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TECHNICAL NOTE R-52

A FORTRAN PROGRAM FOR COMPUTING REFLECTION, TRANSMISSION, AND ABSORPTION COEFFICIENTS FOR AN INHOMOGENEOUS PLASMA LAYER

Prepared By

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J. M. Scarborough

June, 1963

BROWN ENGINEERING COMPANY INC.
HUNTSVILLE, ALABAMA

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June, 1963

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ABSTRACT

This report describes a FORTRAN computer program for computing the transmission, reflection and absorption coefficients of an inhomogeneous plasma layer. A modified Runge-Kutta integration scheme is used to solve Maxwell's equations for the electric and magnetic fields at the boundary of the plasma layer.

Approved:

Raymond C. Watson, Jr.

Raymond C. Watson, Jr.
Director of Scientific Research

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LIST OF SYMBOLS

A	absorption coefficient
$\vec{B}$	time dependent magnetic induction vector
c	speed of light in vacuum = 2.9979576×10^8
$\vec{D}$	time dependent electric displacement vector
$\vec{E}$	time dependent electric intensity vector
$E(z)$	complex magnitude of electric field intensity within plasma
E_i	imaginary part of $E(o)$
E^I	complex magnitude of INCIDENT ~~transmitted~~ electric field intensity
E_r	real part of $E(o)$
E^T	complex magnitude of transmitted electric field intensity
e	base of Napierian logarithms
$E_o{}^I$	complex amplitude of incident electric field intensity
$\vec{H}$	time dependent magnetic field intensity vector
$H(z)$	complex magnitude of incident magnetic field intensity
H_i	imaginary part of $H(o)$
H^I	complex magnitude of incident magnetic field intensity
H_r	real part of $H(o)$
H^T	complex magnitude of transmitted magnetic field intensity
i	$\sqrt{-1}$
$H_o{}^I$	complex amplitude of INCIDENT ~~transmitted~~ magnetic field intensity
$\vec{J}$	time dependent true current density
K_r	relative permittivity of plasma

LIST OF SYMBOLS (Continued)

K_1	dimensionless conductivity of plasma
k	propagation constant for dielectric window
k_o	free space propagation constant
R	reflection coefficient
S	parameter defined by Equation (21)
T	transmission coefficient
t	time
z	Cartesian co-ordinate normal to surface of plasma layer
z_o	value of z at "outer" surface of plasma layer

Greek Symbols

α	propagation constant of plasma
β	attenuation constant of plasma
δ	parameter defined by Equation (22)
ϵ_m	permittivity of dielectric adjoining plasma layer
ϵ_o	permittivity of free space (8.854×10^{-12} farad/meter)
ϵ	effective permittivity of plasma
λ	wavelength in plasma
λ_o	free space wavelength
μ_o	permeability of free space ($4\pi \times 10^{-7}$ henry/meter)
σ	effective conductivity of plasma
ω	angular frequency of incident wave

LIST OF FORTRAN SYMBOLS

AM	absorption coefficient	
E	electric field	
E2	error limits in the integration	
F_{150}	CODE - 0 on all cases but last	
F_{151}	A for conductivity	
F_{152}	B for conductivity	
F_{153}	C for conductivity	
F_{154}	D for conductivity	
F_{155}	E for conductivity	
F_{156}	A for permittivity	
F_{157}	B for permittivity	
F_{158}	C for permittivity	
F_{159}	D for permittivity	
F_{160}	E for permittivity	
F_{161}	M_1	
F_{162}	M_2	
F_{163}	M_3	
F_{164}	M_4	
F_{165}	M_5	Dielectric Constants of the Antenna Window
F_{166}	M_6	
F_{167}	M_7	
F_{168}	M_8	
F_{169}	M_9	
F_{170}	M_{10}	

F_{174}	MB	conductivity equation numbers
F_{175}	J	permittivity equation numbers
H	magnetic field	
OMA	omega	
PHIE	ϕ_E	phase of the electric field at $z = 0$
PHIH	ϕ_H	phase of the magnetic field at $z = 0$
RM	reflection coefficient	
TE	end of range	
TM	transmission coefficient	
TT	beginning of range	
Y1	real component of electric field	
Y2	imaginary component of electric field	
Y3	real component of magnetic field	
Y4	imaginary component of magnetic field	

INTRODUCTION

The program described herein uses a method for computing reflection and transmission coefficients for a plane-parallel inhomogeneous isotropic layer of plasma in which the plasma properties are functions only of distance along a normal to the surface of the layer with normal incidence assumed, as presented in a paper by Scarborough.[1]

This report presents a direct and expedient method for computing these coefficients for a wide variety of distributions of both permittivity and conductivity which takes full advantage of available digital computers. The method involves the direct numerical integration of Maxwell's equations within the plasma by a modified Runge-Kutta integration process that allows accuracy control in the solution of differential equations.

ANALYSIS OF INHOMOGENEOUS PLASMA LAYER

Maxwell's equations for a stationary medium containing no free charges are

$$\nabla \cdot \vec{D} = 0 \quad , \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \quad , \tag{2}$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad , \tag{3}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad . \tag{4}$$

As is shown in Reference 1, Maxwell's equation can be reduced to the following form for an inhomogeneous plasma layer in which the plasma properties are functions of a single Cartesian co-ordinate z normal to the surface of the layer:

$$\frac{dEr}{dz} = -\omega\mu_o H_i \tag{5}$$

$$\frac{dE_i}{dz} = \omega\mu_o H_r \tag{6}$$

$$\frac{dH_r}{dz} = - \omega \epsilon_o \left(K_i E_r + K_r E_i\right) \quad , \tag{7}$$

$$\frac{dH_i}{dz} = \omega \epsilon_o \left(K_r E_r - K_i E_i\right) \quad . \tag{8}$$

In these expressions

$$K_r = \frac{\varepsilon}{\varepsilon_o} \quad , \tag{9}$$

and

$$K_i = \frac{\sigma}{\varepsilon_o \omega} \quad , \tag{10}$$

with ε and σ functions of the co-ordinate z.

A transmitted wave E^T of unit amplitude and phase $(k_o z - k_o z_o)$ is assumed in the free-space region immediately "outside" (i.e., $z > z_o$) the plasma layer as shown in Figure 1. With $E^T(z_o) = E_r(z_o) + iE_i(z_o) = 1 + i0$ and $H^T(z_o) = H_r(z_o) + iH_i(z_o) = \sqrt{\varepsilon_o/\mu_o} + i0$ as initial values, the system of equations (5), (6), (7), and (8) is integrated numerically over the range $0 \leq z \leq z_o$ using the modified Runge-Kutta method described in Appendix A. The integration proceeds backward along the z axis to the origin which is taken at the interface between the plasma layer and the dielectric window of the transmitting antenna. Across the boundary $z = 0$, it is required that the electric and magnetic fields be continuous, and the following equations are derived for reflection and transmission coefficients in terms of the terminal values $E_r(o)$, $E_i(o)$, $H_r(o)$, $H_i(o)$ of the fields:

$$R = \frac{[\sqrt{\varepsilon_m} E_r(o) - \sqrt{\mu_o} H_r(o)]^2 + [\sqrt{\varepsilon_m} E_i(o) - \sqrt{\mu_o} H_i(o)]^2}{[\sqrt{\varepsilon_m} E_r(o) + \sqrt{\mu_o} H_r(o)]^2 + [\sqrt{\varepsilon_m} E_i(o) + \sqrt{\mu_o} H_i(o)]^2} \tag{11}$$

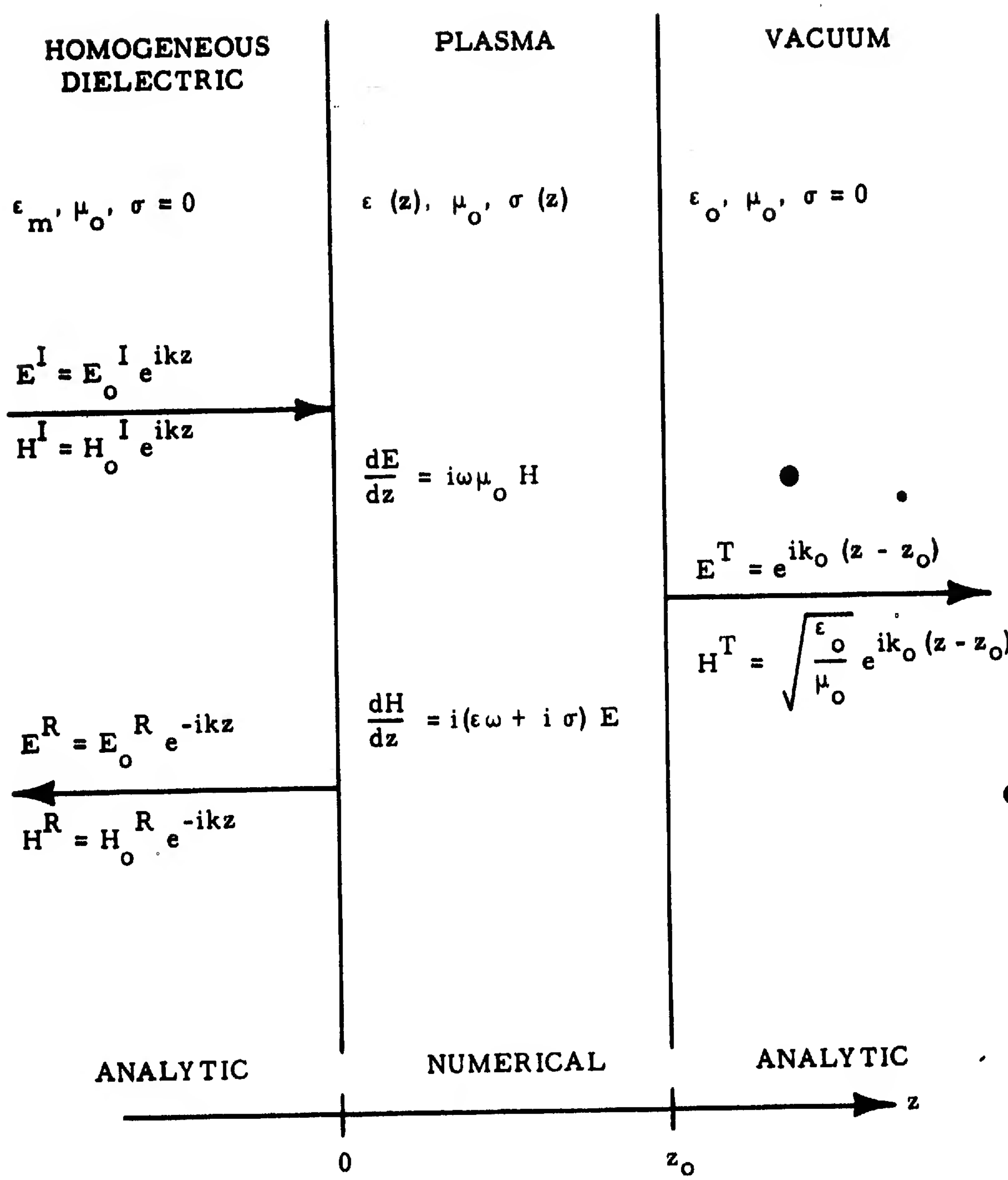

Figure 1. Schematic Representation Showing the Type of Solution Valid in Each Region

$$T = \frac{4\sqrt{\epsilon_m \epsilon_o}}{[\sqrt{\epsilon_m}\, E_r(o) + \sqrt{\mu_o}\, H_r(o)]^2 + [\sqrt{\epsilon_m}\, E_i(o) + \sqrt{\mu_o}\, H_i(o)]^2} \quad . \tag{12}$$

The absorption coefficient, A, is then given by

$$A = 1 - (R + T) \quad . \tag{13}$$

The expressions for $K_r(z)$ and $K_i(z)$ are chosen independently from the following list and these choices are specified in the input to the program.

$$K(z) = Az^4 + Bz^3 + Cz^2 + Dz + E \ , \tag{14}$$

$$K(z) = Az^{-4} + Bz^{-3} + Cz^{-2} + Dz^{-1} + E \ , \tag{15}$$

$$K(z) = Ae^{Bz} + Ce^{-Dz} + E \ , \tag{16}$$

$$K(z) = Ae^{-\frac{(z+B)^2}{C}} + D + E \ . \tag{17}$$

The constants A, B, C, D, and E are chosen to give the best fit to the true distributions over a given range of z. Several expressions may be used to obtain a piecewise fit over the entire range $0 \leq z \leq z_o$ as explained in Appendix B.

The choice of location of the boundaries separating the various ranges is largely arbitrary, but the following conditions should be observed: all boundaries should be located at z values which are integral multiples of

$z_0/100$. If this is not possible, an effort should be made to locate the boundaries at points just less than an integral multiple of $z_0/100$ as the program will in effect shift the boundary to the next integral multiple of $z_0/100$ greater than the boundary specified. In most cases this poses no serious limitation upon the accuracy or the usefulness of the program.

TESTS

In order to determine the accuracy of the program and to investigate the effects of variations in certain parameters on accuracy, a number of test cases were computed and the results compared with those found by other methods. The tests performed were divided into three groups: (1) homogeneous non-conducting layers, (2) homogeneous conducting layers, and (3) inhomogeneous conducting layers. Results of the first two groups of tests were compared with values computed using analytic solutions for these simple cases; results of the third group were compared with those obtained by Albini and Jahn[2] for trapezoidal distributions of electron densities.

Group I. <u>Homogeneous Non-conducting Layers</u>

In these tests reflection coefficients were computed using the following values for the various parameters:

$$K_i = 0 \qquad K_r = 1, 2, 4$$

$$\text{Layer Thickness} = z_o = 1 \text{ m}, 0.1 \text{ m}, 0.01 \text{ m}$$

$$\frac{z_o}{\lambda} = 0.125, 0.250, 0.375, 0.500, 0.675, 0.750, 0.875, 1.000, 1.500, 2.000, 4.000$$

$$\frac{\epsilon_m}{\epsilon_o} = 1, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5$$

Values computed using the formula

$$R = \frac{\left\{\left[\left(\sqrt{\epsilon_m/\epsilon_o} - \sqrt{K_r}\right)\left(\sqrt{K_r} + 1\right) + \left(\sqrt{\epsilon_m/\epsilon_o} + \sqrt{K_r}\right)\left(\sqrt{K_r} - 1\right)\right]^2 - 4\left(\frac{\epsilon_m}{\epsilon_o} - K_r\right)(K_r - 1)\sin^2\frac{2\pi z_o}{\lambda}\right\}}{\left\{\left[\left(\sqrt{\epsilon_m/\epsilon_o} + \sqrt{K_r}\right)\left(\sqrt{K_r} + 1\right) + \left(\sqrt{\epsilon_m/\epsilon_o} - \sqrt{K_r}\right)\left(\sqrt{K_r} - 1\right)\right]^2 - 4\left(\frac{\epsilon_m}{\epsilon_o} - K_r\right)(K_r - 1)\sin^2\frac{2\pi z_o}{\lambda}\right\}} \qquad (18)$$

were used for comparison.

Group II. Homogeneous Conducting Layers

Reflection and transmission coefficients were computed for the following conditions:

$$\frac{K_i}{1 - K_r} = 0,\ 0.1$$

$$\frac{K_i^2 + 1}{1 - K_r} = 0.5,\ 1.0,\ 4.0$$

$$\frac{z_o}{\lambda} = 0.5,\ 3.0$$

$$z_o = 0.1,\ 0.01$$

$$\frac{\epsilon_m}{\epsilon_o} = 1$$

These results were compared with those obtained from a second computer program which computes R and T from the formulas:[3]

$$R = \frac{\sin^2 \alpha z_o + \sinh^2 \beta z_o}{\sin^2 (\alpha z_o + \delta) + \sinh^2 (\beta z_o + S)} , \tag{19}$$

$$T = \frac{\sin^2 \delta + \sinh^2 S}{\sin^2 (\alpha z_o + \delta) + \sinh^2 (\beta z_o + S)} , \tag{20}$$

where

$$S = \ln\left[\frac{(k_o + \alpha)^2 + \beta^2}{(k_o - \alpha)^2 + \beta^2}\right]^{\frac{1}{2}} , \tag{21}$$

$$\delta = \tan^{-1} \frac{2k_o\beta}{\alpha^2 + \beta^2 - k_o^2} , \tag{22}$$

and

$$\alpha = \frac{k_o}{\sqrt{2}}\left(\sqrt{K_r^2 + K_i^2} + K_r\right)^{\frac{1}{2}} , \tag{23}$$

$$\beta = \frac{k_o}{\sqrt{2}}\left(\sqrt{K_r^2 + K_i^2} - K_r\right)^{\frac{1}{2}} \tag{24}$$

Group III. Inhomogeneous Conducting Layers

In Reference 2, Albini and Jahn present in graphical form the results of computations of reflection and transmission coefficients for plasma layers having electron density distributions of trapezoidal form (i.e., distributions which are constant over the central region of the layer and decrease linearly to zero electron density at the surfaces). A constant collision frequency is assumed. These computations were based on analytical expressions obtained by matching plane wave solutions in the uniform central region with appropriate Airy function solutions in the inhomogeneous regions.

