UNCLASSIFIED

AD <u>411455</u>

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION
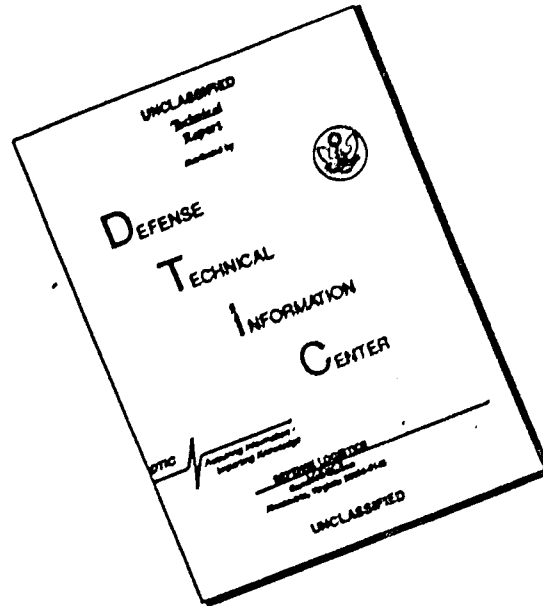
CAMERON STATION, ALEXANDRIA, VIRGINIA

UNCLASSIFIED

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST
QUALITY AVAILABLE. THE COPY
FURNISHED TO DTIC CONTAINED
A SIGNIFICANT NUMBER OF
PAGES WHICH DO NOT
REPRODUCE LEGIBLY.

MICROWAVE MEASUREMENT OF ELECTRON DENSITY
IN SEEDED ROCKET EXHAUST GASES

by

Harold M. Larsen
and
Charles H. Turner

Propulsion Development Department

ABSTRACT. The initial phase of a microwave diagnostics
program involving seeded rocket exhaust gases is presented.
The theory of the microwave determination of electron
density and collision frequency is discussed as it applies
to the temperature and density regimes of both thermonuclear
and chemically-produced plasmas. The procedure used to
predict the electron density in an equilibrium seeded
rocket exhaust gas is outlined, as well as a possible
method for temperature measurement using microwave radi-
ation. The experimental apparatus and procedure are
described, and the results of ten measurements of electron
density are compared with predicted values. The improve-
ments in experimental equipment and technique planned for
future investigations are discussed.

U.S. NAVAL ORDNANCE TEST STATION

China Lake, California

June 1963

FOREWORD

Microwave attenuation measurements were made of the number density of electrons in seeded rocket exhaust gases to verify, in part, the analytic phase of current investigations at the U. S. Naval Ordnance Test Station. Extension of this work, performed during the summer of 1962, to include microwave diagnostics of arbitrary combustion-product gases in conjunction with a coordinated analytic program promises a means for understanding the basic mechanism of microwave interaction with partially-ionized gases, with particular application to the problem of radar attenuation in rocket exhaust plumes.

This report was reviewed for technical accuracy by Robert G. Corzine, and is transmitted for information purposes only. It does not represent the official views or final judgement of the Naval Ordnance Test Station, and the Station assumes no responsibility for action taken on the basis of its contents.

ii

CONTENTS

—

Figures:

NEGATIVE NUMBERS OF ILLUSTRATIONS

Fig. 1, LO 77625; Fig. 2, LO 77621; Fig. 3, LO 77622; Fig. 4, LO 77623; Fig. 5, LO 77626; Fig. 6, LO 78121; Fig. 7, LO 76717.

NOMENCLATURE

A^* Cross-sectional area at sonic throat, area
A_e Cross-sectional area at nozzle exit, area
A_p Cross-sectional area at probed region, area
c Velocity of light, $3 \cdot 10^{10}$ cm/sec (cgsu)
E_s Ionization potential, volts
e Electronic charge, $4.8 \cdot 10^{-10}$ esu (cgsu)
f Microwave frequency, cycles/sec
f_p Plasma frequency, cycles/sec
K Plasma dielectric constant, dimensionless
K' Plasma complex dielectric constant, dimensionless
k Boltzmann constant, $1.38 \cdot 10^{-16}$ erg/degree (cgsu)
M_e Exit Mach number, dimensionless
m_e Electron mass, $9.11 \cdot 10^{-28}$ g (cgsu)
n Gas particle number density, number/volume
n_e Electron number density, electrons/volume
n_s Seed atom number density, atoms/volume
P_s Seed atom partial pressure, pressure
Q Total electron collision cross-section, area
R Seeding rate, dimensionless
T Temperature, Kelvin degrees
t Time, seconds
X Mole fraction of seed, dimensionless

α Attenuation coefficient, cm^{-1}
β Phase coefficient, cm^{-1}
γ Ratio of specific heats, dimensionless
Δ Propagation number, cm^{-1}
μ Degree of ionization, dimensionless
ν Electron collision frequency, sec^{-1}
ψ Propagation constant, cm^{-1}
ω Microwave angular frequency, radians/sec
ω_p Plasma frequency, radians/sec

INTRODUCTION

Microwave diagnostics have become an increasingly important experimental technique in the study of partially- and fully-ionized gases. (Ref. 1). The principle advantage in their use lies in the fact that measurements may be made directly of microscopic plasma parameters (eg. electron density and electron collision frequency) at very low power levels without making physical contact with the gas and, hence, without contaminating or disturbing the system. In addition, measurements may be made of short-duration transient phenomena because of the nature of the parameters measured and of the rapid response of the measuring equipment.

The Propulsion Applied Research Group (Code 4506) at the Naval Ordnance Test Station is currently working on several programs involving partially-ionized gases (Ref. 2-3). Preliminary microwave measurements have been made of the electron density in these high-temperature gaseous systems to verify portions of the analytic work and to determine the feasibility of microwave diagnostics as a research tool in the study of chemically-produced plasmas.

ANALYSIS

I. MICROWAVE DIAGNOSTICS

The theory of microwave diagnostics is concerned with the interaction of electromagnetic radiation of short wavelength (typically on the order of centimeters and less) with partially- or fully-ionized plasmas. This interaction can be analyzed by considering the complex dielectric constant, K', of the plasma (Ref. 1):

$$(1) \qquad K' = 1 - (\omega_p/\omega)^2 \, \frac{1 + i\nu/\omega}{1 + (\nu/\omega)^2} \, ,$$

where ω is the radian frequency of the microwave signal, ν is the collision frequency of the electrons in the plasma, and ω_p is the plasma frequency which is given by (cgsu)

$$(2) \qquad \omega_p = (4\pi e^2 \, n_e/m_e)^{1/2} = 5.6 \cdot 10^4 \, n_e^{1/2} \quad \text{(radians/sec)}.$$

In equation (2), e is the magnitude of the electronic charge, n_e is the electron density (number of electrons per unit volume), and m_e is the mass of the electron.

For many systems of interest (for example, thermonuclear plasma), the condition $(\nu/\omega)^2 < 1$ is satisfied so that equation (1) becomes

$$(3) \qquad K' = 1 - (\omega_p/\omega)^2 (1 + i\nu/\omega).$$

In this case, the dielectric constant of the plasma. K, can be approximated by

$$(4) \qquad K = 1 - (\omega_p/\omega)^2.$$

The propagation of the electric field of a microwave within a plasma can be expressed as $E = E_0 \exp\left[i(\Delta x - \omega t)\right]$, where $\Delta = \omega K^{1/2}(1/c)$. In equation (4), whenever $\omega > \omega_p$, K is positive so that Δ is real, and the wave will propagate through the plasma. The phase of the oscillations, however, will be shifted by an amount Δ radians per unit length of transmission path within the plasma. Whenever $\omega < \omega_p$. K is negative so that Δ is purely imaginary, and the waves will be exponentially attenuated. The depth of penetration of microwave radiation of frequency ω into a plasma required to attenuate the signal to $1/e$ of its original value. the so-called skin depth is given by

$$\text{skin depth} = c/(\omega_p^2 - \omega^2)^{1/2}.$$

Thus, essentially complete reflection occurs when the signal transmission path within the plasma is at least several times the skin depth. When $\omega = \omega_p$, microwave cut-off occurs. Under this condition, the frequency of the microwave signal can be related to the density of electrons in the plasma by means of equation (2):

$$(5) \qquad f = f_p = \omega_p/2\pi = 9 \cdot 10^3 \, n_e^{1/2} \; (\text{cycles/sec})$$

Figure 1 shows the relation between electron density and microwave frequency at cut-off for the case where $(\nu/\omega)^2 \ll 1$.

