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TRANSLATION

ONE METHOD OF DETERMINING THE CRITERION OF OPERATIONAL STABILITY OF A LIQUID-PROPELLANT ROCKET ENGINE

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ONE METHOD OF DETERMINING THE CRITERION OF OPERATIONAL STABILITY OF A LIQUID-PROPELLANT ROCKET ENGINE.

by

G. I. Zamuruyev

At the present time the methods for determining the conditions under which the operation of a liquid-propellant rocket engine (LRE) is stable are still not reliable enough. In the published literature the stability regions are determined graphically.

In this article an attempt is made to find a certain new criterion which will allow analytical investigation of the operational stability of LRE.

Liquid-propellant Rocket Engine with a Mono- (Single-component)-propellant

Figure 1 represents a simplified liquid-propellant rocket engine. The subscripts 1 and 2 in Fig. 1 refer to the tank and the feed line, respectively.

Let V, F, v, and d denote the volume, the cross-sectional area, the velocity of the fluid, and the diameter of the propellant tank, respectively, and p_K denote the pressure in the combustion chamber.

The investigation is conducted without taking into account acoustic vibrations in the combustion chamber and without taking into account wave processes in the propellant circuit.

The monopropellant is forced out of tank 1 under constant pressure p_1 through feed pipe 2 and is injected through atomizer 3 into combustion chamber 4, where it burns and the combustion products are ejected outside through jet nozzle 5.

An oscillatory operating regime of the engine can develop in the following manner.

Let us assume that for some accidental reason the flow of propellant through the atomizer suddenly decreases. As a result, the discharge of combustion products will decrease, and therefore the pressure in the chamber will also decrease. The decrease in the pressure in the chamber leads to an increase in the propellant flow, to a corresponding increase in the discharge of combustion products, and then to an increase in the pressure in the chamber and a decrease in the propellant flow. Thus, the system returns to its initial state, and the oscillatory process, if it is sustained, can go on as long as desired. On the whole, the oscillation amplitudes may be constant or variable and may assume larger or smaller values. If they increase in time, the system becomes unstable. As a result, dangerous pressure fluctuations may develop in the chamber or the pipe and may cause destruction of the chamber.

Equation of Motion

In order to set up the differential equation of unsteady one-dimensional motion of a fluid flow in a fuel-supply system, we shall use the continuity equation and the theorem of kinetic-energy change.

For a finite fixed volume of material particles we can write

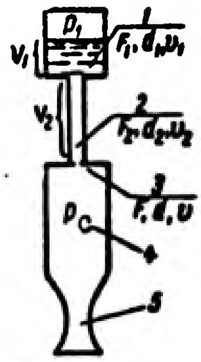

Fig. 1. Simplified scheme of LRE. operating on a mono-(single-component)-propellant. 1) tank, 2) feed pipe, 3) atomizer, 4) combustion chamber, 5) jet nozzle.

$$\left[\frac{d}{dt}\int_V \frac{\rho v^2}{2}\,dV\right]dt=\sum_{i=1}^{n}\delta A_i.$$

or

$$\left[\int_V \frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}\right)dV+\oint_S \frac{\rho v^2}{2}v_n\,dS\right]dt=\sum_{i=1}^{n}\delta A_i. \qquad \text{(A)}$$

where $\int_V \frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}\right)dV$ takes into account the change in kinetic energy inside the fixed volume;

$\oint_S \frac{\rho v^2}{2}v_n\,dS$ accounts for the change in kinetic energy on the surface S;

ρ is the density of the fluid and is assumed to be constant along the feed pipe.

In Fig. 1 there are two regions with constant cross sections.

Obviously, the kinetic energy inside the system corresponding to these two sections after a time interval dt can be represented in the form

$$\left[\int_V \frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}\right)dV\right]dt=\left[V_1\frac{\partial}{\partial t}\left(\frac{\rho v_1^2}{2}\right)+V_2\frac{\partial}{\partial t}\left(\frac{\rho v_2^2}{2}\right)\right]dt;$$

in which case we assume that the partial derivative $\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2}\right)$ at each point of the volume V_1 and V_2 is constant.

The change in kinetic energy on the surface (caused by the different velocities of discharge and intake of fluid particles on the surface) is written as:

$$\left[\oint_S \frac{\rho v^2}{2}v_n\,dS\right]dt=\left[\frac{\rho v^2}{2}Q-\frac{\rho v_1^2}{2}Q_1\right]dt,$$

where $Q_1 = v_1F_1$ and $Q = vF$ is the per-second volume flow of fluid through the tank and atomizer cross sections, respectively.

In Eq. (A) we can easily calculate the sum of the elementary work of all the forces applied to the fluid system (the work of the weight of the fluid is neglected):

the work of normal-pressure forces

$$\sum_{i=1}^{n} \delta A_i = \sum_{i=1}^{n} \bar{P}_i d\bar{l}_i = p_1 F_1 v_1\, dt - p_2 F v\, dt = p_1 Q_1\, dt - p_2 Q\, dt;$$

the work of friction forces

$$\sum_{i=1}^{n} \delta A_i = \sum_{i=1}^{n} \bar{R}_i d\bar{l}_i = -R_1 v_1\, dt - R_2 v_2\, dt - Rv\, dt,$$

where $\bar{P}_1 = \bar{p}_1 F_1$, and $\bar{d}l_1$ are elementary displacements.

Taking into account that the total friction force during the movement of a fluid through a pipe is determined from the well-known formula*

$$R = \zeta \frac{\rho v^2}{2} \frac{l}{d} F,$$

we obtain

$$\sum_{i=1}^{n} \delta A_i = -\zeta \frac{\rho v_1^2}{2} \frac{l_1}{d_1} F_1\, dt - \zeta \frac{\rho v_2^2}{2} \frac{l_2}{d_2} F_2\, dt - \zeta \frac{\rho v^2}{2} \frac{l}{d} F\, dt \approx -\zeta \frac{\rho v_2^2}{2} \frac{l_2}{d_2} F_2,$$

where we have taken

$$\frac{v_1}{v_2} = \frac{F_2}{F_1} = \left(\frac{d_2}{d_1}\right)^2, \quad \frac{v}{v_2} = \frac{F_2}{F} = \left(\frac{d_2}{d}\right)^2$$

and it is assumed that $l_2 \gg l_1$, $l_2 \gg l$ and $d_1 \gg d_2$, $d_2 \approx d$.

