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RECEIVERS FOR LASER RADARS

Project Director – G. F. Smith
Author – R. L. Forward
Quantum Physics Department

Interim Engineering Report No. 1
15 November 1962 through 14 February 1963

Contract AF 33(657)-8769
Task Number 40119

HUGHES RESEARCH LABORATORIES
Malibu, California

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ABSTRACT

The objective of this work is to investigate theoretically and experimentally the properties of a laser preamplifier for an optical Doppler radar receiver.

In order to ascertain the fundamental limitations of various methods of detecting or amplifying optical radiation, a comprehensive literature search was undertaken to include all important papers[1-69] on the quantum theory of noise and quantum mechanical amplifiers. For comparison of the various types of receivers, a particular method was chosen for describing the signal and calculating the uncertainties introduced and was applied uniformly to all the different systems.

The signal-to-noise ratios of an optical signal were calculated after attentuation in a transmission medium and after amplification by four different systems (a photodetector, a single mode laser amplifier, a single mode optical heterodyne, and a laser preamplifier followed by an optical heterodyne).

It was found that an amplifier must be used before the signal becomes too weak (average number of received photons approaches unity) or information will be irretrievably lost. If the system parameters indicate this will happen, a laser amplifier should be used in front of the transmitter (assuming the power handling capabilities are available).

If the number of received signal photons is appreciably larger than unity (or the number of background photons), then any of the amplifiers can be used provided they do not degrade the signal too much. The degradation factor for the laser amplifier is related to the inversion; the degradation factor for the optical heterodyne is the quantum efficiency. Analysis shows that they are mathematically equivalent in their effect on the signal-to-noise ratio. Thus a laser preamplifier with good inversion can be used before an optical heterodyne with a relatively poor quantum efficiency in the photocathode.

I. INTRODUCTION

Before extensive work is started on the construction of laser amplifiers, it is necessary to understand the fundamental limitations imposed on the laser amplifier and possible alternative methods of detection and amplification. This study is a first look at this problem, and it emphasizes the quantum mechanical fluctuations that represent the predominant noise factor in this region of the electromagnetic spectrum.

II. DISCUSSION OF NOISE IN LASER SYSTEMS

A. Signal Fluctuations

The laser has opened up a completely new world to the electronic engineer, and the abrupt change from centimeter wavelengths to micron wavelengths has given us insufficient time to adjust our concepts and methods of attacking problems. Suddenly we have more bandwidth than we can use, narrower antenna beams than we can aim, and circuits without a single conductor in them. A subtle change has also occurred in the nature of the electromagnetic energy that makes up the signal (and the noise). The transition from microwaves to light has also involved a change from classical waves to quantum mechanical photons, from Boltzmann's theory to Planck's, and from thermal noise to fluctuation noise. It is this subtle change in the origin and nature of the noise that we discuss in this report.

Because of the large amount of energy packed into each optical photon and the decrease in energy emitted from a thermal source according to the Planck radiation law, thermal sources almost always will contribute a negligible amount of noise to our new narrow band systems. Although thermal noise is no longer a problem, it has been replaced by another type of noise which is quantum mechanical in origin. No matter what devices are considered, or how the calculations are done, it is always present. It even appears when there are no devices at all, just the signal alone. There is no escaping it, since it is inseparable from one of the most powerful and all-inclusive concepts in quantum mechanics — the uncertainty principle.

In order to study this new phenomenon, we shall describe the progress of a signal through various laser systems. By analyzing the state of the signal and the noise at various points in the system, it is hoped that intuitive pictures can be developed that will be sufficiently accurate to guide the engineer in the selection of the type of laser system suitable for the particular application. The analyses will be short, nonrigorous, very simple and highly heuristic, but they will be backed up by more references than one would normally care to investigate.