When $(\nu/\omega)^2$ is not small compared with unity, equation (5) cannot be used. For this case, the complex dielectric constant can be related to the propagation constant, ψ, of the plasma:

$$(6) \qquad \psi^2 = -\omega^2 K'/c^2.$$

The complex propagation constant is given by

$$(7) \qquad \psi = \alpha + i\beta,$$

where α is the attenuation coefficient and β is the phase coefficient of
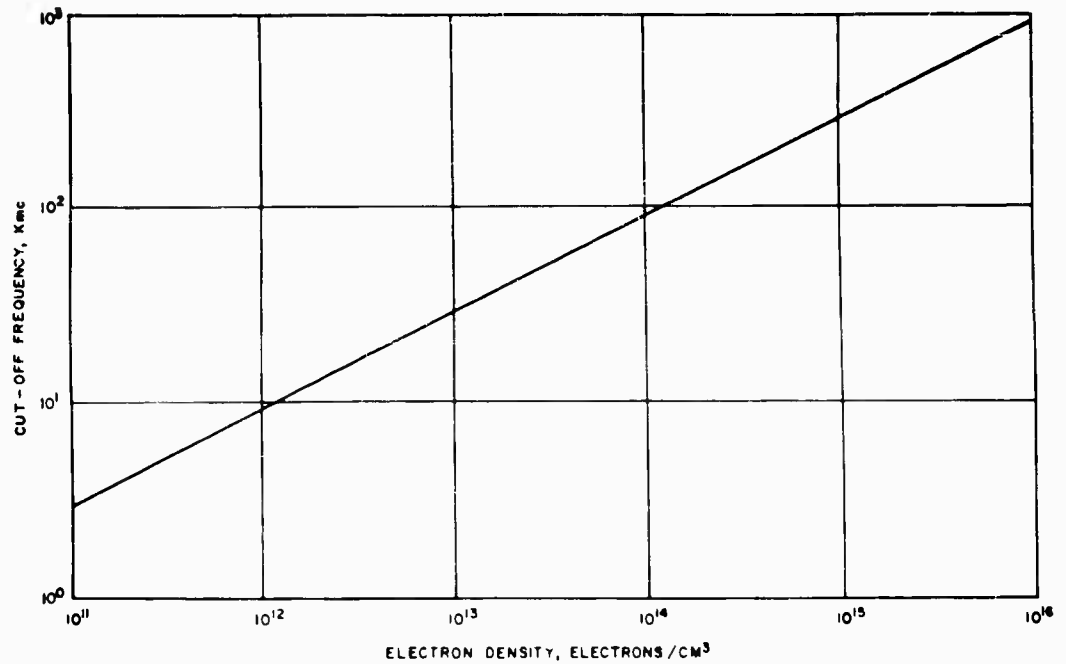
FIG. 1. Microwave Cut-Off Frequency Versus Electron Density.

the microwave signal. Squaring equation (7) and equating real and imaginary parts to equation (6), the following two relations are obtained:

$$\alpha^2 - \beta^2 = - (\omega/c)^2 \text{ Re } (K')$$

and

$$2\alpha\beta = - (\omega/c)^2 \text{ Im } (K').$$

Use of equation (1) then yields the following two equations:

$$(8a) \qquad \alpha^2 - \beta^2 = (\omega/c)^2 \frac{(\omega_p/\omega)^2 - (\nu/\omega)^2 - 1}{1 + (\nu/\omega)^2}$$

and

$$(8b) \qquad 2\alpha\beta = (\omega/c)^2 \frac{(\omega_p/\omega)^2}{\omega/\nu + \nu/\omega}$$

Solving equation (8b) for β and substituting this value into equation (8a), the biquadratic in ω_p/ω is obtained:

$$(9) \qquad (\omega_p/\omega)^4 + A(\omega_p/\omega)^2 - AB = 0,$$

3

where

(10a)
$$A = 4a^2 c^2 (1/\nu^2 + 1/\omega^2)$$

and

(10b)
$$B = \frac{1 + (\nu/\omega)^2}{\omega^2}(a^2 c^2 + \omega^2).$$

The relation between microwave frequency and electron density can then be obtained in terms of the signal attenuation per unit length of transmitted path through the plasma. Since $\omega_p/\omega = f_p/f$, equation (2) can be used to give

(11)
$$n_e = 1.24 \cdot 10^{-8} f^2 (\omega_p/\omega)^2,$$

where the value of $(\omega_p/\omega)^2$ is obtained from the solution of equation (9) in terms of the signal attenuation, signal frequency, and electron collision frequency. In using this method to determine n_e, however, an estimate must be made of the electron collision frequency. Under equilibrium conditions, the collision frequency in the chemical energy range can be written in MKS units (Ref. 4) as

(12)
$$\nu = nQ(8kT/\pi m_e)^{1/2}$$

where n is the number density of gas particles, Q is the total collision cross-section for electrons, k is the Boltzmann constant, and T is the Kelvin temperature. For seeded combustion gases, ν is frequently on the order of 10^{11} sec^{-1} (see Appendix A), so that $\nu \geqslant \omega$ for all but the highest frequency microwave equipment available at present.

Using an experimental arrangement as shown in Fig. 2, the electron density in the plasma (a seeded rocket exhaust in the figure) can be determined by microwave attenuation measurements. If $(\nu/\omega)^2 \ll 1$, equation (5) can be used to compute n_e at the microwave cut-off condition. If $(\nu/\omega)^2$ is not small compared with unity, equation (11) can be used to compute n_e; however, in using this method, the transmitted distance through the plasma and the electron density distribution within the plasma must be known or assumed.

A technique of wider application uses the fact that a microwave signal of given frequency undergoes a phase shift when passing through a plasma. If the attenuation and phase shift of the signal can be measured, equations (8a) and (8b) can be used to compute the values of ω_p/ω and ν/ω. These parameters are most commonly determined by means of a microwave interferometer (Fig. 3). In practice, the bridge circuit is balanced so that no signal is observed when the plasma is absent. When the microwaves are transmitted through the plasma, however, their phase is shifted—this shift unbalances the bridge. A measurement of attenuation and amount of phase shift enables n_e and ν to be determined. This technique is particularly valuable for determining the growth and decay of electron density
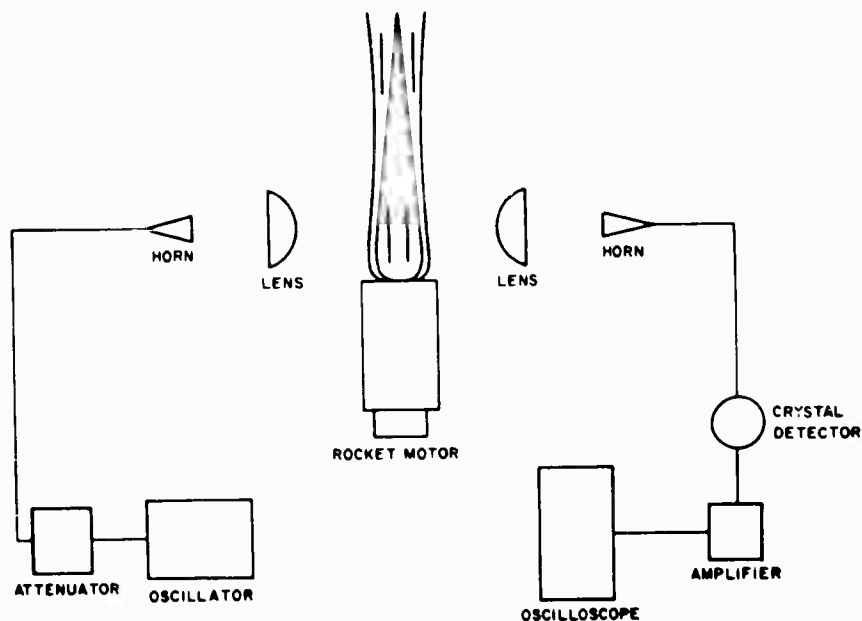
4