Now, taking into account all of the formulas obtained, Eq. (A) can be represented in the form

$$\left[V_1 \frac{\partial}{\partial t}\left(\frac{\rho v_1^2}{2}\right) + V_2 \frac{\partial}{\partial t}\left(\frac{\rho v_2^2}{2}\right)\right] dt + \left[\frac{\rho v^2}{2} Q - \frac{\rho v_1^2}{2} Q_1\right] dt = p_1 Q_1\, dt - p_2 Q\, dt - \zeta \frac{\rho v_2^2}{2} \frac{l_2}{d_2} F_2\, dt.$$

Cancelling out by the time dt and taking into account that

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*In the general case when local hydraulic losses occur, the value of R is determined as $R = \lambda(\rho v^2/2) F$. Here λ is the total coefficient of the losses in the propellant circuit and is related to the given impact pressure.

$V_1 = l_1F_1$ and $V_2 = l_2F_2$, we obtain

$$l_1F_1\rho v_1\frac{dv_1}{dt}+l_2F_2\rho v_2\frac{dv_2}{dt}+\frac{\rho v^2}{2}Q-\frac{\rho v_1^2}{2}Q_1=$$
$$=p_1Q_1-p_eQ-\zeta\frac{\rho v_2^3}{2}\frac{l_2}{d_2}F_2.$$

In view of the validity of the continuity equation $Q_1 = Q_2 = Q = F_1v_1 = F_2v_2$ for an incompressible fluid, we can cancel out the per-second volume flows; then the energy equation of an unsteady one-dimensional fluid flow may finally be written as:

$$\frac{\rho v^2}{2}-\frac{\rho v_1^2}{2}+l_1\rho\frac{dv_1}{dt}+l_2\rho\frac{dv_2}{dt}=p_1-p_e-\zeta\frac{\rho v_2^2}{2}\frac{l_2}{d_2}$$

or

$$p_1-p_e=\frac{\rho v^2}{2}-\frac{\rho v_1^2}{2}+l_1\rho\frac{dv_1}{dt}+l_2\rho\frac{dv_2}{dt}+\zeta\frac{\rho v_2^2}{2}\frac{l_2}{d_2}. \qquad (B)$$

From the continuity equations for an incompressible fluid it follows that $v = v_1(d_1/d)^2 = v_2(d_2/d)^2$. Then Eq. (B) assumes the form

$$p_1-p_e=\frac{\rho v^2}{2}-\frac{\rho v^2}{2}\left(\frac{d}{d_1}\right)^4+l_1\rho\left(\frac{d}{d_1}\right)^2\frac{dv}{dt}+$$
$$+l_2\rho\left(\frac{d}{d_2}\right)^2\frac{dv}{dt}+\zeta\frac{\rho v^2}{2}\left(\frac{d}{d_2}\right)^4\frac{l_2}{d_2}$$

or

$$p_1-p_e=\frac{\rho v^2}{2}\left[1-\left(\frac{d}{d_1}\right)^4\right]+\left[l_1\left(\frac{d}{d_1}\right)^2+l_2\left(\frac{d}{d_2}\right)^2\right]\rho\frac{dv}{dt}+$$
$$+\zeta\frac{\rho v^2}{2}\left(\frac{d}{d_2}\right)^4\frac{l_2}{d_2}.$$

We shall assume that the velocity $\underline{v}$ differs from the velocity of a steady flow v_s by the amount v', i.e., $v = v_s + v'$.

Neglecting terms containing $(d/d_1)^4$ and v'^2, we obtain

$$p_1-p_e=\left[l_1\left(\frac{d}{d_1}\right)^2+l_2\left(\frac{d}{d_2}\right)^2\right]\rho\frac{dv'}{dt}+$$
$$+\rho v_s\left[1+\zeta\left(\frac{d}{d_2}\right)^4\frac{l_2}{d_2}\right]v'+\frac{\rho v_s^2}{2}\left[1+\zeta\left(\frac{d}{d_2}\right)^4\frac{l_2}{d_2}\right].$$

Let us denote the quantity $l_1(d/d_1)^2 + l_2(d/d_2)^2$ by $l'(d/d_2)^2$, then

$$p_1-p_e=\rho l'\left(\frac{d}{d_2}\right)^2\frac{dv'}{dt}+\rho v_s\left[1+\zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]v'+$$
$$+\frac{\rho v_s^2}{2}\left[1+\zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]. \qquad (1)$$

Assuming that a change in velocity causes a change in pressure, the dependence between them can be represented in the form of a Taylor series; confining ourselves to terms of the first order of smallness and expanding $p_c(v_s + v'_\alpha)$ in powers of v'_α, we obtain (in the case of small oscillations)

$$p_c = p_{c.s.} + \left(\frac{\partial p_c}{\partial v}\right)_s v'_\alpha = p_{c.s}\left[1 + \left(\frac{\partial p_c}{\partial v}\right)_s \frac{v_s}{p_{c.s.}} \frac{v'_\alpha}{v_s}\right] = p_{c.s}\left(1 + K\frac{v'_\alpha}{v_s}\right). \qquad (2)$$

where $p_{c.s}$ is the pressure in the chamber during a steady regime;

K is the constant reflecting the dependence of the pressure in the chamber on the velocity of the burned-up propellant; it is assumed that the time that the gases remain in the combustion chamber is negligible in comparison with the gasification lag α;

α is the gasification lag (the time from the moment that a portion of propellant enters the chamber until it is converted into combustion products).

The sign v'_α designates v' calculated for the time $t - \alpha$. The subscript "s" refers to the steady-state process. Substituting Eq. (2) in (1), we obtain

$$p_1 - p_{c.s.} = Kp_{c.s}\frac{v'_\alpha}{v_{c.s.}} + \rho l'\left(\frac{d}{d_2}\right)^2\frac{dv'}{dt} + \rho v_s\left[1 + \zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]v' + \frac{\rho v_s}{2}\left[1 + \zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]. \qquad (3)$$

But for a stable flow $v' = 0$, $v'_\alpha = 0$, $dv'/dt = 0$.

