4417	F	.1878	.0071	.3611	P	.3166	.0744	.4313	F
46	.0342	.0007	.4670	F	.0580	.0024	.4994	F	.1389	.0149	.4866	F	.2181	.0084	.3511	P	.3506	.1202	.4536	F
47	.0240	.0003	.3299	P	.0471	.0012	.3750	P	.1496	.0133	.3937	F	.2129	.0041	.2465	P	.3595	.1737	.4359	F
48	.0234	.0004	.4040	F	.0227	.0002	.3749	P	.1698	.0231	.4960	F	.1569	.0076	.2818	P	.3195	.1251	.5089	F
49	.0283	.0005	.4932	F	.0150	.0005	.7229	F	.1883	.0409	.5000	F	.1357	.0205	.6119	F	.2885	.0979	.5287	F
50	.0286	.0005	.4073	F	.0314	.0019	.7213	F	.1601	.0207	.4713	F	.1472	.0500	.7204	F	.3240	.1873	.6497	F
51	.0207	.0005	.5000	F	.0457	.0039	.7401	F	.1075	.0131	.5348	F	.1047	.0174	.6231	F	.3392	.2235	.6522	F
52	.0147	.0004	.5833	F	.0753	.0064	.6250	F	.0843	.0136	.6297	F	.0551	.0070	.6250	F	.2321	.1666	.6429	F
53	.0112	.0004	.7500	F	.1063	.0181	.6250	F	.0935	.0195	.6246	F	.0886	.0157	.6250	F	.2011	.0536	.5557	F
54	.0086	.0002	.7489	F	.0788	.0066	.6196	F	.1608	.0332	.6189	F	.1306	.0242	.6248	F	.2819	.3443	.7857	F
55	.0188	.0011	.7500	F	.0467	.0047	.6250	F	.1010	.0267	.6663	F	.1957	.0965	.6250	F	.4714	.8035	.7143	F
56	.0331	.0036	.8332	F	.0533	.0057	.7500	F	.0570	.0109	.7500	F	.1740	.0640	.6521	F	.3270	.3067	.7143	F
57	.0190	.0011	.7222	F	.0981	.0289	.7500	F	.1419	.1012	.8873	F	.1575	.1291	.8750	F	.1875	.1506	.7120	F
58	.0047	.0003	1.0000	F	.0683	.0157	.8401	F	.1280	.0606	.8320	F	.4427	.5304	.7500	F	.4521	1.8923	.8476	F
59	.0428	.0115	.8333	F	.0232	.0043	.0000	F	.0926	.0457	.8333	F	.6326	1.7982	.7500	F	.2556	.5356	.7857	F
60	.0846	.0336	.8333	F	.0269	.0058	.0000	F	.1778	.1373	.8333	F	.7706	2.8940	.8355	F	.0835	.0282	.8571	F

TABLE 3b. RANDOMLY CHOSEN SPECTRA.