FIG. 2. Schematic Diagram of Single-Pass Microwave Attenuation Experiment.

in a transient plasma (eg. a shock-ionized medium). In this case the observed signal is composed of interference fringes, the number of which can be related to the time variation in electron density.

While other microwave techniques are applicable to the measurement of various plasma parameters (eg. microwave noise and electron temperature), the attenuation and phase shift measurements are perhaps the most widely used in the density and temperature ranges of chemically-produced plasmas.

II. TEMPERATURE DETERMINATION BY MICROWAVE MEASUREMENTS

The attenuation and phase shift of microwave radiation in a plasma is a function of the number density of electrons in the gas. Under conditions of equilibrium, the electron density due to thermal ionization of atomic and molecular species can be related to the thermodynamical variables of the gas by means of the Saha equation (Ref. 5). Moreover, once the electron density in the gas is known the electrical characteristics, specifically, the electrical conductivity, can be predicted (Ref. 6). Such a relation has been formulated and programmed on a digital computer for a combustion gas which has been seeded with an easily-ionizable material (Ref. 7). This electrical conductivity program is used widely in present work in magnetohydrodynamic (MHD) power generation
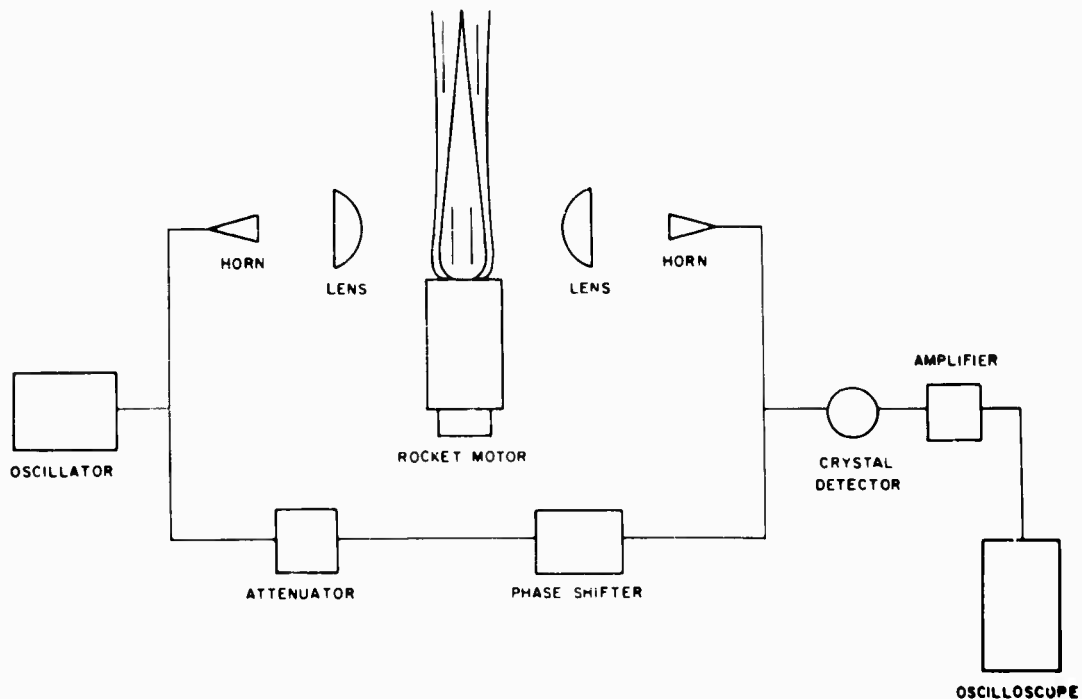
·FIG. 3. Schematic Diagram of Microwave Interferometer.

at NOTS, and applies to an equilibrium system where the electrons are produced by the thermal ionization of a single species in the gaseous mixture. The assumption that one particular species is the sole electron donor is approximately valid when the ionization potential of the seed material is much less than that of any other species present, and when the degree of ionization of the seed material is small. Under these conditions the degree of ionization of the seed material (assumed to be an alkali metal), μ, can be approximated by (Ref. 5)

$$(13) \qquad \mu = (T^{2.5}/P_s)^{1/2} \exp(- 5.8 \cdot 10^3 \ E_s/T - 7.12),$$

where P_s is the partial pressure of the seed material in atmospheres, and E_s is its ionization potential in volts. Refer to Fig. 4 and 5 for the electron density in a seeded rocket exhaust.

By knowing the amount of seed material in a combustion gas and the corresponding pressure, a microwave measurement of the electron density serves to determine the temperature of the system under conditions of equilibrium. It is anticipated that by developing an analytic expression similar to equation (13), but for the case of an arbitrary combustion-product gas (not necessarily seeded), microwave measurements can be

