LAS-TR-199-49
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MULTICOLOR ATMOSPHERIC MODELS

by

Richard J. Kauth

CONTRACT NO. SD-71

Sponsored by
Advanced Research Projects Agency
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THE UNIVERSITY OF CHICAGO

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FOREWORD

This report is one of the series of final reports on studies carried out under Contract No. SD-71 (ARPA).

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This report was illustrated by George Zacharias and prepared for press by Joyce A. Wegner.

Respectfully submitted,

Laboratories for Applied Sciences

Lucien M. Biberman
Associate Director

1. INTRODUCTION

Many of the proposed or existing schemes for discrimination of targets against background depend upon knowledge of the joint probability distribution of one or more parameters. The parameters may have the units of time (e. g., crossing time distributions), space (e. g., position as in the neighborhood modification process), spacial frequency (Weiner spectra), or radiance (point count statistics). Whatever the set of parameters used, whether composite or "pure", if there are n parameters then each observation on an object is represented by a point in an n dimensional space of the parameters. The density of such points as a function of the parameters is the joint probability density function of an object or class of objects. The standard techniques of category recognition then are used to discriminate among classes of objects. Estimates of the density functions for different categories of objects can be gotten from direct observation on known objects, or from assumptions about the laws governing the behavior of the objects in the presence of certain input conditions, combined with estimates of the probability density functions of the input conditions.

In this paper we are considering principally the latter approach applied to the apparent radiance [*], $\mathcal{N}$, of the earth's atmosphere in several wavelength bands.

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[*] Strictly the term "radiance" refers to the self emission of an object, while "brightness" or "apparent radiance" refer to the sum of radiation from all sources, reflected, transmitted and self emitted.

2. PURPOSE

It is the purpose of this paper to prepare the groundwork for the computation of the joint probability density function of apparent radiances in several spectral regions, seen looking down at the atmosphere from above.

The physical situation is as follows:

Radiation from the sun passes through the atmosphere and is absorbed and scattered. It strikes a cloud top and is diffusely reflected back up through the atmosphere to an observer. Towards longer wavelengths emission may also become significant and this contribution may be handled by a separate term involving a diffuse reflector behind an emitter.

The question of probabilities arises because of almost random changes in the atmosphere. Thus the parameters of an atmospheric model are only know probabilistically as are the radiance values.

3. THE MATHEMATICAL FRAMEWORK

The mathematical framework may be specified as follows:

Given, a collection of variates,

$$\left\{x_j\right\} \;, \; j \in n \;, \quad n = \left\{1, \ldots, n\right\}$$

and a collection of dependent variables $y_i = y_i \left\{x_j\right\}$, $i \in n$ and given $p_x \left\{x_j\right\}_{j \in n}$, the joint probability density function of the x_j, then we seek $p_y \left\{y_i\right\}_{i \in n}$, the joint probability function of all the y_i, and further we seek $p_y \left\{y_i\right\}_{i \in m}$, the joint probability density function of the mth subset of the y_i, $m \subset n$.

Certain conditions must be met.[1]

If,

a) $y_i \left\{x_j\right\}$ is everywhere unique and continuous,

b) $\dfrac{\partial\, y_i}{\partial\, x_j}$ finite and continuous except possibly in certain points on an enumerable number of hypersurfaces,

c) $y_i \left\{x_j\right\}$ is a one to one transformation, such that $x_j = x_j \left\{y_i\right\}$ and the x_j are unique,

d) The Jacobain $J = \dfrac{\partial\{x_j\}}{\partial\{y_i\}}$ is different from zero and finite except for points on the exceptional hypersurfaces, then, the probability element of the x_j, $p_x\left\{\underset{j\epsilon n}{x_j}\right\} \cdot \prod\limits_{j\epsilon n} dx_j$, is transformed to

$$p_x \left\{\underset{j\epsilon n}{x_j}\right\} \cdot \prod_{j\epsilon n} dx_j = p_x \left\{\underset{i,\,j\epsilon n}{x_j \left\{y_i\right\}}\right\} \cdot |J| \cdot \prod_{i\epsilon n} dy_i \quad , \tag{1}$$

in which $|J|$ is the absolute value of the Jacobian.

But the probability element of the x_j's covers the same volume of n space as does the probability element of the y_i's, so that

$$p_y \left\{\underset{i\epsilon n}{y_i}\right\} = p_x \left\{\underset{j\epsilon n}{x_j \left\{\underset{i\epsilon n}{y_i}\right\}}\right\} \cdot |J| \; . \tag{2}$$

By definition:

$$p_y \left\{\underset{\substack{i\epsilon m \\ m\subset n}}{y_i}\right\} \equiv \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{[n-m]} p_y \left\{\underset{k\epsilon n}{y_k}\right\} \cdot \prod_{k\notin m} dy_k = \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{[n-m]} p_x \left\{\underset{j,\,k,\,\epsilon n}{x_j \left\{y_k\right\}}\right\} \cdot |J| \cdot \prod_{k\notin m} dy_k \tag{3}$$

If condition c) is not met, then it is still possible to break the space of the x_j into regions in which a one to one transformation does exist and add the contributions to $p_y \left\{ y_i \right\}$ from each region.