	v = 20 kts				v = 25 kts				v = 30 kts				v = 35kts				v = 40 kts			
h	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
0	.0502	.0011	.8333	F	.1082	.0362	1.0000	F	.1529	.0875	.8182	F	.2772	.0909	.7778	F	.2052	.0198	.6250	F
1	.0712	.0034	.4484	F	.1140	.0332	.7048	F	.1852	.0792	.7227	F	.4112	.1519	.6178	F	.3275	.0319	.2751	P
2	.0892	.0096	.5585	F	.1034	.0205	.6061	F	.7042	.0322	.4236	F	.5505	.2782	.6228	F	.4685	.0692	.3349	F
3	.0662	.0050	.5886	F	.0756	.0103	.6049	F	.1591	.0055	.2884	P	.4457	.1722	.7115	F	.4095	.0611	.3347	F
4	.0427	.0013	.5135	F	.0549	.0037	.5469	F	.1053	.0026	.4061	F	.2657	.0572	.6415	F	.2949	.0338	.4033	F
5	.0330	.0009	.5094	F	.0375	.0012	.5038	F	.0649	.0017	.4147	F	.1582	.0173	.4914	F	.1755	.0115	.3807	F
6	.0346	.0011	.3968	F	.0608	.0066	.7271	F	.0605	.0016	.3311	P	.1577	.0136	.4927	F	.1854	.0315	.4250	F
7	.0157	.0001	.4036	F	.0688	.0164	.7492	F	.0373	.0006	.3921	F	.0870	.0040	.4915	F	.1314	.0168	.4136	F
8	.0184	.0006	.4495	F	.1184	.0229	.7482	F	.0249	.0013	.5007	F	.0625	.0019	.4781	F	.2950	.5002	.7725	F
9	.0724	.0185	.7877	F	.5133	.8268	.8701	F	.1227	.0505	.6236	F	.1111	.0039	.3003	P	.7528	4.7431	.8477	F
10	.2917	.2912	.8324	F	1.3116	6.1066	.8750	F	.5533	.9776	.6453	F	.4194	.1446	.5244	F	1.3478	8.3053	.7318	F
11	.7311	1.8756	.8097	F	1.7224	14.1572	.8680	F	1.4092	5.3999	.6395	F	1.2507	1.4264	.6175	F	2.3671	6.9986	.5679	F
12	1.1445	4.0869	.7069	F	1.3358	6.2941	.8295	F	2.1401	7.7128	.5437	F	2.3613	5.6796	.6239	F	3.8317	11.3655	.3571	F
13	1.3629	4.0359	.5833	F	.9798	1.2423	.5074	F	2.3095	5.1930	.6178	F	3.2353	6.1293	.5589	F	4.2785	8.1211	.3437	F
14	1.3704	2.8241	.5822	F	1.2195	1.7238	.5558	F	2.0313	3.1898	.5650	F	3.5451	4.0626	.3758	P	4.2074	5.1759	.3086	P
15	1.2013	1.3379	.5579	F	1.4479	2.6825	.6242	F	1.6898	1.3953	.4479	F	3.2536	4.9054	.3503	P	3.7309	2.6643	.2237	P
16	1.0217	.6250	.4165	F	1.1775	1.6630	.6249	F	1.5112	.6863	.3051	P	2.7583	4.2451	.4112	F	3.1114	1.4975	.2270	P
17	.9817	.4986	.3395	F	.9968	1.2394	.6162	F	1.5544	.5295	.2484	P	2.1298	.9945	.3160	P	2.8154	.9372	.2563	P
18	.8343	.3554	.3333	P	.8292	.7069	.5772	F	1.3927	.2857	.2467	P	1.6890	.0913	.2827	P	2.4846	1.1907	.2838	P
19	.0309	.2137	.3333	P	.5888	.2054	.4721	F	1.1115	.1542	.1880	P	1.6153	.1757	.2281	P	2.0227	.8949	.2398	P
20	.5216	.1711	.3333	P	.4885	.0755	.3655	P	.9966	.1753	.2391	P	1.3948	.2391	.2482	P	1.3912	.3558	.2059	P
21	.4410	.0886	.3658	F	.3883	.0253	.3325	P	.8696	.1847	.2457	P	1.2466	.2829	.4273	F	1.0702	.0777	.2677	P
22	.3668	.0495	.3801	F	.3562	.0182	.3435	P	.8179	.2051	.3561	F	1.2115	.3176	.4364	F	.9509	.0474	.1711	P
23	.3340	.0380	.3285	P	.4202	.0240	.2939	P	.7397	.1494	.3684	F	1.0580	.3120	.5256	F	.7861	.0582	.2071	P
24	.3344	.0347	.3545	F	.3919	.0212	.3243	P	.6047	.0624	.2971	P	.8990	.2615	.4428	F	.6923	.1350	.2675	P
25	.2762	.0259	.3510	F	.2974	.0147	.3979	P	.5524	.0500	.3035	P	.7559	.1664	.3922	P	.6058	.0614	.3320	F
26	.2025	.0119	.3394	F	.2213	.0091	.3717	P	.4783	.0632	.2945	P	.6534	.1388	.4150	F	.5194	.0373	.2015	P
27	.1932	.0089	.3290	P	.2483	.0161	.4475	F	.4033	.0444	.2881	P	.6171	.0915	.3721	P	.5169	.0555	.2967	P
28	.2011	.0120	.3319	P	.2630	.0149	.3119	P	.3355	.0172	.2456	P	.6298	.0959	.3569	P	.5059	.0490	.2323	P
29	.1908	.0111	.3159	P	.2077	.0067	.3909	P	.2585	.0041	.2072	P	.5483	.0614	.3196	P	.4734	.0407	.2355	P
30	.1731	.0066	.2857	P	.1716	.0054	.3229	P	.2381	.0061	.1974	P	.4175	.0344	.2500	P	.4421	.0351	.2438	P

TABLE 3b (continued)