6

used to determine the gas temperature. Accordingly, work has been started on the formulation of this general expression.
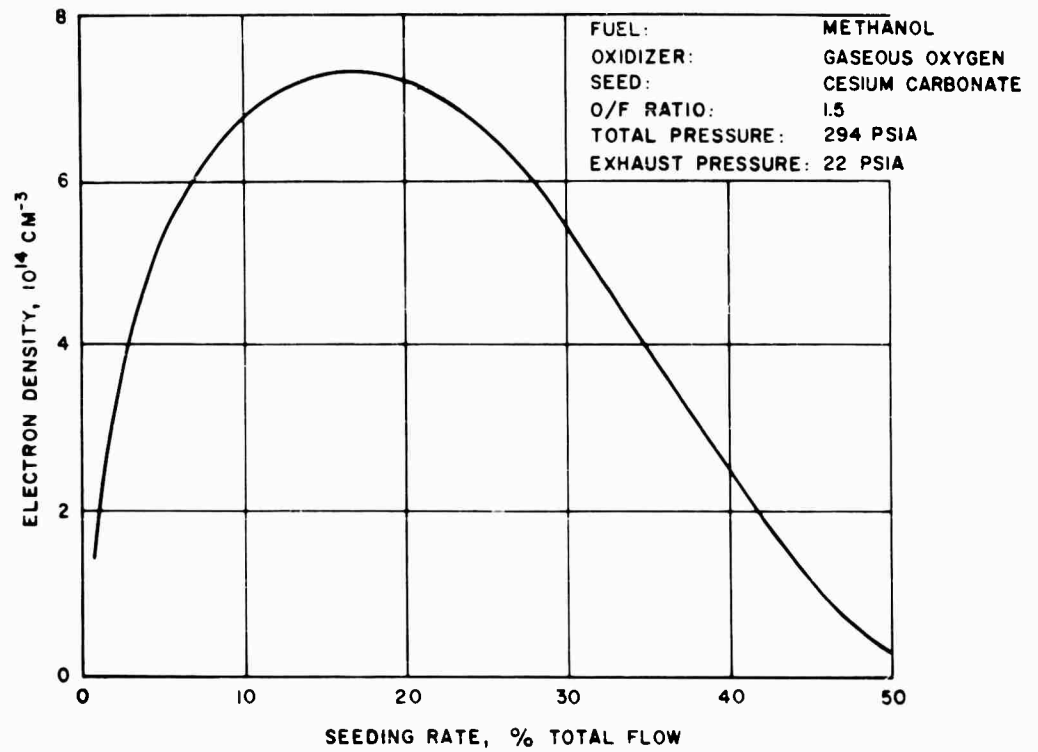
FIG. 4. Electron Density at Nozzle Exit Versus Seeding Rate.
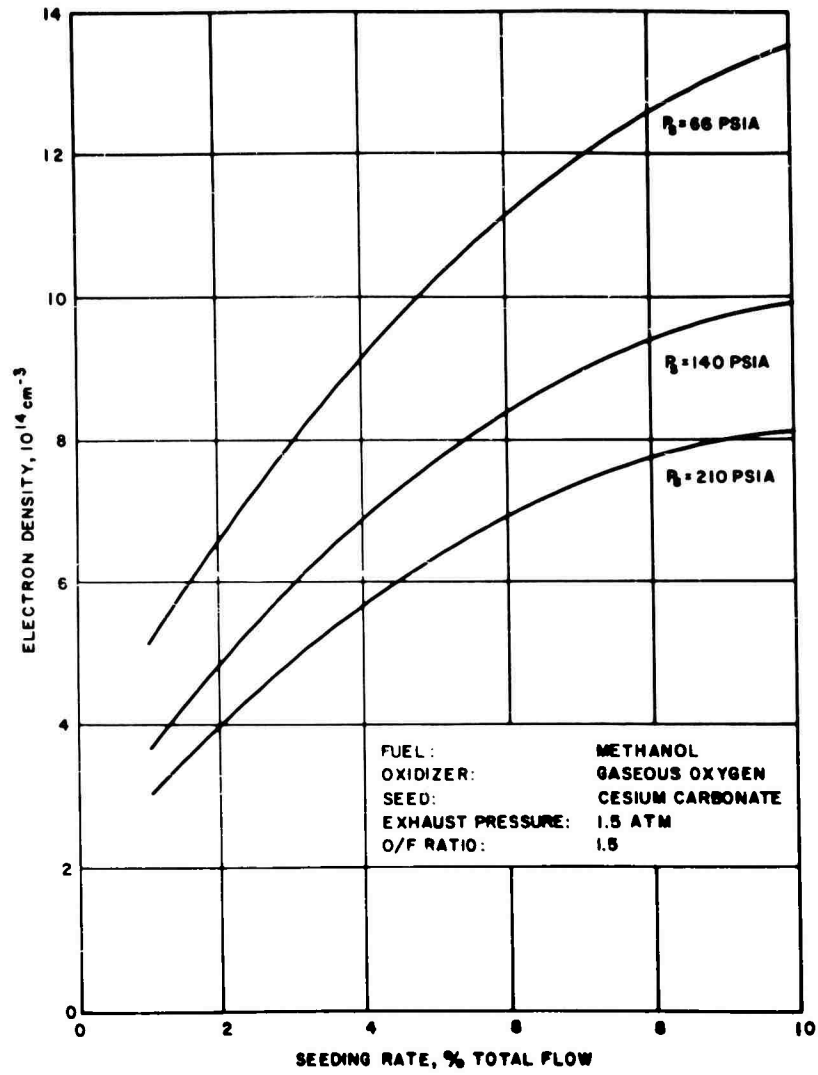
FIG. 5. Electron Density at Nozzle Exit Versus Seeding Rate for Various
Chamber Pressures.

EXPERIMENT

I. EXPERIMENTAL APPARATUS

The majority of the microwave measurements made thus far have been of the electron density in the two-dimensional exhaust of a water-cooled rocket motor. This motor was developed for use in magnetohydrodynamic power generation experiments, and is described more completely in Ref. 8. The rocket burns gaseous oxygen and methanol at a total pressure of from 175 to 300 psia, and at a total temperature in the neighborhood of 3100°K. Ionization is produced by seeding the propellant with cesium carbonate—usually in the amount of from one to eight percent by weight of the total flow. The combustion gases are expanded through a square throat and a two-dimensional nozzle to an exit Mach number of 2.3. Figure 6 shows the motor in operation with its characteristic exhaust fan.

FIG. 6. Rocket Motor with Two-Dimensional Nozzle.

The objective of the microwave experiments was to measure the electron density in a particular region of the rocket exhaust under known rocket motor operating conditions (O/F ratio and total pressure). The single-pass arrangement (Fig. 2) was used in all experiments. For all measurements X-band (8-12 kmc) microwave equipment was used because of its availability.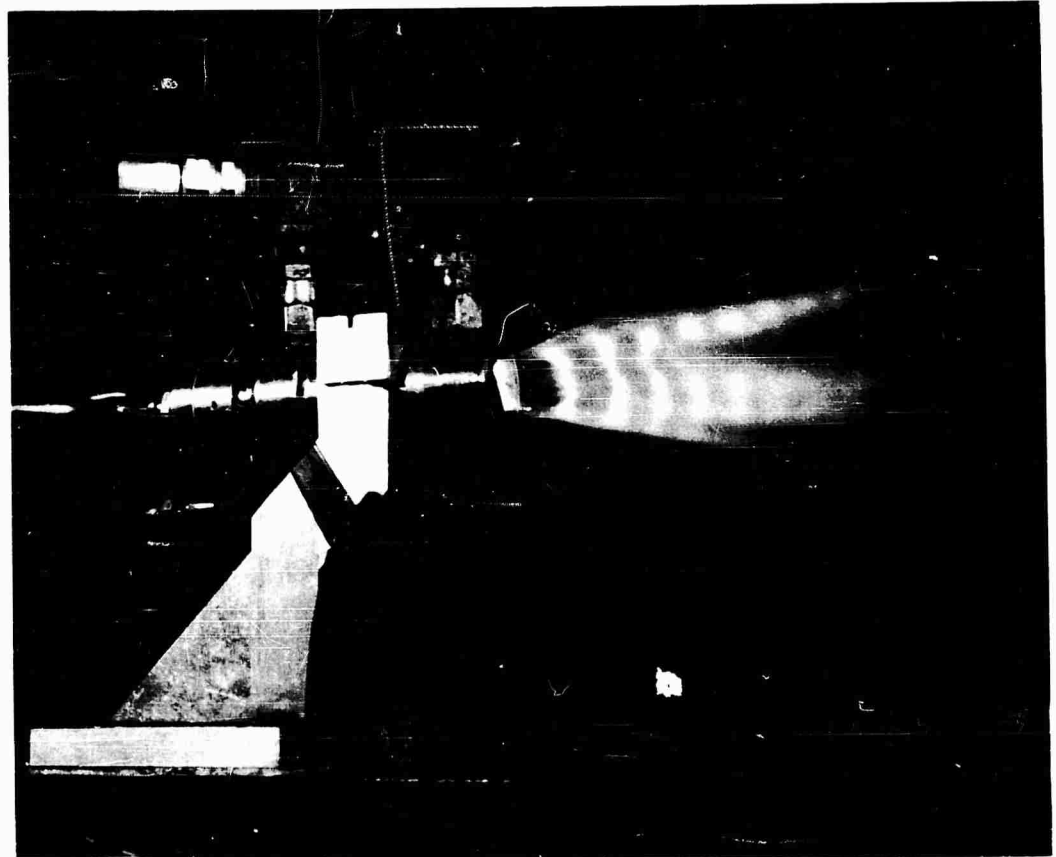 The microwave components were mounted on a rigid beam which attached to the rocket test stand just aft of the rocket motor (Fig. 7), and could be adjusted both vertically and axially. These components consisted of a klystron, variable attenuator, frequency meter, 20-db coupler, wave guide, standard gain horns, and a tunable crystal detector mount. Auxiliary equipment included a klystron power supply and a 1000 cps narrow band amplifier. In the course of the experiments both an oscilloscope and a standing wave ratio (SWR) meter were used to observe the signal.