In general there may be more or less x_j's naturally available than there are y_i's. For instance we may be interested in only two of the y_i, say y_1 and y_2, but still believe that four of the x_j must be allowed to vary. In this case we can with no loss introduce $y_3 = x_3$, $y_4 = x_4$. The Jacobian them simplifies to

$$\frac{\partial\,(x_1, x_2)}{\partial\,(y_1, y_2)} \quad ,$$

since x_3 and x_4 will be explicit functions only of y_3 and y_4 respectively and

$$\frac{\partial\, x_3}{\partial\, y_3} = \frac{\partial\, x_4}{\partial\, y_4} = 1 \; ; \quad \frac{\partial\, x_3}{\partial\, y_{i \neq 3}} = \frac{\partial\, x_4}{\partial\, y_{i \neq 4}} = 0 \; .$$

In the other case, in which the number of y_i's is greater than the number of x_j's, a functional relationship will exist among the y_i's such that $f = f\left\{ y_i \right\} = 0$. Mathematically this implies that one can have perfect discrimination, i.e., if a set of y_i's are observed not satisfying $f\left\{ y_i \right\} = 0$, then one is not observing the object to which the model applies. Physically it means that one must choose a more sophisticated model, i.e., allow variation of more parameters, in order to get a more realistic answer.

4. THE PHYSICAL FRAMEWORK

We wish now to show that the physical situation can be formulated in just this way, i.e., that the apparent radiance in spectral region i, $\mathcal{N}_i$, can be written as:

$$\mathcal{N}_i = \mathcal{N}_i \left\{ x_k \right\} \tag{4}$$

in which the x_k are variates for which $p_x \left\{ x_k \right\}$ can be measured or estimated in some way, and that the conditions a), b), and d) above are satisfied. (It is of course also desirable to keep violations of condition c) to a minimum).

The apparent radiance seen by the observer in some spectral region i is given by:

$$\mathcal{N}_i = S_i R_i T_i + N_i \tag{5}$$

in which,

S_i is the solar irradiance at the top of the earth's atmosphere,

R_i is the cloud diffuse reflectance,

T_i is the transmission from the top of the atmosphere, down to the cloud, and back to the observer, and

N_i is the radiance of the atmosphere above the cloud.

All of the quantities except S_i will be dependent upon the directions of sun and/or observer. These directions are knowable and thus do not enter this problem explicitly.

The assumption that one may simply multiply R_i and T_i is justified by the observation that R varies only slowly with wavelength.

The problem breaks naturally into several areas:

1. Reflection of the clouds.
2. Transmission and radiance of the atmosphere above the clouds.
3. Variates of the model atmosphere used.

A. Reflection of the Clouds

Here there are several approaches available. One may let the R_i become some of the variates x_k. Then it is necessary to estimate from data the joint probability distribution of the R_i and any other parameter which also may be a variate for the transmission and/or the radiance. Such a variate would be h, the cloud height, which will also effect the transmission and radiance of the atmosphere above the cloud. Thus one would estimate $p(\{R_i\}, h)$; and from this $p(h)$ and $p(\{R_i\} | h)$, the marginal probability of h, and the conditional probability of $\{R_i\}$ given h, are available. A second approach is to represent R_i as a function of other variables which then become variates, x_k.

For example

$$R_i = g(\rho)\ f(K_i t) \tag{6}$$

in which ρ is a "roughness coefficient" which describes the random variation of the cloud surface orientation. $f(K_i t)$ is a function of the optical thickness of the cloud, $K_i t$. K_i is the extinction coefficient for the spectral region i and t is the cloud thickness in cm. of water. $f(K_i t)$ can be extracted from the work of C. Bartky[2,3] and H. Brown[4] of these laboratories. It will go asympotically to a maximum as t increases.

On the assumption that the surface orientation of the cloud varies quite rapidly within the field of view of the observer, $g(\rho)$ will be a smooth function of ρ.

It appears that Bartky's work with plane parallel clouds might be extended to develop an approximate model for $g(\rho)$.

B. Transmission and Radiance

The transmission in each spectral region, T_i, is in general a function of the cloud height and of the temperature and the total and partial pressures of the absorbing gases all along the path. The distribution of temperature and pressures as a function of height constitutes a model atmosphere and it is the coefficients of such a model which, in the most general case, must be assigned probability density functions and allowed to vary.

