AD 411599

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

REPORT 402

ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

64 RUE DE VARENNE, PARIS VII

REPORT 402

THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT

by

D. G. GOULD

JULY 1962

NORTH ATLANTIC TREATY ORGANIZATION

REPORT 402

NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT

by

D.G.Gould

This Report was presented at the Twenty-First Meeting of the Flight Mechanics Panel of AGARD, held in Paris, 6-10 July, 1962

SUMMARY

The application of a technique described as the model-controlled method (termed adaptive or semi-adaptive when applied to auotpilots) for providing aircraft with variable stability and control characteristics is described. The development of a variable-stability helicopter using this method is treated in some detail to illustrate the significant design parameters and to demonstrate the advantages of the method over the more customary approach.

The method allows for versatility in regard to simulation of complex equations of motion in that use may be made of standard computing components to construct an electrical analogue of the equations of motion of the simulated aircraft in a manner similar to that employed with ground simulation. An accurate knowledge of the test aircraft stability derivatives and their variation with speed and altitude is not required since the test aircraft response is independent of significant variations in its own characteristics. In the case of application of the method to helicopters, in-flight adjustment of the control loop parameters is not required to ensure this independence over the full speed range of the helicopter. Use of the method for variable-stability fixed-wing aircraft may require in-flight adjustment of the control loop gain if the inherent control sensitivity changes by more than about 50% owing to speed and/or altitude changes during the test.

629.13.014.5:533.6.013.4

629.135.45

SOMMAIRE

L'application d'une technique, dite méthode à contrôle par modèle (désignée méthode adaptive ou semi-adaptive lorsqu'il s'agit des auto-pilotes), permettant de doter les avions de qualités de vol variables, est décrite. La mise au point d'un hélicoptère à stabilité variable grâce à cette méthode, est examinée en détail en vue de mettre en évidence les paramètres de calcul importants et de démontrer les avantages de la méthode sur celle normalement employée.

La méthode permet la versatilité dans la simulation des équations du mouvement complexes, étant donné qu'il est possible de faire appel à des éléments de calcul standard pour construire, par analogie électrique, les équations du mouvement de l'avion simulé d'une façon pareille à celle pour la simulation au sol. Des précisions exactes sur les dérivées de la stabilité de l'avion d'essai et de leur variation en fonction de la vitesse et de l'altitude ne sont pas nécessaires, puisque la réponse de l'avion d'essai est indépendante des variations importantes de ses propres caractéristiques. Dans le cas de l'application de la méthode aux hélicoptères, le réglage en vol des paramètres de la boucle de commande n'est pas requis pour obtenir cette indépendance sur toute la gamme des vitesses de l'hélicoptère. L'utilisation de la méthode pour les avions à aile fixe à stabilité variable peut demander le réglage en vol du gain de la boucle de commande, si la sensibilité inhérente de commande varie de plus de 50% environ par suite de variations de la vitesse et/ou de l'altitude au cours de l'essai.

629.13.014.5:533.6.013.4

629.135.45

CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOTATION

a, A, A_1, A_2, b, B, B_1, B_2	helicopter or aircraft transfer function coefficients (see Equations 1 and 5)
B'	pitching moment of inertia (slug-ft^2)
c, C, D, E	helicopter or aircraft transfer function coefficients (see Equations 1 and 5)
g	acceleration due to gravity (ft/sec^2)
k_H, λ	helicopter or aircraft transfer function coefficients (see Equations 1 and 5)
K	loop component gain
m	mass of helicopter or aircraft (slugs)
M	pitching moment of helicopter or aircraft (lb-ft)
s	Laplace transform variable
q	angular rate (rad/sec)
τ	transport time lag (sec)
k	zero frequency gain of loop component
e_i	closed loop input voltage, proportional to model angular rate
e_0	closed loop feedback voltage, proportional to aircraft angular rate
ϵ	$= e_i - e_0$, error signal (volts)
U_0	initial forward velocity of helicopter or aircraft (ft/sec)
X	longitudinal force (lb)
Z	vertical force, positive downward (lb)

$\left.\begin{array}{l}\alpha\\ \beta\end{array}\right\}$ compensation circuit parameters

γ	servo valve and actuator parameter
δ	control displacement
θ_0	initial pitch angle (rad)
λ	model break-point frequency (rad/sec)
ϕ	loop component phase angle
ω	frequency (rad/sec)
ω_π	ω_0 when $\phi = -\pi$
C(s)	compensation transfer function
G(s)	transducer transfer function
H(s)	helicopter or aircraft transfer function
P(s)	servo valve and actuator transfer function
S(s)	control system transfer function

Subscripts

c	compensation circuit
G	transducer
H	helicopter or aircraft
L	closed loop
m	model
o	open loop
P	servo valve and actuator
S	control system

Subscripts - Aerodynamic derivatives with respect to

q	pitching velocity
u	forward velocity

w vertical velocity

$\dot{w}$ vertical acceleration

δ control displacement

THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT

D.G.Gould*

1. INTRODUCTION

Simulation of physical systems has proved to be invaluable for research and design studies, especially if a human operator forms a part of the control element of the system. Owing to the lack of detailed understanding of the human operator, simulation experiments of this type have had to rely on conjecture about the essence of the real situation. The conclusions from such tests must be treated with some caution and before being generally accepted usually must be substantiated with experiments conducted in the real or very nearly real environment. The variable-stability aircraft has been used in this context as it can provide a facility for reproducing the real situation to a higher degree than ground-based simulators, particularly in regard to the psychological environment.

Significant quantitative differences in the pilots' dynamic behaviour and opinion boundaries between flight and ground simulator experiments have been observed[1-3]. The main differences observed in the pilots' dynamic response have been a reduction in gain by a factor of two, a twofold increase in the 'effective' reaction time delay and a reduction in the linear correlation factor (also by a factor of about two), for measurements in flight compared with those on the ground. Although such experiments demonstrate real differences in behaviour, it is not possible to infer in detail the particular environmental conditions that lead to the variance. Since the tasks were visual tracking exercises, stimulation of the vestibular organs and tactual senses by real motion of the aircraft would be expected to be of secondary importance, and the noted variance in pilots' behaviour is most likely associated with what may be termed loosely as differences in the psychological environment between flight and ground experiments.

Although the variable-stability aircraft can be used as a simulator to give a more realistic psychological environment, one of its distinct disadvantages compared with ground simulators has been the relative difficulty of providing versatility to simulate complex aircraft equations of motion. In the case of ground-based simulators, this versatility is achieved by making use of standard computing components to construct electrical analogues of the equations of motion. Light-weight transistor computing elements and techniques employed in adaptive control processes developed in recent years make it possible to provide the variable-stability aircraft with similar versatility, with regard to aircraft dynamics, to that of ground simulators. One such application of these techniques to variable-stability aircraft and, in particular, to variable-stability helicopters, is discussed in some detail in the following Sections. The method is illustrated by a description of a variable-stability helicopter developed at the N.A.E.

**National Aeronautical Establishment, Ottawa, Ontario, Canada*

2. PHYSICAL MOTION REQUIREMENTS

One of the primary purposes of the variable-stability aircraft, as remarked upon previously, is to provide a facility that reproduces the real situation to a high degree. In spite of its potential, however, it has inherent limitations, particularly in regard to the reproduction of real physical motions in all degrees of freedom (see Section 3). Efforts to simulate comprehensively the real situation can lead to a very complex and unwieldy simulator. Fortunately, recent experiments and our growing, even if rudimentary, understanding of the performance of the human operator have made it possible to establish in a general way the more noteworthy parts of the real motion environment. A number of experiments have been conducted to establish the effect of real motion on the pilots' behaviour and opinion of particular dynamic systems[3-6]. The effects were shown to be a function of both the task and the controlled element dynamics. In general, linear accelerations had little effect on the pilot's response or opinion of a particular system if the accelerations required in the manoeuvres were low (less than 2 to 3g). Angular accelerations, on the other hand, may influence the pilot's opinion of a system, particularly at small values of linear acceleration and for controlled element dynamics of a marginally undesirable character. Data given in Reference 7 show that for simple rectilinear flight tasks a pilot uses angular acceleration as a cue in minimizing the response due to external disturbances.

The reproduction of real linear velocity and displacement becomes important when the pilot is using the external visual environment as his primary reference, as in landing in all types of aircraft or hovering in VTOL types of aircraft.

An important factor in the development of any simulator with reproduction of real motion is the required frequency bandwidth over which the motion must closely approximate that of the system simulated. In the case of systems where a pilot is the primary control element, the required bandwidth is related to the pilot's maximum response frequency. In closed-loop flight control tasks, the highest frequencies of significance in the pilot's response are typically 10 radians per second or less[2]. This results in response times of from 0.3 to 0.4 second for tracking tasks, or minimum pulse periods of from 0.6 to 0.8 second for other common tasks, independent of the response magnitude. A comparison of the desired and actual responses due to a rapid pilot's input of about 0.3 to 0.4 second duration is a meaningful method for assessing the effect of the frequency bandwidth of the simulator.

3. COMPARISON OF MODEL-CONTROLLED METHOD WITH CUSTOMARY APPROACH

The technique for achieving variable stability and control characteristics discussed in detail in this Paper is similar in principle to that used in a current type of autopilot referred to as an adaptive or semi-adaptive system. In order to avoid the controversy as to whether or not this approach can be legitimately classified as adaptive, the present system has been called the model-controlled method.

The advantage of the method can be most easily demonstrated by consideration of some of the difficulties encountered with the customary approach. The model-controlled method resolves these problems by by-passing them, so that the discussion of the more common approach is brief; specifically, only the longitudinal pitch equation is treated.

3.1 Customary Approach

The transfer function relating the pitch response to the longitudinal control input is similar in form for both a helicopter and a fixed-wing aircraft, and for the latter is as follows:

$$\frac{\hat{\theta}}{\hat{\delta}} = \frac{as^2 + bs + c}{As^4 + Bs^3 + Cs^2 + Ds + E} \tag{1}$$

where

$$\left.\begin{aligned}
a &= m(M_{\dot{w}}Z_\delta - Z_{\dot{w}}M_\delta) + m^2M_\delta \\
b &= m(M_wZ_\delta - Z_wM_\delta) + m(M_uX_\delta - X_uM_\delta) + M_{\dot{w}}(Z_uX_\delta - X_uZ_\delta) + \\
&\quad + Z_{\dot{w}}(X_uM_\delta - M_uX_\delta) \\
c &= X_u(M_wZ_\delta - Z_wM_\delta) + Z_u(X_wM_\delta - M_wX_\delta) + M_u(Z_wX_\delta - X_wZ_\delta) \\
A &= mB'(m - Z_{\dot{w}}) \\
B &= mB'(X_u - Z_w) + m(Z_{\dot{w}}M_q - M_{\dot{w}}Z_q) - m^2(U_0M_{\dot{w}} + M_q) - B'X_uZ_{\dot{w}} \\
C &= mX_u(M_q + U_0M_{\dot{w}}) + m(M_qZ_w - Z_qM_w) + m^2(g\theta_0M_{\dot{w}} - U_0M_w) + \\
&\quad + B'(X_uZ_w - Z_uX_w) + X_u(M_{\dot{w}}Z_q - Z_{\dot{w}}M_q) \\
D &= -mg\theta_0X_uM_{\dot{w}} + mg(Z_uM_w - Z_{\dot{w}}M_u) + mU_0(X_uM_w - M_uX_w) + \\
&\quad + m^2g(\theta_0M_w + M_u) + X_u(M_wZ_q - Z_wM_q) + X_w(Z_uM_q - M_uZ_q) \\
E &= mg[\theta_0(X_wM_u - M_wX_u) + (Z_uM_w - Z_wM_u)]
\end{aligned}\right\} \tag{2}$$

and s is the Laplace transform variable. The terms on the right hand side of Equations (2) represent the aircraft mass, the moment of inertia about the lateral axis, and the aerodynamic stability derivatives, in a dimensional form. Within the linear range of response, the coefficients a to E of Equation (1) uniquely determine the response to a particular control input.

In order to make the test aircraft's response in pitch to longitudinal control match that of the simulated aircraft, the coefficients a to E inherent in the test aircraft must be artificially altered to match those of the simulated aircraft. This is normally approximated by sensing particular aircraft motions and aerodynamic parameters (e.g. angular velocity, angle of attack, normal acceleration) and providing for elevator and/or flap (or other lift-producing device) displacement to give pitching moments and vertical forces proportional to the sensed parameters and their derivatives. The difficulties are apparent from a casual inspection of Equations (2). If, for example, the coefficient C of the test aircraft were to be altered to the value corresponding to the simulated aircraft by applying elevator motion proportional to sensed values of pitch rate and angle of attack, the coefficients M_w, M_q are changed wherever they occur in Equations (2) so that the values of b, c, B, D and E are altered along with C. It is difficult therefore to adjust all the coefficients simultaneously to desired values. In extenuation, many of the terms in Equations (2) are small, so that it is usually possible, but difficult, to approximate all the coefficients to the desired accuracy.