Assume that a narrow band modulated laser signal comes from some point in space. The signal could be a pulse from a laser in a communications system or a return from a laser illuminated target in a radar system. The problem is to see the burst of light corresponding to that bit and any others that may follow. Because of the quantum nature of light, the bursts of laser energy consist of a finite, integral number of photons. If the bursts are very intense, they will have millions of photons and the average signal power can be accurately controlled. However, suppose that by the time our bursts have reached the receiver they contain an average of only a few photons. Then the number of photons in the bursts will follow a Poisson distribution which has a statistical variation in the number of photons given by the square root of the average number of photons. The square root of 9 is 3 and of 1 is 1. With these large fluctuations in the number of photons, it is easy to see that quite often a burst designed to designate a bit will not contain enough photons to qualify. And since our receiver has lost part of the original information, we have a finite signal-to-noise ratio, <u>even though we have not yet introduced any noise into the discussion</u>!

This new type of noise (which may be called fluctuation noise, statistical noise, quantum noise, photon noise, etc.) is inherent in the signal due to the limitations imposed on the preparation of the photons by quantum mechanics. It is an unusual noise since it causes drop out only and does not create false alarms. In the optical region it is very important and a much greater problem than thermal noise. To show this, let us now introduce thermal noise by assuming that the laser transmitter or laser-illuminated target is in front of the sun or some other hot background so that photons from the hot object are traveling along with the burst we are interested in detecting. If the burst has an average of $\overline{n}$ photons, then the average energy is $\overline{n}\, h\nu$; if the burst is $2\Delta t = 1/B$ long, then the average signal power is

$$S_s = \overline{n}\, h\nu B \quad . \tag{1}$$

Since the fluctuations in the number of signal photons vary as the square root of the average number of photons, the strength of the signal will vary around the mean by about $\overline{n}^{1/2}$. In reality, a Poisson distribution for small numbers of photons not only has a considerable "skew" and higher order moments, but also does not assume negative values. To properly discuss the effect of the fluctuations on the information contained in the signal would require considerable time. Therefore, for a first estimate, we will assume that the root mean square (rms) value of the fluctuations in the number of photons can be interpreted as a source of "noise," with the average noise power given by

$$N_s = \left(\overline{\delta n^2}\right)^{1/2} h\nu B = \overline{n}^{1/2} h\nu B \quad . \tag{2}$$

Our thermal noise source is also emitting photons, but since we are in the optical region the thermal noise power is not kTB; instead we must use the Planck energy distribution which gives

$$N_t = \frac{h\nu B}{e^{h\nu/kT} - 1} \tag{3}$$

i. e., our signal-to-noise ratio is

$$\left.\frac{S}{N}\right|_s = \frac{S_s}{\bar{n}^{1/2} h\nu B + \frac{h\nu B}{e^{h\nu/kT} - 1}} \approx \bar{n}^{1/2} \quad . \tag{4}$$

Since in the optical region the energy of a single photon is $h\nu = 2 \times 10^{-19}$ Joules, then the quantity $h\nu/k$ has the value of approximately 1.5×10^4 °K. Thus a thermal source would have to be hotter than 10^4 °K before it would contribute an appreciable number of photons. We find that in most cases the fluctuation noise is the major cause of uncertainty in the detection of the signal. For the rest of this preliminary analysis we will assume that background is negligible.

As the number of photons in the signal increases, the fluctuation noise increases. But since it varies as the square root, the signal-to-noise ratio increases with the number of photons in the signal. Thus, for a large number of photons this fluctuation noise is no longer a problem. The important factor is that the signal-to-noise ratio decreases considerably when the number of photons per burst, pulse, or bit approaches unity. The limiting case usually assumed is an average of one photon per bit, or pulse, for then we have one signal photon (on the average) and one "noise" photon.

Even if it were possible to guarantee a given number of photons transmitted per pulse, if a transmission loss exists between the source of the photons and the receiver, then one will find that because the loss mechanism is quantum mechanical in nature and picks the photons off the signal in a random fashion, it takes only a moderate amount of attenuation to give the number of photons per pulse a Poisson distribution. This result applies not only to the loss of photons to an attenuating medium but also to the $1/r^2$ loss in free space where the photons are lost out of the receiver's acceptance cone.

