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A DESCRIPTION OF THE CRITICAL PATH METHOD AND A PROGRAM WHICH APPLIES IT TO LARGE PROJECTS

BY

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ABSTRACT

The Critical Path Method is a technique for planning and scheduling projects. A schedule shows (1) how long a project will take to complete, (2) which activities, of the network constituting the project, are 'critical' in defining the time-length of the project, (3) when each activity can be started, and (4) the tolerance allowed in the starting time of each activity.

The Method is described here, and also a computer program, Pegacrit (4) written by Ferranti Ltd., which applies it to projects.

31 pages

INTRODUCTION

Industrial and scientific projects, in research and development or production, tend to become very large and complex in their structure, and the larger and more complex they become the more difficult it is to see, in precise terms, the relative importance of different activities within the overall pattern of events.

A relatively new technique for analysing, planning and scheduling large projects is the Critical Path Method. This is described in References 1, 2 and 3. Basically, this method determines which jobs or activities in a project are 'critical' in affecting the completion time of the whole project, and how best to schedule all jobs in the project in order to meet a target date with the minimum amount of 'lost' time, and thus at minimum cost.

The Critical Path Method can be applied to a great variety of projects and at different levels of planning, but each project is required to have several characteristics which are essential for Critical Path analysis:

(1) The project consists of a well-defined collection of jobs which, when completed, mark the end of the project.

(2) The jobs may be started and stopped independently of each other, within a given sequence.

(3) The jobs are semi-ordered, i.e. the jobs are arranged in series and parallel series directed from Start to Finish with no loops.

THE METHOD

Although it is not an essential part of the Method, a Project Graph consisting of a complete picture of all the jobs in a project is a valuable guide when drawing up the data for a Critical Path analysis, and will reduce the possibility of ambiguities occurring in sequence relationships of activities.

All the activities (or jobs) in the project should be listed together with their durations and descriptions, the activities being represented on the Graph by arrows. Since each activity is uniquely defined and independent it has a starting event (when the activity begins) and a finishing event (when the activity is completed), these events being represented on the Graph by circles (or nodes). Two consecutive activities will thus have a common node which will be the finishing event of one activity and the starting event of the following activity.

Sequence relationships must be strictly represented, and occasionally 'dummy' activities having zero time content will have to be inserted to make this possible.

The durations of the activities can be written by the corresponding arrows, and the nodes numbered to give identification to activities.

For convenience, all nodes with no predecessors are connected to a node labelled 'Start', and similarly all nodes with no successors are connected to a node labelled 'Finish'.

In this way a project is totally represented graphically.

Thus the graph consists of a number of different 'arrow-paths' from Start to Finish. The total time to complete each path is the sum of the durations of each activity on the path.

The critical path is the path which takes the longest time from Start to Finish. This time is then the shortest time in which the project can be completed.

Only by shortening the duration of 'critical' activities can the total project time be reduced; if there exists more than one critical path then each of these would have to be considered.

Usually about 10% of the jobs are critical; all-out efforts on all the jobs in a project are unnecessary and uneconomic. Of course, if the critical activities are shortened it is likely that the critical path would change its course to include activities which previously were near-critical; it would then be necessary to apply a further Critical Path analysis to the project.

DESCRIPTION OF THE PROGRAM - PEGACRIT (4)

This program, Pegacrit (4), has been written by Ferranti Ltd. for applying the Critical Path Method to large projects. It is written for use on a Pegasus computer, although there is also a program for the Sirius machine. Both programs have been arranged so that the same data tapes can be accepted by either computer. A description of how to use the program is contained in Reference 3.

The program requires a total account of all the activities in the project in terms of their starting and finishing events, durations, and descriptions. With this information an estimate can be made of the total time for each path through the network. The longest path is then the Critical Path.

The program then assesses for each activity in the project the earliest time it can be started and the earliest time it can be finished - taking into account the different paths from the start of the project to the activity, and assuming that all activities follow one another without delay, and similarly the latest time it can be started and the latest time it can be finished - considering the different paths, working backwards, from the terminal event to the activity, and allowing 'non-critical' activities to start as late as possible without delaying the whole project, i.e. without delaying any 'critical' activity.

These times are related as follows:-

For each activity:

early start + duration = early finish

late start + duration = late finish

early start = latest of the early finishes of all the preceding activities

late start = latest of the late finishes of all the preceding activities

And further, for each critical activity:

early start = late start

early finish = late finish.

