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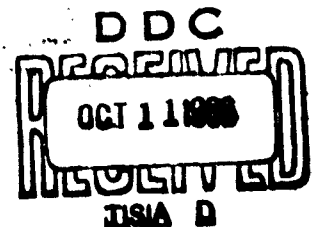CAMERON STATION, ALEXANDRIA, VIRGINIA

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THE ACOUSTIC IMPEDANCE OF A RIGID CIRCULAR PISTON WITHOUT A BAFFLE OR WITH A FINITE CONCENTRIC BAFFLE

BY

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ADMIRALTY RESEARCH LABORATORY

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ABSTRACT

An expression for the self-impedance is derived for any size
of baffle with piston sizes in the range $ka = 0.75 - 2$. The unbaffled
condition, as a particular case, provides a means of checking the expre-
ssion against the few existing results derived by more advanced methods
and good agreement is obtained. Certain sizes of finite baffle raise
the self resistance and reactance somewhat above the infinitely baffled
values.

INTRODUCTION

Values for the acoustic self-impedance of a rigid circular piston source in an infinite baffle are generally available in the literature. The method given here for the cases where the baffle is finite or entirely absent uses the above-mentioned values in conjunction with a principle of superposition. This assumes that the acoustic pressure experienced by the piston when infinitely baffled has two components; namely, the pressure it would experience if the baffle were absent, and that pressure which would result if the baffle were replaced by an infinite (non-uniform) source which at every point on it creates a pressure equal to that actually experienced there by the baffle. If this latter component is evaluated and deducted from the known values of pressure on the piston when infinitely baffled, then the unbaffled values of pressure, and thence self-impedance, will remain.

The replacement of the infinite baffle by a non-uniform source follows, of course, the method of treating problems of reflexion from a rigid wall, where the normal velocity is made to vanish by an appropriate source strength at each point. The present case is rather different in that, since the piston source and infinite baffle are coplanar, there is no incident wave in the sense of one having a normal velocity component. The strength of the fictitious source at each point is therefore determined by the (scalar) pressure occurring at the corresponding point on the infinite baffle.

The case of a finite baffle is treated similarly. Thus, if the finite baffle has outer radius b and inner radius a (the piston radius), the radiation pressure on the piston is assumed to be given by deducting from the infinitely-baffled value a component due to a non-uniform source whose inner radius is b and which extends to infinity. The procedure for dealing with a finite baffle does not take account of diffraction of the sound behind the baffle, nor the thickness of the baffle, and in this respect is an approximation. On the other hand, the application of the method is intended primarily to situations where limited baffling occurs by virtue of the presence of adjacent equipment, frameworks, structures and the like, none of which can usually be reasonably represented by a thin rigid baffle.

2.   THE VELOCITY POTENTIAL IN THE PLANE OF AN INFINITE BAFFLE
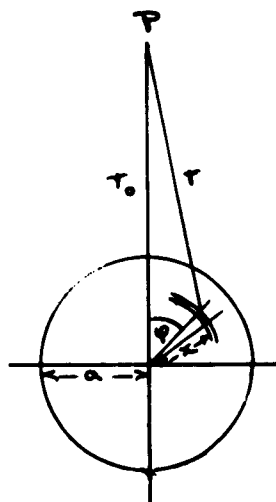
FIG. No. 1

Let ψ_1 be the velocity potential at a point P in the plane of the infinite baffle. Then if a is the piston radius and r_0 is the distance of P from the piston centre (see Fig. No. 1), the potential is given by –

$$\psi_1 = -\frac{u}{\pi} \int_0^a x \, dx \int_0^\pi e^{-ikr} \, d\varphi \qquad (1)$$

– where u is the vibrational velocity of the piston

Since $r = \left(r_0^2 + x^2 - 2 \, r_0 x \cos \varphi \right)^{\frac{1}{2}}$ the first integral in (1) can be evaluated by expressing the integrand as a series of Bessel and Legendre functions [1] –

$$e \frac{-i\kappa r}{r} = \frac{1}{i\,\kappa\,r_0 x} \sum_{n=0}^{a} (2n+1) \, \hat{J}_n(\kappa x) \left\{ \hat{J}_n(\kappa r_0) - i\, \hat{N}_n(\kappa r_0) \right\} P_n(\cos \varphi) \qquad (2)$$

