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by

Edmund J. Appleby

DEPARTMENT OF MECHANICS

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Research

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ABSTRACT

The viscoplastic flow of a long thick-walled tube is investigated. The tube is subjected to internal pressure and has its ends restrained from motion in the axial direction. The material of the tube is rigid-viscoplastic and incompressible. The pressure required to produce a specified expansion of the tube is calculated for two examples. In the former the effect of different viscosity coefficients is observed. In the latter example a comparison is made of the effects of perfect plasticity, viscosity and inertia.

1. INTRODUCTION

A study is made of the mechanical behavior of a long hollow circular cylinder, with its end restrained from motion in the axial direction, when it is subjected to internal pressure. The basic problem of plane strain with rotational symmetry, being one of those which most readily yield to treatment in plasticity, is a standard problem in this field, and has been studied by investigators in a variety of different ways for the ideally plastic and the elastic plastic material. Reviews of such solutions have been given by Hill [1], and Prager and Hodge [2]. The present paper considers an ideal material, a viscoplastic Bingham solid [3], which is undeformable until the stresses reach their yielding values, and then under stresses which are in excess of their yielding values, has strain velocities dependent on this stress excess or overstress. The material does not exhibit work hardening, and as it is known that in the fully plastic state volume changes are negligible, it is assumed to be incompressible.

The solution is obtained by using the viscoplastic constitutive equations due to Hohenemser and Prager [4]. The analysis is valid for a general plastic yield condition. An approach by means of a linearized theory of viscoplasticity, in which the flow is specified by Prager's constitutive equation [5], is equivalent.

A general solution is formulated, but in order to simplify the numerical calculations a specific expansion is imposed on the tube in which the interior boundary has a uniform radial acceleration. The pressure-time variations required to maintain this flow for different viscosity coefficients are compared. A different expansion is then imposed, in which the inner radius of the tube expands to one and one half its initial size, beginning and ending with zero velocity. Comparison is made of the effects of perfect plasticity, viscosity and inertia. If the expansion takes place slowly, the effect of inertia on the required pressure is negligible, and the effects of perfect plasticity and viscosity are comparable. On the other hand if the expansion occurs very quickly the effect of inertia becomes comparable with that of viscosity, and the perfectly plastic contribution to the pressure is negligible.

2. BASIC EQUATIONS

Let the space variables be a system of cylindrical co-ordinates r, θ, z in which the z-axis coincides with the axis of symmetry, then the tube is bounded by the cylinders $r = a$ and $r = b$, where $a < b$. As the pressure inside the tube increases, the stresses in the material nearest the interior boundary will be the first to reach the yield limit. With further increase of pressure the plastic region will extend until its outer boundary coincides with the tube's outer surface. Until this state is reached, the flow of the plastic innermost region of the tube is restricted by a surrounding rigid region and by the conditions of axially symmetric plane strain on the incompressible material. The whole tube therefore remains rigid, and hence the stresses in the innermost plastic region reach but do not exceed the yield limit. When the whole of the tube becomes plastic, the material is about to flow in an unrestricted manner, since any further increase in the internal pressure will then produce overstress in the material of the tube. The time t is measured from this instant, and the values of a and b for $t \le 0$ are denoted by a_0 and b_0.

It is assumed that the tube is sufficiently long to make the stresses and strains independent of the axial co-ordinate, and that at any instant of the flow process each particle of the tube is moving radially outward with a velocity u which depends only on the radial co-ordinate r. The velocity components at a point distance r from the axis at time t are then

$$u_r = u(r,t), \quad u_\theta = 0, \quad u_z = 0. \tag{1}$$

The strain rates are

$$\left.\begin{aligned} &\dot{\epsilon}_r = \frac{\partial u}{\partial r}, \qquad \dot{\epsilon}_\theta = \frac{u}{r}, \quad \dot{\epsilon}_z = 0, \\ &\dot{\gamma}_{\theta z} = 0, \qquad \dot{\gamma}_{zr} = 0, \quad \dot{\gamma}_{r\theta} = 0 \end{aligned}\right\} \tag{2}$$

where the dot denotes differentiation with respect to the time.

