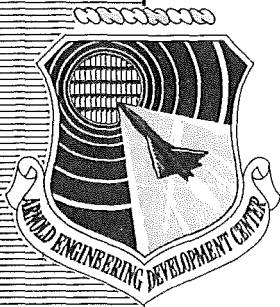


AEDC-TDR-63-143

JUL 15 1963
AUG 1 1963
cy AUG 8 1963
JUN 6 1968



EXPERIMENTAL DETERMINATION OF THE VISCOSITY OF AIR IN THE GASEOUS STATE AT LOW TEMPERATURES AND PRESSURES

By

Daniel R. Grieser and William H. Goldthwaite

Battelle Memorial Institute

Columbus, Ohio

PROPERTY OF U. S. AIR FORCE
AEDC LIBRARY
AE 40(600)1000

TECHNICAL DOCUMENTARY REPORT NO. AEDC-TDR-63-143

June 1963

AFSC Program Area 806A, Project 8951, Task 89603

PROPERTY OF U. S. AIR FORCE
AEDC LIBRARY
AF 40(600)1000

(Prepared under Contract No. AF 40(600)-938 by
Battelle Memorial Institute, Columbus, Ohio.)

ARNOLD ENGINEERING DEVELOPMENT CENTER

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

NOTICES

Qualified requesters may obtain copies of this report from ASTIA. Orders will be expedited if placed through the librarian or other staff member designated to request and receive documents from ASTIA.

When Government drawings, specifications or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

EXPERIMENTAL DETERMINATION OF THE VISCOSITY
OF AIR IN THE GASEOUS STATE AT LOW
TEMPERATURES AND PRESSURES

By

Daniel R. Grieser and William H. Goldthwaite
Battelle Memorial Institute
Columbus, Ohio

(The reproducibles used in the reproduction
of this report were supplied by the authors.)

June 1963

FOREWORD

This report was prepared by the Experimental Physics Division of Battelle Memorial Institute under USAF Contract No. AF 40(600)-938. The work was administered under the direction of the Directorate of Procurement, with First Lt. Leon E. Ring acting as project engineer.

This report covers work conducted from June 12, 1961, to December 11, 1962.

ABSTRACT

A program was undertaken to determine the viscosity of gaseous air at cryogenic temperatures for use in the interpretation of hypervelocity wind-tunnel data. Data were obtained over a temperature range of 92.7 to 198.0 R using an oscillating-disk viscosimeter designed for cryogenic operation. Data were also obtained at greatly reduced pressures where the slip effect becomes important. This was done to aid in the evaluation of the lowest temperature data, where reduced pressures were mandatory in order to avoid condensation.

A fit of the viscosity data to a temperature-viscosity equation of the Sutherland form gives

$$10^7 \eta = 105.8 \frac{(T, ^\circ R)^{3/2}}{(T, ^\circ R) + 178.6}, \text{ poise.}$$

This fit agrees quite well with the data of Johnston and McCloskey and with the low-temperature portion of the equation of F. G. Keyes.

The data obtained at the lowest temperatures deviated from the behavior predicted by the Sutherland Formula. A minor deviation occurred in the vicinity of 140 R, while the major deviation occurred at the lowest temperature of 92.7 R. At this latter temperature, the measured air drag was essentially zero. No substantiated explanation has been given for this latter behavior.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

Larry D. Fitzgerald
 Larry D. Fitzgerald
 Capt, USAF
 Aerospace Sciences Division
 DCS/Research

D. R. Eastman, Jr.
 Donald R. Eastman, Jr.
 DCS/Research

CONTENTS

	<u>Page</u>
1.0 INTRODUCTION	1
2.0 THEORETICAL BACKGROUND.	1
3.0 EXPERIMENTAL INVESTIGATIONS	
3.1 Oscillating-Disk Viscosimeter	3
3.2 Auxiliary Equipment	6
3.3 Supplementary Measurement and Calibration Procedure	7
3.4 Interpretation of Viscosimeter Data	
3.4.1 Analysis of Oscillation Record for Logarithmic Decrement	8
3.4.2 Instrument-Constant Determinations	9
3.5 Experimental Results	
3.5.1 Dry-Air Viscosity	11
3.5.2 Viscosimeter Accommodation Parameter.	14
4.0 CORRELATION OF DATA	15
5.0 DISCUSSION AND CONCLUSIONS.	15
6.0 FUTURE WORK	19
REFERENCES	20

TABLES

1. Instrument Calibration Results	10
2. Experimentally Determined Dry-Air (True) Viscosity	12
3. Experimentally Determined Viscosimeter Accommodation Parameters	14
4. Sutherland Formula Constants	16

ILLUSTRATIONS

Figure

1. Viscosimeter Mounting Arrangement	4
2. Detail of Oscillating-Disk Enclosure	5
3. Dry-Air Viscosity Versus Temperature Data	13

1.0 INTRODUCTION

The interpretation of wind-tunnel data requires calculation of such parameters as Prandl and Reynolds numbers. This demands knowledge of the transport properties of the test medium in the temperature and pressure ranges of the investigations. The expansion of air through the nozzle of a hypersonic wind tunnel results in low air static temperatures and pressures in the testing section. Since under some operating conditions of interest, the air-viscosity values were not known with great confidence, the present program was initiated to perform controlled laboratory measurements of this transport phenomenon. The principal range of interest is in temperatures from 75 R to 200 R and pressures from atmospheric to 0.001 psia. Correlation of the experimental values with empirical relations which have been established at higher pressures and temperatures is also desired.

2.0 THEORETICAL BACKGROUND

Considerable theoretical and experimental study of the transport phenomena of viscosity has been covered as a topic in the field of kinetic theory of gases. Through the efforts of countless investigators, the basic phenomenon has been described to a fairly high degree of precision, both theoretically and experimentally. It has been shown that, to a fairly high precision and up to many atmospheres of pressure, the viscosity of a gas is independent of pressure. At extremely low pressures, however, the viscous drag effects begin to diminish, although the true viscosity remains constant. This behavior occurs when the mean free path of the gas molecules becomes comparable to the characteristic dimensions of the system. Under these conditions, the fluid behavior is no longer that of a classical continuum, but approaches a microscopic free molecular flow behavior where the fluid molecules interact with the solid surfaces in increasing proportion to their interaction among themselves. As a consequence, the surface properties become of increasing importance.

The field of surface phenomena, in spite of extensive effort over the years, remains one of inconsistency. This is partly a result of the weakness of most surface phenomena in the presence of the strong chemical forces on the surfaces. It is also a result of the general lack

of precise techniques for direct measurement of surface materials and properties involving only a few atomic layers in the vicinity of the interface. Although X-ray fluorescence and field-emission microscope techniques are applicable to the identification of surface layers, they are both limited in range of materials and environments that can be studied. As a consequence, the behavior of a system in the free-molecular regime and in the transition between continuous and free-molecular conditions cannot be quantitatively predicted. Empirical equations, which have been developed in many cases to predict behavior in these regimes, can be applied but only after experimental evaluation of constants appropriate to the system of specific interest.