- where $\hat{J}_n(x) = \left(\frac{\pi x}{2}\right)^{\frac{1}{2}} J_{n+\frac{1}{2}}(x)$ and $\hat{N}_n(x) = \left(\frac{\pi x}{2}\right)^{\frac{1}{2}} N_{n+\frac{1}{2}}(x)$

The double integral then becomes -

$$\int_0^a x\,dx \int_0^\pi e \frac{-i\kappa r}{r} = \frac{-1}{\kappa r_0} \sum_{n=0}^{\infty} (2n+1) \left\{ i\, \hat{J}_n(\kappa r_0) + \hat{N}_n(\kappa r_0) \right\} \int_0^a \hat{J}_n(\kappa x)\,dx \int_0^\pi P_n(\cos \varphi)\,d\varphi$$

This vanishes for all odd values of n . Also if κa does not exceed 2 there is very little error entailed by taking only the first two finite terms (i.e. $n = 0,2$), a device which greatly eases the integration problems met below in dealing with the non-uniform source.

$$\psi_1 = \frac{u}{\kappa^2 r_0} \left[\left\{ i\, \hat{J}_0(\kappa r_0) + \hat{N}_0(\kappa r_0) \right\} \int_0^{\kappa a} \hat{J}_0(x)\,dx + \frac{5}{4} \left\{ i\, \hat{J}_2(\kappa r_0) + \hat{N}_2(\kappa r_0) \right\} \int_0^{\kappa a} \hat{J}_2(x)\,dx \right]$$

$$(3)$$

since $\int_0^\pi P_0(\cos \varphi)\,d\varphi = \pi$, and $\int_0^\pi P_2(\cos \varphi)\,d\varphi = \frac{\pi}{4}$

3. THE NON-UNIFORM SOURCE

The infinite baffle is replaced entirely - or that part of it beyond some radius b, as explained above - by a non-uniform source. The source strength at any point is such as to balance the pressure actually experienced by the infinite baffle. If this is p_1 at any point then the required source velocity there can be given, under a limited range of conditions, fairly accurately by the plane-wave relation $v = p_1/\rho c$. The required velocity may be written in terms of ψ_1 ; $v = i\kappa \psi_1$. The range of conditions referred to obviously excludes any relatively rapid change of pressure along the baffle.

If then the potential at a point on the baffle due to the piston is ψ_1, the fictitious source velocity there is $i\kappa \psi_1$ and the velocity potential due in turn to this elementary source is -

$$\psi_{11} = \frac{- i\kappa \psi_1}{2\pi} \cdot e^{- \frac{i\kappa r}{r}} \, dA \qquad (4)$$

- where ψ_1 is expressed by (3), r is the distance from the elementary source and dA is its area. The pressure corresponding to (4) is then -

$$P_{11} = \frac{\rho \omega \kappa \psi_{1_2}}{2\pi} \cdot e^{\frac{-i\kappa r}{r}} \, dA \tag{5}$$

By integrating (5) over the piston, the force thereon due to the elementary source is obtained. thus -

$$df = \frac{\rho \pi \kappa \, dA}{\pi} \int_0^a x \, dx \int_0^\pi \psi_1 \cdot e^{\frac{-i\kappa r}{r}} \, d\varphi \tag{6}$$

referring again to Fig. 1, where P is now the elementary source. On account of the symmetry of the field about the piston axis, it is possible to consider the elementary source as an annulus having uniform source strength and whose radius and width are r_0 and dr_0 respectively (see Fig. 1). That is, $dA = 2\pi r_0 dr_0$ in (6) and a third integration will yield the force on the piston due to a non-uniform source corresponding to an infinite baffle whose inner radius is, say, b . Thus -

$$f = 2 \rho \omega \kappa \int_0^\infty r_0 dr_0 \int_0^a x \, dx \int_0^\pi \psi_1 \, e^{\frac{-i\kappa r}{r}} \, d\varphi \tag{7}$$