The following values of parameters were used to reproduce a particular set of results as presented in Reference 2:

$$\begin{aligned} K_r &= -30.4\,z + 1 \\ K_i &= 4.0\,z \end{aligned} \qquad 0 \leq z \leq 0.025$$

$$\begin{aligned} K_r &= 0.24 \\ K_i &= 0.10 \end{aligned} \qquad 0.025 \leq z \leq z_o - 0.025$$

$$\begin{aligned} K_r &= 30.4\,z + (1 - 30.4\,z_o) \\ K_i &= -4.0\,(z - z_o) \end{aligned} \qquad (z_o - 0.025) \leq z \leq z_o$$

$$z_o = 0.06,\ 0.08,\ 0.10,\ 0.12,\ 0.14,\ 0.16,\ 0.18,\ 0.20$$

$$\omega = 1.8836723 \times 10^{12} \qquad \frac{\epsilon_m}{\epsilon_o} = 1$$

RESULTS

In order to conserve space, only a few results typical of their particular groups are presented.

Group I.

Reflection coefficients for the case $K_r = 2$, $K_i = 0$, and $\epsilon_m/\epsilon_o = 3$ are listed below for eleven values of z_o/λ and two values of z_o. For convenience in comparison, the corresponding values of R as computed from equation (18) are listed in the adjacent column.

$z_o = 1$ m

z_o/λ	R (program)	R (equation 18)
0.125	3.9630627×10^{-2}	3.96305×10^{-2}
0.250	5.1548061×10^{-3}	5.15478×10^{-3}
0.375	3.9630426×10^{-2}	3.96305×10^{-2}
0.500	7.1796888×10^{-2}	7.17968×10^{-2}
0.625	3.9630644×10^{-2}	3.96305×10^{-2}
0.750	5.1548134×10^{-3}	5.15478×10^{-3}
0.875	3.9635804×10^{-2}	3.96305×10^{-2}
1.000	7.1796971×10^{-2}	7.17968×10^{-2}
1.500	7.1796929×10^{-2}	7.17968×10^{-2}
2.000	7.1796910×10^{-2}	7.17968×10^{-2}
4.000	7.1796868×10^{-2}	7.17968×10^{-2}

$z_o = 0.01$ m

z_o/λ	R (program)	R (equation 18)
0.125	3.9891224×10^{-2}	3.96305×10^{-2}
0.250	5.1592200×10^{-3}	5.15479×10^{-3}
0.375	3.9632806×10^{-2}	3.96305×10^{-2}
0.500	7.1796898×10^{-2}	7.17968×10^{-2}
0.625	3.9626767×10^{-2}	3.96305×10^{-2}
0.750	5.1548193×10^{-3}	5.15478×10^{-3}
0.875	3.9635924×10^{-2}	3.96305×10^{-2}
1.000	7.1796883×10^{-2}	7.17968×10^{-2}
1.500	7.1796945×10^{-2}	7.17968×10^{-2}
2.000	7.1796888×10^{-2}	7.17968×10^{-2}
4.000	7.1796868×10^{-2}	7.17968×10^{-2}

Group II.

The following results apply to the case $K_i/(1 - K_r) = 0.1$, $z_o/\lambda = 0.5$, $z_o = 0.1$ m, $\varepsilon_m/\varepsilon_o = 1$, for three values of $(K_i^2 + 1)/(1 - K_r)$: (Values of R and T as computed from equations (19) and (20) respectively are listed for comparison.)

Reflection Coefficients

$\dfrac{K_i^2 + 1}{1 - K_r}$	R (program)	R (equation 19)
0.5	3.2831956×10^{-2}	3.2820612×10^{-2}
1.0	4.9112033×10^{-1}	4.9111955×10^{-1}
4.0	8.9099815×10^{-1}	8.9100520×10^{-1}

Transmission Coefficients

$\frac{K_i^2 + 1}{1 - K_r}$	$\dot{T}$ (program)	T (equation 20)
0.5	2.5536399×10^{-1}	2.5531482×10^{-1}
1.0	2.0548905×10^{-1}	2.0548882×10^{-1}
4.0	5.3000023×10^{-5}	5.2958208×10^{-5}

Group III.

Results of the inhomogeneous conducting layer tests are tabulated below for comparison with values read from the curve of Reference 2. Due to the limitations inherent in reading values from such curves, the last digit in each of these numbers is doubtful.

z_o/λ_o	R (program)	R (Reference 2)
0.6	1.6026404×10^{-1}	1.61×10^{-1}
0.8	1.5735421×10^{-1}	1.57×10^{-1}
1.0	8.7658395×10^{-2}	8.76×10^{-2}
1.2	4.1034775×10^{-2}	4.04×10^{-2}
1.4	5.9441907×10^{-2}	5.90×10^{-2}
1.6	9.3593028×10^{-2}	9.36×10^{-2}
1.8	9.7007352×10^{-2}	9.67×10^{-2}
2.0	7.7091658×10^{-2}	7.73×10^{-2}

CONCLUSIONS

From the results of the Group I tests, it is seen that the program yields values in good agreement with the theoretical values of reflection coefficient. Accuracy seems to be generally poorest for combinations of small z_0 and (relatively) large λ. For other combinations, the program yields results consistently accurate to five significant figures with occasional seven-place accuracy. In no case tested was the accuracy less than 1%.

Results of the Group II tests indicate that the introduction of conductivity into the homogeneous layer does not degrade the accuracy of the program, and transmission coefficients are computed with comparable accuracy.

Results of the third group of tests indicate that inhomogeneities in K_r and K_i requiring piecewise fitting over several ranges of z may be successfully treated. No conclusions concerning the accuracy of the program in this case may be drawn from these results, since in this case, the program is probably more accurate than the values used for comparison.

It should be pointed out that in applications of the relation

$$\omega = \frac{2\pi c}{\lambda_0}$$

to determine the frequency required to produce a wave of given length λ_o in free space, the speed of light c was taken as 2.9979576×10^8 meters/second in order to be consistent with the relation $c = (\epsilon_0 \mu_0)^{-\frac{1}{2}}$.

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APPENDIX A

LIST OF SYMBOLS FOR SUBROUTINE RUNGKT

E	maximum relative error
$E_{n,\,i+1}$	error estimate for each of the 4 dependent variables
f_n	a function
h	step size
i	refers to the old value of the independent variable
$i + 1$	refers to the solution at the new value of the independent variable
I	order of integration (2, 3, or 4)
n	values 1-4 inclusive - referring to the 4 equations
$R_{n,\,i+1}$	relative error
y_n	dependent variables
$\dot{y}_n$	first time derivative of the dependent variables
$y_{n,\,i}$	old values of dependent variables
$y_{n,\,i+1}$	new values of dependent variables
z_i	specific value of the independent variable
z_0	point on z axis in free space region
(I)	one-step approximation
(II)	two-step approximation

SUBROUTINE RUNGKT

The method used by SUBROUTINE RUNGKT is briefly described as follows:[4]

1. Use initial conditions for the 4 dependent variables as solutions at $z = z_0$ and adopt the maximum step size permissible.
2. Perform a Runge-Kutta integration to find the single step solution for the new value of the independent variable, z.
3. Halve the integration step size.
4. Resolve the problem from the last convergent point using two applications of the reduced step size.
5. Compute an extrapolated solution making use of both solutions.
6. Compare the two-step solution with the extrapolated solution.
7. If the two solutions are sufficiently close, the procedure is continued using the extrapolated solution as initial values at the updated point z.
8. If the two solutions are not sufficiently close, the reduced step size is again halved and the procedure restarted from step 4 using the conditions at the last convergent point.

9. If the two solutions agree too closely, then the basic step is doubled and the procedure restarted at step 1 using the extrapolated solution at the new value of the independent variable.

The above briefly describes the method; however, additional details are given below.

The one-step approximation is found by a Runge-Kutta numerical integration.

The two-step approximation is found by halving the step size and computing a one-step approximation at point $\left(z_i + \frac{h}{2}\right)$. This approximation is then used as the starting point for another one-step approximation again using the halved value of step size. This two-step approximation then can be thought of as two one-step approximations.

The procedure used for automatic error control is as follows:[5]

1. An error estimate for each of the four dependent variables using extrapolation to zero step size is expressed

$$E_{n,i+1} = \frac{y^{(II)}_{n,i+1} - y^{(I)}_{n,i+1}}{2^I - 1}$$

where I is the order of integration.

2. These values of $E_{n,i+1}$ are added to the two-step results to obtain an extrapolated solution.

(II)

$$y_{n,i+1} = y_{n,i+1} + E_{n,i+1} \; .$$

3. A relative error term is next defined and computed for each variable.

$$R_{n,i+1} = \frac{E_{n,i+1}}{y_{n,i+1}}$$

4. The maximum relative error $(R_{n,i+1})_{max}$ is designated E and is tested against two bounds E1 and E2. E1 is to keep the integration step size from remaining too small and E2 is used to keep the interval from becoming too large. The program then halves the interval and repeats the step, continues at the same interval, or continues at twice the interval.

5. The subscript n of the maximum relative error is recorded in INDEX. The value of index and the value of the nonconvergent term is printed when no convergence occurs. The program will try to restart the solution five times.

Given the initial conditions of the dependent variables at point z_i, this routine[6] will calculate a one-step approximation for each of the dependent variables at point $z_i + h$ where h is the step size.

Given the function,

$$\dot{y}_n = f_n (z, y_1, y_2, y_3, \ldots, y_n) , \tag{A-1}$$

and the initial values

$$z_i; y_{1,i}; y_{2,i}; y_{3,i}; \ldots; y_{n,i} \tag{A-2}$$

then a second order Runge-Kutta solution gives a one-step approximation of

$$y_{n,i+1} = y_{n,i} + \frac{1}{2} (K_{1,n} + K_{2,n}) , \tag{A-3}$$

where

$$K_{1,n} = h f_n (z_i, y_{1,i}; y_{2,i}; \ldots; y_{n,i}) , \tag{A-4}$$

$$K_{1,n} = h f_n (z_{i+1}; y_{1,i} + K_{0,i}; y_{2,i} + K_{0,2}; \ldots; y_{n,i} + K_{0,n}) .$$

The third order Runge-Kutta solution gives an approximation of

$$y_{n,i+1} = y_{n,i} + \frac{1}{6} (K_{1,n} + 4 K_{2,n} + K_{3,n}) , \tag{A-5}$$

where

$$K_{1,n} = h f_n (z_i; y_{1,i}; y_{2,i}; \ldots; y_{n,i}) , \tag{A-6}$$

$$K_{2,n} = h f_n \left(z_i + \frac{h}{2}; y_{1,i}; + \frac{1}{2} K_{0,1}; y_{2,i} + \frac{1}{2} K_{0,2}; \ldots; y_{n,i} + \frac{1}{2} K_{0,n}\right) ,$$

$$K_{3,n} = h f_n (z_i + h; y_{1,i} + 2 K_{1,1} - K_{0,1}; y_{2,i} + 2 K_{1,2} - K_{0,2};$$

$$\ldots; y_{n,i} + 2 K_{1,n} - K_{0,n}) ,$$

and the fourth order Runge-Kutta solution[7] gives a one-step approximation of

$$y_{n,i+1} = y_{n,i} + \frac{1}{6}(K_{1,n} + 2\,K_{2,n} + 2\,K_{3,n} + K_{4,n}) \quad , \qquad \text{(A-7)}$$

where

$$K_{1,n} = h\, f_n\,(z_i;\ y_{1,i};\ y_{2,i};\ldots;\ y_{n,i}) \quad , \qquad \text{(A-8)}$$

$$K_{2,n} = h\, f_n\left(z_i + \frac{h}{2};\ y_{1,i} + \frac{1}{2}K_{0,1};\ y_{2,i} + \frac{1}{2}K_{0,2};\ \ldots;\ y_{n,i} + \frac{1}{2}K_{0,n}\right) ,$$

$$K_{3,n} = h\, f_n\left(z_i + \frac{h}{2};\ y_{1,i} + \frac{1}{2}K_{1,1};\ y_{2,i} + \frac{1}{2}K_{1,2};\ \ldots;\ y_{n,i} + \frac{1}{2}K_{1,n}\right) ,$$

$$K_{4,n} = h\, f_n\,(z_i + h;\ y_{1,i} + K_{2,1};\ y_{2,i} + K_{2,2};\ldots;\ y_{2,i} + K_{2,n}) \quad .$$

The program proceeds one step at a time until the end of run (TE) is reached at which time it returns control to the calling program for the calculation of R_n, T_n, A_n .

SUBROUTINE DERIV

This routine is used to obtain a numerical evaluation of the differential equations at the last valid point calculated. It also allows the choice of equations for permittivity (K_r) and/or conductivity (K_i) for the plasma layer.

These equations for K_r and K_i are

$$K = Az^4 + Bz^3 + Cz^2 + Dz + E \quad , \tag{A-9}$$

$$K = Ae^{Bz} + Ce^{-Dz} + E \quad , \tag{A-10}$$

$$K = Az^{-4} + Bz^{-3} + Cz^{-2} + Dz^{-1} + E \quad , \tag{A-11}$$

$$K = Ae^{-\left(\frac{z - B}{C}\right)} + D + E \quad . \tag{A-12}$$

The derivatives that this subroutine evaluates are

$$\frac{dE_r}{dz} = - \omega \mu_o H_i \quad , \tag{A-13}$$

$$\frac{dE_i}{dz} = \omega \mu_o H_r \quad , \tag{A-14}$$

$$\frac{dH_r}{dz} = - \omega \varepsilon_o (K_i E_r + K_r E_i) \quad , \tag{A-15}$$

$$\frac{dH_i}{dz} = \omega \varepsilon_o (K_r E_r - K_i E_i) \quad . \tag{A-16}$$

Subroutine DERIV is under the control of subroutine RUNGKT.

SUBROUTINE PRINT

This is a dummy routine necessary for Subroutine RUNGKT to function properly. If it is ever desired to print the values of the fields during the integration process, the print statements should be placed in this routine. The routine will be called 100 times for every complete data case.

APPENDIX B

INPUTS TO THE PROGRAM

The initial conditions are read into the computer from 10 input cards containing the following information in the MKS system of units:

Card #1 --- OMEGA, z_0, M_1

OMEGA - frequency in radians/sec of signal under study

z_0 - thickness of plasma layer in meters

M_1 - relative permeability

Card #2 --- M_2, M_3, M_4

M_2 - relative permeability

M_3 - relative permeability

M_4 - relative permeability

Card #3 --- M_5, M_6, M_7

M_5 - relative permeability

M_6 - relative permeability

M_7 - relative permeability

Card #4 --- M_8, M_9, M_{10}

M_8 - relative permeability

M_9 - relative permeability

M_{10} - relative permeability

Card #5 --- CODE

CODE - if CODE = 0, a complete new data case is read;

if CODE $\neq$ 0, the program will print end of job.