The main problem encountered in using X-band microwaves to probe the exhaust of the small (80 lb nominal thrust) rocket motor was that the dimension of the horns (and, hence, of the radiating beam) was larger than that of the rocket exhaust gases. In order to probe an area small compared to the gas stream, the microwave beam had to be reduced in size. Several different methods were tried with varying results. A metal plate with a rectangular aperture was first placed in front of the transmitting horn. Although the beam cross-section was thereby reduced considerably, definition of the beam became very difficult—probably due to diffraction effects. The beam was then successfully collimated by passing it through a small opening in microwave absorbant material; however, the consequent loss in beam power was found excessive. A zone plate lens was then constructed to focus the beam (Ref. 9). This lens was designed to have a focal length of 5 inches, and consisted of four transparent zones (Ref. 10). By use of this lens, the cross-sectional dimensions of the beam were reduced from 3 by 5 inches to 1 by 2 inches; however, the actual focal length was much less than the design value. To achieve the desired focal length, so that the lens could be placed a safe distance from the rocket exhaust, it appeared that the lens diameter would have to be increased by an impractical amount. A dielectric (Lucite) lens (Ref. 11-12) was then constructed which successfully reduced the microwave beam to approximately 1 inch by 1 inch. This lens had a plano-convex geometry, a 3.5-inch focal length, and a constant radius of curvature of 2.25 inches. Two such lenses were made—one to be used with the transmitting horn, and the other for the receiving horn. All measurements reported were made using these dielectric lenses.

The klystron was tunable over the frequency range 8-12 kmc; however, all measurements were made at one frequency, 11 kmc. The klystron was modulated with a 1000 cps square wave. The detector output was amplified with a narrow band (±20 cps) 1000 cps amplifier—in the experiment the SWR meter was used as the amplifier. Both the SWR meter and an oscilloscope were used to observe the amplified signal, although the meter was found to be more convenient in measuring microwave attenuation.
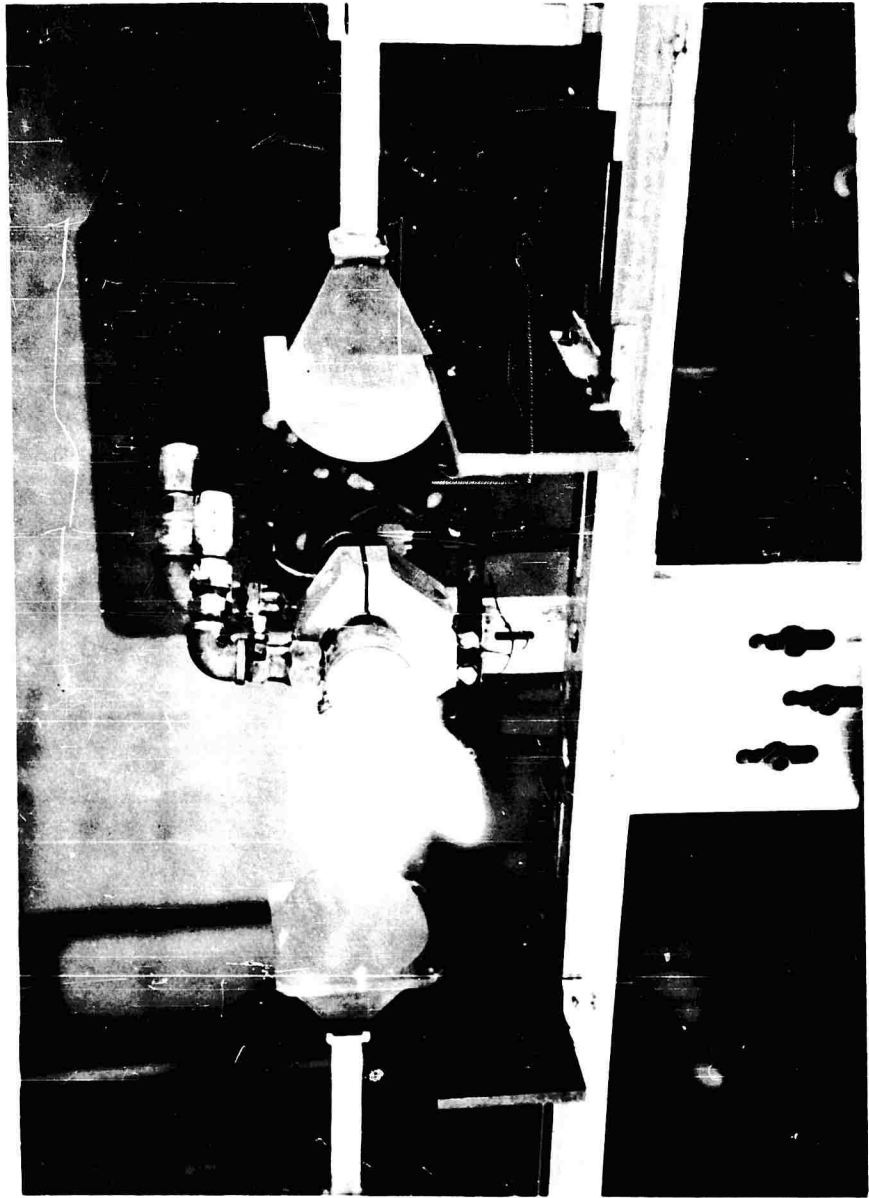
FIG. 7. Microwave Attenuation Measurement

II. PROCEDURE

The seed material (cesium carbonate) was pre-mixed with the methyl alcohol fuel prior to firing, thus enabling the concentration of seed to be precisely determined. Standard ASME orifice plates with manometer readout made possible accurate measurement of fuel and oxidizer (thus, O/F ratio and seeding rate) flow rates. With the microwave beam focused on a known portion of the rocket exhaust, determined prior to firing, the rocket motor was fired and the design combustion chamber pressure reached. The O/F ratio and, hence, the electron density in the probed region of the rocket exhaust, would be varied until a significant amount (8-10 db) of microwave attenuation would be observed. At this point, the signal attenuation, fuel and oxidizer flow rates, and combustion chamber pressure would be recorded. Since firing durations were typically on the order of minutes, several such measurements could be made in the course of each experiment.

The values of the thermodynamical variables (pressure, temperature, and molecular species) in the probed region can be computed, since the rocket motor chamber conditions and geometry are known. This information can then be used to calculate the electron density by means of the electrical conductivity computer program. Predicted values of the electron density can then be correlated with those determined by microwave attenuation measurements. A sample calculation of the electron density in the probed region of the rocket exhaust is given in Appendix B.

RESULTS

A series of four experiments were performed in which ten measurements were made of the electron density in the rocket motor exhaust. In all these experiments, the electron density in the exhaust gas was determined by attenuation measurements to be between $3 \cdot 10^{13}$ and $13 \cdot 10^{13}$ electrons per cubic centimeter, based on a value of $3 \cdot 10^{19}$ square meter for the total electron collision cross-section. The rocket motor operating conditions and the corresponding electron densities (both predicted and measured) are presented in Table 1.