Assume a transmission of an average of $\bar{n}_T$ photons with a variance of $\overline{\delta n_T^2}$. After an attenuation of $L < 1$ the average number of photons received and its variance is given by[59]

$$\bar{n}_R = L\bar{n}_T \qquad (5)$$

$$\overline{\delta n_R^2} = L(1 - L)\bar{n}_T + L^2\overline{\delta n_T^2} \qquad (6)$$

$$\approx L\bar{n}_T = \bar{n}_R \qquad L << 1$$

Thus, despite the nature of the distribution of the transmitted photons, after attenuation the received photons have a Poisson distribution, and the signal-to-noise ratio of the received signal is simply

$$\left.\frac{S}{N}\right|_R = \bar{n}_R^{\,1/2} \quad . \qquad (7)$$

This result reveals an important difference between signal-to-noise calculations in the microwave region and in the optical region. In the microwave region a cold (0°K) attenuator affects signal and noise in the same way, so that the signal-to-noise ratio at the output of the cold attenuator is the same as that at the input. However, in the optical region a cold attenuator causes an increase in the relative fluctuation of the signal and information is irretrievably lost, so that the signal-to-noise ratio at the output of a cold optical attenuator is less than that at the input.

It should be mentioned here that our discussion of signal and "noise" photons assumes that we have been using a measuring instrument, a perfect quantum counter, to detect the photons. Whether the photons "exist" before they are measured or whether the electromagnetic energy is "really" a continuous wave and the process of measurement "quantizes" it is a philosophical question outside of quantum mechanics. Calculations using either viewpoint are valid if carried out properly and should lead to the same results.

B. Analysis of a Photodetector

The properties of a photodetector are well known. A good photodetector will have negligible dark current and if a good optical system is used to cut down sky background, then its only limitation is its quantum efficiency. The photomultiplier process introduces some noise due to the fluctuations in the amplification, but this is usually less than 15% of the fluctuation noise.

If the received signal at the input to the photodetector is represented by a mean and a variation given by

$$\bar{n}_R; \ \overline{\delta n_R^2} \ (= \bar{n}_R \text{ for Poisson distribution}) \ ,$$

then the mean number of output photoelectrons to be amplified is

$$\bar{n}_D = \epsilon \, \bar{n}_R \tag{8}$$

where ϵ is the quantum efficiency. The noise at the output results from the fluctuation of the output which has a mean square value of

$$\overline{\delta n_D^2} = \epsilon^2 \overline{\delta n_R^2} + \epsilon \, (1 - \epsilon) \bar{n}_R \quad . \tag{9}$$

For a Poisson distribution of the received photons $\overline{\delta n_R^2} = \bar{n}_R$, the fluctuations at the output of the photodetector have the value

$$\overline{\delta n_D^2} = \epsilon \bar{n}_R = \bar{n}_D \ , \tag{10}$$

which is also a Poisson distribution.

Since each photoelectron has been given a certain kinetic energy by the incident photon or the collecting fields, the power per electron can be expressed as

$$P = qVB \tag{11}$$

where q is the electronic charge, V is the equivalent potential, and B is the bandwidth of the signal. If we define a signal power by

$$S_D = \bar{n}_D \, qVB = \epsilon \bar{n}_R \, qVB \tag{12}$$

and a measure of the noise power by the rms value of the fluctuations in the number of detected photons

$$N_D = \left(\overline{\delta n_D}\right)^{1/2} qVB = (\epsilon \, n_R)^{1/2} qVB, \tag{13}$$

then our signal-to-noise ratio can be defined as

$$\left.\frac{S}{N}\right|_D = (\epsilon n_R)^{1/2} = \epsilon^{1/2} \left.\frac{S}{N}\right|_R \quad . \tag{14}$$

We find that the signal-to-noise ratio at the input of the receiver has been degraded by the square root of the quantum efficiency of the photocathode.