Consequently, from Eq. (3) we find that

$$p_1 - p_{c.s.} = \frac{\rho v_s^2}{2}\left[1 + \zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]. \qquad (4)$$

Using Eq. (4), from Eq. (3) we obtain

$$\frac{dv'}{dt} + \frac{v_s}{l'}\left(\frac{d_2}{d}\right)^2\left[1 + \zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right]v' = -\frac{K}{\rho l'}\left(\frac{d_2}{d}\right)^2\frac{p_{c.s}}{v_s}v'_\alpha$$

or

$$\frac{dv'}{dt} + Ev' = -Cv_a', \tag{5}$$

where

$$E = \frac{v_2}{l'}\left(\frac{d_2}{d}\right)^2\left[1 + \zeta\frac{l_2}{d_2}\left(\frac{d}{d_2}\right)^4\right] > 0;$$

$$C = \frac{K}{\rho l'}\left(\frac{d_1}{d}\right)^2\frac{p_{k0}}{v_0} > 0;$$

here we have taken $K > 0$.

Equation (5) thus obtained is a homogeneous differential equation of the first degree with a delayed argument α for a single-line system.

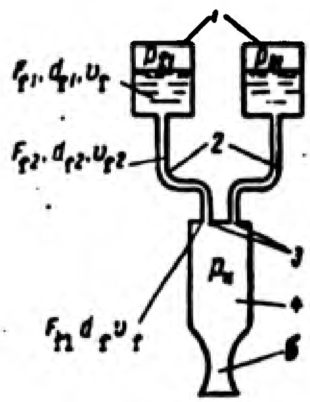

Fig. 2. LRE scheme operating on a two-component propellant. 1) tanks containing fuel and oxidizer, 2) feed pipes, 3) atomizers, 4) combustion chamber, 5) jet nozzle.

Let us extend the results obtained to a double-line system, where the fuel and oxidizer are supplied separately (Fig. 2).

LRE with a double-line propellant-supply system

Figure 2 presents an ordinary double-line system. We shall use the same designations as before. The index "f" will correspond to the parameters of the fuel-supply line, the index "0" to those of the oxidizer-supply line.

We shall assume that the lag time α for both main lines is identical, i.e.,

$$\alpha_o = \alpha_f = \alpha.$$

From a consideration of the energy, as before, we find:

for the fuel

$$p_{f1} - p_{c.s.} = \frac{1}{2}\rho_f v_{f.s.}^2\left[1+\zeta_f\frac{l_{f2}}{d_{f2}}\left(\frac{d_f}{d_{f2}}\right)^4\right]; \tag{6a}$$

for the oxidizer

$$p_{o1} - p_{c.s.} = \frac{1}{2}\rho_o v_{o.s.}^2\left[1+\zeta_o\frac{l_{o2}}{d_{o2}}\left(\frac{d_o}{d_{o2}}\right)^4\right]. \tag{6b}$$

These equations, (6a) and (6b), completely correspond to Eq. (4).

As before, let us expand the pressure p_c in a Taylor series, but now for a function of two variables:

$$p_c = p_{c.s.}\left(1 + K_f\frac{v'_{cf}}{v_{f.s.}} + K_o\frac{v'_{co}}{v_{o.s.}}\right). \tag{7}$$

Then for any moment of time Eq. (1) for the fuel is written as:

$$p_{f1} - p_{c.s.}\left(1 + K_f\frac{v'_{cf}}{v_{f.s.}} + K_o\frac{v'_{co}}{v_{o.s.}}\right) = \rho_f l'_f\left(\frac{d_f}{d_{f2}}\right)^2\frac{dv'_f}{dt} +$$
$$+ \rho_f v_{f.s.}\left[1+\zeta_f\frac{l_{f2}}{d_{f2}}\left(\frac{d_f}{d_{f2}}\right)^4\right]v'_f + \frac{\rho_f v_{f.s.}^2}{2}\left[1+\zeta_f\frac{l_{f2}}{d_{f2}}\left(\frac{d_f}{d_{f2}}\right)^4\right].$$

Thus, taking into account Equalities (6a) and (6b), we obtain for the fuel the equation

$$\frac{dv'_f}{dt} + E_f v'_f = -C_f v'_{cf} - L_f v'_{co}, \tag{8a}$$

where

$$E_f = \frac{v_{f.s.}}{l'_f}\left(\frac{d_{f2}}{d_f}\right)^2\left[1+\zeta_f\frac{l_f}{d_{f2}}\left(\frac{d_f}{d_{f2}}\right)^4\right];$$
$$C_f = \frac{K_f}{\rho_f l'_f}\left(\frac{d_{f2}}{d_f}\right)^2\frac{p_{c.s.}}{v_{f.s.}};$$
$$L_f = \frac{K_o}{\rho_f l'_f}\left(\frac{d_{f2}}{d_f}\right)^2\frac{p_{c.s.}}{v_{o.s.}}.$$

Changing subscripts, we obtain the equation for the oxidizer

$$\frac{dv_o'}{dt}+E_o v_o' = -C_o v'_{\alpha o} - L_o v'_{\alpha f}, \qquad (8b)$$

where

$$E_o=\frac{v_{o0}}{l_o'}\left(\frac{d_{o2}}{d_o}\right)^2\left[1+\zeta_o\frac{l_o}{d_{o2}}\left(\frac{d_o}{d_{o2}}\right)^4\right];$$

$$C_o=\frac{K_o}{\rho_o l_o'}\left(\frac{d_{o2}}{d_o}\right)^2\frac{p_{c.s.}}{v_{o.s.}};$$

$$L_o=\frac{K_f}{\rho_o l_o'}\left(\frac{d_{o2}}{d_o}\right)^2\frac{p_{c.s.}}{v_{f.s.}}$$

These equations, (8a) and (8b), correspond to Eq. (5) of a single line system.

From Eq. (7), it is obvious that

$$K_f=\frac{v_{f.s}}{p_{c.s}}\left(\frac{\partial p_c}{\partial v_f}\right)_s;$$

$$K_o=\frac{v_{o.s}}{p_{c.s}}\left(\frac{\partial p_c}{\partial v_o}\right)_s.$$

Thus, we obtain two equations, (8a) and (8b), of the first order with constant coefficients of the disturbances v_f' and v_o', but with delayed arguments, these equations being reducible to one differential equation of the second order in the following manner.