Since the material is assumed incompressible

$$\frac{\partial u}{\partial r} + \frac{u}{r} = 0. \tag{3}$$

This differential equation has the solution

$$u = \frac{\dot{\Psi}(t)}{r}, \tag{4}$$

in which $\dot{\Psi}(t)$ is an arbitrary function of the time. Expressing the velocity u of a particle, distance r from the axis at time t as dr/dt, and integrating (4), we find that a particle initially at distance r_0 from the axis is at distance $\sqrt{r_0^2 + 2\Psi(t)}$ after a time t. The radial and circumferential strain rates can now be written

$$\dot{\epsilon}_r = -\frac{\dot{\Psi}}{r^2}, \quad \dot{\epsilon}_\theta = \frac{\dot{\Psi}}{r^2}. \tag{5}$$

By the rotational symmetry of the flow field, the shearing stresses with respect to the cylindrical co-ordinates are zero, so the only equation of motion which is not identically satisfied is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = D\,\frac{du}{dt}, \tag{6}$$

where D is the density of the material. When u is replaced by the expression in (4), the above equation can be written

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \frac{D}{r}\left[\ddot{\Psi} - \frac{\dot{\Psi}^2}{r^2}\right] \tag{7}$$

The boundary conditions throughout the yielding process are

$$\begin{aligned} \sigma_r &= -p \ \text{ at } \ r = a, \\ \sigma_r &= \ o \ \text{ at } \ r = b; \end{aligned} \tag{8}$$

it being assumed that there is no pressure on the external boundary.

The analysis so far is independent of the constitutive equations, and is the same for all materials.

For any incompressible isotropic material in plane strain Geiringer [6] has shown that a general yield condition can be expressed as a function of the single variable $\sigma_1 - \sigma_2$; σ_1 and σ_2 being the principle stresses in the plane z = const. For the rotationally symmetric problem the principle stresses are σ_r and σ_θ, so that the yield function F can always be written in the form

$$F = |\sigma_\theta - \sigma_r| - 2k, \tag{9}$$

where k is the yield stress in shear. In particular if $\sigma_\theta \geq \sigma_r$, an assumption which may be verified <u>a posterior</u>, (9) can be replaced by

$$F = \sigma_\theta - \sigma_r - 2k \tag{10}$$

The viscoplastic flow rule of Hohenemser and Prager can be written in the general form

$$\lambda \dot{\epsilon}_{ij} = \langle F \rangle \frac{\partial F}{\partial \sigma_{ij}}, \tag{11}$$

where the notation is defined by:

$$\langle F \rangle = F \text{ if } F \geq 0,$$

$$\langle F \rangle = 0 \text{ if } F < 0,$$

λ is the viscosity coefficient of the material, and F is the yield function. For the yield function (10), therefore, the only nonvanishing strain rate components for $t > 0$ are given by

$$\lambda \dot{\epsilon}_r = -(\sigma_\theta - \sigma_r - 2k), \qquad \lambda \dot{\epsilon}_\theta = \sigma_\theta - \sigma_r - 2k. \quad (12)$$

It is interesting to note that since (10) is a linear function, (12) may also be obtained as a special case of the piecewise-linear visco-plastic flow law proposed by Prager [5].