C. Analysis of a Laser Preamplifier

The properties of a laser amplifier have received a considerable amount of study. The work of Shimoda, Takahasi, and Townes[20] is used as a reference here. If the signal at the input of the receiver is represented by a mean and variation given by

$$\bar{n}_R; \ \overline{\delta n_R^2} (= \bar{n}_R \text{ for Poisson distribution}),$$

then the mean number of output photons is given by the sum of the amplified signal photons and the amplified spontaneous emission photons

$$\bar{n}_o = \bar{n}_s + \bar{n}_{sp} = G\bar{n}_R + \frac{n_2}{n_2 - n_1}(G - 1) \tag{15}$$

$$= G\bar{n}_R + K(G - 1)$$

where $$K = \frac{n_2}{n_2 - n_1} \quad .$$

However, this spontaneous emission is not noise since it is merely a constant output added to the signal output. If it were really constant and the output did not fluctuate about the mean, it could be subtracted out. The noise at the output results from the fluctuation of the output which is given by

$$\overline{\delta n_D^2} = G^2 \delta n_R^2 + \left(\frac{n_2 + n_1}{n_2 - n_1}\right) G(G-1)\bar{n}_R + \frac{n_2(n_2 G - n_1)}{(n_2 - n_1)^2}(G-1) \tag{16}$$

$$= G^2 \overline{\delta n_R^2} + (2K-1)G(G-1)\bar{n}_R + K(G-1)\left[K(G-1)+1\right] .$$

If the output contains more than one mode of the laser, then there is an additional term of $MK(G-1)$ in the mean number of output photons where M is the number of extra modes. This additional spontaneous noise will increase the fluctuations at the output; the calculation of the effect is not straightforward, however, because the poor inversion in the signal mode not only introduces K spontaneous photons, but also broadens the signal fluctuation due to random absorption processes; the extra modes only add spontaneous noise and do not affect the signal directly. We will assume a single mode laser.

For high gain, which does not necessarily mean that $n_2 >> n_1$, eq. (16) reduces to

$$\delta n_o^2 = \left[\delta n_R^2 + \left(\frac{n_2 + n_1}{n_2 - n_1}\right)\bar{n}_R + \left(\frac{n_2}{n_2 - n_1}\right)^2\right] G^2 \tag{17}$$

$$= \left[\overline{\delta n_R^2} + (2K-1)\bar{n}_R + K^2\right] G^2$$

If the input has a Poisson distribution, then this reduces further to

$$\overline{\delta n_o^2} = \left[2\left(\frac{n_2}{n_2 - n_1}\right) n_R + \left(\frac{n_2}{n_2 - n_1}\right)^2\right] G^2 \tag{18}$$

$$= \left[2Kn_R + K^2\right] G^2 .$$

If we define a signal power by

$$S_L = \bar{n}_s h\nu B = G\bar{n}_R h\nu B \tag{19}$$

and a noise power by

$$N_L = \left(\overline{\delta n_o^2}\right)^{1/2} h\nu B = G\left[2Kn_R + K^2\right]^{1/2} h\nu B \quad , \tag{20}$$

then the signal-to-noise power ratio for a Poisson distribution of received photons and a high gain laser is given by

$$\left.\frac{S}{N}\right|_L = \frac{\bar{n}_R}{\left[2Kn_R + K^2\right]^{1/2}} \quad . \tag{21}$$

In the limit where the number of received photons is larger than the number of spontaneous photons $(n_R >> K)$, the fluctuations of the signal are much larger than the fluctuations of the spontaneous emission and we have

$$\left.\frac{S}{N}\right|_L = \left[\frac{n_R}{2K}\right]^{1/2} \tag{22}$$

Thus even a high gain laser needs good inversion or it will degrade the signal-to-noise ratio

$$\left.\frac{S}{N}\right|_L = \frac{1}{(2K)^{1/2}} \left.\frac{S}{N}\right|_R \tag{23}$$

Even for a perfect laser with high inversion $(K = 1)$, the initial signal-to-noise ratio is degraded due to the fluctuations in the amplifying process

$$\left.\frac{S}{N}\right|_L = \left[\frac{n_R}{2}\right]^{1/2} = \frac{1}{2^{1/2}} \left.\frac{S}{N}\right|_R \tag{24}$$

D. Analysis of an Optical Heterodyne

The properties of the optical heterodyne have received considerable study, but the results have been obtained using the concepts of noise that were developed in the frequency range where thermal noise is the major component of noise. Gabor[69] has pointed out that many people confuse the thermal energy or power with the fluctuations in the thermal energy or power. Until now it has lead to no problems since the average thermal energy is given by kT, and when the rms fluctuations of the thermal energy are calculated, they also turn out to have the value kT. This has led to the use of the nonsignal power, rather than the fluctuations in the nonsignal power, as the noise power. If the nonsignal power had no fluctuations, we could subtract it from the output to recover the signal.