This procedure requires also an accurate knowledge of the individual inherent aerodynamic stability parameters of the test aircraft. If the changes in dimensional stability coefficients with speed or altitude are large, only small variations in speed or altitude of the test aircraft are tolerable without affecting the desired response.

3.2 Model Controlled Approach

3.2.1 General Description

A full description of the model-controlled autopilot in use at the N.A.E. is given in Reference 8. In this approach a simple feedback loop is formed for each degree of freedom that is to be controlled (see Figure 1). Each loop consists of a compensation network (generally a phase-lead network), a control actuation servo, the test aircraft dynamics relating response of that particular degree of freedom to the control affecting the response, and an appropriate motion-sensing transducer. In the case of a conventional fixed-wing aircraft, the four degrees of freedom that may be controlled without auxiliary controls are pitch, roll and yaw rotation, and longitudinal translation, using the elevator, aileron, rudder and throttle respectively. In the case of a helicopter, the four degrees of freedom that may be simply controlled are pitch, roll and yaw rotation, and vertical translation, using longitudinal cyclic, lateral cyclic, tail rotor collective and main rotor collective pitch controls. In each case, the response of the uncontrolled degrees of freedom will depend upon the inherent test aircraft dynamics. If the two uncontrolled degrees of freedom are to be controlled, auxiliary controls must be provided, for example - actuation of flaps to control the vertical translation degree of freedom of fixed-wing test aircraft.

The pilot's control commands are fed to an electrical analogue of the equations of motion to be simulated, the output of which is a voltage proportional to the desired response of the motion variable that is sensed by the transducer in the appropriate closed loop. The desired response is compared with the measured response, and the closed loop acts to minimize the difference.

If good closed-loop performance can be maintained over the desired frequency bandwidth, the test aircraft motion in the degrees of freedom being controlled will follow closely the motion that the simulated aircraft would exhibit in the real situation. Furthermore, under these conditions, the test aircraft response in each of the controlled degrees of freedom is independent of its inherent response characteristics, and dependent only upon the equations of motion simulated by the electrical analogue.

This allows for versatility in regard to the form, and independent variation of the values of the coefficients, of the individual transfer functions defining the response of the simulated aircraft about the controlled degrees of freedom.

The method requires that good performance of each of the closed loops be maintained over the range of altitude and speed covered by the test aircraft during the particular simulation. (This range need not, of course, be that of the aircraft simulated unless control of the translational degrees of freedom is necessary.) The significant test aircraft variable that affects the closed-loop performance, as shown in Section 4 of this Paper, is the control sensitivity. The approach is, therefore, particularly suitable to variable-stability helicopters, in that the variation in control sensitivity is sufficiently small that a constant electrical loop gain (K_m) gives good loop response over a wide range of speed and altitude. In the case of fixed-wing aircraft, in-flight adjustment of the electrical gain may be necessary in some cases, to allow for changes in control sensitivity of the test aircraft with altitude and speed.

3.2.2 Description of NAE Variable-Stability Helicopter

A photograph of the variable-stability helicopter developed at the N.A.E. (a Bell H-13G on loan from the U.S. Army) is shown in Figure 2. In this aircraft, only the three angular degrees of freedom are controlled. Control of vertical translation has not yet been attempted because of the very restricted payload capabilities of the machine, and because it was considered to be of lesser importance than control of the angular degrees of freedom for the particular investigations planned. A detailed description of this variable-stability aircraft is given in Reference 9.

The evaluation pilot's controls located on the right hand side are connected to potentiometers to provide the input to an electrical analogue of the equations of motion of the simulated aircraft. Variable spring feel is used for the longitudinal, lateral and directional controls, and the longitudinal control is provided with a servo motor to alter the spring bias position to simulate longitudinal trim changes. The evaluation pilot's instrument panel contains engine instrumentation, an artificial horizon, a needle and ball, a directional gyro, an ILS indicator and direct-current meters for simulated air speed, altitude and rate of change of height.

The electrical analogue for simulation of the desired equations of motion consists of commercially available transistorized analogue computing elements and is mounted externally on the right hand side of the aircraft.

The motion transducer in each of the three rotational degree-of-freedom control loops is an angular rate gyro. The servos are pneumatic and are connected in parallel to the normal mechanical system with approximately full authority for each control. The servo torques are low enough for the safety pilot on the left hand side to overpower the actuators and safely fly the aircraft using the normal mechanical control system. Switches on both control sticks allow for instant disengagement of the servo actuators.

A fourteen-channel galvanometer recorder is located externally on the left side of the aircraft to record pilot's control motions, aircraft angular response, output of the electrical analogue or error signals (q_m - q_H), and the error signal from the ILS system in the aircraft.

Photographs of the installation of the various components of the system are shown in Figure 3 (a, b and c).

4. CONTROL LOOP REQUIREMENTS AND PERFORMANCE

In common with most process designs, specification of the required closed-loop performance in clear terms is difficult. There are upwards of a dozen specifications for this purpose, each of which has a clear meaning for only particular instances. Whether or not the performance is adequate can only be ascertained by a detailed knowledge of the particular loop and its effect on the complete process performance. An analysis of the control loops for the variable-stability helicopter developed at the N.A.E. is given in the following Sections to demonstrate the significant parameters in the design. The analysis and the significance of parameters are characteristic of most applications to helicopters and, to a certain extent, of applications to fixed-wing aircraft.

Each of the control loops (Fig.1) can be separated into component transfer functions describing the dynamics of the motion transducer, G(s), the aircraft, H(s), the control system, S(s), the servo actuator, P(s), and the compensation circuit, C(s). If the open-loop gains and phase angles of these components are designated K_G, K_H, K_S, K_p, K_C and ϕ_G, ϕ_H, ϕ_S, ϕ_P, ϕ_C respectively, the closed-loop response representing the ratio q_m/q_H as a function of frequency is given by

$$\frac{q_m}{q_H} = \frac{K_0}{\sqrt{\{(K_0 + \cos\phi_0)^2 + \sin^2\phi_0\}}} e^{i\phi_L} \tag{3}$$

$$\left.\begin{aligned} \text{where} \quad \phi_L &= \tan^{-1} \frac{\sin\phi_0}{K_0 + \cos\phi_0} \\ \phi_0 &= \phi_G + \phi_H + \phi_S + \phi_P + \phi_C \qquad \text{(total open-loop phase angle)} \\ K_0 &= K_G \times K_H \times K_S \times K_P \times K_C \qquad \text{(open-loop gain).} \end{aligned}\right\} \tag{4}$$

Over the range of frequency where $K_0 \gg 1$, ϕ_L is small and $q_m/q_H = 1$, so that the helicopter response q_H follows closely the desired response q_m. Since K_0 must be less than unity at $\phi_0 = -\pi$ or the closed loop will oscillate, q_m/q_H is approximately unity and ϕ_L is small only for values of ω somewhat less than ω_π, the value of ω where $\phi_0 = -\pi$. All the components, with the exception of the compensation circuit, produce phase lag and gain attenuation at high frequencies. The compensation circuit is designed to give phase lead and increase the frequency at which ϕ_0 becomes equal to $-\pi$.

4.1 Pitch Control Loop

4.1.1 Motion Transducer, G(s)

The motion transducer for each of the angular degrees of freedom is an angular rate gyro, with a sufficiently large natural frequency such that ϕ_G is negligible and $K_G = -K_m$ = constant over the range of frequency of interest.

4.1.2 *Aircraft Dynamics,* H(s)

The pitch rate to longitudinal control transfer function of the helicopter (exclusive of the blade response mode which is included in the control system transfer function S(s)) is of the form

$$H(s) = \frac{k_H(s + \lambda)(s^2 + as + b^2)}{(s^2 + A_1s + B_1{}^2)(s^2 + A_2s + B_2{}^2)} \qquad (5)$$

The coefficients of this transfer function for the NAE helicopter were obtained from transient response tests at two speeds, one near maximum and the other near hovering, and are given in Table I. The two modes defining this response, a long-period lightly or neutrally damped mode (coefficients A_2 and B_2^2) and a short-period well damped or subsidence mode (coefficients A_1 and B_1^2) are characteristic of most helicopters. The most significant coefficient, from the point of view of the closed-loop performance, k_H the control sensitivity, changes little with forward speed. The phase angle ϕ_H and gain K_H for the high speed case are shown as a function of frequency in Figures 4 and 5.

4.1.3 *Control System Dynamics,* S(s)

The control system transfer function in this analysis contains an approximation to the blade response mode and the mechanical play in the control system. The blade response is normally a high-frequency well-damped oscillatory mode. It was found from the transient response tests that the control system could be adequately approximated by a transport time lag having a value between 0.05 and 0.1 second. This approximation ($S(s) = e^{-\tau s}$) is valid at frequencies below about 10 radians per second but over-estimates the phase lag at higher frequencies. ϕ_s for $\tau = 0.05$ second is shown in Figure 4 as a function of frequency.

4.1.4 *Servo Valve and Actuator Dynamics*

For reasons that become clear on examination of the closed-loop performance given in a following section, an integrating servo actuator is usually most suitable. The transfer function for such actuators is of the form

$$P(s) = \frac{\gamma\, k_p}{s(s + \gamma)} \qquad (6)$$

The servo used was pneumatic, designed for use with light aircraft. The servo valve is dithered with a square wave oscillation at 25 cycles per second. The square-wave oscillation is pulse width modulated by the input signal to the servo. The function

$$P(s) = \frac{5k_p}{s(s + 5)} \qquad (7)$$

is a good linear approximation to the servo valve and actuator dynamics for frequencies less than 30 radians per second. The phase angle ϕ_p and gain K_p are given in Figures 4 and 5.

4.1.5 *Compensation Circuit Dynamics*

The compensation circuit was an electrical phase-lead network with a transfer function of the form

$$C(s) = \alpha k_C \frac{s + \beta}{s + \alpha\beta} \tag{8}$$

for single-stage compensation, or

$$C(s) = \alpha^2 k_C \frac{(s + \beta)^2}{(s + \alpha\beta)^2} \tag{9}$$

for double-stage compensation.

The value of β was chosen as a function of α and the sum of the three fixed component phase lags, $\phi_P + \phi_S + \phi_H$, such that ω_π (the frequency for $\phi_0 = -\pi$) was maximum. As α is increased the limit phase-lead angle is $\pi/2$ or π for single or double-stage compensation respectively.

The ratio of high-frequency to low-frequency gain is α or α^2 for single or double-stage compensation, placing an upper limit on the value of α in order to prevent amplifier saturation due to high-frequency noise in the loop. The phase lead ϕ_C and gain K_C are given in Figures 4 and 5 for a two-stage compensation circuit with β chosen to give a value of $\omega_\pi = 16$ radians per second at a value of $\alpha = 11$.

4.1.6 *Closed-Loop Dynamics*

The closed-loop frequency response given by Equations (3) and (4) for the loop component dynamics of Figures 4 and 5 is shown in Figure 6 for a gain ratio of 1.45 (the inverse of the total open-loop gain K_0 at $\omega = \omega_\pi$). The curves are typical of those for the longitudinal loop with compensation designed to give values of ω_π in the range from 10 to 18, at the same gain ratio (see Fig.7 for $\omega_\pi = 12$).

The amplitude ratio peaks to a value just less than 3 at a frequency near ω_π and the closed-loop phase angle is $-\pi$ at $\omega = \omega_\pi$. The amplitude ratio falls off to about 0.95 at low frequency, owing to the small static gain of the helicopter (see Fig.5). The reason for using an integrating type servo actuator is shown by the curves of Figures 4, 5 and 6. The amplitude ratio of the helicopter reduces to near zero at low frequencies and the phase angle approaches $+\pi$. There is a possibility of low-frequency loop oscillation if the open-loop phase angle approaches $+\pi$. The integrating servo introduces $\pi/2$ phase lag at low frequencies, ensuring good low-frequency open-loop phase margin. Furthermore, the servo gain tends to infinity at low frequency to offset the reduction in gain introduced by the helicopter, giving higher values of K_0 and corresponding good low-frequency closed-loop characteristics.

The broken lines in Figure 6 show the closed-loop performance for a gain ratio of 2.90 (that is, a 50% reduction in open-loop gain).

The closed-loop frequency response for high-speed flight (see Table I) with compensation to make $\omega_\pi = 12$ and electrical gain K_m to give a gain ratio of 1.45 is given in Figure 7. The corresponding curves for low-speed flight are also shown for the same values of compensation circuit parameters and electrical gain, K_m.

4.2 Lateral Control Loop

The components of the lateral control loop are the same as those for the longitudinal loop with the exception of the helicopter dynamics, the compensation circuit values of α and β, and the electrical gain K_m. α, β and K_m were chosen as for the longitudinal loop.