3. SOLUTION

Equations (12) can alternatively be written

$$\frac{\lambda \dot{\Psi}}{r^2} = \sigma_\theta - \sigma_r - 2k. \tag{13}$$

The substitution of equation (13) into the equation of motion (7) gives

$$\frac{\partial \sigma_r}{\partial r} = \frac{2k}{r} + \frac{\lambda \dot{\Psi}}{r^3} + \frac{D}{r}\left[\ddot{\Psi} - \frac{\dot{\Psi}^2}{r^2}\right]. \tag{14}$$

Integrating this equation with respect to r, and employing the boundary condition, $\sigma_r = 0$ when $r = b$, to evaluate the arbitrary function of time furnishes the stress components:

$$\left.\begin{aligned}\sigma_r &= 2k \log \frac{r}{b} + D\ddot{\Psi} \log \frac{r}{b} + \frac{\lambda\dot{\Psi}}{2}\left[\frac{1}{b^2} - \frac{1}{r^2}\right] - \frac{D\dot{\Psi}^2}{2}\left[\frac{1}{b^2} - \frac{1}{r^2}\right],\\ \sigma_\theta &= 2k\left[1 + \log \frac{r}{b}\right] + D\ddot{\Psi} \log \frac{r}{b} + \frac{\lambda\dot{\Psi}}{2}\left[\frac{1}{b^2} + \frac{1}{r^2}\right] - \frac{D\dot{\Psi}^2}{2}\left[\frac{1}{b^2} - \frac{1}{r^2}\right].\end{aligned}\right\} \tag{15}$$

Since a and b can be expressed in terms of their initial values a_0 and b_0 and the function $\Psi(t)$, equations(15) furnish the stress distribution at any instant in terms of Ψ and its first and second derivatives. The functions Ψ and p must satisfy the boundary condition $\sigma_r = - p$ on $r = a$. Hence, by using the expression

$r = \sqrt{r_0^2 + 2\Psi}$,

$$\ddot{\Psi} D \log\left[\frac{a_0^2 + 2\Psi}{b_0^2 + 2\Psi}\right] + \frac{\lambda\dot{\Psi}(a_0^2 - b_0^2)}{(a_0^2 + 2\Psi)(b_0^2 + 2\Psi)} + \frac{\dot{\Psi}^2 D(b_0^2 - a_0^2)}{(a_0^2 + 2\Psi)(b_0^2 + 2\Psi)}$$

$$= -2p - 2k \log\left[\frac{a_0^2 + 2\Psi}{b_0^2 + 2\Psi}\right]; \qquad (16)$$

the conditions at $t = 0$ being $\Psi = 0$ and $\dot{\Psi} = 0$, that is $\sqrt{r_0^2 + 2\Psi}$ is initially r_0 and the initial radial velocity is zero.

If the effects of inertia and viscosity had been neglected, the left hand side of equation (16) would be zero, and hence $p = k \log [1 + (b_0^2 - a_0^2)/a^2]$, which is the internal pressure required in the case of an ideally plastic tube to maintain it in a state of unrestricted flow (cf. [2], page 118). If, on the other hand, the inertia term alone is neglected and p is regarded as a constant internal pressure, equation (16) gives a formula for $\dot{\Psi}$ in terms of Ψ which does not contain t explicitly, namely

$$\dot{\Psi} = \frac{2(a_0^2 + 2\Psi)(b_0^2 + 2\Psi)}{\lambda(b_0^2 - a_0^2)}\left\{p + \log\left[\frac{a_0^2 + 2\Psi}{b_0^2 + 2\Psi}\right]\right\}. \qquad (17)$$

Integration of equation (17) by means of several substitutions yields

$$\left.\begin{aligned} \Psi &= \frac{b_0^2 - a_0^2 \exp(x)}{2[\exp(x) - 1]}, \\ \text{where} \quad x &= \frac{p}{k}[1 - \exp(kt/\lambda)] + \exp(kt/\lambda) \log \frac{b_0^2}{a_0^2}. \end{aligned}\right\} \qquad (18)$$

The fields of radial velocity and radial and circumferential strain rate can now be written down from equations (4) and (5). The stress field is obtained by substituting for Ψ and $\dot{\Psi}$ as functions of the time in the following equations:

$$\left.\begin{aligned}\sigma_r &= k \log \frac{r^2}{(b_0^2 + 2\Psi)} + \frac{\lambda\dot{\Psi}}{2}\left[\frac{1}{(b_0^2 + 2\Psi)} - \frac{1}{r^2}\right],\\ \sigma_\theta &= 2k\left[1 + \frac{1}{2}\log\frac{r^2}{(b_0^2 + 2\Psi)}\right] + \frac{\lambda\dot{\Psi}}{2}\left[\frac{1}{(b_0^2 + 2\Psi)} + \frac{1}{r^2}\right].\end{aligned}\right\} \quad (19)$$