We will go through an analysis of the optical heterodyne using the terminology of Oliver[40] and Gordon,[51] but with a different concept of noise. We will include the fluctuations in the input signal which they assumed zero, or at worst simply additive.

If the signal at the input of the receiver is represented by a mean and a variation given by

$$\bar{n}_R \,; \qquad \overline{\delta n_R^2} \qquad (= \bar{n}_R \text{ for Poisson Distribution})$$

Then the dc component of the current in the tube by the local oscillator is given by

$$I_{DC} = \frac{\epsilon q}{h\nu} P_{LO} = q B \epsilon n_{LO} \tag{25}$$

where ϵ is the quantum efficiency, n_{LO} is the number of local oscillator photons, and q is the electronic charge.

We are assuming here that the local oscillator and signal directions have been defined by the optical system so that we have only a "single mode" heterodyne. This condition can be imagined by assuming that the photocathode is on the focal plane, and the local oscillator and the signal are both focused onto the same diffraction limited spot. Any local oscillator power coming from other angles will be focused at different points on the photocathode, and this will increase the shot noise fluctuations without amplifying the signal. (In practice, the other modes will probably be limited by a diaphragm at some other focal plane and then the signal and local oscillator mode defocused before being placed on the cathode to prevent burning by the LO power.) The problems of obtaining "single mode" operation in the optical heterodyne and in the laser preamplifier seem to be identical.

The mean square signal current due to the square law mixing of the signal photons and LO photons is given by

$$\overline{I_s^2} = 2\left(\frac{\epsilon q}{h\nu}\right)^2 P_s P_{LO} = 2q^2 B^2 \epsilon n_{LO} \epsilon n_R \quad . \tag{26}$$

Equation (26) indicates that the signal power is proportional to the number of received signal photons, as in the photon beam. The number of signal electrons in the output has a mean

$$\bar{n}_s = \frac{\overline{I_s^2}}{2q^2 B^2} = (\epsilon n_{LO}) \epsilon \bar{n}_R = g\epsilon \bar{n}_R \tag{27}$$

where $g = \epsilon n_{LO}$ is a "conversion gain" factor.

Since the electrons are not so dense that Fermi statistics must be used, the variation in the number of signal electrons in the output is

$$\overline{\delta n_s^2} = \bar{n}_s = g\epsilon \, \bar{n}_R \quad . \tag{28}$$

However, we also have shot currents resulting from the creation of the dc beam in the tube by the local oscillator. This shot current has a mean square current of

$$\overline{I_{shot}^2} = 2qI_{DC} B = 2q^2 B^2 \epsilon n_{LO} \quad . \tag{29}$$

The number of electrons in the output contributing power in the bandpass of the amplifier due to this shot current has an average of

$$\bar{n}_{shot} = \frac{\overline{I_{shot}^2}}{2q^2 B^2} = \epsilon n_{LO} \equiv g \quad . \tag{30}$$

and a fluctuation around the average of

$$\overline{\delta n_{shot}^2} = \bar{n}_{shot} = g \quad . \tag{31}$$

The mean number of electrons in the output is given by the sum of the signal electrons and the equivalent shot electrons

$$\bar{n}_o = \bar{n}_s + \bar{n}_{shot} = g\epsilon n_R + g \quad . \tag{32}$$

The noise at the output of an optical heterodyne system results from the fluctuations of the output (the sum of the amplified fluctuations of the signal photoelectrons plus the fluctuations in the amplified signal output and the fluctuations in the shot current power) (see (28) and (31)):

$$\overline{\delta n_o^2} = g\left[g\overline{\delta n_D^2} + g\epsilon\bar{n}_R + g\right] \quad . \tag{33}$$