As in the article by D. Gander and D. Friant [1], let us differentiate Eq. (8a) with respect to t:

$$\frac{d^2v_f'}{dt^2}+E_f\frac{dv_f'}{dt}=-C_f\frac{dv'_{\alpha f}}{dt}-L_f\frac{dv'_{\alpha o}}{dt}. \qquad (9)$$

Let us write Eqs. (8a) and (8b) for the moment of time $t_1 = t-2\alpha$ (i.e., we shall delay the argument still more by the amount α):

$$\frac{dv'_{\alpha f}}{dt}=-E_f v'_{\alpha f}-C_f v'_{2\alpha f}-L_f v'_{2\alpha o}; \qquad (10a)$$

$$\frac{dv'_{\alpha o}}{dt}=-E_o v'_{\alpha o}-C_o v'_{2\alpha o}-L_o v'_{2\alpha f}, \qquad (10b)$$

in which $v'_{2\alpha f}$ plays the role of the velocity v_f', related to the period $t-2\alpha$.

From Eq. (10a) we have

$$v'_{2\alpha o}=-\frac{1}{L_f}\left(\frac{dv'_{\alpha f}}{dt}+E_f v'_{\alpha f}+C_f v'_{2\alpha f}\right) \qquad (11)$$

and from (8a)

$$v'_{\alpha o}=-\frac{1}{L_f}\left(\frac{dv_f'}{dt}+E_f v_f'+C_f v'_{\alpha f}\right). \qquad (12)$$

Let us substitute Eqs. (11) and (12) in Eq. (10b):

$$L_f\frac{dv'_{\alpha o}}{dt}=E_o\left(\frac{dv_f'}{dt}+E_f v_f'+C_f v'_{\alpha f}\right)+ \qquad (13)$$

$$+C_o\left(\frac{dv'_{\alpha f}}{dt}+E_f v'_{\alpha f}+C_f v'_{2\alpha f}\right)-L_f L_o v'_{2\alpha f}.$$

Let us substitute Eq. (13) in Eq. (9):

$$\frac{d^2v'_f}{dt^2}+E_f\frac{dv'_f}{dt}=-C_f\frac{dv'_{of}}{dt}-E_o\left(\frac{dv'_f}{dt}+E_fv'_f+C_fv'_{of}\right)-$$
$$-C_o\left(\frac{dv'_{of}}{dt}+E_fv'_{of}+C_fv_{2of}\right)+L_fL_ov'_{2of}$$

here there is no velocity with the subscript "0".

Grouping the terms, we obtain

$$\frac{d^2v'_f}{dt^2}+(E_f+E_o)\frac{dv'_f}{dt}+E_oE_fv'_f=-(C_f+C_o)\frac{dv'_{of}}{dt}-$$
$$-(E_fC_o+E_oC_f)v'_{of}-(C_fC_o-L_fL_o)v'_{2of} \qquad (14)$$

and since $C_fC_o - L_fL_o = 0$ (which is easily verified, if we substitute the expressions for each term of the equation), then

$$\frac{d^2v'_f}{dt^2}+(E_f+E_o)\frac{dv'_f}{dt}+E_oE_fv'_f=$$
$$=-(C_f+C_o)\frac{dv'_{of}}{dt}-(E_fC_o+E_oC_f)v'_{of}$$

or

$$\frac{d^2v'_f}{dt^2}+\mu\frac{dv'_f}{dt}+\nu v'_f+\sigma\frac{dv'_{of}}{dt}+\tau v'_{of}=0, \qquad (15)$$

where

$$\mu=E_f+E_o>0;\quad \sigma=C_f+C_o;$$
$$\nu=E_fE_o>0;\quad \tau=E_fC_o+E_oC_f.$$

Equation (15) is a differential equation of the second order with a delayed argument, in which case the argument does not figure in the highest derivative.

Investigation of the Equation of Disturbed Motion

A necessary condition for stability of the system is that the differential equation obtained (15) should have no partial solutions which increase infinitely in time.

These partial solutions can be made up of terms of the type

$v'_f = Ce^{\frac{zt}{a}}$; consequently

$$v'_{af} = v'_{t-a} = Ce^{\frac{z(t-a)}{a}} = Ce^{\frac{zt}{a}}e^{-z},$$

where the disturbances v'_f are taken in the interval $-\alpha < t < 0$.

After substituting these expressions for v'_f and $v'_{\alpha f}$ in Eq. (15), we obtain the following characteristic equation, which the value $\underline{z}$ must satisfy ($\underline{z}$ is the root of the characteristic equation):

$$\frac{z^2}{a^2} + \mu\frac{z}{a} + \nu + \sigma\frac{z}{a}e^{-z} + \tau e^{-z} = 0$$

or, multiplying by α^2, we obtain

$$z^2 + Mz + N + e^{-z}(Sz + T) = 0, \tag{16}$$

where $M = a\mu > 0;\ N = a^2\nu > 0;\ S = a\sigma > 0;\ T = a^2\tau > 0.$

The expressions for M, N, S, and T can be written differently. Let us introduce the notations

$$B_f = aE_f;\quad B_o = aE_o;\quad H_f = aC_f;\quad H_o = aC_o.$$

Using (8a) and (8b), let us write

$$\left.\begin{aligned}
B_f &= v_{f.s.}\frac{a}{l_f'}\left(\frac{d_{f2}}{d_f}\right)^2\left[1 + \zeta_f\frac{l_f}{d_{f2}}\left(\frac{d_f}{d_{f2}}\right)^4\right];\\
B_o &= v_{o.s.}\frac{a}{l_o'}\left(\frac{d_{o2}}{d_o}\right)^2\left[1 + \zeta_o\frac{l_o}{d_{o2}}\left(\frac{d_o}{d_{o2}}\right)^4\right];\\
H_f &= \frac{K_f}{\rho_f}\frac{a}{l_f'}\left(\frac{d_{f2}}{d_f}\right)^2\frac{p_{c.s.}}{v_{f.s.}};\\
H_o &= \frac{K_o}{\rho_o}\frac{a}{l_o'}\left(\frac{d_{o2}}{d_o}\right)^2\frac{p_{c.s.}}{v_{o.s.}}
\end{aligned}\right\} \tag{17}$$

or else, in accordance with Eqs. (6) and (17)

$$\left.\begin{aligned}
B_f &= 2\frac{gaF_{f2}}{G_{f.s.}l_f'}\Delta p_f; & B_o &= 2\frac{gaF_{o2}}{G_{o.s.}l_o'}\Delta p_o;\\
H_f &= K_f\frac{gaF_{f2}}{G_{f.s.}l_f'}p_{c.s.}; & H_o &= K_o\frac{gaF_{o2}}{G_{o.s.}l_o'}p_{c.s.}
\end{aligned}\right\} \tag{18}$$

where $G_{f.s.} = g\rho_f v_{f.s.}F_f$ is the per-second weight flow of fuel;

