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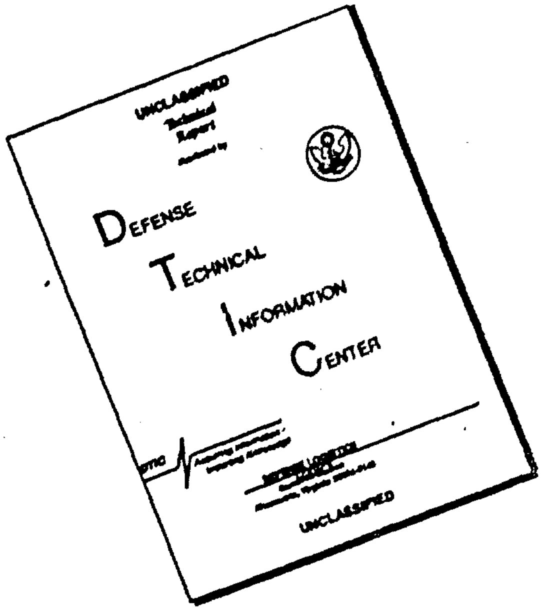


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AFCRL 62-892

# RESEARCH IN GEODESY AND GRAVITY

## Computer Programs for Orbit Correction and Station Location

C. G. Hilton
J. E. Evans
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64-5

Astrosciences Department
AERONUTRONIC DIVISION
Ford Motor Company
Newport Beach, California

Publication No. U-1490

Contract No. AF 19(604)-7253

Final Report
(Revised)
August 1962

Prepared
for

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH (USAF)
LAURENCE G. HANSCOM FIELD
BEDFORD, MASSACHUSETTS

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Project No. 8607
Task No. 86071

Contract No. AF 19(604)-7253


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# ABSTRACT

This report describes two computer programs. The first corrects the orbital elements of a geodetic satellite. The second uses residuals generated by the first program to correct the geocentric coordinates of the observing sensors and of the origin of the datum by which a set of such sensors are connected.

CONTENTS


| SECTION | | PAGE |
|---|---|---|
| 1 | INTRODUCTION | 1 |
| 2 | ORBIT CORRECTION PROGRAM | 4 |
| | 2.1 Theory | 5 |
| | 2.2 Formulation | 7 |
| | 2.3 Flow Charts | 32 |
| | 2.4 Input and Output Formats | 49 |
| | 2.5 Operating Procedure | 58 |
| 3 | STATION LOCATOR PROGRAM | |
| | 3.1 Theory | 62 |
| | 3.2 Logical Program | 64 |
| | 3.3 Flow Diagrams | 76 |
| | 3.4 Input and Output Formats | 98 |
| | 3.5 Operating Procedure | 103 |
| | 3.6 Experimentation | 104 |
| 4 | MODIFICATIONS | 108 |

# SECTION 1

# INTRODUCTION

Under contract AF 19(604)-7253, Aeronutronic has developed two computer programs to perform the following tasks:

(1) Orbit Correction Program

To compute a satellite ephemeris to high precision, using optical, radar slant range and doppler observations. High precision is understood to mean that errors no greater than 20 feet shall arise during any one day portion of the orbit from the effects of zonal harmonics or computational procedures for the case of a spherical satellite with high mass to surface area ratio in a circular orbit approximately 500 statute miles above the earth's surface. This error does not include errors introduced by air drag, radiation pressure, longitudinal variations of gravity, observational errors, or station errors. Air drag, however, is accounted for by a model atmosphere. Radiation pressure and longitudinal variations of gravity can be introduced by modifications of the program.

(2) Station Locator Program

To compute, from the output of the Orbit Correction Program, a geocentric translation vector for each station location such as to minimize the residuals from that station and, for each datum, a geocentric datum translation vector, which minimizes the station correction vectors for all the stations on that datum. This program includes means of weighting observations and station coordinates according to criteria determined externally to the program.

The decision to correct the station location in a separate least squares process was taken partly on the basis of expediency. It was thought to be simpler and quicker to write and check out the two programs. Such a procedure was also considered to provide the user with greater flexibility in the use of the programs. The following practical advantages may be cited in favor of separate programs:

(a) The matrices are kept small and easily manageable.

(b) The orbit computation can be limited to a few revolutions in order to limit the effect of round off errors. Yet, the residuals obtained from several such orbital arcs can be used to correct station location.

(c) The observations from several different satellites could be combined in the Station Locator Program. This may require a change to accomodate several ephemerides.

(d) A different form of orbit computation or prediction could be used to provide input to the Station Locator Program.

(e) The weighting factors may be changed between the programs and for different runs of the Station Locator Program. This advantage vanishes when the evaluation of these weights is formalized and can, therefore, be made inside the program or programs.

On the other hand, certain theoretical considerations indicate advantages of simultaneous correction of both orbit and station errors. The principal argument is that what is desired is a best fit to the observations of the total physical model. This model includes both the orbit and the observing net. Errors in station coordinates could be misinterpreted by the Orbit Correction Program as errors in orbital elements and remaining errors in orbital elements could be interpreted as station errors by the Station Locator Program. There is a justifiable fear that these misinterpretations could limit the accuracy of the process. That is, the programs could not converge to as close an approximation to the true model as could aprogram which corrects all quantities simultaneously.

The same problem can occur in the combined program also. That is, the confusion of error source can be attributed to a similarity, analytical or numerical, of the partial derivatives which express the relationship between the error and its possible sources. If such a similarity exists, there is a danger that the large matrix may be nearly singular and, therefore, unstable.

None of the above argues against combining the two programs in such a way that separate solutions are possible and that much of the intermediate output is available for inspection. This procedure will assure the greatest possible efficiency since the convergence will generally be much faster in the combined program.

Other desirable modifications will be discussed in Section 4. These include the introduction of weighting by accuracy of observations in the Orbit Correction Program, the use of these accuracies to give uncertainties in the elements, the inclusion of the effects of additional perturbations and to improve the precision of the model for atmospheric drag.

This report is intended principally as a description of the two programs developed, and does not include all of the mathematical development. Those who wish to know why as well as how are referred for theoretical background of differential correction techniques and of the variation of parameters to Aeronutronic Publication U-880, <u>Astrodynamic Analysis for the National Space Surveillance Control Center</u>, June 1960. The variation of parameters method can be found in Vol. 1, Appendix 3D, "Efficient Precision Orbit Computation Techniques," and the differential correction procedure is shown in Vol. II, Appendix 4A, "Differential Correction."

# SECTION 2

# ORBIT CORRECTION PROGRAM

The Orbit Correction Program generates satellite ephemerides for the computation of acquisition coordinates for sensors as well as correcting the orbital elements of the satellite. The first use is referred to hereafter as simulation. This feature is incidental to the tasks of this contract. It has been used in the checkout of the programs and may prove valuable also, as a means of producing look angles, scheduling and evaluation of sensor sites.

The primary purpose of the program is to correct the orbital elements by means of observations of slant range, range rate, right ascension and declination, and/or azimuth and elevation angle. After the last orbit correction, the program prepares the residuals in the observations for use in the Station Locator Program and a tape containing position and velocity vectors for the satellite at the time of each observation.

It should be noted that the observation time shall be the time that the electromagentic radiation left the satellite. That is, the time of reception of the signal at the station must be corrected for the time it takes light to travel from the satellite to the station. For an active radar observation, this can be done by averaging the times of transmission and reception as well as differencing them, but with passive doppler and angle observations one requires an approximation of slant range in order to find this "light time" correction. Neglect of this effect produces a bias of about 25 parts per million of the slant range in the direction of satellite motion. This is a bias of 40 feet at a slant range of 350 statute miles.

## 2.1 THEORY

The Orbit Correction Program employs two computational techniques which speed the computation without loss of accuracy:

(1) The satellite motion is numerically integrated by the variation of parameters formulation, thereby eliminating from the integrands the large central acceleration term.
(2) The differential correction uses analytical differential expressions, thereby eliminating the need for more than one numerical integration over the observation period.

By developing the differential equations of motion in terms of parameters which remain invariant in the absence of perturbations to the two-body motion, the dominant two-body term is supressed. The parameters employed in this formulation are:

| | |
|---|---|
| $\underline{a} = e\underline{P}$ | a vector defining eccentricity and perigee location |
| $\underline{h} = \sqrt{p}\,\underline{W}$ | the orbital angular momentum |
| $L_o = M_o + \omega + \Omega$ | the mean longitude of the object at some epoch |

These parameters are valid, in the formulation employed here, for all satellite eccentricities and for all inclinations, including zero in either case.

The differential characteristics of a slightly-perturbed satellite orbit are, to a first order, identical to those of the osculating orbit. Thus the cause-and-effect linear relationships needed for differential correction may be developed analytically, rather than by the alternate "variant calculation" procedure where neighboring ephemerides are integrated, in each case with one of the parameters modified by a small amount. The parameters employed in the development of the differential expressions resemble those used in the variation-of-parameters ephemeris program, i.e.:

| | |
|---|---|
| $\underline{a} = e\underline{P}$ | a vector defining eccentricity and perigee location |
| $a$ | semi-major axis |
| $U_o = M_o + \omega$ | the mean argument of the latitude at some epoch |
| $\Omega$ | nodal longitude |
| $i$ | inclination |

The adoption of $\Omega$ and i to describe the orbit plane orientation and of $U_o$ to denote initial position, restricts the development to non-equatorial orbits. In addition, the vector a is described in terms of two components in the orbit plane to avoid redundancy; these components are designated $a_{xN}$ and $a_{yN}$, with the former in the direction of the ascending node.

Any observation $0_i$ by an earth-fixed observer may be expressed in terms of these six parameters or elements describing the orbit and the time. First order differential expressions relating observation and parameter follow from the leading term in a Taylor expansion, i.e.,

$$\Delta 0_i = \sum_j \frac{\partial 0_i}{\partial X_j} \Delta X_j \qquad (1)$$

where the $X_j$ are the six orbit parameters. Where there are m observations available to define the six parameters, a set of m differential expressions may be written; in matrix form, this set is

$$(\Delta 0_i) = (C_{ij}) (\Delta X_j) \qquad (2)$$

where $(C_{ij})$ is the m x 6 matrix with typical element,

$$C_{ij} = \frac{\partial 0_i}{\partial X_j} \qquad (3)$$

The $(\Delta X_j)$ is a six component vector, and $(\Delta 0_j)$ is an m component vector.

The elements of the $(C_{ij})$ matrix are the partial derivatives of the observed quantities with respect to the orbit parameters and may be determined analytically or by variant trajectory calculations, wherein the parameters are varied, one by one, and the resulting changes in the observations are noted.

If there are more observed quantities $0_i$ than parameters $X_j$, that is, when $m > 6$, the system is overdetermined and the number of equations may be reduced by the method of least squares. The solution takes the form

$$(\Delta X_j) = \left[ (C_{ij})^T (C_{ij}) \right]^{-1} (C_{ij})^T (\Delta 0_i) \qquad (4)$$

where the -1 and T superscripts denote inverse matrix and transpose matrix, respectively. The bracketed quantity in (4) is the so-called least square matrix, N:

$$N = (C_{ij})^T \quad (C_{ij}) \tag{5}$$

In the interest of achieving efficiency, the $C_{ij}$ are evaluated from analytical expressions, which are detailed in the following section presenting the formulation of the program.

## 2.2 FORMULATION

The Orbit Correction Program can be conveniently divided into three parts, one concerned with the correction process, another concerned with the generation of different types of ephemerides, and, most important, that portion necessary for both, (mainly concerned with the integration of the equations of motion). These parts will be documented below in the inverse order.

### 2.2.1 VARIATIONS OF THE PARAMETERS

The major portion of the Orbit Correction Program is spent in generating position and velocity of the satellite at successive times by numerically integrating the equations of motion. This integration process is started by generating the initial values of the integrands and other quantities:

Given $a_{xN_o}$, $a_{yN_o}$, $\underline{h}_o$, $L_o$ (subscript o indicates epoch), the following procedure is common to both the simulation and differential correction portions of the program:

(a) Compute Greenwich sidereal time at epoch, $\Theta_{gr}$:

$$\Theta_{gr} \text{ (in degrees)} = D\,(\dot{\Theta} - 360^{o}) + \dot{\Theta} f + \Theta'_{gr}$$

where D is epoch day number, f is the fraction of a day elapsed from start of epoch day to epoch, $\Theta'_{gr}$ is Greenwich sidereal time at the beginning of epoch year, and $\dot{\Theta} = 360^{o}.9856472$, the rotation rate of the earth in degrees per mean solar day.

(b) Compute the semi-latus rectum:

$$p_o = \underline{h}_o \cdot \underline{h}_o$$

(c) Compute the orientation vector, $\underline{W}_o$:

$$\underline{W}_o = \underline{h}_o / \sqrt{p_o}$$

(d) Compute the orientation angles, $i_o$, $\Omega_o$, and $U_o$:

$$\sin i_o = \sqrt{1 - W_{z_o}^{2}}$$

$$\cos i_o = W_{z_o}$$

$$i_o = \tan^{-1}\left(\frac{\sin i_o}{\cos i_o}\right) \qquad 0 \leq i_o < \pi$$

$$\sin \Omega_o = W_{x_o} / \sin i_o$$

$$\cos \Omega_o = -W_{y_o} / \sin i_o$$

$$\Omega_o = \tan^{-1}\left(\frac{\sin \Omega_o}{\cos \Omega_o}\right) \qquad 0 \leq \Omega_o < 2\pi$$

$$U_o = L_o + \Omega_o \text{ if } W_{z_o} < 0 \quad \text{(retrograde motion)}$$

$$U_o = L_o - \Omega_o \text{ if } W_{z_o} > 0 \quad \text{(direct motion)}$$

(e) Compute the equatorial coordinates of $\underline{a}_o$:

$$\underline{a}_o \begin{cases} a_{x_o} = a_{xN_o} \cos \Omega_o - \cos i_o \, a_{yN_o} \sin \Omega_o \\ a_{y_o} = a_{xN_o} \sin \Omega_o + \cos i_o \, a_{yN_o} \cos \Omega_o \\ a_{z_o} = a_{yN_o} \sin i_o \end{cases}$$

(f) Compute the eccentricity, $e_o$, and the semi-major axis, $a_o$:

$$e_o^2 = a_{x_o}^2 + a_{y_o}^2 + a_{z_o}^2$$

$$a_o = p_o / 1 - e_o^2$$

Also common to both simulation and differential correction is the numerical integration. The equations to be integrated are of the form:

$$\frac{dy_i}{dt} = f_i\,(t, y_1, y_2, \ldots, y_6, y_7) \qquad i = 1, 2, \ldots, 7$$

where the $y_i$ equal $a_x$, $a_y$, $a_z$, $h_x$, $h_y$, $h_z$, and L.

The numerical integration scheme used is based on the following fourth order Runge-Kutta method:

$$y_i^{n+1} = y_i^n + \frac{\Delta t}{6} \left(K_{1_i} + 2K_{2_i} + 2K_{3_i} + K_{4_i}\right)$$

where $K_{1_i} = f_i\ (t^n, y_1^n, \dots y_7^n)$

$$K_{2_i} = f_i\ \left(t^n + \frac{\Delta t}{2}, y_1^n + \frac{K_{11}}{2}, \dots , y_7^n + \frac{K_{17}}{2}\right)$$

$$K_{3_i} = f_i\ \left(t^n + \frac{\Delta t}{2}, y_1^n + \frac{K_{21}}{2}, \dots , y_7^n + \frac{K_{27}}{2}\right)$$

$$K_{4_i} = f_i\ (t^n + \Delta t, y_1^n + K_{31}, \dots , y_7^n + K_{37})$$

As can be seen, it is necessary to compute $\frac{d\underline{a}}{dt}$, $\frac{d\underline{h}}{dt}$, and $\frac{dL}{dt}$ from $\underline{a}$, $\underline{h}$, and L, several times at each integration step. The first step is to compute position and velocity, $\underline{r}$, $\dot{\underline{r}}$ from $\underline{a}$, $\underline{h}$, and L.

Given $\underline{a}$, $\underline{h}$, and L at some time t, the following procedure is used to derive position, $\underline{r}$, and velocity, $\dot{\underline{r}}$. Note that whenever the anomalies v, E, and M are used, they appear either in a sum with $\omega$ or in products with the coefficient e. Thus, no indeterminacy exists for zero eccentricity. In its present form, precisely equatorial orbits cannot be integrated, since the ascending node is employed as a reference direction.

(a) Compute p, e, a, n:

$$p = \underline{h} \cdot \underline{h} = h_x^2 + h_y^2 + h_z^2$$

$$e^2 = \underline{a} \cdot \underline{a} = a_x^2 + a_y^2 + a_z^2$$

$$a = p / 1-e^2$$

$$n = K_e\sqrt{\mu} \big/ a^{3/2} \quad \text{where } K_e\sqrt{\mu} = .07436574$$

(b) Compute the orientation vectors $\underline{W}$, $\underline{M}$, and $\underline{N}$:

$$\underline{W} = \underline{h}/\sqrt{p} \qquad \text{(note } W_z = \cos i)$$

$$M_z = \sqrt{1 - W_z^2} = \sin i$$

$$N_x = {-W_y}/{M_z} = \cos \Omega$$

$$M_y = N_x W_z = \cos \Omega \cos i$$

$$N_y = {}^{W_x}\!/_{M_z} = \sin \Omega$$

$$M_x = -N_y W_z = -\sin \Omega \cos i$$

$$N_z = 0$$

(c) Compute the components of $\underline{a}$ in the orbit plane, $a_{xN}$ and $a_{yN}$:

$$a_{xN} = \underline{a} \cdot \underline{N}$$

$$a_{yN} = \underline{a} \cdot \underline{M}$$

(d) Compute the orientation angles, $\Omega$ and U:

$$\Omega = \tan^{-1}\left({}^{N_y}\!/_{N_x}\right) \quad 0 \leq \Omega < 2\pi$$

$$U = L + \Omega \text{ if } W_z < 0 \qquad \text{(retrograde motion)}$$

$$U = L - \Omega \text{ if } W_z > 0 \qquad \text{(direct motion)}$$

(e) Solve Kepler's equation for $E + \omega$ by iteration, using a first guess of U (mod $2\pi$):

$$E + \omega = U + a_{xN} \sin (E + \omega) - a_{yN} \cos (E + \omega)$$

(f) Compute $\underline{r}$ and $\dot{\underline{r}}$:

$$e \cos E = a_{xN} \cos (E + \omega) + a_{yN} \sin (E + \omega)$$

$$e \sin E = a_{xN} \sin (E + \omega) - a_{yN} \cos (E + \omega)$$

$$r = a (1 - e \cos E)$$

$$\dot{r} = \frac{\sqrt{\mu a}}{r} \; e \sin E$$

$$r\dot{v} = \sqrt{\frac{\mu a}{r}} \sqrt{1 - e^2}$$

$$\cos u = \frac{a}{r}\left[\cos (E+\omega) - a_{xN} + a_{yN}\left(\frac{e \sin E}{1 + \sqrt{1-e^2}}\right)\right]$$

$$\sin u = \frac{a}{r}\left[\sin (E+\omega) - a_{yN} - a_{xN}\left(\frac{e \sin E}{1 + \sqrt{1-e^2}}\right)\right]$$

$$\underline{U} = \cos u\, \underline{N} + \sin u\, \underline{M}$$

$$\underline{V} = -\sin u\, \underline{N} + \cos u\, \underline{M}$$

$$\underline{r} = r\underline{U}$$

$$\underline{\dot{r}} = \dot{r}\underline{U} + r\dot{v}\underline{V}$$

From the position and velocity, it is possible to compute the perturbative accelerations, specifically the bulge perturbation, $\underline{\dot{r}}_B$:

$$\dot{x}_B = \frac{x}{r^5} J'(5U_z^2 - 1) + \frac{xz}{r^7} H'(7U_z^2 - 3)$$

$$+ \frac{x}{6r^7} K'(42U_z^2 - 63U_z^4 - 3) + \frac{21xU_z}{8r^8} J_5 a_e^5 \mu (5 - 30 U_z^2 + 33U_z^4)$$

$$\dot{y}_B' = {}^{y}/_{x}\ \dot{x}_B'$$

$$\dot{z}_B^{\backprime} = \frac{z}{r^5} J' (5U_z^2 - 3) + \frac{3}{5r^5} H' \left(\frac{35}{3} U_z^4 - 10U_z^2 + 1\right)$$

$$+ \frac{z}{6r^7} K' (-63U_z^4 + 70 U_z^2 - 15) + \frac{3}{8r^7} J_5 a_e^5 \mu (-5 + 105U_z^2$$

$$- 315 U_z^4 + 231 U_z^6)$$

where $U_z = \frac{z}{r}$ and $J' = .001\ 623\ 41$

$H' = -.000\ 006$

$K' = .000\ 009\ 09$

$J_5\ a_e^5/\mu = -.000\ 000\ 2$

At each point during the integration at which the derivatives dL/dt, $d\underline{h}/dt$, $d\underline{a}/dt$, are evaluated, the perturbative accelerations due to drag $\dot{\underline{x}}_D^{\backprime}$, $\dot{y}_D^{\backprime}$, $\dot{z}_D^{\backprime}$, must be evaluated and added to the bulge accelerations, $\dot{x}_B^{\backprime}$, $\dot{y}_B^{\backprime}$, $\dot{z}_B^{\backprime}$, to obtain the total perturbative accelerations, $\dot{x}^{\backprime}$, $\dot{y}^{\backprime}$ and $\dot{z}^{\backprime}$.

Given $\underline{r}$ and $\dot{\underline{r}}$, and tabulated values of the density ratio and atmospheric molecular weight versus altitude:

(1) Compute the relative air speed vector,:

$$\gamma_x = \dot{x} + y\,\dot{\theta}$$

$$\gamma_y = \dot{y} - x\,\dot{\theta}$$

$$\gamma_z = \dot{z}$$

Also compute the magnitude of the relative air speed vector

$$\nu = \sqrt{\nu_x^2 + \nu_y^2 + \nu_z^2}\ .$$

$\dot{\theta}$ in the above equations, is the angular rotation rate of the Earth.

(2) Compute the altitude above the oblate spheroid in Earth radii:

$$H = r - 1 - \frac{3}{2} f^2 \left(\frac{z}{r}\right)^4 + \left(f + \frac{3}{2} f^2\right) \left(\frac{z}{r}\right)^2,$$

where f is the flattening of the Earth = 1/298.3.

(3) Look up the density ratio $\sigma = \sigma(H)$ and the molecular weight $M_e = M_e(H)$ from the tabulated data (see Appendix C for the programmed tables), and calculate the atmospheric density :

$$\sigma = \rho / \rho_o$$

where $\rho_o$ is the sea level value of the atmospheric density = 0.001225 gm/cm$^3$.

(4) Compute the skin temperature of the vehicle:

$$T_s = \left[ \frac{\rho C_D (V_{co})^3 \gamma^3}{4 \epsilon \sigma_s} + (300)^4 \right]^{1/4}$$

where

$\sigma_s$ = Stefan-Boltzmann constant = $5.672 \times 10^{-5}$,

and

$\epsilon$ = emissivity of the satellite = .9.

(5) Compute the drag coefficient, $C_D$, by first computing the auxiliary quantity

$$C = \frac{6.972 \times 10^9 \gamma d}{C_{Do} \sqrt{(M_e T_s)}} ,$$

and then

$$C_D = C_{Do} (1 + 1.1739130 \ e^{-C\sigma}) ,$$

$C_{Do}$ being a reference value of the drag coefficient = 0.92.

(6) Compute the drag terms:

$$\dot{x}_D^{\backprime} = \nu_x \left[ C_D \, \rho \, \nu \left( - \frac{K \pi d^2}{8m} \right) \right] \qquad x \longrightarrow y,\ z,$$

where d is the diameter of the satellite, m is its mass, and K is a constant relating the units. The program value of $K = 2.504742 \times 10^9$.

Finally we are ready to calculate the integrands $d\underline{a}/dt$, $d\underline{h}/dt$, and $dL/dt$:

(a) Determine the total perturbations $\underline{\dot{r}}^{\backprime}$

$$\underline{\dot{r}}^{\backprime} = \underline{\dot{r}}_B^{\backprime} + \underline{\dot{r}}_D^{\backprime}$$

(b) Compute:

$$r\,\dot{r}^{\backprime} = \underline{r} \cdot \underline{\dot{r}}^{\backprime}$$

$$\dot{s}\,\dot{s}^{\backprime} = \underline{\dot{r}} \cdot \underline{\dot{r}}$$

$$r\,\dot{r} = \underline{r} \cdot \underline{\dot{r}}$$

and the auxiliary quantities:

$$D = \frac{r\,\dot{r}}{\sqrt{\mu}}$$

$$D^{\backslash} = \frac{r\,\dot{r}^{\backslash}}{\sqrt{\mu}}$$

$$\dot{D}^{\backslash} = \frac{2\dot{s}\;\dot{s}^{\backslash}}{\sqrt{\mu}}$$

(c) Compute $n^{\backslash}$, $r\dot{b}^{\backslash}$, $\ell^{\backslash}$, $\underline{a}^{\backslash}$, $ev^{\backslash}$, and $L^{\backslash}$:

$$n^{\backslash} = -\frac{3}{2}\,na\,\frac{\dot{D}^{\backslash}}{\sqrt{\mu}}$$

$$r\dot{b}^{\backslash} = \underline{W} \cdot \underline{\dot{r}}^{\backslash}$$

$$\ell^{\backslash} = \frac{z\,(r\dot{b}^{\backslash})}{(1 + W_z)\sqrt{\mu\,p}}$$

$$\underline{a}^{\backslash} = \dot{D}^{\backslash}\underline{r} - D\underline{\dot{r}}^{\backslash} - D^{\backslash}\underline{\dot{r}}$$

$$e\underline{Q} = \underline{W} \times \underline{a}$$

$$-\,e^{2}v^{\backslash} = e\underline{Q} \cdot \underline{a}^{\backslash}$$

$$\grave{L} = -\frac{2\grave{D}}{\sqrt{a}} - \frac{e^{2}\grave{v}}{1 + \sqrt{1 - e^{2}}}$$

(d) Compute $\grave{\underline{h}}$:

$$\grave{\underline{h}} = \frac{\underline{r} \times \grave{\dot{\underline{r}}}}{\sqrt{\mu}}$$

(e) Compute the derivatives:

$$\frac{dL}{dt} = k_{e}\grave{L} + n$$

$$\frac{d\underline{a}}{dt} = k_{e}\grave{\underline{a}}$$

$$\frac{d\underline{h}}{dt} = k_{e}\grave{\underline{h}}$$

### 2.2.2 EPHEMERIDES

The word ephemeris is defined as a table of the positions of celestial bodies at regular intervals of time. As used here in reference to the Earth satellite ephemerides, the intervals of time are integration steps. The ephemerides are of three kinds.

The first type of ephemeris shows geocentric position and velocity vectors. No additional formulation is needed to produce this.

The second type of ephemeris shows the position of the satellite as latitude, longitude, and height above the geoid. This is the subsatellite track.

The third type of ephemeris shows acquisition coordinates for a number of designated sensors. These "look angles" show the coordinates at which the satellite would appear to the station if it had started with the given boundary conditions at the epoch. These acquisition coordinates are produced for every integration step at which the satellite is above the horizon of the designated station.

a. Computation of Sub-Satellite Track:

If the option to compute a sub-satellite track consisting of latitude, $\phi$ , East longitude, $\lambda_E$, and height above the earth, H, is chosen, then these quantities are computed at each point of the ephemeris as follows:

$$\phi = \tan^{-1}\left[\frac{U_z}{(1-f)^2\sqrt{1-U_z^{\,2}}}\right] \quad -90^\circ \leq \phi \leq 90^\circ$$

where $f = \frac{1}{298.3}$ is the flattening of the Earth

$$\lambda_E \text{ (in degrees)} = \theta - \dot{\theta}\,(t-t_o) - \theta_{gr}, \quad \lambda_E \geq 0^\circ$$

where $\Theta = \tan^{-1} (\frac{y}{x}) \qquad 0^\circ \leq \Theta < 360^\circ$

and $\dot{\Theta} = .250\ 684\ 48$ is the rotation rate of the Earth in degrees per solar minute.

$$H \text{ (in Earth radii)} = r - 1 + (\frac{3}{2} f^2 + f)\, U_z^2 - \frac{3}{2} f^2\, U_z^4$$

b. Simulation of Acquisition Coordinates:

This part of the program simulates "observations" of the satellite in the orbit specified by the input parameters. Given $\phi$, the latitude of a station in degrees, $\lambda_E$, the East longitude of the station in degrees, H, the height of the station above sea level in meters, and $\Theta_{gr}$, Greenwich Sidereal Time at the initial time, $t_o$, in degrees, the following procedure computes $\alpha$, $\delta$, A, h, $\rho$, and $\dot{\rho}$ for time t:

(1) Convert $\phi$, $\lambda_E$, and $\Theta_{gr}$ to radians and H to Earth radii

(2) Compute:

$$C = (1 - \epsilon^2 \sin^2 \phi)^{-1/2}$$

where $\epsilon^2 = 2f - f^2$, $f = \frac{1}{298.3}$ = the flattening of the Earth.

$$S = C\ (1 - \epsilon^2)$$

(3) Compute:

$$\Theta = .004\ 375\ 269\ 1\ (t - t_o) + \Theta_{gr} + \lambda_E$$

(4) Compute the station vector, $\underline{R}$:

$$X = -(C + H)\cos\phi\cos\Theta$$

$$Y = -(C + H)\cos\phi\sin\Theta$$

$$Z = -(S + H)\sin\phi$$

(5) Compute the slant range $\rho$:

$$\underline{\rho} = \underline{r} + \underline{R}$$

$$\rho = \sqrt{\underline{\rho}\cdot\underline{\rho}} = \sqrt{(x+X)^2 + (y+Y)^2 + (z+Z)^2}$$

(6) Compute azimuth, A, and elevation angle, h, of object as "seen" from the observation station:

$$L_{xh} = \frac{(x+X)\cos\Theta\sin\phi + (y+Y)\sin\Theta\sin\phi - (z+Z)\cos\phi}{\rho}$$

$$L_{yh} = \frac{-(x+X)\sin\Theta + (y+Y)\cos\Theta}{\rho}$$

$$L_{zh} = \frac{(x+X)\cos\Theta\cos\phi + (y+Y)\sin\Theta\cos\phi + (z+Z)\sin\phi}{\rho}$$

$$h = \tan^{-1}\left(\frac{L_{zh}}{\sqrt{1-L_{zh}^2}}\right), \quad -\pi/2 \leq h \leq \pi/2$$

If $h < 0$ (i.e., the object is below the local horizon) the output for this time is omitted.

$$A = \tan^{-1}\left(\frac{L_{yh}}{-L_{xh}}\right) \quad 0 \leq A < 2\pi$$

The quadrant is determined from an examination of the sign of the numerator and denominator.

(7) Compute topocentric right ascension, $\alpha$, and declination, $\delta$:

$$\underline{L} \quad (L_x, L_y, L_z) = \frac{\underline{\rho}}{\rho}$$

$$\delta = \tan^{-1}\left(\frac{L_z}{\sqrt{1-L_z^2}}\right) \qquad -\pi/2 \leq \delta \leq \pi/2$$

$$\alpha = \tan^{-1}\left(\frac{L_y}{L_z}\right) \qquad 0 \leq \alpha < 2\pi$$

The quadrant is determined from sign of numerator and denominator.

(8) Compute the slant range rate, $\dot{\rho}$ :

$$\left.\begin{aligned} \dot{X} &= -Y\dot{\theta} \\ \dot{Y} &= X\dot{\theta} \\ \dot{Z} &= 0 \end{aligned}\right\} \dot{\theta} = 0.058\ 834\ 47$$

$$\dot{\underline{\rho}} = \dot{\underline{r}} + \dot{\underline{R}} \quad (\dot{X}, \dot{Y}, \dot{Z})$$

$$\dot{\rho} = \underline{L} \cdot \dot{\underline{\rho}} = L_x\ (\dot{x} + \dot{X}) + L_y\ (\dot{y} + \dot{Y}) + L_z\ (\dot{z} + \dot{Z})$$

### 2.2.3 DIFFERENTIAL CORRECTION OF ORBITAL ELEMENTS:

This part of the program relates residuals in the observations at time, t, to corrections to be applied to the initial orbital parameters at time $t_o$.

The procedure calculates the orbital parameters and quantities associated with the station coordinates at the observation time. It combines these quantities to obtain the coefficients of the linear relationships relating residuals to any combination of $\Delta a_o / a_o$, $\Delta a_{xN_o}$, $\Delta a_{yN_o}$, $\Delta U_o$, $\Delta \Omega_o$, $\Delta i_o$ and represents the observations to determine the residuals. Finally, the corrections to be applied to the parameters are determined by solving the (usually overdetermined) system of linear correction equations.

a. Forming the Linear Correction Equations

Given:

(1) $\phi$, the latitude in degrees
(2) $\lambda_E$, the longitude in degrees
(3) H, meters above sea level

} of the observing station

(4) $\Theta_{gr_o}$ Greenwich Sidereal time at epoch in degrees

(5) t, the time of the observation in minutes, and

(6) one or more observed quantities at this time,

the following will compute one line of the above-mentioned system for each observed quantity.

(1) Repeat steps 1 through 4 under simulation to obtain $\Theta$ and $\phi$ in radians and the station vector $\underline{R}$.