Introducing again the relationship expressed in (2)

$$\int_0^a x \, dx \int_0^\pi \psi_1 \, e^{-\frac{i\kappa r}{r}} \, d\varphi = \frac{\psi_1 \pi}{\kappa^2 r_0} \left[\left\{ i \, \hat{J}_0(\kappa r_0) + \hat{N}_0(\kappa r_0) \right\} \int_0^{\kappa a} \hat{\bar{J}}_0(z) \, dz + \frac{5}{4} \left\{ i \, \hat{J}_2(\kappa r_0) + \right. \right.$$

$$\left. \left. + \hat{N}_2(\kappa r_0) \right\} \int_0^{\kappa a} \hat{\bar{J}}_2(z) \, dz \right] =$$

$$= \frac{-\pi u}{\kappa^2 r_0^2} \left[\left\{ i \hat{J}_0(\kappa r_0) + \hat{N}_0(\kappa r_0) \right\} \int_0^{\kappa a} \hat{\bar{J}}_0(z) \, dz + \frac{5}{4} \left\{ i \hat{J}_2(\kappa r_0) + \hat{N}_2(\kappa r_0) \right\} \times \right.$$

$$\left. \times \int_0^{\kappa a} \hat{\bar{J}}_2(z) \, dz \right] \tag{8}$$

- where again only the first two finite terms in (2) are used. By discarding the term containing

$$\left(\int_0^{\kappa a} \hat{\bar{J}}_2(z) \, dz \right)^2 \quad ,$$

which is evidently a relatively very small quantity provided that
$\kappa a \neq 2$, the third integral in (7) is easily evaluated and the total
force on the piston due to the non-uniform source can be expressed as -

$$f = \frac{2\pi\rho cu}{\kappa^2} \left[\left(\int_0^{\kappa a} \hat{J}_0(z)\, dz \right)^2 \left\{ \int_{2\kappa b}^{\infty} \frac{\cos \upsilon}{\upsilon}\, d\upsilon - i\int_{2\kappa b}^{\infty} \frac{\sin \upsilon}{\upsilon}\, d\upsilon \right\} + \right.$$

$$\left. + \frac{2}{4}\int_0^{\kappa a} \hat{J}_0(z)\, dz \left\{ \left| -\frac{6\cos \upsilon}{\upsilon^2} \right|_{2\kappa b}^{\infty} - \int_{2\kappa b}^{\infty} \frac{\cos \upsilon}{\upsilon}\, d\upsilon + \left| i\,\frac{6\sin \upsilon}{\upsilon^2} \right|_{2\kappa b}^{\infty} + i\int_{2\kappa b}^{\infty} \frac{\sin \upsilon}{\upsilon}\, d\upsilon \right\} \right]$$

(9)

- where $\upsilon = 2\kappa r_0$. The integrals with respect to both υ and s are
readily obtainable from tables. It is perhaps worth emphasising that
this comparatively straightforward expression for a quantity normally
apprehended by much more sophisticated methods is made possible by
using the simple plane-wave pressure-velocity relation to obtain source
velocity. This accordingly restricts the use of the method to values
of κa not less than 0.75, the upper limit being 2, as indicated earlier.
Some justification for putting forward a method covering a limited range
of piston sizes is the fortunate circumstance that transducers currently
under development at A.R.L. are comfortably within this range. The
possibility of extending the range of κa values will be studied. At the
lower end, i.e. $\kappa a < 0.75$, the good approximation to a rigid piston given
by a hemispherical source might be exploited, whilst values exceeding 2
will involve the consideration of additional terms in the series (2).

There is, as far as the writer is aware, rather little reliable data
available with which the results of the present method may be compared.
The work of S. Hanish[2] provides values of self resistance and reactance
derived by much more advanced methods, but which apply only to the
completely unbaffled condition and then only for values of κa not exceed-
ing unity. The unbaffled condition is given in the present method by
putting $b = a$; that is, by making the lower limit of the integrals with
respect to υ equal to $2\kappa a$ in (9). For example, putting $\kappa a = 1, b = a$,
and dividing (9) by $\pi a^2 u\rho c$ to give specific impedance, and finally de-
ducting the result from the known infinitely-baffled self-impedance
value, the value for the unbaffled condition becomes $Z = 0.2372 + i\,0.6108$
as compared with $0.2322 + i\,0.5812$ by Hanish. Repeating this with
$\kappa a = 0.75$ gives $Z = 0.136 + i\,0.4633$ against Hanish's $0.1337 + i\,0.4512$.
Below $\kappa a = 0.75$ all resemblance between the present results and those of
Hanish rapidly disappears, the latter being evidently above suspicion if
only because at those sizes they should and do conform to the simple
spherical source equivalent.

Since the unbaffled condition is given here merely by using one
particular value of κb in a general expression, the agreement shown above
permits some confidence in the results for partial baffling. The latter
are shown graphically in Fig. 2 for $\kappa a = 0.75$, 1 and 2 and are in agreement

with some results given by Nimura and Watanabe[3] in showing that certain finite sizes of baffle produce values of self resistance and reactance greater than those obtained with an infinite baffle.