Activities not on the critical path usually have different earliest and latest starting (or finishing) times, the difference being the total amount of time the activity can be delayed before interfering with the scheduled times of the critical activities, which would delay the project completion time. This delay time is called the Total Float available to the activity.

Two other kinds of float are computed by the program: Free Float and Independent Float.

The Free Float is the amount of time an activity can be delayed without delaying any following activity. It is defined as the difference between the activity's early finishing time and the early starting time of all its immediate successors.

The Independent Float is the amount of time available without affecting the schedules of following and preceding activities. It is defined as the difference between the earliest start time of immediate successors and the latest finishing time of predecessors, less the duration of the activity. This type of float is infrequent (i.e. infrequently a positive quantity).

The importance of floats is that they indicate clearly which activities deserve second priority to the critical activities.

The most important facility of the program is that amendments to the initial data can be made without the necessity to punch an entire new data tape. This facility is described in Reference 4. Amendments are of three types: changes in durations of activities, removals of activities, and insertions of new activities.

After a period of time a project can be 'updated' by making a series of amendments to the previous set of data. Activities which have been completed are removed, those which are in progress are given revised completion times, and new activities occurring are inserted. Event times are rescheduled by the program relative to the new starting event of the project which is then the current time of review.

The amendment facility makes the program very easy to run after the initial schedule has been computed. It will be seen from each new schedule whether the critical path consists of the same activities as before, or if new activities have become critical, in which case the project plan can be revised to include the new priorities.

PREPARATION OF THE DATA TAPES

Initially three data tapes are required, a numerical tape, a title tape, and an amendment tape.

The numerical tape contains a list of all the activities in the project, each activity being described by its starting event, finishing event, and duration. The activities can be listed in any order and need not be consecutive, with the one condition that activities having the same starting event must be grouped together.

The title tape supplements the numerical tape by containing the descriptions of all the activities. The descriptions must be arranged in the same order as their corresponding activities in the numerical tape.

The amendment tape contains a list of all the amendments necessary to the updating of the project. For the first schedule from the initial set of data, though there will be no amendments a tape must still be prepared as it is required for the running of the program. For each subsequent run the previous output tape is used as input, replacing the numerical and title tapes. The only work involved for the updating of the data is the preparation of an amendment tape.

Amendments are of three types, viz. changes of activity times, removals of activities, and insertions of new activities. Each amendment is written in the form of an activity (i.e. starting event S, finishing event F, and duration D) and is preceded by a code letter. Amendments are written as follows:

Change of time	:	original S and F and new D
Removal	:	original S, F and D
Insertion	:	new S, F and D followed by its description

The data is fed into the computer by inserting the numerical tape (or output tape) in Tape Reader 0 and the amendment tape in Tape Reader 1. The program reads the first amendment and compares it with successive activities on the numerical tape until the necessary agreement is found for the amendment to be made. Thus:-

(i) for a change of time or removal, the starting events and finishing events under both tape readers must be the same,

(ii) for an insertion, either the starting events or finishing events must be the same.

If an insertion is made, the next amendment is read and compared with the activity still in TR0, as above, otherwise the amendment is read and the program immediately begins searching the numerical tape for the next activity to be amended. Thus any number of insertions can be made at one point on the numerical tape (providing condition (ii) above holds).

The amendments for changes of time and removals of activities should be arranged in an order identical to that of the activities on the numerical tape (or previous output tape) to which they refer. New activities, to be inserted, are more difficult to place in the amendment tape as they may "split" a group of activities on the numerical tape having a common starting event; this could happen if the insertion is made by a comparison of finishing events. If this is the case and it is necessary to insert an activity in the middle of the group, then in order to preserve the group it will be necessary to insert at the beginning of the group all the activities (in the group) following the place of insertion, to insert the new activity and then remove all those which have been re-inserted at the beginning. An example of this is contained in the Amendment list to the Example Network in the Appendix.

Normally, an insertion involves just one amendment order, though it may sometimes be necessary to precede it by a 'pseudo-amendment' - a change-of-time amendment having the same time as the activity it refers to - to place the insertion correctly. For instance, if several consecutive activities have the same starting or finishing event as the one to be inserted, a pseudo-amendment to the activity immediately before the place for the new activity will enable the insertion to be made in the right order.