The helicopter transfer function coefficients (see Equation (5) for definition of coefficients) are given in Table I. The closed-loop frequency response curves for the lateral loop, with compensation for an $\omega_\pi = 12$ and an electrical gain setting of K_m to give a gain ratio of 1.45 for the high-speed case, are shown in Figure 8. The curves are similar to those for the longitudinal loop.

4.3 Directional Control Loop

The directional control loop differs from the longitudinal and lateral cases only in the helicopter dynamics, compensation circuit parameters, and the control system dynamics. There is no equivalent of the time lag due to the blade response modes in the directional case. A transport time lag of 0.05 second was assumed in order to allow for control system play. The helicopter transfer function coefficients are given in Table I.

The closed-loop performance is shown in Figure 9 for the high and low-speed flight cases for values of α, β and K_m fixed at the values chosen for the high-speed flight case.

4.4 Criteria for Assessing Closed-Loop Performance

The criterion used for assessing the adequacy of the closed-loop performance was to compare the helicopter response, q_H, with the desired response, q_m, for a control input typical of the most rapid pilot's response. The limited response frequency bandwidth of the pilot results in S-shaped inputs with small overshoots for closed-loop tracking tasks with a minimum rise time of from 0.3 to 0.4 second (motor impulse responses, see Refs.2 and 7). The pilot's input used is shown in the upper part of Figures 10, 11 and 12.

The pilot's input is modified by the electrical analogue of the equations of motion of the simulated aircraft to form the input to the closed loop, so that the differences between the desired and actual responses are dependent upon the characteristics of the aircraft simulated. The amplitude of higher-frequency components of the output of the electrical analogue for typical simulated aircraft is greatest for first-order models with a high break-point frequency λ, since the high frequencies are only attenuated proportionally to the inverse of the frequency. The variable-stability helicopter discussed in this analysis was developed for V/STOL handling qualities studies, so that the λ is typically less than 2. In particular instances and in cases where high-speed aircraft are to be simulated, a value of λ of about 5 is typical.

A comparison of the desired and actual responses to a pilot's rapid motor impulse type input for values of λ of 2 and 5 are shown for various values of ω_π of the closed loop in Figures 10 and 11. For the lowest values of ω_π shown (5 rads/sec) the actual response lags the desired response by about 0.3 second for $\lambda = 2$ (Fig.10) and by about 0.4 second for $\lambda = 5$ (Fig.11), and the actual response overshoots the desired response by a significant amount. The overshoot could be reduced by reducing the loop gain, but a further increase in lag time would result. Both the lag time and the amount of overshoot are decreased by increasing the value of ω_π of the closed loop. For values of ω_π greater than 12 for a λ of 2, and greater than 16 for a λ of 5, the lag time is less than 0.1 second and there is no noticeable overshoot. Lag times less than 0.1 second are generally not discernible by the pilot. Qualitative studies using a fixed-base simulator were used to confirm the adequacy of this criterion.

The response curves shown in Figures 10 and 11 were obtained for a gain ratio of 1.45. The curves given in Figure 12 show the effect of increasing gain ratio (decreasing open-loop gain K_0) for a value of $\omega_\pi = 14$ and $\lambda = 2$. At the lowest gain ratio the rate of rise of the actual response was greater than that of the desired response to compensate for the initial lag. As the gain ratio was increased to 3 (a reduction of open-loop gain of about 50%), the two responses became similar with a constant lag time of about 0.1 second throughout the rise time. For larger gain ratios, with the same value of ω_π, the lag time increases beyond 0.1 second, making it necessary to increase the value of ω_π to ensure satisfactory response to rapid pilot's inputs.

4.5 Limitations of Closed Loop Performance

The criterion used in the foregoing section showed that it is necessary to make the value of ω_π of the closed loop equal to or greater than 12, and to maintain the gain ratio greater than about 1.5 and less than about 3. (If the gain ratio is less than about 1.5, external disturbances to the helicopter result in lightly damped loop oscillations.) The component phase angle curves shown in Figure 4 demonstrate the limitations imposed on the closed-loop performance by the different system components. These limitations are typical of the control loops for the longitudinal, lateral and directional controls.

In the range of frequencies of interest ($12 < \omega < 20$) the helicopter phase lag is nearly 90° and does not increase in magnitude significantly with increasing frequency. The phase lag of the servo actuator used is 60° to 80° greater than that of a perfect integrating servo. The phase lag due to mechanical play in the control system and the approximation to the blade response mode ranges from 30° to 60°. A double-stage compensation circuit capable of giving phase-lead angles greater than those achievable with a single stage of compensation and/or a servo actuator that does not introduce phase lags greater than 90° in the required frequency range is necessary to achieve the required closed-loop frequency bandwidth. Double-stage compensation was chosen for the system developed at the N.A.E. Although it is theoretically possible to achieve phase-lead angles approaching 180° with two-stage compensation, the high-frequency amplification necessarily associated with phase-lead networks limited the available phase-lead angle to about 130°. The major noise components were in the range of 10 to 12 cycles per second, originating from twice per rotor revolution mechanical vibration and from the oscillators supplying the servo valve dither frequency.

The amplification of this noise (relative to the zero frequency gain) through the compensation circuit is shown in Figure 13 as a function of ω_π for values of control system transport lags of 0.1 and 0.05 second for the servo used and also for a high-frequency electro-hydraulic servo. Amplification of this noise by a factor greater than about 200 caused low-frequency signal distortion due to amplifier saturation. This set the upper limit of phase lead that could be used with double-stage compensation. The advantage of a faster servo actuator is demonstrated by the broken curves in Figure 13. Value of ω_π greater than 12 can be achieved with only small amplification of noise. A disadvantage of a high-frequency response servo is that high-frequency electrical and mechanical noise in the loop is passed through the servo with very little amplitude attenuation, giving rise to possible problems associated with servo actuator and blade structural dynamics.

4.6 Effects of Variation of Helicopter Dynamics with Changing Flight Conditions

The curves shown in Figure 4 showed that the phase lag of the helicopter was approaching 90° and was increasing only slowly with frequency for frequencies greater than 10 radians per second. This is typical of the phase angle in this range of frequency for the helicopter response about all three axes, over the full speed range. Since the only component of the closed loop that changes with flight conditions is that of helicopter dynamics, the influence of this change on the frequency at which the total open-loop phase angle reaches -180° (ω_π) is small for values of ω_π greater than 12 radians per second.

The helicopter gain at high frequencies (see Equation 5 and the transfer function coefficients in Table I) is proportional to the control sensitivity k_H. Since the change of control sensitivity with speed is small, the open-circuit gain and hence the gain ratio for values of ω_π greater than 12 only changes an insignificant amount with changing flight conditions.

The effect of other changes in the transfer function coefficients of the helicopter with speed on the closed-loop frequency response curves is small, as shown by comparisons of the two sets of curves in Figure 7 for the longitudinal loop, Figure 8 for the lateral loop, and Figure 9 for the directional loop.

4.7 Vertical Translation Control

Control of vertical translation has not yet been attempted with the NAE variable-stability helicopter because of the very restricted payload capabilities of the machine, and because it was considered to be of lesser importance than control of the angular degrees of freedom for simulation of V/STOL aircraft (see Section 2). For helicopters having a greater excess of power, and particularly those powered by turbine engines, control of the vertical translation degree of freedom should not introduce any particularly difficult problems.

4.8 Application to Fixed-Wing Aircraft

The transfer functions relating the angular response of aircraft to control inputs are similar to those for helicopters except for the values of the coefficients. The longitudinal modes for an aircraft are a long-period mode which does not significantly affect the closed-loop performance and a short-period mode, usually well damped. The frequency of the short-period mode is dependent upon speed and altitude. Since this frequency is typically less than 6 radians per second, the longitudinal phase lag is approaching 90° at the frequencies critical to the closed loop ($\omega_{\pi} > 16$). It is likely, therefore, that no adjustment of the compensation circuit parameters would be required with changing forward speed and altitude. The control sensitivity of an aircraft changes significantly with forward speed. If these changes are greater than about 50%, adjustment of the loop gain in flight is necessary in order to maintain sufficiently good closed-loop response.

The lateral-directional modes of an aircraft are a short-period oscillatory mode with frequencies typically less than 6 radians per second, a strong subsidence mode (with a high break-point frequency) and a mild subsidence or divergence. Once again, if the short-period oscillatory-mode frequency and the break-point frequency of the strong subsidence mode are small relative to the required values of ω_{π}, the compensation circuit parameters could be held constant. Changes in aileron and rudder control sensitivity may require in-flight adjustment of the open-loop gain.

5. CONCLUDING REMARKS

The model-controlled method of providing aircraft with variable-stability characteristics can provide for versatility in regard to the simulation of complex equations of motion in that use may be made of standard electrical computing components in a manner similar to that employed with ground simulators.

The difficulties encountered with more customary approaches when independent variation of transfer function coefficients is attempted are circumvented by using the model-controlled method.

The method does not require an accurate knowledge of the test aircraft transfer function coefficients and their variation with speed and altitude. Significant changes in the aircraft response parameters can be tolerated with no noticeable effect on the desired response. In the case of the application of the method to variable-stability helicopters, no in-flight changes of the control-loop parameters are necessary to ensure that the response accurately follows the desired response over the full speed range of the test helicopter. In the case of use of the method for variable-stability fixed-wing aircraft, in-flight adjustment of the control-loop gain may be required if the control sensitivity changes by more than about 50% with speed and/or altitude variation during the test.

REFERENCES

1. Seckel, E. et alii	*Human Pilot Dynamic Response in Flight and Simulator.* WADC Tech. Report 57-520, August 1958.
2. McRuer, D.T. et alii	*A Systems Analysis View of Longitudinal Flying Qualities.* WADD Tech. Report 60-43, January 1960.
3. Rathert, G.A., Jr. et alii	*The Use of Piloted Flight Simulators in General Research.* Ames Research Centre, N.A.S.A. (Paper presented to the Fluid Dynamics and Flight Mechanics Panels of A.G.A.R.D., Brussels), April 1961.
4. McWeeney, R.J. Davison, F.E.	*Flight Controls - Human Dynamic Response Study.* U.S. Air Force, Bureau of Aeronautics, Report AE-61-6.
5. Douvillier, J.G., Jr. et alii	*Effects of Flight Simulator Motion on Pilots' Performance of Tracking Tasks.* NASA TN D-143, February 1960.
6. Sadoff, M. et alii	*A Study of Longitudinal Control Problems at Low and Negative Damping and Stability with Emphasis on Motion Cues.* NASA TN D-348, January 1961.
7. Kuehnel, H.A.	*In-Flight Measurement of the Time Required for a Pilot to Respond to an Aircraft Disturbance.* NASA TN D-221, March 1960.
8. Daw, D.W. Ursel, L.V.	*A Model-Controlled Autopilot for an Airborne VTOL Simulator.* NRC Aeronautical Report LR-302, February 1961.
9. Daw, D.W. McGregor, D.M.	*Development of a Model-Controlled V/STOL Airborne Simulator.* NRC Aeronautical Report (to be published).