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PHGC/EH

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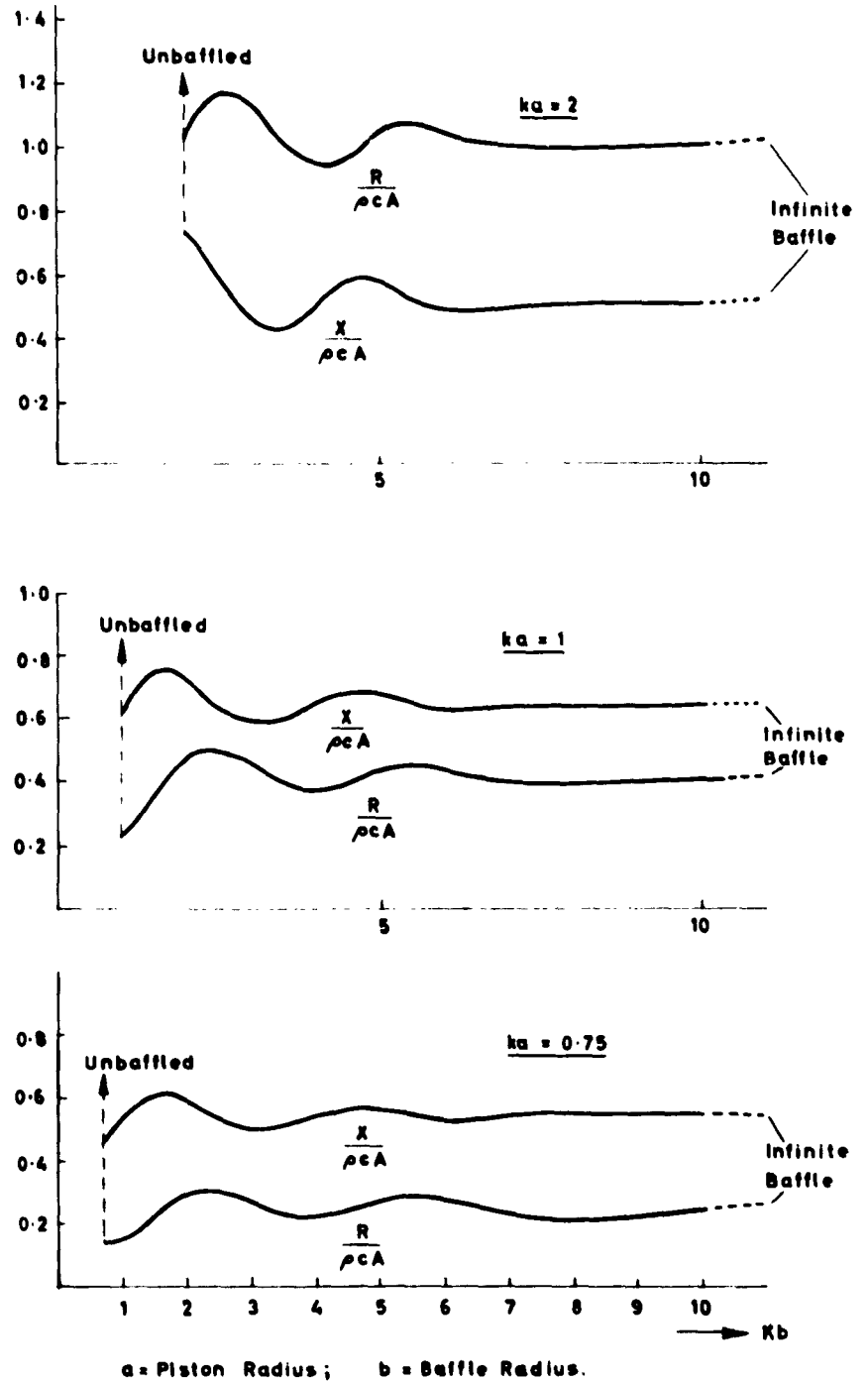3. NIMURA, T. and WATANABE, Y., Effect of a finite Circular Baffle Board on Acoustic Radiation, J.A.S.A. 25 76 (1953)

FIG. 2   VARIATION OF ACOUSTIC SELF
IMPEDANCE WITH BAFFLE RADIUS

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