Card #6 --- Equation No. A, B

Equation No. - number 1, 2, 3, or 4 depending upon which equation for permittivity is being used

A - constant of the equations for permittivity

B - constant of the equations for permittivity

Card #7 --- C, D, E

C - constant of the equations for permittivity

D - constant of the equations for permittivity

E - constant of the equations for permittivity

Card #8 --- Equation No. A, B

Equation No. - number 1, 2, 3, or 4 depending upon which equation for conductivity is being used

A - constant of the equation for conductivity

B - constant of the equation for conductivity

Card #9 --- C, D, E

C - constant of the equation for conductivity

D - constant of the equation for conductivity

E - constant of the equation for conductivity

Card #10 --- Break Point

Break Point - If the break point = 0, then program proceeds with its calculations, but if the break point > 0, program reads new data with cards (1) through (5) omitted for a change in K_r and/or K_i for the plasma layer.

NOTE: Format is 3E15. 8.

If it is desired to change equations during the integration process, set break point equal to the point at which it is desired to change to a new equation and place the constants on a new group of input cards (Cards six (6) through ten (10)). Set the break point of these cards to a new change point or to zero.

OUTPUT OF THE PROGRAM

The quantities shown on the sample print-out are defined as follows:

- OMEGA - frequency in radians/sec of signal under study
- z - thickness of plasma layer in meters $(= z_o)$
- EQ. NO. - number of equations being used
- RANGE - portion of medium in which above equations are used
- E - electric field in volts per meter
- ϕ_E - phase of E with respect to phase at z_o
- H - magnetic field intensity in ampere-turns per meter
- ϕ_H - phase of H with respect to phase at z_o
- RM - reflection coefficient for a specific value of M
- TM - transmission coefficient for a specific value of M
- AM - absorption coefficient for a specific value of M
- M - relative permittivity of dielectric window

This program was written for an IBM 7040 computer which uses eight significant figures. Each case ran in approximately one minute and forty seconds on this machine.

SP - 67

```
$JOB            SP 67
$IBJOB SP67     MAP
$IBFTC SP67
      DIMENSION Y%25¤,DY%25¤,F%175¤
      COMMON Y,DY,F,P,TT,TP,TE,E2,I,NCI,NN,T
    2 READ 1,OMA   ,TT,%F%MA¤,MA # 161,170¤,F%150¤
      E2 # .1E-03
    1 FORMAT%3E15.8¤
      I # 4
      TP #-TT/100.
      P # TP
      NN # 4
      NCI #4
      Z # TT
      F%171¤ # .8854E-11
      F%172¤ # .12566371E-05
      F%173¤ # OMA
      EO # F%171¤
      XMU # F%172¤
      Y%3¤ # SQRT %EO/XMU¤
```

SP - 67

```
      Y%4¤ # 0.
      Y%2¤ # 0.
      Y%1¤ # 1.
      WRITE %6,6¤OMA,TT
    6 FORMAT%1H1,23X,86HREFLECTION , TRANSMISSION AND ABSORPTION  COEFFI
     1CIENTS FOR INHOMOGENEOUS PLASMA LAYER ///24X.7HOMEGA #,E15.8,33X
     2,                    5HZ    #E15.8////21X,3HEQ. 9X,      5HRANGE12X,
     31HA,12X,1HB,12X,1HC,12X,1HD,12X,1HE// ¤
    5 READ 1,F%175¤,%F%MA¤,MA #156,160¤,F%174¤,%F%MA¤,MA # 151,155¤,TE
      R1  # TT
      R2  # TE
      J # F%175¤
      MB# F%174¤
      PRINT7,J,TT,TE,%F%MA¤,MA#156,160¤,MB     ,%F%MA¤,MA # 151,155¤
    7 FORMAT%1H ,5X,12HPERMITTIVITYI5,F10.4,2HTOF6.4,5X,E11.4,4E13.4//
     16X,12HCONDUCTIVITYI5,21X,5E13.4///¤
      CALL RUNGKT
      TP # P
      IF%TE¤108,108,5
  108 SMU # SQRT %XMU¤
```

SP - 67

```
      E# SQRT % Y%1¤**2&Y%2¤**2¤
      H # SQRT % Y%3¤**2&Y%4¤**2¤
      PHIE # ATAN2%Y%2¤ , Y%1¤¤ * 180./3.1415927
      PHIH # ATAN2%Y%4¤ , Y%3¤¤ * 180./3.1415927
      WRITE%6,107¤ %Y%IX¤,IX # 1,4¤
  107 FORMAT%1H ,///15X,6HE%R¤ #E15.8,5X,6HE%I¤ #E15.8,10X,6HH%R¤ #,
     1E15.8,10X,6HH%I¤ #E15.8 ¤
      PRINT 109,E,PHIF,H,PHIH
  109 FORMAT%1H ,3X,///15X,3HE #E15.8, 8X,8HPHI%E¤ #E15.8, 8X,3HH #E15.8
     1,13X,8HPHI%H¤ #E15.8/////25X,1HM26X,2HRM,27X,2HTM,26X,2HAM///¤
      DO4M # 1,10
      M6 # M & 160
      XM # F%M6¤
      SEM # SQRT %XM*E0¤
      RM #%%SEM*Y%1¤-SMU*Y%3¤¤**2&%SEM*Y%2¤-SMU*Y%4¤¤**2¤/%%SEM*Y%1¤&SMU
     1 *Y%3¤¤**2&%SEM*Y%2¤& SMU*Y%4¤¤**2¤
      TM # 4.0*XM*E0/%%SEM*Y%1¤&SMU*Y%3¤¤**2&%SEM*Y%2¤&SMU*Y%4¤¤**2¤
     1/SQRT %XM¤
      AM # 1.-TM - RM
    4 PRINT 8,XM,RM,TM ,AM
```

SP - 67

```
    8 FORMAT%1H ,    4E30.8¤
      IF%F%150¤¤111,2,111
  111 PRINT 112
      STOP
  112 FORMAT %1H1, 5X////////////////25X , 10HEND OF JOB/1H1¤
      END
$IBFTC D67
      SUBROUTINE DERIV
      DIMENSION Y%25¤,DY%25¤,F%175¤
      COMMON Y,DY,F,P,TT,TP,TE,E2,I,NCI,NN,T
      J # F%175¤
      MA # F%174¤
      OMA # F%173¤
      EO # F%171¤
      XMU # F%172¤
      GO TO %71,72,73,74¤,J
   71 XR# F%156¤*T**4 &F%157¤*T**3  & F%158¤*T**2  &F%159¤*T & F%160¤
      GO TO 115
   72 XR# F%156¤/T**4 &F%157¤/T**3  & F%158¤/T**2  & F%159¤/T & F%160¤
      GO TO 115
```

SP - 67

```
 73 XR # F%156¤*EXP %F%157¤*T¤ &  F%158¤*EXP %F%159¤*%-T¤¤& F%160¤
    GO TO 115
 74 XR # F%156¤*EXP %-%T-F%157¤¤/F%158¤¤&F%159¤
115 GO TO %75,76,77,78¤,MA
 75 XI # F%151¤*T**4 &F%152¤*T**3 &F%153¤*T**2 &F%154¤*T & F%155¤
    GO TO 119
 76 XI #F%151¤/T**4 &F%152¤/T**3 &F%153¤/T**2 & F%154¤/T & F%155¤
    GO TO 119
 77 XI #F%151¤*EXP %F%152¤*T¤&F%153¤*EXP %-F%154¤*T¤ & F%155¤
    GO TO 119
 78 XI #F%151¤*EXP %-%T-F%152¤¤/F%153¤¤ & F%154¤
119 DY%1¤ # -OMA*XMU*Y%4¤
    DY%2¤ #  OMA*XMU*Y%3¤
    DY%3¤ # -OMA*%XI*Y%1¤& XR*Y%2¤¤  *EO
    DY%4¤ # OMA *%XR*Y%1¤- XI*Y%2¤¤ *EO
    RETURN
    END
$IBFTC P67
    SUBROUTINE PRINT
    RETURN
```

SP - 67

```
 73 XR # F%156¤*EXP %F%157¤*T¤ &  F%158¤*EXP %F%159¤*%-T¤¤& F%160¤
    GO TO 115
 74 XR # F%156¤*EXP %-%T-F%157¤¤/F%158¤¤&F%159¤
115 GO TO %75,76,77,78¤,MA
 75 XI # F%151¤*T**4 &F%152¤*T**3 &F%153¤*T**2 &F%154¤*T & F%155¤
    GO TO 119
 76 XI #F%151¤/T**4 &F%152¤/T**3 &F%153¤/T**2 & F%154¤/T & F%155¤
    GO TO 119
 77 XI #F%151¤*EXP %F%152¤*T¤&F%153¤*EXP %-F%154¤*T¤ & F%155¤
    GO TO 119
 78 XI #F%151¤*EXP %-%T-F%152¤¤/F%153¤¤ & F%154¤
119 DY%1¤ # -OMA*XMU*Y%4¤
    DY%2¤ #  OMA*XMU*Y%3¤
    DY%3¤ # -OMA*%XI*Y%1¤& XR*Y%2¤¤  *EO
    DY%4¤ # OMA *%XR*Y%1¤- XI*Y%2¤¤ *EO
    RETURN
    END
$IBFTC P67
    SUBROUTINE PRINT
    RETURN
```

SP - 67

```
      END
$IBFTC RUNG
      SUBROUTINE  RUNGKT
C
       DIMENSION Y%25¤,DY%25¤,F%175¤
       COMMON Y,DY,F,P,TT,TP,TE,E2,I,NCI,NN,T
      E1 # E2/100.
      NCII # 0
      N #NN
      L # 4
      DI # TP
      TP # DI & TT
  800 T # TT
      GO TO %75 ,200,300,400¤,L
   75 IG # IG
      GO TO %101,102¤,IG
  101 J # 1
      L # 2
      M # 0
      TS # T
```

SP - 67

```
      DO 106 K # 1,N
      K1 # K & N * 3
      K2 # K1 & N
      K3 # N  & K
      F%K1¤ # Y%K¤
      F %K3¤ # F%K1¤
  106 F%K2¤ # DY%K¤
      GO TO 402
  102 GO TO 60
   99 J # J & 1
      IF%J-I¤ 103,103,104
  103 L # 1
      GO TO 402
  104 M # M & 1
  105 GO TO %110,120,130¤,M
  110 DO 111 K # 1,N
      K1 # K & N & N
  111 F%K1¤ # Y%K¤
  112 DO 113 K # 1,N
      K1 # K & 3*N
```

SP - 67

```
      K2 # K1 & N
      K3 # N  & K
      Y%K¤ # F%K1¤
      F%K3¤# F%K1¤
113   DY%K¤ # F%K2¤
      T # TS
      IF%P¤ 114,116,114
114   IF %ABS %H/P¤-.0000001¤ 115,115,116
115   M # 0
      L # 4
      GO TO 402
116   DT # .5*H
      M # 1
      J # 1
      GO TO 300
120   DO 121 K # 1,N
      K1 # K & N
121   F%K1¤ # Y%K¤
      M # 2
      J # 1
```

SP - 67

```
      IG # 2
      L # 1
      GO TO 402
  130 DO 131 K # 1,N
      K1 # K & 2*N
      F%K¤ # %Y%K¤-F%K1¤¤/%2.**I-1.¤
      Y%K¤ # Y%K¤ & F%K¤
      IF%ABS %F%K¤¤-.00001¤139,139,140
  139 F%K¤# 0.
      GO TO 131
  140 F%K¤ # ABS %F%K¤/Y%K¤¤
  131 CONTINUE
      E # F%1¤
      INDEX # 1
      IF %N-1¤1335,1335,1315
 1315 DO 133 K # 2,N
      IF%E-F%K¤¤132,133,133
  132 INDEX # K
      E # F%K¤
  133 CONTINUE
```

SP - 67

```
1335 IF%E-E1¤134,135,135
 134 H # H & H
1345 DT # H
     GO TO 401
 135 IF%E-E2¤1345,1345,136
 136 DO 137 K # 1,N
     K1 # K & N
     K2 # K & N & N
 137 F%K2¤ # F%K1¤
 138 H # .5*H
     GO TO 112
 200 MU # MU
     GO TO %203,204¤,MU
 203 H#AMAX1%H,H1,H2¤
     MU # 2
 204 H1 # ABS %H¤
     IF%P¤205,206,206
 205 H # -H1
     GO TO 207
 206 H # H1
```

SP - 67

```
207 IF%ABS %P¤-H1¤208,209,209
208 H # P
209 T2 # TP - T
    IF% ABS%T2¤ - .1E-08¤ 212,210,210
210 H2 # ABS %T2¤
211 IF%ABS %T2/DI¤-.00001¤ 212,213,213
212 T # TP
    L # 3
    GO TO 402
213 M # 0
    J # 1
    IF%H1-H2¤ 215,215,214
214 MU # 1
    H # T2
215 DT # H
300 IG # 2
    GO TO 102
400 MU # 2
    H # P
    DT # P
```

SP - 67

```
      N # NN
 401  IG # 1
      L # 1
 402  TT # T
      GO TO % 902,903,904,905 ¤,L
 903  GO TO 800
 904  CONTINUE
       CALL PRINT
      IF%TP-TE¤901,901,801
 801  TP # TP & DT
      GO TO 800
 905  WRITE%6,909¤INDEX,TT,Y%INDEX¤
 909  FORMAT%1H0,///5X,I2,25HDOES NOT CONVERGE AT T # ,F14.8,25HCURRENT
     1VALUE OF Y%I¤ IS ,E15.8///¤
      IF%NCI-NCII¤901,901,908
 908  NCII # NCII & 1
      J # 1
      IG # 1
      DT #  H
      M # 0
```

SP - 67

```
      GO TO 800
   60 DO 100 K # 1,N
      K1#K
      K2#K&5*N
      K3#K2&N
      K4#K&N
      GOTO %999,85,95,95¤,I
   85 GOTO %86,2,999,999¤,J
   86 F%K1¤#DY%K¤*DT
      Y%K¤#F%K4¤&F%K1¤
      GO TO 100
   95 GOTO %1,2,3,4¤,J
    1 F%K1¤#DY%K¤*DT
      Y%K¤#F%K4¤&.5*F%K1¤
      GO TO 100
    2 F%K2¤#DY%K¤*DT
      GOTO %999,22,23,24¤,I
    3 F%K3¤#DY%K¤*DT
      GOTO %999,33,33,34¤,I
    4 Y%K¤#F%K4¤&%F%K1¤&2.*%F%K2¤&F%K3¤¤&DY%K¤*DT¤/6.
```