PUNCHING RULES

1. Both the numerical tape and title tape must be headed by the code sequence Jv1, then followed by the main title of the project. This title must not exceed 48 chs., including shifts, etc.

The tapes are headed thus:

CRLF λ J ϕ v1

CRLF λ N ϕ

CRLF (Title) ϕϕ ..... ϕ

and are then punched as follows:-

(a) Numerical tape

Each activity consists of three numbers, starting event, finishing event, and activity time. The activities are punched on separate lines and are grouped in pages of 100 each. Each page is headed by a page number on a separate line, and each line by a line number (1-100). The lines of data and page numbers must begin with CRLF and all numbers must be followed by a space.

A line of data is punched:

CRLF	line no.	sp	starting event	sp	finishing event	sp	activity time	sp

(b) <u>Title tape</u>

This tape contains the descriptions of all the activities and must be punched in the same order as the activities on the numerical tape. As on the numerical tape, the 'titles' are grouped in pages of 100 each, each page being headed by a page number, and each line by a line number (1-100). The line number is followed by a space, two dots, and the description, which should not exceed 24 characters (not including shifts). A line of data is punched:

CRLF	line no.	sp	..	(Title)	øø ..... ø

There must be <u>two</u> dots before the title and at least two figure-shifts after it (i.e. blank tape).

2. <u>Amendment tape</u>

The amendment tape is headed by a subsidiary title, referring to the set of data after it has been amended, which will be printed at the head of every page of output. The title must be followed by blank tape.

The amendments are punched in the order described in the previous section.

A line is punched:

CRLF	code letter	starting event	sp	finishing event	sp	activity time	sp

If an activity is to be removed, the code letter is R

If an activity time is to be changed, the code letter is C, and new time replaces the old one

If a new activity is to be inserted, the code letter is I, and the line of data is continued with

.. (description) øø .... ø

<u>NOTE</u> If there are no amendments a tape must still be punched, it will contain just the subsidiary title followed by blank tape.

3. <u>All</u> tapes must be terminated by

CRLF -1 CRLF.

OUTPUT

The output tape is headed by the following:-

S followed by the number of the first event in the network

T followed by the number of the last event in the network

E followed by the number of events in the network

A followed by the number of activities in the network

Faulty data representing a network with more than one initial or terminal node will be indicated by the printing of more than one S or T. A "split group", occurring either through faulty preparation of the data or a wrongly placed insertion, will be indicated by the letter G followed by the common starting event number of this group.

After the main and subsidiary headings, the schedule is arranged in lines of data, each referring to the activity defined by the first three numbers of the line - starting event, finishing event and duration. The line continues with the early and late starting times, early and late finishing times, total, free and independent floats, and description of the activity. Activities which are on the Critical Path are indicated by the printing of 'dashes' between the latter numbers on the line. The estimated total duration of the project is equal to the early or late finishing time of the critical activity whose finishing event is the terminal event of the project (indicated by T at the head of output).

The lines of data are arranged in pages of 100 each.

COMPUTER OPERATION

1. Suppress Optional Printing.

2. Insert Pegacrit (4) in TR0. Start/Run. This ends with a 77-stop.

3. Insert numerical tape in TR0 and amendment tape in TR1. Operate the Run key.

4. When the 77-stop is reached, place title tape in TR0 and re-insert amendment tape in TR1. Operate the Run key. The schedule is punched out as the title tape is read in. The run ends with a 77-stop.

The next schedule may be run by repeating these operations, starting from operation 3.

UPDATING RUNS

Updating runs are operated as above except that the previous output tape is used instead of the numerical and title tapes, the output tape thus being read in twice.

SIZE OF PROJECTS

A project must conform to the following limits if it is to be analysed by the program:

Maximum number of events	1280
Maximum number of activities	2000
Maximum duration of the project or any one activity	9999 time units

Any time units can be used, but they must be consistent throughout the project and must be expressed as integers. Events can be numbered from 0 to 1279. The numbers can be in any sequence and need not run consecutively.

TIMING

The following approximate times will give a rough guide as to how long a schedule will take to compute:-

To read in program - Pegacrit (4) : ½ min.

To read in numerical tape : ¾ min. for every 100 activities.

Computing time : 1½ mins. for every 100 activities.

To read in title tape and output the schedule : 4 mins. for every 100 activities.