In the present study of the phenomenon of viscous drag, two general equations have been chosen as well representative of the behavior of gaseous air. The first relates the true gas viscosity to absolute temperature and is of the Sutherland Form:

$$\eta = A \frac{T^{3/2}}{T + B} ,$$

where

η = true viscosity,
 T = absolute temperature, and
 A, B = constants.

The second relates the apparent viscosity, as determined from the viscous drag, to the true viscosity and the absolute gas pressure. This equation evolved by Kundt and Warburg (1875) is:

$$\eta' = \frac{\eta p}{p + \alpha \eta} ,$$

where

η' = apparent or measured viscosity,
 η = true viscosity,
 p = absolute gas pressure, and
 α = constant.

The value of the constant α , called the accommodation parameter, is dependent upon the mean free path of the gas molecules, the gas-surface momentum interchange, and the characteristic dimension of the drag system. In most laboratory systems this parameter is small so that, at pressures above a few hundredths of psia, the apparent and true viscosity are equal for all practical purposes. On the other hand, at pressures below a few ten-thousandths of a psia, the apparent viscosity is independent of the true viscosity and is a function only of the interface phenomena and of the absolute pressure. (The latter behavior has been used as the basis for the operation of viscosity gages applicable to pressure measurements in the moderate-vacuum range.)

In the present work, the value of the accommodation parameter was determined at room temperature and two temperatures in the cryogenic

range. These determinations are of minor engineering importance because they relate specifically to the silver disks used in the present viscosimeter. The disks were chemically treated to promote the radiative transfer of heat between them and to the enclosure walls. The determinations were used, primarily, to aid in the calculation of air viscosity from drag data obtained at the lowest temperatures where the pressure was drastically reduced to avoid condensing air constituents on the disks.

3.0 EXPERIMENTAL INVESTIGATIONS

3.1 OSCILLATING-DISK VISCOSIMETER

The oscillating-disk viscosimeter design was chosen for making the present determinations. This choice was based upon its substantial use for gaseous viscosity determinations at cryogenic temperatures by past investigators and because the theory of operation has been well documented. This type of viscosimeter has been shown capable of giving measurements reproducible within narrow limits and is readily adapted to measurements at low pressures as well as low temperatures.

The viscosimeter, mounting base, and dewars are illustrated in Figure 1. As shown, the heavy angle-iron frame supporting the viscosimeter-dewar system was mounted upon a second horizontal iron frame with specially designed rubber pads between the two frames and between the horizontal frame and the concrete floor. Sand-bag loading was applied to each frame to obtain the maximum vibration isolation capability of the rubber pads. Because the viscosimeter was designed to have a high sensitivity to small forces in order to be capable of measuring the small damping expected from air at low temperatures and pressures, considerable effort was applied to minimizing the perturbing effects of external vibration sources. With all of the precautions employed, it was still possible to see the effects of street traffic 100 feet distant, and therefore, the operating schedule was chosen to avoid periods of peak traffic flow. Shown also in Figure 1 is the glass bell jar atop the rig which contains the torsion wire mount and electrical transducer used to measure the angular twist of the oscillating disk. An 8.9-inch length of 3.8-mil diameter annealed Fe-Cr-Al alloy wire was used to provide the restoring torque to the oscillating disk. The wire was maintained almost continuously at 110 F throughout the program by means of a surrounding electrical tape heater in an effort to obtain a constancy in restoring force and internal-frictional damping.*

Details within the disk enclosure are depicted in Figure 2. Shown are the two fixed disks with the oscillating disk between them.

* This effort appeared quite successful since no significant change in wire damping was noticed over a period of over 6 months. This contrasts sharply with the difficulties experienced by Johnston and McCloskey^{(1)**} in which the wire damping decreased appreciably from week to week.