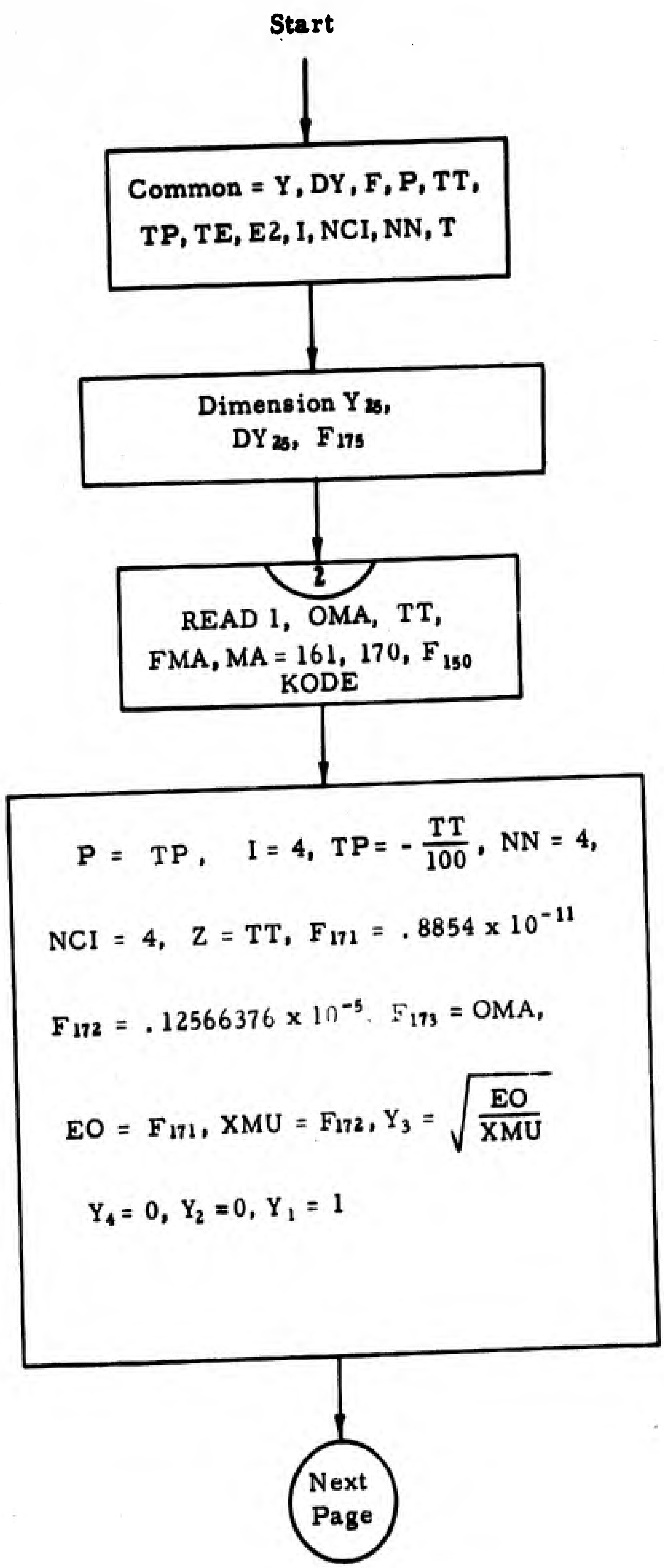
SP - 67

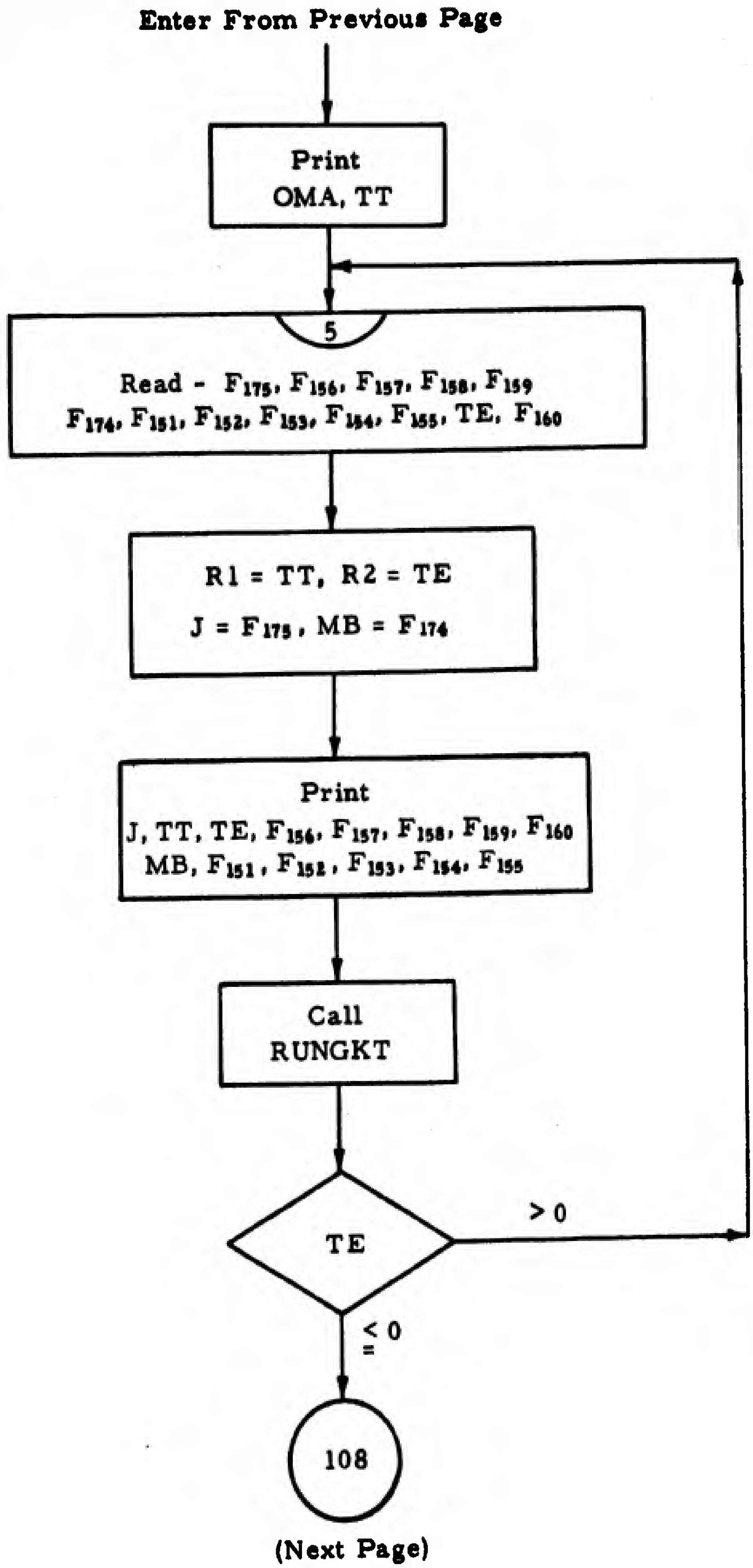
```
      GOTO100
   22 Y%K¤#.5*%F%K1¤&F%K2¤¤
      GOTO25
   23 Y%K¤#2.*F%K2¤-F%K1¤
      GOTO25
   24 Y%K¤#.5*F%K2¤
   25 Y%K¤#Y%K¤&F%K4¤
      GOTO100
   33 Y%K¤#F%K4¤&%F%K1¤&4.*F%K2¤&F%K3¤¤/6.
      GOTO100
   34 Y%K¤#F%K4¤&F%K3¤
  100 CONTINUE
      GO TO  %50,61,62,58 ¤,J
   50 GO TO %999,56,57,57¤,I
   61 GO TO %999,58,57,58¤,I
   62 GO TO %999,58,58,57¤,I
   56 T # DT & T
      GO TO 58
   57 T # T  & .5* DT
   58 GO TO 99
```

SP - 67

```
999 CALL DUMP
    GO TO 58
902 CONTINUE
     CALL DERIV
    GO TO 800
901 RETURN
    END
```

MAIN PROGRAM




Enter From Previous Page
Print
OMA, TT
5
Read - F_{175}, F_{156}, F_{157}, F_{158}, F_{159}
F_{174}, F_{151}, F_{152}, F_{153}, F_{154}, F_{155}, TE, F_{160}
R1 = TT, R2 = TE
J = F_{175}, MB = F_{174}
Print
J, TT, TE, F_{156}, F_{157}, F_{158}, F_{159}, F_{160}
MB, F_{151}, F_{152}, F_{153}, F_{154}, F_{155}
Call
RUNGKT
TE
> 0
≤ 0
108
(Next Page)

Enter from Previous Page

108

$$SMU = \sqrt{XMU}, \quad E = \sqrt{Y_1^2 + Y_2^2}$$

$$H = \sqrt{Y_3^2 + Y_4^2}, \quad PHIE = TAN^{-1} \frac{Y_2}{Y_1}$$

$$PHIE = TAN^{-1} \frac{Y_4}{Y_3}$$

Print E,
PHIE, H, PHIH

DO 4
M = 1, 10

$$M6 = M + 160, \quad XM = F_{M6}, \quad SEM = \sqrt{(XM)(EO)}$$

$$RM = \frac{(SEM \cdot Y1 - SMU \cdot Y3)^2 + (SEM \cdot Y2 - SMU \cdot Y4)^2}{(SEM \cdot Y1 + SMU \cdot Y3)^2 + (SEM \cdot Y2 + SMU \cdot Y4)^2}$$

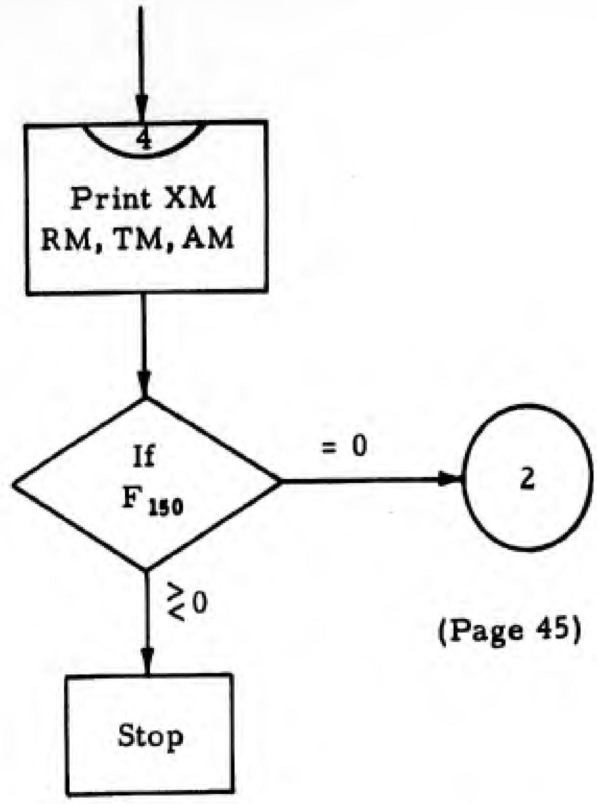
$$TM = \frac{4 \cdot EO \cdot XM}{(SEM \cdot Y1 + SMU \cdot Y3)^2 + (SEM \cdot Y2 + SMU \cdot Y4)^2}$$

$$AM = 1 - TM - RM$$

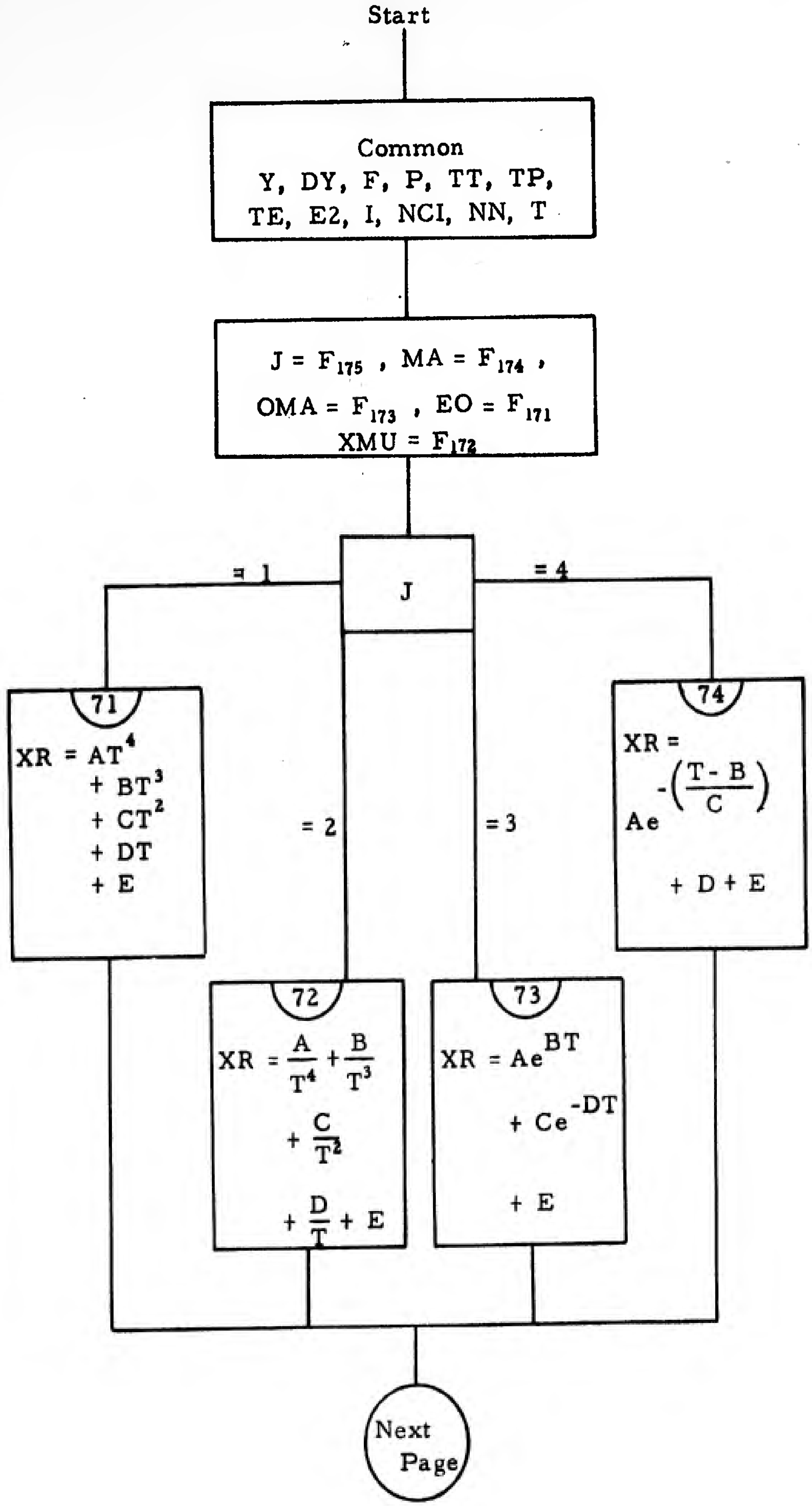
4

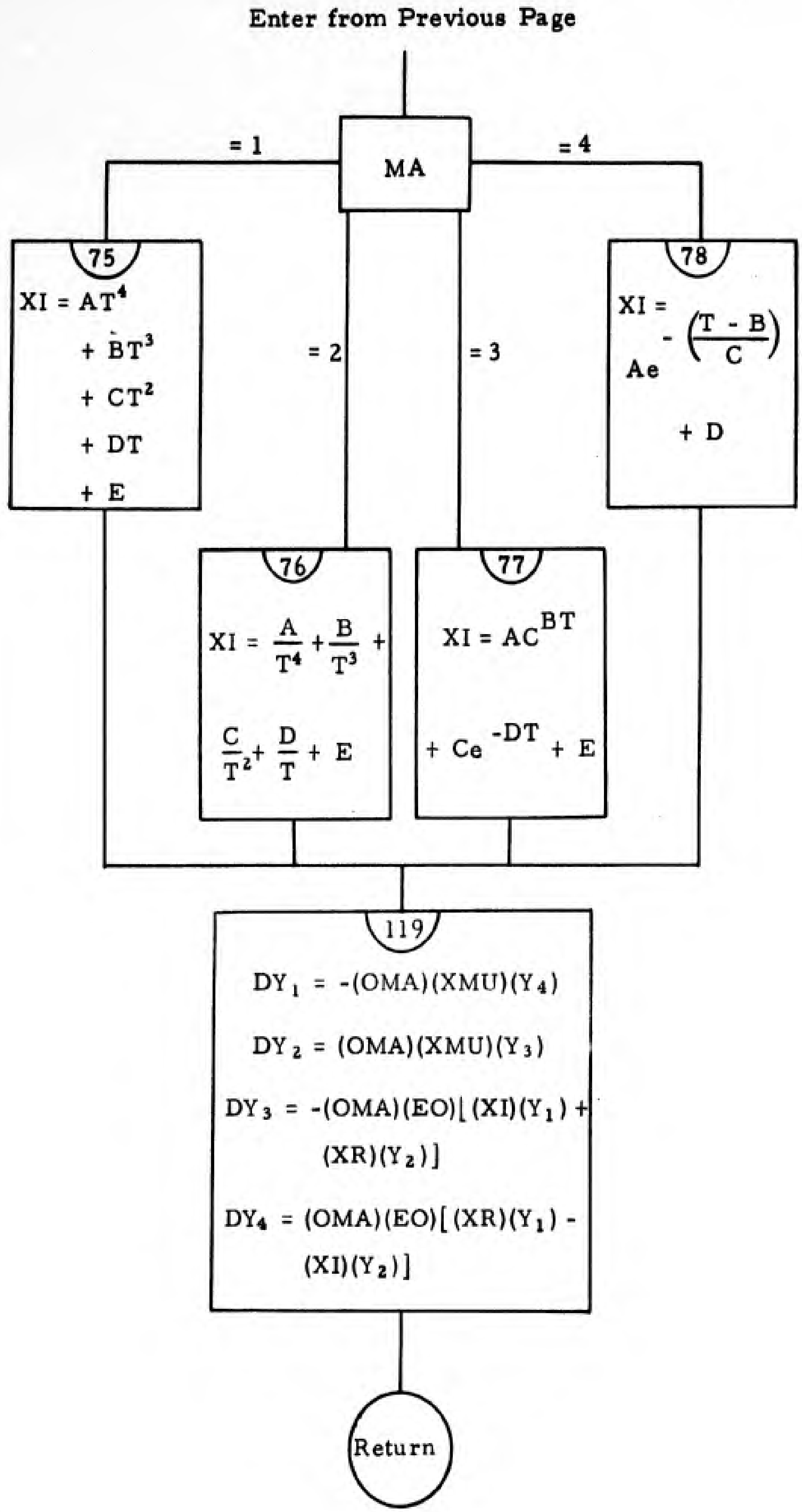
(Next Page)