h	v = 20 kts				v = 25 kts				v = 30 kts				v = 35 kts				v = 40 kts			
	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.	Mean	Var.	D_n	P.F.
31	.1518	.0037	.3214	P	.1463	.0036	.3306	P	.2434	.0079	.2941	P	.3708	.0378	.3952	P	.4114	.0406	.2675	P
32	.1562	.0044	.2618	P	.1396	.0043	.4261	F	.2798	.0121	.2946	P	.3553	.0246	.3716	P	.3674	.0252	.2764	P
33	.1497	.0051	.2994	P	.1470	.0074	.4135	F	.2910	.0153	.2977	P	.3544	.0357	.4114	F	.3101	.0232	.2441	P
34	.1268	.0045	.3559	F	.1436	.0073	.4605	F	.2459	.0113	.3081	P	.3382	.0634	.4934	F	.2556	.0146	.3013	P
35	.1312	.0030	.2619	P	.1352	.0059	.3658	P	.2048	.0042	.2158	P	.2878	.0399	.4722	F	.2522	.0206	.2539	P
36	.1450	.0057	.3727	F	.1173	.0038	.4203	F	.1929	.0045	.2463	P	.2502	.0257	.3703	P	.2708	.0302	.3401	F
37	.1293	.0046	.3529	F	.0965	.0013	.3297	P	.1960	.0160	.3259	P	.2621	.0272	.4809	F	.2882	.0335	.3143	F
38	.1098	.0036	.3398	F	.0959	.0015	.4024	P	.1831	.0241	.4283	F	.3094	.0230	.3682	P	.2467	.0192	.3275	F
39	.1027	.0041	.3965	F	.0934	.0022	.4389	F	.1592	.0198	.5814	F	.3164	.0187	.3332	P	.1974	.0134	.2573	P
40	.0886	.0040	.4533	F	.0773	.0011	.4236	F	.1517	.0106	.4175	F	.3046	.0191	.3551	P	.2125	.0103	.3385	F
41	.0784	.0029	.3717	F	.0651	.0011	.3626	P	.1489	.0078	.3411	F	.2865	.0198	.3837	P	.2549	.0345	.4026	F
42	.0804	.0026	.3639	F	.0634	.0013	.3882	P	.1400	.0086	.4038	F	.2614	.0138	.3619	P	.2770	.0367	.3218	F
43	.0878	.0072	.5442	F	.0668	.0016	.5677	F	.1213	.0079	.4882	F	.2403	.0110	.3298	P	.2532	.0183	.3068	P
44	.0912	.0115	.5868	F	.0752	.0039	.5764	F	.1078	.0055	.4208	F	.2320	.0119	.3352	P	.2133	.0110	.3741	F
45	.0944	.0110	.5799	F	.0874	.0073	.6489	F	.1038	.0062	.3624	F	.2284	.0191	.4594	F	.1648	.0092	.2667	P
46	.0943	.0097	.5611	F	.0837	.0048	.5010	F	.1026	.0051	.3273	P	.2232	.0176	.4646	F	.1263	.0166	.4256	F
47	.0930	.0093	.5643	F	.0596	.0026	.5457	F	.1089	.0018	.2458	P	.2195	.0205	.4674	F	.1122	.0146	.4995	F
48	.0833	.0099	.5832	F	.0405	.0023	.4999	F	.1036	.0017	.2082	P	.2194	.0562	.6488	F	.0893	.0101	.5507	F
49	.0649	.0063	.6126	F	.0280	.0007	.4335	F	.0801	.0098	.5595	F	.1995	.0590	.6250	F	.0724	.0078	.5542	F
50	.0432	.0029	.5638	F	.0339	.0022	.6228	F	.0954	.0232	.5395	F	.0783	.0096	.7046	F	.0823	.0052	.4292	F
51	.0321	.0018	.6184	F	.0416	.0045	.8380	F	.1135	.0248	.5358	F	.0347	.0035	.7563	F	.0937	.0103	.4848	F
52	.0369	.0024	.6511	F	.0466	.0054	.8468	F	.0864	.0204	.6657	F	.0226	.0019	.8750	F	.1068	.0197	.5885	F
53	.0415	.0029	.6221	F	.0428	.0035	.6911	F	.0654	.0149	.6667	F	.0753	.0213	.7481	F	.1173	.0244	.5714	F
54	.0404	.0033	.6620	F	.0340	.0023	.6932	F	.0651	.0067	.5833	F	.2370	.0766	.6066	F	.1059	.0379	.7143	F
55	.0587	.0092	.7457	F	.0365	.0021	.6250	F	.0811	.0214	.5816	F	.3012	.1110	.6250	F	.0810	.0405	.7663	F
56	.0534	.0096	.8277	F	.0398	.0047	.7500	F	.1114	.0295	.6667	F	.2428	.1188	.7493	F	.1115	.0491	.8109	F
57	.0165	.0010	.8333	F	.0307	.0034	.8750	F	.1254	.0530	.6667	F	.2111	.1362	.7500	F	.1071	.0671	.7823	F
58	.0137	.0015	.9167	F	.0456	.0166	1.0000	F	.0703	.0354	.8034	F	.3303	.4558	.8750	F	.1062	.0274	.6429	F
59	.0083	.0005	.9167	F	.0357	.0033	.7500	F	.1604	.0594	.7321	F	.2408	.1666	.8223	F	.0797	.0351	.7857	F
60	.0209	.0052	1.0000	F	.0416	.0093	.8750	F	.3226	.2436	.7500	F	.2867	.1927	.6250	F	.0943	.0777	.8571	F

over a band 1/180 sec^{-1} wide centered on the above frequency. The average wind speeds for each of the synoptically chosen subsets are 20, 25.3, 29.9, 35.3, and 40.1 knots. Included in Table 3a are the results (P = Pass, F = Fail) of the Kolmogorov-Smirnov test applied at each frequency for each subset. The explanation of the columns designated by D_n is given below.

The K-S test was applied in order to test the hypothesis that the spectral values for the synoptically chosen spectra came from a Chi-square distribution with 19.33 degrees of freedom with a mean adjusted to the mean of each subset. The synoptically chosen subset is still a random sample of all possible spectra that might be observed under the required synoptic weather conditions. In the original spectral analyses, 20 degrees of freedom were used. This procedure is adequate for the spectral estimates; however, 19.33 degrees of freedom are necessary for refinement.

The Kolmogorov-Smirnov test (K-S test) determines the maximum deviation of a sample cumulative distribution $F_n(x)$, from the assumed cumulative distribution $F(x)$. That is,

$$D_n = \text{maximum } |F_n(x) - F(x)|.$$

If D_n is smaller than the Kolmogorov statistic, determined by the number of samples (in this case the number of spectra in each subset) and the probability level used, we accept the hypothesis that the samples come from the assumed population. If D_n is larger than this Kolmogorov statistic, we reject the hypothesis. The probability level used in this test was 0.90. For a detailed description of the K-S test the reader is referred to any of a number of textbooks dealing with mathematical statistics.*

From Table 3a it can be seen that the lower and higher frequencies fail to pass the K-S test. These results are not unexpected, since the validity of the results at the higher frequencies, from the use of the Tucker recorder, are in doubt. Nonlinear effects at the very low frequencies and the steepness of the forward faces of the spectra are probably reasons for the failure of the K-S test at these frequencies.