For the radiance, N_i, similar statements hold, with one possible addition. Because the self absorption of emitted radiation is very strong, it is a good assumption that but little if any self emitted radiation will reach a cloud and be reflected back to an observer. The observer will simply see a contribution from the self emission of the atmosphere above the cloud.

Now the apparent radiance in the spectral region i may be rewritten as:

$$\mathcal{N}_i = S_i \, g(\rho) \, f(K_i t) \; T_i \left(\left\{ a_j \right\}, h \right) + N_i \left\{ a_j \right\} ,$$

in which

$\left\{ a_j \right\}$ = the collection of the coefficients or parameters of a model atmosphere, and h = the cloud top height.

T_i is clearly a one to one unique transformation with respect to h,[5/] but not necessarily with respect to all of the a_j, and one may be forced to break the computation into pieces as mentioned on page 4. In either case, $\mathcal{N}_i$ may be written formally as:

$$\mathcal{N}_i = \mathcal{N}_i \left(\rho, t, h, \left\{ a_j \right\} \right) = \mathcal{N}_i \left\{ x_k \right\} . \qquad (7)$$

A variety of assumptions are available which simplify the problem of estimating p_x. For example, assume that the distribution of the a_j is independent of ρ, t or h, but that the distributions of ρ and t depend separately on h. Thus,

$$p_x \left\{x_k\right\} = p_a \left\{a_j\right\} \cdot p_h(h) \cdot p_t(t \mid h) \cdot p_\rho(\rho \mid h) . \tag{8}$$

If in addition the collection a_j has been reduced to its minimum size by recourse to common causes [6/] of pressure and temperature dependencies then the a_j's may be assumed independent and,

$$p_a \left\{a_j\right\} = \prod_j p(a_j)$$

Depending on the data or theory available, p_ρ and p_t may be written as they appear in equation 8, or as

$$\frac{p_\rho(\rho, h)}{p\ (h)} \qquad \text{and} \qquad \frac{p_t(t, h)}{p\ (h)} \qquad \text{respectively.}$$

Finally, if the direct obserable is $p\ (\left\{R_i\right\}, h)$ then Eqs. 7 and 8 may be reformulated to take account of that fact. In all cases the system of equations will satisfy the physical requirements implied by conditions a), b), and d), the conditions of uniqueness and continuity on $\mathcal{N}_i$, of finiteness and continuity on $\frac{\partial \mathcal{N}_j}{\partial x_k}$ and of non-zero and finiteness on J.

C. Other Modifications

The constant assumption of this section has been that the transmission (and also the radiance) model used would require knowledge of the pressures and temperature all along the path.

The only difficulty is that no exact <u>and</u> <u>usable</u> model for either T_i or N_i exist. Gates[7/] has done a rigorous computation for

T_i for water vapor and CO_2 for a homogeneous path. He has also computed the transmission for long slant paths using the average pressures and temperature in the path and achieved excellent agreement with the CARDE [*] data. On this basis E. Walbridge [8] of these laboratories has used Gates program for transmission and Bartky's results for plane homogeneous clouds to compute the apparent radiance seen by an observer in the 1.9μ and 2.7μ bands, for a variety of cloud heights and sun and observer angles. His average pressure and temperature through the path above the cloud is computed from the ARDC atmosphere. As pointed out on page 4 this approach using two dependent variables but only one independent variate, h, gives rise to a functional dependence between the dependent variables, $\mathcal{N}_{1.9}$ and $\mathcal{N}_{2.7}$. Additional quantities might be used as variates from Walbridge's work, for instance the sun and observer angles. Then $p_{\mathcal{N}}(\mathcal{N}_{1.9}, \mathcal{N}_{2.7})$ can be computed for a given $p_h(h)$ and a given scan geometry. If however one wishes to consider these quantities (sun and observer angles) to be known, then for a more realistic estimate of $p_{\mathcal{N}}$ some additional atmospheric parameters will have to be allowed as variates.

Attempting to extend Gate's approach by admitting the average temperature and pressure as variates, using the techniques implied by Eq. 3, would multiply an already long computer program. A useful compromise might be to use the average pressures and the average temperature as variates in conjunction with one of the many less rigorous models available [9-12]. This would give a reasonable number of variates (from 4 to 6 variates depending on whether ρ and t, were included) which should give reasonable estimates for a similar order joint probability density function. The main advantage of

[*] Referred to by Gates, Reference 7.

using one of the less rigorous models is that a much greater part of the work might be done analytically before numerical computations are undertaken.

5. CONCLUSION

This paper has attempted to demonstrate that the computation of joint density functions of apparent radiance (as opposed to their direct measurement) is not an unreasonable task. It should be pointed out that even if direct measurements are made, these are always limited in extent by costs and therefore serve as tests of the atmospheric models used.

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