** References are listed on page 27.

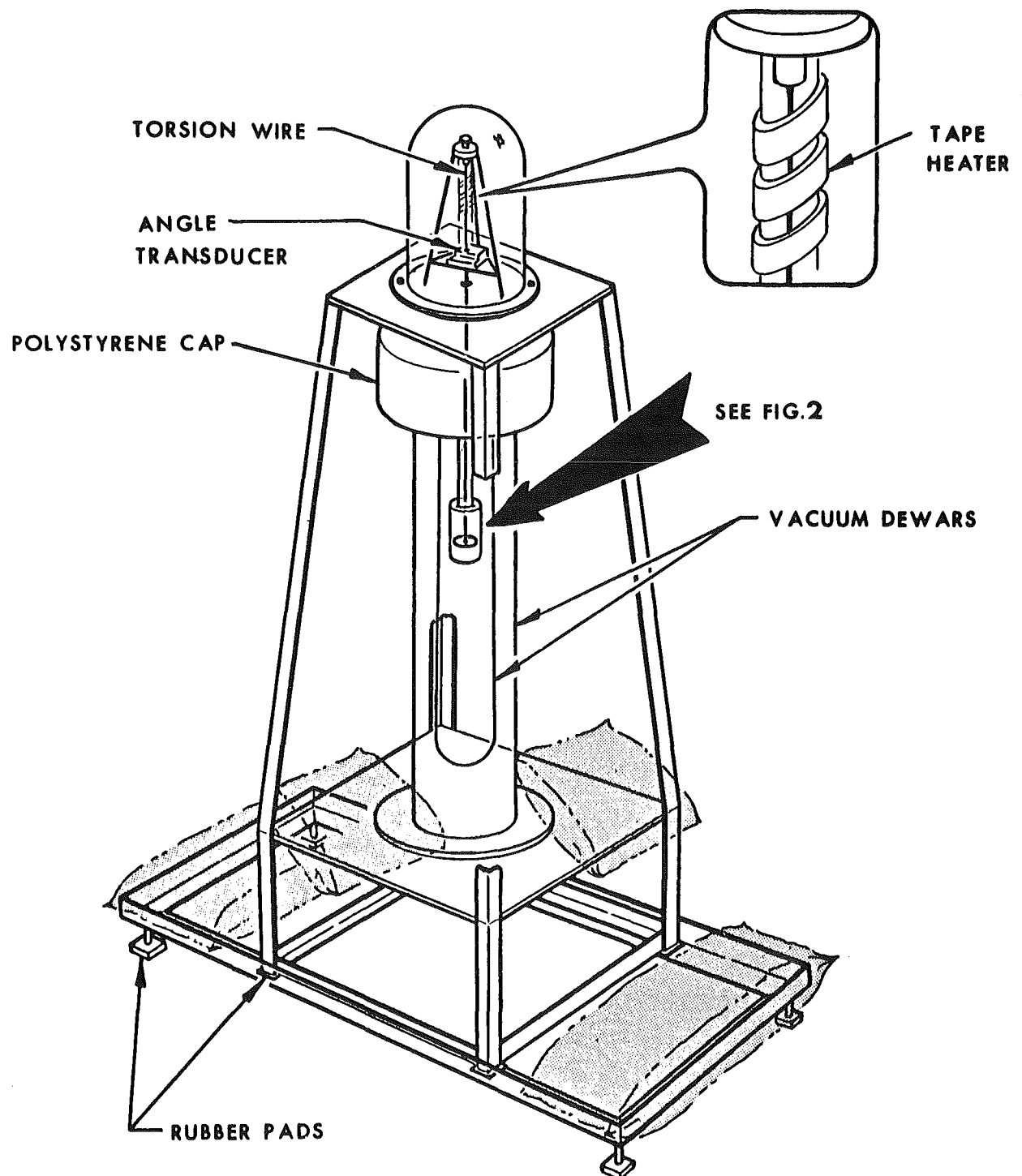


Fig. 1 Viscosimeter Mounting Arrangement

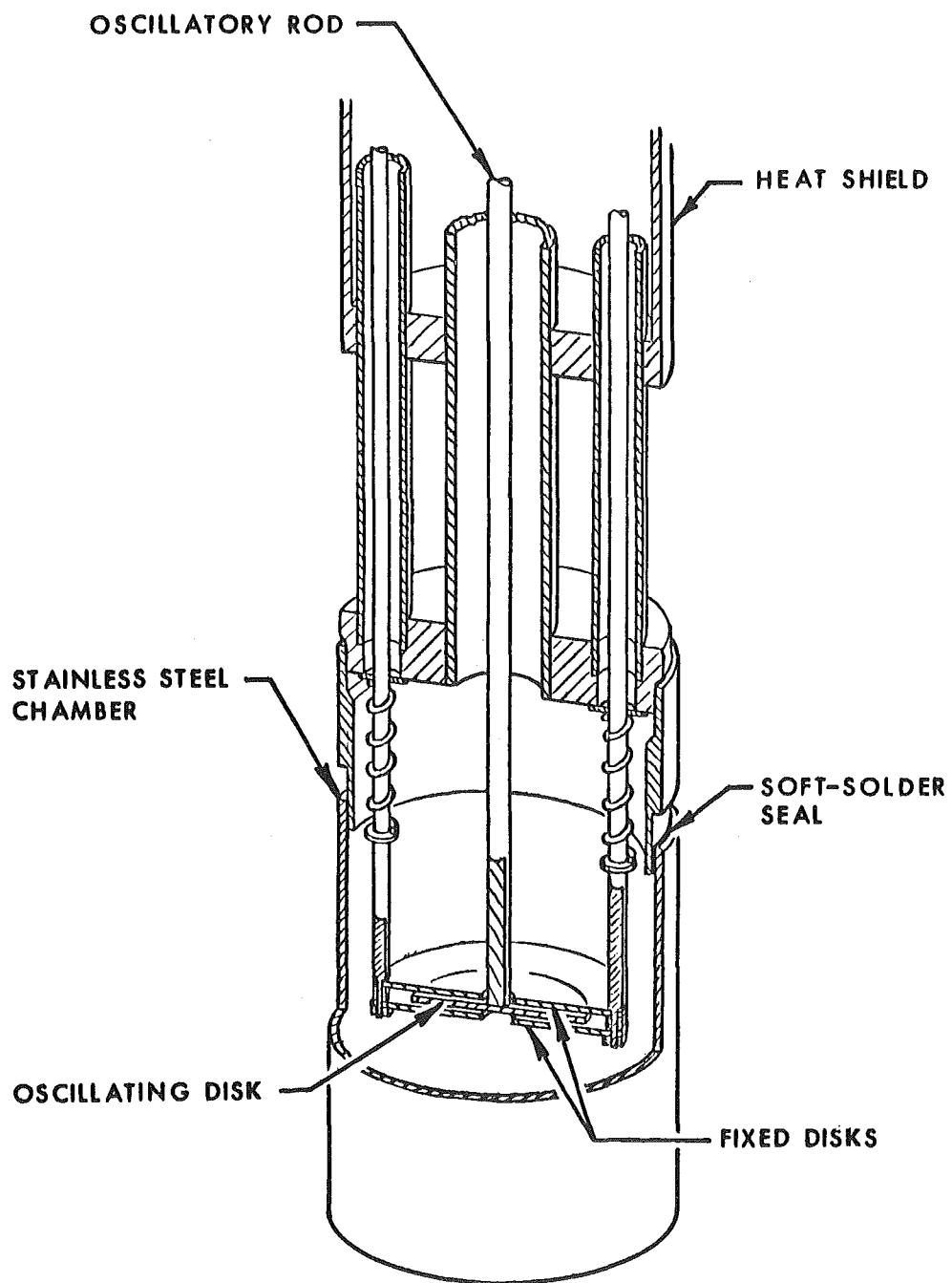


Fig. 2 Detail of Oscillating-Disk Enclosure

The oscillating disk, 2.33 inches in diameter and 0.035 inch thick, was fastened to the bottom end of a brass rod, which in turn was suspended from the torsion wire at the top end. Mounted at the top of the oscillatory rod was the angle transducer armature; about 6 inches below this was a short transverse slug of iron for electromagnetic starting of the torsional oscillations; and a plastic joint was mounted about midway in the 30-inch-long rod to suppress the heat conductance down the rod. A fine copper wire across the plastic joint was used to electrically connect the lower and upper halves of the oscillatory rod in order to avoid a charge buildup on the oscillating disk which could alter the drag unpredictably. All three disks were made of silver in an effort to reduce thermal gradients within the disks and to promote more rapid equilibration of the air temperature between the disks. A copper-constantan thermocouple bead was imbedded in the upper and in the lower fixed disk for determination of the local temperature.

The viscous drag between the moving and fixed disks was a function of the top and bottom spacing as well as of the viscosity of the gas. Because the disks are inaccessible for optical micrometer measurement of spacing while at cryogenic temperatures, the fixed-disk support system was designed to minimize the effects of differential expansivity. Rods supporting the fixed disks were extended out the top of the insulation and fastened to the top of tubes extending up into the room-temperature region. Springs near the lower end of each of the four fixed-disk support rods held them snugly against the adjustment nuts at the upper ends. An open can container was incorporated above the disk enclosure which, when filled with liquid nitrogen, served as a shield to decrease the heat flow down the fixed-disk support tubes and the central tube.