The randomly chosen subsets

From all the available data, records were then selected at random for which the wind speeds as determined from the ships' logs were exactly 20, 25, 30, 35, and 40 knots. In the entire set of data, there were 23 spectra for 20 knots, 23 spectra for 25 knots, 30 spectra for 30 knots, 28 spectra for 35 knots, and 35 spectra for 40 knots. The identifications for all of the spectra at a particular wind speed were put into a hat and the number of cases drawn was exactly the same as for the synoptically chosen subsets. No criteria were set on the synoptic conditions. The only criterion used for this selection was the exactness of the wind speed. The spectra that were chosen at random are presented in Table 2b. The averages of the randomly chosen spectra and the results of the K-S test applied to these subsets are given in Table 3b.

Comparison

In Table 4 the mean spectra for both the synoptically chosen and randomly chosen subsets are divided into three frequency bands.

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*See Whitney, D. R., 1959, Elements of Mathematical Statistics. Henry Holt and Company, New York.

The frequency bands are given in terms of the lag number, H, (f = H/180). The number of frequencies in each band which pass and fail the K-S test are also given as the percentage of passes for the dominant band of frequencies. The frequency bands are not the same for each wind speed since the dominant range of frequencies changes with wind speed. The total number of passes for the dominant frequency bands of the synoptically chosen subsets is 119 from a total of 142. This gives a total passing percentage of 84%. At the 90% level the number of expected passes is $142 \times 0.90 \pm 1.96\sigma$, where σ is the standard deviation. The standard deviation is computed by

$$\sigma = \sqrt{npq} = \sqrt{142 \times 0.90 \times 0.10} = 3.6$$

The minimum number of expected passes is therefore, 128 - 7.06 or approximately 121. Although these results place the synoptically chosen spectra outside the realm of acceptance, the dividing line is extremely thin.

The major frequency bands for the randomly chosen subsets produce only 87 passes from the same total of 142. This gives a total passing percentage of 61%. The total number of passes for the random subsets far from equals the minimum number of 121 required. The number of passes for the randomly chosen subsets is more than 11 standard deviations from the expected value, and thus the odds are very small that the subsets are from a homogeneous population.

If the means of the Chi-square distribution had been adjusted to the means of each of the synoptically chosen subsets and the K-S test then applied to the randomly chosen subsets, the total number of passes for the randomly chosen subsets would be still lower.

Table 4. **Results of the K-S test for synoptic and random subsets for different wind speeds.**

Lag	V	N	Pass	Fail	% pass
Synoptic	20	12			
0-19			1	19	
20-43			22	2	92
44-60			1	16	
Random		12			
0-19			2	18	
20-43			10	14	42
44-60			0	17	
Synoptic	25	8			
0-16			0	17	
17-42			21	5	81
43-60			4	14	
Random		8			
0-16			0	17	
17-42			16	10	62
43-60			0	18	
Synoptic	30	12			
0-12			4	9	
13-42			23	7	77
43-60			0	18	
Random		12			
0-12			2	11	
13-42			20	10	57
43-60			3	15	
Synoptic	35	8			
0-12			5	8	
13-42			24	6	80
43-60			6	12	
Random		8			
0-12			1	12	
13-42			19	11	63
43-60			2	16	
Synoptic	40	14			
0-10			2	9	
11-42			29	3	91
43-60			0	18	
Random		14			
0-10			1	10	
11-42			22	10	68
43-60			2	16	

In figure 2, the mean spectra of the synoptically chosen subsets are plotted against the mean spectra of the randomly chosen subsets. The percentage of passes for the central range of frequencies for the two curves and this range is shown as given in Table 4. From a comparison of the graphs it may be concluded that (in general) from wind considerations only, the state of the sea may be difficult to describe. Decaying seas and swell can produce non-representative spectra as is evident in the graphs for 20 and 25 knots. This effect is also noticeable in the graph for 30 knots but not to the same extent. The two spectra for 35 knots are very similar. The locations of the Ocean Weather Ships, from which the wave records were obtained, are in those areas of the North Atlantic where wind speeds of 30 and 35 knots occur quite frequently. Since most of the analyzed records for these wind speeds, had had fairly long durations, it is not too surprising that the mean spectra for both the synoptic and random subsets are not very different from one another. In the 40 knot case it is evident that the records that comprise the randomly chosen subsets did not have durations of such length as to produce a saturated sea state for this wind speed.

To summarize briefly, the synoptically chosen spectra appear to come from a more homogeneous population than do the randomly chosen spectra. There is probably some variability in the data not explanable that caused the synoptically chosen spectra to pass the K-S test fewer times than would be expected. Much of this variability could be attributed to the lack of precision in the wind reports, and the spread in the wind values that were used.