4. COMPARISON OF THE VISCOUS EFFECT FOR DIFFERENT VISCOSITY COEFFICIENTS

It will now be assumed that p varying with the time, is the internal pressure required to keep the expanding tube flowing unrestrictedly in a certain manner. In order to simplify the problem the condition that the interior boundary of the tube expands with a uniformly increasing speed is imposed. The pressure $p(t)$ which is required to produce this effect is found. The expansion of a is of the form

$$a = a_0 + ct^2, \tag{20}$$

where c is a positive constant. The function $\Psi(t)$ is found to satisfy

$$\left.\begin{aligned} & 2\Psi = c^2t^4 + 2a_0\, ct^2, \\ \text{giving}\quad & \dot{\Psi} = 2ct\,(a_0 + ct^2)\,, \\ \text{and}\quad & \ddot{\Psi} = 2c\,(a_0 + 3ct^2). \end{aligned}\right\} \tag{21}$$

Omitting details of the calculation, we find that equation (16) now furnishes

$$P(h) = \frac{1}{2}\log\left[1 + \frac{(\alpha_0^2 - 1)}{(1+h^2)^2}\right] + \frac{R}{B}\left\{\frac{(3h^2+1)}{2}\log\left[1 + \frac{(\alpha_0^2 - 1)}{(1+h^2)^2}\right] + \frac{(1-\alpha_0^2)h^2}{(\alpha_0^2 + h^4 + 2h^2)}\right\} - \frac{1}{2B}\,\frac{(1-\alpha_0^2)h}{(\alpha_0^2 + h^4 + 2h^2)(1+h^2)}\,, \tag{22}$$

where $\alpha_0 = b_0/a_0$,

$h = t\,(c/a_0)^{\frac{1}{2}}$, a dimensionless time, and $P(h) = p(t)/2k$, the dimensionless pressure. Also R and B are two dimensionless parameters analogous to the Reynolds number and the Bingham number and defined by

$$R = \frac{D\,a_0 c}{\lambda}\left(\frac{a_0}{c}\right)^{\frac{1}{2}}, \qquad B = \frac{k}{\lambda}\left(\frac{a_0}{c}\right)^{\frac{1}{2}}.$$

For the present purpose it will be more convenient to utilize less significant parameters defined by

$$R_p = \frac{D\,a_0 c}{k}, \qquad B_p = \frac{\lambda}{2k}\left(\frac{c}{a_0}\right)^{\frac{1}{2}} \tag{23}$$

The pressure can then be written

$$P(h) = \frac{1}{2}\log\left[1 + \frac{(\alpha_0^2 - 1)}{(1+h^2)^2}\right] + R_p\left\{\frac{(3h^2+1)}{2}\log\left[1 + \frac{(\alpha_0^2-1)}{(1+h^2)^2}\right] + \frac{(1-\alpha_0^2)h^2}{(\alpha_0^2 + h^4 + 2h^2)}\right\} - B_p\frac{(1-\alpha_0^2)h}{(\alpha_0^2+h^4+2h^2)(1+h^2)} \tag{24}$$