(2) Compute the coefficients R and U where:

$$R_u = \left(a^2/r\right) e \sin E$$

$$R_a = r - \frac{3}{2} (U - U_o) R_u$$

$$R_{xN} = \left(a^2/r\right) \left[a_{xN} - \cos (E + \omega)\right]$$

$$R_{yN} = \left(a^2/r\right) \left[a_{yN} - \sin (E + \omega)\right]$$

$$U_u = \left(a^2/r\right)\sqrt{1 - e^2}$$

$$U_a = -\frac{3}{2} (U - U_o) U_u$$

$$U_{xN} = \frac{a^2}{r} \left\{ (1 + \frac{r}{a}) \sin (E+\omega) + a_{xN}\, e \sin E \right.$$

$$\left. \left[ \frac{e^2 - (1 + \sqrt{1 - e^2}) e \cos E}{\sqrt{1 - e^2}\, (1 + \sqrt{1 - e^2})^2} \right] - \frac{a_{yN}}{1 + \sqrt{1 - e^2}} \right\}$$

$$U_{yN} = \frac{a^2}{r} \left\{ - (1 + \frac{r}{a}) \cos (E + \omega) + a_{yN}\, e \sin E \right.$$

$$\left. \left[ \frac{e^2 - (1 + \sqrt{1 - e^2})\, e \cos E}{\sqrt{1 - e^2}\, (1 + \sqrt{1 - e^2})^2} \right] + \frac{a_{xN}}{1 + \sqrt{1 - e^2}} \right\}$$

(3) Compute:

$$\underline{\rho}_c = \underline{r} + \underline{R}$$

$$\rho_c = \sqrt{\underline{\rho}_c \cdot \underline{\rho}_c}$$

$$\underline{L}_c = \frac{\underline{\rho}_c}{\rho_c}$$

(4) If $\rho$, the slant range is observed, compute

$$\Delta\rho = \rho - \rho_c$$

Form the coefficients:

$$C_{\frac{\Delta a}{a}} = (\underline{L}_c \cdot \underline{U})\, R_a + (\underline{L}_c \cdot \underline{V})\, U_a$$

$$C_{\Delta a_{xN}} = (\underline{L}_c \cdot \underline{U})\, R_{xN} + (\underline{L}_c \cdot \underline{V})\, U_{xN}$$

$$C_{\Delta a_{yN}} = (\underline{L}_c \cdot \underline{U})\, R_{yN} + (\underline{L}_c \cdot \underline{V})\, U_{yN}$$

$$C_{\Delta U_o} = (\underline{L}_c \cdot \underline{U})\, R_u + (\underline{L}_c \cdot \underline{V})\, U_u$$

$$C_{\Delta\Omega} = (\underline{L}_c \cdot \underline{V})\ r \cos i - (\underline{L}_c \cdot \underline{W})\ r \sin i \cos u$$

$$C_{\Delta i} = (\underline{L}_c \cdot \underline{W})\ r \sin u$$

Enter the following linear correction equation into the system of such equations:

$$\Delta\rho = C_{\frac{\Delta a}{a}} \frac{\Delta a_o}{a_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o}$$

$$+ C_{\Delta U_o} \Delta U_o + C_{\Delta\Omega} \Delta\Omega_o + C_{\Delta i} \Delta i_o$$

(5) If A, azimuth, and h, elevation angle are observed, compute:

$$\left.\begin{aligned} S_x &= \sin\phi \cos\Theta \\ S_y &= \sin\phi \sin\Theta \\ S_z &= -\cos\phi \end{aligned}\right\} \underline{S}$$

$$\left.\begin{aligned} E_x &= -\sin\Theta \\ E_y &= \cos\Theta \\ E_z &= 0 \end{aligned}\right\} \underline{E}$$

$$\left.\begin{aligned} Z_x &= \cos\phi \cos\Theta \\ Z_y &= \cos\phi \sin\Theta \\ Z_z &= \sin\phi \end{aligned}\right\} \underline{Z}$$

$$\left.\begin{aligned} L_{xh} &= -\cos A \cos h \\ L_{yh} &= \sin A \cos h \\ L_{zh} &= \sin h \end{aligned}\right\} \quad \underline{L}_h$$

$$\left.\begin{aligned} A_{xh} &= \sin A \\ A_{yh} &= \cos A \\ A_{zh} &= 0 \end{aligned}\right\} \quad \tilde{\underline{A}}_h$$

$$\left.\begin{aligned} D_{xh} &= c\cos A \sin h \\ D_{yh} &= --\sin A \sin h \\ D_{zh} &= \cos h \end{aligned}\right\} \quad \tilde{\underline{D}}_h$$

$$\underline{L}_{obs} = L_{xh}\,\underline{S} + L_{yh}\,\underline{E} + L_{zh}\,\underline{Z}$$

$$\tilde{\underline{A}}_{obs} = A_{xh}\,\underline{S} + A_{yh}\,\underline{E} + A_{zh}\,\underline{Z}$$

$$\tilde{\underline{D}}_{obs} = D_{xh}\,\underline{S} + D_{yh}\,\underline{E} + D_{zh}\,\underline{Z}$$

Compute $\underline{\Delta L} = \underline{L}_{obs} - \underline{L}_c$

Form the coefficients as in (4) with $\tilde{\underline{A}}_{obs}$ replacing $\underline{L}_c$ and enter the following linear correction equation into the system of such equations:

$$\rho_c \underset{\sim}{\tilde{A}}_{obs} \cdot \underline{\Delta L} = C_{\frac{\Delta a}{a}} \frac{\Delta a_c}{a_c} + C_{\Delta a_{xN}} \Delta a_{xN_o}$$

$$+ C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o + C_{\Delta \Omega} \Delta \Omega_o + C_{\Delta i} \Delta i_o$$

Again form the coefficients as in (4), this time with $\underset{\sim}{D}_{obs}$ replacing $\underset{\sim}{L}_c$, and enter the following linear correction equation into the system of such equations:

$$\rho_c \underset{\sim}{\tilde{D}}_{obs} \cdot \underline{\Delta L} = C_{\frac{\Delta a}{a}} \frac{\Delta a_o}{a_o} + C_{\Delta a_{xN}} \Delta a_{xN_o}$$

$$+ C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o + C_{\Delta \Omega} \Delta \Omega_o + C_{\Delta i} \Delta i_o$$

(6) If $\alpha$, topocentric right ascension, and $\delta$, topocentric declination are observed, compute:

$$\left.\begin{aligned} L_x &= \cos\delta \cos\alpha \\ L_y &= \cos\delta \sin\alpha \\ L_z &= \sin\delta \end{aligned}\right\} \quad \underset{\sim}{L}_{obs}$$

$$\left.\begin{aligned} A_x &= -\sin\alpha \\ A_y &= \cos\alpha \\ A_z &= 0 \end{aligned}\right\} \quad \underset{\sim}{A}_{obs}$$

$$\left.\begin{aligned} D_x &= -\sin\delta\cos\alpha \\ D_y &= -\sin\delta\sin\alpha \\ D_z &= \cos\delta \end{aligned}\right\} \underline{D}_{obs}$$

Compute $\underline{\Delta L} = \underline{L}_{obs} - \underline{L}_c$

Form the coefficients and compute the linear correction equations as in (5), substituting $\underline{A}_{obs}$ for $\underline{\widetilde{A}}_{obs}$ and $\underline{D}_{obs}$ for $\underline{\widetilde{D}}_{obs}$

(7) If $\dot{\rho}$, the slant range rate, is observed, compute:

$$\left.\begin{aligned} \dot{X} &= -Y\dot{\Theta} \\ \dot{Y} &= X\dot{\Theta} \\ \dot{Z} &= 0 \end{aligned}\right\} \underline{\dot{R}} \quad \text{where } \dot{\Theta} = 0.058\ 834\ 47$$

$$\underline{\dot{\rho}}_c = \underline{\dot{r}} + \underline{\dot{R}}$$

$$\dot{\rho}_c = \underline{L}_c \cdot \underline{\dot{\rho}}_c$$

$$ex_\omega = a\,(e\cos E - e^2)$$

$$ey_\omega = a\sqrt{1-e^2}\;e\sin E$$

$$\dot{v} = \frac{r\dot{v}}{r}$$

Compute the coefficients $\dot{R}$ and $\dot{U}$ where:

$$\dot{R}_u = \sqrt{\mu}\;a^{3/2}\;ex_\omega/r^3$$

$$\dot{R}_a = -\dot{r}/2 - 3/2\,(U - U_o)\,\dot{R}_u$$

$$\dot{R}_{xN} = (\sqrt{\mu}\, a^{5/2}/r^3) \left\{ \sin (E + \omega) - a_{xN}\, e \sin E - a_{yN} \right\}$$

$$\dot{R}_{yN} = (\sqrt{\mu}\, a^{5/2}/r^3) \left\{ - \cos (E + \omega) - a_{yN}\, e \sin E + a_{xN} \right\}$$

$$\dot{U}_u = -\sqrt{\mu}\, a^{3/2} e y_\omega / r^3$$

$$\dot{U}_a = \frac{-r\dot{v}}{2} - \frac{3}{2} (U - U_o)\, \dot{U}_u$$

$$\dot{U}_{xN} = (\sqrt{\mu}\, a^{5/2}/r^3) \sqrt{1 - e^2} \left\{ \cos (E + \omega) - a_{xN}\left(1 + \frac{r^2}{ap}\right) \right\}$$

$$\dot{U}_{yN} = (\sqrt{\mu}\, a^{5/2}/r^3) \sqrt{1 - e^2} \left\{ \sin (E + \omega) - a_{yN}\left(1 + \frac{r^2}{ap}\right) \right\}$$

(8) Form the coefficients:

$$C_{\frac{\Delta a}{a}} = (\underline{L}_c \cdot \underline{U})\left[\rho_c (\dot{R}_a - \dot{v}\, U_a) - \dot{\rho}_c R_a\right] + (\dot{\underline{\rho}}_c \cdot \underline{U})\, R_a$$

$$+ (\underline{L}_c \cdot \underline{V})\left[\rho_c \left(\dot{U}_a + \frac{\dot{r}}{r} U_a\right) - \dot{\rho}_c U_a\right] + (\dot{\underline{\rho}}_c \cdot \underline{V})\, U_a$$

$$C_{\Delta a_{xN}} = (\underline{L}_c \cdot \underline{U})\left[\rho_c (\dot{R}_{xN} - \dot{v}\, U_{xN}) - \dot{\rho}_c R_{xN}\right] + (\dot{\underline{\rho}}_c \cdot \underline{U})\, R_{xN}$$

$$+ (\underline{L}_c \cdot \underline{V})\left[\rho_c \left(\dot{U}_{xN} + \frac{\dot{r}}{r} U_{xN}\right) - \dot{\rho}_c U_{xN}\right] + (\dot{\underline{\rho}}_c \cdot \underline{V})\, U_{xN}$$

$$C_{\Delta a_{yN}} = (\underline{L}_c \cdot \underline{U})\left[\rho_c (\dot{R}_{yN} - \dot{v}\, U_{yN}) - \dot{\rho}_c R_{yN}\right] + (\dot{\underline{\rho}}_c \cdot \underline{U})\, R_{yN}$$

$$+ (\underline{L}_c \cdot \underline{V})\left[\rho_c \left(\dot{U}_{yN} + \frac{\dot{r}}{r} U_{yN}\right) - \dot{\rho}_c U_{yN}\right] + (\dot{\underline{\rho}}_c \cdot \underline{V})\, U_{yN}$$

$$C_{U_o} = (\underline{L}_c \cdot \underline{U})\left[\rho_c (\dot{R}_u - \dot{v}\, U_u) - \dot{\rho}_c R_u\right] + (\dot{\underline{\rho}}_c \cdot \underline{U})\, R_u$$

$$+ (\underline{L}_c \cdot \underline{V})\left[\rho_c \left(\dot{U}_u + \frac{\dot{r}}{r} U_u\right) - \dot{\rho}_c U_u\right] + (\dot{\underline{\rho}}_c \cdot \underline{V})\, U_u$$

$$C_{\Delta\Omega} = -(\underline{L}_c \cdot \underline{U})\rho_c r\dot{v} \cos i + (\underline{L}_c \cdot \underline{V}) \cos i\left[\rho_c \dot{r} - \dot{\rho}_c r\right]$$
$$+ (\dot{\underline{\rho}}_c \cdot \underline{V}) r \cos i + (\underline{L}_c \cdot \underline{W}) \sin i \left[\rho_c (r\dot{v} \sin u - \dot{v} \cos u)\right.$$
$$\left. + \dot{\rho}_c r \cos u\right] - (\dot{\underline{\rho}}_c \cdot \underline{W}) r \sin i \cos u$$

$$C_{\Delta i} = (\underline{L}_c \cdot \underline{W})\left[\rho_c (r\dot{v} \cos u + \dot{r} \sin u) - \dot{\rho}_c r \sin u\right]$$
$$+ (\dot{\underline{\rho}}_c \cdot \underline{W}) r \sin u$$

Compute $\Delta\dot{\rho} = \dot{\rho}_{obs} - \dot{\rho}_c$

Enter the following linear correction equation into the system of such equations:

$$\rho_c \Delta\dot{\rho} = C_{\frac{\Delta a}{a}} \frac{\Delta a_o}{a_o} + C_{\Delta a_{xN}} \Delta a_{xN_o} + C_{\Delta a_{yN}} \Delta a_{yN_o} + C_{\Delta U_o} \Delta U_o$$
$$+ C_{\Delta\Omega} \Delta\Omega_o + C_{\Delta i} \Delta i$$

b. <u>Computing the Corrected</u> $L_o$, $a_{xN_o}$, $a_{yN_o}$, $h_{x_o}$, $h_{y_o}$, $h_{z_o}$

Let $\sum_{j=1}^{N} C_{ij} \Delta X_j = \Delta 0_i$, $i = 1, 2, 3, \ldots$ represent all of the linear correction equations (i.e , the $C_{ij}$'s are the coefficients, the $\Delta X_j$'s are the corrections to the orbital parameters at time $t_o$, the $\Delta 0_i$ being corrected). The following matrix equation is solved to give the corrections, in a least squares sense, to the orbital parameters at time $t_o$.

$$\begin{bmatrix} \sum c^2_{i1} & \sum c_{i1} c_{i2} & \cdots & \sum c_{i1} c_{iN} \\ \sum c_{i1} c_{i2} & \sum c^2_{i2} & \cdots & \sum c_{i2} c_{iN} \\ \vdots & \vdots & & \vdots \\ \sum c_{i1} c_{iN} & \sum c_{i2} c_{iN} & & \sum c^2_{iN} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_N \end{bmatrix} = \begin{bmatrix} \sum c_{i1} \Delta 0_i \\ \sum c_{i2} \Delta 0_i \\ \\ \sum c_{iN} \Delta 0_i \end{bmatrix}$$

These corrections are applied as follows (a prime means that the element is a corrected element):

$$a'_{xN_o} = a_{xN_o} + \Delta a_{xN_o}$$

$$a'_{yN_o} = a_{yN_o} + \Delta a_{yN_o}$$

$$a'_o = a_o \left(1 + \frac{\Delta a_o}{a_o}\right)$$

$$\Omega'_o = \Omega_o + \Delta \Omega_o$$

$$i'_o = i_o + \Delta i_o \qquad 0 \leq i_o < \pi$$

$$L'_o = L_o + \Delta U_o - \Delta \Omega_o \text{ if } \cos i'_o < 0$$

$$L'_o = L_o + \Delta U_o + \Delta \Omega_o \text{ if } \cos i'_o > 0$$

$$\underline{W}'_o \begin{cases} W'_{x_o} = \sin i'_o \sin \Omega'_o \\ W'_{y_o} = -\sin i'_o \cos \Omega'_o \\ W'_{z_o} = \cos i'_o \end{cases}$$

$$e_o'^2 = a_{xN_o}'^2 + a_{yN_o}'^2$$

$$p_o' = a_o (1 - e_o'^2)$$

$$\underline{h}_o' = \sqrt{p_o'}\ \underline{W}_o'$$

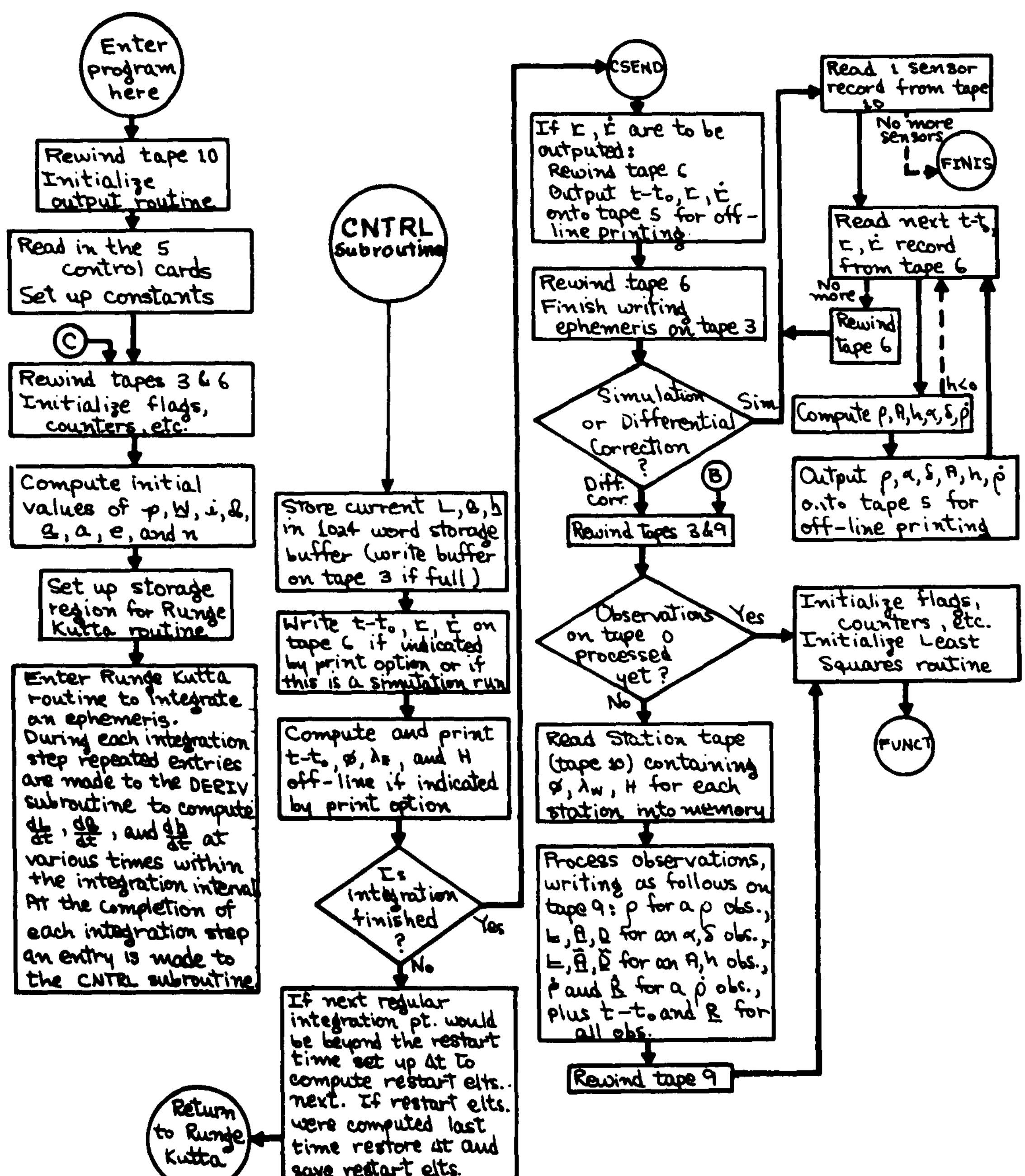
$$\underline{a}_o' \begin{cases} a_{x_o}' = - a_{yN_o}' \cos i_o' \sin \Omega_o' - a_{xN_o}' \cos \Omega_o' \\ a_{y_o}' = a_{yN_o}' \cos i_o' \cos \Omega_o' + a_{xN_o}' \sin \Omega_o' \\ a_{z_o}' = a_{yN_o}' \sin i_o' \end{cases}$$

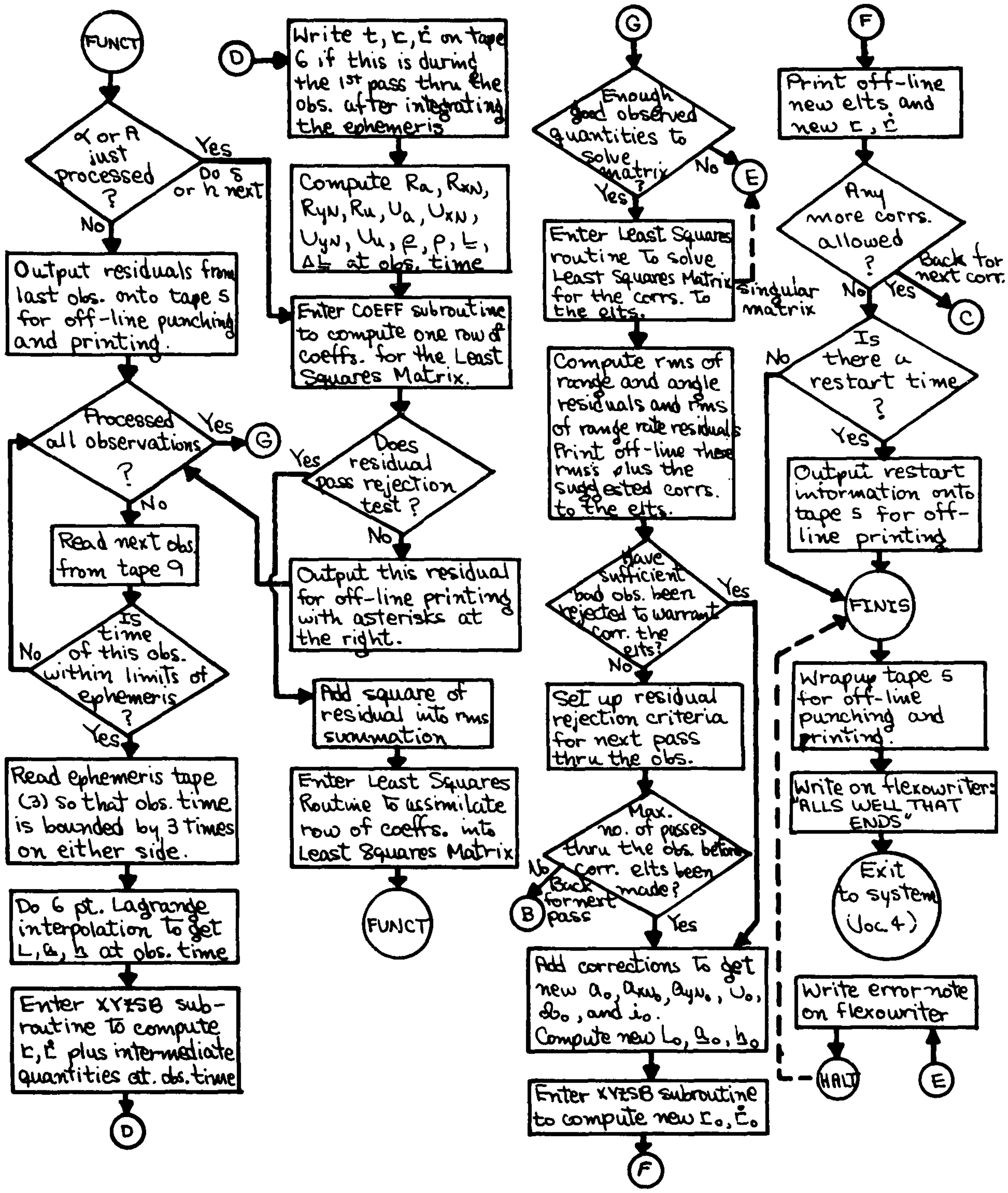
## 2.3 FLOW CHARTS

The following diagrams indicate the computational procedures in diagramatic form. The first two pages illustrate the general flow of computation. The succeeding pages contain a more detailed view of the program.

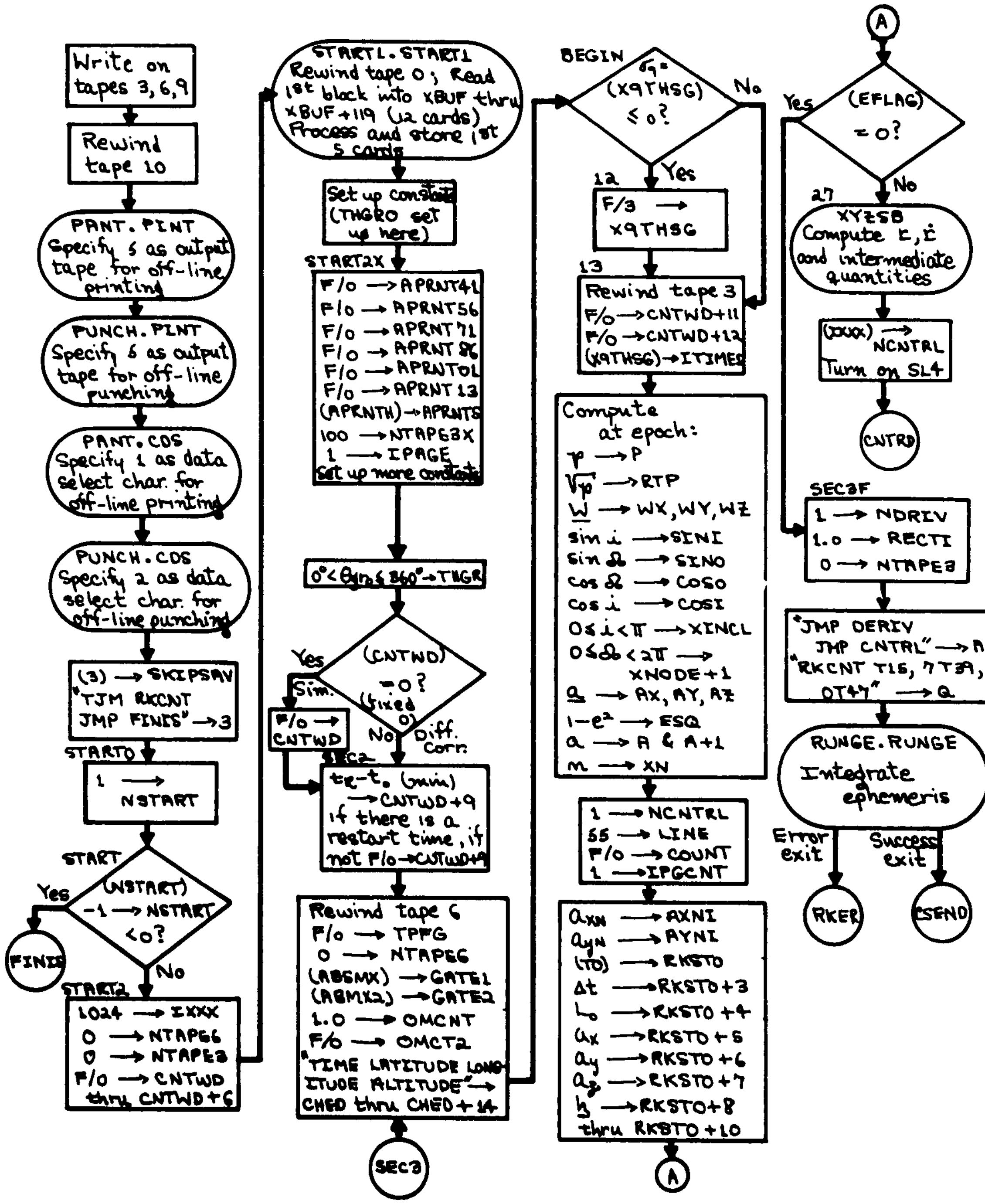
In the diagrams, ovals indicate subroutines of the program. They are identified by their symbolic location in the oval. The rectangle blocks contain other computations, bookkeeping instructions, etc. Circles indicate connectors. In the detailed flow the circles usually show the symbolic location names. The single letters do not correspond to actual program locations. Diamonds are used for logical decisions made in the program. The conditions causing the program to proceed in the different directions are given beside the appropriate arrow.

## GENERAL FLOWCHART OF ORBIT CORRECTION PROGRAM

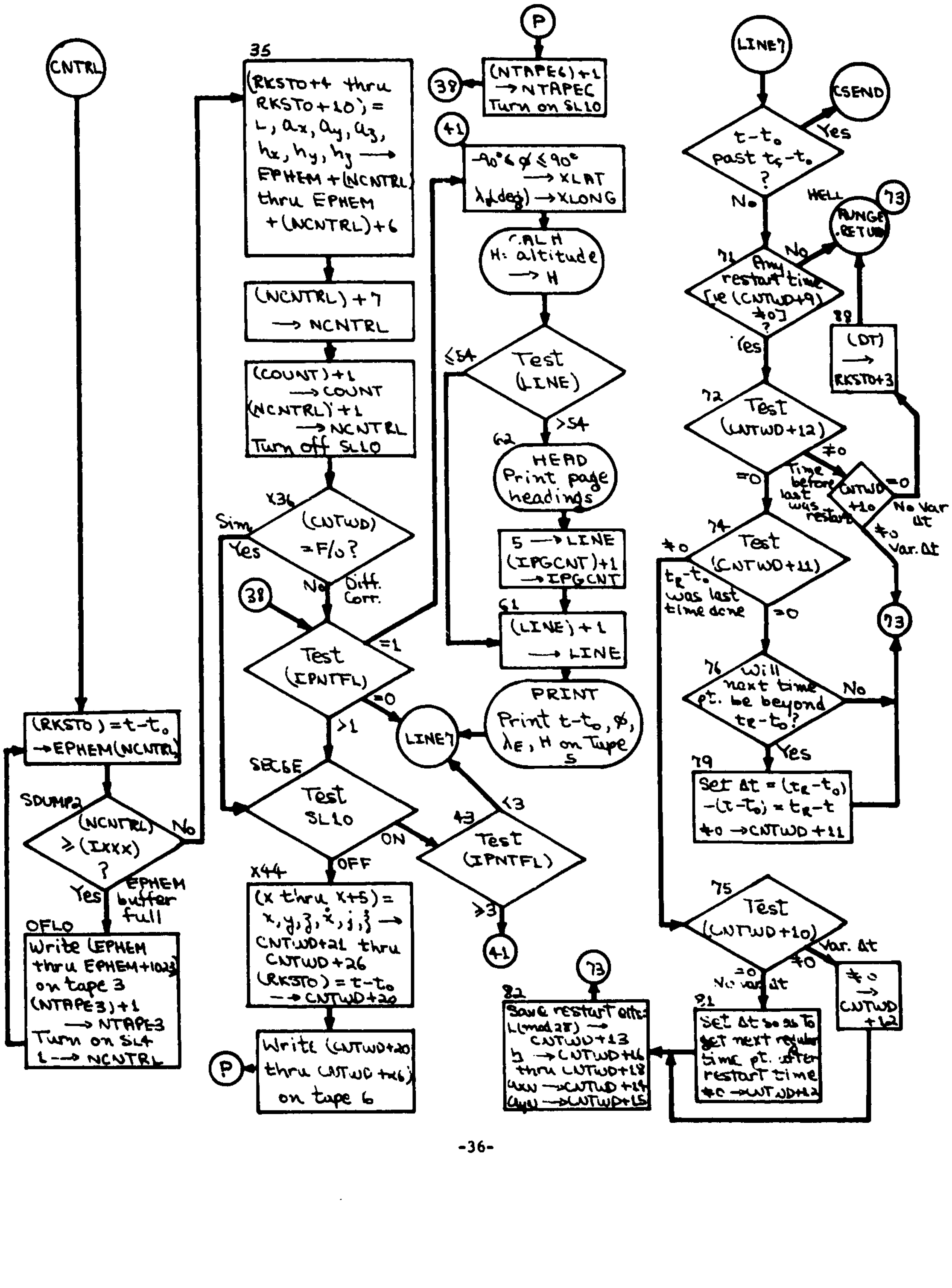


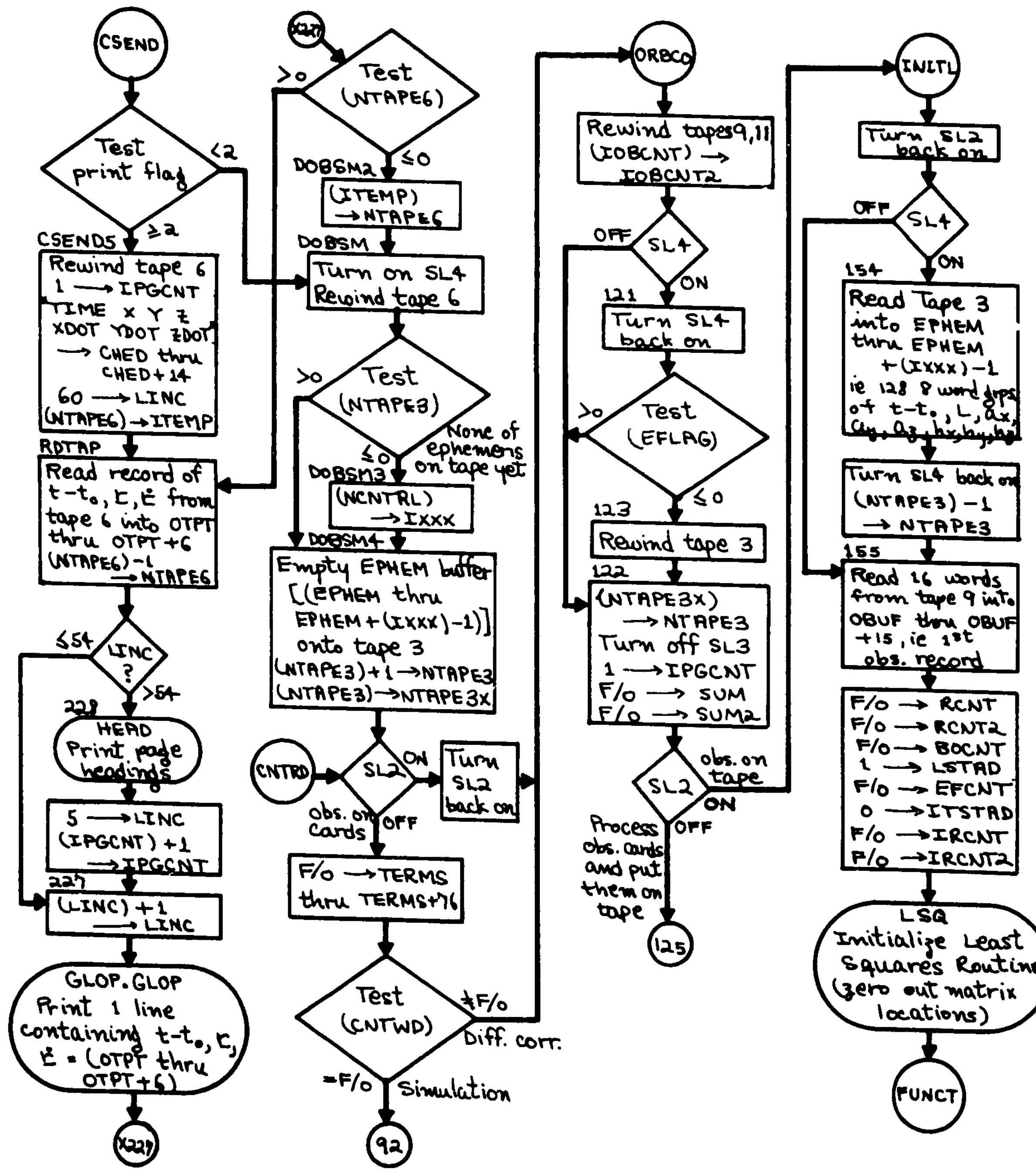

FUNCT
α or R just processed ?
Yes
Do S or h next
No
Output residuals from last obs. onto tape 5 for off-line punching and printing.
Processed all observations ?
Yes
G
No
Read next obs. from tape 9
Is time of this obs. within limits of ephemeris ?
No
Yes
Read ephemeris tape (3) so that obs. time is bounded by 3 times on either side.
Do 6 pt. Lagrange interpolation to get L, a, h at obs. time
Enter XYZSB sub-routine to compute r, ṙ plus intermediate quantities at obs. time
D
D
Write t, r, ṙ on tape 6 if this is during the 1st pass thru the obs. after integrating the ephemeris
Compute Ra, RxN, RyN, Ru, Ua, UxN, UyN, Uu, ρ, ρ̇, t, Δt at obs. time
Enter COEFF subroutine to compute one row of coeffs. for the Least Squares Matrix.
Does residual pass rejection test ?
Yes
No
Output this residual for off-line printing with asterisks at the right.
Add square of residual into rms summation
Enter Least Squares Routine to assimilate row of coeffs. into Least Squares Matrix
FUNCT
G
Enough good observed quantities to solve matrix ?
No
E
Yes
Enter Least Squares routine to solve Least Squares Matrix for the corrs. to the elts.
singular matrix
Compute rms of range and angle residuals and rms of range rate residuals. Print off-line these rms's plus the suggested corrs. to the elts.
Have sufficient bad obs. been rejected to warrant corr. the elts?
Yes
No
Set up residual rejection criteria for next pass thru the obs.
Max. no. of passes thru the obs. before corr. elts been made ?
No
Back for next pass
B
Yes
Add corrections to get new a₀, axN₀, ayN₀, u₀, Ω₀, and i₀. Compute new L₀, a₀, h₀
Enter XYZSB subroutine to compute new r₀, ṙ₀
F
F
Print off-line new elts and new r, ṙ
Any more corrs. allowed ?
Yes
Back for next corr.
C
No
Is there a restart time ?
No
Yes
Output restart information onto tape 5 for off-line printing
FINIS
Wrapup tape 5 for off-line punching and printing.
Write on flexowriter: "ALLS WELL THAT ENDS"
Exit to system (loc. 4)
Write error note on flexowriter
HALT
E

DETAILED FLOWCHART OF ORBIT CORRECTION PROGRAM



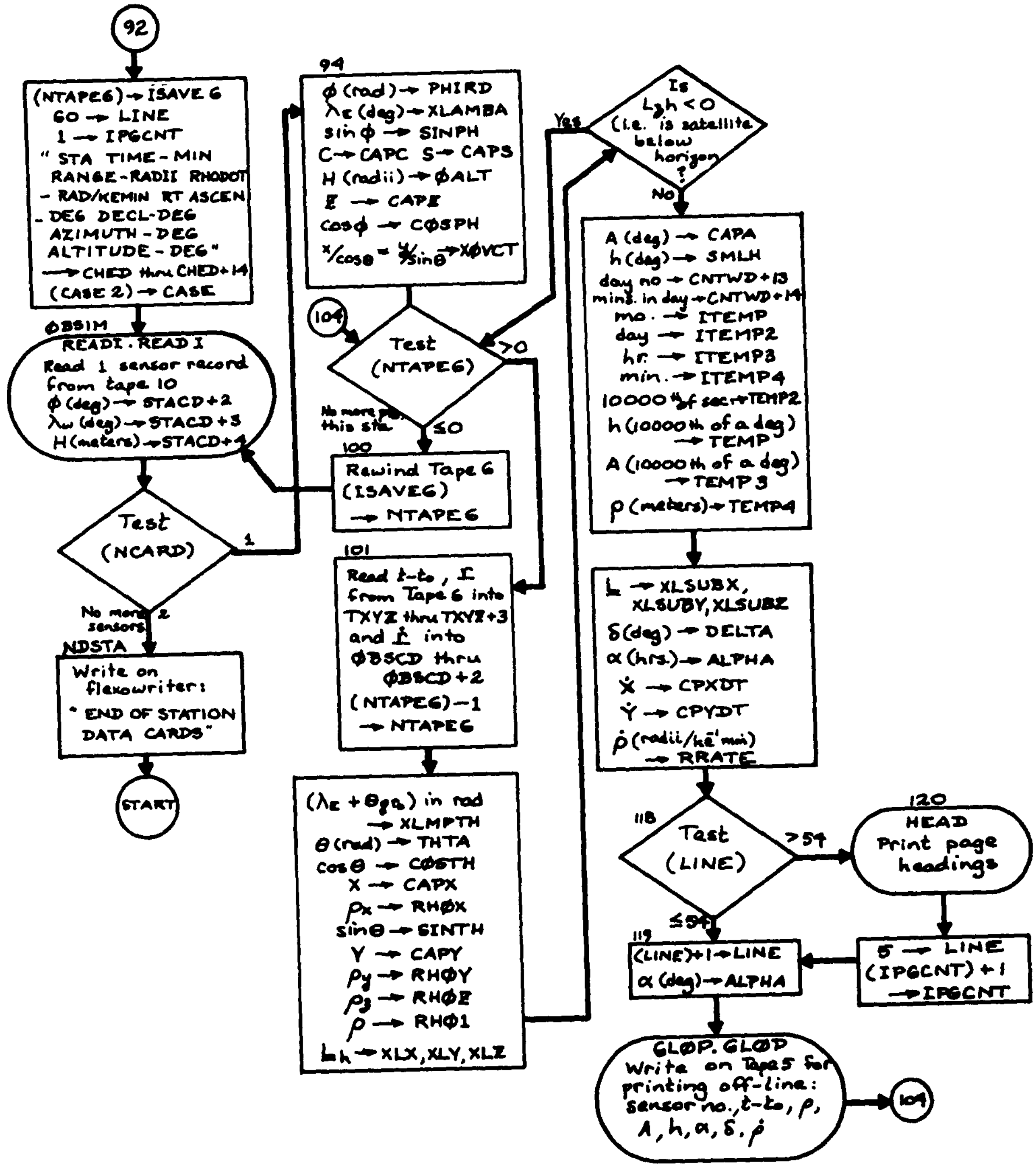
## CONTROL ROUTINE FOR EPHEMERIS INTEGRATION