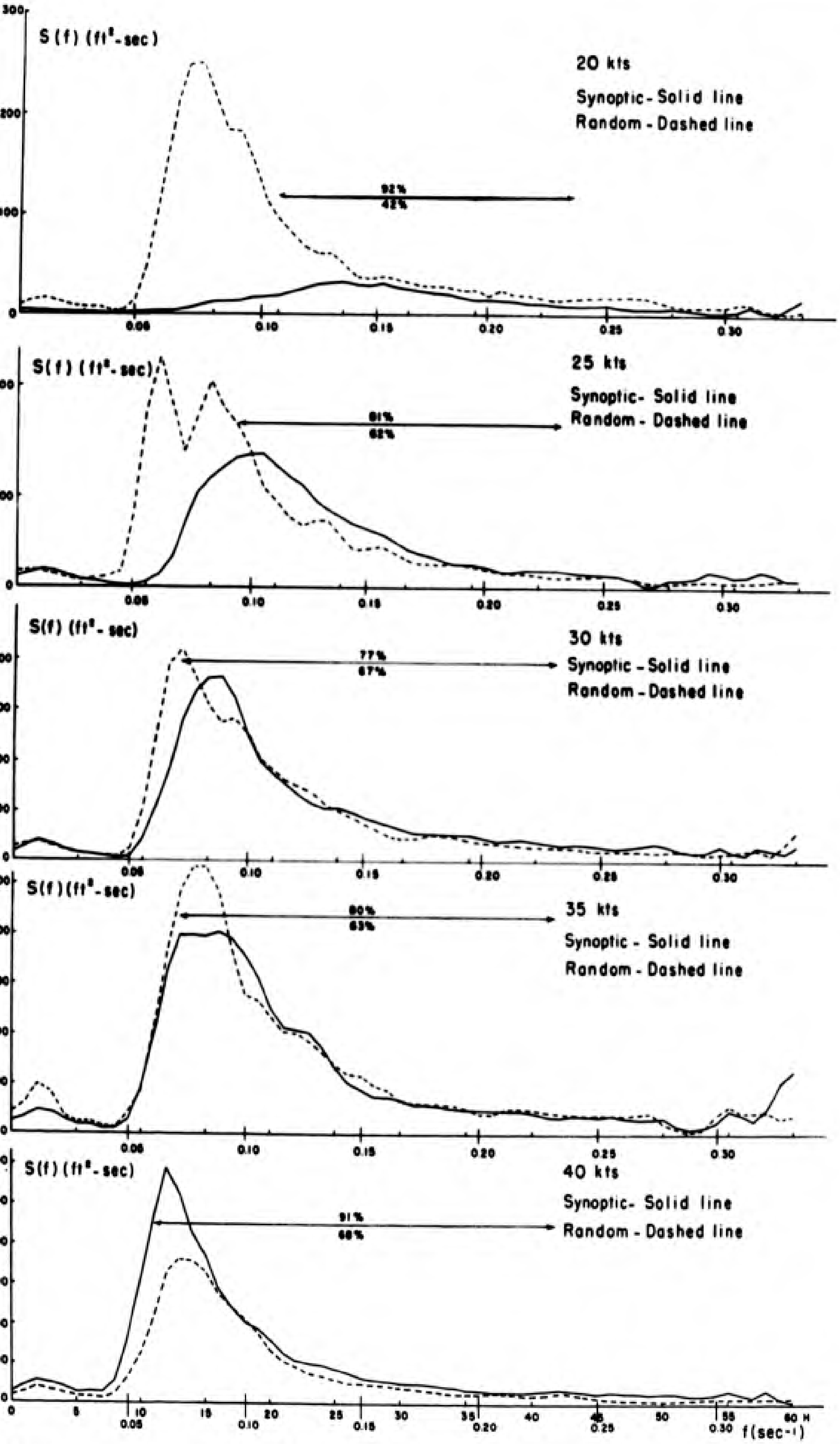

FIG. 2 GRAPHS OF THE CHOSEN AND RANDOM SPECTRA FOR EACH WIND SPEED SUBSET

Higher resolution spectra

Another question might arise as to the shape of the spectra obtained from the use of the digital techniques of Blackman and Tukey (1958). These techniques produce spectra that have been convolved with a spectral window that is fairly wide. The synoptically chosen spectra were recomputed from the original digitized wave records using 180 lags instead of 60 lags for the covariance function and the same frequency range, thus obtaining three times the resolution at the expense of the sampling variability. It was suspected that due to the use of higher resolution the forward faces of the spectra might be steeper and the peaks might be shifted toward the lower frequencies. In figure 3 the spectral estimates are plotted at the two different resolutions. The results tend to show that only at the higher wind speeds is it possible to detect a shift toward lower frequencies of the spectral peaks. It is also possible to detect a slight steepening of the forward faces of the spectra, at these wind speeds.

Additional results

From the mean spectra given in Table 3a and some of the Russian work on ocean wave spectra (Kitaigorodskii, 1961; Kitaigorodskii and Strekalov, 1962) an equation for the wave spectrum was determined (Pierson and Moskowitz, 1963) as described in that paper. The general purpose of this work is the development of a numerical wave forecasting technique. The spectral form obtained is valid only for fully developed seas and deep water and is not valid for coastal regions. It is hoped that this equation may eventually be modified to include the effects of shorter fetch and duration.