DISCUSSION

The agreement between the microwave measurements of the electron density in the rocket exhaust and those predicted by the analytical procedure is encouraging. However, the latitude in experimental accuracy due to lack of knowledge concerning the electron collision frequency should be pointed out. A microwave interferometer should be used in future experiments to obtain a direct measurement of this parameter. In addition, the dimensions of the probed region should be made as small as practicable, certainly small compared to the characteristic size of the

gas stream. Improved beam optics and the use of higher frequency micro-waves should achieve this result—increasing the size of the rocket motor is undesirable. With these improvements in experimental equipment and technique it appears that rather precise measurements can be made of the electron density in rocket exhaust gases, thus providing one method for determining the validity of the theory of combustion gas ionization.

TABLE 1. Summary of Experimental Conditions with Predicted and Measured Electron Densities

Measurement	O/F Ratio	Chamber Pressure (psia)	Seeding Rate (% total flow)	Probe[a] (in.)	Predicted Electron Density ($\times 10^{13}$ cm^{-3})	Measured Electron Density ($\times 10^{13}$ cm^{-3})
1	1.35	196	1.06	1.25	3.9	
2	1.53	224	0.99	1.25	5.4	
3	1.48	192	1.00	0.50	12.4	
4	2.00	186	0.83	0.50	8.3	
5	1.59	194	0.97	1.00	6.7	$3 \leq n_e \leq 13$
6	1.93	174	0.86	1.00	4.5	
7	1.98	179	0.84	1.00	4.4	
8	1.62	192	0.96	1.00	7.9	
9	1.59	197	1.93	1.00	9.3	
10	1.46	182	2.03	1.00	9.0	

[a]Figures refer to distance of probed region downstream from nozzle exit.

Appendix A

CALCULATION OF ELECTRON COLLISION FREQUENCY

In the collision-dominant regime of chemical rocket combustion processes, the electron collision frequency can be expressed by means of equation (12):

$$\nu = nQ(8kT/\pi\, m_e)^{1/2}.$$

By using Dalton's law for an equilibrium system, this can be put into the more convenient form

$$\nu = PQ(8/\pi\, m_e KT)^{1/2},$$

where P is the gas pressure. With the exception of Q, all factors in this expression are constants or measurables. For the case of a cesium-seeded rocket exhaust gas under equilibrium conditions, a value of $3 \cdot 10^{-19}$ square meter can be used for the total collision cross-section (Ref. 13). At a pressure of one atmosphere and a temperature on the order of 2000°K (the approximate conditions of the experiment), the electron collision frequency is found to be about $3 \cdot 10^{11}$ sec^{-1}.

Appendix B

SAMPLE CALCULATION OF ELECTRON DENSITY

The following is an example of the procedure used to compute the electron density in the probed region of the rocket motor exhaust. The rocket burns gaseous oxygen and methanol with cesium carbonate added to the fuel prior to firing. Typically, the seeding rate would be such that the salt would comprise 1% by weight of the total flow when the motor was operated at the stoichiometric mixture ratio, an O/F ratio of 1.5. At the time the microwave measurement is made, the following data are recorded: combustion chamber pressure, 194 psia; indicated fuel flow rate, 0.120 lb/sec; oxygen flow rate, 0.187 lb/sec. For a 1% seeding rate at an O/F ratio of 1.5, approximately 2.53% of the measured fuel flow contains cesium carbonate. Therefore, about 97.47% of the indicated fuel flow is the actual flow rate of met'anol, 0.117 lb/sec. The actual O/F ratio is then 1.6, and the seeding rate is 0.96% of total flow by weight.

From the known values for nozzle throat diameter and exit diameter, together with the measured chamber pressure and calculated O/F ratio, a value of 1.19 is chosen for γ (the average ratio of specific heats in the rocket exhaust) based upon IBM propellant evaluation data under conditions of isentropic expansion and shifting equilibrium. For this area ratio data and value for γ, the nozzle exit Mach number, M_e, is found (Ref. 14) to be 2.3—the exit static pressure is correspondingly found to be 15.5 psia. Using the IBM data, graphs have been constructed of the chamber and exit temperatures for various O/F ratios, chamber pressures, and expansion ratios. From these graphs, the chamber and exit temperatures are found to be $3157^{\circ}K$ and $2560^{\circ}K$, respectively. These temperatures, however, correspond to an unseeded gaseous oxygen-methanol propellant combination, and do not take into account the cooling effect of the seed. An empirical expression to correct for this condition is $T = T' (1 - XR)$, where the prime denotes the "unseeded" temperature, R is the percent seeding rate, and X is a constant equal to $1.77 \cdot 10^{-3}$. Using this expression the adjusted temperatures are found to be $3152^{\circ}K$ and $2556^{\circ}K$, respectively.

At this point, the chamber and exit conditions are known; however, the region of the rocket exhaust probed by the microwaves is located approximately 1 inch downstream from the exit plane of the motor. The static temperature and pressure of the gas must be known in the probed region, and some assumptions need be made concerning changes in the

16

thermodynamic state of the rocket exhaust in expanding from the exit plane to the probed region. Examination of photographs of the operating rocket motor (Fig. 6) reveals that the gases tend to continue expanding aft of the nozzle exit at approximately the same rate as within the nozzle, that is, with a divergence half-angle of 22°; and, further, that the first shock band is located about one nozzle-width downstream from the exit plane. The assumptions are then made that the gases expand isentropically with shifting equilibrium from the nozzle exit plane (with a 22° expansion half-angle) to the probed region, and that this region lies upstream from the first shock band.

The nozzle of the rocket is two-dimensional, having a square throat (0.5 inch by 0.5 inch) and a rectangular exit (0.5 inch by 1.35 inches). With this information and the measured distance from the nozzle exit to the probed region, the ratio of the stream area in the probed region to the stream area at the nozzle throat is found:

$$A_p/A^* = (A_p/A_e)(A_e/A^*) = 1 - \frac{2 \tan 22^\circ}{1.35} (1.35/0.5) = 4.32.$$

where the subscript p refers to the probed region, the asterisk to the sonic throat, and e to the exit plane. With the area ratio known and the value for γ assumed constant at 1.19, the Mach number of the flow in the probed region is found to be 2.66—the Mach number then serves to determine the static pressure as 7.9 psia. The corresponding static temperature is $2291^\circ K$.

The mole fraction of seed material, X_s, in the form of monatomic cesium (the only species assumed to contribute electrons) in the gas is approximated by $X_s = 0.0061 R$, based upon IBM data for this propellant combination. Therefore, 0.0059 moles of monatomic cesium are present per 100 grams of gas at the seeding rate of 0.96%. The molecular weight of the gas is found to be 27, again using the IBM data. The partial pressure of monatomic cesium in the rocket exhaust is given by $P_s = CX_s$, which C is the conversion factor for mole fraction to partial pressure in the NOTS Propellant Evaluation Program, and is given by the product of molecular weight and gas pressure divided by one hundred. The partial pressure is thus found to be $8.56 \cdot 10^{-4}$ atmosphere.

All required parameter values have now been determined for use in the Saha equation. The degree of ionization of the monatomic cesium is, from equation (13),

$$\mu = (2556^{2.5}/8.56 \cdot 10^{-4})^{1/2} \exp(- 5.8 \cdot 10^3 \times 3.87/2556 - 7.12)$$

$$= 0.0243.$$

The electron density is then

$$n_e = \mu n_s = \mu P_s/kT = 6.7 \cdot 10^{13} \text{ electrons/centimeter}^3.$$

REFERENCES

1. Glasstone, Samuel, and Ralph H. Lovberg. Controlled Thermonuclear Reactions. Princeton, N. J. Van Nostrand, 1960. P. 170.