3.2 AUXILIARY EQUIPMENT

The disk enclosure and bell-jar chamber connected above it were pumped through a bellows seal vacuum valve by an oil-diffusion pump and a mechanical fore and roughing pump. Attached to the bell-jar chamber were three vacuum gages, a precision bourdon gage, a thermocouple gage, and a McLeod gage. The bourdon gage was used to measure pressures from atmospheric to about 0.4 psia and the McLeod gage from about 0.5 psia down to about 0.00002 psia. The thermocouple gage was used primarily as a vacuum monitor at the lower pressures since the McLeod gage can be used only to make discontinuous measurements. Also attached to the bell jar was a gas expansion train for introduction of small quantities of air. Included in the air inlet were Molecular Sieve and Ascarite filter bottles for reducing the water and carbon dioxide content of the room air to insignificant levels. Additional plumbing in this system was used for introduction of helium or argon.

All of the copper-constantan thermocouples were connected through a crushed-ice reference bottle to a K-3 precision potentiometer for the determination of temperatures.

The angular transducer was powered by a Brush strain-gage amplifier, and the transducer output was recorded on a 10-inch-strip chart recorder.

3.3 SUPPLEMENTARY MEASUREMENT AND CALIBRATION PROCEDURE

Measurements were made initially of disk spacing and thermocouple response. With the bottom section of the enclosure removed, the upper and lower disk spacings were measured at room temperature by means of a traveling microscope, and the fixed rods were adjusted to obtain equal spacing, amounting to 0.043 inch, above and below the oscillatory disk and to parallelize the fixed and moving disks. The two disk thermocouples and six external couples, all from the same spools of copper constantan wire were immersed in liquid nitrogen for establishing their individual responses. The exterior couples were subsequently immersed in liquid hydrogen to obtain a second calibration point. Comparison of these calibrations showed that individual thermocouples differed by as much as 2 R from the average. There was also an unpredictable tendency for long-time drift amounting to as much as 2 R measured over repeated thermal cycling between ambient and cryogenic temperatures. Although the liquid-nitrogen calibration of the two disk thermocouples showed an agreement within 0.2 F of the average of all of the exterior couples, an absolute accuracy of better than 2 R cannot be accepted with high confidence because of the possibility of drift subsequent to calibration.

Throughout the program of viscosity determination, zero (or vacuum) damping runs were repeated to establish the value for subtraction from the total observed drag in order to obtain the drag due solely to the enclosed gas. As remarked in the previous section, the zero damping remained essentially constant through the program and was found to be equivalent to a gas viscosity of $31.5 \pm 1.7 \times 10^{-7}$ poise.

3.4 INTERPRETATION OF VISCOSIMETER DATA

The principal equation relating the observed logarithmic decrement of decay of the disk oscillations and the viscous drag of the fluid between the disks is given by:

$$\eta' = eC \left(\frac{\lambda}{\tau} - \frac{\lambda_0}{\tau_0} \right)$$

where

- η' = apparent viscosity,
- e = edge effect and thermal expansion correction factor,
- C = instrument constant,
- λ = observed logarithmic decay decrement,
- λ_0 = vacuum logarithmic decrement, and
- τ, τ_0 = observed period of oscillation with gas and vacuum, respectively.

As noted previously, the internal friction in the torsion wire contributes to the over-all observed drag and must be subtracted to obtain the drag of the gas alone. The instrument constant is a function of the interdisk spacing, moving disk radius, and the amount of inertia of the oscillating system. Although the viscosimeter was designed to minimize thermal-expansion effects upon the instrument constant, a minor effect remained which amounted to a calculated difference of 1.2 per cent between the room temperature and the 150 R values and was accounted for in the data analysis.

The drag experienced by a finite disk is made up of two principal parts. The viscous shear of the gas between the plane surfaces of the fixed and moving disks is the major component, while a minor, but significant, drag arises from the shear at the outer edge of the moving disk. The original derivation of the basic equation as described by Maswood⁽²⁾ has been restudied by Kestin and Wang⁽³⁾ to include an assessment of this edge effect. The graphical solution given in Reference (3) was used to evaluate this factor. Over the range of temperatures and pressures studied in the present investigation, this edge effect varied by a maximum of 2.5 per cent.

3.4.1 Analysis of Oscillation Record for Logarithmic Decrement

The angular transducer/recorder system yielded a graph of torsion angle versus time, and the damping produced a decay envelope upon the magnitude of the oscillations. Since the decay decrement is a ratio of oscillatory amplitudes, no absolute angular calibration was necessary. In order to span the fairly large range of decrements expected during this program, a total of 100 oscillations was chosen for each damping run. A uniform analytical technique was developed to facilitate the handling of the large number of determinations anticipated. This technique consisted of listing the maximum and minimum chart reading for every fifth oscillation of a 100-oscillation run and subtracting each minimum from its corresponding maximum. Each of these differences is the relative breadth of the decay envelope at that particular oscillation. The oscillation arbitrarily chosen as the start for the analysis was numbered "0" and each successive one was numbered 5, 10, 15, ... 100. The logarithm of each of the differences was taken and plotted as a function of oscillation number, and the graph used as a check for erratic data, errors, and a rough check of the decrement. The logarithmic decrements actually used to calculate the viscous drag were determined by a root-mean-square analytical procedure. It was assumed that the equation relating the magnitude of the oscillation to the assigned oscillation number is linear and is of the form:

$$y = \beta + \lambda X,$$

where

y = logarithm of the oscillation magnitude,
 X = oscillation number,
 λ = logarithmic decrement,
 β = constant,

and therefore the rms fit of the data may be obtained by solution of the following:

$$\begin{aligned}\Sigma y_i &= \beta n + \lambda \Sigma X_i \\ \Sigma X_i y_i &= \beta \Sigma X_i + \lambda \Sigma X_i^2,\end{aligned}$$

where

$$\begin{aligned}y_i X_i &= \text{data points} \\ n &= \text{total number of data pairs.}\end{aligned}$$

By eliminating β , these two equations may be solved for λ , giving

$$\lambda = \frac{n \Sigma X_i y_i - \Sigma X_i \Sigma y_i}{n \Sigma X_i^2 - (\Sigma X_i)^2}.$$

Because of the uniform procedure adopted in this program for analyzing the data, this equation could be simplified by inserting the values of $n = 21$, and $X_i = 0, 5, 10, 15, \dots, 100$, to give

$$\lambda = \frac{\Sigma X_i y_i - 50 \Sigma y_i}{19,250}.$$

Although the chart records cannot be read to a precision of better than about 3 per cent under the most unfavorable circumstances, it is estimated that the statistical procedure described above yields a decay decrement that is accurate to within 0.5 per cent.