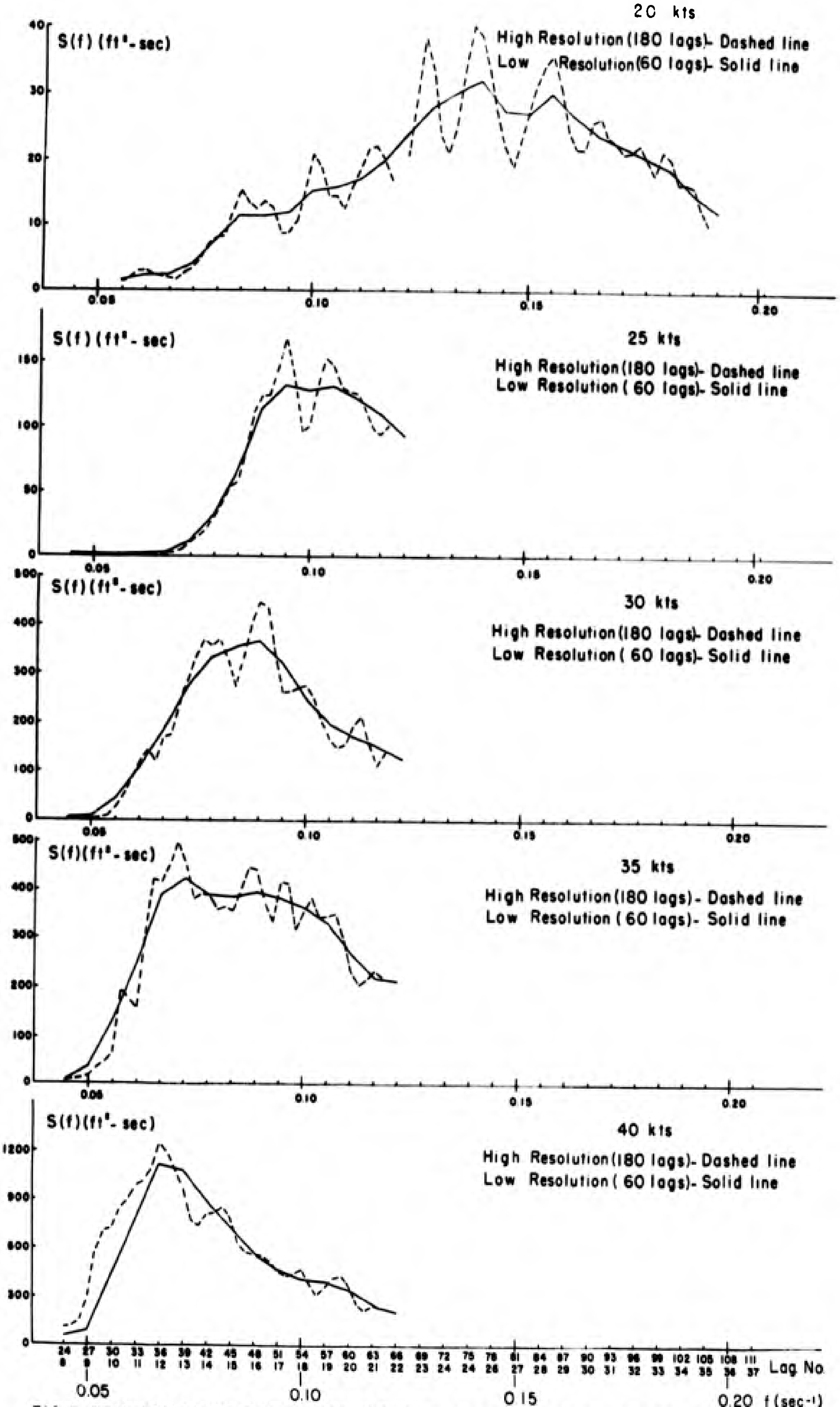

FIG. 3 COMPARISON OF SYNOPTICALLY CHOSEN SPECTRA USING 180 AND 60 LAGS

<u>The significant height as a function of wind speed</u>

From the tabulated spectral values given in Table 3a, the significant heights for each of the synoptically chosen subsets were determined using the equation

$$\bar{H}_{1/3} = 4\sqrt{\text{Total variance}} \tag{1}$$

The spectral values for the first six tabulated values were not used because they are probably not related to the wind sea.

Several techniques were used in order to determine the relationship between significant wave height and wind speed. The first technique was a least square fit applied to the equation

$$\bar{H}_{1/3} = a\,v^2 \tag{2}$$

in order to determine the coefficient a. This is accomplished by requiring that

$$\Sigma[H_i - a\,v_i^2]^2 = \text{minimum} \tag{3}$$

where H_i and v_i are the significant wave heights and average wind speeds respectively of each of the subsets. Second, the value of b in

$$\bar{H}_{1/3} = b\,v^{2.5} \tag{4}$$

was obtained by using the same technique. Third, the values of d and n (n not necessarily an integer) in

$$\bar{H}_{1/3} = d\,v^n$$

was found in a similar manner. In this case, the minimization yields

$$d = \frac{\Sigma v_i^{n_1} H_i}{\Sigma v_i^{2n_1}} = \frac{\Sigma v_i^{n_1-1} H_i}{\Sigma v_i^{2n_1-1}} \tag{5}$$

Various values of n from less than 2 to 2.5 were assumed, and the values of the two fractions were compared until they became equal. At this point the exponent and coefficient were determined that provided a minimum mean square error.