ERRORS

The following loop-stops, indicating errors on the data tapes, can occur during a run:-

During the reading-in of the numerical tape:

3.3 + 661	wrong page number
3.0 + 661	wrong line number

During computing:

1.3 + 660	loop detected in network

During reading-in of the title tape:

2.0 + 661	wrong page number
2.5 + 661	space and two dots not punched before description

On the amendment tape:

0.3 +	661	character ≠ I, C, R, or -1
1.7	560	insertion read that is already on the numerical tape

The project graph, data tapes, and output schedules of a small example network for an initial run and an updated run are given in the Appendix.

D. G. Cumming (A.E.O.)

DGC/CMJ

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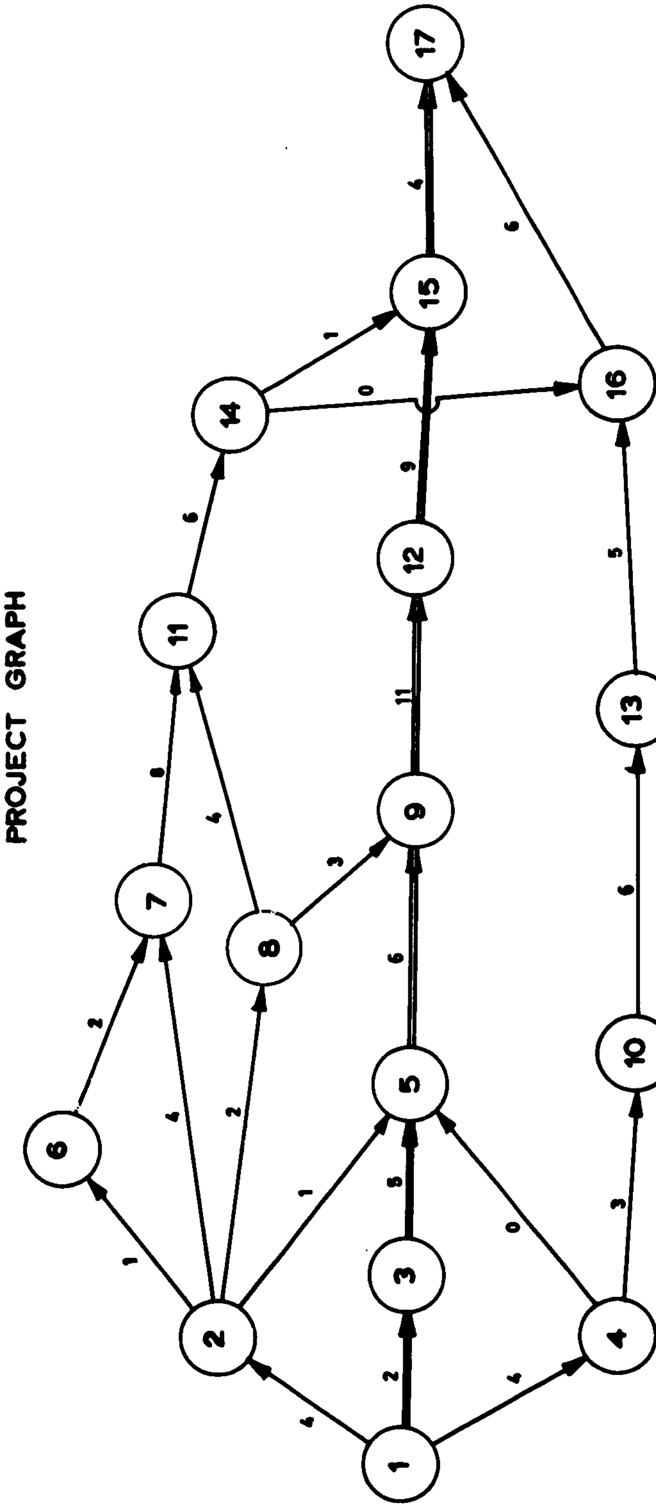
2. LEVY, F. K., THOMPSON, G. L. and WIEST, J. D., Critical Path Method - A New Tool for Management, ONR No. 86 Graduate School of Industrial Administration Carnegie Institute of Technology 1962

3. The Critical Path Schedule and Review Programme, Ferranti Ltd. List CS 344 October 1962

4. Critical Path Programmes Sirius and Pegasus - Rules for Amendments Ferranti Ltd.

APPENDIX 1.