TABLE 1

HELICOPTER TRANSFER FUNCTION COEFFICIENTS

$$H(s) = \frac{k_H(s + \lambda)(s^2 + as + b^2)}{(s^2 + A_1 s + B_1{}^2)(s^2 + A_2 s + B_2{}^2)}$$

Degree of Freedom	Speed	k_H	λ	a	b^2	A_1	$B_1{}^2$	A_2	$B_2{}^2$
Long.	High	3.12	0	0.125	-0.0055	2.00	0	0.041	0.206
	Low	3.14	2.99	0.137	0.0553	2.74	4.61	0.030	0.276
Lat.	High	0.431	0	0.531	0	0	0	3.31	1.57
	Low	0.419	0	0.954	0	0	0	3.39	3.63
Dir.	High	0.773	0	0.287	0	0	0	1.132	4.86
	Low	0.710	0	0.930	-0.124	6.14	0	0.042	0.168

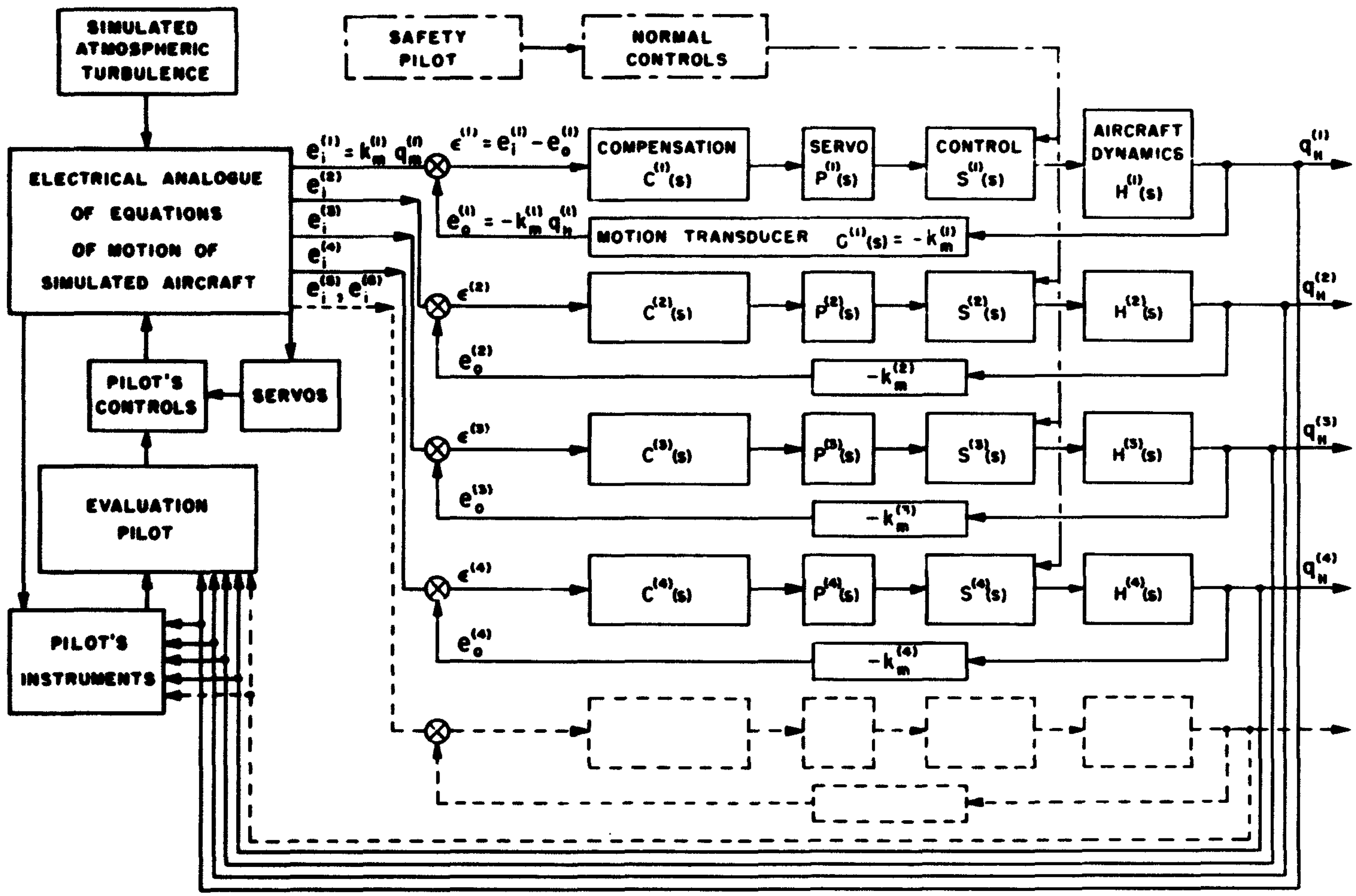

Fig. 1 Symbolic sketch of model-controlled variable-stability system

Fig. 2 Variable-stability helicopter developed at the N.A.E.

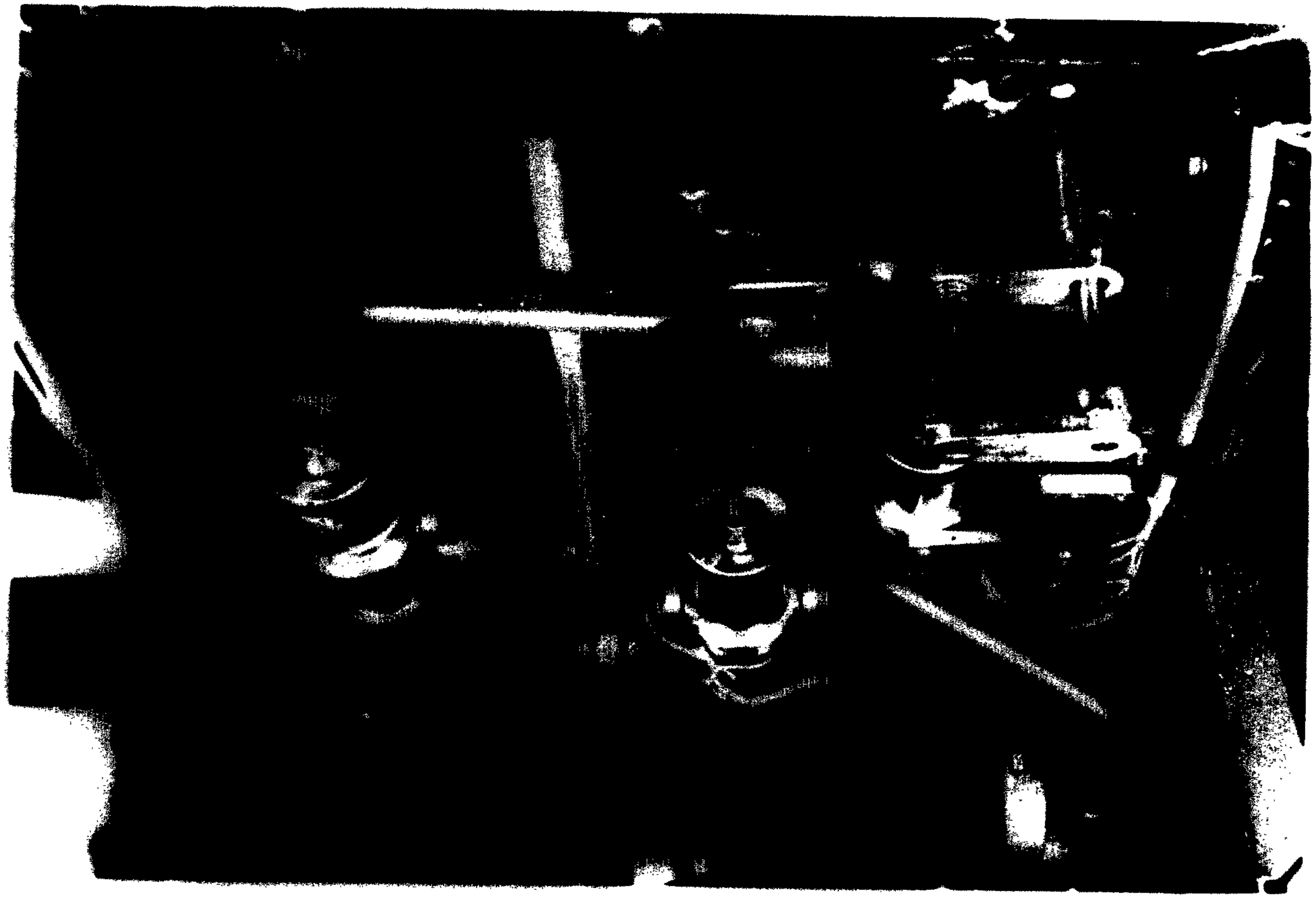

Fig. 3(a) Electric stick and rudder spring feel system with servo-driven spring bias