Equation (24) gives the dimensionless pressure required to produce an unrestricted flow of the tube in which the interior boundary has a uniform acceleration. The stress field is then given by:

$$\frac{\sigma_r}{2k} = \frac{1}{2}\log\left[1 + \frac{(\rho_0^2 - \alpha_0^2)}{(\alpha_0^2 + h^4 + 2h^2)}\right] + R_p\left\{\frac{(3h^2+1)}{2}\log\left[1 + \frac{(\rho_0^2-\alpha_0^2)}{(\alpha_0^2+h^4+2h^2)}\right] - \frac{h^2(h^2+1)^2(\rho_0^2 - a_0^2)}{(a_0^2+h^4+2h^2)(\rho_0^2+h^4+2h^2)}\right\} + \frac{B_p h(1+h^2)(\rho_0^2 - a_0^2)}{(a_0^2+h^4+2h^2)(\rho_0^2+h^4+2h^2)} \tag{25}$$

$$\frac{\sigma_\theta}{2k} = 1 + \frac{1}{2}\log\left[1 + \frac{(\rho_0^2 - \alpha_0^2)}{(\alpha_0^2 + h^4 + 2h^2)}\right] + R_p\left\{\frac{(3h^2+1)}{2}\log\right.$$

$$\left.\left[1+\frac{(\rho_0^2 - \alpha_0^2)}{(\alpha_0^2 + h^4 + 2h^2)}\right] - \frac{h^2(h^2+1)^2(\rho_0^2 - \alpha_0^2)}{(\alpha_0^2 + h^4 + 2h^2)(\rho_0^2 + h^4 + 2h^2)}\right\}$$

$$+ \frac{B_\nu h(1+h^2)(\rho_0^2 + \alpha_0^2 + 2h^4 + 4h^2)}{(\alpha_0^2 + h^4 + 2h^2)(\rho_0^2 + h^4 + 2h^2)}, \qquad (25)$$

and the velocity field u is found to satisfy

$$u^2 = \frac{4(1+h^2)^2}{(\rho_0^2 + h^4 + 2h^2)} \quad a_0 ch^2, \qquad (26)$$

where $\rho_0 = r_0/a_0$.

As estimate of the effect of viscosity on plastic flow of the type specified above was obtained from equation (24). The numerical values used in the investigation were as follows in c. g. s. units: $a_0 = 5, \alpha_0 = 2$, $c = 0.1$; $D = 8.5$; and $k = 23.5 \times 981 \times 10^5$, appropriate for a thick walled brass (Zn 30 percent, Cu 70 percent) tube. The viscosity coefficient of the metal in these units would be of order 10^{10}. Two values $\lambda = 5 \times 10^{10}$ and $\lambda = 10^{11}$ were chosen, and the results compared with that for $\lambda = 0$ corresponding to zero viscosity. With the above parameters the second term in (24), the inertia term, becomes comparable with the other terms only in the final stage of yielding, and for all practical purposes is negligible. The variations of the dimensionless pressure with dimensionless time, for the three assigned values of the viscosity, are shown in Fig. 1. It is seen that the viscosity has a considerable effect during the initial stages of the yield process, and that in order to maintain the same yield with greater viscosity, the initial rate of loading must be correspondingly increased. The required pressure attains its maximum and may be allowed to decrease at a rate more nearly comparable with the rate of unloading in the nonviscous case. In each case the required pressure approaches zero asymptotically with increasing time, as might be expected in the absence of fracture, since the thickness of the tube is steadily decreasing.

5. COMPARISON OF PLASTIC, VISCOUS AND INERTIA EFFECTS

It is also of interest to compare the effects of perfect plasticity, viscosity, and inertia on the expanding tube. For this purpose an expansion

$$a = (a_0/4)\ [\ 5 + \sin(\omega t - \pi/2)] \tag{27}$$

of the inner radius of the tube is imposed during the time interval $t = 0$ to $t = \pi/\omega$ When $t = 0$, $a = a_0$ and $da/dt = 0$; when $t = (\pi/\omega)$, $a = 3a_0/2$ and $da/dt = 0$. See Fig. 2. Hence (27) corresponds to an expansion of the inner radius to one and a half times its initial value in a time $T = \pi/\omega$, starting and finishing with zero velocity. It can then easily be shown that