The fluctuations of the signal photoelectrons in turn depend upon the attenuated fluctuations of the incident signal photons due to poor quantum efficiency and the fluctuations in the detected signal output due to the random nature of the attenuation process.

$$\overline{\delta n_D^2} = \epsilon^2\,\overline{\delta n_R^2} + (\epsilon - \epsilon^2)\,\bar{n}_R \tag{34}$$

when (34) is substituted into (33) we obtain

$$\overline{\delta n_o^2} = g^2\epsilon^2\left[\overline{\delta n_R^2} + \left(\frac{2}{\epsilon} - 1\right)\bar{n}_R + \frac{1}{\epsilon^2}\right] \quad . \tag{35}$$

If we assume that the fluctuations of the input signal are given by a Poisson distribution, then (35) reduces to

$$\overline{\delta n_o^2} = g^2\epsilon^2\left[\frac{2\bar{n}_R}{\epsilon} + \frac{1}{\epsilon^2}\right] \quad . \tag{36}$$

If we define a signal power by

$$S_H = \overline{I_s^2}\,R_{eq} = n_s\,2q^2B^2R_{eq} = g\epsilon\,n_R\,2q^2B^2R_{eq} \tag{37}$$

and a noise power by

$$N_H = \left(\overline{\delta n_o^2}\right)^{1/2} 2q^2B^2R_{eq} \quad , \tag{38}$$

then the signal-to-noise power ratio at the output is given by

$$\left.\frac{S}{N}\right|_H = \frac{\epsilon\, n_R}{\left[2\epsilon\, n_R + 1\right]^{1/2}} = \frac{n_R}{\left[2\,\frac{n_R}{\epsilon} + \frac{1}{\epsilon^2}\right]^{1/2}} \quad , \tag{39}$$

which is completely analogous to the case of the laser for high gain. In the limit where the number of received photons is large we have

$$\left.\frac{S}{N}\right|_H = \left[\frac{\epsilon\, n_R}{2}\right]^{1/2} \quad . \tag{40}$$

Even for a perfect optical heterodyne receiver ($\epsilon = 1$), the initial signal-to-noise ratio is degraded due to the fluctuations in the mixing process

$$\left.\frac{S}{N}\right|_{\substack{H \\ \epsilon = 1}} = \left[\frac{n_R}{2}\right]^{1/2} = \frac{1}{2^{1/2}} \left.\frac{S}{N}\right|_R \quad . \tag{41}$$

E. Analysis of a Laser Preamplifier Followed by an Optical Heterodyne

We shall now use the equations developed in the previous sections to discuss a system which uses a laser preamplifier in front of an optical heterodyne in an attempt to make up for the poor quantum efficiency of the photocathode. At the transmitter we have a signal given by a mean and a variance of

$$\bar{n}_T \quad ; \quad \overline{\delta n_T^2} \quad .$$

Regardless of the statistics or method of preparation, after an amount of attenuation sufficient to lower the number of photons per wavelength below one, the statistics of the photons will be described by a Poisson distribution with a mean and a variance given by

$$\bar{n}_R \quad ; \quad \overline{\delta n_R^2} = \bar{n}_R \quad . \tag{42}$$

If the signal-to-noise ratio of the signal itself is defined as

$$\left.\frac{S}{N}\right|_R = \frac{\bar{n}_R\, h\nu B}{\bar{n}_R^{1/2}\, h\nu B} = \bar{n}_R^{1/2} \quad , \tag{43}$$

then it will reach its lowest value at the entrance to the receiver. If the fluctuations have caused the number of photons representing a certain bit to drop to zero, the information in that bit is lost and can never be retrieved by any amount of amplification or background elimination.

If we amplify the signal with a high gain laser amplifier, then the number of signal photons at the output will be

$$\bar{n}_s = G\,\bar{n}_R \tag{44}$$

and the fluctuations in the number of output photons will be

$$\overline{\delta n_o^2} = G^2\left[2K\,\bar{n}_R + K^2\right] \tag{45}$$

Note that this is <u>not</u> a Poisson distribution referred to the signal.