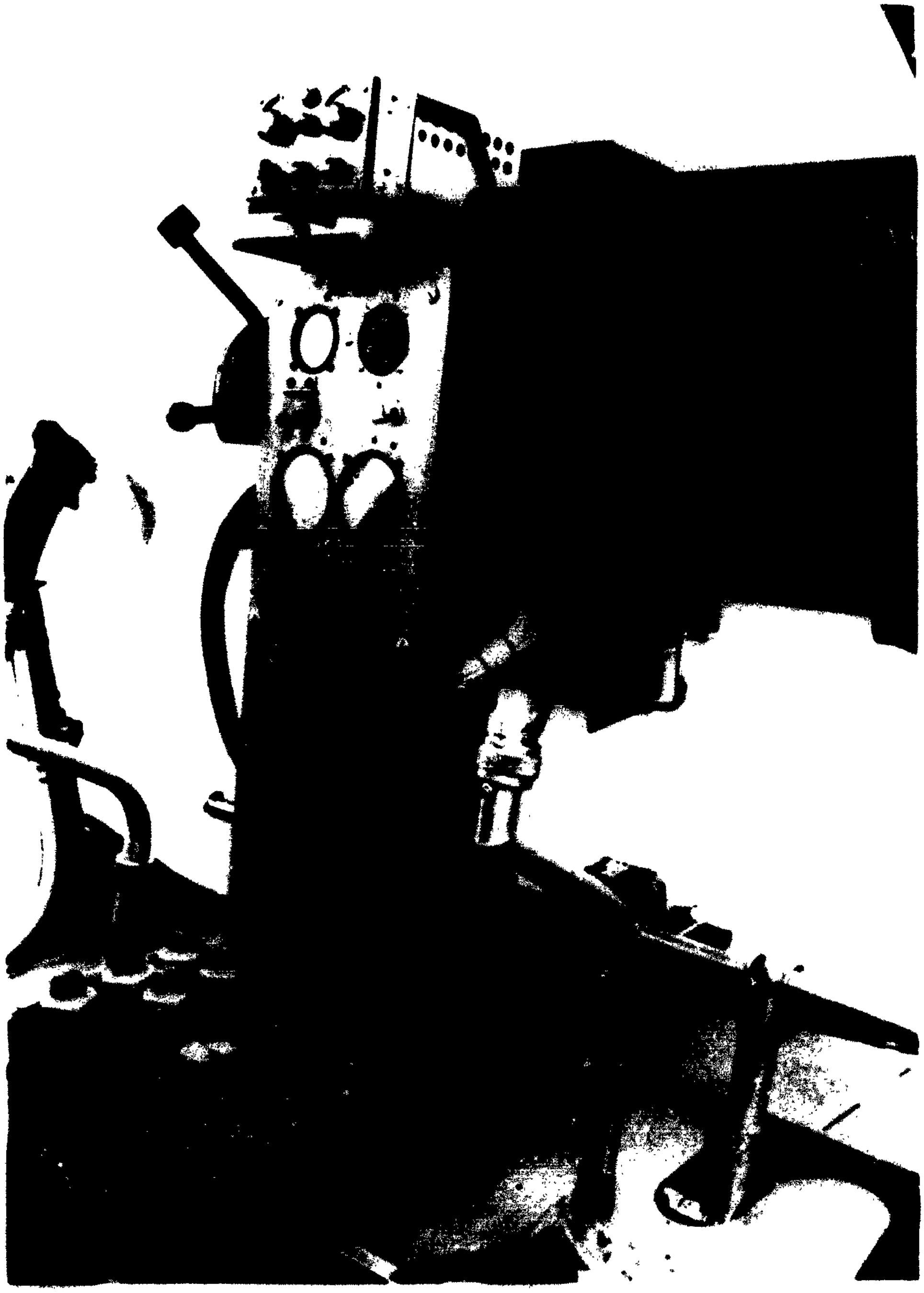

Fig. 3(b) Evaluation pilot's controls and instrument panel

Fig.3(c) Lateral control loop servo valve and actuator

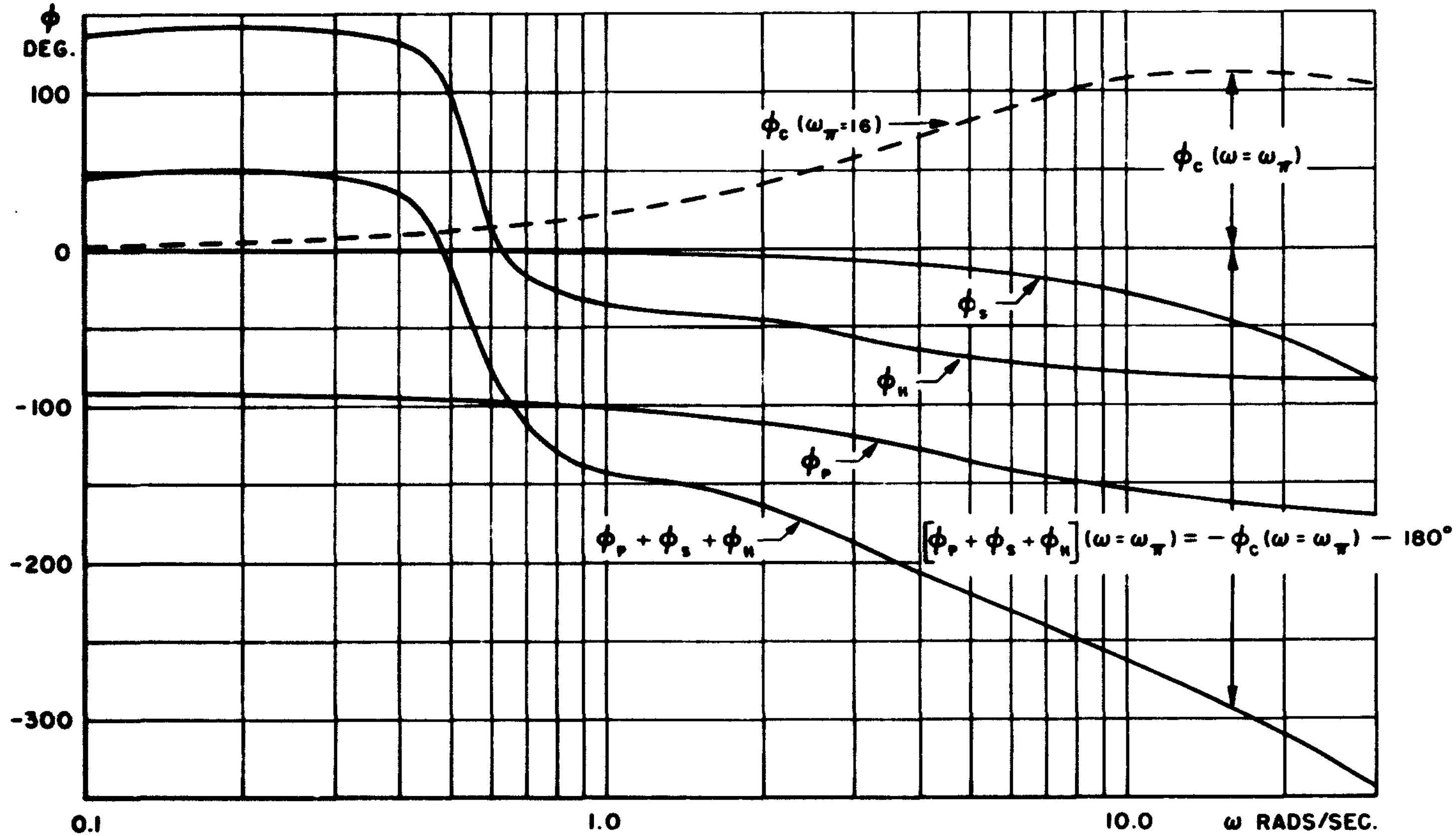

Fig. 4 Phase angle of longitudinal loop components, high speed case
($\omega_\pi = 16,\ \alpha = 11.0,\ \beta = 4.82,\ \tau = 0.05$)

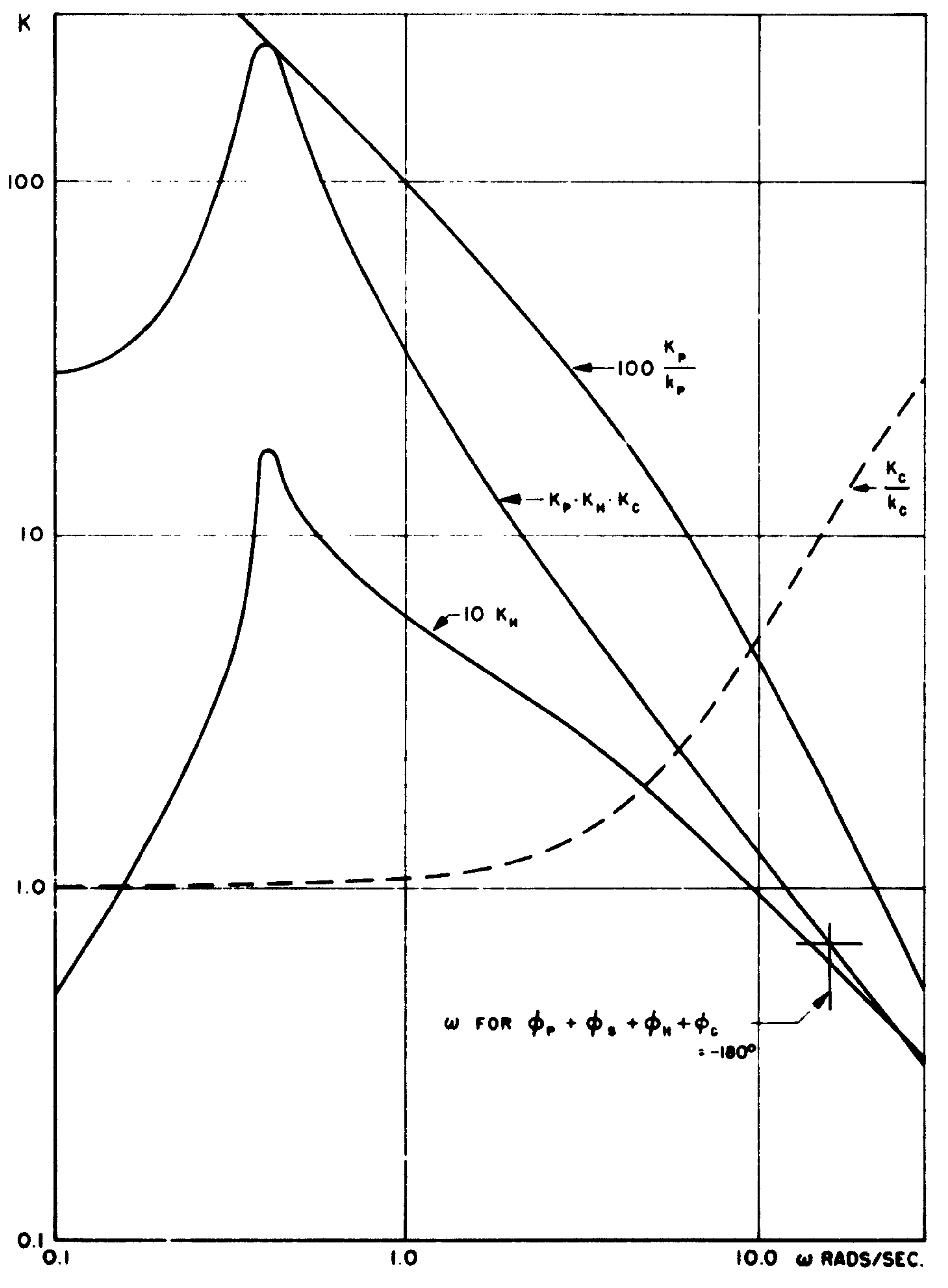

Fig. 5 Gain of longitudinal loop components, high speed case
(ω_π = 16, α = 11.0, β = 4.82, $K_0(\omega = \omega_\pi)$ = 1/1.45)

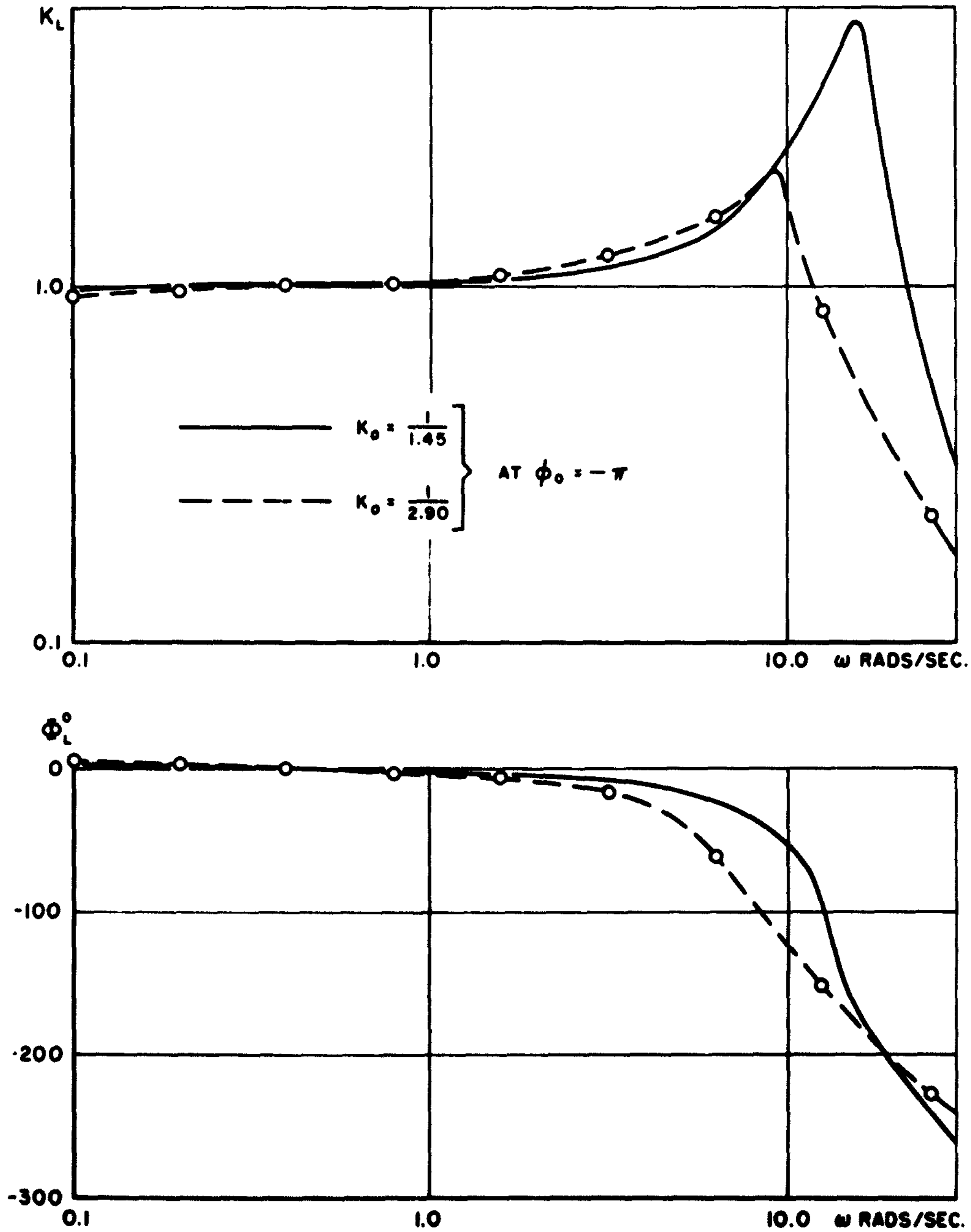

Fig. 6 Closed-loop gain and phase angle for longitudinal loop, high speed case (ω_π = 16, α = 11.0, β = 4.82)

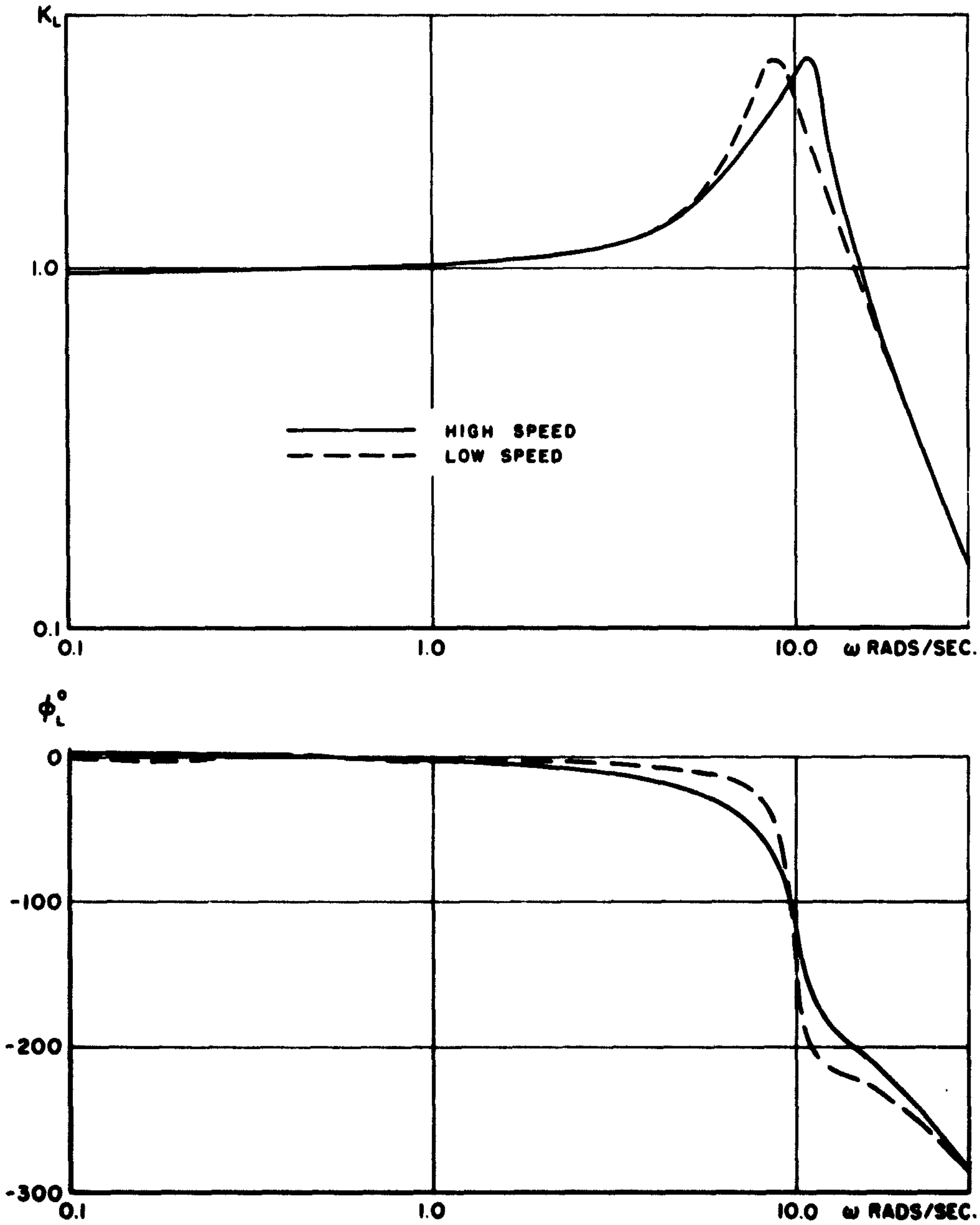

Fig. 7 Closed-loop gain and phase angle for longitudinal loop ($\omega_n = 12$, $\alpha = 6.4$, $\beta = 4.73$, $K_0(\omega = \omega_n) = 1/1.45$) for high speed case