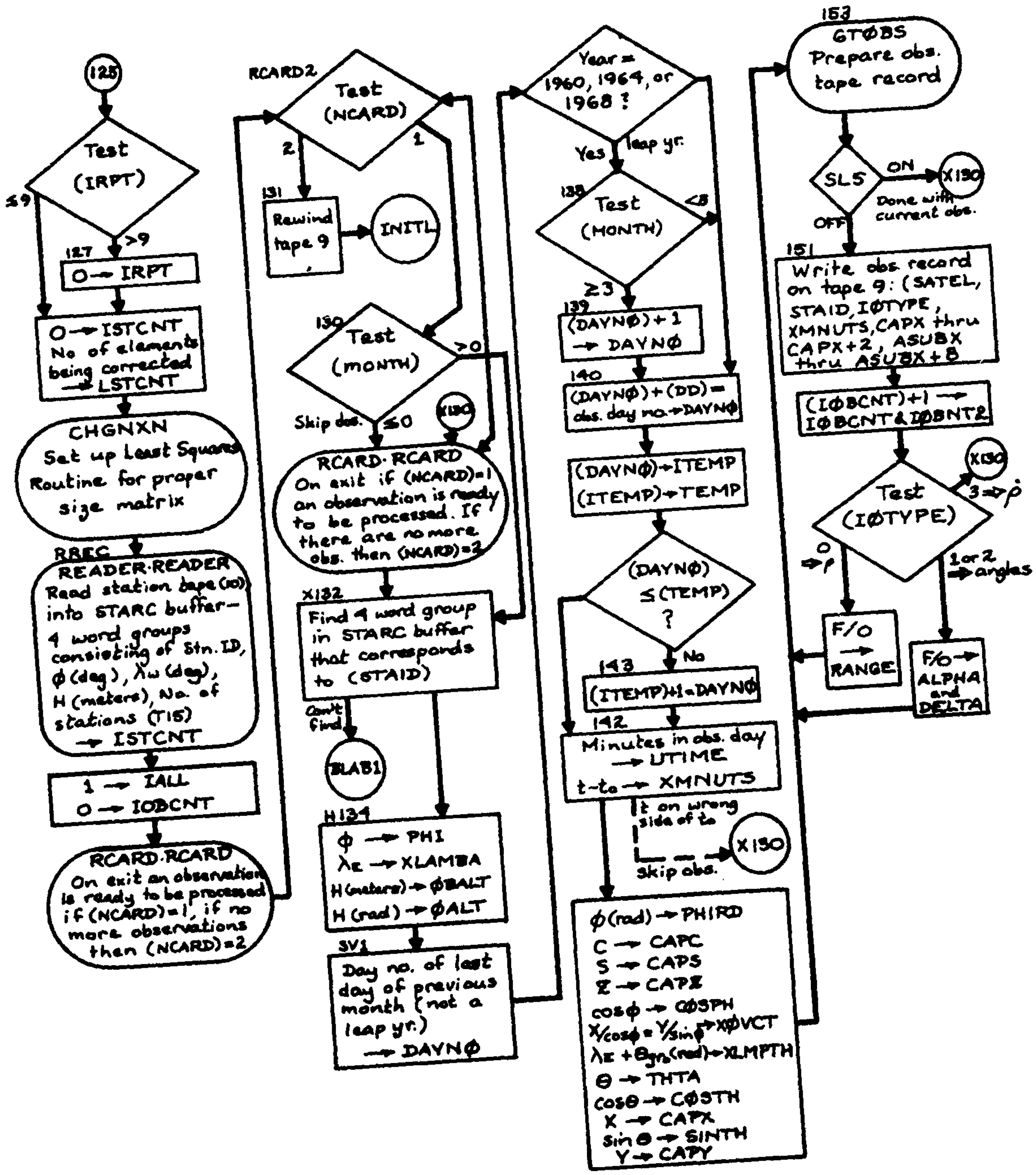


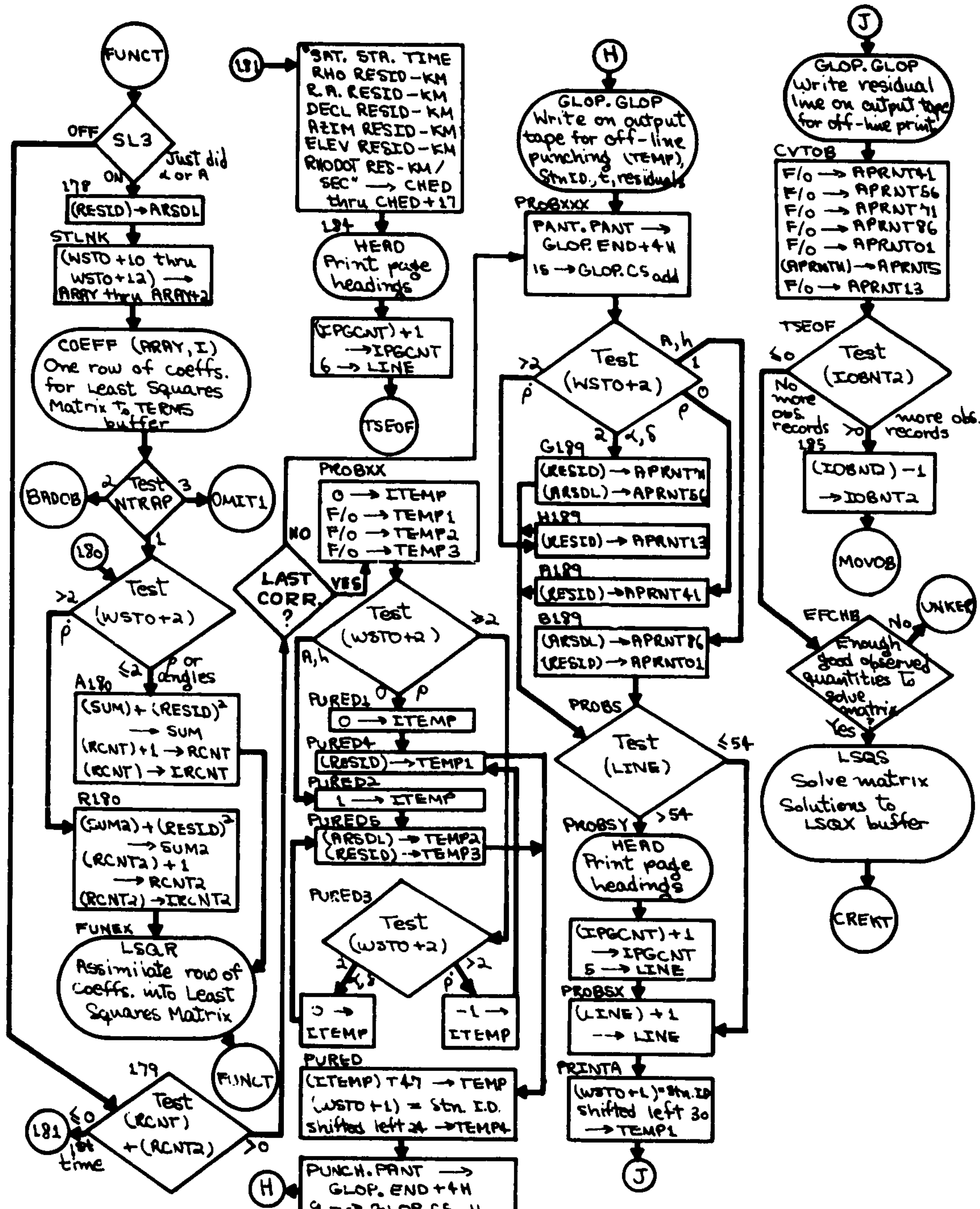
CSEND
Test print flag
<2
≥2
CSENDS
Rewind tape 6
1 → IPGCNT
TIME X Y Z XDOT YDOT ZDOT → CHED thru CHED+14
60 → LINC
(NTAPE6) → ITEMP
RDTAP
Read record of t-t₀, τ, τ̇ from tape 6 into OTPT thru OTPT+6
(NTAPE6)-1 → NTAPE6
LINC ?
≤54
>54
228
HEAD
Print page headings
5 → LINC
(IPGCNT)+1 → IPGCNT
227
(LINC)+1 → LINC
GLOP.GLOP
Print 1 line containing t-t₀, τ, τ̇ = (OTPT thru OTPT+6)
X227
X227
Test (NTAPE6)
>0
≤0
DOBSM2
(ITEMP) → NTAPE6
DOBSM
Turn on SL4
Rewind tape 6
Test (NTAPE3)
>0
≤0
None of ephemeris on tape yet
DOBSM3
(NCNTRL) → IXXX
DOBSM4
Empty EPHEM buffer [(EPHEM thru EPHEM+(IXXX)-1)] onto tape 3
(NTAPE3)+1 → NTAPE3
(NTAPE3) → NTAPE3X
CNTRD
SL2
ON
Turn SL2 back on
obs. on cards
OFF
F/0 → TERMS thru TERMS+76
Test (CNTWD)
≠F/0
Diff. corr.
=F/0
Simulation
92
ORBCO
Rewind tapes 9, 11
(IOBCNT) → IOBCNT2
SL4
OFF
ON
121
Turn SL4 back on
Test (EFLAG)
>0
≤0
123
Rewind tape 3
122
(NTAPE3X) → NTAPE3
Turn off SL3
1 → IPGCNT
F/0 → SUM
F/0 → SUM2
SL2
obs. on tape
ON
OFF
Process obs. cards and put them on tape
125
INITL
Turn SL2 back on
SL4
OFF
ON
154
Read Tape 3 into EPHEM thru EPHEM +(IXXX)-1
ie 128 8 word grps of t-t₀, L, ax, ay, az, bx, by, bz
Turn SL4 back on
(NTAPE3)-1 → NTAPE3
155
Read 16 words from tape 9 into OBUF thru OBUF+15, ie 1st obs. record
F/0 → RCNT
F/0 → RCNT2
F/0 → BOCNT
1 → LSTAD
F/0 → EFCNT
0 → ITSTAD
F/0 → IRCNT
F/0 → IRCNT2
LSQ
Initialize Least Squares Routine (zero out matrix locations)
FUNCT

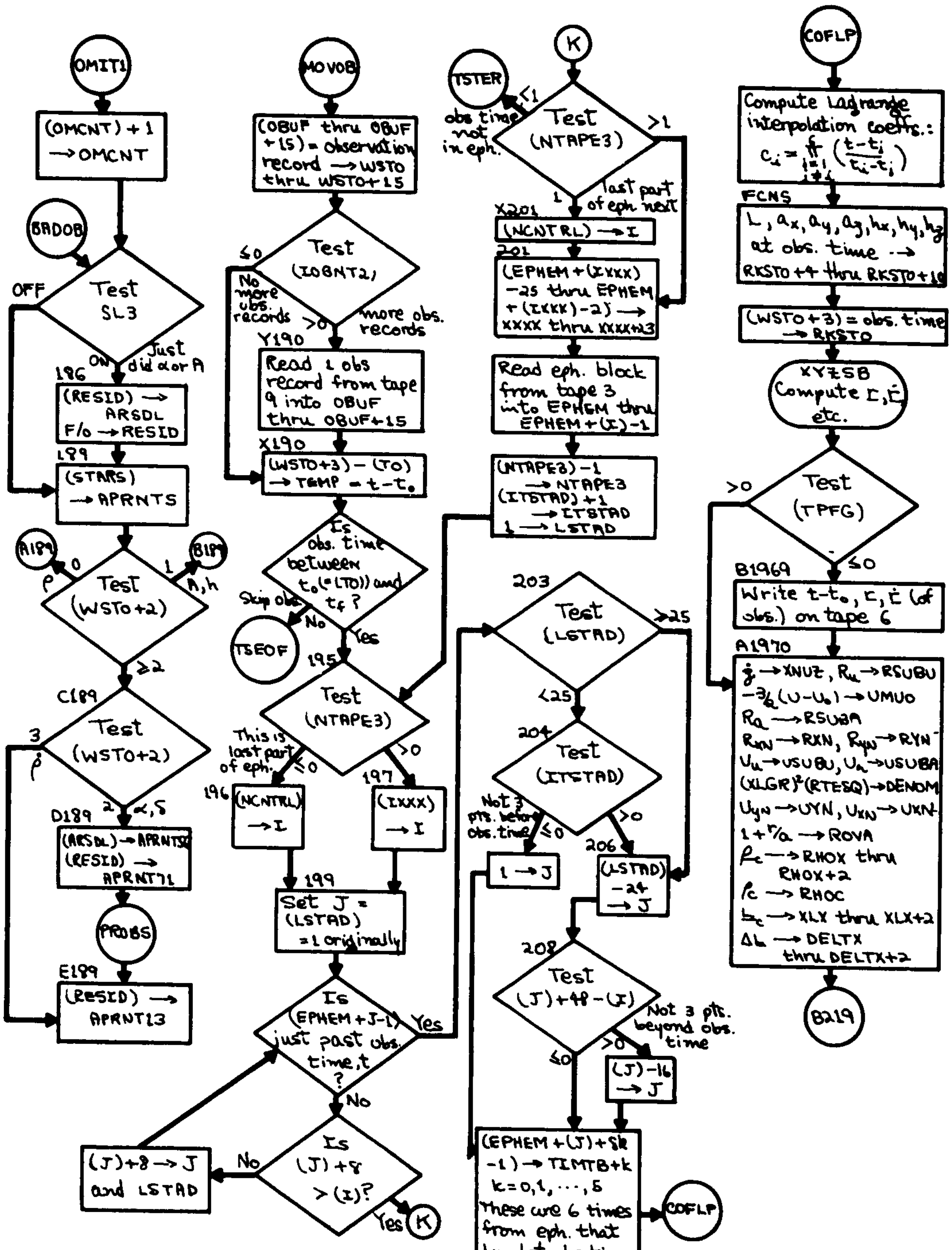
## SIMULATION

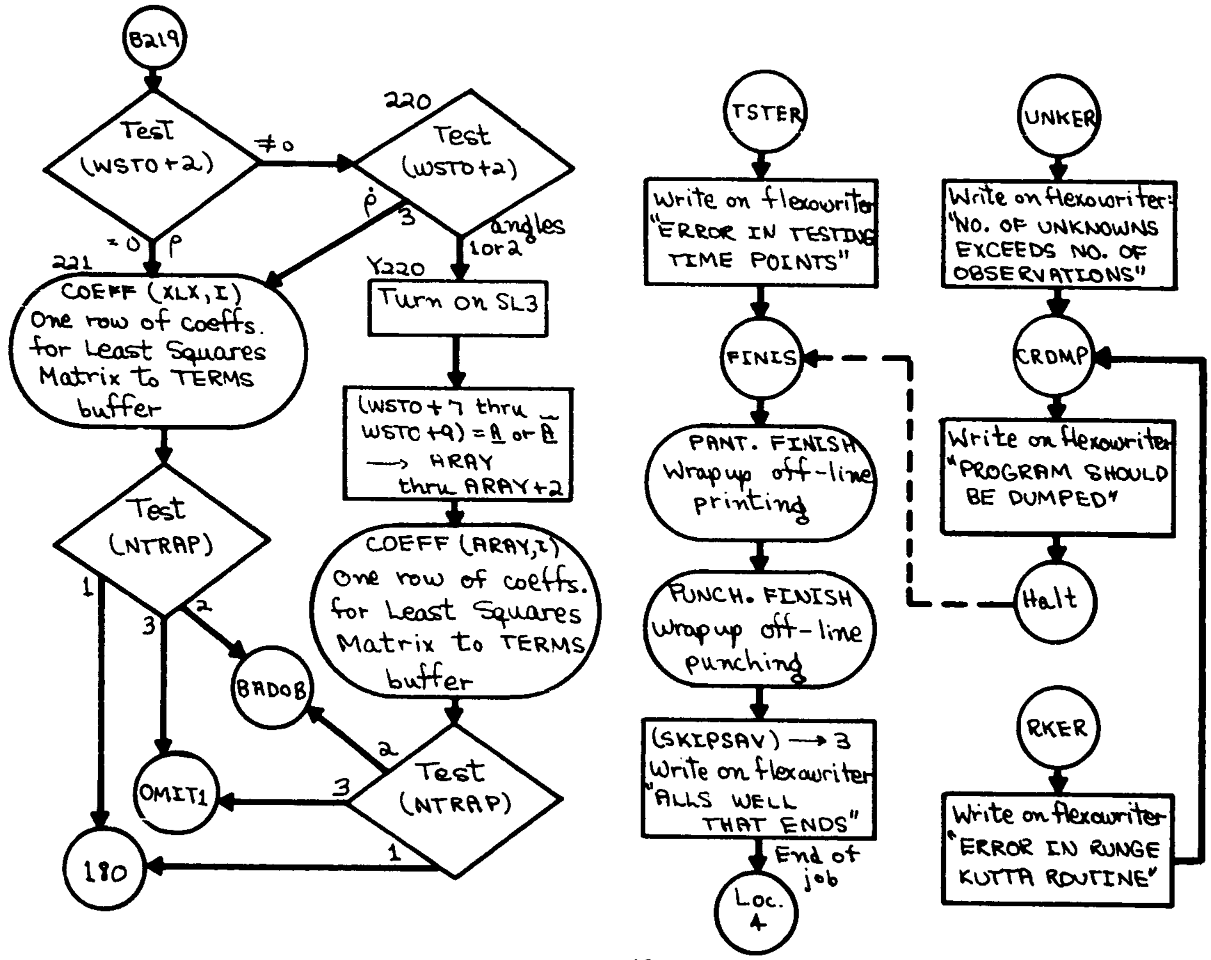

92
(NTAPE6) → ISAVE 6
60 → LINE
1 → IPGCNT
" STA TIME-MIN RANGE-RADII RHODOT - RAD/KEMIN RT ASCEN - DEG DECL-DEG AZIMUTH - DEG ALTITUDE - DEG "
→ CHED thru CHED+14
(CASE 2) → CASE
ØBSIM
READI. READ I
Read 1 sensor record from tape 10
φ (deg) → STACD+2
λw (deg) → STACD+3
H (meters) → STACD+4
Test (NCARD)
1
No more sensors
2
NDSTA
Write on flexowriter: " END OF STATION DATA CARDS "
START
94
φ (rad) → PHIRD
λE (deg) → XLAMBA
sin φ → SINPH
C → CAPC S → CAPS
H (radii) → ØALT
E → CAPE
cos φ → CØSPH
x/cosθ = y/sinθ → XØVCT
104
Test (NTAPE6)
>0
No more pass this sta.
≤0
100
Rewind Tape 6 (ISAVE6) → NTAPE6
101
Read t-to, r from Tape 6 into TXYZ thru TXYZ+3 and r into ØBSCD thru ØBSCD+2
(NTAPE6)−1 → NTAPE6
(λE + θgo) in rad → XLMPTH
θ (rad) → THTA
cos θ → CØSTH
X → CAPX
ρx → RHØX
sin θ → SINTH
Y → CAPY
ρy → RHØY
ρz → RHØZ
ρ → RHØ1
Lh → XLX, XLY, XLZ
Is Lzh < 0 (i.e. is satellite below horizon ?
Yes
No
A (deg) → CAPA
h (deg) → SMLH
day no → CNTWD+13
mins. in day → CNTWD+14
mo. → ITEMP
day → ITEMP2
hr. → ITEMP3
min. → ITEMP4
10000ths of sec → TEMP2
h (10000th of a deg) → TEMP
A (10000th of a deg) → TEMP3
ρ (meters) → TEMP4
L → XLSUBX, XLSUBY, XLSUBZ
δ (deg) → DELTA
α (hrs.) → ALPHA
Ẋ → CPXDT
Ẏ → CPYDT
ρ̇ (radii/ke⁻¹min) → RRATE
118
Test (LINE)
>54
≤54
120
HEAD
Print page headings
119
(LINE)+1 → LINE
α (deg) → ALPHA
5 → LINE
(IPGCNT)+1 → IPGCNT
GLØP. GLØP
Write on Tape5 for printing off-line: sensor no., t-to, ρ, A, h, α, δ, ρ̇
104

# PREPARE OBSERVATION TAPE

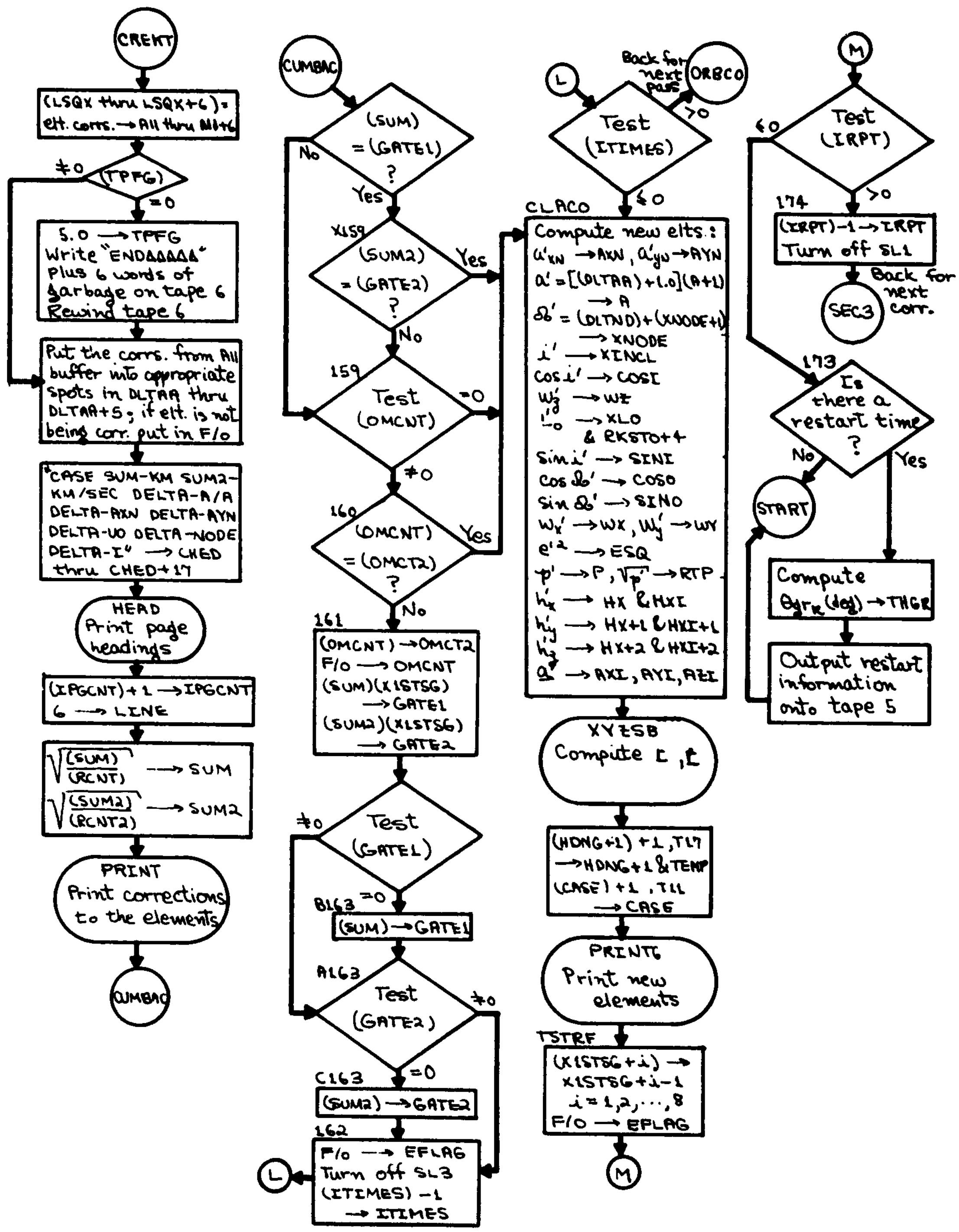



FUNCT
SL3
OFF
ON
Just did α or A
178
(RESID) → ARSOL
STLNK
(WSTO+10 thru WSTO+12) → ARRY thru ARRY+2
COEFF (ARRY, I) One row of coeffs. for Least Squares Matrix to TERMS buffer
BADOB
Test NTRAP
OMIT1
180
Test (WSTO+2)
A180
(SUM) + (RESID)² → SUM (RCNT)+1 → RCNT (RCNT) → IRCNT
R180
(SUM2) + (RESID)² → SUM2 (RCNT2) + 1 → RCNT2 (RCNT2) → IRCNT2
FUNEX
LSQR Assimilate row of coeffs. into Least Squares Matrix
FUNCT
179
Test (RCNT) + (RCNT2)
181
1st time
181
SAT. STA. TIME RHO RESID-KM R.A. RESID-KM DECL RESID-KM AZIM RESID-KM ELEV RESID-KM RHODOT RES-KM/SEC" → CHED thru CHED+17
184
HEAD Print page headings
(IPGCNT)+1 → IPGCNT 6 → LINE
TSEOF
PROBXX
0 → ITEMP F/0 → TEMP1 F/0 → TEMP2 F/0 → TEMP3
LAST CORR. ?
NO
YES
Test (WSTO+2)
PURED1
0 → ITEMP
PURED4
(RESID) → TEMP1
PURED2
1 → ITEMP
PURED5
(ARSDL) → TEMP2 (RESID) → TEMP3
PURED3
Test (WSTO+2)
0 → ITEMP
-1 → ITEMP
PURED
(ITEMP) T47 → TEMP (WSTO+1) = Stn. I.D. shifted left 24 → TEMP4
H
PUNCH. PANT → GLOP. END+4H 9 → GLOP.CS add.
H
GLOP. GLOP Write on output tape for off-line punching (TEMP), Stn.ID., ε, residuals
PROBXXX
PANT. PANT → GLOP. END+4H 15 → GLOP.CS add
Test (WSTO+2)
G189
(RESID) → APRNT11 (ARSDL) → APRNT56
H189
(RESID) → APRNT13
A189
(RESID) → APRNT41
B189
(ARSDL) → APRNT86 (RESID) → APRNT01
PROBS
Test (LINE)
PROBSY
HEAD Print page headings
(IPGCNT)+1 → IPGCNT 5 → LINE
PROBSX
(LINE)+1 → LINE
PRINTA
(WSTO+1) = Stn. ID shifted left 30 → TEMP1
J
J
GLOP. GLOP Write residual line on output tape for off-line print
CVTOB
F/0 → APRNT41 F/0 → APRNT56 F/0 → APRNT71 F/0 → APRNT86 F/0 → APRNT01 (APRNT11) → APRNT5 F/0 → APRNT13
TSEOF
Test (IOBNT2)
No more obs. records
more obs. records
185
(IOBNT2) - 1 → IOBNT2
MOVOB
EFCHB
Enough good observed quantities to solve matrix
No
Yes
UNKEP
LSQS Solve matrix Solutions to LSQX buffer
CREKT

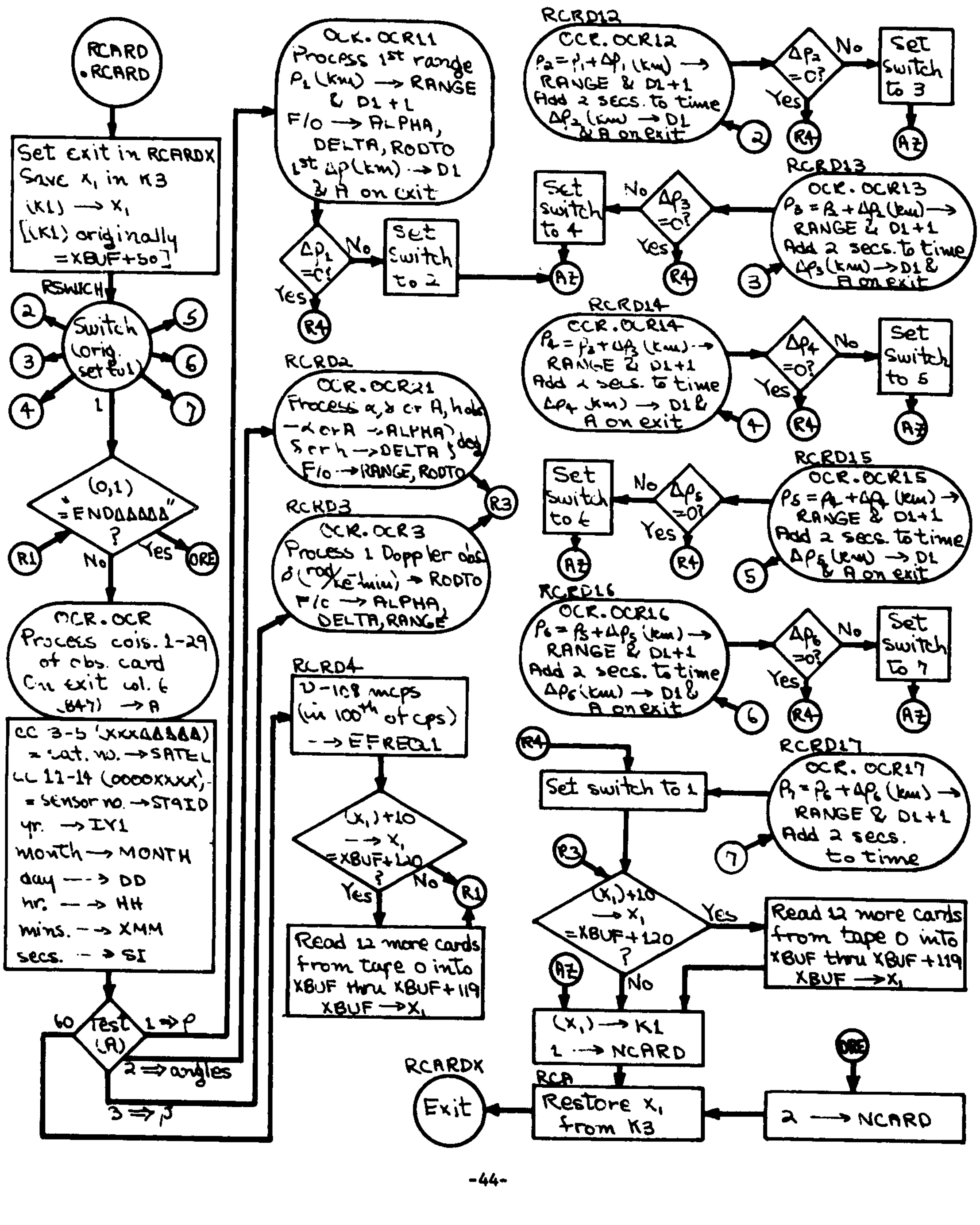

OMIT1
(OMCNT) + 1 → OMCNT
BADOB
Test SL3
OFF
ON
Just did α or A
196
(RESID) → ARSDL
F/o → RESID
199
(STARS) → APRNTS
A189
B189
Test (WSTo+2)
0
1
ρ
A,h
≥2
C189
Test (WSTo+2)
3
ρ̇
2
α,δ
D189
(ARSDL) → APRNTS
(RESID) → APRNT71
PROBS
E189
(RESID) → APRNT13
MOVOB
(OBUF thru OBUF+15) = observation record → WSTO thru WSTO+15
Test (IOBNT2)
≤0
No more obs. records
>0
more obs. records
Y190
Read 1 obs record from tape 9 into OBUF thru OBUF+15
X190
(WSTO+3) − (TO) → TEMP = t−t₀
Is obs. time between t₀(=(TO)) and t_f ?
Skip obs.
No
Yes
TSEOF
195
Test (NTAPE3)
This is last part of eph.
≤0
>0
196
(NCNTRL) → I
197
(IXXX) → I
199
Set J = (LSTAD) = 1 originally
Is (EPHEM+J−1) just past obs. time, t ?
Yes
No
Is (J)+8 > (I)?
No
Yes
(J)+8 → J and LSTAD
K
TSTER
K
Test (NTAPE3)
<1
obs time not in eph.
>1
1
last part of eph next
X201
(NCNTRL) → I
201
(EPHEM+(IXXX) −25 thru EPHEM+(IXXX)−2) → XXXX thru XXXX+23
Read eph. block from tape 3 into EPHEM thru EPHEM+(I)−1
(NTAPE3)−1 → NTAPE3
(ITSTAD)+1 → ITSTAD
1 → LSTAD
203
Test (LSTAD)
>25
<25
204
Test (ITSTAD)
Not 3 pts. before obs. time
≤0
>0
206
1 → J
(LSTAD) −24 → J
208
Test (J)+48−(I)
Not 3 pts. beyond obs. time
≤0
>0
(J)−16 → J
(EPHEM+(J)+8k−1) → TIMTB+k
k = 0,1,···,5
These are 6 times from eph. that bracket obs. time
COFLP
COFLP
Compute Lagrange interpolation coeffs.:
$c_i = \prod_{j=1, j\neq i} \left(\frac{t-t_j}{t_i-t_j}\right)$
FCNS
L, aₓ, a_y, a_z, hₓ, h_y, h_z at obs. time → RKSTO+4 thru RKSTO+10
(WSTO+3) = obs. time → RKSTO
XYZSB
Compute r, ṙ etc.
Test (TPFG)
>0
≤0
B1969
Write t−t₀, r, ṙ (of obs.) on tape 6
A1970
ġ → XNUZ, Rᵤ → RSUBU
−³⁄₂(U−U₀) → UMUO
Rₐ → RSUBA
R_xN → RXN, R_yN → RYN
Uᵤ → USUBU, Uₐ → USUBA
(XLGR)²(RTESQ) → DENOM
U_yN → UYN, U_xN → UXN
1+r/a → ROVA
ρ_c → RHOX thru RHOX+2
ρ_c → RHOC
L_c → XLX thru XLX+2
ΔL → DELTX thru DELTX+2
B219



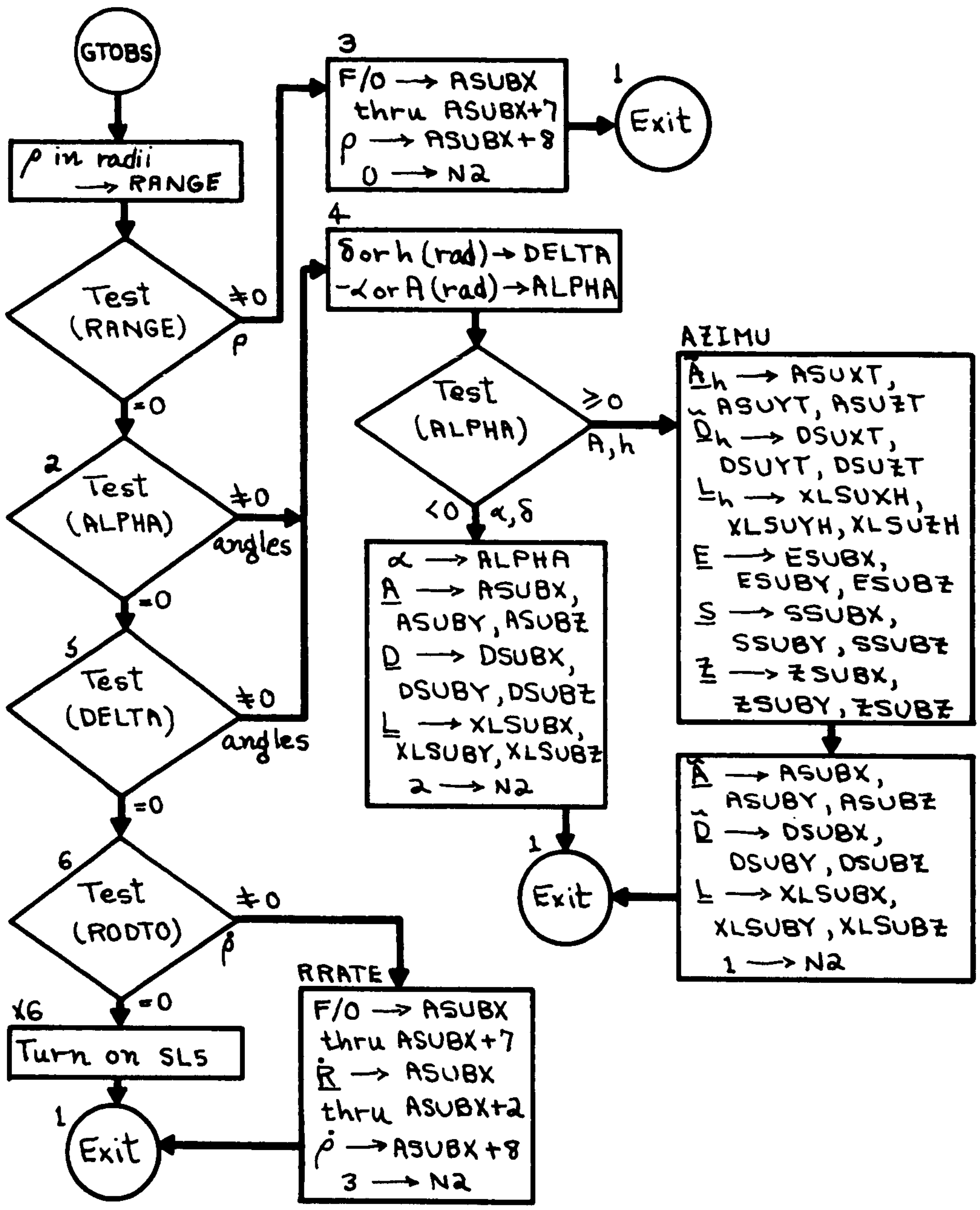
B219
Test (WSTO+2)
≠0
=0
ρ
220
Test (WSTO+2)
ρ̇
3
angles 1 or 2
221
COEFF (XLX,I) One row of coeffs. for Least Squares Matrix to TERMS buffer
Y220
Turn on SL3
(WSTO+7 thru WSTO+9) = A or Ā → ARAY thru ARAY+2
COEFF (ARAY,I) One row of coeffs. for Least Squares Matrix to TERMS buffer
Test (NTRAP)
1
2
3
BADOB
OMIT1
180
Test (NTRAP)
TSTER
Write on flexowriter "ERROR IN TESTING TIME POINTS"
FINIS
PANT. FINISH Wrap up off-line printing
PUNCH. FINISH Wrap up off-line punching
(SKIPSAV) → 3 Write on flexowriter "ALLS WELL THAT ENDS"
End of job
Loc. 4
UNKER
Write on flexowriter: "NO. OF UNKNOWNS EXCEEDS NO. OF OBSERVATIONS"
CROMP
Write on flexowriter "PROGRAM SHOULD BE DUMPED"
Halt
RKER
Write on flexowriter "ERROR IN RUNGE KUTTA ROUTINE"