PROJECT GRAPH



CRITICAL PATH IS SHOWN BY DOUBLE LINES

APPENDIX 2

NUMERICAL TAPE

```
JV 1
N

EXAMPLE  NETWORK

1
1 1 2 4
2 1 3 2
3 1 4 4
4 2 5 1
5 2 6 1
6 2 7 4
7 2 8 2
8 3 5 5
9 4 5 0
10 4 10 3
11 5 9 6
12 6 7 2
13 7 11 8
14 8 9 3
15 8 11 4
16 9 12 11
17 10 13 6
18 11 14 6
19 12 15 9
20 13 16 5
21 14 15 1
22 14 16 0
23 15 17 4
24 16 17 6
-1

********
```

APPENDIX 3

TITLE TAPE

```
JV1
N

EXAMPLE  NETWORK


1
1 ..1
2 ..2
3 ..3
4 ..4
5 ..5
6 ..6
7 ..7
8 ..8
9 ..9
10 ..10
11 ..11
12 ..12
13 ..13
14 ..14
15 ..15
16 ..16
17 ..17
18 ..18
19 ..19
20 ..20
21 ..21
22 ..22
23 ..23
24 ..24
-1
XXXXXXXX
```

APPENDIX 4

AMENDMENT TAPE FOR INITIAL RUN

N

FIRST SCHEDULE

-z

zzzzzzzz

APPENDIX 5

FIRST SCHEDULE

S 1
T 17
E 17
A 24
JU2
N

EXAMPLE NETWORK

N

FIRST SCHEDULE

EVENTS S	EVENTS F	JOB TIME	START E	START L	FINISH E	FINISH L	FLOATS T	FLOATS F	FLOATS I	DESCRIPTION
1	2	4	0	2	4	6	2	0	0	..1
1	3	2	0	0	2	2	0	0	0	..2
1	4	4	0	3	4	7	3	0	0	..3
2	5	1	4	6	5	7	2	2	0	..4
2	6	1	4	14	5	15	10	0	0	..5
2	7	4	4	13	8	17	9	0	0	..6
2	8	2	4	8	6	10	4	0	0	..7
3	5	5	2	2	7	7	0	0	0	..8
4	5	0	4	7	4	7	3	3	0	..9
4	10	3	4	17	7	20	13	0	0	..10
5	9	6	7	7	13	13	0	0	0	..11
6	7	2	5	15	7	17	10	1	0	..12
7	11	8	8	17	16	25	9	0	0	..13
8	9	3	6	10	9	13	4	4	0	..14
8	11	4	6	21	10	25	15	6	2	..15
9	12	11	13	13	24	24	0	0	0	..16
10	13	6	7	20	13	26	13	0	0	..17
11	14	6	16	25	22	31	9	0	0	..18
12	15	9	24	24	33	33	0	0	0	..19
13	16	5	13	26	18	31	13	4	0	..20
14	15	1	22	32	23	33	10	10	1	..21
14	16	0	22	31	22	31	9	0	0	..22
15	17	4	33	33	37	37	0	0	0	..23
16	17	6	22	31	28	37	9	9	0	..24

-1

APPENDIX 6

AMENDMENT TAPE FOR UPDATING RUN

```
N

SECOND SCHEDULE

C1 2 2
R1 3 2
C1 4 1
I1 5 1 ..A
I2 8 2 ..B
I2 18 3 ..C
I18 7 4 ..D
R2 7 4
R2 8 2
R3 5 5
C4 10 2
I6 11 3 ..E
R6 7 2
C9 12 7
-1

********
```