Enter 4 From Previous Page

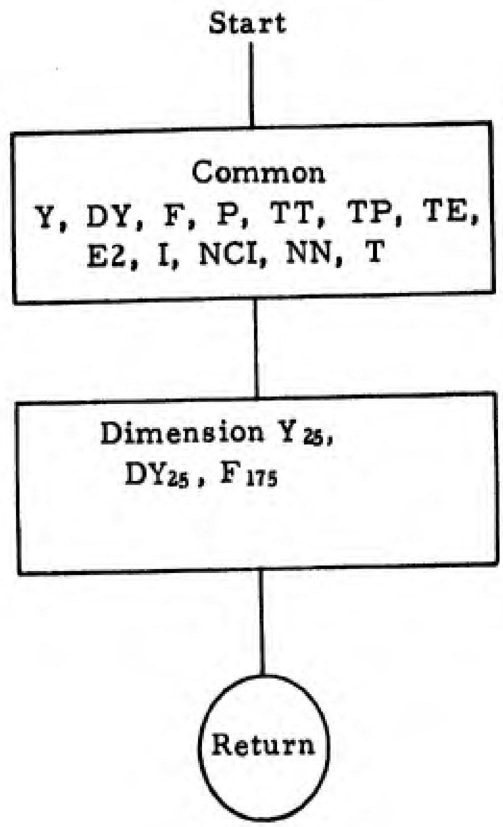


SUBROUTINE DERIV

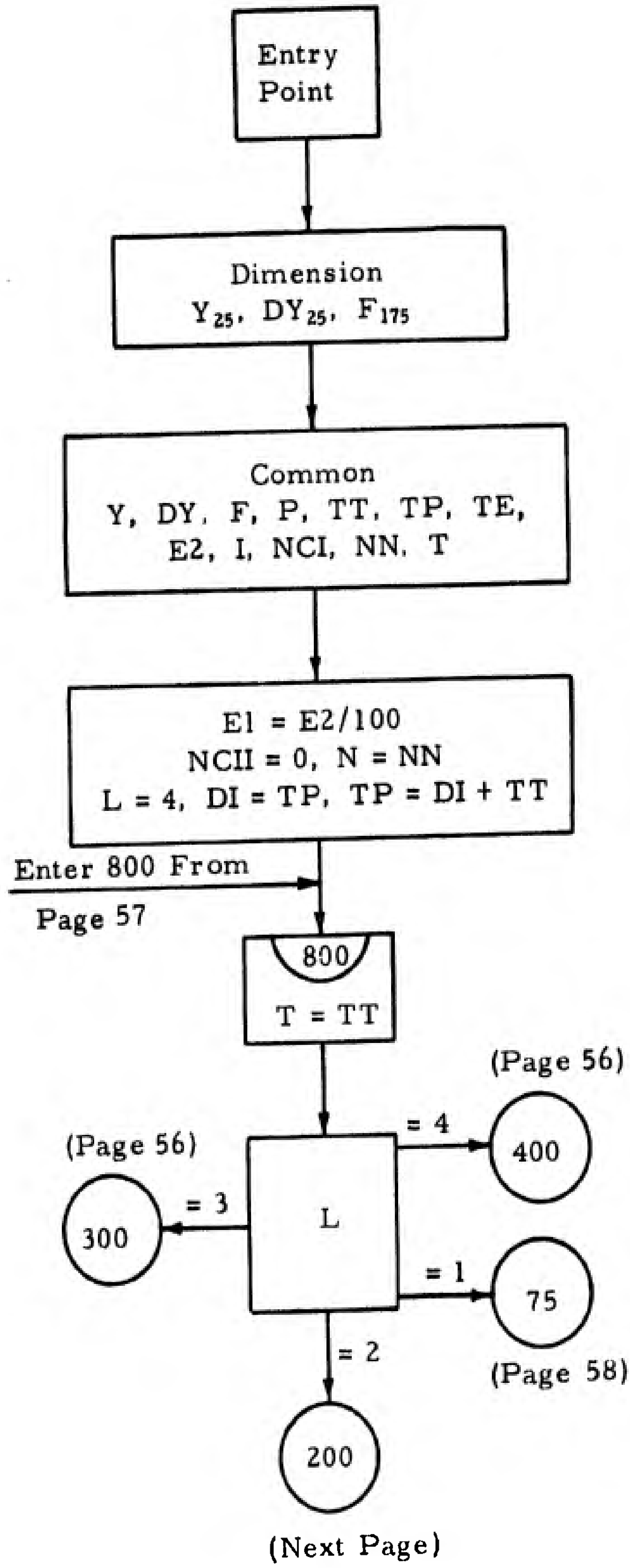



Enter from Previous Page
MA
= 1
= 2
= 3
= 4
75
$XI = AT^4 + BT^3 + CT^2 + DT + E$
76
$XI = \frac{A}{T^4} + \frac{B}{T^3} + \frac{C}{T^2} + \frac{D}{T} + E$
77
$XI = AC^{BT} + Ce^{-DT} + E$
78
$XI = Ae^{-\left(\frac{T - B}{C}\right)} + D$
119
$DY_1 = -(OMA)(XMU)(Y_4)$
$DY_2 = (OMA)(XMU)(Y_3)$
$DY_3 = -(OMA)(EO)[(XI)(Y_1) + (XR)(Y_2)]$
$DY_4 = (OMA)(EO)[(XR)(Y_1) - (XI)(Y_2)]$
Return

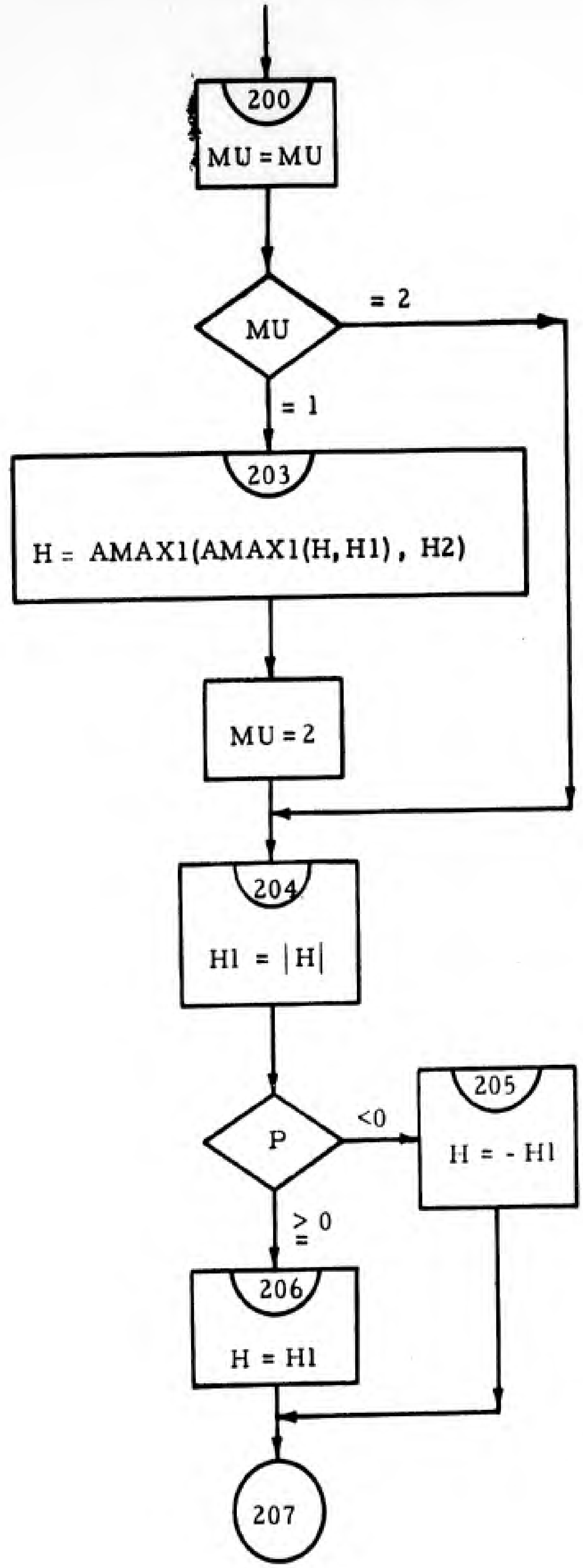
SUBROUTINE PRINT



SUBROUTINE RUNGKT

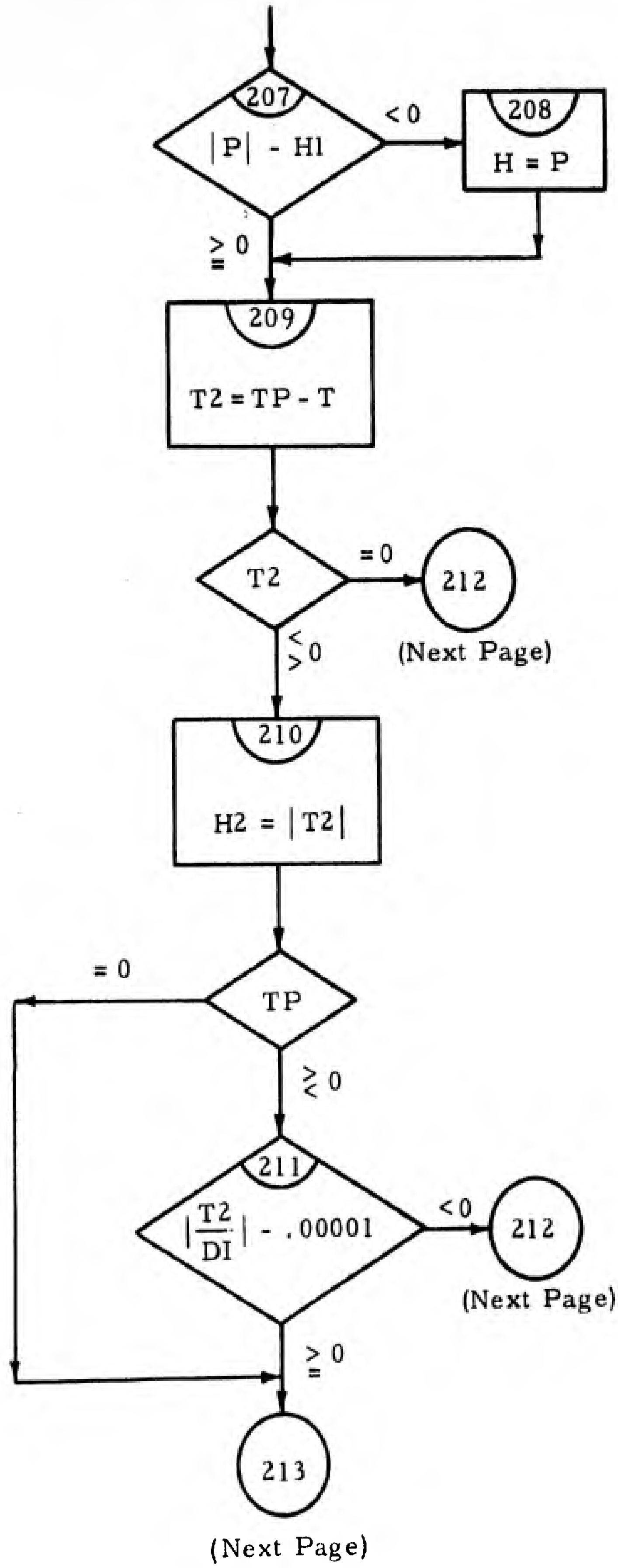


Enter 200 from Previous Page

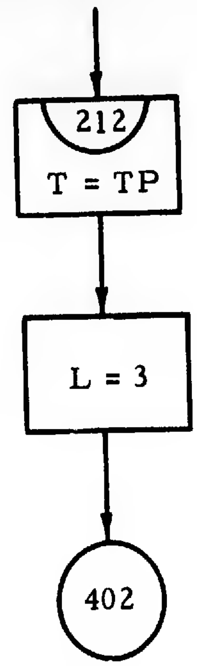


(Next Page)

Enter 207 from Previous Page

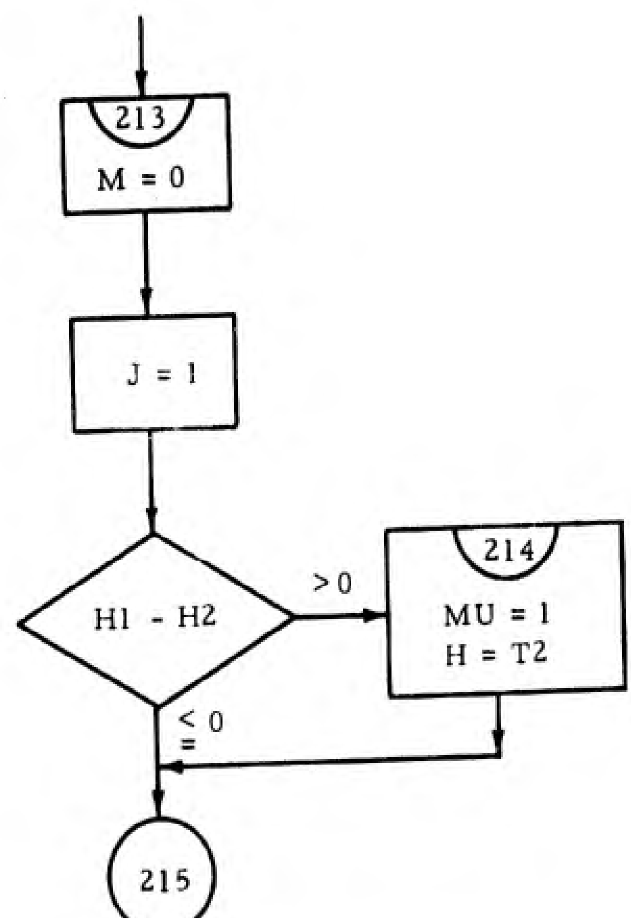


Enter 212 from Previous Page

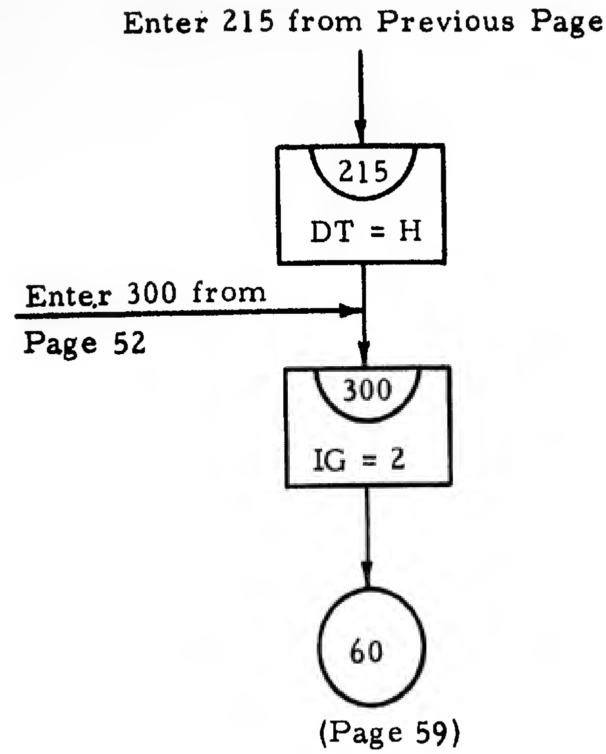


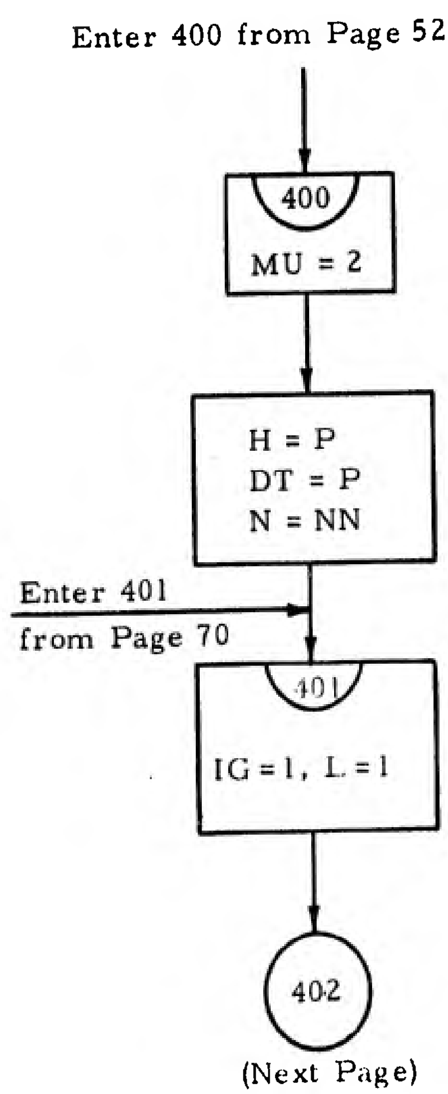
(Page 57)