$G_{o.s.} = g\rho_o v_{o.s.}F_o$ is the per-seond weight flow of oxidizer;

$\Delta p_o = p_{o1} - p_{c.s.}$ is the pressure drop on the oxidizer atomizer;

$\Delta p_f = p_{f1} - p_{c.s.}$ is the pressure drop on the fuel atomizer;

$$K_f = \frac{G_{f.s}}{p_{c.s}}\frac{\partial p_c}{\partial G_f}; \quad K_o = \frac{G_{o.s}}{p_{c.s}}\frac{\partial p_c}{\partial G_o}.$$

Finally, we obtain

$$\left.\begin{aligned} M &= B_f + B_o; \quad S = H_f + H_o; \\ N &= B_f B_o; \quad T = B_f H_o + B_o H_f. \end{aligned}\right\} \tag{19}$$

For the solution of Equation (16) let us assume $z = x + iy$; then the expression for v_f' will assume the form

$$v_f' = Ce^{\frac{x+iy}{a}t} = Ce^{\frac{x}{a}t}e^{\frac{iy}{a}t} = Ce^{\frac{x}{a}t}\left(\cos\frac{y}{a}t + i\sin\frac{y}{a}t\right).$$

Thus it is obvious that three cases of oscillations are possible:

1) $x/a > 0$ - the oscillations of the system increase infinitely (the disturbances v_f' increase), and consequently the system is unstable;

2) $x/a < 0$ - the oscillations are damped; the system is stable;

3) $x/a = 0$ - there occur periodic oscillations, which we shall not investigate in this article, in view of their complexity, since, owing to insignificant disturbances, a transfer of the root of Eq. (16) from the imaginary axis to the left or right half-plane is possible.

Examining the three cases of oscillations presented above, we can conclude that if Eq. (16) has a root $\underline{z}$ with a positive real part, then the system is unstable; if $\underline{z}$ has a negative real part, then the system is stable.

Thus, for stability it is necessary that Eq. (16) have no roots in the right half-plane.

Replacing $\underline{z}$ by the number iz and expressing e^{iz} in terms of cot $z/2$, let us transform Eq. (16).

According to Euler's formulas

$$\cos\frac{z}{2} = \frac{e^{i\frac{z}{2}} + e^{-i\frac{z}{2}}}{2}$$

and

$$\sin\frac{z}{2}=\frac{e^{i\frac{z}{2}}-e^{-i\frac{z}{2}}}{2i},$$

therefore

$$\cot\frac{z}{2}=i\frac{e^{i\frac{z}{2}}+e^{-i\frac{z}{2}}}{e^{i\frac{z}{2}}-e^{-i\frac{z}{2}}}\frac{e^{i\frac{z}{2}}}{e^{i\frac{z}{2}}}=i\frac{e^{iz}+1}{e^{iz}-1},$$

whence

$$e^{iz}=\frac{\cot\frac{z}{2}+i}{\cot\frac{z}{2}-i}.$$

After replacing z by the number iz, let us write Eq. (16) in the form

$$-z^2+Miz+N+(Siz+T)\,e^{-iz}=0.$$

Expressing e^{iz} in terms of the cotangent, we find that

$$\cot\frac{z}{2}+i\frac{(N-z^2-T)+i(M-S)z}{(N-z^2+T)+iz(M+S)}=0$$

or

$$\cot\frac{z}{2}+Q(z)\equiv F(z)=0, \qquad (20a)$$

where

$$Q(z)=i\frac{(N-z^2-T)+i(M-S)z}{(N-z^2+T)+i(M+S)z}.$$

Separating the real and imaginary parts of the function Q(z), we obtain

$$Q(z)=\varphi(z)+i\psi(z),$$

where

$$\varphi(z)=\frac{2z\,[TM-S(N-z^2)]}{(N-z^2+T)^2+z^2(M+S)^2},$$

$$\psi(z)=\frac{(N-z^2)^2-T^2+z^2(M^2-S^2)}{(N-z^2+T)^2+z^2(M+S)^2}.$$

The functions $\varphi(z)$ and $\psi(z)$ are rational functions assuming real values for real values of z.

Equation (16) can now be represented in the form

$$\cot\frac{z}{2}+\varphi(z)+i\psi(z)\equiv F(z)=0. \qquad (20b)$$

When z is replaced by the number iz, Eq. (16) is replaced by

Eq. (20b); in this case the roots located in the right half-plane pass respectively into the lower half-plane.

Thus the determination of the number of roots of Eq. (16) lying in the right half-plane, rational with respect to e^z and $\underline{z}$ (e^z enters only in the first power), reduces to a determination of the number of roots of Eq. (20b) lying in the lower half-plane, and therefore to ascertain the circumstances under which the system is stable reduces to the question: under what conditions does Eq. (20b) not have roots in the lower half-plane?

In order to determine the number of roots of Eq. (20b) lying in the lower half-plane, let us consider the contour depicted in Fig. 3. On the real axis the function $F(z)$ will have as poles the points where cot $z/2$ goes to infinity, i.e., the points $\pm 2\pi$, 4π, 6π, etc. The poles of the function cot $z/2$ are by-passed by this contour by means of semicircles of small radius δ.

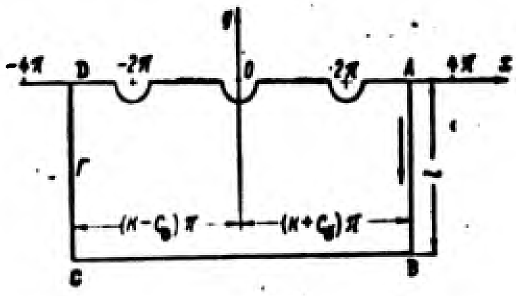

Fig. 3.

The number of roots of the function $F(z)$ located in the lower half-plane is equal to the limit to which the number of complete revolutions described by the vector representing the function $F(z)$ during a passage around the contour tends, when the numbers $\underline{l}$ and $\underline{k}$ tend to infinity and the radius δ of the semicircle tends to zero, plus the number of poles of the function F(z) lying in the lower half-plane.