2. U. S. Naval Ordnance Test Station. Magnetohydrodynamic Power Generation From a Supersonic Rocket Exhaust: An Interim Progress Report, by Harold M. Larsen and Leroy J. Krzycki. China Lake, Calif., NOTS, 10 July 1962. (NOTS IDP 1615)

3. ------. Transfer of Heat Reduced Magnetically (THERMA). China Lake, Calif., NOTS, June 1962. (NOTS Propulsion Development Department Review, Chap. 1, pp. 3-4.)

4. Goldberg, Philip A. "Electrical Properties of High-Altitude Shock Waves," in Plasma Physics, ed. by James E. Drummond. New York, McGraw-Hill, 1961. P. 245.

5. Laderman, A. J., and others. "Flame Ionization During the Development of Detonation," in Eighth Symposium (International) on Combustion. Baltimore, Md., Williams and Wilkins, 1962. P. 199.

6. Fay, J. A. "Electrical Conductivity of Combustion Gases," in Engineering Magnetohydrodynamics, MIT Summer Program. Cambridge, Mass., MIT, 1961. P. 3.4-1.

7. U. S. Naval Ordnance Test Station. Formulation of an Electrical Conductivity Expression for Seeded Combustion Gases, by Harold M. Larsen, Informal Memorandum. China Lake, Calif., NOTS, 26 September 1961. (Reg. 4506-25)

8. Larsen, Harold M., and Leroy J. Krzycki. "The Electrical Properties of Seeded Rocket Exhaust Gases and Their Application in Magnetohydrodynamic Power Generation," Propulsion Development Department, NOTS. Paper to be presented at the Western States Section Meeting of the Combustion Institute, Sacramento, California, November 1962.

9. Van Buskirk, L. F., and C. E. Hendrix. "The Zone Plate as a Focusing Element," NOTS. Presented at Western Electronic Show and Convention, Los Angeles, California, August 1960.

10. Jenkins, F. A., and H. E. White. Fundamentals of Optics, 3rd ed. New York, McGraw-Hill, 1957.

11. Naval Research Laboratory. Electromagnetic Probes for Near-Zone Measurements, by D. C. Rohfs. Washington, D. C., NRL, October 1959. (NRL Report 5378)

12. ------. Converging Lens Dielectric Antennas, by F. E. Boyd. Washington, D. C., NRL, April 1951. (NRL Report 3780)

13. Brown, S. C. Basic Data of Plasma Physics. New York, Wiley, and The Technology Press of MIT, 1959.

14. Dailey, C. L., and F. C. Wood. Computation Curves for Compressible Fluid Problems. New York, Wiley, 1949.

INITIAL DISTRIBUTION

30 Chief, Bureau of Naval Weapons

CS (1)	RAPP (1)	RMMP-4 (1)
DLI-31 (2)	RM (1)	RMMP-42 (1)
DMP-33 (1)	RM-1 (1)	RRMA-221 (1)
FWAM (1)	RM-3 (1)	RRRE (1)
FWWS-3 (1)	RM-35 (1)	RRRE-5 (1)
P-1 (1)	RMMP (1)	RRRE-6 (1)
PMA-3 (1)	RMMP-12 (1)	RT (1)
R-12 (1)	RMMP-2 (1)	RUME-11 (1)
R-5 (1)	RMMP-24 (1)	RUSD-23 (1)
RAAV-34 (1)	RMMP-33 (1)	

5 Chief of Naval Operations
   Deputy Chief for Air (1)
   Operations Evaluation Group (1)
2 Chief of Naval Research
   Code 104 (1)
   Code 463 (1)
1 Assistant Secretary of the Navy (Material) (W. H. Moore)
1 Fleet Anti-Air Warfare Training Center, San Diego (Guided Missile
   Section)
1 Naval Air Development Center, Johnsville (Library)
1 Naval Air Engineering Center, Philadelphia
1 Naval Air Force, Atlantic Fleet
1 Naval Air Force, Pacific Fleet
1 Naval Air Test Center, Patuxent River (Aeronautical Publications
   Library)
1 Naval Avionics Facility, Indianapolis (Library)
1 Naval Ordnance Laboratory, White Oak (Library)
2 Naval Propellant Plant, Indian Head
1 Naval Research Laboratory (Code 2021)
2 Naval Underwater Ordnance Station, Newport
1 Naval Weapons Laboratory, Dahlgren (Technical Library)
2 Naval Weapons Services Office
1 Operational Test and Evaluation Force
1 Bureau of Naval Weapons Fleet Readiness Representative, Pacific
1 Bureau of Naval Weapons Representative, Azusa
1 Naval Liaison Officer, Tactical Air Command, Langley Air Force Base
4 Chief of Ordnance
   ORDTA (1)
   ORDTB (1)
   ORDTS (1)
   ORDTU (1)
2 Aberdeen Proving Ground
   Ballistic Research Laboratories (1)
   Development and Proof Services (1)

7 Army Missile Command, Redstone Arsenal
   ABMA (1)
   ORDDW-IDE (1)
   ORDXR-ODA (1)
   Redstone Scientific Information Center (4)
1 Diamond Ordnance Fuze Laboratory (Library)
2 Frankford Arsenal
   ORDBA-1421 (1)
   Pitman-Dunn Laboratory (1)
1 Picatinny Arsenal (Library)
1 Rock Island Arsenal
3 White Sands Missile Range (ORDBS-Technical Library)
1 Headquarters, U. S. Air Force (AFDRD-CC)
1 Tactical Air Command, Langley Air Force Base (TPL-RQD-M)
1 Ogden Air Materiel Area, Hill Air Force Base (OOYD)
1 Aeronautical Systems Division, Wright-Patterson Air Force Base
   (ASAPRD-Dist)
1 Air Force Cambridge Research Laboratories, Laurence G. Hanscom Field
1 Air Force Flight Test Center, Edwards Air Force Base
1 Air Force Office of Scientific Research (Librarian)
1 Air Proving Ground Center, Eglin Air Force Base (PGEM)
1 Air University Library, Maxwell Air Force Base
1 Holloman Air Force Base
1 Advanced Research Projects Agency
10 Armed Services Technical Information Agency (TIPCR)
1 Weapons Systems Evaluation Group
1 National Aeronautics and Space Administration (Chief, Liquid
   Propulsion, RPL 152)
1 Lewis Research Center (Library)
2 Manned Spacecraft Center, Houston
   David Hamock (1)
   Library (1)
2 Scientific and Technical Information Facility, Bethesda (NASA
   Representative (S-AK/DL)
1 Advanced Kinetics, Inc., Costa Mesa, Calif. (Dr. R. W. Waniek)
1 AeroChem Research Laboratories, Inc., Princeton, N. J.
   (Dr. H. F. Calcote)
1 Aerojet-General Corporation, Azusa, Calif. (Librarian) via BuWepsRep
1 Aerojet-General Corporation, Sacramento (Librarian) via BuWepsRRep
1 Aeronautical Research Associates of Princeton, Princeton, N. J.
   (Librarian)
1 Acrospace Corporation, Los Angeles
1 Allegany Ballistics Laboratory, Cumberland, Md.
2 Applied Physics Laboratory, JHU, Silver Spring (Document Library)
1 Arthur D. Little, Inc., Cambridge (W. H. Varley)
1 Atlantic Research Corporation, Alexandria, Va.
1 Avco Research Laboratory, Everett, Mass. (Librarian)
1 Bermite Powder Company, Saugus, Calif. (L. LeFiego)
1 C. A. Parsons and Company, Ltd., Nuclear Research Center, Fossway,
   Newcastle Upon Tyne 6, England (Dr. B. C. Lindley), via BuWeps (DSC)