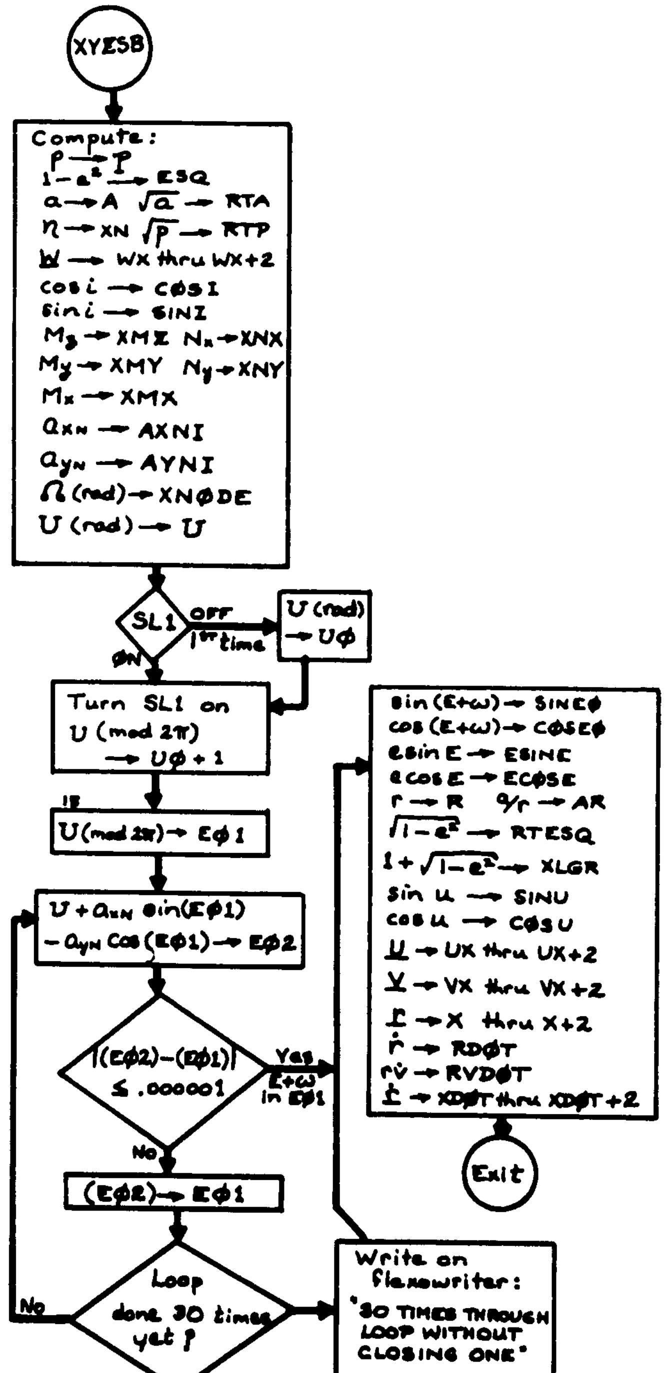

CREKT
(LSQX thru LSQX+6)= elt. corrs. → A11 thru A11+6
(TPFG)
≠0
=0
5.0 → TPFG
Write "ENDΔΔΔΔΔ" Plus 6 words of garbage on tape 6
Rewind tape 6
Put the corrs. from A11 buffer into appropriate spots in DLTAA thru DLTAA+5; if elt. is not being corr. put in F/0
"CASE SUM-KM SUM2-KM/SEC DELTA-A/A DELTA-AXN DELTA-AYN DELTA-U0 DELTA-NODE DELTA-I" → CHED thru CHED+17
HEAD
Print page headings
(IPGCNT)+1 → IPGCNT
6 → LINE
√((SUM)/(RCNT)) → SUM
√((SUM2)/(RCNT2)) → SUM2
PRINT
Print corrections to the elements
CUMBAC
CUMBAC
(SUM) = (GATE1) ?
No
Yes
X159
(SUM2) = (GATE2) ?
Yes
No
159
Test (OMCNT)
=0
≠0
160
(OMCNT) = (OMCT2) ?
Yes
No
161
(OMCNT) → OMCT2
F/0 → OMCNT
(SUM)(X1STSG) → GATE1
(SUM2)(X1STSG) → GATE2
Test (GATE1)
≠0
B163
=0
(SUM) → GATE1
A163
Test (GATE2)
≠0
C163
=0
(SUM2) → GATE2
162
F/0 → EFLAG
Turn off SL3
(ITIMES) − 1 → ITIMES
L
L
Back for next pass
ORBCO
Test (ITIMES)
>0
≤0
CLACO
Compute new elts:
a'xn → AXN, a'yn → AYN
a' = [(DLTAA)+1.0](A+1) → A
☊' = (DLTND)+(XNODE+1) → XNODE
i' → XINCL
cos i' → COSI
W'z → WZ
l'o → XLO & RKSTO+4
Sin i' → SINI
Cos ☊' → COSO
Sin ☊' → SINO
W'x → WX, W'y → WY
e'² → ESQ
p' → P, √p' → RTP
h'x → HX & HXI
h'y → HX+1 & HXI+1
h'z → HX+2 & HXI+2
a' → AXI, AYI, AZI
XYZSB
Compute r, ṙ
(HDNG+1)+1, T17 → HDNG+1 & TEMP
(CASE)+1, T11 → CASE
PRINTG
Print new elements
TSTRF
(X1STSG+i) → X1STSG+i−1
i = 1, 2, ..., 8
F/0 → EFLAG
M
M
Test (IRPT)
≤0
>0
174
(IRPT)−1 → IRPT
Turn off SL1
Back for next corr.
SEC3
173
Is there a restart time ?
No
Yes
START
Compute θgrK (deg) → THGR
Output restart information onto tape 5

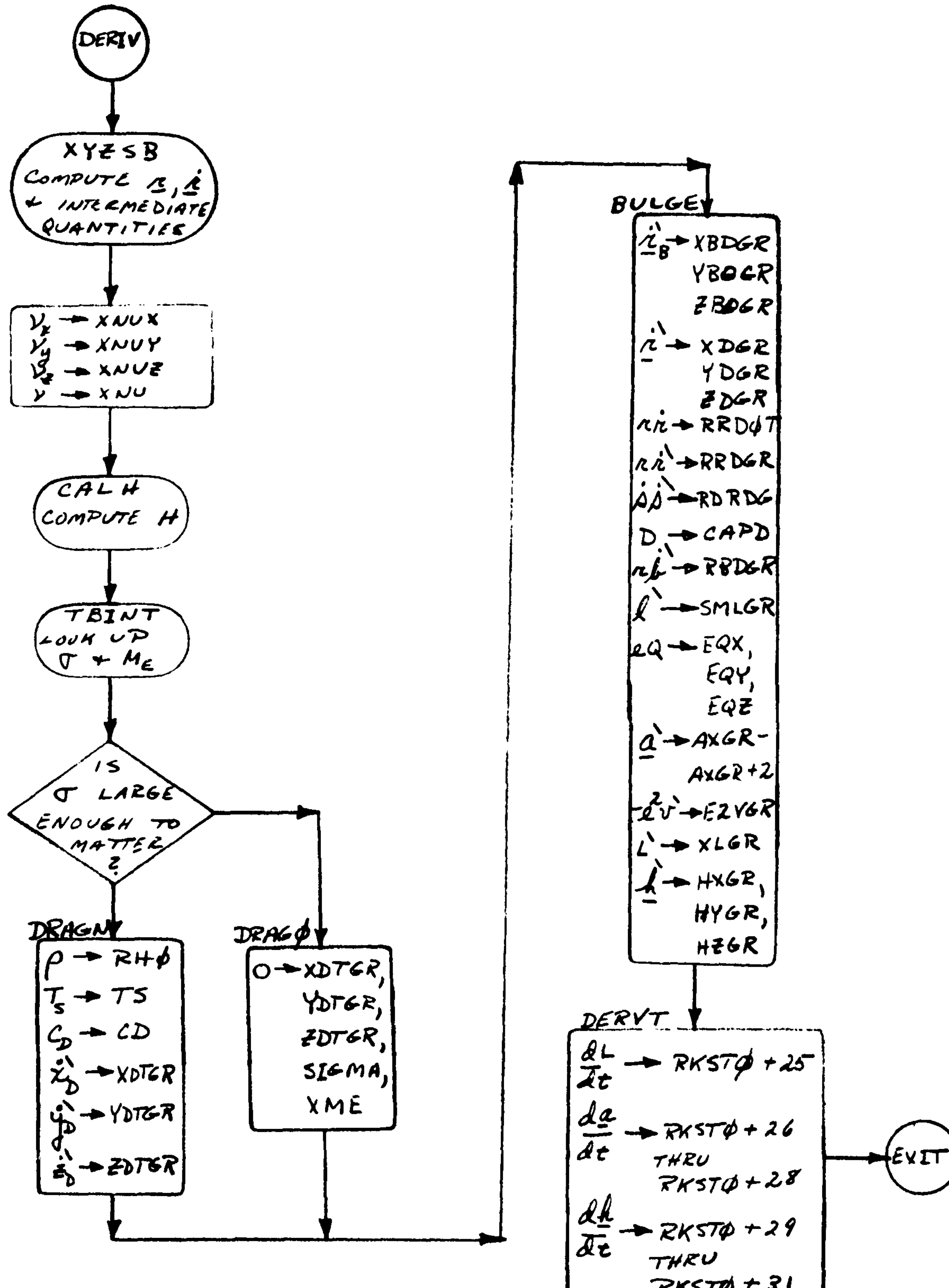
ROUTINE TO READ OBSERVATION CARDS



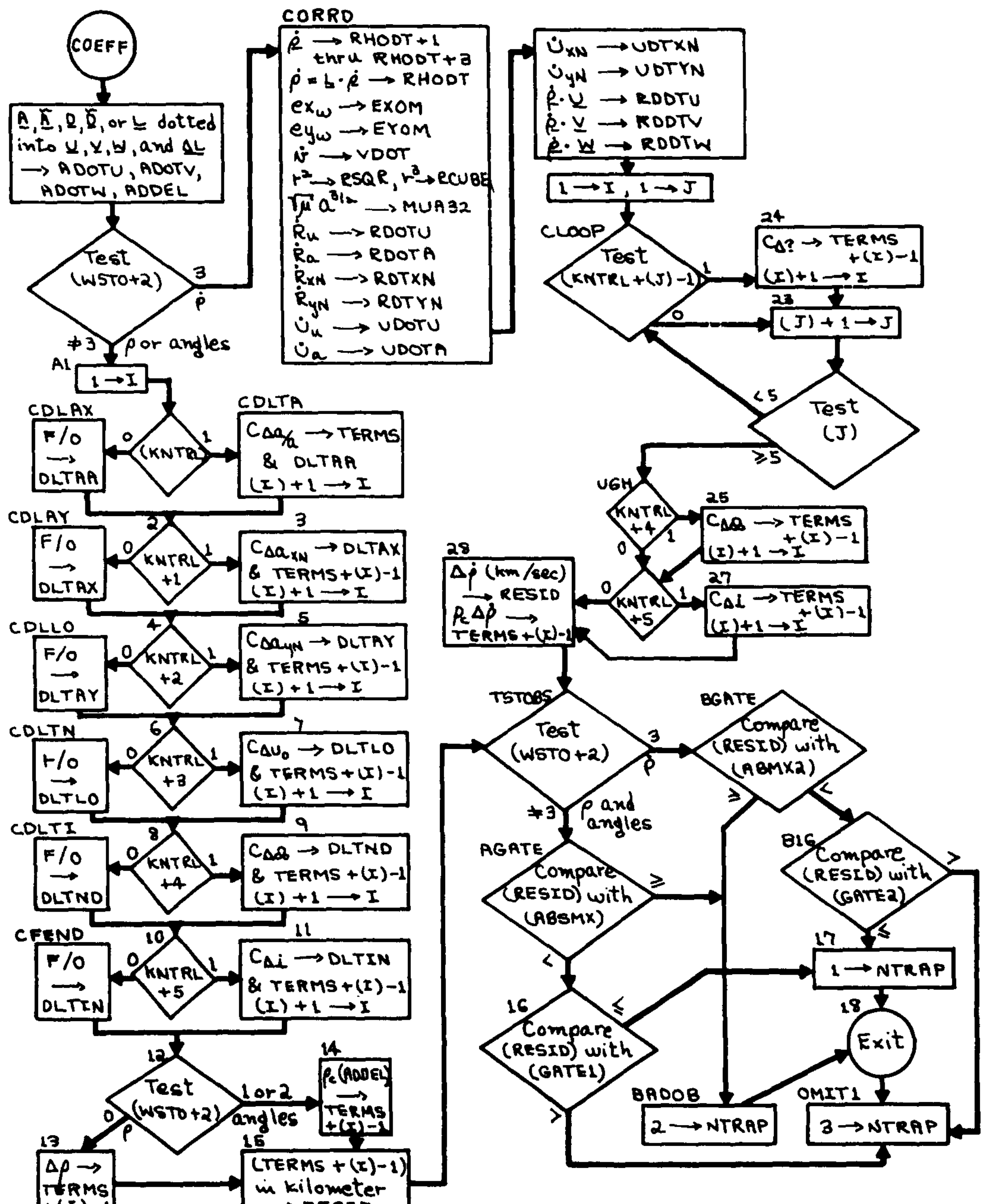
## ROUTINE TO PREPARE OBSERVATION RECORD




XYESB
Compute:
p → P
1−e² → ESQ
a → A √a → RTA
n → XN √p → RTP
W → WX thru WX+2
cos i → CØSI
sin i → SINI
Mz → XMZ Nx → XNX
My → XMY Ny → XNY
Mx → XMX
axN → AXNI
ayN → AYNI
Ω (rad) → XNØDE
U (rad) → U
SL1
OFF
1st time
U (rad) → UØ
ØN
Turn SL1 on
U (mod 2π) → UØ+1
U (mod 2π) → EØ1
U + axN sin(EØ1) − ayN cos(EØ1) → EØ2
|(EØ2)−(EØ1)| ≤ .000001
Yes
E+ω in EØ1
No
(EØ2) → EØ1
Loop done 30 times yet?
No
Write on flexowriter: "30 TIMES THROUGH LOOP WITHOUT CLOSING ONE"
sin (E+ω) → SINEØ
cos (E+ω) → CØSEØ
esin E → ESINE
ecos E → ECØSE
r → R a/r → AR
√1−e² → RTESQ
1+√1−e² → XLGR
sin u → SINU
cos u → CØSU
U → UX thru UX+2
V → VX thru VX+2
r → X thru X+2
ṙ → RDØT
rv̇ → RVDØT
ṙ → XDØT thru XDØT+2
Exit


DERIV
XYZSB
COMPUTE r, ṙ + INTERMEDIATE QUANTITIES
ν_x → XNUX
ν_y → XNUY
ν_z → XNUZ
ν → XNU
CALH
COMPUTE H
TBINT
LOOK UP σ + M_E
IS σ LARGE ENOUGH TO MATTER ?
DRAGN
ρ → RHØ
T_s → TS
C_D → CD
ẋ_D → XDTGR
ẏ_D → YDTGR
ż_D → ZDTGR
DRAGØ
0 → XDTGR, YDTGR, ZDTGR, SIGMA, XME
BULGE
ṙ_B → XBDGR YBDGR ZBDGR
ṙ → XDGR YDGR ZDGR
rṙ → RRDØT
rṙ → RRDGR
ρ̇ρ → RDRDG
D → CAPD
rρ̇ → RBDGR
ℓ → SMLGR
eQ → EQX, EQY, EQZ
a → AXGR-AXGR+2
e²ν → E2VGR
L → XLGR
h → HXGR, HYGR, HZGR
DERVT
dL/dt → RKSTØ + 25
da/dt → RKSTØ + 26 THRU RKSTØ + 28
dh/dt → RKSTØ + 29 THRU RKSTØ + 31
EXIT

## ROUTINE TO COMPUTE ONE ROW OF COEFFICIENTS FOR LEAST SQUARES MATRIX



## 2.4 INPUT AND OUTPUT FORMATS

The Orbit Correction Program uses two input information tapes, Control and Observation Cards on logical tape unit 0 and Station Information on logical tape unit 10. The output is available as printed output of various types on logical tape unit 5, the residuals can be punched from the tape on logical unit 5 with data select 2. The tape on logical tape unit 6 contains the positions and velocities at all observation times for use in the Station Locator Program.

### 2.4.1 INPUT

Before transferring control to this program, the data cards described below must be put on logical tapes 0 and 10. This is usually done by appropriate control cards preceding these data cards in the deck submitted at execution time.

a. Control and Observation Cards on Logical Tape 0:

Four cards for beginning the ephemeris computation are needed plus 1 control card to specify the use to be made of the program. These five cards are all that are necessary for Simulation but if Differential Correction is desired, these five cards are followed by any number of observation cards which are followed by an END card (a card with "END△△△△△" in columns 1-8).* The formats of these cards are described below.

Control Cards:

| Card No. | Card Cols. | Contents |
|---|---|---|
| 1 | 1 | Print option:<br>0 means print off-line none of the following.<br>1 means print off-line $t-t_o$ (time since epoch) in minutes, $\phi$ (latitude) in degrees, $\lambda_E$ (East longitude) in degrees, and H (height above earth) in meters for each point of the ephemeris.<br>2 means print off-line $t-t_o$ in minutes, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ for each point of the ephemeris. x, y, z are in earth radii and $\dot{x}$, |

* means blank

| Card No. | Card Cols. | | Contents |
|---|---|---|---|
| 1 | 1 | | $\dot{y}$, $\dot{z}$ are in earth radii per $k_e^{-1}$ min. |
| | | 3 | means print off-line the output for both 1 and 2. |
| | 2-6 | | △ CASE |
| | 7-12 | | △0XXX△ -- where 0XXX is the case number. |
| | 13-72 | | First line of heading for each page of output. |
| | 73-80 | | Not used. |
| 2 | 1-72 | | Second line of heading for each page of output. |
| | 73-80 | | Not used. |
| 3 | 1-12 | floating point (see note 2) | +00000000+00 |
| | 13-24 | | $\Delta t$ in minutes--time interval used in Runge Kutta integration of ephemeris |
| | 25-36 | | $t_f - t_o$ in minutes--time interval between epoch time and final time point in ephemeris integration. |
| | 37-48 | | d in meters--the caliber or reference diameter of the vehicle. |
| | 49-60 | | m in kilograms--the weight of the vehicle. |
| | 61-80 | | Not used. |
| 4 | 1-12 | floating point (see note 2) | $L_o$ in radians--mean longitude at epoch. |
| | 13-24 | | $a_{xN_o} = e_o \cos \omega_o$ } Components in orbit plane at epoch of $\underline{a} = e\underline{P}$ |
| | 25-36 | | $a_{yN_o} = e_o \sin \omega_o$ |
| | 37-48 | | $h_{x_o}$ } Components of angular momentum vector at epoch, $\underline{h}_o$ |
| | 49-60 | | $h_{y_o}$ |
| | 61-72 | | $h_{z_o}$ |
| | 73-80 | | Not used. |

| Card No. | Card Cols. | Contents |
|---|---|---|
| 5 | 1-6 | XXXXXX--these characters determine which parameters are to be corrected. Each character may be a "0" or a "1" where a zero indicates that the corresponding parameter is not to be corrected and a one indicates that correction is desired. All zeros indicate that simulation of acquisition coordinates is desired. The order of the parameters corresponding to the 6 characters is: $a_o$, $a_{xN_o}$, $a_{yN_o}$, $U_o$, $\Omega_o$, $i_o$. |
| | 7-10 | Epoch, day of year--fixed point integer. |
| | 11-18 | Time from beginning of epoch day to epoch in minutes--fixed point. |
| | 19 | Not used. |
| | 20 | Number of times to repeat correction. |
| | 21 | 0 |
| | 22-24 | $\sigma_1$ (see note 1)--fixed point. |
| | 25-28 | Restart, day of year--fixed point integer. |
| | 29-37 | Time from beginning of restart day to restart time in minutes--fixed point. |
| | 38-42 | Not used. |
| | 43-48 | ABSMX = absolute maximum for angle and range residuals--fixed point. (see note 1) |
| | 49-51 | $\sigma_2$ |
| | 52-54 | $\sigma_3$ |
| | 55-57 | $\sigma_4$ |
| | 58-60 | $\sigma_5$ fixed point (see note 1) |
| | 61-63 | $\sigma_6$ |
| | 64-66 | $\sigma_7$ |
| | 67-69 | $\sigma_8$ |

| Card No. | Card Cols. | Contents |
|---|---|---|
| 5 | 70-72 | $\sigma_9$ = Maximum number of times to pass through the observations before actually correcting the parameters. $\sigma_9$ is set to 3 internally if input as 0. (see note 1) fixed point. |
| | 73-78 | ABMX2 = absolute maximum for range rate residuals--fixed point. (see note 1) |
| | 79-80 | Not used. |

Note 1

During differential correction there is an automatic feature for rejection of bad observations which works as follows: usually at least two passes are made through all the observations before actually correcting the orbit parameters. During the first pass, before correcting the parameters the $i^{th}$ time, the residuals are compared with ABSMX for range and for angle observations and ABMX2 for range rate observations. All the residuals whose absolute values are less than these limits are used in the formation of two root-mean-square values: $rms_1 = \sqrt{\frac{1}{N}\sum_{j=1}^{N} r_j^2}$ where the $r_j$ are the N range and angle residuals that pass the test, and $rms_2 = \sqrt{\frac{1}{M}\sum_{j=1}^{M} s_j^2}$ where the $s_j$ are the M range rate residuals that pass the test. During the second and subsequent passes through the observations, before correcting the parameters the $i^{th}$ time, the range and angle residuals are compared with $\sigma_i$ times the $rms_1$ of the previous pass, while the range rate residuals are compared with $\sigma_i$ times the $rms_2$ of the previous pass. As before, only the residuals that pass the test are used in computing the next two rms values. Only the observations whose residuals pass the test are used in the computation of the corrections to the orbital parameters. The parameters are not actually corrected until either the same number of residuals are rejected on successive passes, or the maximum allowed number of passes ($\sigma_9$) are made.

Note 2

By a "floating point" number is meant a number in the following format:

±XXXXXXXX±YY (this is for a 12 column field)

where ±XXXXXXXX is the mantissa and ±YY is the power of 10 that it is multiplied by. The decimal point is assumed just before the first X, which must be non zero (unless, of course, the number equals zero). For instance, -25 would be written as -25000000+02 and +.0003 would be written as +30000000-03 or 30000000-03.

Observation Cards:

These cards must be ordered with respect to time. There are four different types of observation cards: Radar, Optical, Doppler, and Emitted Frequency cards. The information on the first three of these types of cards in columns 1-29, is the same and is as follows:

| Card Cols. | Contents | |
|---|---|---|
| 1-2 | Year of launch (1958=01) | |
| 3-4 | Satellite Greek letter ($\alpha$ =01, etc.) | Satellite identification |
| 5 | Satellite component number | |
| 6 | Observation type (1 = Radar, 2 = Optical, 3 = Doppler) | |
| 7-10 | Observation or pass number (not used by program) | |
| 11-14 | Station identification | |
| 15 | Year (1958=1) | |
| 16-17 | Month | |
| 18-19 | Day | |
| 20-21 | Hour | Observation time |
| 22-23 | Minute | |
| 24-25 | Second | |
| 26-29 | .0001 second | |

The observation time must be referred to the beginning of the same year as the epoch. Days in January of the next year should be referred to as days of December: 1962 January 15 = 1961 December 46. $\Theta_{gr_o}$ (Greenwich Sidereal time at epoch) is computed within the program. To do this, Greenwich Sidereal time at the beginning of January 0 of epoch year is a constant in the program. At the time of this writing, this constant is for January 0 of 1961 ($99^{\circ}.420937$). Therefore, to use an epoch that is in any other year than 1961, requires a change of this constant.

Radar Observation Card:

| Card Cols. | Contents | |
|---|---|---|
| 1-29 | Identification and time of observation. | |
| 30-34 | Year and day of reduction (not used by program) | |
| 35-42 | Slant range in meters--fixed point integer | |
| 43-47 | Difference in slant range from t to t+2 seconds | fixed point integer |
| 48-52 | Difference in slant range from t+2 to t+4 seconds | fixed point integer |
| 53-57 | Difference in slant range from t+4 to t+6 seconds | fixed point integer |
| 58-62 | Difference in slant range from t+6 to t+8 seconds | fixed point integer |
| 63-67 | Difference in slant range from t+8 to t+10 seconds | fixed point integer |
| 68-72 | Difference in slant range from t+10 to t+12 seconds | fixed point integer |
| 73-80 | Slant range at t+12 seconds (not used by program). | fixed point integer |

Optical Observation Card:

| Card Cols. | Contents |
|---|---|
| 1-29 | Identification and time of observation |
| 30-32 | Right ascension (hours) or azimuth (degrees)--negative sign in column 30 means that this is a right ascension and declination observation, anything else in column 30 means an azimuth and elevation angle observation. |
| 33-34 | Right ascension or azimuth (minutes of time and arc, respectively). |
| 35-36 | Right ascension or azimuth (seconds of time and arc, respectively). |

| Card Cols. | Contents |
|---|---|
| 37-38 | Right ascension or azimuth (.01 seconds of time and arc, respectively). |
| 39-42 | Declination or elevation (degree) (right adjusted). |
| 43-44 | Declination or elevation (minutes). |
| 45-46 | Declination or elevation (seconds). |
| 47 | Declination or elevation (0.1 seconds). |
| 48-58 | Miscellaneous information not used by program. |
| 59-80 | Not used. |

Doppler (Range Rate) Observation Card:

| Card Cols. | Contents |
|---|---|
| 1-29 | Identification and time of observation. |
| 30-37 | Miscellaneous information not used by program. |
| 38-48 | Received frequency, $\nu'$, in hundredths of cycles per second, scaled to 108 megacycles per second, i.e., 108,000,000 must be added to obtain actual received frequency in cycles per second (i.e., this field contains $\nu'$ - 108 mc.). |
| 49-53 | Miscellaneous information not used by program. |
| 54-80 | Not used. |

To compute range rage, $\dot{\rho}$, from the above information, the following formula is used: $\dot{\rho} = -c \frac{(\nu' - \nu)}{\nu}$ where c is the speed of light and $\nu$ is the frequency emitted by the satellite. $\nu$ is obtained from the most recently encountered Emitted Frequency Card (format described next).

Emitted Frequency Card

| Card Cols. | Contents |
|---|---|
| 1-5 | Not used. |
| 6 | Blank--this differentiates this card from the other observation cards. |

| Card Cols. | Contents |
|---|---|
| 7-29 | Not used. |
| 30-40 | Frequency emitted by the satellite, $\nu$, in hundredths of cycles per second scaled to 108 megacycles per second (i.e., this field contains $\nu$ - 108 mc.) |
| 41-80 | Not used. |

b. Station Information Stored on Logical Tape 10:

Any number of station cards in any order followed by an END card (a card with "END$\Delta\Delta\Delta\Delta\Delta$" in columns 1-8), can be read onto tape:

| Card Cols. | Contents |
|---|---|
| 1-4 | Station identification. |
| 5-12 | $\phi$, latitude in one hundred thousandths of a degree, i.e., there is a decimal point assumed between columns 7 and 8. |
| 13-21 | $\lambda_E$, east longitude in one hundred thousandths of a degree, i.e., there is a decimal point assumed between columns 16 and 17. |
| 22-27 | H, height above sea level in meters. |
| 28-80 | Not used. |

For simulation, acquisition coordinates will be computed at all points of the ephemeris, for all stations which can see the vehicle.

During differential correction, if cols. 11-14 of an observation card do not match exactly cols. 1-4 of some station card, the observation will be skipped and "STATION NUMBER 0000XXXX NOT FOUND" will be written on the flexowriter.

## 2.4.2 OUTPUT

The output of the Orbit Correction Program is used to monitor the correction and simulation procedures, to provide restart information when the correction procedure is interrupted, and to provide input to the Station Locator Program. The following describe where each type of information is to be found.

a. Printed Information:

The following printed output is obtained by printing tapes with data select 1. This output depends considerably on whether simulation or differential correction is being done. However, there is some output that is the same in either case. All pages are headed by the information on cards 1 and 2 (see INPUT). If the print option (col. 1, card 1) is equal to 1 or 3, $t-t_o$ (time since epoch) in minutes, $\phi$ (latitude) in degrees, $\lambda_E$ (East longitude) in degrees, and H (height above earth) in meters are printed under appropriate headings for all points of the ephemeris. If the print option is equal to 2 or 3, $t-t_o$ in minutes, x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$ (x, y, z in earth radii and $\dot{x}$, $\dot{y}$, $\dot{z}$ in earth radii per $k_e^{-1}$min) are printed under appropriate headings for all points of the ephemeris.

After Simulation:

In addition to the above, $t-t_o$ in minutes, $\rho$ (range) in earth radii, $\dot{\rho}$ (range rate) in earth radii per $k_e^{-1}$ min, $\alpha$ (right ascension) in degrees, $\delta$ (declination) in degrees, A (azimuth) in degrees, and h (elevation angle) in degrees are printed under appropriate headings for every station for points of the ephemeris. Under each station only those points of the ephemeris are output where $h > 0$ (i.e. the satellite is above the horizon).

After Differential Correction:

For all passes through the observations all residuals are printed. Printed with each residual are the satellite identification (cols. 3-5 of the observation card), the station number (cols. 12-14 of the observation card), and the time since epoch in minutes. All quantities are listed under an appropriate heading. The range and angle residuals are in kilometers, while the range-rate residuals are in kilometers per second. The rejected residuals are flagged at the right of the line by four asterisks (****).

On the next page following the residual information for each pass are printed the case number (cols. 8-11 of ephemeris card 1), the root-mean-square of the good range and angle residuals in kilometers (under the heading SUM-KM), the root-mean-square of the good range-rate residuals in kilometers per second (under the heading SUM2-KM/SEC), and the corrections to the orbital parameters, $\Delta a_o/a_o$, $\Delta a_{xN_o}$, $\Delta a_{yN_o}$, $\Delta U_o$, $\Delta \Omega_o$, and $\Delta i_o$, based on the good observations from this pass.

After the final pass through the observations when these corrections are actually added to the orbital parameters, the following information is printed just after these corrections. The case number (which is increased by one in the left-most position at each correction) and the new orbital parameters at epoch, $L_o$, $a_{xN_o}$, $a_{yN_o}$, $h_{x_o}$, $h_{y_o}$, and $h_{z_o}$. Also x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$ at epoch are printed.

After all the corrections have been done, if there is a restart time specified in the control card, then the orbital parameters at this restart time are printed.

b. Punched Information:

The punched output is obtained by punching tape 11 with data select 2. There is no punched output for simulation. In the differential correction, all the good residuals from the last correction are punched and may be used as input to the Station Locator Program. The format is described in the Station Locator Program input specifications (Section 3.4).

On Binary Tape:

At the end of differential correction, the tape on logical tape unit 6 contains $t-t_o$ in minutes, x, y, z, $\dot{x}$, $\dot{y}$, and $\dot{z}$ at all the observation times (x, y, z are in earth radii and $\dot{x}$, $\dot{y}$, $\dot{z}$ are in earth radii per $k_e^{-1}$ min). This tape is to be used as input, in conjunction with the above residual cards, to the Station Locator Program.

## 2.5 OPERATING PROCEDURE:

The deck submitted at execution time has the following make up:

| Starting in Col. 17 | Col. 25 | Purpose |
|---|---|---|
| JOB | XXXXX... | Signals the start of a new job. Cols. 25 on, are typed on the flexowriter at the beginning of the program and may contain any alphanumeric information. |

| Col. 17 | Col. 25 | Purpose |
|---|---|---|
| REWIND | 0,3,6,9,10 | Rewinds the indicated tapes. |
| RPL | 1,DATA,GO | Reads routine named DATA from tape 1 into memory and transfers control to it. |
| TAPE | 0 | Control instruction for DATA,telling it which tape to put the following data on. |
| - - - - | | |
| - - - - | | |
| - - - - | | Data to be put on tape 0 (cf. Sec. 2.4.1) |
| - - - - | | |
| - - - - | | |
| ENDDATA | | Control instruction for DATA telling it that there is no more data to be put on tape 0. |
| REWIND | 0 | Rewinds tape 0. |
| JMP | * | Transfers control to the program in memory (DATA). (See note) |
| TAPE | 10 | Control instruction for DATA telling it which tape to put the following data on. |
| - - - - | | |
| - - - - | | |
| - - - - | | Data to be put on tape 10 (cf. Sec. 2.4.1) |
| - - - - | | |
| - - - - | | |
| ENDDATA | | Control instruction for DATA telling it that there is no more data to put on tape 10. |
| REWIND | 10 | Rewinds tape 10. |
| RPL | 4,XXX...XXX,GO | XXX...XXX is the identity of the Orbit Correction Program (up to 16 characters). It is read into memory from tape 4 and control is transferred to it. |

Note:

The card pertaining to tape 10 may be omitted during differential correction if a previously prepared station tape has been mounted on logical tape 10.

Tapes Used:

| Logical Tape No. | Purpose |
|---|---|
| 0 | Data input. |
| 3 | Ephemeris tape--used to save $t-t_o$, L, $\underline{a}$, $\underline{h}$, at all ephemeris points. |
| 4 | RPL tape containing Orbit Correction Program. |
| 5 | Output tape. |
| 6 | Used during computation of ephemeris, during simulation or during differential correction, if the print option is 2 or 3, to save $t-t_o$, $\underline{r}$, $\dot{\underline{r}}$ at observation times to be used by Station Locator Program. |
| 9 | Observation tape --observations are processed and saved on this tape. |
| 10 | Data input. (Station tape) |
| 11 | Last correction residuals |

To Use the Orbit Correction Program with the Station Locator Program:

(1) Run the Orbit Correction Program with nominal station coordinates.
(2) Save tape 6--binary tape containing $t-t_o$, $\underline{r}$, and $\dot{\underline{r}}$ at observation times.
(3) Print tape 5 off-line, using DS1 (data select 1) for printed output from Orbit Correction Program.

(4) Punch tape 5 off-line using DS2 (data select 2) for the general information card ($\theta_{gr_o}$) for input to the Station Locator Program.

(5) Punch tape 11 off-line, using DS2 (data select 2) for punched output from Orbit Correction Program. The residuals are punched for the last pass through the observations.

(6) If desired, punch (by hand) weights in the residual cards. If none are punched, 1's will be assumed by the Station Locator Program.

(7) Punch additional input cards for Station Locator Program (station information card(s) ), and set up input deck.

(8) Run the Station Locator Program with the binary tape containing $t-t_o$, $\underline{r}$ and $\dot{\underline{r}}$ at observation times prepared by the Orbit Correction Program on logical tape 3.

(9) Print tape 5 off-line, using DS0 (data select 0) for printed output from Station Locator Program.

(10) If desired, repeat Orbit Correction with new station coordinates, to obtain new elements and new residuals.

(11) Then, if desired, repeat Station Locator, to refine station corrections.

# SECTION 3

# STATION LOCATOR PROGRAM

The purpose of the Station Locator Program is to compute from observed deviations by a geodetic satellite from a good model of its orbit, corrections to the geocentric coordinates of the stations observing the satellite and to the geocentric coordinates of the origin of any datum to which a group of such stations are tied. The good orbital model results from the Orbit Correction Program and the observed deviations are the residuals produced by that program.

Since the station coordinates have been determined already, with a certain accuracy from previous information (e.g., surveys), it is not desirable to base the determination of the corrections entirely on the observations of one geodetic satellite. Therefore, limits are placed upon the magnitude of the station corrections. Furthermore, there is provision for weighting the stations on the same datum to account for different strengths of their ties to the datum origin. The individual observational quantities may be weighted at will, to account for observing conditions, etc.

## 3.1 THEORY

Assuming the orbit of an earth satellite is known, any observation of the satellite can be expressed in terms of the time and three coordinates describing the location of the observer. Equations (1), (2), and (3) of Section 2.1, are again applicable where the $X_j$ are now the three station coordinates and $(C_{ij})$ is an m x 3 matrix. The elements of this matrix are now the partial derivatives of the observed quantities with respect to the station coordinates. The solution again takes the form of equation (4), and thus the iterative process would result in improvements to the station coordinates.