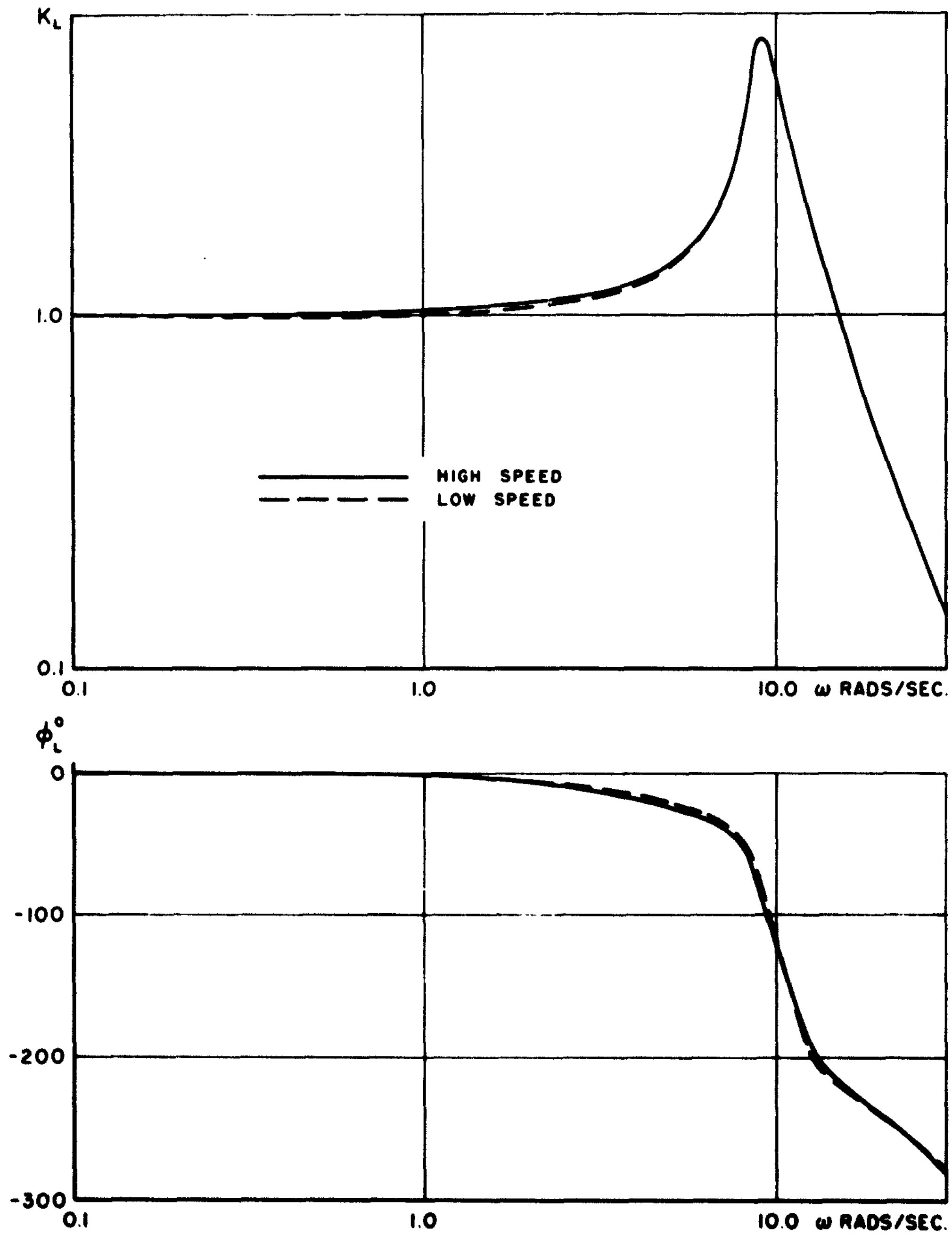

Fig. 8 Closed-loop gain and phase angle for lateral control loop

($\omega_n = 12$, $\alpha = 5.51$, $\beta = 5.10$, $K_0(\omega = \omega_n) = 1/1.45$)
for high speed case

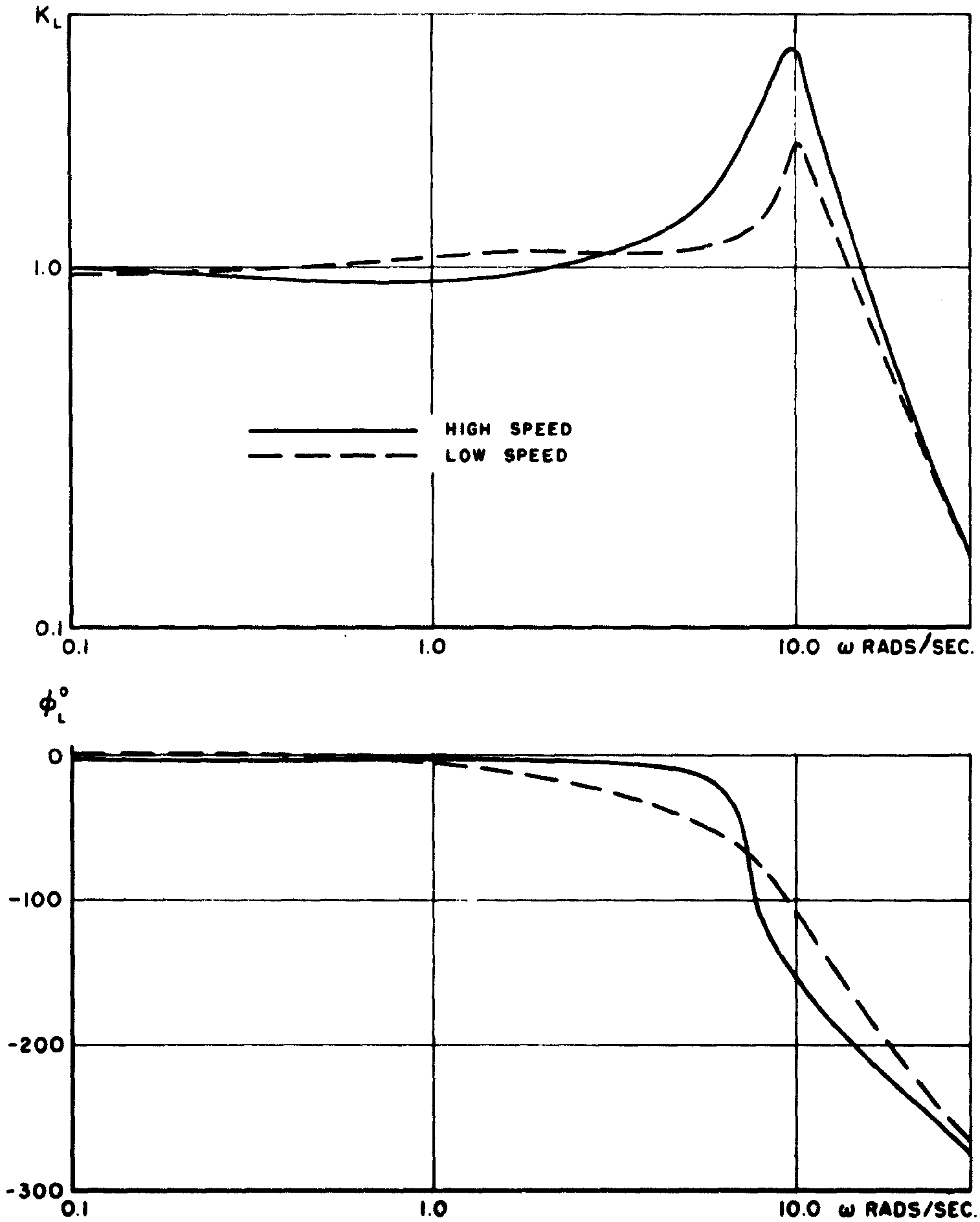

Fig.9 Closed-loop gain and phase angle for directional control loop (ω_n = 12, α = 7.30, β = 4.44, $K_0(\omega = \omega_n)$ = 1/1.45) for high speed case

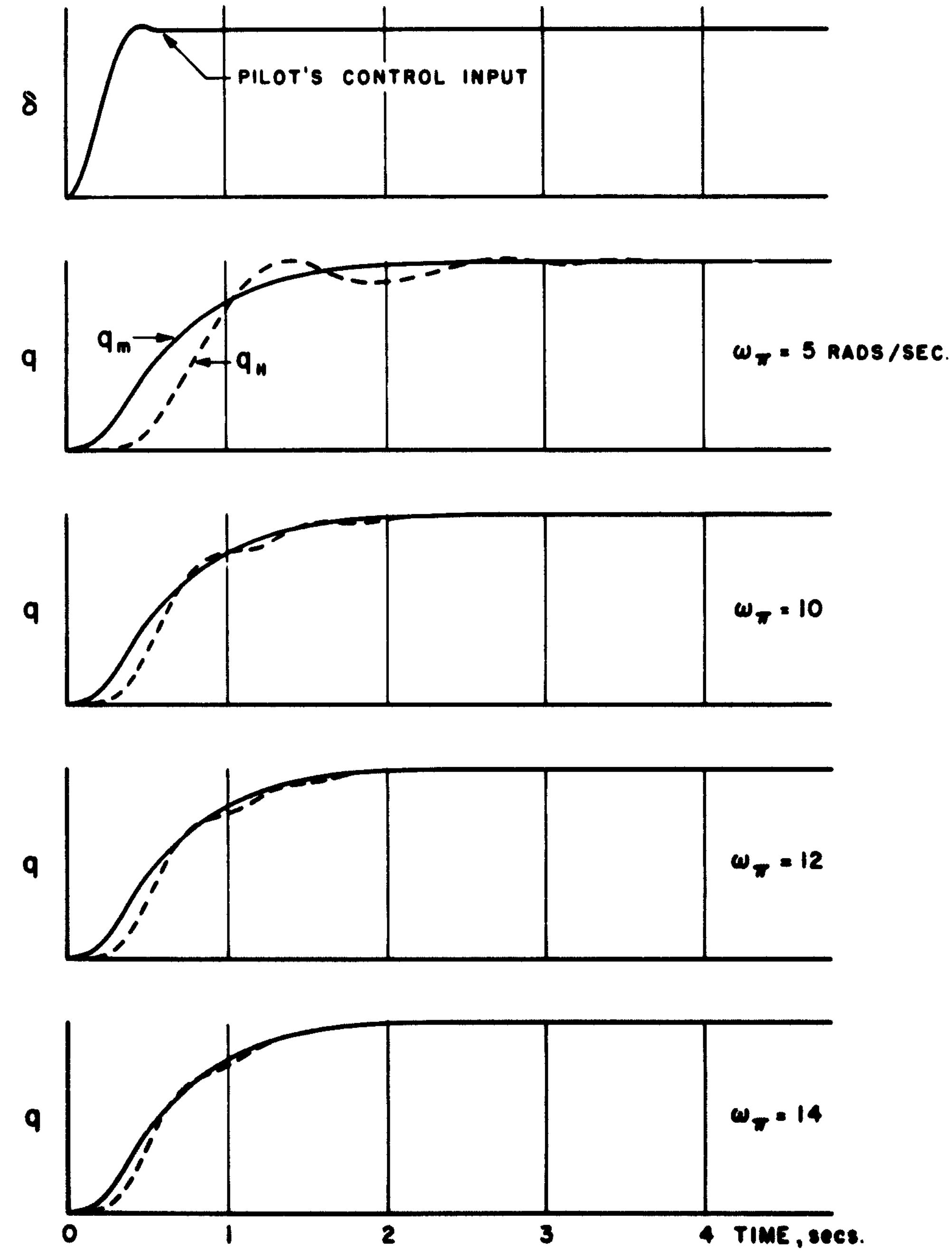

Fig. 10(a) Comparison of helicopter (q_H) and desired response (q_m) for a rapid pilot's input, ω_π = 5 to 14 rads./sec.