Enter 213 from Previous Page

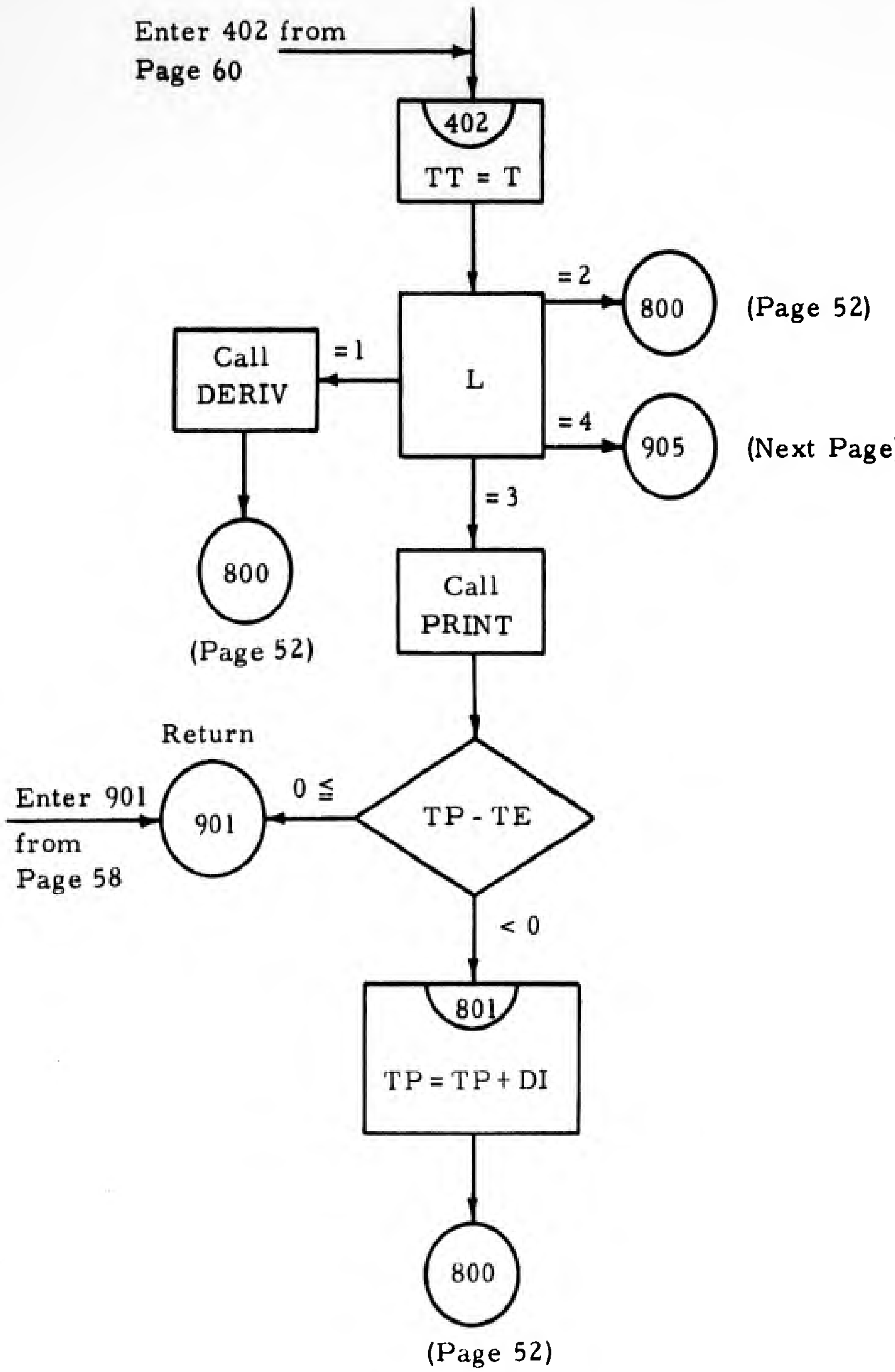


(Next Page)

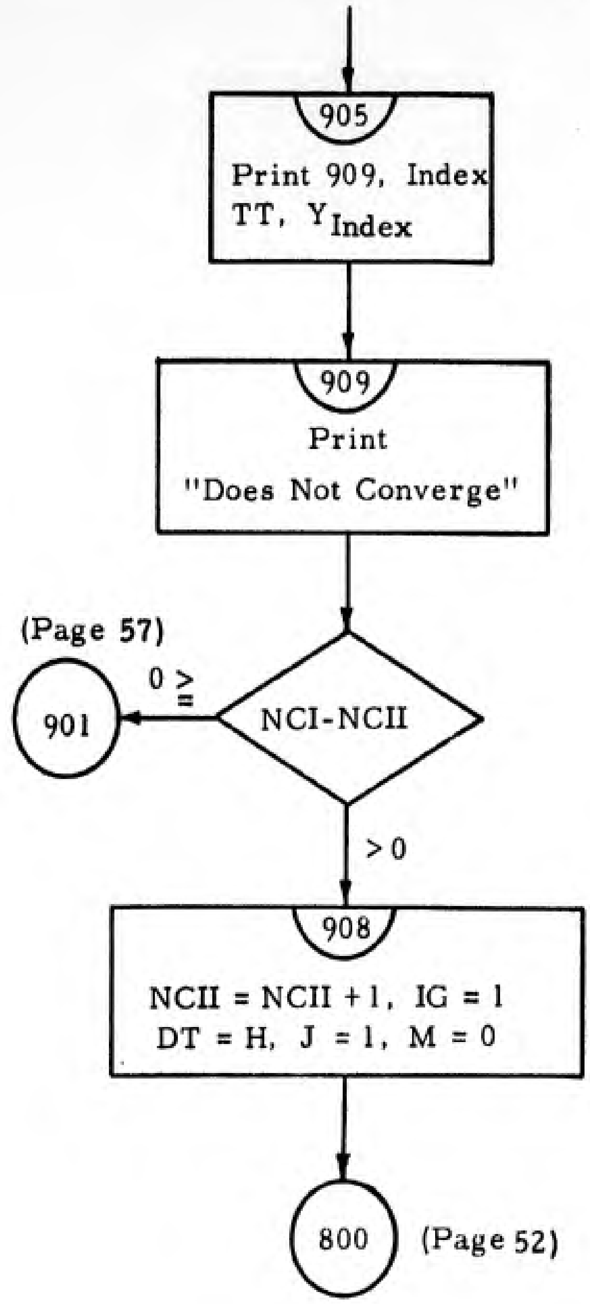

Enter 215 from Previous Page
215
DT = H
Enter 300 from
Page 52
300
IG = 2
60
(Page 59)


Enter 400 from Page 52
400
MU = 2
H = P
DT = P
N = NN
Enter 401
from Page 70
401
IG = 1, L = 1
402
(Next Page)

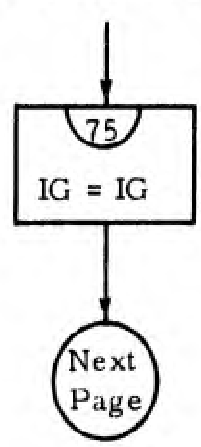
Enter 402 from Previous Page

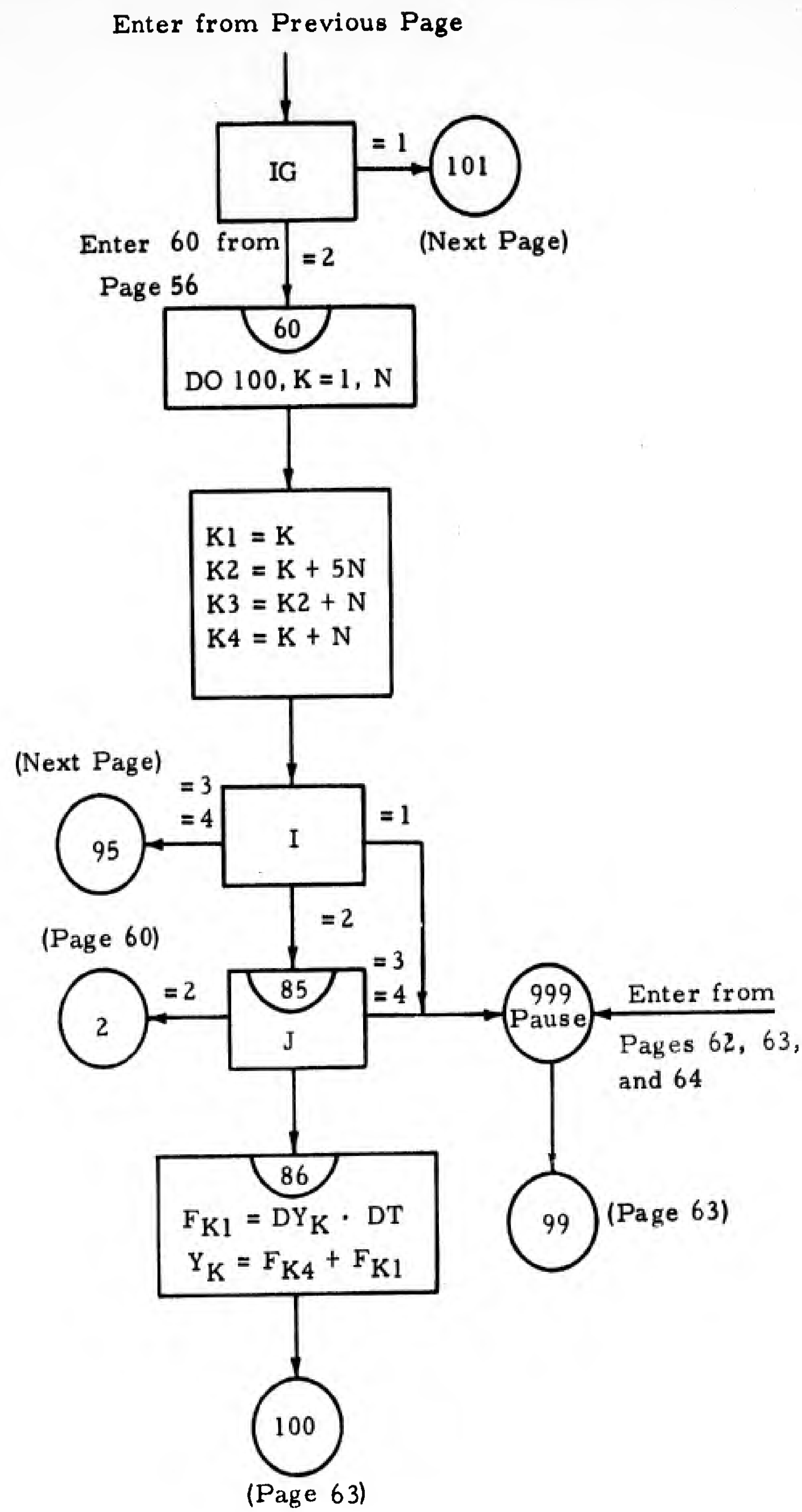


Enter 905 from Previous Page

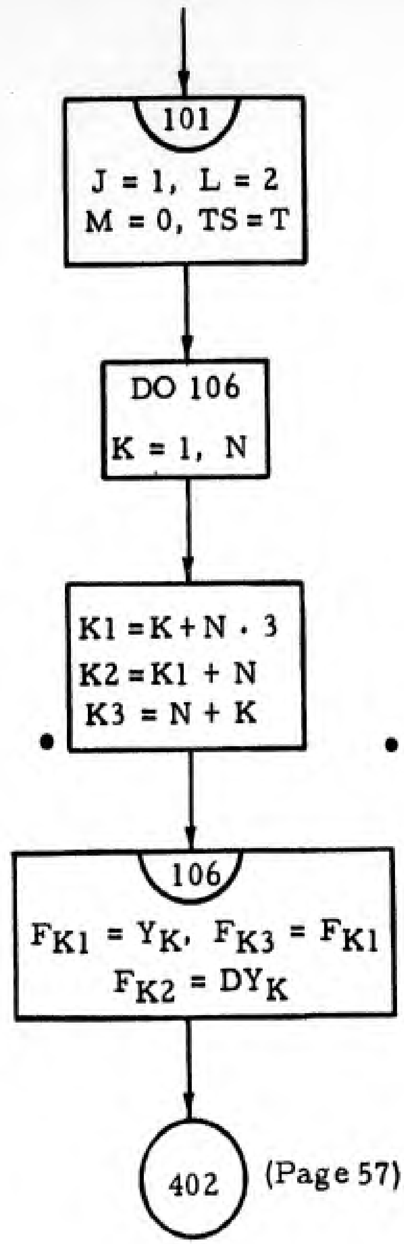


Enter 75 from Page 52

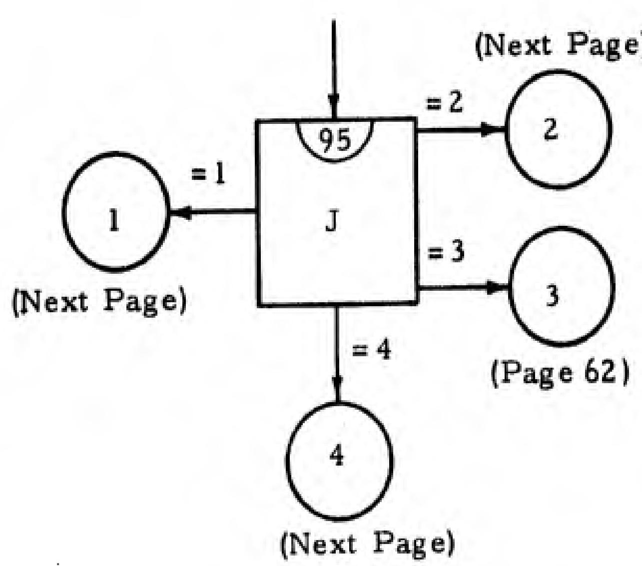


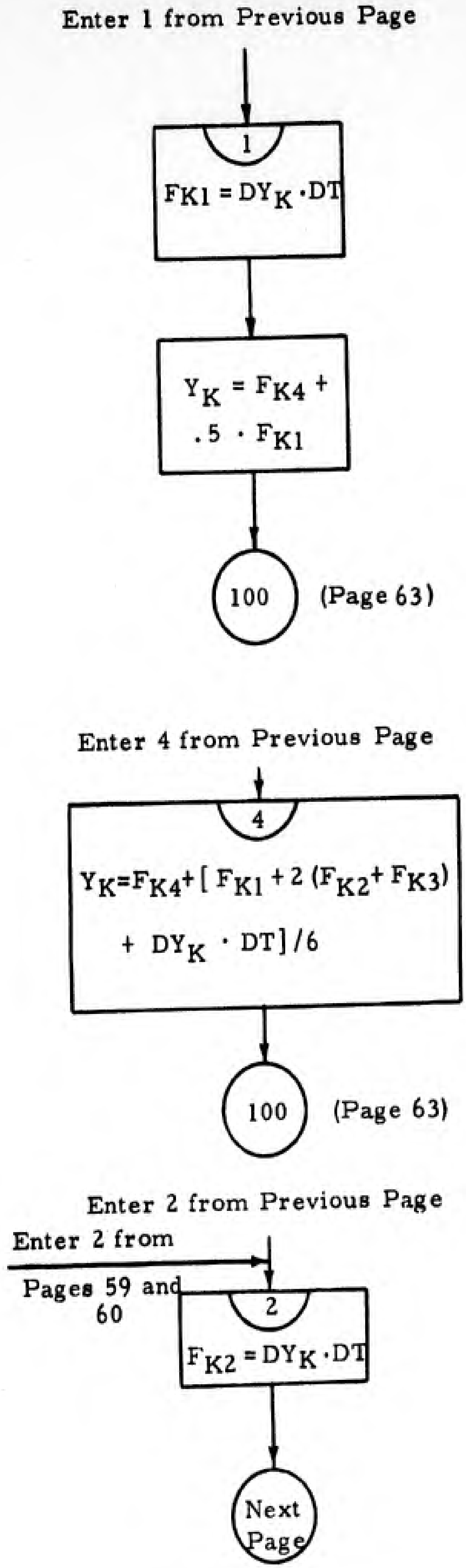

Enter from Previous Page
IG
= 1
101
(Next Page)
Enter 60 from Page 56
= 2
60
DO 100, K = 1, N
K1 = K
K2 = K + 5N
K3 = K2 + N
K4 = K + N
(Next Page)
= 3
= 4
95
I
= 1
= 2
(Page 60)
2
= 2
85
J
= 3
= 4
999 Pause
Enter from Pages 62, 63, and 64
86
$F_{K1} = DY_K \cdot DT$
$Y_K = F_{K4} + F_{K1}$
99
(Page 63)
100
(Page 63)