As was first shown by I. V. Svirskiy [3], and then specified more accurately by Ye. M. Yesipovich [2], the formula for the calculation of the number of zeros of the function F(z) in the lower half-plane, appears as:

$$N = \frac{1}{2}(j - n) + m + P, \tag{21}$$

in this case, for stability of the system, Expression (21) should be equal to zero. In this formula

- N is the unknown number of zeros (for a stable system $N = 0$; for an unstable one $N \neq 0$);
- j is the order of the pole at an infinitely distant point of the rational part Q(z) of the function F(z);

$$Q(z) = \varphi(z) + i\psi(z) = \sum_{k=-\infty}^{j} a_k z^k,$$

in this case

- j is the number of intersections of the real axis and the vector F(z) when passing around contour G along ABCD (the number j is equal to the difference between the powers of the numerator and the denominator of the function Q(z); in our problem $j = 0$);
- n is the number of intersections of the real axis and the vector F(z) during movement from D to A along the rectilinear segments of contour G;

$$n = \sum_i \operatorname{sign}\left\{\frac{1}{R_e F(t)} \frac{d\,ImF(t)}{dt}\right\} = \sum_i \operatorname{sign}\left\{\frac{1}{\cot\frac{t}{2} + \varphi(t)} \frac{d\psi(t)}{dt}\right\}_{t=t_i},$$

where t_1 are the real roots of the equation $\psi(z) = 0$;

$$\varphi(z)=\varphi(t)=\frac{2t\,[S(N-t^2)-TM]}{(N-t^2+T)^2+t^2(M+S)^2};$$

$$\psi(z)=\psi(t)=\frac{(N-t^2)^2-T^2+t^2(M^2-S^2)}{(N-t^2+T)^2+t^2(M+S)^2};$$

<u>m</u> is the number of poles of cot z/2 at which $\psi(2k\pi) < 0$ (when passing in a semicircle around each such pole we obtain two intersections of the real axis, i.e., one complete revolution of the vector $F(z)$;

P is the number of poles of the function $Q(z)$ in the lower half-plane; it is equal to the number of roots of its denominator located in the lower half-plane.

Formula (21) is correct, when the function $Q(z)$ does not have zeros or poles on the real axis.

Let us carry out an investigation of the function $Q(z)$. The function $Q(z)$ on the real axis ($y = 0$) vanishes provided that:

$$\varphi(z)=\varphi(x)=\frac{2x\,[TM-S(N-x^2)]}{(N-x^2+T)^2+x^2(M+S)^2}=0; \tag{22}$$

$$\psi(z)=\psi(x)=\frac{(N-x^2)^2-T^2+x^2(M^2-S^2)}{(N-x^2+T)^2+x^2(M+S)^2}=0. \tag{23}$$

Simultaneous solution of Eqs. (22) and (23) gives the following conditions of applicability of Formula (21):

$$N \neq T \text{ и } (M^2-S^2)(T^2+NS^2-MTS) \neq 0.$$

Now let us show that the function $Q(z)$ does not have any poles on the real axis ($y = 0$). For this purpose, we shall equate the denominator of the function Q/z to zero and we shall solve the biquadratic equation

$$x^4+[(M+S)^2-2N]x^2+N^2+T^2=0,$$

whence we obtain

$$(x^2)_{1,2}=\frac{(M+S)^2}{2}-N\pm\sqrt{(M+S)^2\left[\frac{(M+S)^2}{4}-N\right]-T^2}.$$

It is difficult to tell from the appearance of the expression thus obtained whether the roots of the equation will have zero solutions. However, we note that the function Q/z on the real axis has poles in very rare cases.

Let us analyze another possible case, where the numerator and denominator of Q/z both vanish simultaneously. In this case $\lim_{x\to\infty} \varphi(x) = 0$ and $\lim_{x\to\infty} \psi(z) = 1$, and, consequently, the function Q/z does not have any poles when $z \to \infty$.

Let us determine the values P, m, and n, which enter into Expression (21).

1. Determination of the Number of Poles P in the Lower Half-plane of the Function F(z). As was shown in the article by I. V. Svirskiy, the function cot z/2 does not have any poles in the lower half-plane; therefore in Eq. (20a) only function Q/z is subject to investigation.

Let us write out the value of this function;

$$Q(z) = i\frac{(N - z^2 - T) + i(M - S)z}{(N - z^2 + T) + i(M + S)z}.$$

Let us see where the poles of this function are located. For this purpose let us equate to zero the denominator of the fraction $(N - z^2 + T) + i(M + S)z = 0$, whence

$$z = \frac{M+S}{2}i \pm \sqrt{N+T-\left(\frac{M+S}{2}\right)^2}. \tag{24}$$

Here two values of the root z are possible:

1) $N+T > \left(\frac{M+S}{2}\right)^2$; then the vectors z_1 and z_2 will be located in the upper half-plane and, consequently, there are no roots in the lower half-plane. Therefore, we shall consider the number of poles P located in the lower half-plane to be equal to zero;

2) $N+T<\left(\frac{M+S}{2}\right)^2$; then both terms of the expression for z are imaginary.

In this case, if the vectors z_1 and z_2 are located in the upper half-plane; then the number of poles P is equal to zero; if one of the vectors (z_1 or z_2) is located in the lower half-plane, then the number of poles P is equal to unity.

2. Determination of the value of m. In order to determine the real roots (t_1) of the function $\psi(t) = 0$, let us equate its numerator to zero; then we obtain the following biquadratic equation:

$$(N-t^2)^2-T^2+t^2(M^2-S^2)=0,$$

$$(t^2)_{1,2}=\frac{2N+S^2-M^2}{2}\pm\sqrt{\left(\frac{2N+S^2-M^2}{2}\right)^2+T^2-N^2}$$

or

$$\left.\begin{aligned} t_{11}&=+\sqrt{\frac{2N+S^2-M^2}{2}+\sqrt{\left(\frac{2N+S^2-M^2}{2}\right)^2+T^2-N^2}};\\ t_{21}&=+\sqrt{\frac{2N+S^2-M^2}{2}-\sqrt{\left(\frac{2N+S^2-M^2}{2}\right)^2+T^2-N^2}};\\ t_{12}&=-t_{11};\\ t_{22}&=-t_{21}.\end{aligned}\right\} \qquad (25)$$

It is possible to have cases where all four roots (25) will be real, or two roots will be real and two imaginary, or all roots will be imaginary.