3.4.2 Instrument-Constant Determinations

The instrument constant, C , was determined experimentally by measurement of the logarithmic decrement for gases under conditions for which the viscosity is well known. Multiple measurements were made both with dry air at 69.8 F and with argon at 68.0 F as the calibration gases. Multiple runs were made with helium at 69.8 F and 144.0 R in order to check the validity of correction factors used to predict the low-temperature instrument constant from the room-temperature calibrations. The results of these calibration runs are given in Table 1. As can be seen from the table, the instrument constant as determined for the different gases agreed quite well, and the maximum discrepancy, which occurred between the argon and the low-temperature helium values, amounted to only 0.90 per cent, which is not significantly different from the individual standard deviations.

TABLE 1
INSTRUMENT CALIBRATION RESULTS

Gas	Number of Runs	Average Temperature, °R	Assumed Viscosity, 10 ⁻⁷ poise	Experimental Instrument Constant C, 10 ⁻⁴ poise-sec	Standard Deviation of C, per cent
Dry Air	5	529.5	1820.0	835.0	0.60
Argon	5	527.7	2224.0	842.3	0.76
Helium	5	529.5	1986.9	840.9	0.92
Helium	5	144.0	855.6	834.8	0.65

3.5 EXPERIMENTAL RESULTS

3.5.1 Dry-Air Viscosity

The viscosity determinations fell into two rather natural temperature groups divided at 144 R, the normal boiling point of nitrogen. At temperatures above this temperature, liquid nitrogen was used as a system coolant, while at temperatures below 144 R, liquid hydrogen was employed. The large majority of data was obtained in the range above 144 R since it proved vastly simpler to transfer liquid nitrogen and to establish an operating temperature that was fairly stable for the 18 minutes required for a complete run of 100 oscillations. In addition, at enclosure-wall temperatures much below 100 R, the air condensed out on the walls to the detriment of further cooling of the disks. Accordingly, a low-temperature procedure was established of introducing a small amount of helium gas into the enclosure to aid in disk cooling and then vacuum pumping the helium from the system and replacing with dry air in preparation for a viscosity determination. The vibration of the rig introduced by this manipulation and the time required for helium pumping, air introduction, and pressure measurement resulted in numerous abortive attempts in which either the temperature would not remain sufficiently stable or the vibration proved excessive.

In temperature ranges of 144 R to 200 R, two types of runs were made. During the first series of determinations, the dry air was maintained at a fixed pressure, at about $1/2$ atmosphere, so that condensation would be avoided with certainty. Five runs were made at each selected temperature level. A second series of determinations at fixed temperature levels was made in which the absolute pressure of the dry air was varied over the range for which a marked slip behavior was anticipated. The latter data were used to evaluate the accommodation parameter of the viscosimeter in anticipation of the low-temperature viscosity determinations in which the low vapor pressure, characteristic of air at temperatures below its normal solidus point, would force operation in the slip regime. The second series of determinations was also used to calculate true viscosity values for inclusion with those at other temperatures obtained at pressures well above the slip regime. An additional series of individual runs made at various different temperatures and reduced pressures was made as a check and is included in the list of results.

Table 2 lists all of the true dry-air viscosity values determined during this program. These values are arranged in order of descending temperature. The table also includes the absolute pressure or pressure range over which the determinations were made. The percentage standard deviation was calculated for multiple determinations.

The viscosity values of Table 2 have been plotted in Figure 3 as a function of temperature along with a plot of the Sutherland equation fitted to selected viscosity values obtained in the present program.

TABLE 2
EXPERIMENTALLY DETERMINED DRY-AIR (TRUE) VISCOSITY

Temperature, R	True Viscosity, 10^7 poise	Absolute Pressure, psia	Number of Determinations	Percentage Standard Deviation For Replicate Determinations
529.56	1820.0	(Arbitrarily selected standard value)		
<i>K</i> 529.6	1830.0	9×10^{-5} to 11×10^{-3}	11	
110 198.0	1.63 781.2	6.76	5	1.00
108 194.4	1.626 778.3	7.2×10^{-3}	1	--
102.6 184.6	1.535 734.8	5.5×10^{-3}	1	--
100 180.0	1.472 704.8	6.71	5	1.84
98.3 176.9	1.483 709.9	3.7×10^{-3}	1	--
96.5 173.7	1.416 678.0	1.2×10^{-3}	1	--
95.0 171.0	1.410 675.2	1.3×10^{-3} to 1.02×10^{-2}	5	1.70
90.5 162.9	1.345 644.1	3.8×10^{-3}	1	--
90.4 162.8	1.380 660.8	1.3×10^{-3}	1	--
89.0 160.2	1.321 632.5	6.61	5	1.99
85.0 153.0	1.255 601.0	9.6×10^{-4} to 8.0×10^{-3}	5	1.66
82 147.6	1.250 598.3	0.49 to 5.86	5	0.85
80 144.0	1.248 597.6	6.65	5	1.02
75 135.0	1.140 545.6	6.5×10^{-4}	1	--
71.9 129.4	1.066 510.5	4.3×10^{-3}	1	--
67.9 122.2	0.979 468.9	12.7×10^{-3}	1	--
92.7	2.8	8.3×10^{-5}	1	--