Enter 101 from Previous Page



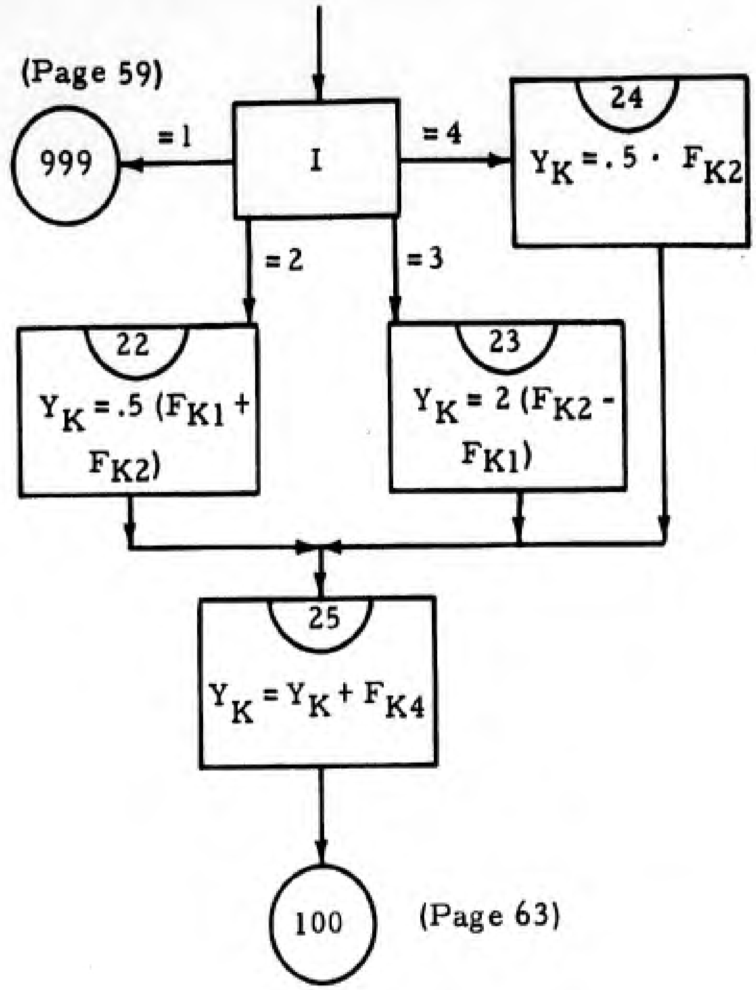
Enter 95 from Previous Page



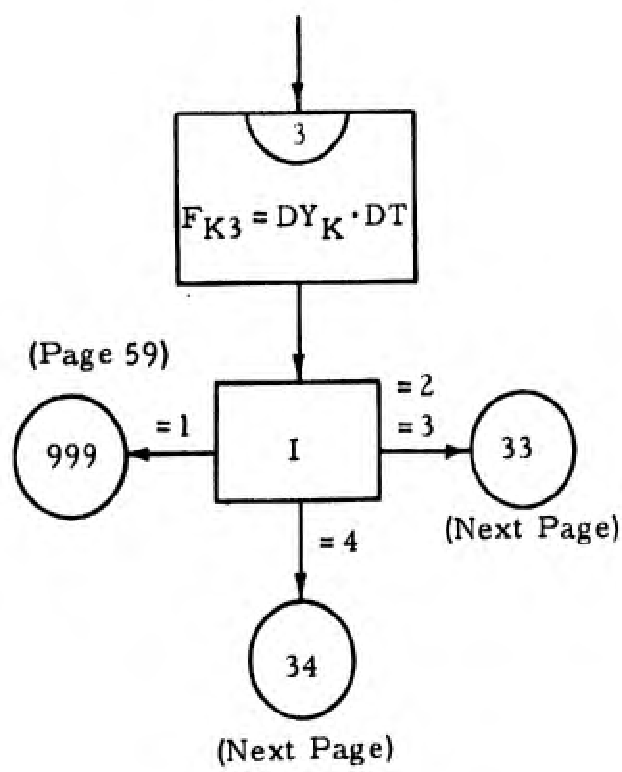


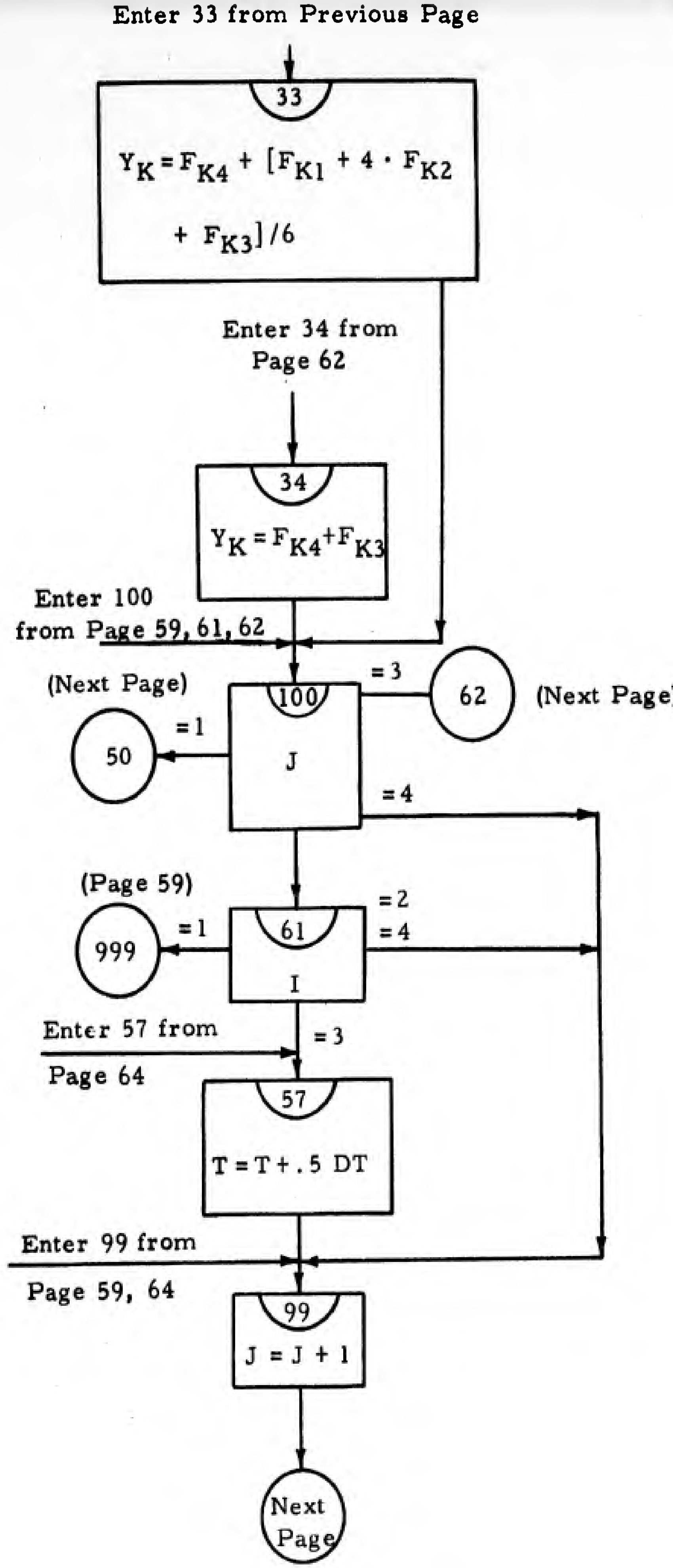
Enter 1 from Previous Page
1
FK1 = DYK ·DT
YK = FK4 +
.5 · FK1
100
(Page 63)
Enter 4 from Previous Page
4
YK=FK4+ [FK1 + 2 (FK2+ FK3)
+ DYK · DT] /6
100
(Page 63)
Enter 2 from Previous Page
Enter 2 from
Pages 59 and
60
2
FK2 = DYK ·DT
Next
Page

Enter from Previous Page



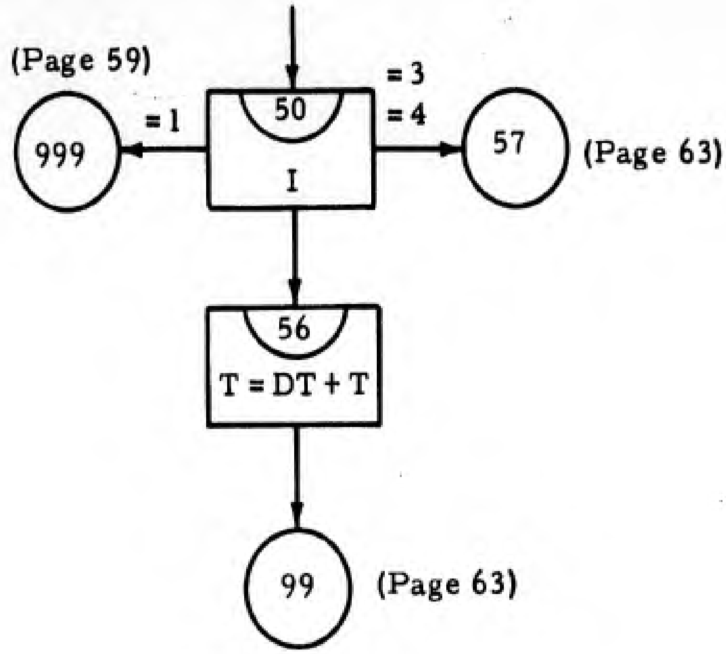
Enter 3 from Page 60



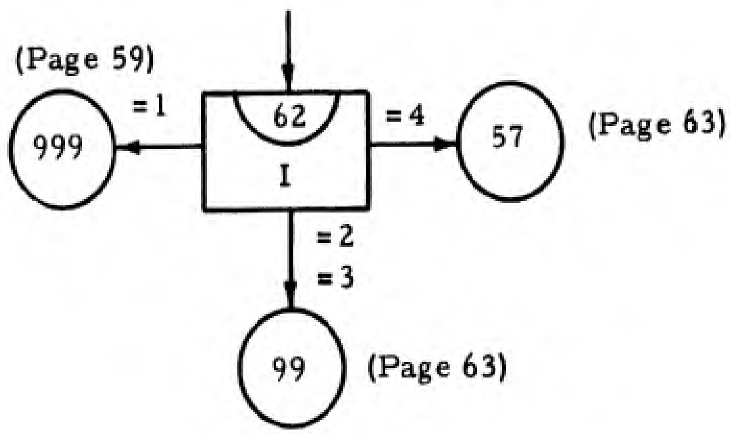


Enter 33 from Previous Page
33
$Y_K = F_{K4} + [F_{K1} + 4 \cdot F_{K2} + F_{K3}]/6$
Enter 34 from Page 62
34
$Y_K = F_{K4} + F_{K3}$
Enter 100 from Page 59, 61, 62
(Next Page)
=1
50
100
J
=3
62
(Next Page)
=4
(Page 59)
999
=1
61
I
=2
=4
=3
Enter 57 from Page 64
57
T = T + .5 DT
Enter 99 from Page 59, 64
99
J = J + 1
Next Page