1 California Institute of Technology, Pasadena (Engineering Library)
1 Central Electricity Research Laboratories, Cleeve Road, Leatherhead,
   Surry, England (Mr. Swift-Hook), via BuWeps (DSC)
1 Continental Aviation and Engineering Corporation, Detroit (Librarian)
1 Cornell Aeronautical Laboratory, Inc., Buffalo (Engineering Library)
1 Curtiss Wright Corporation, Wright Aeronautical Division, Wood-Ridge, N. J.
1 Douglas Aircraft Company, Inc., Long Beach (Chief Engineer)
1 Experiment, Inc., Richmond, Va.
1 General Electric Company, Missile and Space Vehicle Department,
   Philadelphia (Dr. George Sutton)
1 Hughes Aircraft Company, Culver City, Calif. (Research and Development
   Library)
1 Institute for Defense Analyses
1 Jet Propulsion Laboratory, CIT, Pasadena (Dr. W. H. Pickering)
1 Lockheed Aircraft Corporation, Missiles and Space Division, Palo Alto,
   Calif. (Librarian)
1 Lockheed Aircraft Corporation, Missiles and Space Division, Van Nuys,
   Calif. (Library)
1 MHD Research, Inc., Newport Beach, Calif. (Dr. V. H. Blackman)
1 Martin Marietta Corporation, Baltimore (Librarian)
1 Martin Marietta Corporation, Orlando Aerospace Division, Orlando, Fla.
   (Advanced Technology Staff, MP-233)
1 Massachusetts Institute of Technology, Cambridge (Engineering Library)
1 Naval Warfare Research Center, Stanford Research Institute, Menlo Park,
   Calif. (Librarian)
1 North American Aviation, Inc., Atomics International Division, Canoga
   Park, Calif. (Librarian)
1 North American Aviation, Inc., Space and Information Systems Division,
   Downey, Calif. (Technical Information Center, 4-096-314)
1 Northrop Corporation, Norair Division, Hawthorne, Calif.
1 Northwestern University, Evanston, Ill. (Technological Institute, Gas
   Dynamics Laboratory)
1 Polytechnic Institute of Brooklyn, Brooklyn (Engineering Library)
1 Princeton University, Princeton, N. J. (Engineering Library)
1 Princeton University, Forrestal Research Center, Princeton, N. J.
1 Purdue University, Lafayette, Ind. (Engineering Library)
1 Rensselaer Polytechnic Institute, Troy, N. Y. (Engineering Library)
1 Republic Aviation Corporation, Farmingdale, N. Y. (Librarian)
1 Research Analysis Corporation, Bethesda, Md. (Document Control Office)
1 Rocketdyne, Canoga Park, Calif. (Librarian)
2 Rohm and Haas Company, Redstone Arsenal Research Division (Librarian)
1 Ryan Aerospace, A Division of Ryan Aeronautical Company, San Diego
   (C. F. Friend)
1 Space Technology Laboratories, Inc., Redondo Beach, Calif. (Librarian)
1 Stanford University, Stanford, Calif. (Engineering Library)
1 Temco Electronics and Missiles Company, Dallas (Correspondence Control)
1 The Boeing Company, Scientific Research Laboratories, Seattle (Librarian)
1 The Rand Corporation, Santa Monica, Calif.
1 The University of Michigan, Ann Arbor (Engineering Library)

ABSTRACT CARD

U. S. Naval Ordnance Test Station
  Microwave Measurement of Electron Density in
Seeded Rocket Exhaust Gases, by Harold M. Larsen and
Charles H. Turner. China Lake, Calif., NOTS, June
1963.   19 pp. (NAVWEPS Report 8059, NOTS TP 3062),
UNCLASSIFIED.
  ABSTRACT. The initial phase of a microwave diag-
nostics program involving seeded rocket exhaust gases
is presented. The theory of the microwave determina-
tion of electron density and collision frequency is

                                        (Over)
                    1 card, 4 copies

U. S. Naval Ordnance Test Station
  Microwave Measurement of Electron Density in
Seeded Rocket Exhaust Gases, by Harold M. Larsen and
Charles H. Turner. China Lake, Calif., NOTS, June
1963.   19 pp. (NAVWEPS Report 8059, NOTS TP 3062),
UNCLASSIFIED.
  ABSTRACT. The initial phase of a microwave diag-
nostics program involving seeded rocket exhaust gases
is presented. The theory of the microwave determina-
tion of electron density and collision frequency is

                                        (Over)
                    1 card, 4 copies

U. S. Naval Ordnance Test Station
  Microwave Measurement of Electron Density in
Seeded Rocket Exhaust Gases, by Harold M. Larsen and
Charles H. Turner. China Lake, Calif., NOTS, June
1963.   19 pp. (NAVWEPS Report 8059, NOTS TP 3062),
UNCLASSIFIED.
  ABSTRACT. The initial phase of a microwave diag-
nostics program involving seeded rocket exhaust gases
is presented. The theory of the microwave determina-
tion of electron density and collision frequency is

                                        (Over)
                    1 card, 4 copies

U. S. Naval Ordnance Test Station
  Microwave Measurement of Electron Density in
Seeded Rocket Exhaust Gases, by Harold M. Larsen and
Charles H. Turner. China Lake, Calif., NOTS, June
1963.   19 pp. (NAVWEPS Report 8059, NOTS TP 3062),
UNCLASSIFIED.
  ABSTRACT. The initial phase of a microwave diag-
nostics program involving seeded rocket exhaust gases
is presented. The theory of the microwave determina-
tion of electron density and collision frequency is

                                        (Over)
                    1 card, 4 copies

NAVWEPS Report 8059

discussed as it applies to the temperature and densi-
ty regimes of both thermonuclear and chemically-pro-
duced plasmas. The procedure used to predict the
electron density in an equilibrium seeded rocket
exhaust gas is outlined, as well as a possible
method for temperature measurement using microwave
radiation. The experimental apparatus and proce-
dure are described, and the results of ten measure-
ments of electron density are compared with predict-
ed values. The improvements in experimental equip-
ment and technique planned for future investigations
are discussed.

NAVWEPS Report 8059

discussed as it applies to the temperature and densi-
ty regimes of both thermonuclear and chemically-pro-
duced plasmas. The procedure used to predict the
electron density in an equilibrium seeded rocket
exhaust gas is outlined, as well as a possible
method for temperature measurement using microwave
radiation. The experimental apparatus and proce-
dure are described, and the results of ten measure-
ments of electron density are compared with predict-
ed values. The improvements in experimental equip-
ment and technique planned for future investigations
are discussed.

NAVWEPS Report 8059

discussed as it applies to the temperature and densi-
ty regimes of both thermonuclear and chemically-pro-
duced plasmas. The procedure used to predict the
electron density in an equilibrium seeded rocket
exhaust gas is outlined, as well as a possible
method for temperature measurement using microwave
radiation. The experimental apparatus and proce-
dure are described, and the results of ten measure-
ments of electron density are compared with predict-
ed values. The improvements in experimental equip-
ment and technique planned for future investigations
are discussed.

NAVWEPS Report 8059

discussed as it applies to the temperature and densi-
ty regimes of both thermonuclear and chemically-pro-
duced plasmas. The procedure used to predict the
electron density in an equilibrium seeded rocket
exhaust gas is outlined, as well as a possible
method for temperature measurement using microwave
radiation. The experimental apparatus and proce-
dure are described, and the results of ten measure-
ments of electron density are compared with predict-
ed values. The improvements in experimental equip-
ment and technique planned for future investigations
are discussed.