APPENDIX 7

SECOND SCHEDULE

S 1
T 17
E 17
A 24
JV2
N

EXAMPLE NETWORK

N

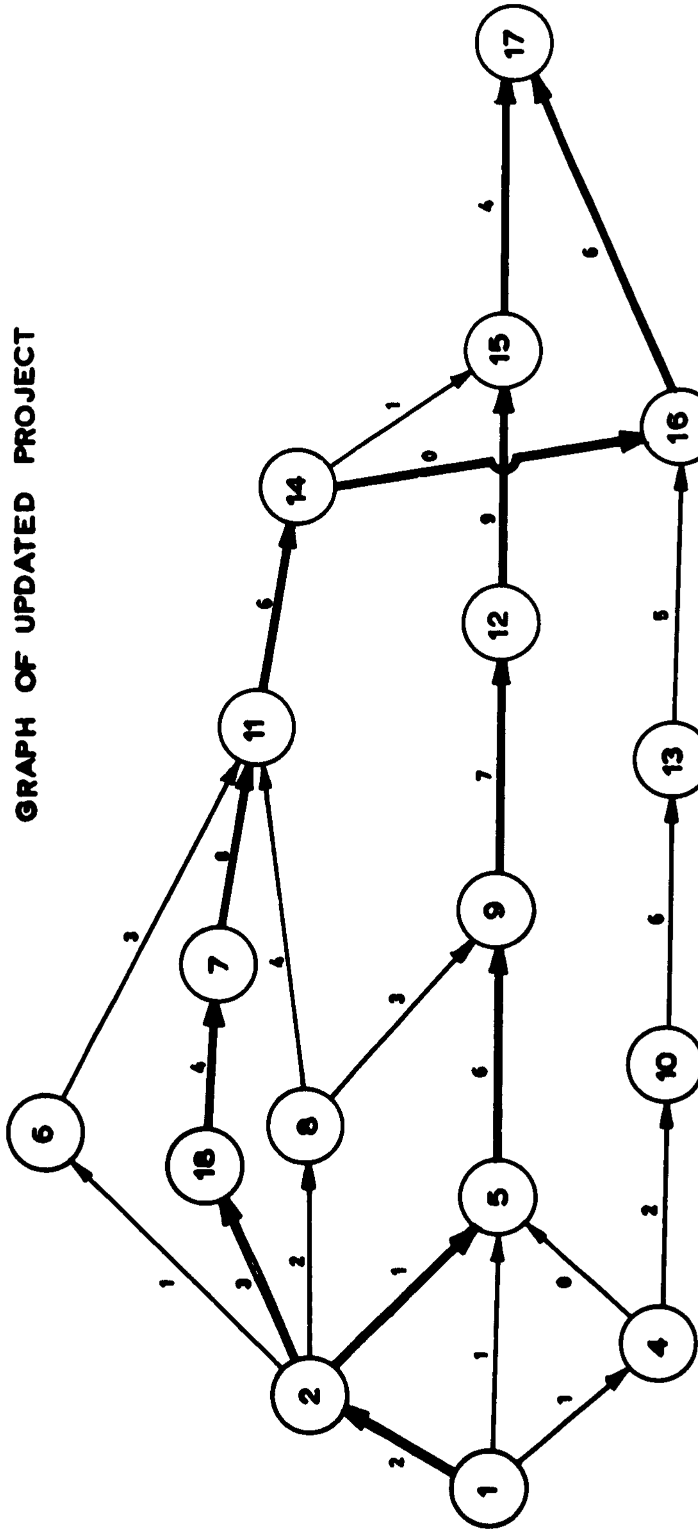
SECOND SCHEDULE

EVENTS S	EVENTS F	JOB TIME	START E	START L	FINISH E	FINISH L	FLOATS T	FLOATS F	FLOATS I	DESCRIPTION
1	2	2	0	0	2	2	0	0	0	1
1	4	1	0	2	1	3	2	0	0	3
1	5	1	0	2	1	3	2	2	2	A
2	8	2	2	4	4	6	2	0	0	B
2	18	3	2	2	5	5	0	0	0	C
2	5	1	2	2	3	3	0	0	0	4
2	6	1	2	13	3	14	11	0	0	5
18	7	4	5	5	9	9	0	0	0	D
4	5	0	1	3	1	3	2	2	0	9
4	10	2	1	10	3	12	9	0	0	10
5	9	6	3	3	9	9	0	0	0	11
6	11	3	3	14	6	17	11	11	0	E
7	11	8	9	9	17	17	0	0	0	13
8	9	3	4	6	7	9	2	2	0	14
8	11	4	4	13	8	17	9	9	7	15
9	12	7	9	9	16	16	0	0	0	16
10	13	6	3	12	9	18	9	0	0	17
11	14	6	17	17	23	23	0	0	0	18
12	15	9	16	16	25	25	0	0	0	19
13	16	5	9	18	14	23	9	9	0	20
14	15	1	23	24	24	25	1	1	1	21
14	16	0	23	23	23	23	0	0	0	22
15	17	4	25	25	29	29	0	0	0	23
16	17	6	23	23	29	29	0	0	0	24

-1

APPENDIX 8.

GRAPH OF UPDATED PROJECT



CRITICAL PATH IS SHOWN BY DOUBLE LINES

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