Similar techniques using log fits were employed so as to minimize the ratios of deviations from the fitted curve instead of squared absolute distances. The first of these equations is

$$m \ln a_1 + n_2 \Sigma \ln v_i = \Sigma \ln H_i \tag{6}$$

where m represents the number of subsets (5), n_2 is equal to 2, and $\ln a_i$ determines a different value for the a of equation (2). Equation (6) was also solved setting $n_2 = 2.5$ and solving for $\ln b_1$. The last technique required solving equation (6) and equation (7) simultaneously for an unknown n_1 and $\ln d_1$. Equation (7) is given by

$$[\Sigma \ln v_i] \ln d_1 + n_1 [\Sigma (\ln v_i)^2] = \Sigma (\ln v_i \ln H_i) \tag{7}$$

The results of these computations produced a v^2 law of the form

$$\overline{H}_{1/3} = 0.02\, v^2 \quad \text{(H in feet, v in knots)} \tag{8}$$

The $v^{2.5}$ law did not fit the data as well as the v^2 law. The results from equation (4) verified a v^2 law and not a $v^{2.5}$ law. The use of the logarithmic fitting procedures verified the results just given and resulted in only minor changes in the constant.

These data have much less scatter than the usual data fitted to a v^2 law and since alternate possibilities were tried and rejected, it is believed that they provide a valid observational verification of the law.

The relationship of significant wave height to the wind speed, as given in equation (8), was still to be modified because in its derivation the spectral values at the high frequencies were used in the determination of the significant heights. Since there is doubt as to their true values, the spectral form given by Pierson and Moskowitz (1963) was used to provide an equation for the significant height of a fully developed sea that took into account the too high values in Table 2a at high frequencies. This equation is given by

$$\overline{H}_{1/3} = 0.0182\ v^2 \tag{9}$$

This equation is then taken to be the relationship of the significant wave height (in feet) of a fully developed sea to the wind speed (in knots). The constant is 9% lower than the one determined by the least square fits.

Figure 4 shows a plot of the above v^2 law and the wave height-wind speed relationships of PNJ (1955), Darbyshire (1959), and Sverdrup Munk Bretschneider (SMB) (as given in Neumann and Pierson, 1957). The relationships for these various curves are:

$$\text{PNJ} \quad \overline{H}_{1/3} = 4.426 \times 10^{-3}\ v_{7.5}^{2.5}$$

$$\text{SMB} \quad \overline{H}_{1/3} = 0.0233\ v_{10-12}^{2}$$

$$\text{Darbyshire} \quad \overline{H}_{1/3} = 0.0133\ v^{2}$$

$$\text{Moskowitz} \quad \overline{H}_{1/3} = 0.0182\ v_{19.5}^{2}$$

where $\overline{H}_{1/3}$ is given in feet and v is in knots.