(Model break-point frequency, λ = 2; closed-loop gain ratio, 1.45)

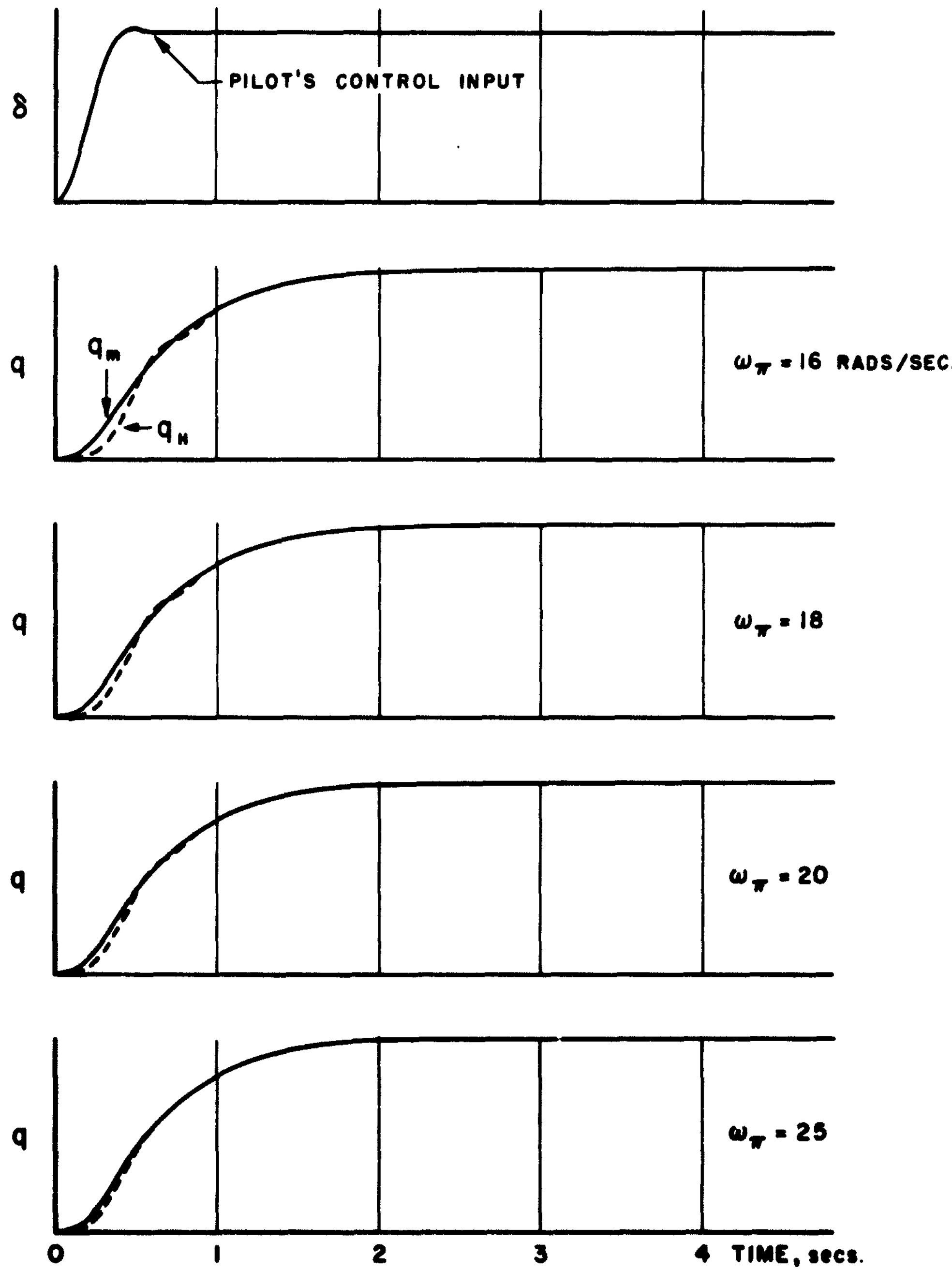

Fig. 10(b) Comparison of helicopter (q_H) and desired response (q_m) for a rapid pilot's input, ω_π = 16 to 25 rads./sec.

(Model break-point frequency, λ = 2; closed-loop gain ratio, 1.45)

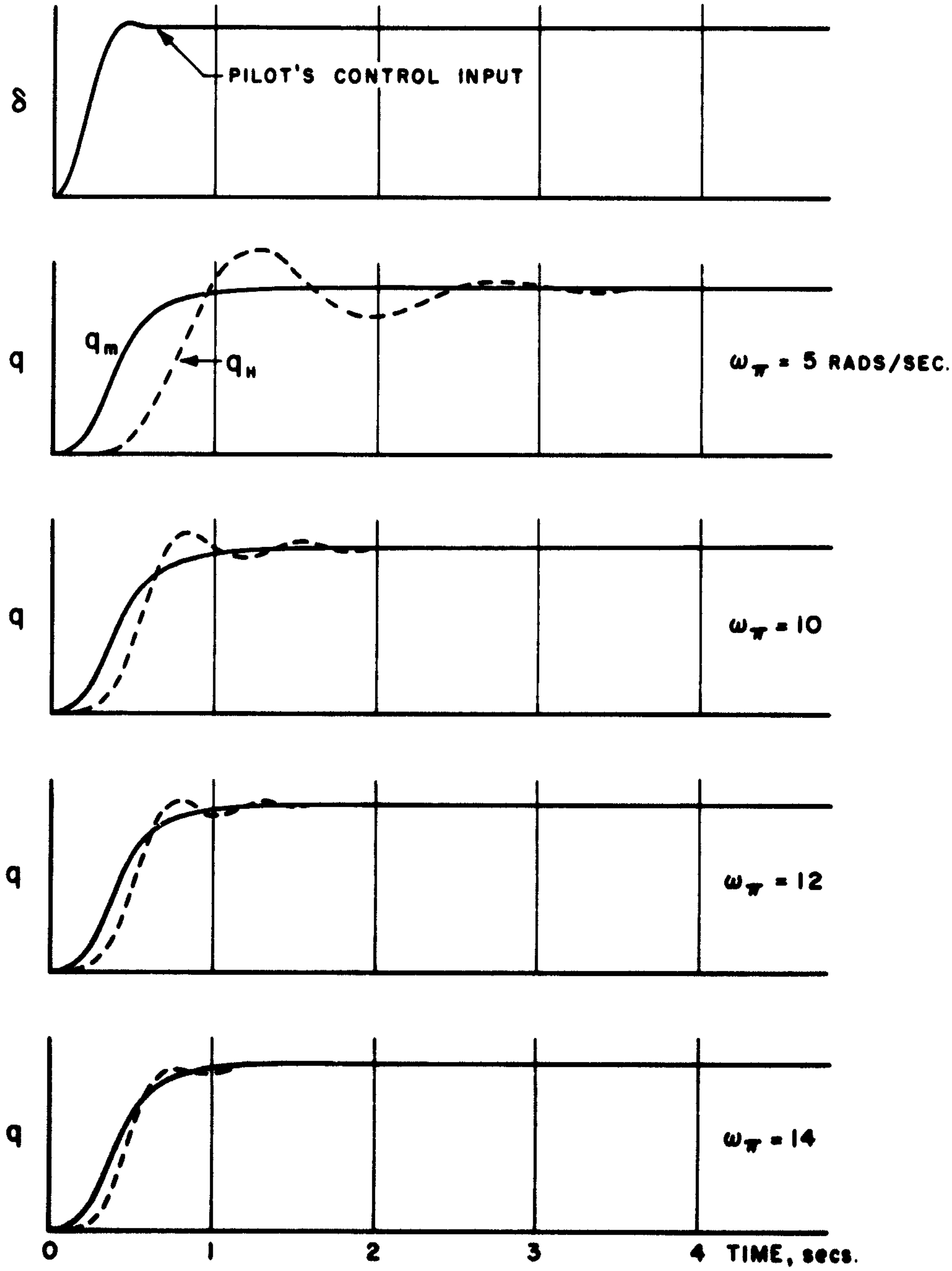

Fig. 11(a) Comparison of helicopter (q_H) and desired response (q_m) for a rapid pilot's input, ω_π = 5 to 14 rads./sec.

(Model break-point frequency, λ = 5; closed-loop gain ratio, 1.45)

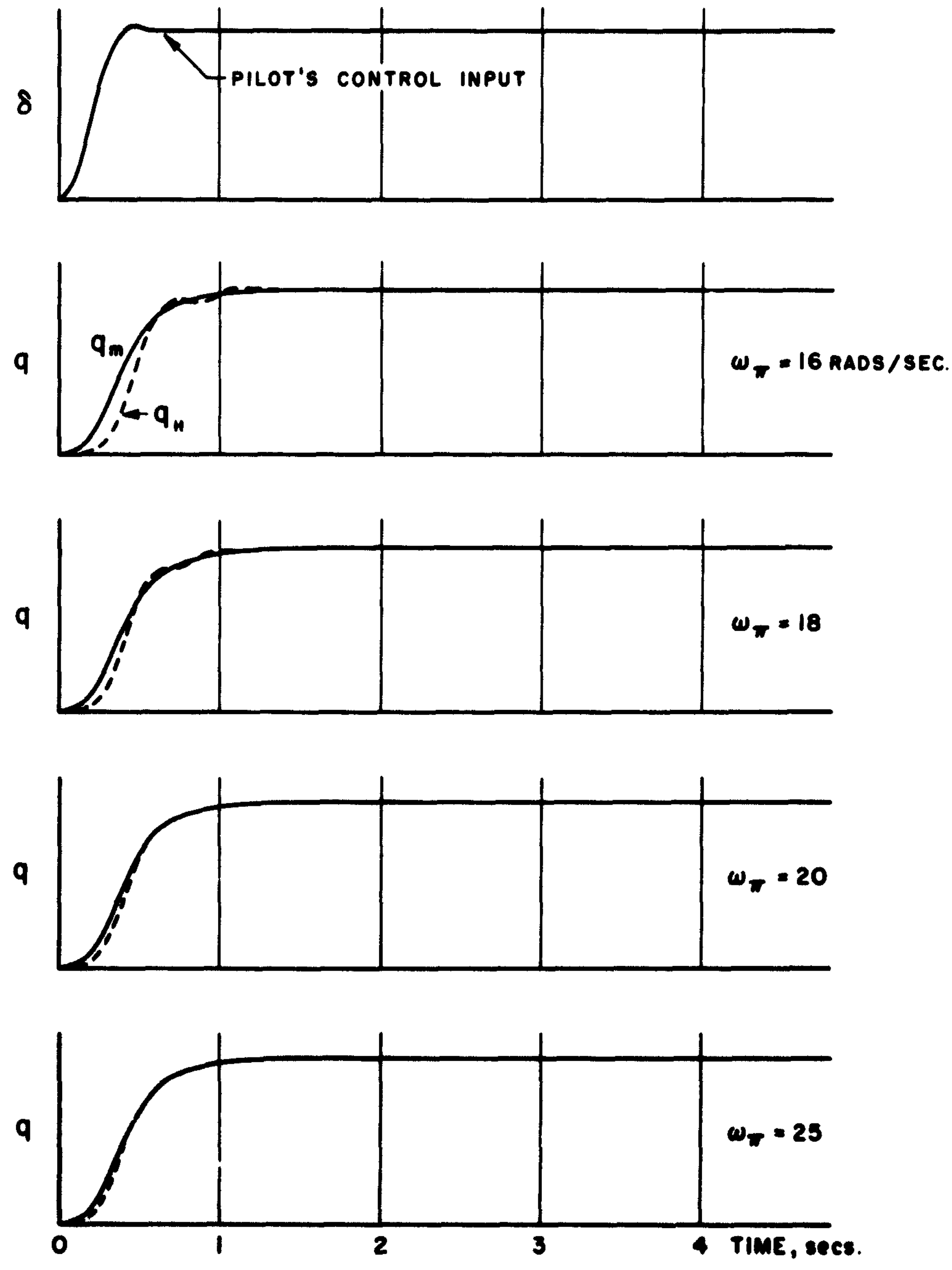

Fig. 11(b) Comparison of helicopter (q_H) and desired response (q_m) for a rapid pilot's input, ω_π = 16 to 25 rads./sec.