When weighting is desired in the determination to account for different types of tracking data or different tracking instruments, the least squares matrix can be constructed according to

$$N = (C_{ij})^T \quad (P_{ip}) \quad (C_{ij}) \tag{6}$$

where $(P_{ip})$ is the diagonal weight matrix, defined as

$$P_{ip} = \omega_{oi} \qquad i = p$$

$$P_{ip} = 0 \qquad i \neq p$$

The $\omega_{oi}$ is the weighting factor of the observation $0_i$. The solution now takes the form:

$$(\Delta X_j) = N^{-1} (C_{ij})^T (P_{ip}) \quad (\Delta o_i) \tag{7}$$

The $\Delta X_j$ are compared against limits, chosen with regard to the uncertainty in that coordinate prior to the geodetic satellite observations. If one of the corrections, say $\Delta X_3$, exceeds its limit, the correction is set equal to the limit with the sign of the discarded excessive correction. The result is to transform the equations of condition from

$$\Delta 0_i = \frac{\partial 0_i}{\partial X_1} \Delta X_1 + \frac{\partial 0_i}{\partial X_2} \Delta X_2 + \frac{\partial 0_i}{\partial X_3} \Delta X_3 \tag{8}$$

to

$$\Delta 0_i - \frac{\partial 0_i}{\partial X_3} \Delta X_3 = \frac{\partial 0_i}{\partial X_1} \Delta X_1 + \frac{\partial 0_i}{\partial X_2} \Delta X_2 \tag{9}$$

This set of equations can be solved for $\Delta X_1$ and $\Delta X_2$ as previously shown, where the $(C_{ij})$ matrix is now an m x 2 matrix, and the least squares matrix N becomes a 2 x 2 matrix.

In the iterative process, the corrections to the two-station coordinates, as obtained from equation (9), would be compared to their limits. This test determines whether the solution is acceptable or whether a "least squares" solution in terms of only one of the station coordinates is more desirable. Such a one coordinate solution amounts to a weighted average.

The following example shows a fictitious case. The units are given in the last column.

| | Limits | Solutions 1 | 2 | 3 | |
|---|---|---|---|---|---|
| $\Delta X_1 = \Delta \phi$ | $\pm 0.003$ | $+ 0.00177$ | $+ 0.00279$ | - - | degrees |
| $X_2 = \cos \phi \Delta \lambda$ | $\pm 0.003$ | $+ 0.00045$ | $+ 0.00192$ | - - | degrees |
| $\Delta X_3 = \Delta H$ | $\pm 100$ | $- 327$ | $- 100$ | - - | meters |

$\Delta X_3$ from the first solution exceeds its limit. It is set equal to that limit with the appropriate sign. Since neither $\Delta X_1$ nor $\Delta X_2$, from the second solution exceed their limits, the third solution is not necessary and the second solution is adopted.

## 3.2 LOGICAL PROGRAM

The cartesian station coordinates are calculated for each station under the condition that the Greenwich meridian is aligned with the great circle passing through the poles and the vernal equinox (i.e., $\Theta_{gr} = 0$)

$$X_s\ (L) = - x_c \cos \lambda$$

$$Y_s\ (L) = - x_c \sin \lambda$$

$$Z_s\ (L) = - y_c$$

where $y_c = (S + H) \sin \phi$

and $x_c = (C + H) \cos \phi$

The quantities C and S are computed as follows:

$$C = a_e (1 - e^2 \sin^2 \phi)^{-\frac{1}{2}} \qquad S = (1 - e^2)C$$

The constants $a_e$ (semi-major axis of the ellipsoid) and $e^2$ (e is the eccentricity of the ellipsoid) are now constants contained within the program:

$$a_e = 1.000\ 000\ 000\ 0$$

$$e^2 = 0.006\ 693\ 421\ 6$$

A logical change is necessary at this point to allow for a look-up of $a_e$ and e according to the datum number. It is advisable that these "constants" correspond to the ellipsoid used to tie the datum together. For each station the constants, CK1, CK2, CK3, and CK4 are calculated as follows:

$$CK1 = -(C + H) + e^2C^3 \cos^2\phi a_e^{-2}$$

$$CK2 = (S + H) + e^2SC^2 \sin^2\phi a_e^{-2}$$

$$CK3 = x_c \tan\phi - e^2C^3 \cos^2\phi \sin\phi a_e^{-2}$$

$$CK4 = y_c \cot\phi + SC^2e^2 \sin^2\phi \cos\phi a_e^{-2}$$

Calculation of $\theta$, the sidereal time, from the observation time, t, gives the components of the station location vector at this time: X1, Y1, and Z1.

$$\theta = \theta_{gr} + \dot{\theta}(t - t_o) + \lambda$$

Where $\theta_{gr}$ is the angle between the vernal equinox the the Greenwich meridian at epoch time $t_o$, $\lambda$ is the earth-fixed longitude (i.e., its reference is the Greenwich meridian) and $\dot{\theta}$ is the rotation rate of the earth.

The station components at this longitude are:

$$X1 = -x_c \cos \theta$$

$$Y1 = -x_c \sin \theta$$

$$Z1 = -y_c$$

a. Calculation of the Components of the Topocentric Range Vector

The range vector, $\rho$, has components

$\rho_x = x + X1$

$\rho_y = y + Y1$

$\rho_z = z + Z1$

from which the direction cosines are:

$L_x = \rho_x / \rho$

$L_y = \rho_y / \rho$ where $\rho = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}$

$L_z = \rho_z / \rho$

The geometric quantities x, y, z, the geocentric components of the radius vector and the satellite, at time t, were read from a binary tape, prepared by the Orbit Correction Program.

b. Calculation of the Weighted Differential Coefficients for the Different Types of Residuals

The residual in range, $\Delta\rho$, is related to errors in $\phi$, $\lambda$, and H as follows:

$$\Delta\rho = R_1 \Delta\phi + R_2 \cos \Delta\lambda + R_3 \Delta H \qquad (10)$$

Where

$$R1 = (L_x \cos \theta + L_y \sin \theta) \, CK3 - (L_z) \, CK4$$

$$R2 = (C + H) (L_x \sin \theta - L_y \cos \theta)$$

$$R3 = - \cos \phi \, (L_x \cos \theta + L_y \sin \theta + L_z \tan \phi)$$

At this point the weighting factor must be taken into account, and equation (10) becomes

$$\sqrt{W_\rho}\,\Delta\rho = \sqrt{W_\rho}\left(R_1\Delta\phi + R_2\cos\phi\,\Delta\lambda + R_3\,\Delta H\right)$$

The coefficients in this weighted equation are the quantities CE(1), CE(2), CE(3) and RES, defined as:

$$CE(1) = \sqrt{W_\rho}\;R_1$$

$$CE(2) = \sqrt{W_\rho}\;R_2$$

$$CE(3) = \sqrt{W_\rho}\;R_3$$

$$RES = \sqrt{W_\rho}\;\Delta\rho$$

These quantities are then utilized to construct the least squares matrix. This is done in the portion of the program referred to as subroutine Sub 1. Utilizing the same theory as stated above, the following relationships are obtained for the residuals $\rho\Delta\alpha\cos\delta$, $\rho\Delta\delta$, $\rho\Delta A\cos h$, $\rho\Delta h$ and $\Delta\dot{\rho}$. The quantities CE(1), CE(2), CE(3) and RES are calculated in precisely the same manner as they were calculated for the residual of $\Delta\rho$, except that the differential coefficients depend upon the type of residual being utilized, and will thus vary from equation (10).

For the residual $\rho\Delta\alpha\cos\delta$:

$$\sqrt{W_\alpha}\,\rho\Delta\alpha\cos\delta = \sqrt{W_\alpha}\left(AL1\,\Delta\phi + AL2\,\cos\phi\,\Delta\lambda + AL3\,\Delta H\right)$$

or

$$CE(1) = \sqrt{W_\alpha}\quad AL1$$

$$CE(2) = \sqrt{W_\alpha}\quad AL2$$

$$CE(3) = \sqrt{W_\alpha}\quad AL3$$

$$RES = \sqrt{W_\alpha}\,\rho\Delta\alpha\cos\delta$$

where

$$AL1 = CK3\left(A_x\cos\theta + A_y\sin\theta\right)$$

$$AL2 = (C + H)\left(A_x\sin\theta - A_y\cos\theta\right)$$

$$AL3 = -\cos\phi\left(A_x\cos\theta + A_y\sin\theta\right)$$

and: $A_x = \dfrac{-L_y}{\sqrt{1 - L_z^2}}$

$A_y = \dfrac{L_x}{\sqrt{1 - L_z^2}}$

For the residual $\rho \Delta \delta$:

$$\sqrt{W_\delta}\ \rho \Delta \delta = \sqrt{W_\delta} \left( DL1\ \Delta \phi + DL2\ \cos\phi\ \Delta \lambda + DL3\ \Delta H \right)$$

or $CE(1) = \sqrt{W_\delta}\ DL1$

$CE(2) = \sqrt{W_\delta}\ DL2$

$CE(3) = \sqrt{W_\delta}\ DL3$

$RES = \sqrt{W_\delta}\ \rho \Delta \delta$

where $DL1 = -CK4\ (D_z) + CK3\ (D_x \cos\Theta + D_y \sin\Theta)$

$DL2 = (C + H)\ (D_x \sin\Theta - D_y \cos\Theta)$

$DL3 = -\cos\phi\ (D_x \cos\Theta + D_y \sin\Theta + D_z \tan\phi)$

and: $D_x = -L_z A_y$

$D_y = L_z A_x$

$D_z = \sqrt{1 - L_z^2}$

For the residual $\rho \Delta A \cos h$:

$$\sqrt{W_A}\ \rho \Delta A \cos h = \sqrt{W_A} \left( AZ1\ \Delta \phi + AZ2\ \cos\phi\ \Delta \lambda + AZ3\ \Delta H \right)$$

where: $AZ1 = \widetilde{A}_{xh} \left( \sin\phi\ (CK3) + \cos\phi\ (CK4) \right)$

$AZ2 = -(C + H)\ \widetilde{A}_{yh}$

$AZ3 = 0$

$$S_x = \sin\phi \cos\theta$$

$$S_y = \sin\phi \sin\theta$$

$$S_z = -\cos\phi$$

$$E_x = -\sin\theta$$

$$E_y = \cos\theta$$

$$Z_x = \cos\phi \cos\theta$$

$$Z_y = \cos\phi \sin\theta$$

$$Z_z = \sin\phi$$

$$A_{xh} = \frac{L_{yh}}{\sqrt{1 - L_{zh}^2}}$$

$$A_{yh} = \frac{-L_{xh}}{\sqrt{1 - L_{zh}^2}}$$

where

$$L_{xh} = L_x S_x + L_y S_y + L_z S_z$$

$$L_{yh} = L_x E_x + L_y E_y$$

$$L_{zh} = L_x Z_x + L_y Z_y + L_z Z_z$$

For the residual of $\rho\Delta h$:

$$\sqrt{W_h}\,\rho\Delta h = \sqrt{W_h}\left(EL1\,\Delta\phi + EL2\cos\phi\,\Delta\lambda + EL3\,\Delta H\right)$$

or

$$CE(1) = \sqrt{W_h}\ EL1$$

$$CE(2) = \sqrt{W_h}\ EL2$$

$$CE(3) = \sqrt{W_h}\ EL3$$

$$RES = \sqrt{W_h}\ \rho \Delta h$$

where

$$EL1 = CK3\ (\widetilde{D}_{xh} \sin\phi + \widetilde{D}_{zh} \cos\phi) - CK4\ (-\widetilde{D}_{xh} \cos\phi + \widetilde{D}_{zh} \sin\phi)$$

$$EL2 = -(C + H)\ \widetilde{D}_{yh}$$

$$EL3 = -\widetilde{D}_{zh}$$

and

$$\widetilde{D}_{xh} = \widetilde{A}_{yh}\ L_{zh}$$

$$\widetilde{D}_{yh} = -\widetilde{A}_{xh} L_{zh}$$

$$\widetilde{D}_{zh} = \sqrt{1 - L_{zh}^2}$$

For the residual of $\Delta\dot{\rho}$:

$$\sqrt{W_{\dot{\rho}}}\ \Delta\dot{\rho} = \sqrt{W_{\dot{\rho}}}\ \left( RD1\ \Delta\phi + RD2\ \cos\phi\ \Delta\lambda + RD3\ \Delta H \right)$$

or

$$CE(1) = \sqrt{W_{\dot{\rho}}}\ RD1$$

$$CE(2) = \sqrt{W_{\dot{\rho}}}\ RD2$$

$$CE(3) = \sqrt{W_{\dot{\rho}}}\ RD3$$

$$RES = \sqrt{W_{\dot{\rho}}}\ \Delta\dot{\rho}$$

where

$$RD1 = -\ CK3\ \cos\theta \left( \rho_y \dot{\theta} + \dot{x} - \dot{\rho} L_x \right) + CK3\ \sin\theta \left( \rho_x \dot{\theta} - \dot{y} + \dot{\rho} L_y \right)$$

$$+ CK4$$

$$RD2 = (C + H)\left[\cos\theta\,(\rho_x\dot{\theta} - \dot{y} + \dot{\rho}L_y) + \sin\theta\,(\rho_y\dot{\theta} + \dot{x} - \dot{\rho}L_x) + x_c\dot{\theta}\right]$$

$$RD3 = \cos\phi\left[-\cos\theta\,(\rho_y\dot{\theta} + \dot{x} - \dot{\rho}L_x) + \sin\theta\,(\rho_x\dot{\theta} - \dot{y} + \dot{\rho}L_y)\right] + \sin\phi\left[-\dot{z} + \dot{\rho}L_z\right]$$

$$\dot{X}1 = -Y1\,\dot{\theta}$$

$$\dot{Y}1 = X1\,\dot{\theta}$$

$$\dot{\rho} = (\rho_x\dot{\rho}_x + \rho_y\dot{\rho}_y + \rho_z\dot{\rho}_z)/\rho$$

$$\dot{\rho}_x = \dot{x} + \dot{X}1$$

$$\dot{\rho}_y = \dot{y} + \dot{Y}1$$

$$\dot{\rho}_z = \dot{z}$$

c. <u>The Least Squares Matrix</u>

The least squares matrix, Q, has the form:

$$\begin{bmatrix} Q(1) & Q(2) & Q(3) \\ Q(2) & Q(4) & Q(5) \\ Q(3) & Q(5) & Q(6) \end{bmatrix}$$

The normal equations are:

$$\begin{vmatrix} P(1) \\ P(2) \\ P(3) \end{vmatrix} = \left[Q\right] \begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \\ \Delta H \end{vmatrix}$$

These equations are solved for $\Delta\phi$, $\cos\phi\,\Delta\lambda$ and $\Delta H$ by pre-multiplying both sides of the equation by $[Q]^{-1}$.

In case one, two, or all three of the original calculated corrections, $\Delta\phi$ and $\cos\phi\,\Delta\lambda$, or $\Delta H$ are greater than a specified limit, the logical procedure is as follows: Take, as an example, the case when $\Delta H$ fails to pass the test. That is; $\Delta\phi < \lim\Delta\phi$, $\cos\phi\,\Delta\lambda < \lim \cos\phi\,\Delta\lambda$, and $\Delta H > \lim\Delta H$. In this case we replace the magnitude of $\Delta H$ with its limit value, $\lim\Delta H$, and proceed to solve for the remaining corrections.

From the complete least squares formulation we have:

$$\begin{vmatrix} P(1) \\ P(2) \\ P(3) \end{vmatrix} = \begin{bmatrix} Q(1) & Q(2) & Q(3) \\ Q(2) & Q(4) & Q(5) \\ Q(3) & Q(5) & Q(6) \end{bmatrix} \begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \\ \Delta H \end{vmatrix}$$

Now replace the quantity $\Delta H$ by its limit and the sign of $\Delta H$ and then rearrange:

$$P(1) - \frac{\Delta H}{|\Delta H|} \lim\ \Delta H\, Q(3) = Q(1)\,\Delta\phi + Q(2)\cos\phi\,\Delta\lambda \qquad (11)$$

$$P(2) - \frac{\Delta H}{|\Delta H|} \lim\ \Delta H\, Q(5) = Q(2)\,\Delta\phi + Q(4)\cos\phi\,\Delta\lambda \qquad (12)$$

$$P(3) - \frac{\Delta H}{|\Delta H|} \lim\ \Delta H\, Q(6) = Q(3)\,\Delta\phi + Q(5)\cos\phi\,\Delta\lambda \qquad (13)$$

Note that the quantities on the left hand side are known and that only $\Delta\phi$ and $\cos\phi\,\Delta\lambda$ remain as unknowns.

The last of the series of three equations, equation (13), can be neglected.

This can be verified by repeating the least squares procedure for two unknown.

There are now two equations in two unknowns -- (11) and (12)-- which have the form

$$\begin{vmatrix} P(1) \\ P(2) \end{vmatrix} = \begin{bmatrix} Q(1) & Q(2) \\ Q(2) & Q(4) \end{bmatrix} \begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \end{vmatrix}$$

$$\begin{vmatrix} P(1) \\ P(2) \end{vmatrix} = \begin{bmatrix} A(1) & A(2) \\ A(3) & A(4) \end{bmatrix} \begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \end{vmatrix} \qquad = \begin{bmatrix} A \end{bmatrix} \begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \end{vmatrix}$$

where

$$A(1) = Q(1)$$

$$A(2) = Q(2)$$

$$A(3) = Q(2)$$

$$A(4) = Q(4)$$

from which it is possible to solve for $\Delta\phi$ and $\cos\phi\Delta\lambda$

$$\begin{vmatrix} \Delta\phi \\ \cos\phi\,\Delta\lambda \end{vmatrix} = \begin{bmatrix} A \end{bmatrix}^{-1} \begin{vmatrix} P(1) \\ P(2) \end{vmatrix}$$

The values of $\Delta\phi$ and $\cos\phi\Delta\lambda$ obtained in this manner are the best values, in a least squares sense, that accommodate the limiting value of $\Delta H$. This same procedure would be followed should either $\Delta\phi$ or $\cos\phi\Delta\lambda$ fail the limit check.

What happens if two of the calculated quantities $\Delta\phi$, $\cos\phi\Delta\lambda$, or $\Delta H$ fail to pass the limiting test? Again, assume $\Delta H$ to have failed but also assume $\Delta\phi$ to have failed.

The least squares equations become:

$$P(1) - Q(1)\frac{\Delta\phi}{|\Delta\phi|}\lim\Delta\phi - Q(3)\frac{\Delta H}{|\Delta H|}\lim\Delta H = Q(2)\cos\phi\Delta\lambda \quad (14)$$

$$P(2) - Q(2)\frac{\Delta\phi}{|\Delta\phi|}\lim\Delta\phi - Q(5)\frac{\Delta H}{|\Delta H|}\lim\Delta H = Q(4)\cos\phi\Delta\lambda \quad (15)$$

$$P(3) - Q(3)\frac{\Delta\phi}{|\Delta\phi|}\lim\Delta\phi - Q(6)\frac{\Delta H}{|\Delta H|}\lim\Delta H = Q(5)\cos\phi\Delta\lambda \quad (16)$$

These equations are all equivalent to the least squares equation for finding one unknown from a group of measures, equation (15). Finally we have

$$\cos\phi\,\Delta\lambda = \frac{P(2) - Q(2)\dfrac{\Delta\phi}{|\Delta\phi|}\lim\Delta\phi - Q(5)\dfrac{\Delta H}{|\Delta H|}\lim\Delta H}{Q(4)}$$

which is a weighted average for $\cos\phi\,\Delta\lambda$, given the limiting values of $\Delta\phi$ and $\Delta H$. A similar procedure is followed regardless of which two fail the test.

If all three of the original corrections fail the test, then:

$$\Delta\phi = \frac{\Delta\phi}{|\Delta\phi|}\lim\Delta\phi$$

$$\cos\phi\,\Delta\lambda = \frac{\cos\phi\,\Delta\lambda}{|\cos\phi\,\Delta\lambda|}\lim\cos\phi\,\Delta\lambda$$

$$\Delta H = \frac{\Delta H}{|\Delta H|}\lim\Delta H$$

If one or two of the original calculated corrections fail to pass the test and new least square values are calculated for the quantities, these new values are also checked against the limiting values and, if they fail, a procedure similar to that just described is followed. The flow diagram explains what happens if the newly calculated values fail the test in a reasonable amount of detail.

d. Corrections to the Geocentric Station Location Vector:

The corrections to the geocentric station location vector, for station L, are calculated as follows:

$$\Delta X_s(L) = (C + H)\sin\lambda\,(S2) - \cos\lambda\left[(CK1)\sin\phi\,(S1) + \cos\phi\,(53)\right]$$

$$\Delta Y_s(L) = -(C + H)\cos\lambda\,(S2) - \sin\lambda\left[(CK1)\sin\phi\,(S1) + \cos\phi\,(53)\right]$$

$$\Delta Z_s(L) = -(CK2)\cos\phi\,(S1) + \sin\phi\,(53)$$

where:

$$S1 = \Delta\phi \text{ or } \frac{\Delta\phi}{|\Delta\phi|} \lim \Delta\phi$$

$$S2 = \cos\phi\ \Delta\lambda \text{ or } \frac{\cos\phi\ \Delta\lambda}{|\cos\phi\ \Delta\lambda|} \lim \cos\phi\ \Delta\lambda$$

$$S3 = \Delta H \text{ or } \frac{\Delta H}{|\Delta H|} \lim \Delta H$$

The values of $\Delta X_s(L)$, $\Delta Y_s(L)$ and $\Delta Z_s(L)$ are obviously governed by the value of S1, S2, or S3 selected. The proper value of S1, S2, or S3 is in turn governed by the checking procedure used on the original calculated values of $\Delta\phi$, $\cos\phi\ \Delta\lambda$, and $\Delta H$. This procedure is illustrated in the flow diagram and in the preceding subsection (c).

e. Calculation of Final Station Coordinates

The final values of the station coordinates are calculated as follows:

$$\phi = \phi + S1$$

$$\lambda = \lambda + S2/\cos\phi$$

$$H = H + S3$$

where S1, S2, and S3 are the same as those described in (d) above. The manner in which the proper value of S1, S2, or S3 is selected is identical to the method explained there.

f. Datum Translation Vector

The datum translation vector is calculated in the following manner:

$$\Delta X_D = \frac{\sum_L \Delta X_S(L) \cdot W_{X_S}(L)}{\sum_L W_{X_S}(L)}$$

$$\Delta Y_D = \frac{\sum_L \Delta Y_S(L) \cdot W_{Y_S}(L)}{\sum_L W_{Y_S}(L)}$$

$$\Delta Z_D = \frac{\sum_L \Delta Z_S(L) \cdot W_{Z_S}(L)}{\sum_L W_{Z_S}(L)}$$

where $W_{X_S}$, $W_{Y_S}$, and $W_{Z_S}$ are the weighting factors for each component of the station location correction vector (i.e., $\Delta X_S$, $\Delta Y_S$, $\Delta Z_S$).

g. Correction to Station Location Vector

The correction to each individual station, to determine its location with respect to the datum, is calculated as follows:

$$\Delta X_{S/D}(L) = \Delta X_S(L) - \Delta X_D(L)$$

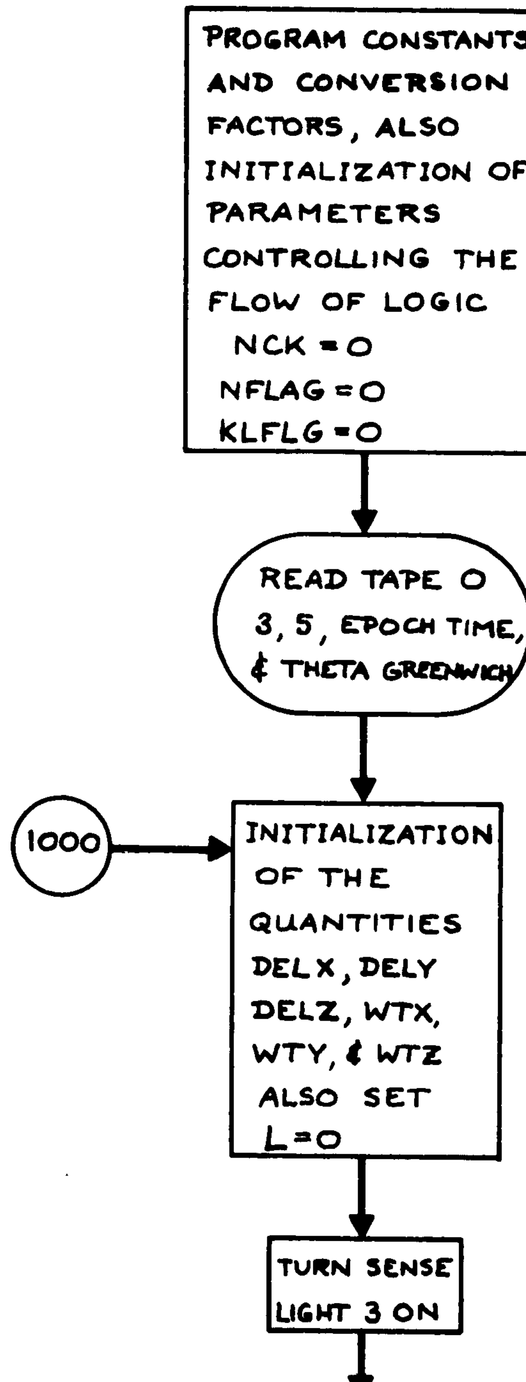
$$\Delta Y_{S/D}(L) = \Delta Y_S(L) - \Delta Y_D(L)$$

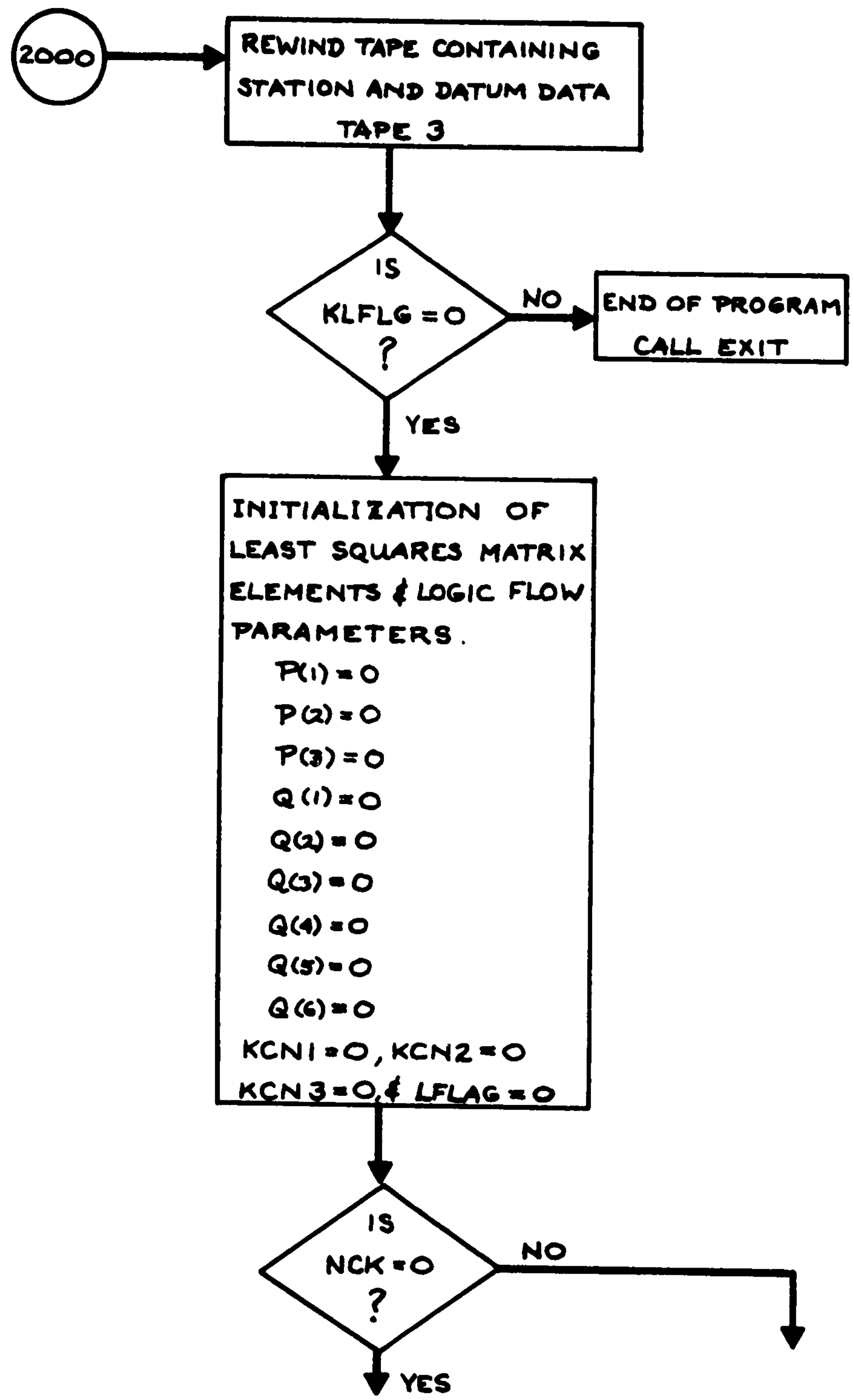
$$\Delta Z_{S/D}(L) = \Delta Z_S(L) - \Delta Z_D(L)$$

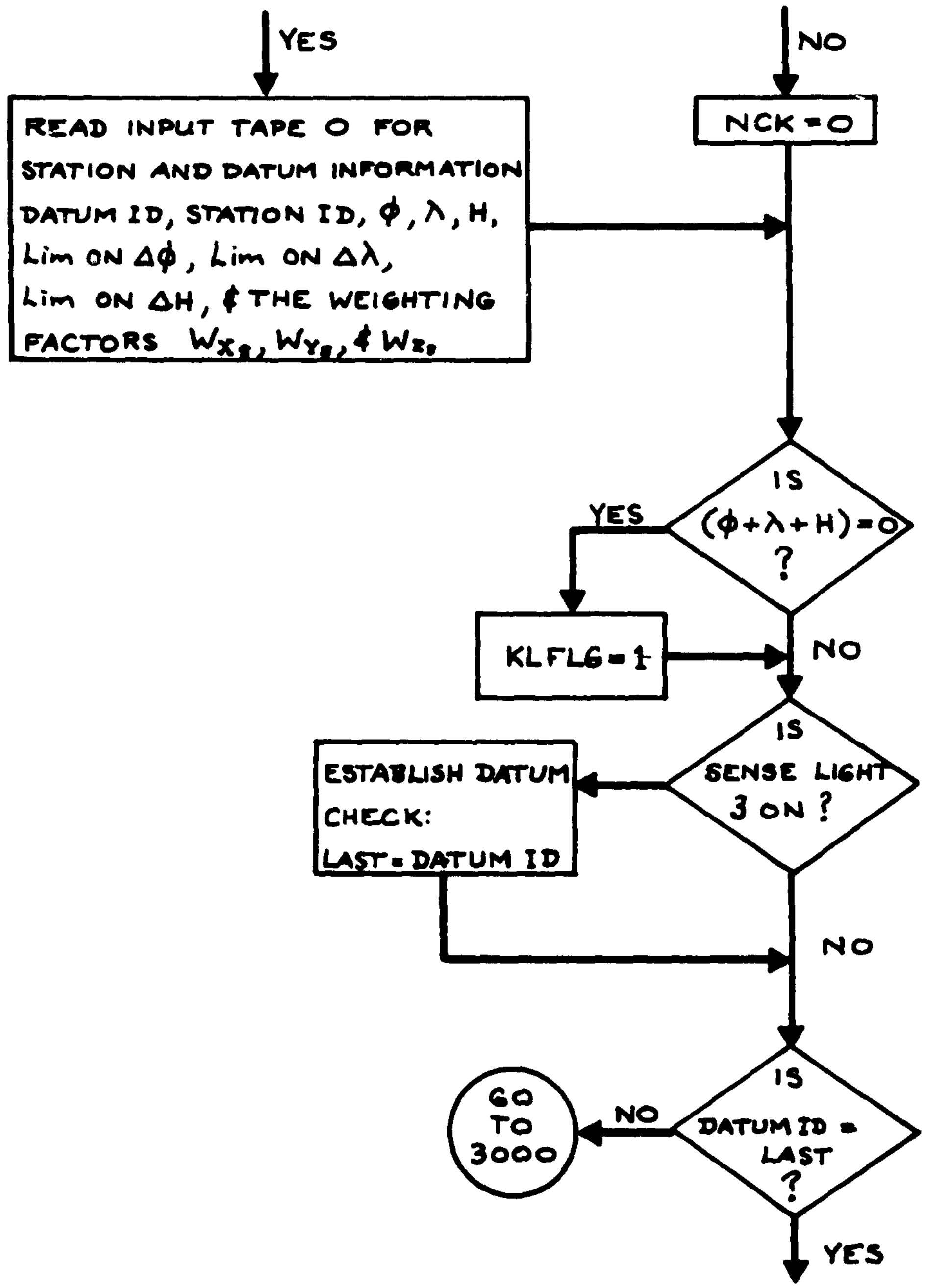
where the subscript S/D implies "station with respect to the datum" and, as before, the subscripts S and D imply station and datum where both are referenced to a geocentric coordinate system.

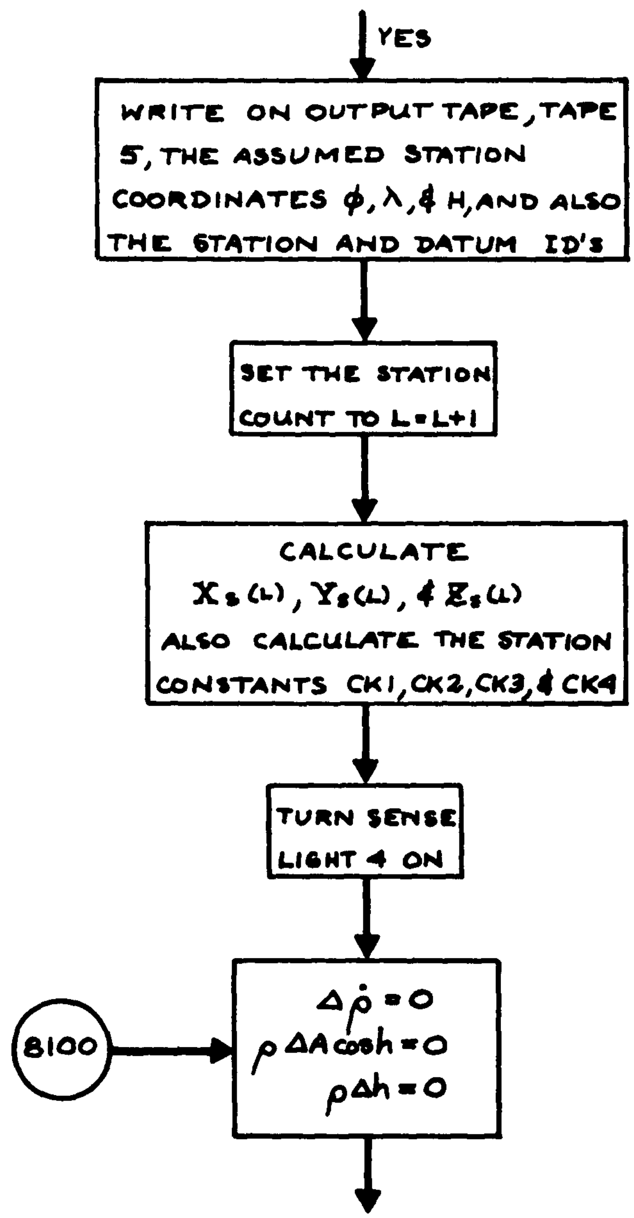
## 3.3 FLOW DIAGRAMS

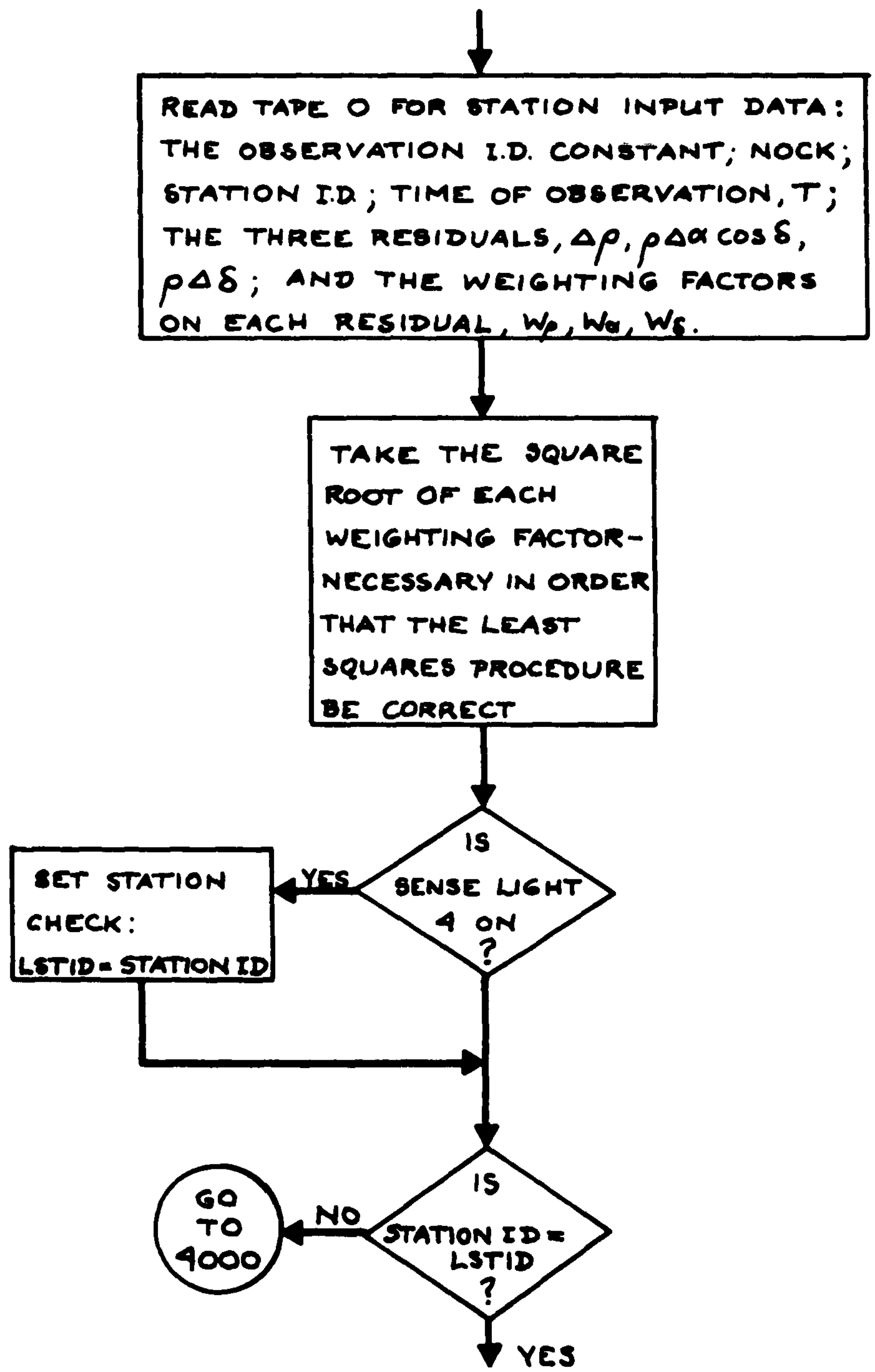
The following charts are similar to those of Section 2.3. Since the Station Locator Program was written in FORTRAN and ALTAC, the symbolic program location names used in the connectors are numbers rather than letters. Arrows at the bottom of one page indicate that the flow is continued as indicated by the corresponding arrow at the top of the next page.