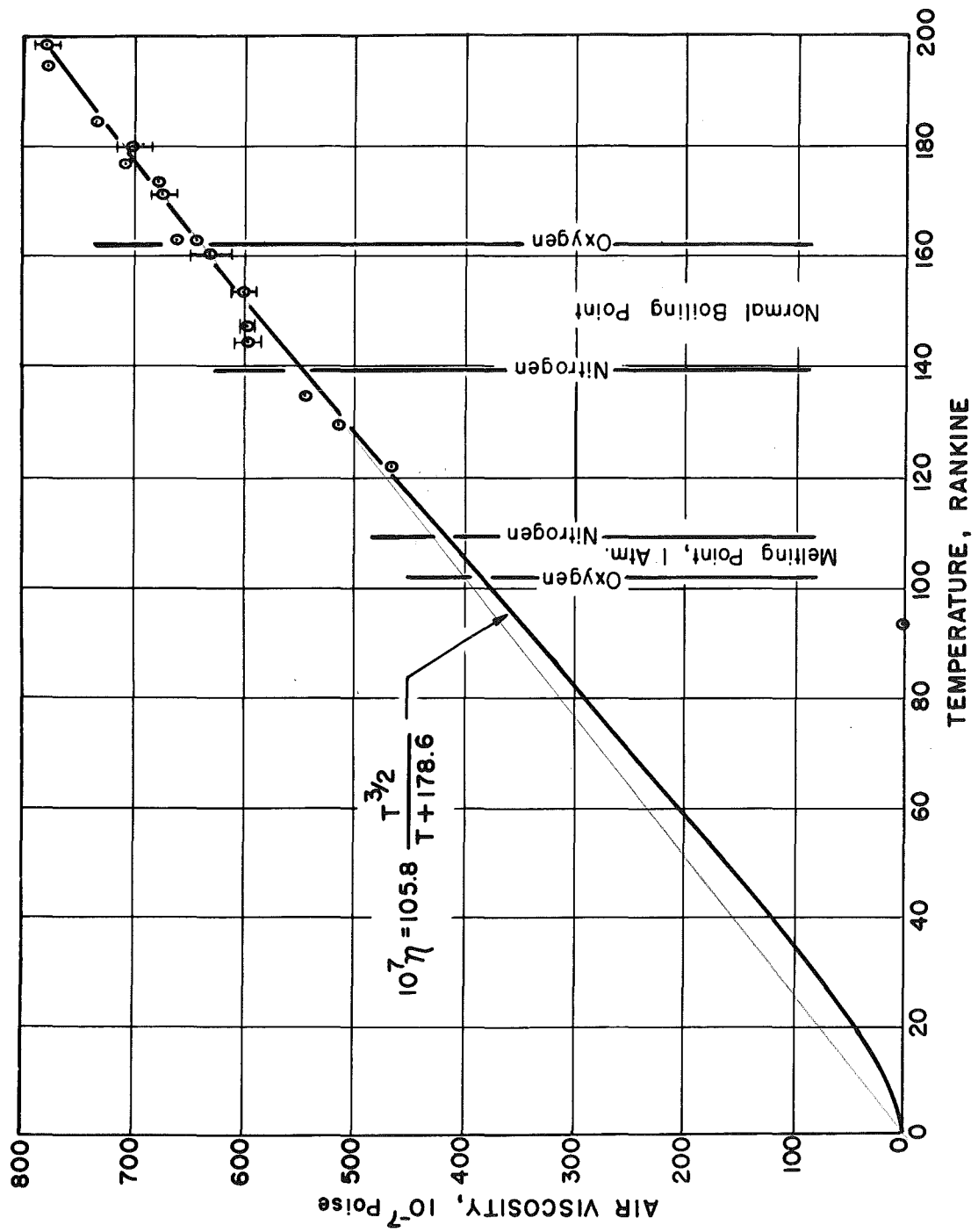


Fig. 3 Dry-Air Viscosity versus Temperature Data

3.5.2 Viscosimeter Accommodation Parameter

Table 3 lists the accommodation parameters determined for the viscosimeter from the series of determinations in which the pressure was varied over low levels at selected temperatures.

TABLE 3
EXPERIMENTALLY DETERMINED VISCOSIMETER
ACCOMMODATION PARAMETERS

Temperature, R	α , Accommodation Parameter, psia/poise
529.6	10.50
171.0	4.50
153.0	3.52

The measured accommodation parameters, when fitted to an equation of the form

$$\alpha = k\sqrt{T} \quad ,$$

where

α = accommodation parameter,
 k = constant, and
 T = absolute temperature,

yielded a value of 0.403 psia/(poise $\times$ (R)^{1/2}) for k . The fit was not very good and the lowest temperature value was almost 30 per cent lower than calculated from this equation. This reflects the probability that the surface accommodation coefficient (not to be confused with accommodation parameter), which is incorporated in k , is not constant but is actually dependent upon temperature. This would agree somewhat with the results of Itterbeck and Van Paemel⁽⁴⁾ in which they found a variation with temperature. Their experiments, however, showed that the accommodation coefficient was apparently larger at lower temperatures, while in the present work the opposite appears to be the case.

The accommodation parameter values listed in Table 3 and the corresponding true viscosity values were obtained by fitting the apparent viscosity values and absolute pressures to a linear form of the slip equation which, when rearranged, becomes

$$\frac{1}{\eta'} = \alpha \left(\frac{1}{p} \right) + \frac{1}{\eta} \quad .$$

A root-mean-square statistical procedure was applied to the data to obtain the slope, α , and the intercept, η^{-1} . An examination of the plotted data showed a reasonably good fit with the equation. The

maximum deviation amounted to about 4 per cent and generally occurred at the lowest pressure where the uncertainty in the pressure was of this same magnitude.

4.0 CORRELATION OF DATA

A root-mean-square curve-fitting procedure was used to obtain the constants of the Sutherland equation relating true viscosity to temperature. The average viscosity values obtained at 144.0, 147.6, 160.2, 180.0, 198.0 R were used for this analysis. Subsequent to this analysis, the remaining data presented in Table 2 were obtained and, with the exception of the 92.7 R viscosity value, appeared to fit the same constants consistent with the average standard deviation of ± 1.5 per cent measured for the replicate determinations. Because all of the present measurements were based upon the arbitrarily chosen value of 1820.0×10^{-7} poise for dry-air viscosity at 68.9 F, the curve fitting was designed to make the equation fit this value exactly. Each investigator cited has chosen a different standard value as a base-line viscosity for his data. The constants presented in Table 4 have all been adjusted to the present base line and units in order to facilitate intercomparison. Included also in the table is the viscosity value at 162.0 R calculated from each set of constants. This was done to further facilitate comparison, since the formula of F. G. Keyes is not of quite the same form as the others, and the Temperature ranges over which the constants were determined are not the same.

5.0 DISCUSSION AND CONCLUSIONS

As can be seen in Table 4, the dry-air viscosity data obtained in the present program compare far more favorably with the data obtained by Johnston and McCloskey and with the empirical formula of F. G. Keyes than with the NBS/NACA formula. At 162.0 R, the NBS/NACA equation yields a viscosity value about 3.0 per cent lower than predicted by either the present work or by the other two sources, whereas the latter three agree within 0.3 per cent. The F. G. Keyes equation, however, being of a slightly different form than the common Sutherland equation, predicts a viscosity at 198 R which is 0.6 per cent lower than predicted by the present work. Even though this is poorer agreement than at 162.0 R, it is still far better than the agreement with NBS/NACA.

Since the average standard deviation of the present data is 1.5 per cent and the temperature range comparatively limited, it is not possible to deduce with confidence from these data alone what the form of the viscosity-temperature equation should be. However, based on the large temperature range of validity claimed by F. G. Keyes for the modified form and the excellent fit with the data of Johnston and McCloskey and with the present work, it is recommended that this modified form and constants be accepted for calculation of dry-air

TABLE 4
SUTHERLAND FORMULA CONSTANTS

Investigator	Temperature Range, R	Constants ^(a)		Calculated Viscosity at 162.0 R, 10 ⁻⁷ poise	Ratio of Calculated Value to Present Value
		A	B		
Present work	144-530	105.8	178.6	640.6	--
NBS/NACA ⁽⁵⁾	180-3420	108.7	198.7	621.5	0.970
Johnston and McCloskey ^(b)	162-540	105.4	176.2	642.7	1.003
F. G. Keyes ⁽⁶⁾	142-3321	110.6	211.4 ^(c)	641.8	1.002
			@529.6 R		
		193.	193.4 @162.0 R		

(a) Constants of formula: $10^7 \eta = A \frac{(T,R)^{3/2}}{(T,R) + B}$, poise.