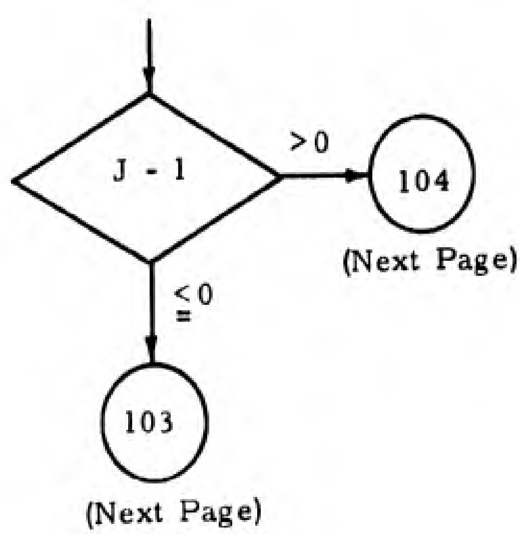
Enter 50 from Previous Page



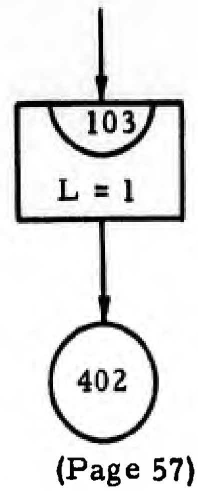
Enter 62 from Previous Page



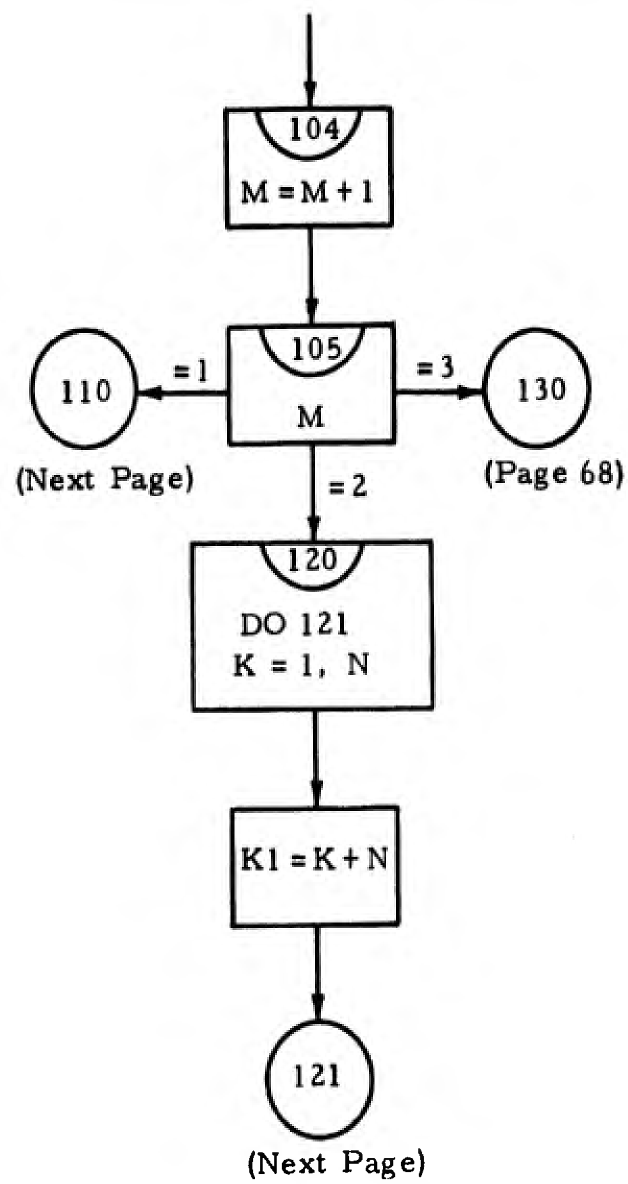
Enter from Previous Page



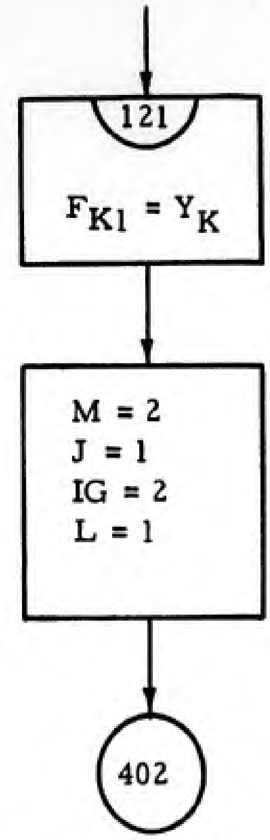
Enter 103 from Previous Page



Enter 104 from Previous Page

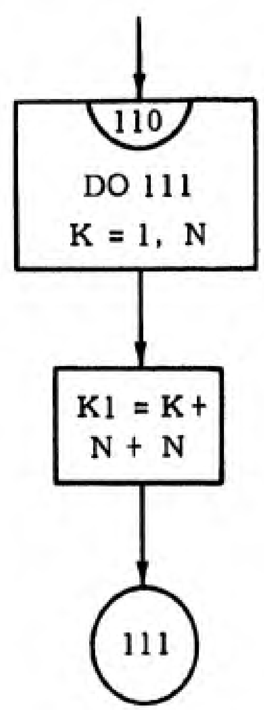


Enter 12 from Previous Page

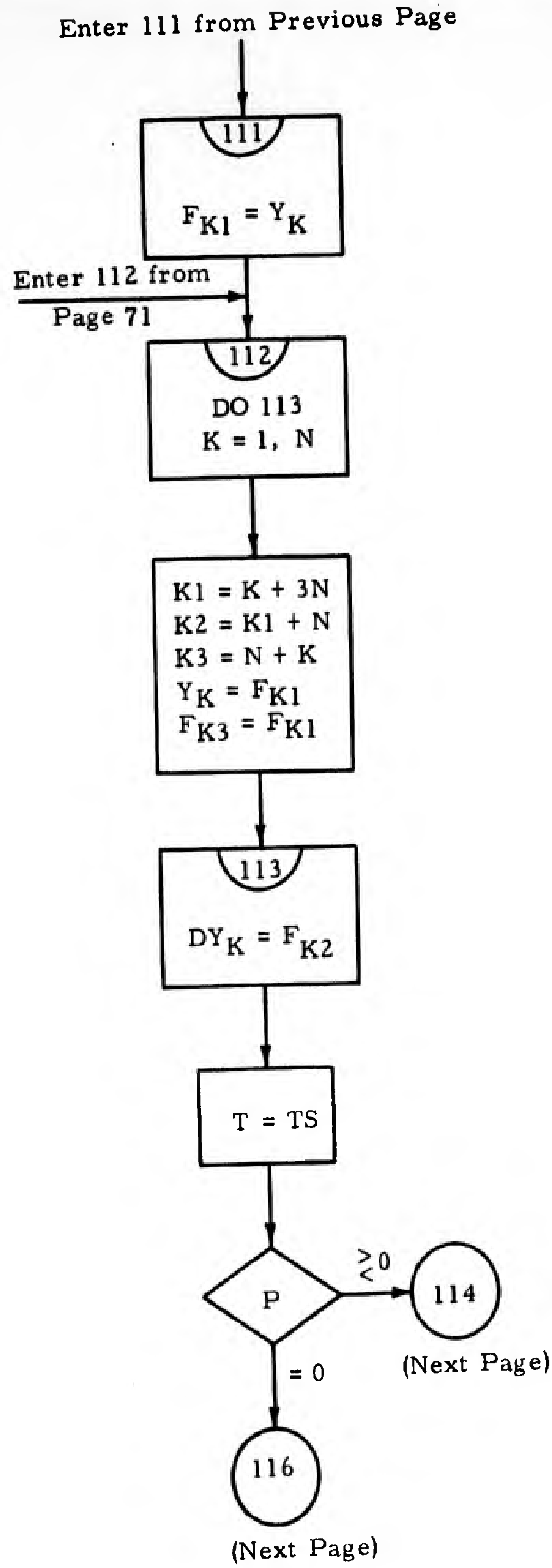


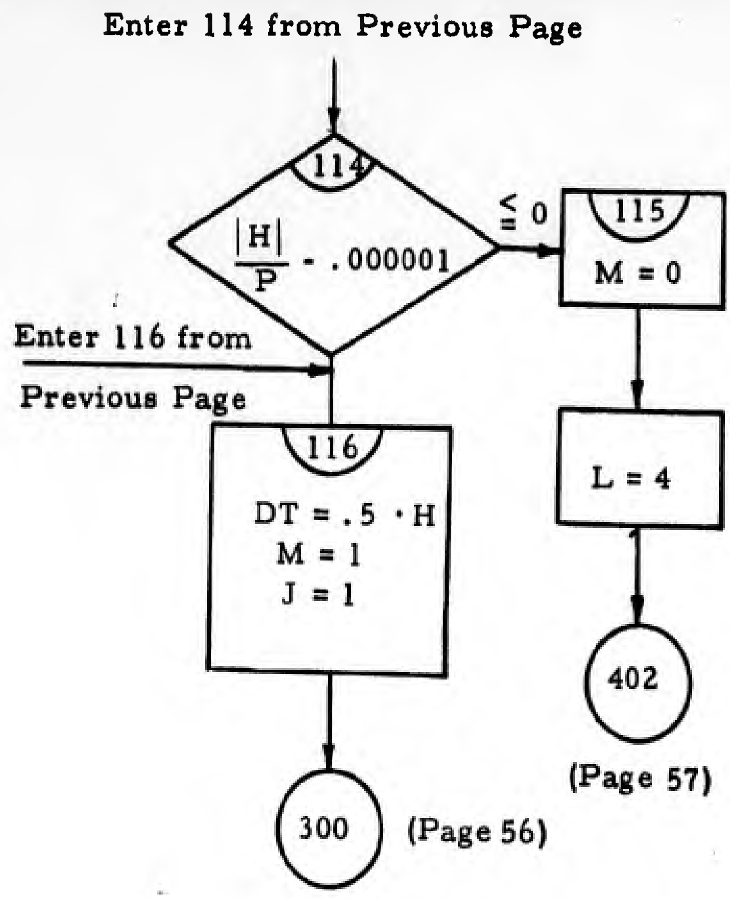
(Page 57)

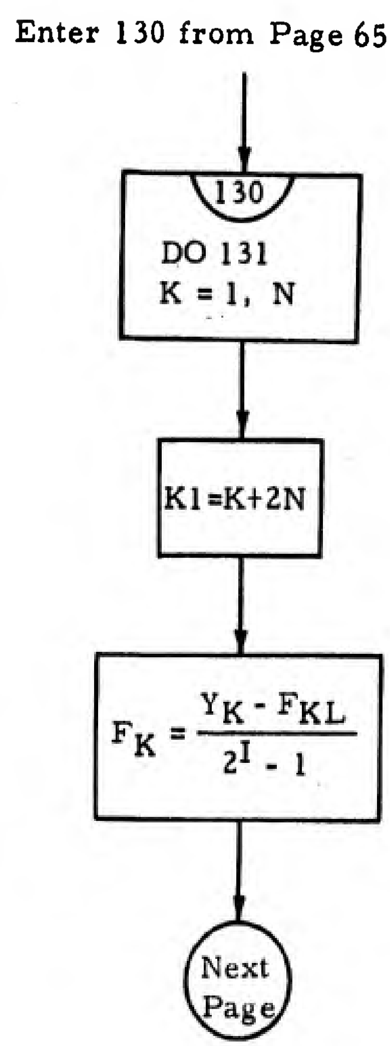
Enter 110 from Previous Page



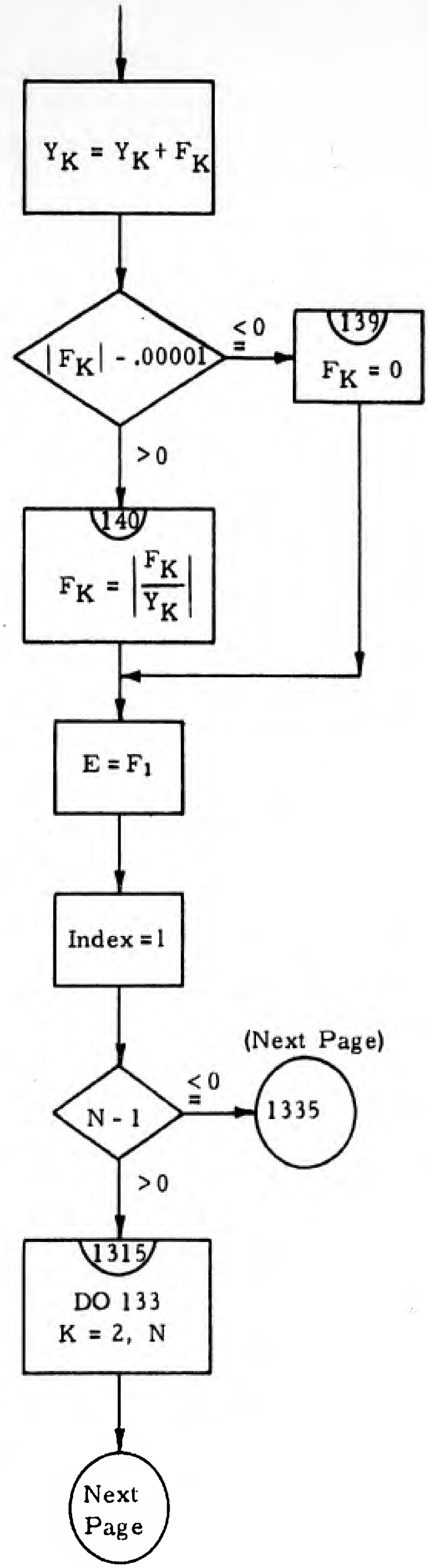
(Next Page)

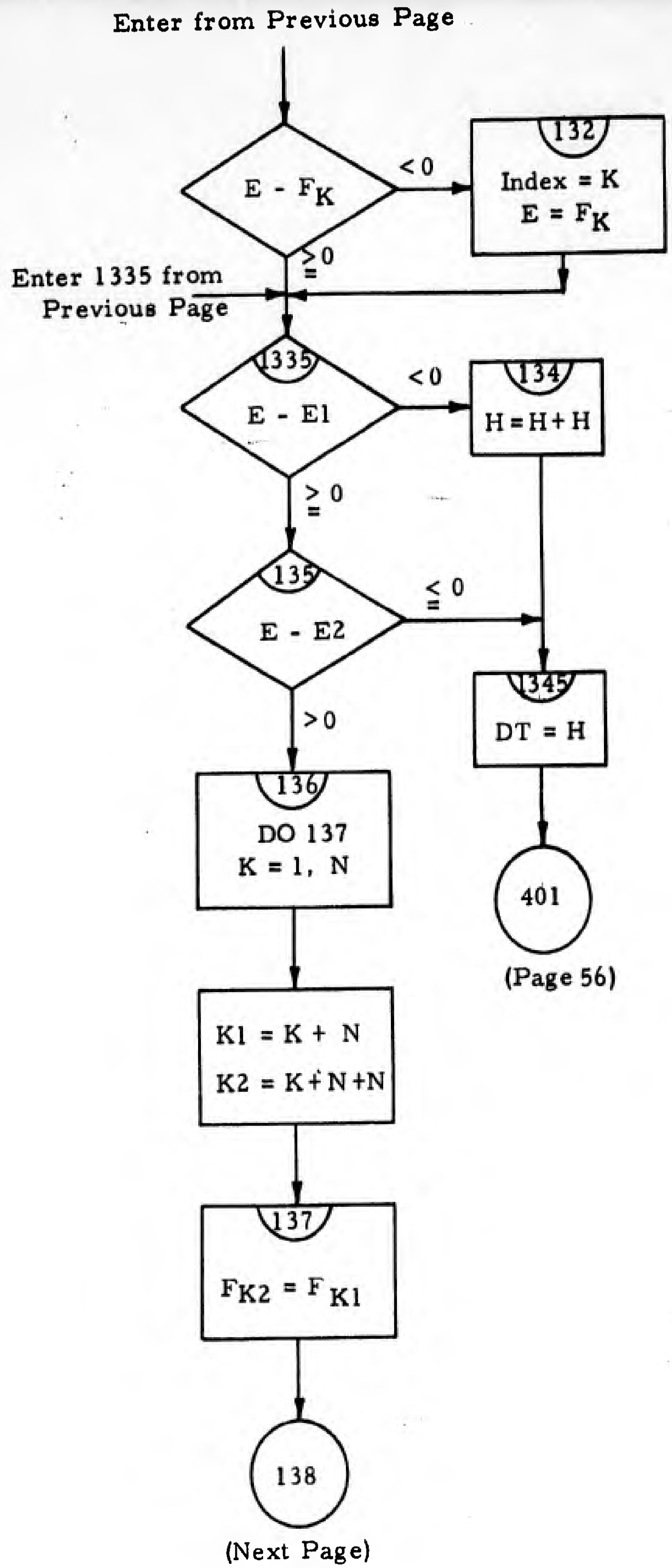

Enter 111 from Previous Page
111
$F_{K1} = Y_K$
Enter 112 from Page 71
112
DO 113
K = 1, N
$K1 = K + 3N$
$K2 = K1 + N$
$K3 = N + K$
$Y_K = F_{K1}$
$F_{K3} = F_{K1}$
113
$DY_K = F_{K2}$
T = TS
P
$\gtrless 0$
114
(Next Page)
= 0
116
(Next Page)


Enter 114 from Previous Page
114
|H| / P - .000001
≦ 0
115
M = 0
L = 4
402
(Page 57)
Enter 116 from Previous Page
116
DT = .5 · H
M = 1
J = 1
300
(Page 56)

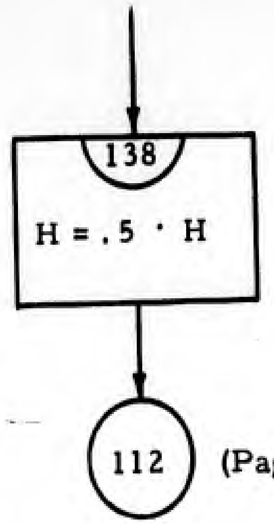

Enter 130 from Page 65
130
DO 131
K = 1, N
K1 =K+2N
$F_K = \frac{Y_K - F_{KL}}{2^I - 1}$
Next Page

Enter from Previous Page




Enter from Previous Page
E - F_K
< 0
132
Index = K
E = F_K
≥ 0
Enter 1335 from Previous Page
1335
E - E1
< 0
134
H = H + H
≥ 0
135
E - E2
≤ 0
1345
DT = H
401
(Page 56)
> 0
136
DO 137
K = 1, N
K1 = K + N
K2 = K + N + N
137
F_K2 = F_K1
138
(Next Page)

Enter 138 from Previous Page



(Page 67)

END OF PROGRAM

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