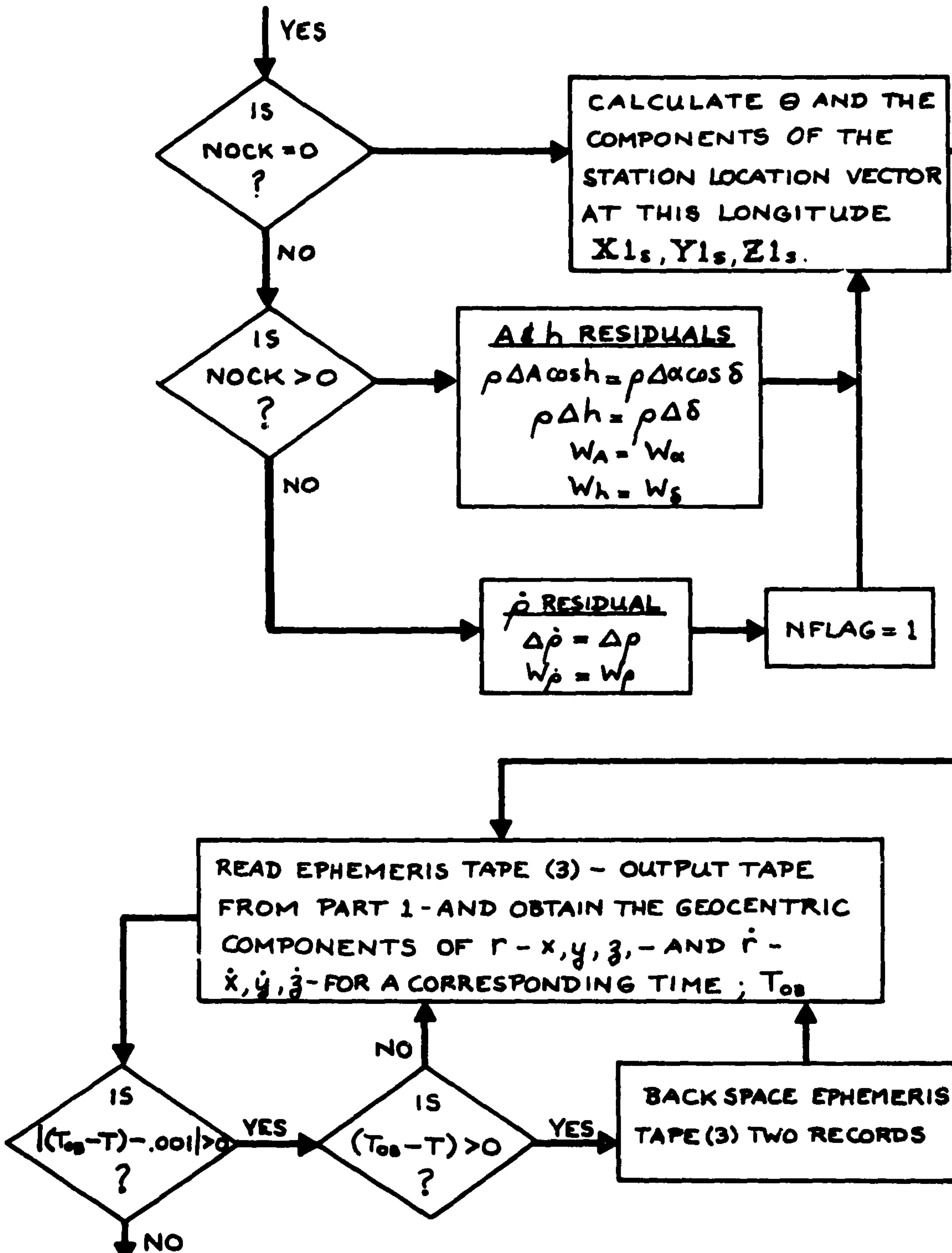

PROGRAM CONSTANTS AND CONVERSION FACTORS, ALSO INITIALIZATION OF PARAMETERS CONTROLLING THE FLOW OF LOGIC
NCK = 0
NFLAG = 0
KLFLG = 0
READ TAPE 0 3, 5, EPOCH TIME, & THETA GREENWICH
1000
INITIALIZATION OF THE QUANTITIES DELX, DELY DELZ, WTX, WTY, & WTZ ALSO SET L=0
TURN SENSE LIGHT 3 ON

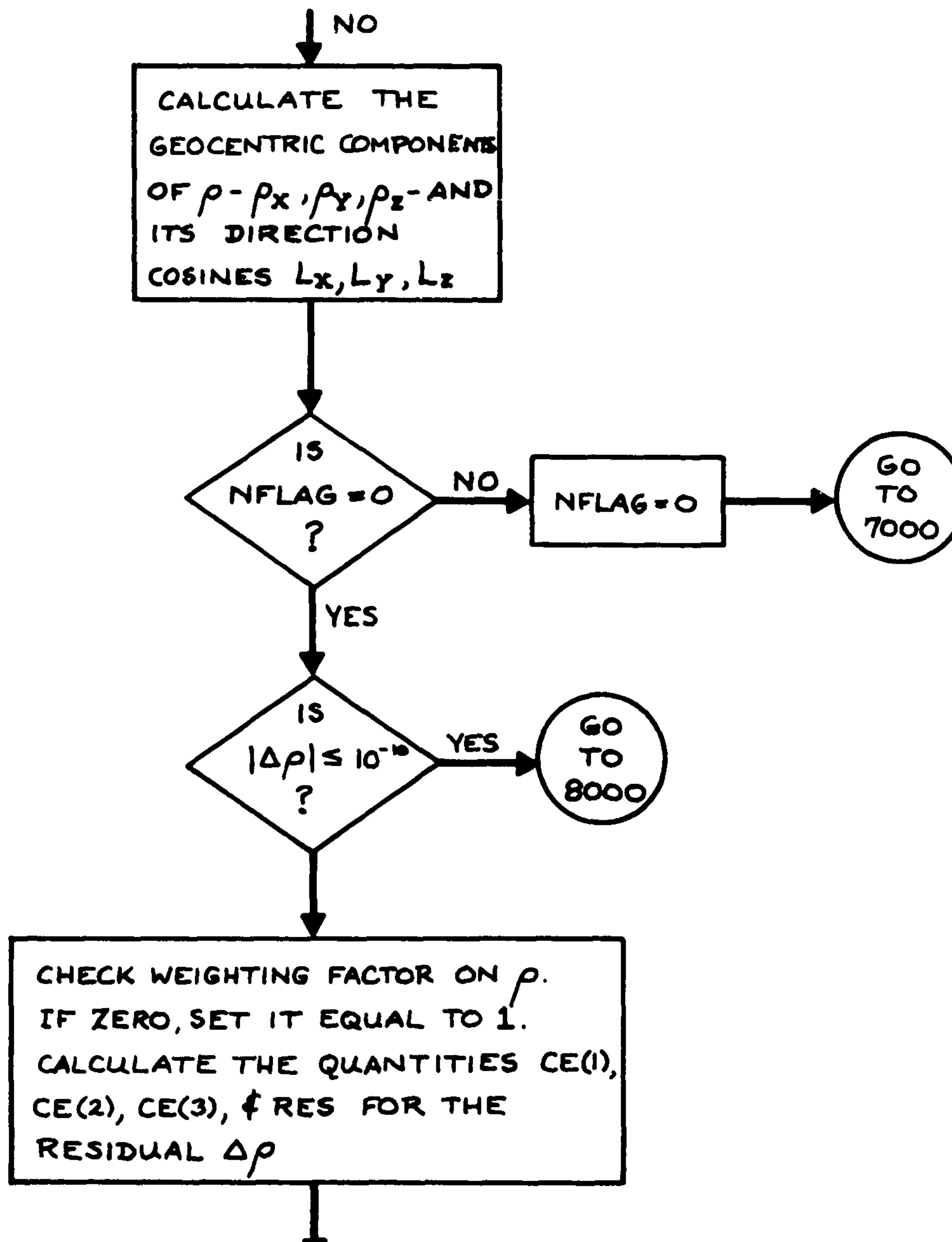

2000
REWIND TAPE CONTAINING
STATION AND DATUM DATA
TAPE 3
IS
KLFLG = 0
?
NO
END OF PROGRAM
CALL EXIT
YES
INITIALIZATION OF
LEAST SQUARES MATRIX
ELEMENTS & LOGIC FLOW
PARAMETERS.
P(1) = 0
P(2) = 0
P(3) = 0
Q(1) = 0
Q(2) = 0
Q(3) = 0
Q(4) = 0
Q(5) = 0
Q(6) = 0
KCN1 = 0, KCN2 = 0
KCN3 = 0, & LFLAG = 0
IS
NCK = 0
?
NO
YES

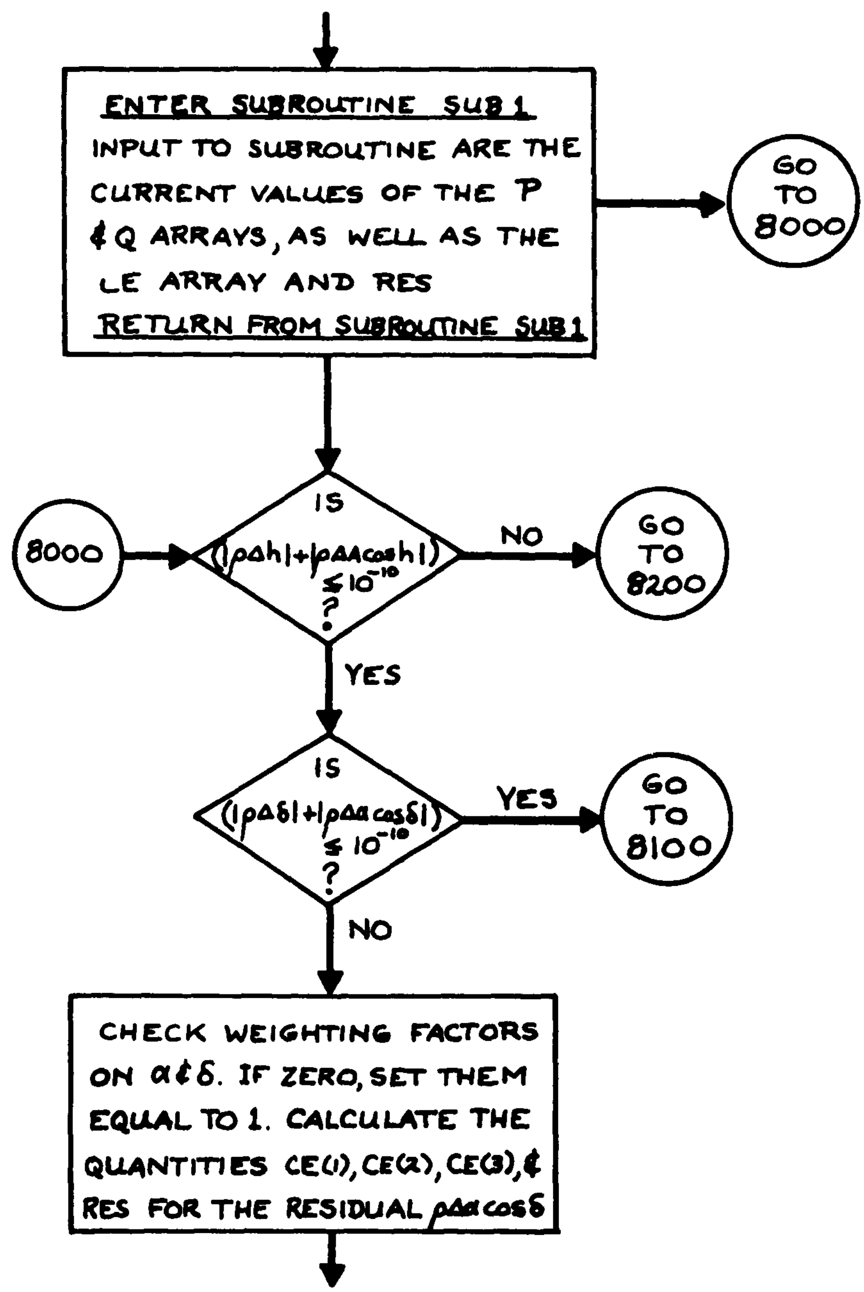

YES
NO
READ INPUT TAPE O FOR STATION AND DATUM INFORMATION DATUM ID, STATION ID, φ, λ, H, Lim ON Δφ, Lim ON Δλ, Lim ON ΔH, & THE WEIGHTING FACTORS Wx, Wy, & Wz
NCK = O
IS (φ+λ+H) = O ?
YES
KLFLG = 1
NO
IS SENSE LIGHT 3 ON ?
ESTABLISH DATUM CHECK: LAST = DATUM ID
NO
IS DATUM ID = LAST ?
NO
GO TO 3000
YES

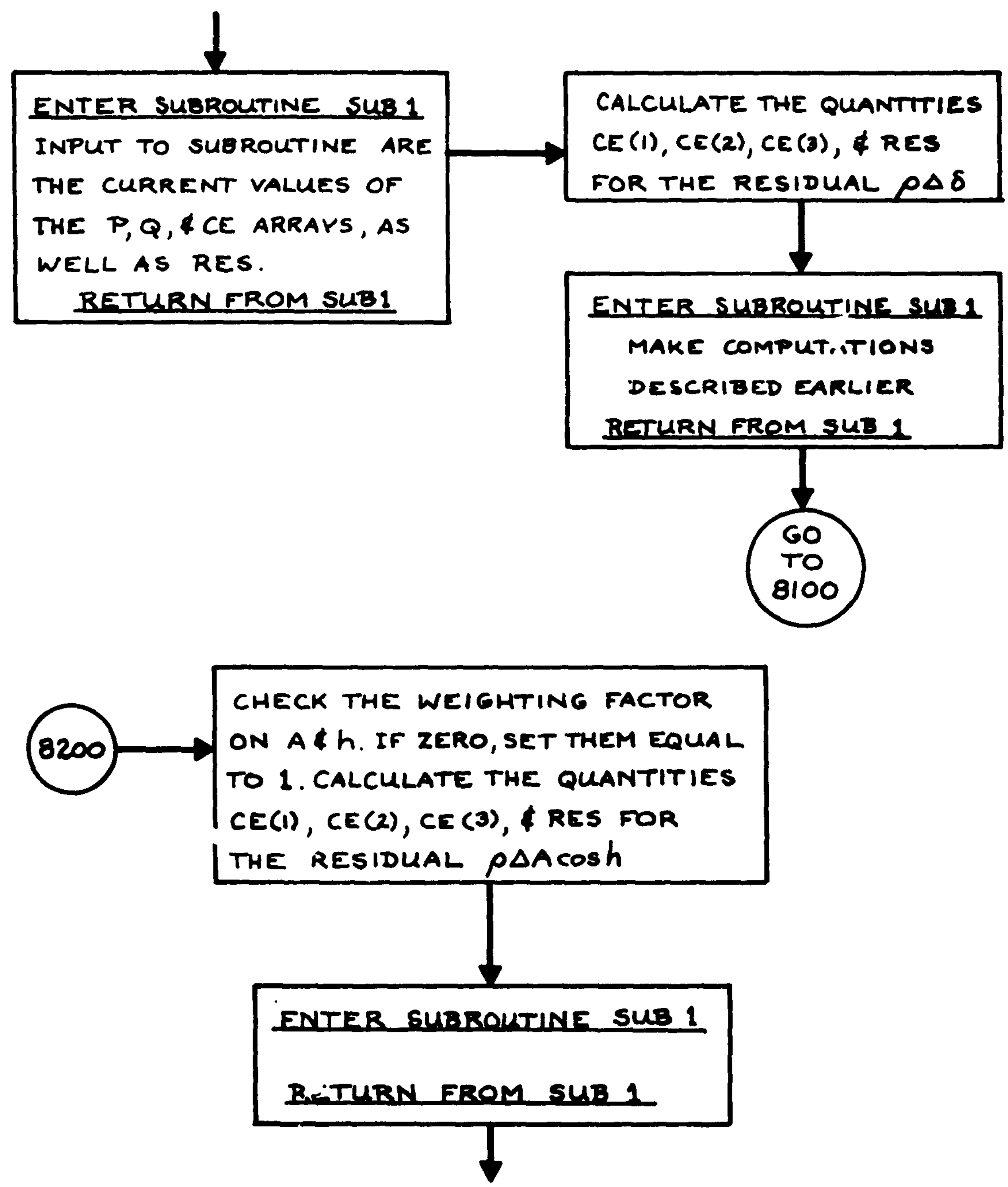

YES
WRITE ON OUTPUT TAPE, TAPE 5, THE ASSUMED STATION COORDINATES $\phi$, $\lambda$, & H, AND ALSO THE STATION AND DATUM ID'S
SET THE STATION COUNT TO L=L+1
CALCULATE $X_s(L)$, $Y_s(L)$, & $Z_s(L)$ ALSO CALCULATE THE STATION CONSTANTS CK1, CK2, CK3, & CK4
TURN SENSE LIGHT 4 ON
8100
$\Delta\dot{\rho}=0$
$\rho\Delta A\cos h=0$
$\rho\Delta h=0$

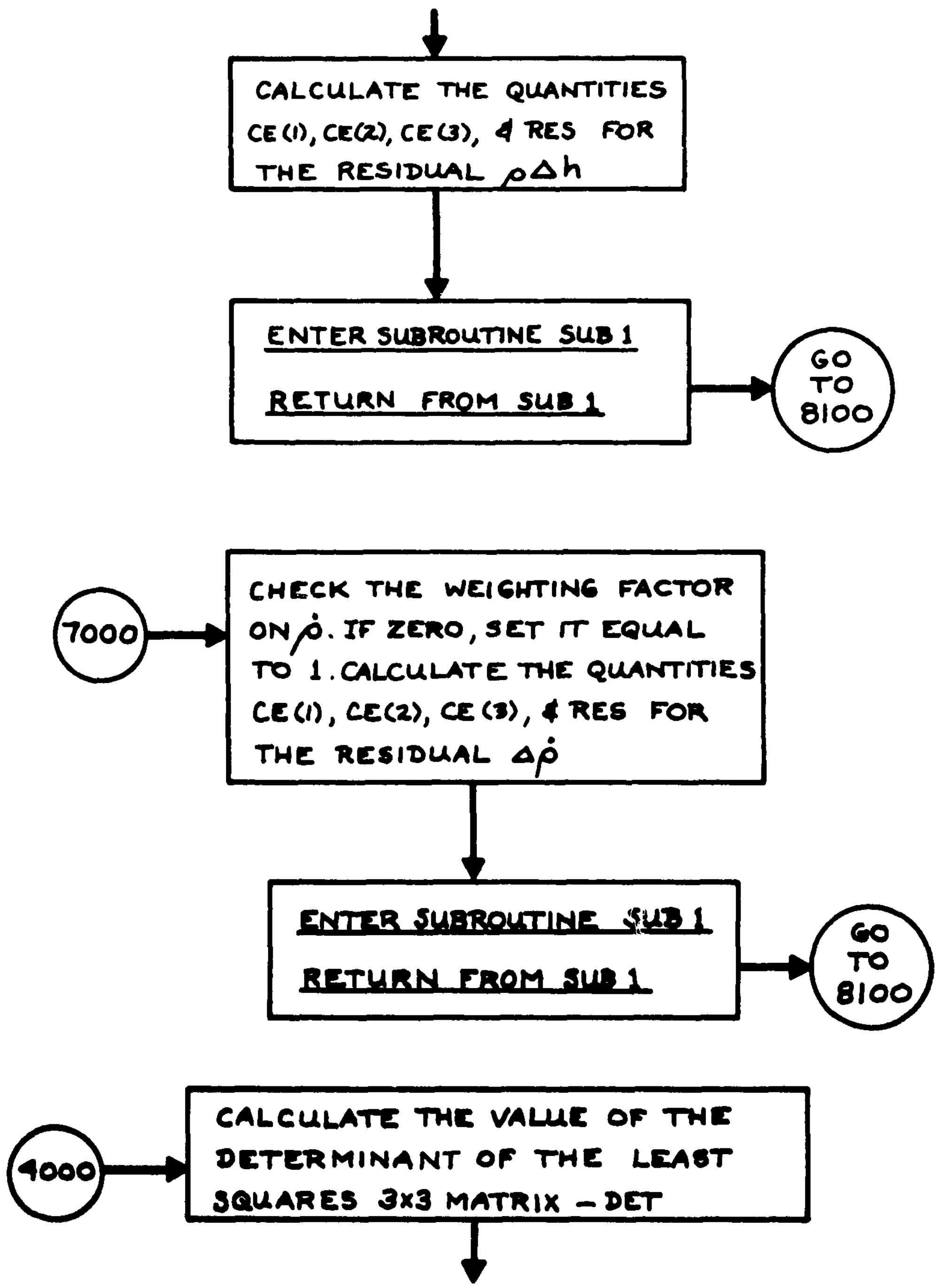

READ TAPE 0 FOR STATION INPUT DATA: THE OBSERVATION I.D. CONSTANT; NOCK; STATION I.D.; TIME OF OBSERVATION, T; THE THREE RESIDUALS, $\Delta\rho$, $\rho\Delta\alpha\cos\delta$, $\rho\Delta\delta$; AND THE WEIGHTING FACTORS ON EACH RESIDUAL, $W_\rho$, $W_\alpha$, $W_\delta$.
TAKE THE SQUARE ROOT OF EACH WEIGHTING FACTOR- NECESSARY IN ORDER THAT THE LEAST SQUARES PROCEDURE BE CORRECT
IS SENSE LIGHT 4 ON ?
YES
SET STATION CHECK: LSTID = STATION ID
IS STATION ID = LSTID ?
NO
GO TO 4000
YES

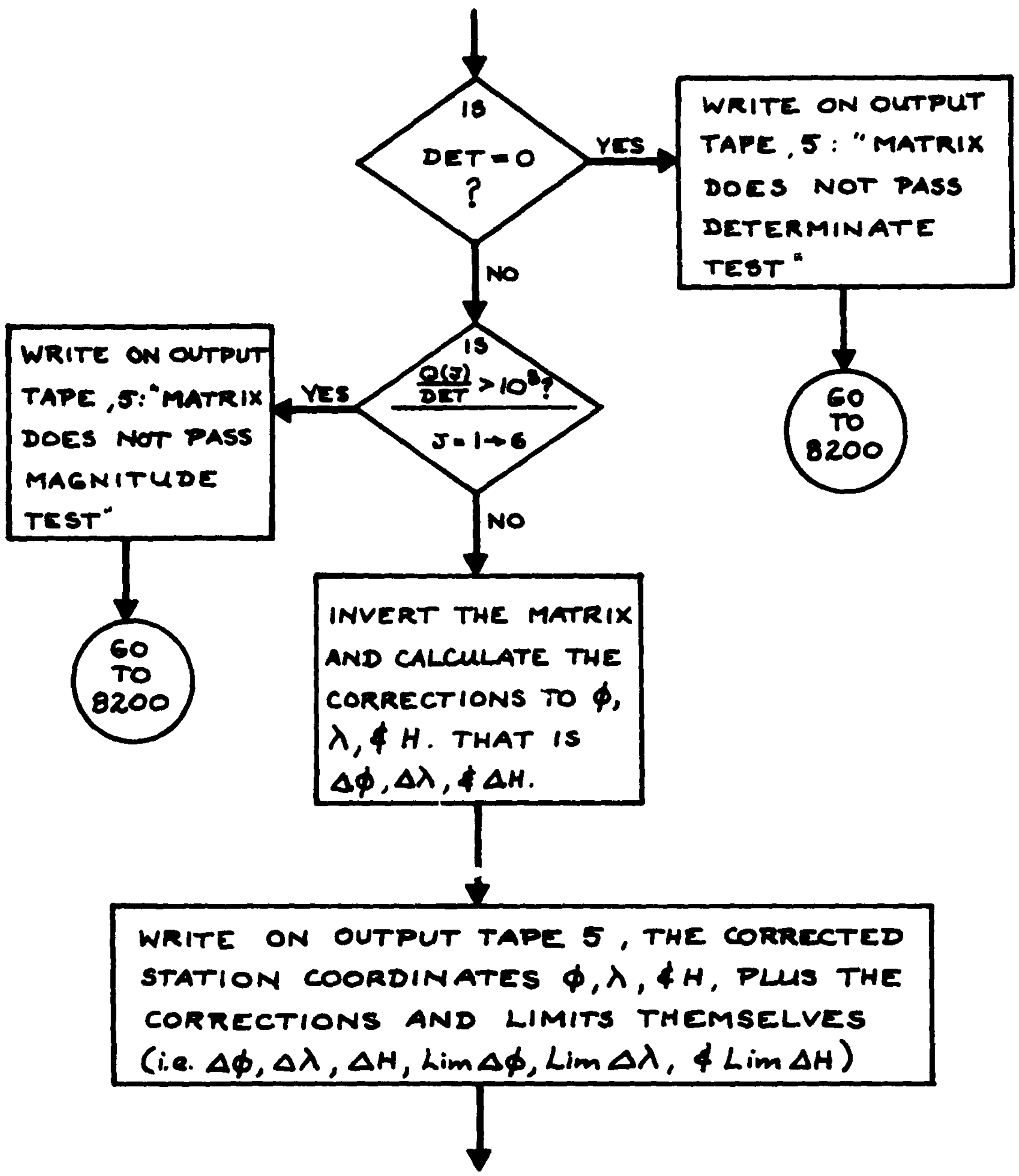

YES
IS NOCK = 0 ?
CALCULATE Θ AND THE COMPONENTS OF THE STATION LOCATION VECTOR AT THIS LONGITUDE $X1_s, Y1_s, Z1_s$.
NO
IS NOCK > 0 ?
A & h RESIDUALS
$\rho \Delta A \cos h = \rho \Delta \alpha \cos \delta$
$\rho \Delta h = \rho \Delta \delta$
$W_A = W_\alpha$
$W_h = W_\delta$
NO
$\dot{\rho}$ RESIDUAL
$\Delta \dot{\rho} = \Delta \rho$
$W_{\dot{\rho}} = W_\rho$
NFLAG = 1
READ EPHEMERIS TAPE (3) - OUTPUT TAPE FROM PART 1 - AND OBTAIN THE GEOCENTRIC COMPONENTS OF $r$ - $x, y, z$, - AND $\dot{r}$ - $\dot{x}, \dot{y}, \dot{z}$ - FOR A CORRESPONDING TIME ; $T_{OB}$
IS $|(T_{OB}-T)-.001|>0$ ?
YES
NO
IS $(T_{OB}-T)>0$ ?
YES
BACK SPACE EPHEMERIS TAPE (3) TWO RECORDS
NO

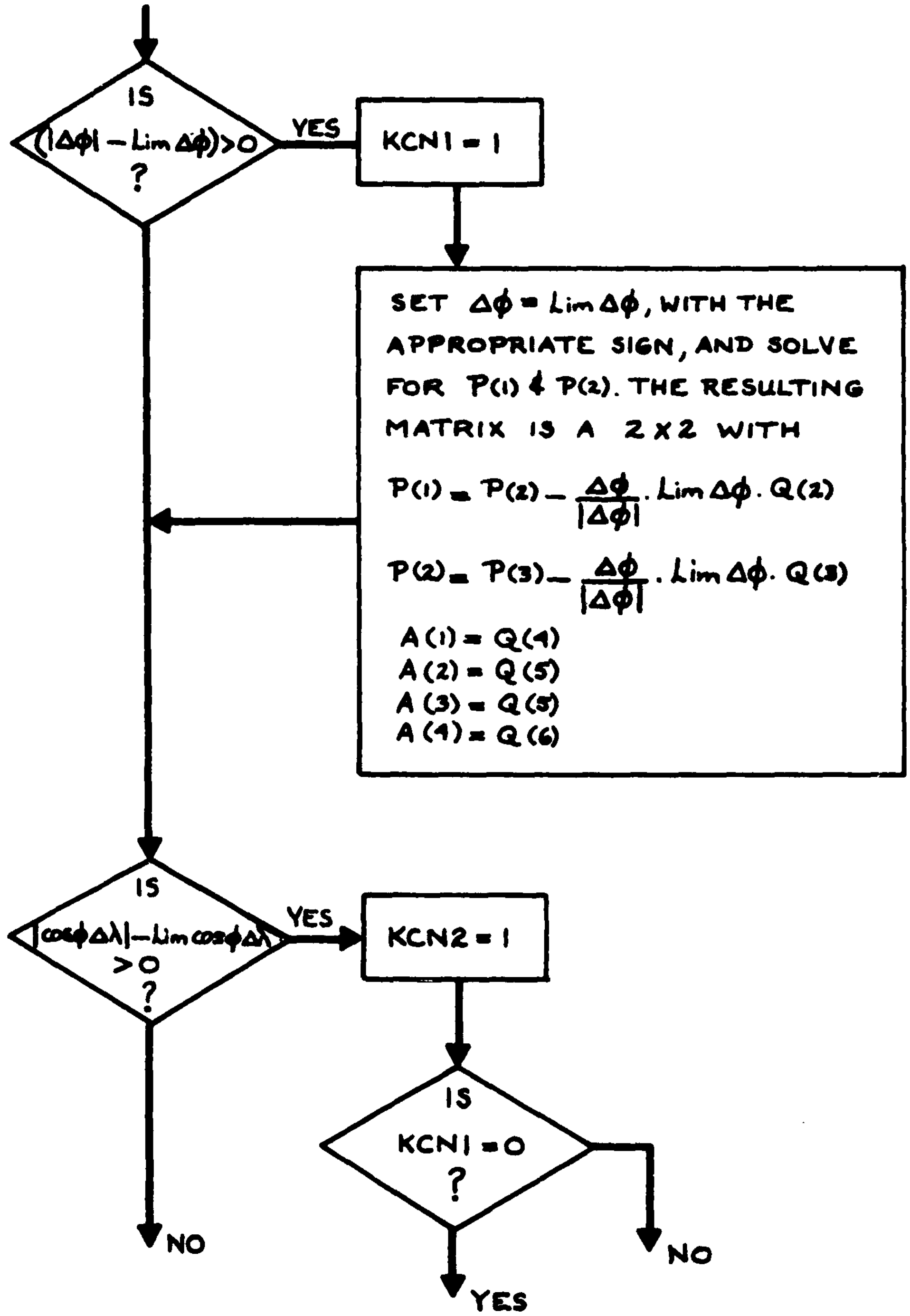

NO
CALCULATE THE GEOCENTRIC COMPONENTS OF $\rho$ - $\rho_X$, $\rho_Y$, $\rho_Z$ - AND ITS DIRECTION COSINES $L_X, L_Y, L_Z$
IS NFLAG = 0 ?
NO
NFLAG = 0
GO TO 7000
YES
IS $|\Delta\rho| \leq 10^{-10}$ ?
YES
GO TO 8000
CHECK WEIGHTING FACTOR ON $\rho$. IF ZERO, SET IT EQUAL TO 1. CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL $\Delta\rho$


ENTER SUBROUTINE SUB 1
INPUT TO SUBROUTINE ARE THE CURRENT VALUES OF THE P & Q ARRAYS, AS WELL AS THE LE ARRAY AND RES
RETURN FROM SUBROUTINE SUB 1
GO TO 8000
8000
IS $(|\rho\Delta h| + |\rho\Delta\alpha \cos h|) \leq 10^{-10}$ ?
NO
GO TO 8200
YES
IS $(|\rho\Delta\delta| + |\rho\Delta\alpha \cos\delta|) \leq 10^{-10}$ ?
YES
GO TO 8100
NO
CHECK WEIGHTING FACTORS ON $\alpha$ & $\delta$. IF ZERO, SET THEM EQUAL TO 1. CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL $\rho\Delta\alpha \cos\delta$

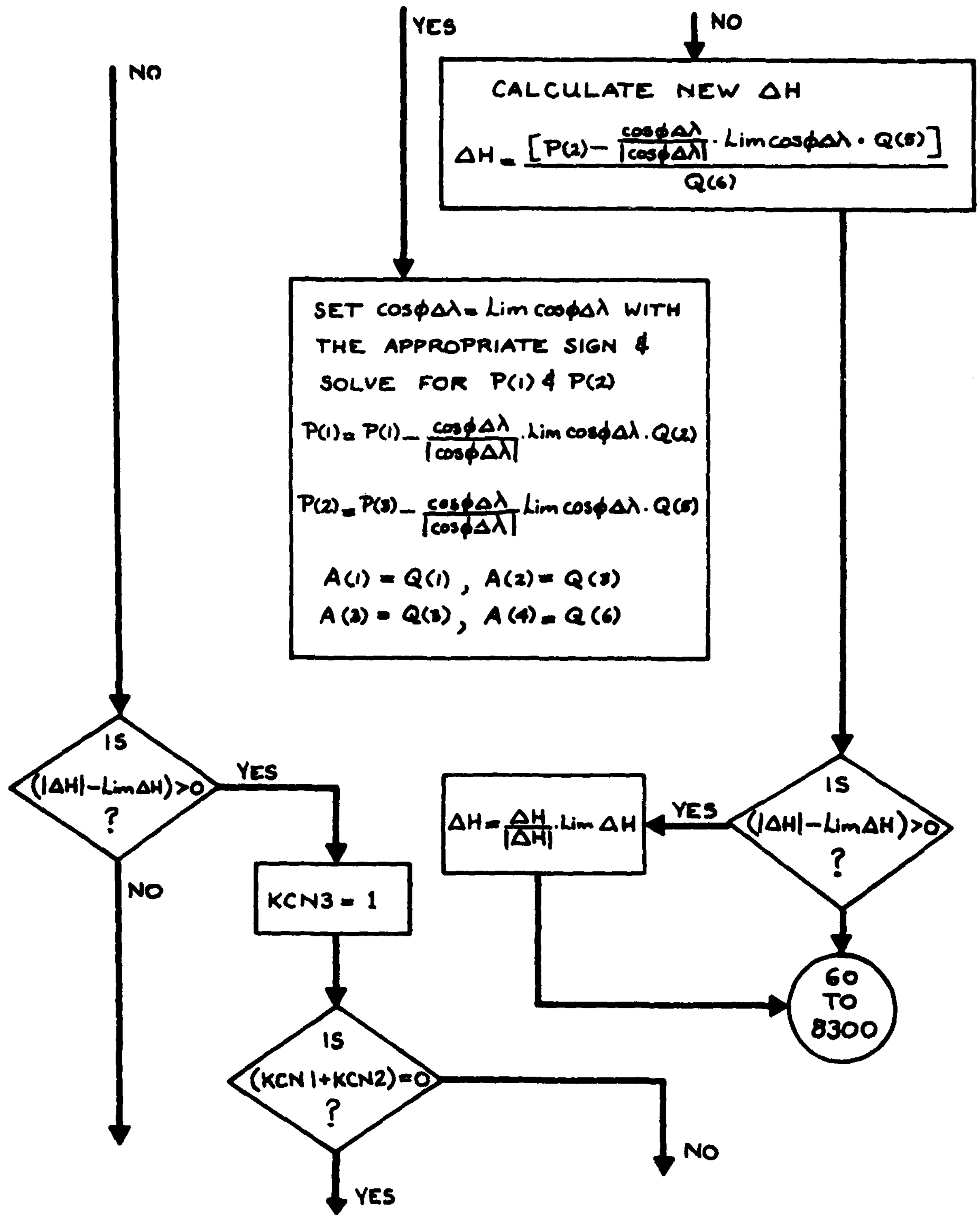

ENTER SUBROUTINE SUB 1
INPUT TO SUBROUTINE ARE THE CURRENT VALUES OF THE P, Q, & CE ARRAYS, AS WELL AS RES.
RETURN FROM SUB1
CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL ρΔδ
ENTER SUBROUTINE SUB 1
MAKE COMPUTATIONS DESCRIBED EARLIER
RETURN FROM SUB 1
GO TO 8100
8200
CHECK THE WEIGHTING FACTOR ON A & h. IF ZERO, SET THEM EQUAL TO 1. CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL ρΔAcosh
ENTER SUBROUTINE SUB 1
RETURN FROM SUB 1