Bearing in mind that m is the number of all poles of cot z/2 lying within intervals where the function $\psi(t)<0$, we obtain the following values of m for various cases:

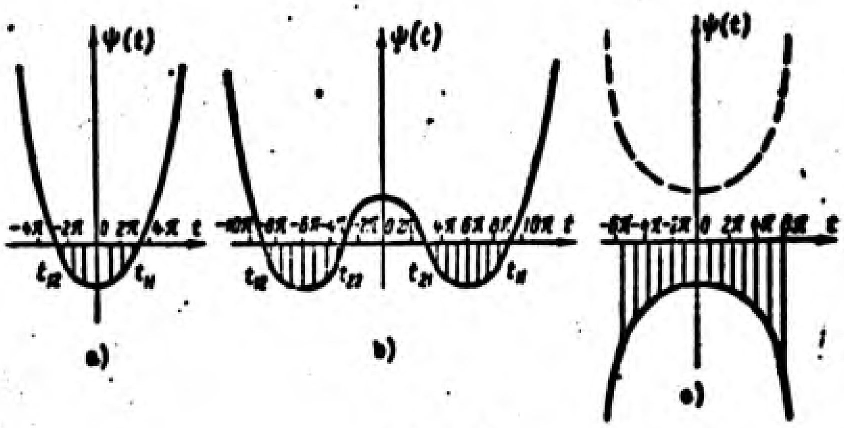

Fig. 4.

1) if the biquadratic equation has two real roots (see Fig. 4a), then

$$m=2\left[\frac{t_{11}}{2\pi}\right]+1; \tag{26a}$$

2) if the biquadratic equation has four real roots (see Fig. 4b), then

$$m=2\left[\frac{t_{11}-t_{21}}{2\pi}\right]; \tag{26b}$$

The sign of the integral part is [ ] (for example, $[3\frac{1}{2}] = 3$);

3) if all four roots of the biquadratic equation are imaginary, then it is necessary to see where the function $\psi(t)$ is located: if $\psi(t) > 0$, then $m = 0$ (see Fig. 4c, broken line) and from Eq. (23) by means of the substitution $x = t = 0$ we obtain

$$N^2 - T^2 > 0. \tag{26c}$$

If $\psi(t) < 0$, then $m = \infty$ (see Fig. 4c, solid line), and from Eq. (23) we obtain

$$N^2 - T^2 < 0 \tag{26d}$$

3. Determination of the value of $\underline{n}$. As was shown in the

article by I. V. Svirskiy,

$$n=\sum_i \operatorname{sign}\left\{\frac{\frac{d}{dt}\psi(t)}{\cot\frac{t}{2}+\varphi(t)}\right\}_{t=t_i},$$

where

$$\varphi(t)=\frac{2t[S(N-t^2)-TM]}{(N-t^2+T)^2+t^2(M+S)^2};$$

$$\psi(t)=\frac{(N-t^2)^2-T^2+t^2(M^2-S^2)}{(N-t^2+T)^2+t^2(M+S)^2}.$$

Consequently,

$$\frac{d}{dt}\psi(t)=2t\frac{\begin{gathered}[(N-t^2+T)^2+t^2(M+S)^2][(M^2-S^2)-2(N-t^2)]-\\-[(N-t^2)^2-T^2+t^2(M^2-S^2)][(M+S)^2-2(N-t^2+T)]\end{gathered}}{[(N-t^2+T)^2+t^2(M+S)^2]^2}=$$

$$=4t\frac{\begin{gathered}t^4(MS+S^2-T)+2t^2T(N+T)+\\+(N+T)[M^2T+MS(T-N)-N(S^2+T)-T^2]\end{gathered}}{[(N-t^2+T)^2+t^2(M+S)^2]^2}.$$

Sbubstituting $\varphi(t)$, $\psi(t)$, and $(d/dt)\ \psi(t)$ in the expression for n and discarding in the expression newly obtained after appropriate transformations the quantity

$$4\frac{(N-t^2+T)^2+t^2(M+S)^2}{[(N-t^2+T)^2+t^2(M+S)^2]^2},$$

which does not influence the sign change, we finally write

$$n=\sum_i \operatorname{sign}\left\{t\frac{\begin{gathered}t^4(MS+S^2-T)+2t^2T(N+T)+\\+(N+T)[M^2T+MS(T-N)-N(S^2+T)-T^2]\end{gathered}}{[(N-t^2+T)^2+t^2(M+S)^2]\cot\frac{t}{2}+2t[S(N-t^2)-TM]}\right\}_{t=t_i}. \tag{27}$$

From Formula (27) it is obvious that the number of sign changes can vary depending on the values of M, N, S, and T, and also on the number and values of the real roots (25).

Using the expressions for m and n and taking into account that in the case of our data j = 0, let us write in final form the criterion of stability of LRE (21) for the three cases:

1) the case where the biquadratic equation $(N-t^2)^2-T^2+t^2(M^2-S^2)=0$ has two real roots:

$$P+2\left[\frac{t_{11}}{2\pi}\right]+1-\frac{1}{2}\sum_{i}\operatorname{sign}\times \tag{28a}$$

$$\times\left\{t\frac{t^4(MS+S^2-T)+2t^2(N+T)T+ +(N+T)[M^2T+MS(T-N)-N(S^2+T)-T^2]}{[(N+T-t^2)^2+t^2(M+S)^2]\cot\frac{t}{2}+2t[S(N-t^2)-TM]}\right\}_{t=t_i}=0;$$

2) the case where this biquadratic equation has four real roots:

$$P+2\left[\frac{t_{11}-t_{21}}{2\pi}\right]-\frac{1}{2}\sum_{i}\operatorname{sign}\times \tag{28b}$$

$$\times\left\{t\frac{t^4(MS+S^2-T)+2t^2(N+T)T+ +(N+T)[M^2T+MS(T-N)-N(S^2+T)-T^2]}{[(N+T-t^2)^2+t^2(M+S)^2]\cot\frac{t}{2}+2t[S(N-t^2)-TM]}\right\}_{t=t_i}=0;$$

3) the case where all four roots of this biquadratic equation are imaginary (in this case Expression (27) assumes the value $n = 0$).