(b) Tabulated smoothed data of Johnston and McCloskey were fit to Sutherland equation at 160 and 540 R.

(c) Keyes uses a modified form in which B is a function of temperature, i.e., $B = 219.8 \times 10^{-9}/T, R$.

viscosity at temperatures down to at least 153 R. An accuracy of 0.5 per cent or better in the cryogenic range may be accepted with moderate confidence.

Below 153 R, the present data show a marked departure from either the Sutherland or modified viscosity predictions. Most startling of all is the almost complete lack of viscous drag in the lone determination made at 92.7 R. Because of the large, relative uncertainty in this experiment, it is not possible to conclude that any viscous drag, other than the internal torsion-wire friction, was measured in the viscosimeter. Calculation of the apparent viscosity that would be expected for the temperature and pressure conditions of this experiment gives a value of 95×10^{-7} poise. This is more than 30 times the value that was measured.

No reasonable explanation has yet been advanced for the low drag observed at 92.7 R. The fact that a damping equivalent to the viscosity of hydrogen gas was observed at 36 R when a leak developed in the evacuated enclosure during early attempts to operate with liquid hydrogen as a heat-sink supports the belief that this low-temperature behavior is not an abnormality of the viscosimeter. Drag determinations made at almost identical pressure but at room temperature did not show this anomalous behavior, suggesting that this was not a result of gross inaccuracy of the McLeod gage or a problem associated with flow of the gas into the region between the disks. Indeed all reasonable disturbances such as condensation on the disks and shift in disk centering would tend to yield a larger value of drag than would be predicted for normal operation.

As mentioned earlier, a small amount of helium was admitted to the chamber to facilitate cooling prior to this experiment. Although this helium was evacuated before dry air was admitted and pressure monitoring during the evacuation showed that no significant amount of helium remained, it is of interest to see what effect residual helium would have had on the drag measurement. Since the true viscosity of helium gas at 92.7 R is higher than that of air, this alone would not explain the lower observed drag. Although the slip effect for helium is greater than for air, a crude estimate of this effect based upon accommodation coefficients cited in the literature indicates that pure helium would have exhibited a drag at least 10 times that observed.

Air may actually exhibit superfluidity at temperatures below its normal liquidus range, or more probably, this lack of viscous drag was the result of an unexpectedly large slip at the disk surfaces. However, since this behavior was observed for only a single rather isolated determination, it is not possible to come to a valid conclusion. There is a trend indicated by the data at the four temperature levels above this toward a more rapid drop in viscosity with decreasing temperature than predicted by the equations at higher temperatures. However, because of the lack of replicate determinations below the normal boiling point of nitrogen, the validity of this trend is also open to question. Unfortunately the low vapor pressure of air at temperatures approaching the normal solidus range presents difficulties in avoiding condensation of the air in the region between the disks. In the present apparatus, this was combated by

providing that the enclosure walls be colder than the disks. Attempts to make determinations in the 75 to 90 R range, however, were foiled by the nearly complete freezing of the air on the walls and consequent lack of sufficient gaseous molecules to perform valid drag determinations. Accurate pressure measurements are also difficult to make under these circumstances because of the thermo-molecular pressure difference between the gage and the enclosure which occurs at large temperature differences and low absolute pressures. Where this correction was required in the present work, the equation for nitrogen given by Low and Fergusson⁽⁷⁾ was applied, and never amounted to more than a 20 per cent correction.

In addition to the anomalous behavior over the liquidus-to-solidus transition range, there also appeared a deviate behavior in the region of the gas-liquid transition. The two sets of replicate determinations at 144.0 and 147.6 R indicate that the true air viscosity is greater than predicted by the ordinary viscosity-temperature relations. Since these temperatures approach the normal boiling point of nitrogen of 139.5 R, it is not completely unexpected that the common kinetic theory might fail to predict this behavior. Very little experimental data have been uncovered on the viscous behavior of gases in the close proximity of phase transitions. A small graph of the viscosity of water vapor versus temperature appearing in an article by Richter⁽⁸⁾ shows that a deviation of the character revealed by the present work may indeed be real. No experimental data are given, but the graph shows that as the saturation line is approached, the gaseous viscosity exhibits an increase in the immediate vicinity of the saturation temperature over that which would be predicted by kinetic theory. The effect with water, however, appears marked only at conditions approaching the critical point. The present work, on the other hand, was conducted under conditions well below the critical temperature and density of both oxygen and nitrogen. Also, the present data obtained at 147.6 R made under different reduced pressures showed no significant density dependence. This is not in accord with the behavior that might be expected since these deviations in the vicinity of the phase transition should be affected by density. Accordingly, the observed behavior appears reasonable in character if not in magnitude or in detail.

The data obtained from the series of runs at 153.0, 171.0, and 529.6 R conducted in the slip regime confirmed the form of the second equation on page 2 relating the true viscosity and the absolute pressure. These results substantiate those of Van Itterbeck and Van Paemel⁽⁴⁾ for helium, hydrogen, deuterium, and neon at lower temperatures. However, because of the present lack of data on slip behavior of air at the six lowest temperatures examined and the observed anomalous behavior at 92.7 R, some doubt must be retained as to the universality of this relation for air below 153 R.

6.0 FUTURE WORK

From the results of the present investigation, it would appear that air viscosity above about 153 R can be predicted with a high degree of confidence. Below this temperature, and particularly in the liquidus-solidus and liquidus-vaporous transition ranges, the present work indicates that the viscous drag exhibits anomalous behavior. Additional investigation may be necessary if precise air-viscosity values in the 90 to 150 R range are required. These investigations should entail measurements of damping at several temperature levels in this range as a function of absolute pressure. The pressure levels should be chosen to span the slip regime as much as possible to provide data from which accommodation information may be extracted. The disk spacing may be varied to provide additional information on the slip phenomena. Different disk materials should be studied to determine accommodation coefficients for air on engineering material of interest at cryogenic temperatures.

Many of the difficulties involved in a more thorough investigation of this transitional range can be anticipated. The extremely low vapor pressure of air at the lower end of this range dictates careful attention to thermomolecular effects upon pressure measurements. Wall temperatures must be carefully controlled to avoid excessive cryopumping of the test region, and vibration effects must be minimized so that extremely low damping can be measured with a fair degree of precision.