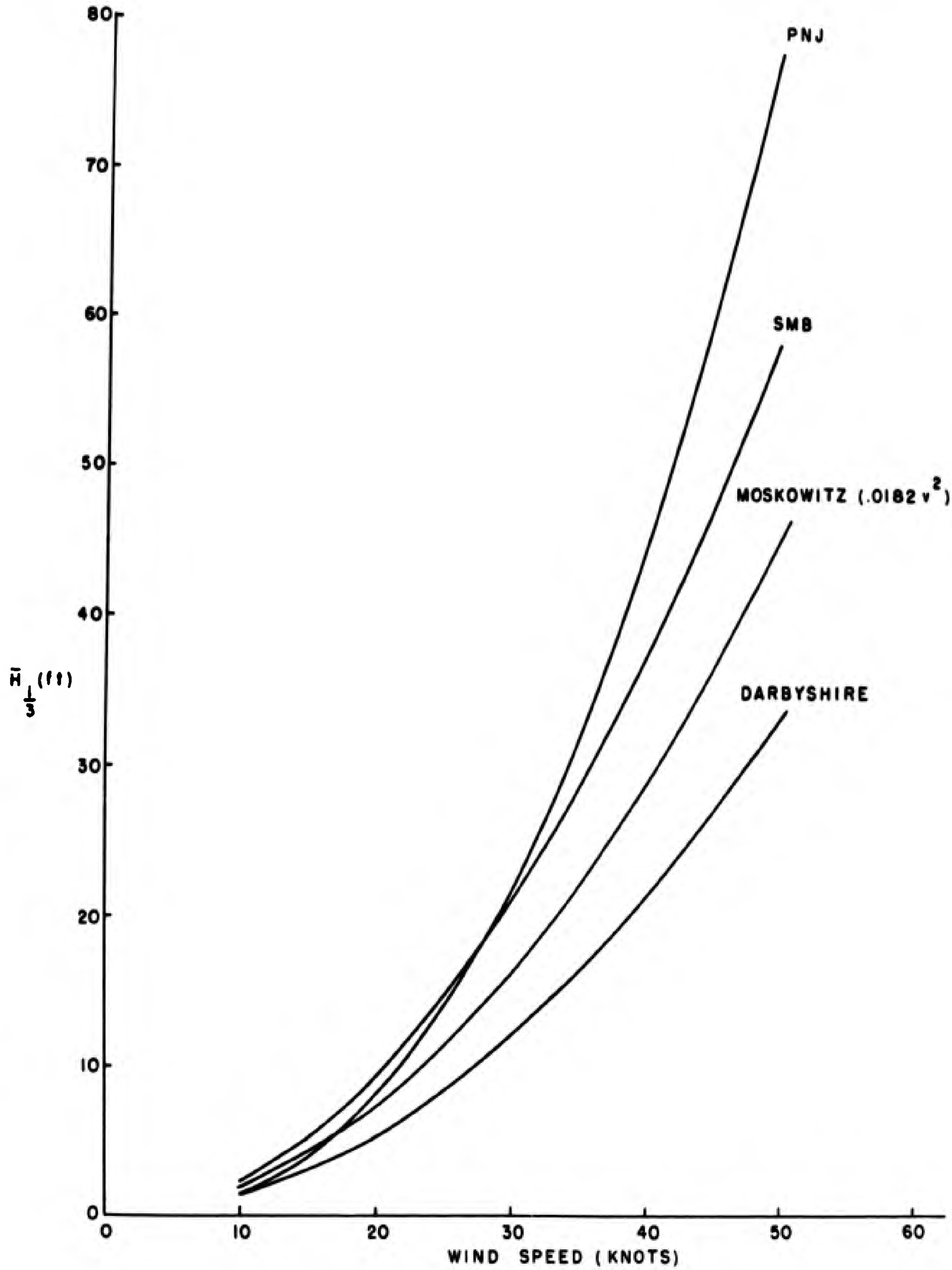

FIG. 4 GRAPH OF SIGNIFICANT HEIGHT - WIND SPEED RELATIONSHIPS

Subscripts indicate the height in meters at which the wind is measured. As will be shown by Pierson, the PNJ, SMB, and Moskowitz curves are in substantial agreement when the variation in anemometer height is considered.

Comments on wind speed data

The results of the K-S test suggest scme weaknesses in the reported wind speeds. For those ships equipped with anemometers the reported wind speeds are one minute averages. For those ships not equipped with anemometers the wind speeds are estimated from the sea state by "experienced observers". It is known that such experienced people are able to approximate the wind speed from the sea state fairly well. It is also possible to show many examples where the wave heights at different times can be far apart for a given wind speed and an experienced observer may be in error.

Certainly a one minute average of the wind speed is not long enough to describe the wind conditions best for a wind sea. The improvement of the techniques for reporting wind speed is becoming more important in many aspects of the study of air sea boundary processes. Although the reporting of wind speeds is of great importance, the measurement of waves, of which little has been said, is of prime importance. Shipborne wave recorders on all the weather ships would help in refining these spectra and improving wave forecasts.

With the preceding discussion in mind, and with the aid of the derived spectral equation, it was possible to determine a modified wind speed that gave a good fit between the analytic spectrum mentioned above and these wind speeds are 19.5, 25.8, 31.7, 33.6, and 40 knots as described by Pierson and Moskowitz (1962). If these wind speeds are substituted into the spectral equation and the results plotted against the spectral values given in Table 3a, the curves are very much alike.

Conclusions

The criteria used in the selection of the wave records appear to be many, if not all, of the criteria necessary for the selection of wave records (from synoptic considerations) when the seas were fully developed.

The Kolmogorov-Smirnov test applied to the synoptically chosen spectra for each mean wind speed showed that the basic assumptions used in the data reduction were valid. The results of the K-S test when applied to a randomly chosen subset for each of the mean wind speeds as compared to the synoptically chosen spectra allow two conclusions to be drawn. First, criteria must be applied and extreme care must be taken when selecting synoptic situations where fully developed seas for various wind speeds might be found. Second, wind speed determinations from the observed sea state may not be accurate especially if higher winds persisted before observation time. Stated differently, given a wind speed one cannot accurately describe the sea state at all times.

From the results given in this paper, it was possible to determine an equation to describe the spectra. This equation is not too different from previous studies. The results allowed a comparison to be made of the various significant wave height-wind speed relationships. The results produce a curve that is a compromise between PNJ (1955), Darbyshire (1959) and the Sverdrup Munk Bretschneider method.

In areas of the North Atlantic where the Ocean Weather Ships are located, it is not uncommon to find very intense cyclones, very strong winds, and mountainous sea conditions in the winter months. It is a very rare situation, however, to find cases of fully developed seas for wind speeds greater than 40 knots. During the five year period (April 1955 -March 1960) only one case for 45 knots and one case for 50 knots could be found that approached the fully developed state. Perhaps, if wave records taken in the southern hemisphere, especially in the regions of the roaring forties or howling sixties, were available, more information could be obtained for the higher wind speeds. Wave statistics for light winds are also needed. However, winds less than 20 knots were not studied in this report.

From the analyzed records the role stability plays in wave generation is not obvious. No effects due to air sea temperature differences could be detected. The winter months produce the more interesting and higher sea conditions than do the summer months. This is probably due to the frequency of occurrence and intensity of the transient cyclones and the stability.

The general problems in reporting wind speeds at sea are a direct hindrance to the verification of wave forecasting techniques. When better techniques for wind observations are adopted and put into practice, it should be possible to repeat a more refined analysis of selected wave records to eliminate the source of variability in these results due to the lack of precision in measuring the wind speeds.

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