(Model break-point frequency, λ = 5; closed-loop gain ratio, 1.45)

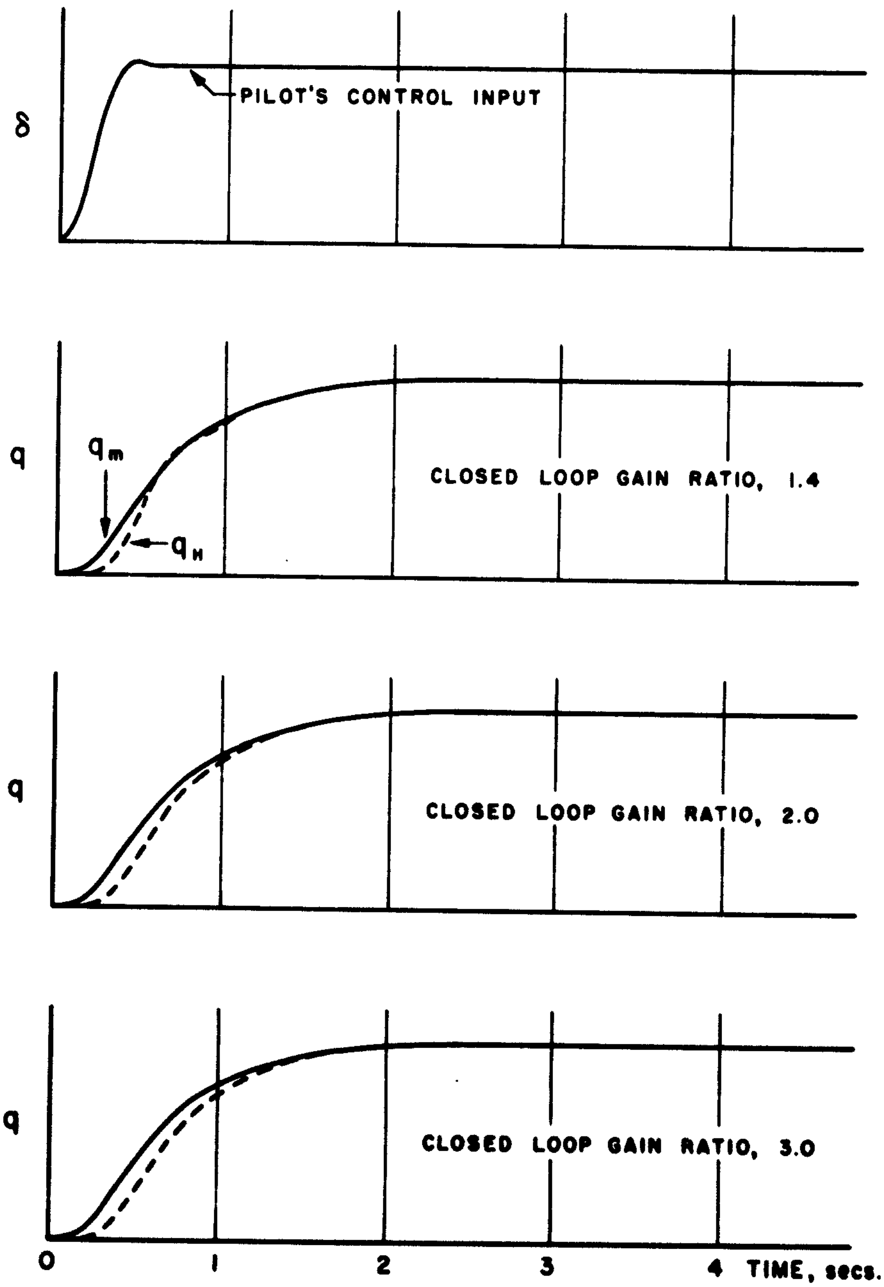

Fig. 12 Comparison of helicopter (q_H) and desired response (q_m) for a rapid pilot's input

(Model break-point frequency, λ = 2; ω_n = 14 rads./sec.)

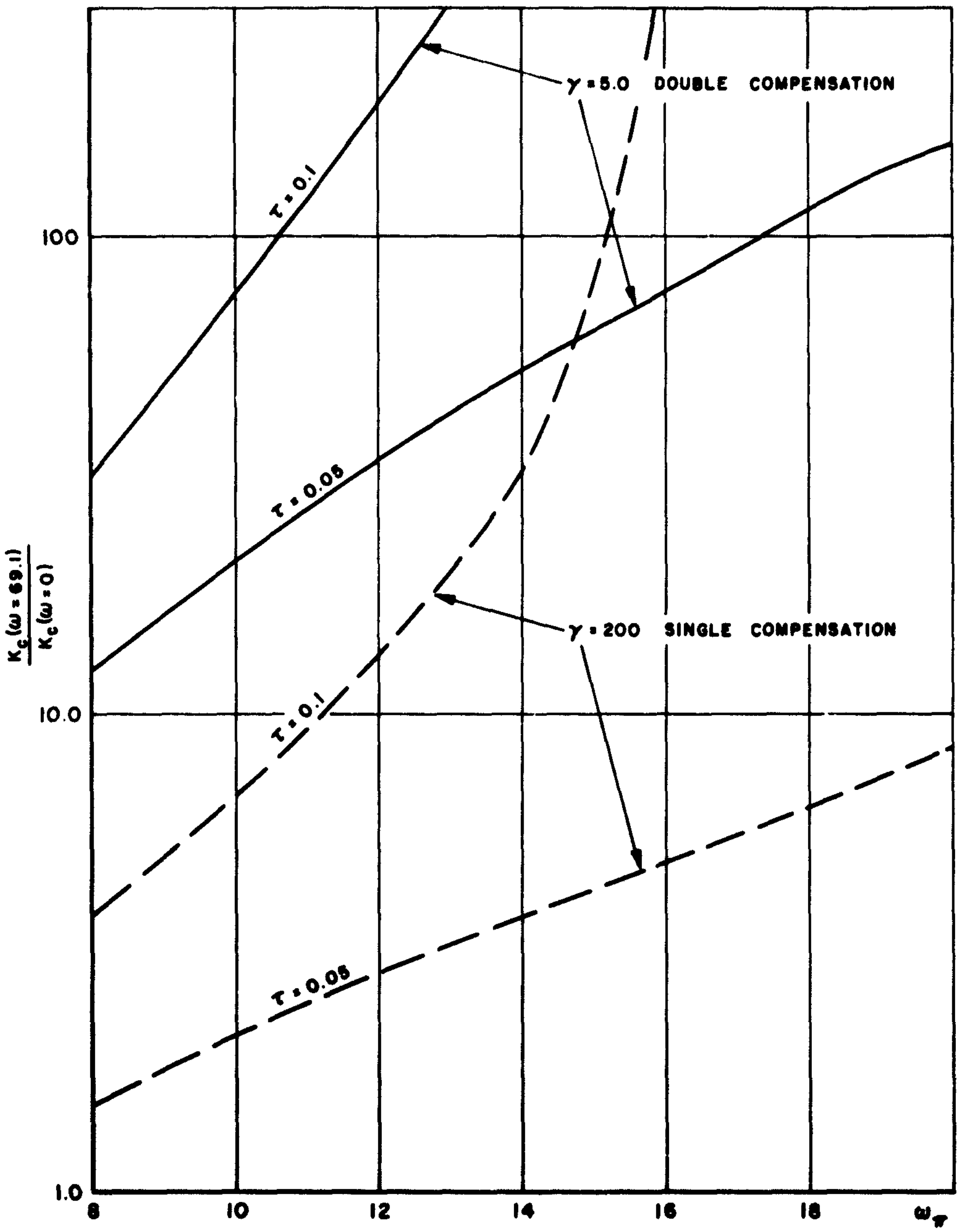

Fig. 13 Amplification of 11 c.p.s. (twice per rotor revolution) noise through compensation components

(Longitudinal loop, high speed case)

DISTRIBUTION

Copies of AGARD publications may be obtained in the various countries at the addresses given below.

On peut se procurer des exemplaires des publications de l'AGARD aux adresses suivantes.

BELGIUM BELGIQUE	Centre National d'Etudes et de Recherches Aéronautiques 11, rue d'Egmont, Bruxelles
CANADA	Director of Scientific Information Service Defense Research Board Department of National Defense 'A' Building, Ottawa, Ontario
DENMARK DANEMARK	Military Research Board Defense Staff Kastellet, Copenhagen Ø
FRANCE	O.N.E.R.A. (Direction) 25, Avenue de la Division Leclerc Châtillon-sous-Bagneux (Seine)
GERMANY ALLEMAGNE	Wissenschaftliche Gesellschaft für Luftfahrt Zentralstelle der Luftfahrtdokumentation Munchen 64, Flughafen Attn: Dr. H.J. Rautenberg
GREECE GRECE	Greek National Defense General Staff B. MEO Athens
ICELAND ISLANDE	Director of Aviation c/o Flugrad Reykjavik
ITALY ITALIE	Ufficio del Generale Ispettore del Genio Aeronautico Ministero Difesa Aeronautica Roma
LUXEMBURG LUXEMBOURG	Obtainable through Belgium
NETHERLANDS PAYS BAS	Netherlands Delegation to AGARD Michiel de Ruyterweg 10 Delft

AGARD Report 402
North Atlantic Treaty Organization, Advisory Group for Aeronautical Research and Development
THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT
D.G.Gould
1962
31 pages, incl. 9 refs., 13 figs.

629.13.014.5
533.6.013.4
629.135.45

The application of a technique described as the model-controlled method (termed adaptive or semi-adaptive when applied to autopilots) for providing aircraft with variable stability and control characteristics is described. The development of a variable-stability helicopter using this method is treated in some detail to illustrate the significant design parameters and to demonstrate the advantages of the method over the more customary approach.

This Report was presented at the Twenty-First Meeting of the Flight Mechanics Panel of AGARD, held in Paris, 6-10 July, 1962.

AGARD Report 402
North Atlantic Treaty Organization, Advisory Group for Aeronautical Research and Development
THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT
D.G.Gould
1962
31 pages, incl. 9 refs., 13 figs.

629.13.014.5
533.6.013.4
629.135.45

The application of a technique described as the model-controlled method (termed adaptive or semi-adaptive when applied to autopilots) for providing aircraft with variable stability and control characteristics is described. The development of a variable-stability helicopter using this method is treated in some detail to illustrate the significant design parameters and to demonstrate the advantages of the method over the more customary approach.

This Report was presented at the Twenty-First Meeting of the Flight Mechanics Panel of AGARD, held in Paris, 6-10 July, 1962.

AGARD Report 402
North Atlantic Treaty Organization, Advisory Group for Aeronautical Research and Development
THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT
D.G.Gould
1962
31 pages, incl. 9 refs., 13 figs.

629.13.014.5
533.6.013.4
629.135.45

The application of a technique described as the model-controlled method (termed adaptive or semi-adaptive when applied to autopilots) for providing aircraft with variable stability and control characteristics is described. The development of a variable-stability helicopter using this method is treated in some detail to illustrate the significant design parameters and to demonstrate the advantages of the method over the more customary approach.

This Report was presented at the Twenty-First Meeting of the Flight Mechanics Panel of AGARD, held in Paris, 6-10 July, 1962.

AGARD Report 402
North Atlantic Treaty Organization, Advisory Group for Aeronautical Research and Development
THE MODEL-CONTROLLED METHOD FOR THE DEVELOPMENT OF VARIABLE-STABILITY AIRCRAFT
D.G.Gould
1962
31 pages, incl. 9 refs., 13 figs.

629.13.014.5
533.6.013.4
629.135.45

The application of a technique described as the model-controlled method (termed adaptive or semi-adaptive when applied to autopilots) for providing aircraft with variable stability and control characteristics is described. The development of a variable-stability helicopter using this method is treated in some detail to illustrate the significant design parameters and to demonstrate the advantages of the method over the more customary approach.

This Report was presented at the Twenty-First Meeting of the Flight Mechanics Panel of AGARD, held in Paris, 6-10 July, 1962.

NORWAY NORVEGE	Mr. O. Blichner Norwegian Defence Research Establishment Kjeller per Lilleström
PORTUGAL	Col. J.A. de Almeida Viama (Delegado Nacional do 'AGARD') Direcção do Serviço de Material da F.A. Rua da Escola Politecnica, 42 Lisboa
TURKEY TURQUIE	Ministry of National Defence Ankara Attn. AGARD National Delegate
UNITED KINGDOM ROYAUME UNI	Ministry of Aviation T.I.L., Room 009A First Avenue House High Holborn London W.C.1
UNITED STATES ETATS UNIS	National Aeronautics and Space Administration (NASA) 1520 H Street, N.W. Washington 25, D.C.

Printed by Technical Editing and Reproduction Ltd
95 Great Portland St. London, W.1.