If $\psi(t) > 0$, then $m = 0$, and for stability of the system the conditions

$$\left.\begin{aligned} N^2-T^2>0; \\ P=0 \end{aligned}\right\} \tag{28c}$$

must be fulfilled; (if $\psi(t) < 0$, then $m = \infty$ and $N^2 - T^2 < 0$; the system is unstable).

Here

- t_i are the real roots of the biquadratic equation and are determined from Formula (25);
- P is the number of poles and is equal to the number of roots with negative coefficients in the imaginary part, these poles being determined from Formula (24);
- M, N, S, T are quantities entering into Expressions (19) and are determined by the structural parameters and also by the physics of the combustion process α.

Examples. Let us consider the following example (data borrowed from the article by D. Gander and D. Friant [1]):

$$
\begin{aligned}
&a = 8 \cdot 10^{-3}\ \text{sec}; && K_o = 0{,}6; \\
&F_{g2} = 0{,}389\ \text{cm}^2; && l_g = 61\ \text{cm}, \\
&F_{o2} = 0{,}66\ \text{cm}^2; && l_o = 61\ \text{cm}; \\
&\Delta p_g = 3{,}557\ \text{kg/cm}^2; && p_{o.c.} = 35{,}57\ \text{kg/cm}^2; \\
&\Delta p_o = 3{,}557\ \text{kg/cm}^2; && G_g = 0{,}1656\ \text{kg/sec}, \\
&K_g = 0{,}4; && G_o = 0{,}28804\ \text{kg/sec}.
\end{aligned}
$$

From Expressions (18) we obtain

$$
B_g = 2{,}13; \quad H_g = 4{,}26;
$$
$$
B_o = 2{,}08; \quad H_o = 6{,}24.
$$

Consequently, on the basis of (19), we have

$$
M = B_g + B_o = 4{,}21; \qquad S = H_g + H_o = 10{,}5;
$$
$$
N = B_g B_o = 4{,}43; \qquad T = B_g H_o + B_o H_g = 22{,}2.
$$

From Formula (24) we find that $z = 7.36i \pm 5.26i$, whence $z_1 = 12.62i$ and $z_2 = 2.1i$, i.e., both roots are located in the upper half-plane; therefore

$$
P = 0.
$$

From Formulas (25) we determine

$$
t_{11} = 15{,}3; \quad t_{21} = 5{,}66i;
$$
$$
t_{12} = -15{,}3; \quad t_{22} = -5{,}66i.
$$

Since we have obtained two real roots, we shall use Eq. (28a), for which we calculate

$$
2\left[\frac{t_{11}}{2\pi}\right] = 2\,[2{,}44] = 2 \cdot 2 = 4,
$$

when $t_{11} = 15.3$

$$
\operatorname{sign} \frac{t_{11}\,[t_{11}^4(-88{,}15) + 1184 t_{11}^2 + 2663]}{[(26{,}63 - t_{11}^2)^2 + 21{,}7 t_{11}^2] \cot \frac{t_{11}}{2} + 2 t_{11}\,[10{,}5\,(4{,}43 - t_{11}^2) - 93{,}5]} =
$$
$$
= \operatorname{sign} \frac{15{,}3\,[-4\,555\,000]}{48\,090 \cot 23°21' - 74\,900} =
$$
$$
= \operatorname{sign} \frac{-696 \cdot 10^6}{111\,300 - 74\,900} = \operatorname{sign}\,[-19\,320] = -1
$$

and when $t_{12} = -15.3$

$$\operatorname{sign}\frac{t_{12}[t_{12}^4(-88{,}15)+1184t_{12}^2+2663]}{[(26{,}63-t_{12}^2)^2+21{,}7t_{12}^2]\cot\frac{t_{12}}{2}+2t_{12}[10{,}5(4{,}43-t_{12}^2)-93{,}5]}=$$

$$=\operatorname{sign}\frac{696\cdot 10^{6}}{-111300+74900}=\operatorname{sign}(-19320)=-1.$$

The sum of the signs (Σ sign) is equal to $[-1 + (-1)] = -2$.

Substituting all the values found into Eq. (28a), we obtain $6 \neq 0$, i.e., the stability criterion is not fulfilled; therefore the given system is unstable.

Let us change the system parameters in the following manner (leaving the others unchanged):

$$F_{g2}=0{,}162 \text{ cm}^2;\quad \Delta p_g=22{,}032 \text{ kg/cm}^2;$$
$$F_{o2}=0{,}274 \text{ cm}^2;\quad \Delta p_o=22{,}032 \text{ kg/cm}^2;$$

then

$$B_g=5{,}48;\quad H_g=1{,}768;$$
$$B_o=5{,}35;\quad H_o=2{,}59,$$

whence

$$M=10{,}83;\quad S=4{,}36;$$
$$N=29{,}3;\quad T=23{,}7.$$

Moreover,

$P = 0$, since $z = 14.61 \pm 12.61$

(whence $z_1 = 27.21$ and $z_2 = 21$)

$$t_{11}=3{,}3i;\quad t_{21}=5{,}5i;$$
$$t_{12}=-3{,}3i;\quad t_{22}=-5{,}5i.$$

All of the roots prove to be imaginary. According to (28c), we obtain $N^2 - T^2 = 29.3^2 - 23.7^2 > 0$, i.e. $\psi(t) > 0$, which means that $m = 0$.

Therefore in Eq. (21) the quantities m, n and P are equal to zero. Consequently, the stability criterion (21) is fulfilled, and the system has a stable character.

CONCLUSIONS

1. The method presented can be used for determining the

criterion for operating stability of LRE with separate feeding of fuel and oxidizer.

2. If the equalities of Expressions (28 a, b, c) are fulfilled, then the system is stable only in the case of small deviations of the disturbances v' and provided that the following inequalities are maintained:

$$N \neq T;\ (M^2-S^2)(T^2+NS^2-MTS) \neq 0.$$

In the case where all four roots of Eqs. (25) are real, it is necessary to use Expression (28a); when two roots are real and two imaginary, use Expression (28b). In the case where all four roots are imaginary (in the case n = 0), it is necessary to see where the function $\psi(t)$ is located.

If $\psi(t) > 0$, then m = 0, and for stability of the system it is necessary to fulfill the conditions $N^2 - T^2 > 0$ and P = 0.

If $\psi(t) < 0$, then $m = \infty$ and $N^2 - T^2 < 0$; in this case the system is unstable.

3. Examples illustrating the use of these criteria have been given.

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