It should be noted that one must be cautious about applying accommodation information obtained at low surface speeds to the analysis of hypervelocity wind tunnel performance.⁽⁹⁾ Although high-speed data might be obtained with a system similar to that of J. W. Beams⁽¹⁰⁾ in which peripheral velocities of a spinning ball of about 3000 feet per second can be obtained, the measurement and control problems in the present temperature-pressure regime would be formidable. On the other hand, the oscillating-disk technique, although inherently a low-speed device, can be designed for careful control and accurate measurement, and because of the scarcity of information about the accommodation coefficient for air⁽¹¹⁾ and the complete lack of data at cryogenic temperatures, the low-speed investigations would serve as a valuable guide to future investigations.

REFERENCES

- (1) Johnston, H. L., and McCloskey, K. E., "Viscosity of Gases", J. Phys. Chem, 44, 1038 ff (1940).
- (2) Macwood, G. E., "The Theory of the Measurement of Viscosity and Slip of Fluids by the Oscillating-Disk Method", Physica, 5, 374 ff and 763 ff (1938).
- (3) Kestin, J., and Wang, H. E., "Corrections for the Oscillating-Disk Viscosimeter", Brown University Report No. 891/5, AD 82740 (December, 1955).
- (4) Van Itterbeck, A., and Van Paemel, O., "Measurements on the Viscosity of Gases for Low Pressures at Room Temperature and at Low Temperatures", Physica, 7, 273-283 (April, 1940).
- (5) "The NBS-NACA Tables of Thermal Properties of Gases", NBS, Table 2.39 (1951?).
- (6) Keyes, F. G., "The Heat Conductivity, Viscosity, Specific Heat and Prandtl Number for Thirteen Gases", Project Squid, MIT Tech. Report 37 (April 1, 1952).
- (7) Los, J. M., and Fergusson, R. R., "Measurements of Thermomolecular Pressure Differences on Argon and Nitrogen", Trans. Faraday Soc., 48, 730-738 (1952).
- (8) Richter, H., "Wahrscheinliche Werte für die Zähigkeit von Wasserdampf", Brennstoff-Warme-Kraft, 13 (4), 117-120 (April, 1951).
- (9) Wachman, H. Y., "The Thermal Accommodation Coefficient: A Critical Survey", GE-MSVD Report R61SD197, Philadelphia, Pennsylvania, p. 35 (December, 1961).
- (10) Beams, J. W., et al, "Spinning Rotor Pressure Gage", Review of Scientific Instr., 33 (2), 151-155 (February, 1962).
- (11) Hartnett, J. P., "A Survey of Thermal Accommodation Coefficients", The Rand Corporation Res. Mem. RM-2585, Santa Monica, California, pp 33-37 (March 19, 1960).

DRG:WHG/SJH

February 19, 1963

Arnold Engineering Development Center Arnold Air Force Station, Tennessee Rpt. No. AEDC-TDR-63-143. EXPERIMENTAL DETERMINATION OF THE VISCOSITY OF AIR IN THE GASEOUS STATE AT LOW TEMPERATURES AND PRESSURES. June 1963, 24 p. incl 11 refs., illus., tables. Unclassified Report A program was undertaken to determine the viscosity of gaseous air at cryogenic temperatures for use in the interpretation of hypervelocity wind-tunnel data. Data were obtained over a temperature range of 92.7 to 198.0 R using an oscillating-disk viscosimeter designed for cryogenic operation. Data were also obtained at greatly reduced pressures where the slip effect becomes important. This was done to aid in the evaluation of the lowest temperature data, where reduced pressures were mandatory in order to avoid condensation. A fit of the viscosity data to a temperature-viscosity equation of the Sutherland form 	1. Air 2. Viscosity 3. Chemical analysis 4. Low-pressure research 5. Low-temperature research 6. Hypersonic wind tunnels 7. Viscosimeters I. AFSC Program Area 806A, Project 8951, Task 89603 II. Contract AF 40(600)-938 III. Battelle Memorial Institute, Columbus, Ohio IV. D. R. Grieser and W. H. Goldthwaite V. Available from OTS VI. In ASTIA Collection	Arnold Engineering Development Center Arnold Air Force Station, Tennessee Rpt. No. AEDC-TDR-63-143. EXPERIMENTAL DETERMINATION OF THE VISCOSITY OF AIR IN THE GASEOUS STATE AT LOW TEMPERATURES AND PRESSURES. June 1963, 24 p. incl 11 refs., illus., tables. Unclassified Report A program was undertaken to determine the viscosity of gaseous air at cryogenic temperatures for use in the interpretation of hypervelocity wind-tunnel data. Data were obtained over a temperature range of 92.7 to 198.0 R using an oscillating-disk viscosimeter designed for cryogenic operation. Data were also obtained at greatly reduced pressures where the slip effect becomes important. This was done to aid in the evaluation of the lowest temperature data, where reduced pressures were mandatory in order to avoid condensation. A fit of the viscosity data to a temperature-viscosity equation of the Sutherland form 	1. Air 2. Viscosity 3. Chemical analysis 4. Low-pressure research 5. Low-temperature research 6. Hypersonic wind tunnels 7. Viscosimeters I. AFSC Program Area 806A, Project 8951, Task 89603 II. Contract AF 40(600)-938 III. Battelle Memorial Institute, Columbus, Ohio IV. D. R. Grieser and W. H. Goldthwaite V. Available from OTS VI. In ASTIA Collection
gives $10^7 \eta = 105.8 \frac{(T, ^\circ R)^{3/2}}{(T, ^\circ R) + 178.6}, \text{ poise.}$ This fit agrees quite well with the data of Johnston and McCloskey and with the low-temperature portion of the equation of F. G. Keyes. The data obtained at the lowest temperatures deviated from the behavior predicted by the Sutherland Formula. A minor deviation occurred in the vicinity of 140 R, while the major deviation occurred at the lowest temperature of 92.7 R. At this latter temperature, the measured air drag was essentially zero. No substantiated explanation has been given for this latter behavior. 		gives $10^7 \eta = 105.8 \frac{(T, ^\circ R)^{3/2}}{(T, ^\circ R) + 178.6}, \text{ poise.}$ This fit agrees quite well with the data of Johnston and McCloskey and with the low-temperature portion of the equation of F. G. Keyes. The data obtained at the lowest temperatures deviated from the behavior predicted by the Sutherland Formula. A minor deviation occurred in the vicinity of 140 R, while the major deviation occurred at the lowest temperature of 92.7 R. At this latter temperature, the measured air drag was essentially zero. No substantiated explanation has been given for this latter behavior. 	