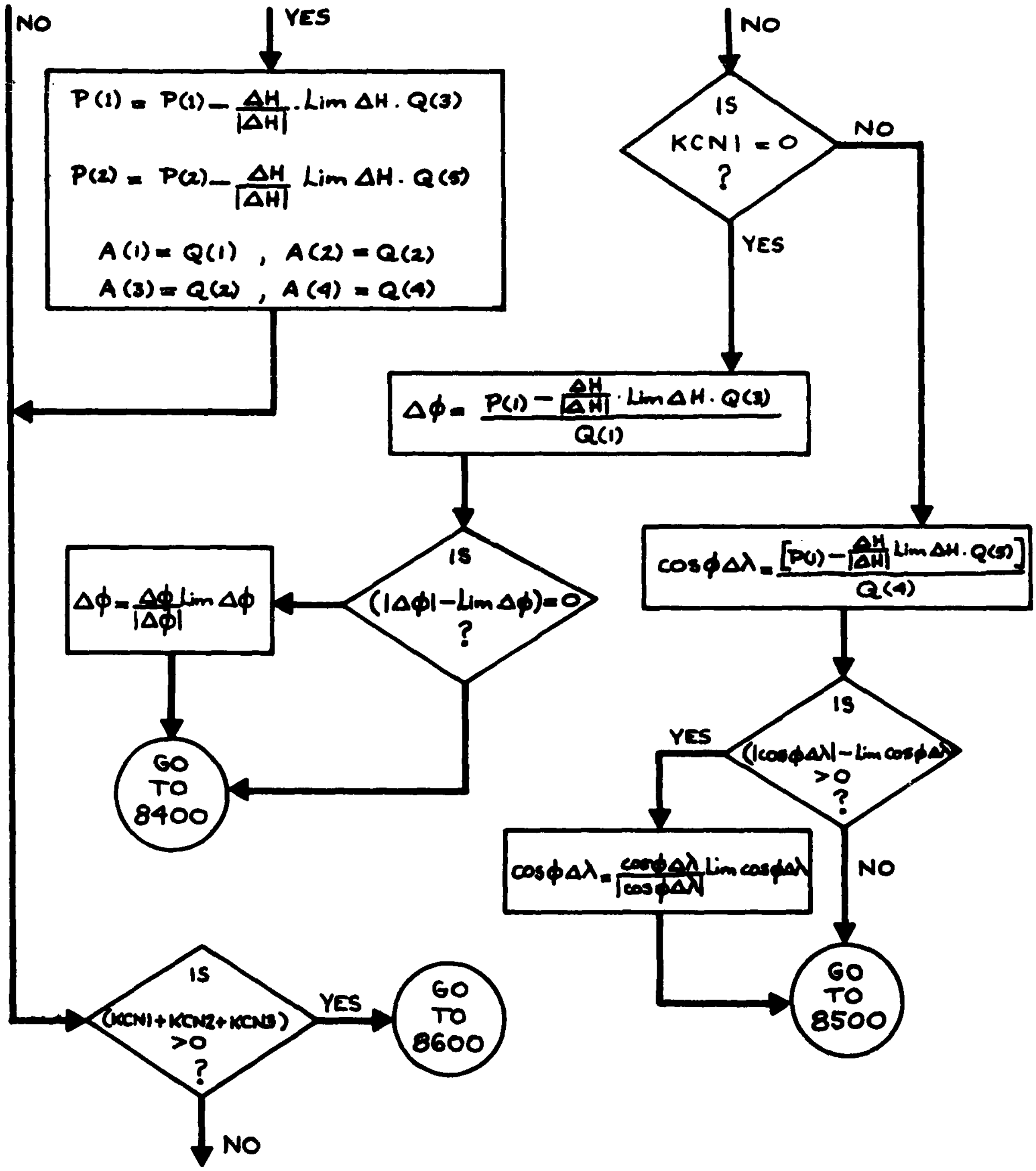

CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL ρΔh
ENTER SUBROUTINE SUB 1
RETURN FROM SUB 1
GO TO 8100
7000
CHECK THE WEIGHTING FACTOR ON ρ̇. IF ZERO, SET IT EQUAL TO 1. CALCULATE THE QUANTITIES CE(1), CE(2), CE(3), & RES FOR THE RESIDUAL Δρ̇
ENTER SUBROUTINE SUB 1
RETURN FROM SUB 1
GO TO 8100
4000
CALCULATE THE VALUE OF THE DETERMINANT OF THE LEAST SQUARES 3x3 MATRIX – DET

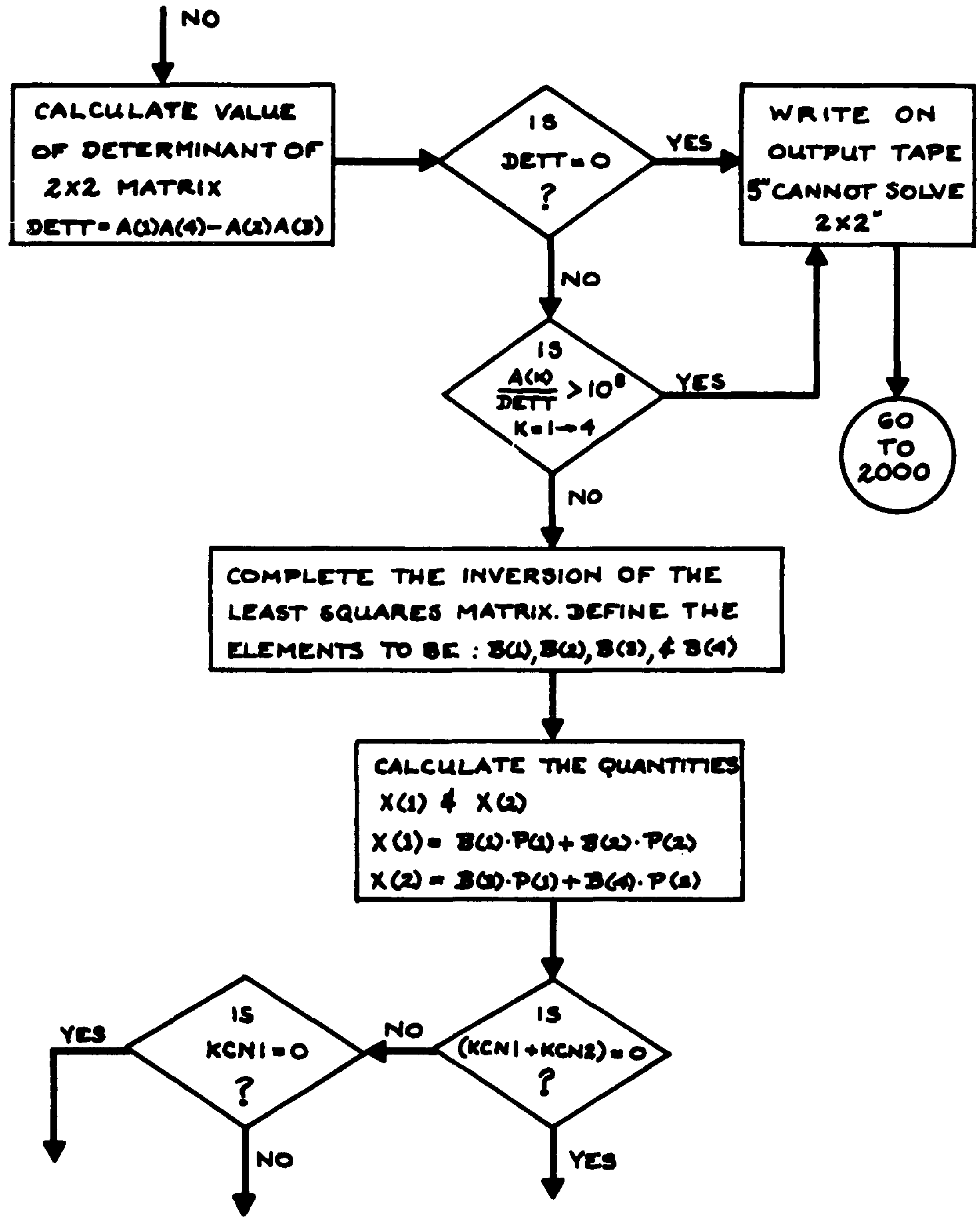

IS DET = 0 ?
YES
WRITE ON OUTPUT TAPE, 5: "MATRIX DOES NOT PASS DETERMINATE TEST"
GO TO 8200
NO
IS $\frac{Q(J)}{DET} > 10^{8}$? J = 1→6
YES
WRITE ON OUTPUT TAPE, 5: "MATRIX DOES NOT PASS MAGNITUDE TEST"
GO TO 8200
NO
INVERT THE MATRIX AND CALCULATE THE CORRECTIONS TO $\phi$, $\lambda$, & H. THAT IS $\Delta\phi$, $\Delta\lambda$, & $\Delta H$.
WRITE ON OUTPUT TAPE 5, THE CORRECTED STATION COORDINATES $\phi$, $\lambda$, & H, PLUS THE CORRECTIONS AND LIMITS THEMSELVES (i.e. $\Delta\phi$, $\Delta\lambda$, $\Delta H$, Lim $\Delta\phi$, Lim $\Delta\lambda$, & Lim $\Delta H$)

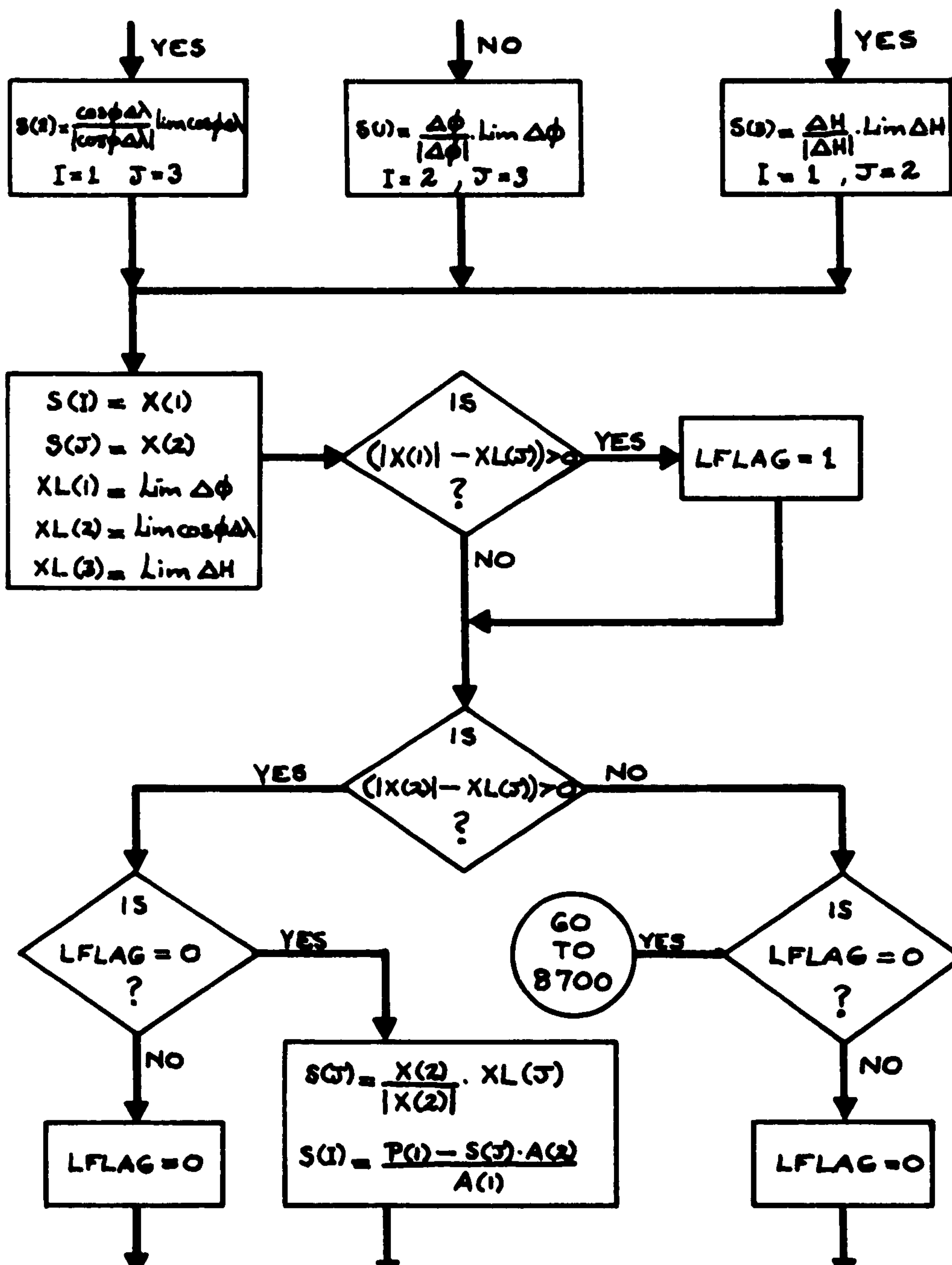


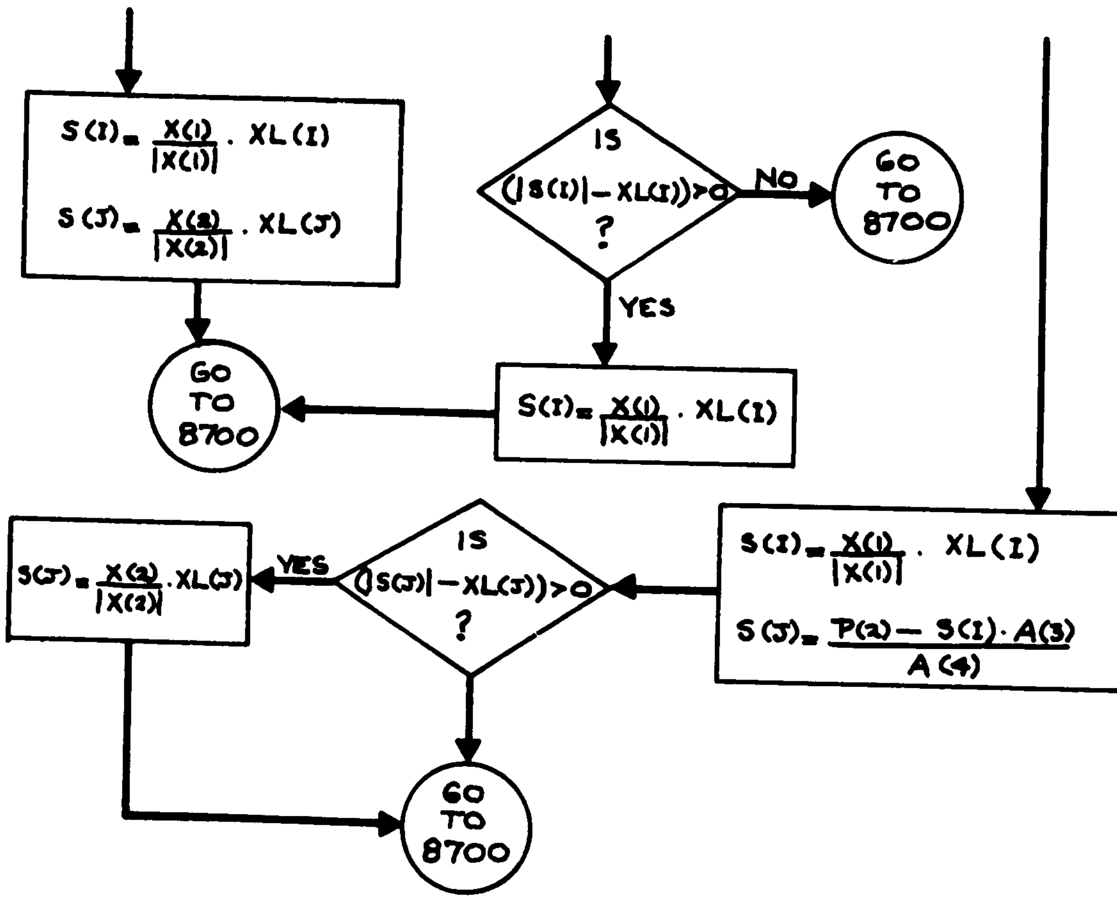
IS
$(|\Delta\phi| - Lim\,\Delta\phi) > 0$
?
YES
KCN1 = 1
SET $\Delta\phi = Lim\,\Delta\phi$, WITH THE APPROPRIATE SIGN, AND SOLVE FOR P(1) & P(2). THE RESULTING MATRIX IS A 2 X 2 WITH
$P(1) = P(2) - \frac{\Delta\phi}{|\Delta\phi|} \cdot Lim\,\Delta\phi \cdot Q(2)$
$P(2) = P(3) - \frac{\Delta\phi}{|\Delta\phi|} \cdot Lim\,\Delta\phi \cdot Q(3)$
A(1) = Q(4)
A(2) = Q(5)
A(3) = Q(5)
A(4) = Q(6)
IS
$|\cos\phi\Delta\lambda| - Lim\,\cos\phi\Delta\lambda$
$> 0$
?
YES
KCN2 = 1
IS
KCN1 = 0
?
NO
YES
NO

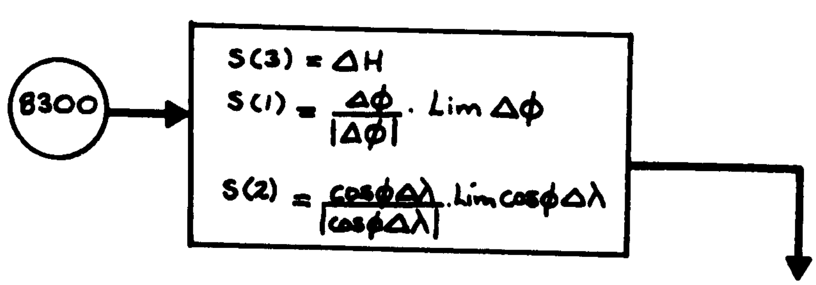

NO
YES
NO
CALCULATE NEW ΔH
$\Delta H = \dfrac{\left[P(2) - \frac{\cos\phi\Delta\lambda}{|\cos\phi\Delta\lambda|} \cdot Lim\cos\phi\Delta\lambda \cdot Q(5)\right]}{Q(6)}$
SET cosφΔλ = Lim cosφΔλ WITH THE APPROPRIATE SIGN & SOLVE FOR P(1) & P(2)
$P(1) = P(1) - \frac{\cos\phi\Delta\lambda}{|\cos\phi\Delta\lambda|} \cdot Lim\cos\phi\Delta\lambda \cdot Q(2)$
$P(2) = P(3) - \frac{\cos\phi\Delta\lambda}{|\cos\phi\Delta\lambda|} \cdot Lim\cos\phi\Delta\lambda \cdot Q(5)$
A(1) = Q(1), A(2) = Q(3)
A(3) = Q(3), A(4) = Q(6)
IS (|ΔH| − LimΔH) > 0 ?
YES
NO
KCN3 = 1
IS (KCN1+KCN2) = 0 ?
NO
YES
$\Delta H = \frac{\Delta H}{|\Delta H|} \cdot Lim\,\Delta H$
YES
IS (|ΔH| − LimΔH) > 0 ?
GO TO 8300

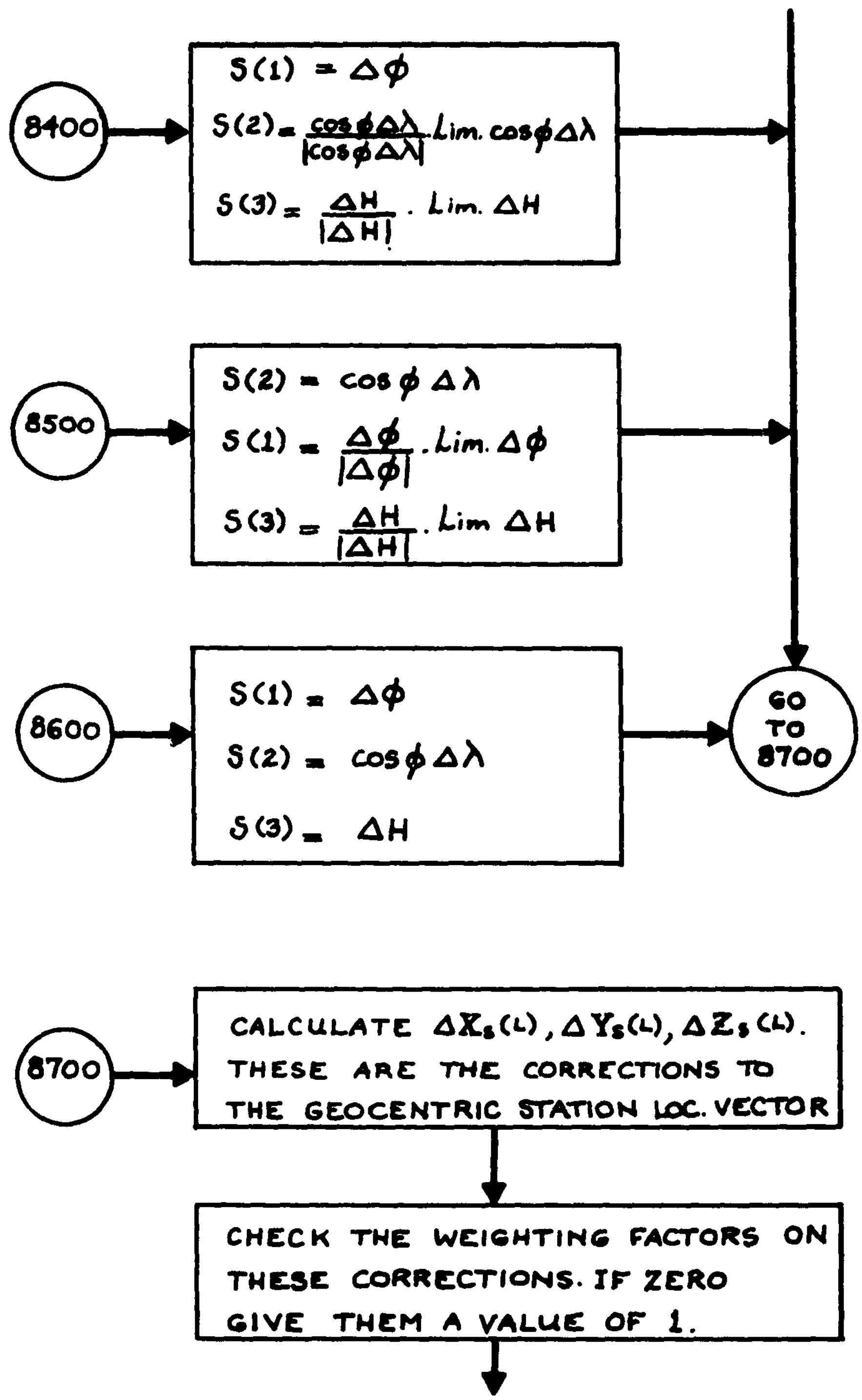

NO
YES
NO
P(1) = P(1) − ΔH/|ΔH| · Lim ΔH · Q(3)
P(2) = P(2) − ΔH/|ΔH| Lim ΔH · Q(5)
A(1) = Q(1) , A(2) = Q(2)
A(3) = Q(2) , A(4) = Q(4)
IS KCN1 = 0 ?
NO
YES
Δφ = [P(1) − ΔH/|ΔH| · Lim ΔH · Q(3)] / Q(1)
cosφΔλ = [P(1) − ΔH/|ΔH| Lim ΔH · Q(5)] / Q(4)
IS (|Δφ| − Lim Δφ) = 0 ?
Δφ = Δφ/|Δφ| Lim Δφ
GO TO 8400
IS (|cosφΔλ| − Lim cosφΔλ) > 0 ?
YES
NO
cosφΔλ = cosφΔλ/|cosφΔλ| Lim cosφΔλ
GO TO 8500
IS (KCN1 + KCN2 + KCN3) > 0 ?
YES
GO TO 8600
NO

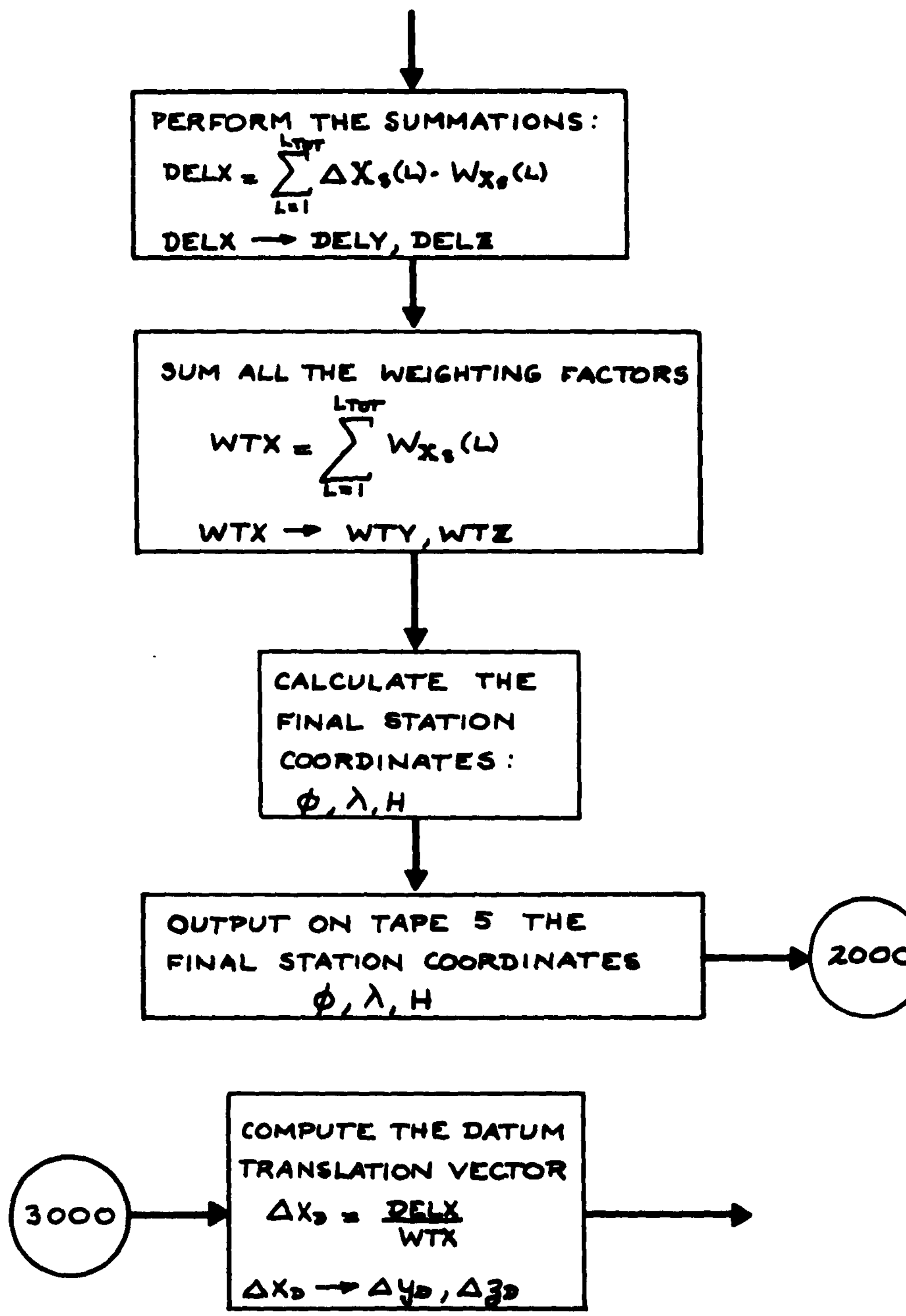

NO
CALCULATE VALUE OF DETERMINANT OF 2X2 MATRIX
DETT = A(1)A(4)−A(2)A(3)
IS DETT = 0 ?
YES
WRITE ON OUTPUT TAPE 5 "CANNOT SOLVE 2X2"
NO
IS A(K)/DETT > 10^8 K=1→4
YES
GO TO 2000
NO
COMPLETE THE INVERSION OF THE LEAST SQUARES MATRIX. DEFINE THE ELEMENTS TO BE : B(1), B(2), B(3), & B(4)
CALCULATE THE QUANTITIES X(1) & X(2)
X(1) = B(1)·P(1) + B(2)·P(2)
X(2) = B(3)·P(1) + B(4)·P(2)
IS (KCN1 + KCN2) = 0 ?
NO
YES
IS KCN1 = 0 ?
YES
NO

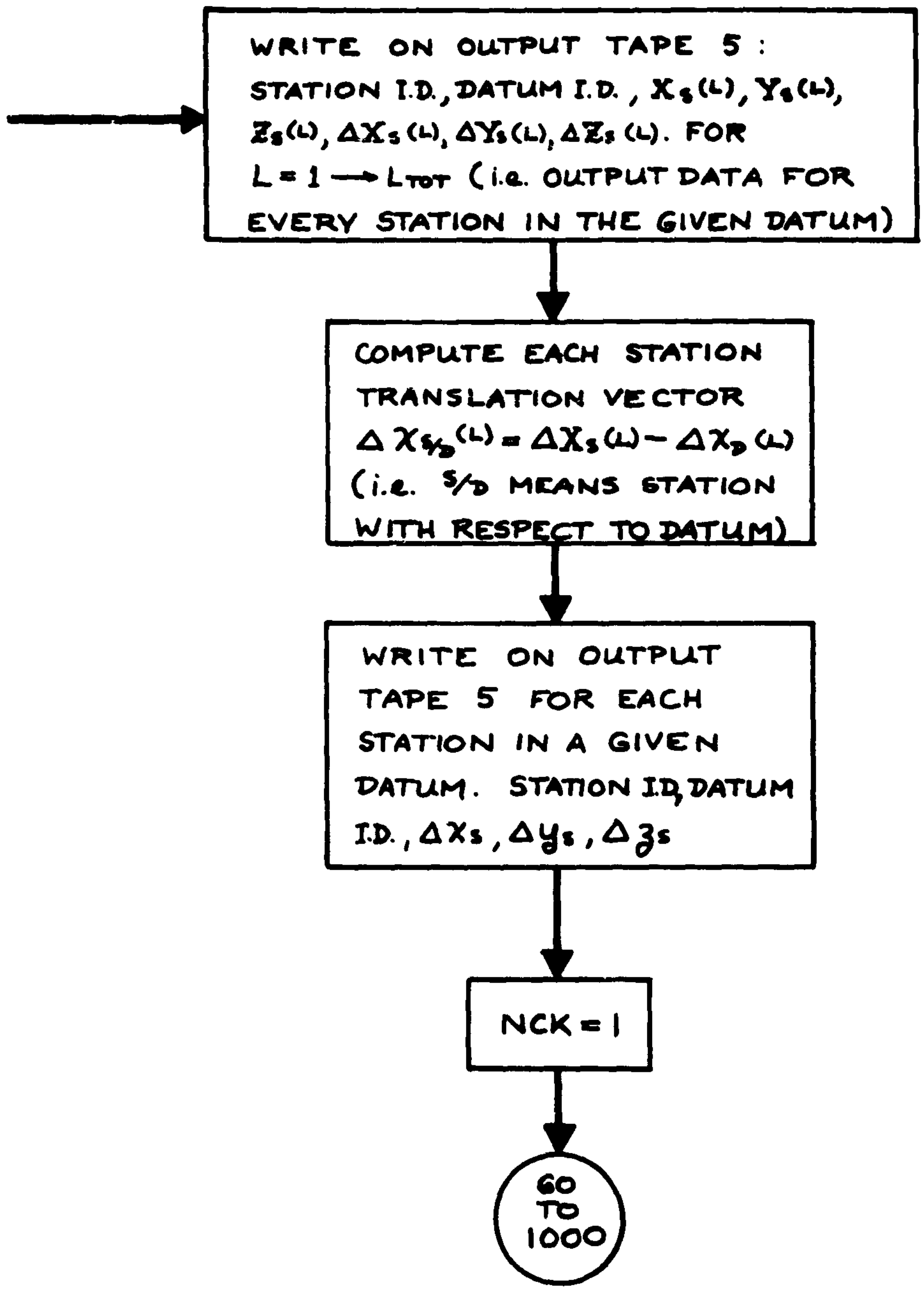

YES
NO
YES
S(2) = cosφΔλ/|cosφΔλ| · Lim cosφΔλ
I = 1 J = 3
S(1) = Δφ/|Δφ| · Lim Δφ
I = 2 , J = 3
S(3) = ΔH/|ΔH| · Lim ΔH
I = 1 , J = 2
S(I) = X(1)
S(J) = X(2)
XL(1) = Lim Δφ
XL(2) = Lim cosφΔλ
XL(3) = Lim ΔH
IS
(|X(1)| − XL(J)) > 0
?
YES
LFLAG = 1
NO
IS
(|X(2)| − XL(J)) > 0
?
YES
NO
IS
LFLAG = 0
?
YES
NO
LFLAG = 0
S(J) = X(2)/|X(2)| · XL(J)
S(I) = (P(1) − S(J)·A(2))/A(1)
GO
TO
8700
YES
IS
LFLAG = 0
?
NO
LFLAG = 0


S(I) = X(I)/|X(I)| . XL(I)
S(J) = X(2)/|X(2)| . XL(J)
GO TO 8700
IS (|S(I)| − XL(I)) > 0 ?
NO
GO TO 8700
YES
S(I) = X(I)/|X(I)| . XL(I)
S(I) = X(I)/|X(I)| . XL(I)
S(J) = (P(2) − S(I) · A(3)) / A(4)
IS (|S(J)| − XL(J)) > 0 ?
YES
S(J) = X(2)/|X(2)| . XL(J)
GO TO 8700

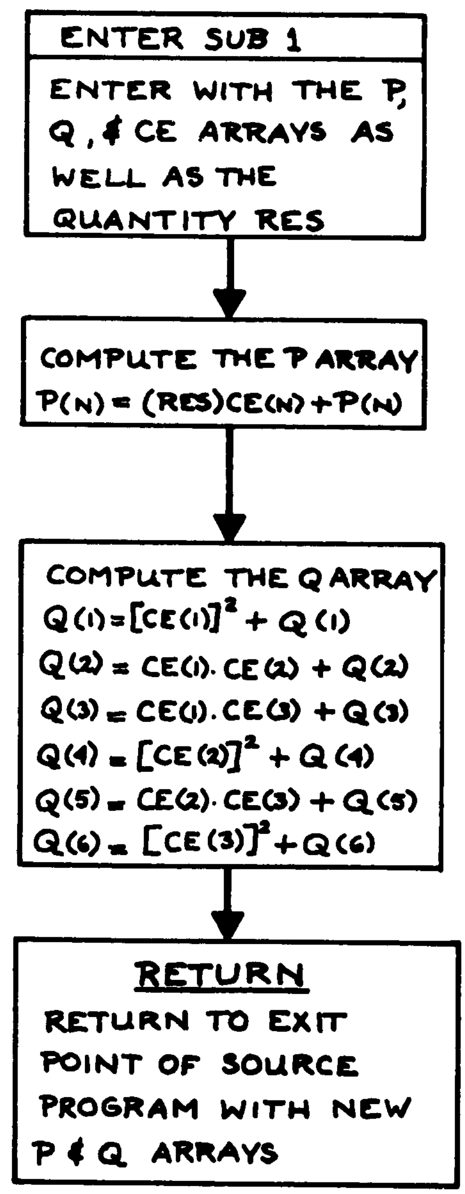

8300
S(3) = ΔH
S(1) = Δφ/|Δφ| . Lim Δφ
S(2) = cosφΔλ/|cosφΔλ| . Lim cosφΔλ


8400
S(1) = Δφ
S(2) = cosφΔλ / |cosφΔλ| . Lim. cosφΔλ
S(3) = ΔH / |ΔH| . Lim. ΔH
8500
S(2) = cos φ Δλ
S(1) = Δφ / |Δφ| . Lim. Δφ
S(3) = ΔH / |ΔH| . Lim ΔH
8600
S(1) = Δφ
S(2) = cosφ Δλ
S(3) = ΔH
GO TO 8700
8700
CALCULATE ΔXs(L), ΔYs(L), ΔZs(L). THESE ARE THE CORRECTIONS TO THE GEOCENTRIC STATION LOC. VECTOR
CHECK THE WEIGHTING FACTORS ON THESE CORRECTIONS. IF ZERO GIVE THEM A VALUE OF 1.