We now put the signal photons into the optical heterodyne system with a given quantum efficiency. Assume that the mean number of spontaneous photons can be considered small compared with the fluctuations of the signal.

The output of the optical heterodyne will contain a signal given by

$$\bar{n}_s = g\epsilon\,\bar{n}_i = Gg\epsilon\,\bar{n}_R \tag{46}$$

and it will have fluctuations

$$\overline{\delta n_o^2} = g^2\,\epsilon^2\left[\overline{\delta n_i^2} + \left(\frac{2}{\epsilon} - 1\right)\bar{n}_i + \frac{1}{\epsilon^2}\right] \quad . \tag{47}$$

In this example however, we must use as the fluctuations of the input signal the fluctuations in the output of the laser amplifier

$$\overline{\delta n_o^2} = g^2 \epsilon^2 \left[G^2 (2K\, \bar{n}_R + K^2) + G\left(\frac{2}{\epsilon} - 1\right) \bar{n}_R + \frac{1}{\epsilon^2} \right]$$

$$= G^2 g^2 \epsilon^2 \left[(2K\, \bar{n}_R + K^2) + \frac{1}{G}\left(\frac{2}{\epsilon} - 1\right) \bar{n}_R + \frac{1}{G^2 \epsilon^2} \right] \tag{48}$$

If we define a signal power by

$$S = \bar{n}_s\, 2q^2 B^2 \mathrm{Req} \tag{49}$$

and a noise power by

$$N = \left(\overline{\delta n_o^2}\right)^{1/2} 2q^2 B^2 \mathrm{Req} \quad , \tag{50}$$

then the signal-to-noise ratio at the output is given by

$$\left.\frac{S}{N}\right|_{LH} = \frac{n_R}{\left[(2Kn_R + K^2) + \frac{1}{G}\left(\frac{2}{\epsilon} - 1\right) \bar{n}_R + \frac{1}{G^2 \epsilon^2} \right]^{1/2}} \quad ; \tag{51}$$

in the limit of high laser gain this reduces to the signal-to-noise ratio at the output of the laser, independent of the quantum efficiency ϵ of the photomixer

$$\left.\frac{S}{N}\right|_{LH} = \frac{n_R}{(2Kn_R + K^2)^{1/2}} \approx \left(\frac{n_R}{2K}\right)^{1/2} = \left.\frac{S}{N}\right|_{L} . \tag{52}$$

III. CONCLUSIONS

It has been found that the laser preamplifier and the optical heterodyne receiver, the two methods of coherent detection, have the same fundamental limitations on their noise performance. They both have internally generated noise, they both have signal degradation factors, and they both require good optical systems to limit their operation to a minimum number of spatial modes. The choice of one or the other will depend upon the particular system requirements and their unique characteristics of signal handling (one is a mixer and the other is a fix tuned amplifier); the choice may ultimately depend upon which one can most closely approach the ideal performance.

IV. RECOMMENDATIONS

The future theoretical work should involve a formulation of a simple parameter that will best describe the "signal-to-noise" performance of a receiver. The present concepts and definitions of noise are not completely satisfactory. The primary problem is that we are dealing with Poisson distributions rather than Gaussian ones. For instance, information theory is valid only for Gaussian distributions.

Experimental work should eventually involve the construction of infrared laser amplifiers and the measurement of the inversion factor. Present photo surfaces have a very poor quantum efficiency in this region, and a laser preamplifier with moderately good inversion would be of great value in increasing system performance. However, there may be some merit in constructing a laser preamplifier in the visible range in order to determine how closely the ideal amplifier performance can be approached.

Ruby or other visible region power amplifiers (for use at the transmitter) would be of more value in the visible region where quantum efficiencies are moderately high and are improving. Here it is valuable to increase the number of transmitted photons per unit of information so that the mean number at the receiver will be sufficiently high to ensure a good detection probability.

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