PERFORM THE SUMMATIONS:
$DELX = \sum_{L=1}^{LTOT} \Delta X_S(L) \cdot W_{X_S}(L)$
DELX → DELY, DELZ
SUM ALL THE WEIGHTING FACTORS
$WTX = \sum_{L=1}^{LTOT} W_{X_S}(L)$
WTX → WTY, WTZ
CALCULATE THE FINAL STATION COORDINATES:
$\phi, \lambda, H$
OUTPUT ON TAPE 5 THE FINAL STATION COORDINATES
$\phi, \lambda, H$
2000
3000
COMPUTE THE DATUM TRANSLATION VECTOR
$\Delta X_D = \frac{DELX}{WTX}$
$\Delta X_D \rightarrow \Delta Y_D, \Delta Z_D$


WRITE ON OUTPUT TAPE 5 : STATION I.D., DATUM I.D., $X_S(L)$, $Y_S(L)$, $Z_S(L)$, $\Delta X_S(L)$, $\Delta Y_S(L)$, $\Delta Z_S(L)$. FOR $L = 1 \longrightarrow L_{TOT}$ (i.e. OUTPUT DATA FOR EVERY STATION IN THE GIVEN DATUM)
COMPUTE EACH STATION TRANSLATION VECTOR $\Delta X_{S/D}(L) = \Delta X_S(L) - \Delta X_D(L)$ (i.e. S/D MEANS STATION WITH RESPECT TO DATUM)
WRITE ON OUTPUT TAPE 5 FOR EACH STATION IN A GIVEN DATUM. STATION I.D., DATUM I.D., $\Delta X_S$, $\Delta Y_S$, $\Delta Z_S$
NCK = 1
GO TO 1000

# SUBROUTINE SUB 1



## 3.4 INPUT AND OUTPUT FORMATS

Residual Information

| Columns | Quantity and Explanation | Format |
|---|---|---|
| 1-4 | Fixed Point Constant (FLAG)<br>0 implies $\alpha$, $\delta$ residuals<br>+ N implies A, h residuals<br>- N implies $\dot{\rho}$ residuals | I 4 |
| 5-8 | Station Identification Number (fixed point constant) | I 4 |
| 9-21 | Time (min) - Time of Observation | E 13.8 |
| 22-34 | $\Delta\rho$ or $\Delta\dot{\rho}$ - (Km or KM/sec) | E 13.8 |
| 35-47 | $\rho \cos h \, \Delta A$ or $\rho \cos\delta \, \Delta\alpha$ - (Km) | E 13.8 |
| 48-60 | $\Delta h$ or $\Delta\delta$ - (Km) | E 13.8 |
| 61-64 | $W_\rho$ or $W_{\dot{\rho}}$ | F 4.2 |
| 65-68 | $W_A$ or $W_\alpha$ | F 4.2 |
| 69-72 | $W_h$ or $W_\delta$ | F 4.2 |

Note 1

Range residuals can be processed with either type of angles $\alpha$, $\delta$ or A, h; range-rate information must be processed separately.

Note 2

Residuals and weighting factors must be entered in the designated locations. If a residual does not exist for a particular case, the corresponding location should be blank.

## General Information - Input

| Columns | Quantity and Explanation | Format |
|---|---|---|
| 1-14 | Theta Greenwich ($\theta_{gr}$) - the Greenwich sidereal time at epoch time $t_o$ (degrees) | F 14.0* |
| 15-72 | Blank | |

*Recall that a punched decimal will override the decimal stated in the F format.

## Station Information - Input

| Columns | Quantity and Explanation | Format |
|---|---|---|
| 1-4 | Datum identification number | I 4 |
| 5-8 | Station identification number | I 4 |
| 9-17 | Latitude of station $\phi$ - degrees | F 9.0 |
| 18-27 | Longitude of station $\lambda_E$ - degrees | F 10.0 |
| 28-33 | Height of station H - meters | F 6.0 |
| 34-40 | Limit of $\Delta\theta$ - degrees | F 7.0 |
| 41-48 | Limit of cos $\theta\Delta\lambda$ - degrees | F 8.0 |
| 49-54 | Limit of $\Delta H$ - meters | F 6.0 |
| 55-60 | Weighting Factor for X component of station location vector - $W_{X_S}$ | F 6.0 |

Station Information - Input (continued)

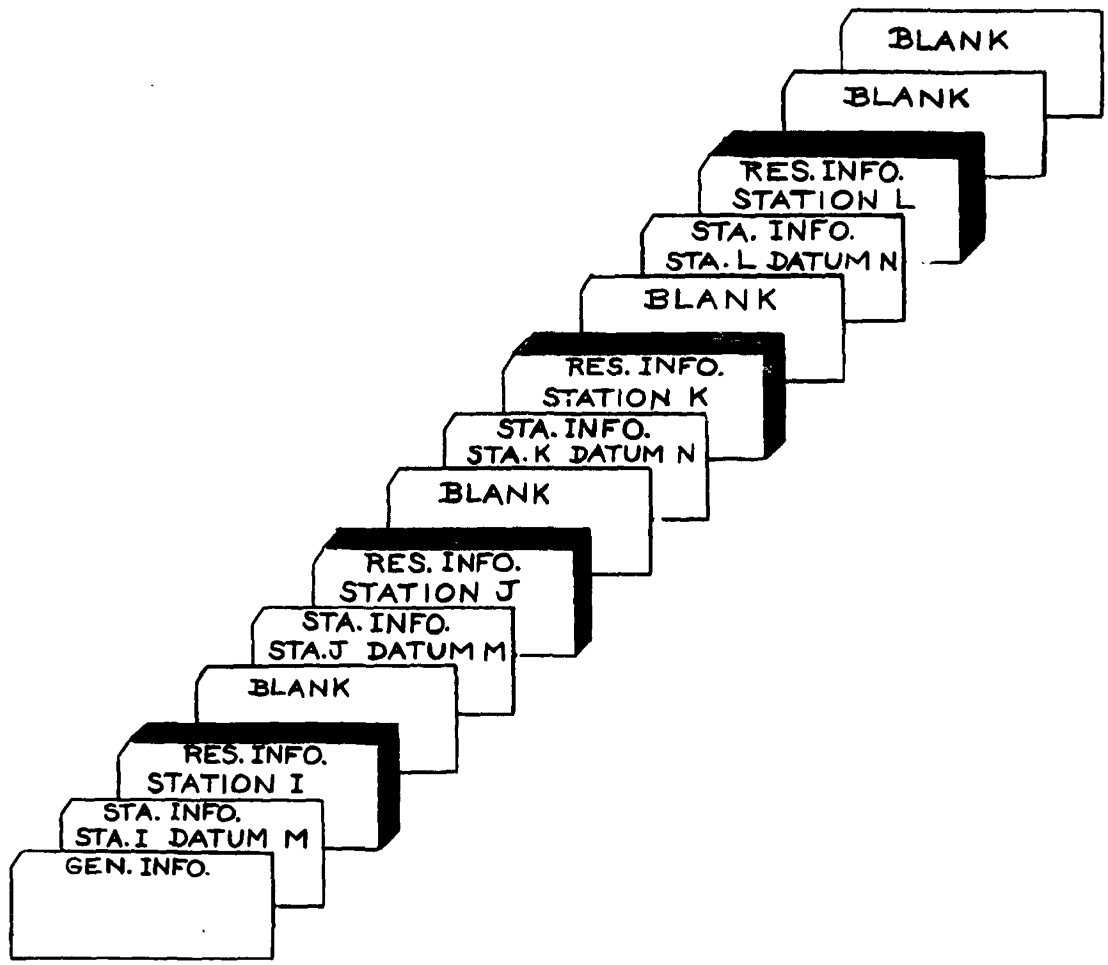
| Columns | Quantity and Explanation | Format |
|---|---|---|
| 61-66 | Weighting factor for Y component of station location vector - $W_{Y_s}$ | F 6.0 |
| 67-72 | Weighting factor for Z component of station location vector - $W_{Z_s}$ | F 6.0 |

Position and Velocity Data

The ephemeris data is input to this program as a binary tape. This tape is output in the proper format from the Ephemeris Program of Task 1. Its format is as follows: time of observation, position of satellite, and velocity of satellite. Position and velocity are given in a geocentric coordinate system with units of Earth radii and Earth radii per kemin. Time is given in minutes. In symbolic form, the input format is t, x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$.

The position and velocity data is input on a binary tape through drive number 3.

The quantities -- residual information, general information, and station information-- are input into the present program from tape unit number 0. This tape can be prepared from punched cards of the formats listed above. Their order of input -- more than one datum and station -- is as follows:




The stations of a particular datum do not have to be in numerical order; however, all the observation residuals of a station and all of the various stations of a datum must be sorted and input immediately after the corresponding station and datum information.

The quantities in the output are as follows:

Page Output for Each Station

1 Datum ID ...... NDATID

2 Station ID .... NSTID

3 Assumed station latitude (degrees) ...... $\phi$

4 Assumed station longitude (degrees) ..... $\lambda$

5 Assumed station height (meters) ......... H

6 Corrected station latitude (degrees) .... $\phi$

7 Corrected station longitude (degrees) ... $\lambda$

8 Corrected station height (meters) ....... H

9 Correction to assumed station latitude (degrees) ..... $\Delta\phi$

10 Correction to assumed station longitude (degrees) .... $\cos\phi\,\Delta\lambda/\cos\phi$

11 Correction to assumed station height (meters) ........ $\Delta H$

12 Limit on correction to assumed station latitude (degrees) ... lim $\Delta\phi$

13 Limit on correction to assumed station longitude (degrees) .. lim $\cos\phi\,\Delta\lambda/\cos\phi$

14 Limit on correction to assumed station height (meters) ......lim $\Delta H$

15 Final station latitude (degrees) ....... $\phi$

16 Final station longitude (degrees) ...... $\lambda$

17 Final station height (meters) .......... H

Output for each Datum

18 The assumed rectangular coordinates of each station and their corrections (megameters) - $X_s(L)$, $Y_s(L)$, $Z_s(L)$, $\Delta X_s(L)$, $\Delta Y_s(L)$, and $\Delta Z_s(L)$

19 Datum correction vector (megameters) .... $\Delta X_D$, $\Delta Y_D$, $\Delta Z_D$

Output for each Datum (continued)

20 Station translation vector with respect to the datum for each station. (megameters) $\Delta X_s$, $\Delta Y_s$, $\Delta Z_s$.

This output is obtained by printing tape 5 with data select 0.

3.5 OPERATING PROCEDURE

The deck submitted at execution time has the following make up:

| Starting in Col. 17 | Col. 25 | Purpose |
|---|---|---|
| JOB | XXXXX... | Signals the start of a new job. Cols.25 on are typed on the flexowriter at the beginning of the program and may contain any desired alphanumeric information. |
| REWIND | 0,3 | Rewinds tapes 0 and 3. |
| RPL | 1, DATA,GO | Reads routine named DATA from tape 1 into memory and transfers control to it. |
| TAPE | 0 | Control instruction for DATA telling it which tape to put the following data on. |
| - - - - | | Data to be put on tape 0. |
| - - - - | | |
| - - - - | | |
| - - - - | | |
| - - - - | | |
| ENDDATA | | Control instruction for DATA telling it that there is no more data to put on tape 0. |
| REWIND | 0 | Rewinds tape 0 |
| RPL | 4,XXX...XXX,GO | XXX...XXX is the identity of the Station Locator Program (up to 16 characters). It is read into memory from tape 4 and control is transferred to it. |

Tapes Used:

| Logical Tape No. | Use |
|---|---|
| 0 | Data input. |
| 3 | Binary input tape containing $t-t_o$, $\underline{r}$, and $\dot{\underline{r}}$ at observation times (output from Orbit Correction Program). |
| 4 | RPL tape containing Station Locator Program. |
| 5 | Output tape. |

The use of the Station Locator Program, together with the Orbit Correction Program is outlined in Section 2.5.

## 3.6 EXPERIMENTATION

Two test cases were used to check the use of the Station Locator Program, in conjunction with the Orbit Correction Program.

(1) Using the Differential Correction Program, observations were simulated from two stations for the following orbit:

$L_o$ = 4.890 210 9 $\qquad$ $h_{x_o}$ = - .618 213 08

$a_{xN_o}$ = -.007 371 516 8 $\qquad$ $h_{y_o}$ = .484 673 16

$a_{yN_o}$ = .007 750 635 7 $\qquad$ $h_{z_o}$ = .704 418 46

$t_o$ = 0.0 minutes, 98 days, 1961

The coordinates of the two stations were:

| Station 44 | Station 3 |
|---|---|
| $\phi = 27\overset{\circ}{.}020\ 3$ | $\phi = 35\overset{\circ}{.}707\ 2$ |
| $\lambda_E = 279\overset{\circ}{.}886\ 9.$ | $\lambda_E = 139\overset{\circ}{.}491\ 7$ |
| H = 12 meters | H = 0 meters |

Using these observations and the same orbital elements, the Orbit Correction Program was used to compute residuals by altering the station coordinates. The residuals generated were four slant ranges, four pairs of right ascension and declination and four pairs of azimuth and elevation angle for Station 3 and three slant ranges, three pairs of right ascension and declination and four pairs of azimuth and elevation angle for Station 44. The root mean square of the residuals in these observations caused by "moving" the stations was 0.1714 km. The amount of the station displacements is shown in Table 1 below.

These residuals were used in the Station Locator Program to correct the station coordinates. The limits on the correction were set at very large numbers so that all three coordinates would be corrected for both stations. These corrected station coordinates were fed back into the Orbit Correction Program to get new residuals which were again fed into the Station Locator Program to further correct the station locations. The results are summarized in the following table.

TABLE 1

RESULTS OF EXPERIMENTATION

| | Displaced Station | After First Correction | After Second Correction |
|---|---|---|---|
| Station 44 | | | |
| $\phi$ - degrees | 27.021 8 | 27.020 241 | 27.020 299 |
| $\lvert\Delta\phi\rvert$ - meters | 166.7 | 6.6 | .1 |
| $\lambda_E$ - degrees | 279.885 4 | 279.887 01 | 279.886 90 |
| $\lvert\Delta\lambda\rvert$ - meters | 148.5 | 10.9 | 0 |
| H - meters | 112.0 | 17.109 621 | 12.266 |
| $\lvert\Delta H\rvert$ - meters | 100.0 | 5.1 | .3 |
| Station 3 | | | |
| $\phi$ - degrees | 35.705 2 | 35.706 880 | 35.707 172 |
| $\lvert\Delta\phi\rvert$ - meters | 222.2 | 35.6 | 3.1 |
| $\lambda_E$ - degrees | 139.493 2 | 139.491 62 | 139.491 69 |
| $\lvert\Delta\lambda\rvert$ - meters | 135.4 | 7.2 | .9 |
| H - meters | 200.0 | - 27.791 097 | - 6.695 |
| $\lvert\Delta H\rvert$ - meters | 200.0 | 27.8 | 6.7 |

(2) Using the same simulated observations as before and the same displaced station coordinates as before, but with the following orbital elements:

$$L_o = 4.891 \qquad h_{x_o} = -.618\ 2$$

$$a_{xN_o} = -.007\ 37 \qquad h_{y_o} = .484\ 7$$

$$a_{yN_o} = .007\ 75 \qquad h_{z_o} = .704\ 4$$

four differential corrections were made with the Orbit Correction Program to correct the orbital elements as much as possible before correcting the station locations. The final orbital elements were:

$$L_o = 4.890\ 224\ 84 \qquad h_{x_o} = -.618\ 183\ 83$$

$$a_{xN_o} = -.007\ 377\ 960 \qquad h_{y_o} = .484\ 681\ 51$$

$$a_{yN_o} = .007\ 746\ 031 \qquad h_{z_o} = .704\ 438\ 41$$

The initial root mean square of the range and angle residuals was 20.377 km. and for the range-rate residuals it was .030 982 km/sec, while the final root mean square for the range and angle residuals was .141 km and for the range-rate residuals it was .000 177 km/sec. The final residuals were fed into the Station Locator Program to correct the station coordinates. Four observations of range rate from each station were used, in addition to the same observations used in the above experimentation. The results are summarized in Table 2.

TABLE 2

RESULTS OF EXPERIMENTATION

| | Before Station Correction | After Station Correction |
|---|---|---|
| Station 44 | | |
| $\phi$ - degrees | 27.021 8 | 27.020 296 |
| $\lvert\Delta\phi\rvert$ - meters | 166.7 | .4 |
| $\lambda_E$ - degrees | 279.885 4 | 279.887 01 |
| $\lvert\Delta\lambda\rvert$ - meters | 148.5 | 10.9 |
| H - meters | 112.0 | 19.247 218 |
| $\lvert\Delta H\rvert$ - meters | 100.0 | 7.2 |
| Station 3 | | |
| $\phi$ - degrees | 35.705 2 | 35.706 857 |
| $\lvert\Delta\phi\rvert$ - meters | 222.2 | 38.1 |
| $\lambda_E$ - degrees | 139.493 2 | 139.491 71 |
| $\lvert\Delta\lambda\rvert$ - meters | 135.4 | .9 |
| H - meters | 200.0 | - 32.768 020 |
| $\lvert\Delta H\rvert$ - meters | 200.0 | 32.8 |

# SECTION 4

# MODIFICATIONS

This section is devoted to suggested modifications to the programs described above and to subsidiary programs which may prove desirable.

The programs heretofore described use a single Earth ellipsoid for all datums. The station coordinates produced by the Station Locator Program will be correct in cartesian coordinates and consistent with that ellipsoid in latitude, longitude and height. All station coordinates should, in the near future, be recomputed on the basis of such a world-wide datum or, alternative, available in a cartesian system.

There exist, however, good observations (e.g., surveys) which are not used by the present programs. They connect many more stations than can observe the geodetic satellite. The presence of such ties is felt in the Station Locator Program in the limits imposed on the station coordinate corrections. An alternative method of accounting for such additional information as the geodetic ties is to represent it by equations of condition which state that each station is where the surveys put it. Initially these equations will say for every station coordinate:

$$\Delta X_j = 0 .$$

Each equation should be weighted according to the accuracy of the previous determination of that coordinate. After the first correction, the right sides of the equations would no longer be zero. Each equation then would be equal to the negative sum of all the previous corrections to that coordinate. The advantages of this procedure include:

(1) No station position is ever underdetermined.

(2) Because the limits are not used, there is but one solution.

(3) A new uncertainty can be generated on the basis of both the old ties and the new geodetic satellite observations.

The third feature can be added to the present Station Locator Program only insofar as the new observations are concerned. If valid estimates of the uncertainties in each observation are used, the uncertainty in each station coordinate can be generated.

The uncertainties in the observations can also be translated into uncertainties in orbital elements. This feature can be added to the Orbit Correction Program. At the same time the observational uncertainties can be used to weight the corresponding equations of the Orbit Correction Program. These features will improve the solution for the elements, give each element a figure of merit and enable the program to be used to evaluate various geodetic satellite orbits with respect to their accuracy in locating a given station.

If these weighting factors are introduced into the Orbit Correction Program, the criteria for the rejection of discordant observations should be re-examined. Rejection and/or weighting on the basis of the distribution and numbers of observations should be considered.

Other changes could be introduced into the satellite orbit model to improve accuracy. These include the introduction of the perturbative acceleration due to high-order zonal and sectorial harmonics of the Earth's gravitational potential and due to the pressure of solar radiation, as well as refinements in the computation of aerodynamic drag. The effects of lunar and solar attraction and of magnetic forces should be evaluated and introduced if necessary.

As indicated in the Introduction, the two programs should be combined in a flexible manner so that only orbital elements or orbital elements and station coordinates simultaneously could be corrected.

The values of the coefficients of the harmonics of the Earth's gravitational potential may be determined by differential correction processes also. Whether this should be done simultaneously with other corrections is subject to the same sort of argument as presented in the Introduction.

## Appendix A

## Glossary of Symbols, Constants, and Units

| Symbol | Definition | Units |
|---|---|---|
| $a$ | Mean distance or semi-major axis of elliptical orbit | earth radii |
| $a_e$ | Earth's equatorial radius = 6378.150 meters | earth radii |
| $\underline{a}$ | Vector directed to perigee having magnitude e, and with orbit plane components: $a_{xn}$, along $\underline{N}$; and $a_{yn}$, along $\underline{M}$ | dimensionless |
| $A$ | Azimuth; measured in a positive sense from the north point eastward in the horizon plane | radians |
| $\underline{A}$ | Unit vector in the equator system forming an orthogonal set with $\underline{L}$ and $\underline{D}$; components are $A_x$, $A_y$, $A_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ coordinate system | dimensionless |
| $\underline{\tilde{A}}_h$ | Unit vector in the horizon system forming an orthogonal set with $\underline{L}$ and $\underline{\tilde{D}}_h$; components are $A_{xh}$, $A_{yh}$, $A_{zh}$ in the $\underline{S}$, $\underline{E}$, $\underline{Z}$ coordinate system | dimensionless |
| $C_D$ | Atmospheric drag coefficient | dimensionless |
| $C_{D_o}$ | Reference atmospheric drag coefficient = 0.92 | dimensionless |
| $d$ | Diameter of satellite | meters |
| $\underline{D}$ | Unit vector in the equator system forming an orthogonal set with $\underline{L}$ and $\underline{A}$; components are $D_x$, $D_y$, $D_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| $\underline{\tilde{D}}_h$ | Unit vector in the horizon system forming an orthogonal set with $\underline{L}$ and $\underline{\tilde{A}}_h$; components are $D_{xh}$, $D_{yh}$, $D_{zh}$ in the $\underline{S}$, $\underline{E}$, $\underline{Z}$ system | dimensionless |
| $e$ | Eccentricity of elliptical orbit; also used as the earth's meridional eccentricity, $e^2 = 2f - f^2$ where f is the flattening | dimensionless |

| Symbol | Definition | Units |
|---|---|---|
| E | Eccentric anomaly | radians |
| $\underline{E}$ | Unit vector in horizon system directed toward east | dimensionless |
| f | Flattening of the spheriod (Earth) = $\frac{1}{298.3}$ | dimensionless |
| h | Elevation angle; angular distance above (positive) or below (negative) the horizon | radians |
| $\underline{h}$ | A vector related to the angular momentum (per unit mass), being equal to $(\underline{r} \times \dot{\underline{r}})/\sqrt{\mu}$ | radii $^2/k_e^{-1}$min m$^{1/2}$ |
| H | Height above sea level, or height above the geoid | earth radii |
| H' | A coefficient of the 3rd order, describing the perturbative acceleration resulting from the 3rd harmonic of the Earth's potential = −.00000575 | dimensionless |
| i | Inclination of the orbit plane to the plane of the equator | radians |
| $\underline{I}$ | Unit vector in the inertial framework directed toward the vernal equinox | dimensionless |
| J' | Coefficient of the 2nd order zonal harmonic of the Earth's potential = .00162341 | dimensionless |
| $J_5$ | Coefficient of the 5th order harmonic of the Earth's potential = −.0000002 | dimensionless |
| $\underline{J}$ | Unit vector in the inertial framework in the equator plane forming an orthogonal set with $\underline{I}$ and $\underline{K}$ | dimensionless |
| $k_e$ | Geocentric gravitational constant = .07436574 | radii $^{3/2}/(k_e^{-1}$min) |
| K' | Coeffieicnt of the 4th order harmonic involving "flattening" of spheroid and centrifugal acceleration = .00000795 | dimensionless |

| Symbol | Definition | Units |
|---|---|---|
| $\underline{K}$ | Unit vector in the inertial framework directed northward along the polar axis of the Earth | dimensionless |
| l | True orbital longitude, referred to intertial coordinate system $l = v + \Omega + \omega$ | radians |
| L | Mean orbital longitude referred to inertial coordinate system L = M + + | radians |
| $\underline{L}$ | Unit vector directed from observer to to satellite | dimensionless |
| m | Mass of orbiting vehicle | kilograms |
| M | Mean anomaly | radians |
| $M_e$ | Molecular weight of atmosphere | gram mol. wt. |
| $\underline{M}$ | Unit vector in orbit plane directed to a point 90° from node in direction of motion; components $M_x$, $M_y$, $M_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| n | Mean angular motion | radians/min |
| $\underline{N}$ | Unit vector to ascending node; components $N_x$, $N_y$, $N_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| p | Semi-latus rectum | earth radii |
| $\underline{P}$ | Unit vector directed to perigee; components $P_x$, $P_y$, $P_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| $\underline{Q}$ | Unit vector parallel to minor axis and velocity vector at perigee; components $Q_x$, $Q_y$, $Q_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| r | Distance of object from geocenter | earth radii |
| $\dot{r}$ | Radial component of vehicle velocity vector | earth radii/$k_e^{-1}$min |
| $\underline{r}$ | Vector from geocenter to object; components x, y, z in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system and $x_\omega$, $y_\omega$ in the $\underline{P}$, $\underline{Q}$ system | earth radii |
| $\underline{\dot{r}}$ | Velocity vector of object relative to geocenter; components $\dot{x}$, $\dot{y}$, $\dot{z}$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system and $\dot{x}_\omega$, $\dot{y}_\omega$ in the $\underline{P}$, $\underline{Q}$ system | earth radii/$k_e^{-1}$min |

| Symbol | Definition | Units |
|---|---|---|
| $\underline{\dot{r}}^{\backslash}$ | Total perturbative acceleration of vehicle; components $\dot{x}^{\backslash}$, $\dot{y}^{\backslash}$, $\dot{z}^{\backslash}$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii/$(k_e^{-1}$ min$)^2$ |
| $\underline{\dot{r}}^{\backslash}_B$ | Perturbative acceleration of vehicle due to Earth's bulge; components $\dot{x}^{\backslash}_B$, $\dot{y}^{\backslash}_B$, $\dot{z}^{\backslash}_B$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii/$(k_e^{-1}$ min$)^2$ |
| $\underline{\dot{r}}^{\backslash}_D$ | Perturbative acceleration of vehicle due to atmospheric drag; components $\dot{x}^{\backslash}_D$, $\dot{y}^{\backslash}_D$, $\dot{z}^{\backslash}_D$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii/$(k_e^{-1}$ min$)^2$ |
| $r\dot{b}$ | Normal component of vehicle velocity vector | earth radii/$k_e^{-1}$ min |
| $r\dot{v}$ | Transverse component of vehicle velocity vector | earth radii/$k_e^{-1}$ min |
| $\underline{R}$ | Position vector of geocenter with respect to observer; components X, Y, Z in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii |
| $\underline{\dot{R}}$ | Velocity vector of geocenter with respect to observer; components $\dot{X}$, $\dot{Y}$, $\dot{Z}$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii/$k_e^{-1}$ min |
| $\dot{s}$ | Magnitude of $\underline{\dot{r}}$ | earth radii/$k_e^{-1}$ min |
| $\underline{S}$ | Unit vector in horizon system directed toward south; components $S_x$, $S_y$, $S_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| t | Time | minutes |
| $T_s$ | Skin temperature of satellite | degrees Kelvin |
| u | Argument of latitude; $u = v + \omega$ | radians |
| U | Mean argument of latitude; $U = M + \omega$ | radians |
| $\underline{U}$ | Unit vector directed along radius vector; components $U_y$, $U_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| v | True anomaly | radians |
| $V_{co}$ | Circular satellite velocity at unit distance = 7.9048 km/sec | km/sec |

| Symbol | Definition | Units |
|---|---|---|
| $\underline{V}$ | Transverse unit vector perpendicular to radius vector; components $V_x$, $V_y$, $V_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| $\underline{W}$ | Unit vector perpendicular to orbit plane; components $W_x$, $W_y$, $W_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| $\underline{Z}$ | Unit vector in horizon system directed toward zenith; components $Z_x$, $Z_y$, $Z_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | dimensionless |
| $\alpha$ | Right ascension | radians |
| $\delta$ | Declination | radians |
| $\epsilon$ | Emissivity of satellite = .9 | dimensionless |
| $\theta$ | Local sidereal time | radians |
| $\theta_{gr}$ | Greenwich sidereal time | radians |
| $\dot{\theta}$ | Rotational rate of the Earth = .0043752689 | radians/min. |
| $\lambda$ | Longitude | radians |
| $\mu$ | Mass function = $\sqrt{m_1 + m_2}$ (= unity for $m_1 \gg m_2$) | earth mass |
| $\underline{\nu}$ | Velocity vector relative to the Earth's atmosphere; components $\nu_x$, $\nu_y$, $\nu_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | radii/$k_e^{-1}$min |
| $\rho$ | Slant range to vehicle | earth radii |
| $\rho$ | Atmospheric density | gm/cm$^3$ |
| $\dot{\rho}$ | Range rate of vehicle | radii/$k_e^{-1}$min |
| $\underline{\rho}$ | Range vector to vehicle; components $\rho_x$, $\rho_y$, $\rho_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | earth radii |
| $\dot{\underline{\rho}}$ | Velocity vector of vehicle relative to observer; components $\dot{\rho}_x$, $\dot{\rho}_y$, $\dot{\rho}_z$ in the $\underline{I}$, $\underline{J}$, $\underline{K}$ system | radii/$k_e^{-1}$min |

| Symbol | Definition | Units |
| --- | --- | --- |
| $\sigma$ | Atmospheric density ratio = $\rho/\rho_o$ where $\rho_o$ = 0.001225 gm/cm$^3$ | dimensionless |
| $\sigma_s$ | Stefan-boltzmann constant = 5.672 x 10$^{-5}$ | erg cm$^{-2}$deg$^{-4}$sec$^{-1}$ |
| $\phi$ | Geodetic latitude | radians |
| $\Omega$ | Longitude of ascending node of the orbit; angle measured in the equator plane from the vernal equinox | radians |
| $\omega$ | Argument of perigee | radians |

The above symbols are modified in many cases by the following symbols:

| Symbol | Description | Definition |
| --- | --- | --- |
| $\cdot$ | Over dot | Denotes time derivative exclusive of perturbations |
| \ | Grave | Denotes time derivative due to perturbations |
| — | Underscore | Denotes vector quantity |
| o | Zero subscript | Denotes quantity referenced to some epoch or standard value |
| $\Delta$ | Delta prefix | Denotes residual quantity |
| c | Subscript | Denotes computed quantity |
| obs | Subscript | Denotes observed quantity |

## Appendix B

## List of Constants

## Orbit Correction Program and Station Locator Program

| Symbol | Value | Definition |
|---|---|---|
| THGRO | 98.67400833 (1960)<br>99.42093750 (1961)<br>99.18221667 (1962)<br>98.94350000 (1963) | $\Theta_{GR_o}$, Greenwich Sidereal Time at beginning of epoch year |
| SIDRT | .9856472 | $\dot{\Theta}$-360°, where $\dot{\Theta}$ is the rotation rate of the earth in degrees/mean solar day |
| FLP 25 | -.25068448 | The negative of the rotation rate of the earth in degrees/minute |
| THDOT | .05883447 | The rotation rate of the earth in radians/ $k_e^{-1}$ min |
| EMIS | 0.9 | $\epsilon$, The emissivity of the satellite (used in the drag calculation) |
| XK | .2504742 E10 | A constant relating the units used in the drag calculation |
| CDO | .92 | An emperically determined constant used to evaluate the drag coefficient |
| ONEPI | 1.173913 | An empirically determined constant used to evaluate the drag coefficient |
| SIXP9 | 6.972 E9 | An empirically determined constant used to evaluate the drag coefficient |
| CD | 2.0 | Initial approximation to the drag coefficient |
| SIGS | .5672 E-4 | $\sigma_s$, the Stefan-Boltzmann constant |
| XKE | .07436574 | $k_e$, the gravitational constant |
| F | .0033523299 | f, the flattening of the earth |
| EPSQD | 6.6934216 E-3 | $e^2$ (=$2f-f^2$), where e is the eccentricity of the terrestrial ellipsoid |

| Symbol | Value | Definition |
|---|---|---|
| XJMPRM | 1.62341 E-3 | J', the 2nd harmonic of the earth's potential |
| HMPRM | -5.75 E-6 | H', the 3rd harmonic of the earth's potential |
| XKMPRM | 7.95 E-6 | K', the 4th harmonic of the earth's potential |
| XJAY5 | -0.2 E-6 | $J_5$, the 5th harmonic of the earth's potential |
| XMPER<br>XK2ER | .6378150 E7 | Meters per earth radii |
| XKS2RK | 7.9048 | km/sec per earth's radii/ke min |
| VCO3 | .49393823 E18 | $(V_{co})^3$ where $V_{co}$ is the speed of a circular satellite at 1 earth radius, in (cm/sec) |
| ERPM | .15678527 E-6 | Earth radii per meter |
| ER2MM | .15678527 | Earth radii per megameter |
| RHOO | .001225 | $\rho_o$, the atmospheric density at the surface of the earth in gms/$cm^3$ |

Reference for all geodetic constants: Makemson, Baker, and Westrom (AAS Journal, Vol. 8, #1, p.1, Spring 1961)

## Appendix C

### Density Table

(1959 ARDC Model Atmos.)
Used in the Calculation of the Drag Perturbation in the Orbit Correction Program

| H (meters) | $\ln(\rho/\rho_o)$ | $M_E$ |
|---|---|---|
| $50.0 \times 10^3$ | -7.0310587 | 28.966 |
| 55.0 | -7.6032281 | 28.966 |
| 60.0 | -8.1538223 | 28.966 |
| 65.0 | -8.7536457 | 28.966 |
| 70.0 | -9.4124843 | 28.966 |
| 75.0 | -10.143371 | 28.966 |
| 80.0 | -10.964450 | 28.97 |
| 85.0 | -11.969229 | 28.97 |
| 90.0 | -12.972557 | 28.97 |
| 95.0 | -14.027182 | 28.94 |
| 100.0 | -15.003575 | 28.90 |
| 105.0 | -15.911001 | 28.86 |
| 110.0 | -16.843617 | 28.82 |
| 115.0 | -17.625944 | 28.77 |
| 120.0 | -18.231593 | 28.71 |
| 125.0 | -18.725451 | 28.66 |
| 130.0 | -19.142311 | 28.59 |
| 135.0 | -19.503103 | 28.53 |
| 140.0 | -19.820952 | 28.45 |
| 145.0 | -20.125377 | 28.36 |
| 150.0 | -20.361461 | 28.27 |
| 155.0 | -20.595472 | 28.16 |
| 160.0 | -20.802337 | 28.04 |
| 165.0 | -21.003537 | 27.91 |
| 170.0 | -21.157902 | 27.75 |
| 175.0 | -21.305866 | 27.57 |
| 180.0 | -21.434534 | 27.36 |
| 185.0 | -21.561085 | 27.12 |
| 190.0 | -21.685493 | 26.85 |
| 195.0 | -21.807622 | 26.59 |
| 200.0 | -21.927794 | 26.32 |
| $210.0 \times 10^3$ | -22.159273 | 25.80 |
| 220.0 | -22.379078 | 25.29 |
| 235.0 | -22.699344 | 24.56 |
| 250.0 | -23.008858 | 23.87 |
| 270.0 | -23.406146 | 23.03 |

| H (meters) | $Ln(\rho/\rho_o)$ | $M_E$ |
|---|---|---|
| 300.0 | -23.971776 | 21.95 |
| 330.0 | -24.504617 | 21.06 |
| 360.0 | -25.007847 | 20.33 |
| 400.0 | -25.637420 | 19.56 |
| 450.0 | -26.366557 | 18.83 |
| 500.0 | -27.040063 | 18.28 |
| 600.0 | -28.246917 | 17.52 |
| 900.0 | -31.867479 | 16.50 |

($\rho_o$ = 0.001225 gms/cm$^3$)

## Appendix D

### Error Exits and On-Line Comments*

| On-Line Comment | Origination | Action |
|---|---|---|
| ERROR IN NODE COMPUTATION 0/0 | Entered from XYZSB if any XNX = XNY = 0 | Normal system exit (location 4) |
| 30 TIMES THRU LOOP WITHOUT CLOSING ON E | Entered from XYZSB if the iteration on $E+\omega$ does not converge to within $10^{-6}$ after 30 iterations | Continues with 30th value of $E+\omega$ |
| ERROR IN RUNGE KUTTA ROUTINE PROGRAM SHOULD BE DUMPED | Entered from RUNGE subroutine in either of 2 ways: a) if $\Delta t = 0$, the sign of the A register is set positive. b) if there is an error in computing the error control term (in the variable interval mode), the sign of the A register is set negative. | Program halts. If advance bar is pushed, program transfers to FINIS |
| END OF STATION DATA CARDS | Normal exit from program if simulation is being used | Program transfers to FINIS |
| ERROR IN TAN A DIVIDING 0 BY 0 | Entered from OBSIM if: 1. XLX = XLY = 0 (error in computing A), or 2. XLSUBX = XLSUBY = 0 (error in computing $\alpha$) | Program transfers to FINIS |
| STATION NUMBER XXXX NOT FOUND | Entered from ORBCO if the station number on an observation card cannot be associated with the current list of station numbers in the program | Reads in the next observation card |
| NO. OF UNKNOWNS EXCEEDS NO. OF OBSERVATIONS | Entered from FUNCT if there are less good (i.e., not rejected) observations than unknowns | Program transfers to FINIS |

| On-Line Comment | Origination | Action |
|---|---|---|
| (FINIS) ALL'S WELL THAT ENDS | Entered from above referred actions | Clears out PUNCH and PANT buffers and executes the normal system exit (location 4) |

*This table applies only to the Orbit Correction Program; the Station Locator Program has no error exits or on-line comments.

Air Force Cambridge Research Laboratories, Office of Aerospace Research, Laurence G. Hanscom Field, Bedford, Massachusetts.
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Unclassified Report
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2. Orbital Flight Paths
3. Programming
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