$$\left.\begin{aligned}
\Psi &= \frac{a_0^2}{32}\left[5 + \sin\left(\omega t - \frac{\pi}{2}\right)\right]^2 - \frac{a_0^2}{2}\ , \\
\dot{\Psi} &= \frac{a_0^2\,\omega}{32}\ \sin\ 2\left(\omega t - \frac{\pi}{2}\right) + \frac{10 a_0^2\,\omega}{32}\cos\left(\omega t - \frac{\pi}{2}\right), \\
\dot{\Psi} &= \frac{a_0^2\,\omega^2}{16}\ \cos\ 2\left(\omega t - \frac{\pi}{2}\right) - \frac{10\, a_0^2\,\omega^2}{32}\sin\left(\omega t - \frac{\pi}{2}\right).
\end{aligned}\right\} \tag{28}$$

The pressure variation needed to produce expansion (27) can be written as

$$\frac{p(t)}{2k} = P_P(t) + P_I(t) + P_V(t)\ , \tag{29}$$

where $P_P(t)$, $P_I(t)$ and $P_V(t)$ are the contributions due to perfect plasticity, inertia, and viscosity respectively, and given by

$$\left.\begin{aligned}
P_P(t) &= \frac{1}{2}\log\left[\frac{b_0^2 + 2\Psi}{a_0^2 + 2\Psi}\right] , \\
P_I(t) &= \frac{D}{4k}\left\{\ddot{\Psi}\log\left[\frac{b_0^2 + 2\Psi}{a_0^2 + 2\Psi}\right] - \frac{\left(b_0^2 - a_0^2\right)\dot{\Psi}^2}{\left(a_0^2 + 2\Psi\right)\left(b_0^2 + 2\Psi\right)}\right\} , \\
P_V(t) &= \frac{\lambda}{2}\,\frac{\left(b_0^2 - a_0^2\right)\dot{\Psi}}{\left(a_0^2 + 2\Psi\right)\left(b_0^2 + 2\Psi\right)} .
\end{aligned}\right\} \tag{30}$$

The numerical values of a_0, b_0, D and k are chosen the same as previously. The value of λ is now fixed at 5×10^{10}. By varying ω the expansion can be made to take place in any desired time. Three values of ω were chosen. These correspond to an expansion of the tube in 1 sec., 10^{-6} sec., and 10^{-7} sec. respectively. The pressure variation with time required to produce the expansion was calculated in each case from equations (28) through (30). The dimensionless pressure contributions P_P, P_I, and P_V were plotted against t in each case. When T = 1, corresponding to a slow expansion, inertia has a negligible effect on the pressure required, but the perfectly plastic and viscous contributions to this pressure are of the same order. See Fig. 3. When $T = 10^{-6}$ the required pressure is considerably increased and the perfectly plastic contribution is a negligible part, but the inertia effect begins to be apparent. See Fig. 4. For $T = 10^{-7}$ the inertia term in $p(t)/2k$ has a considerably greater effect (see Fig. 5).

It is noted that Figs. 4, 5 show that a negative pressure is required near the end of the period to produce the required expansion. By referring to equations (14), (15) and (16) it is seen that p can be replaced by a pressure $(p_i - p_e)$, where p_i and p_e are an internal pressure and an external pressure respectively. Thus, imposing a negative value of the pressure p is exactly equivalent to an application of external pressure and hence is physically possible.

The order of magnitudes involved in P_V and P_I is apparent from equations (28) and (30). When the time of expansion T is altered by some factor K , the viscous pressure contribution is altered by a factor K^{-1}, and the inertia contribution by a factor K^{-2}; the perfectly plastic pressure contribution is, of course, independent of T. For the type of problem considered, this shows an interesting comparison over the full range of expansion times of the three effects mentioned. In a slow expansion the flow approximates that of the plastic quasi-static theory in which the strain rates are very small and the inertia effects are neglected. As the speed of expansion increases the viscous effect becomes important and then dominant, whereas the inertia effect is still negligible. For even faster expansions the effect of inertia becomes important and finally predominates.

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LIST OF FIGURES

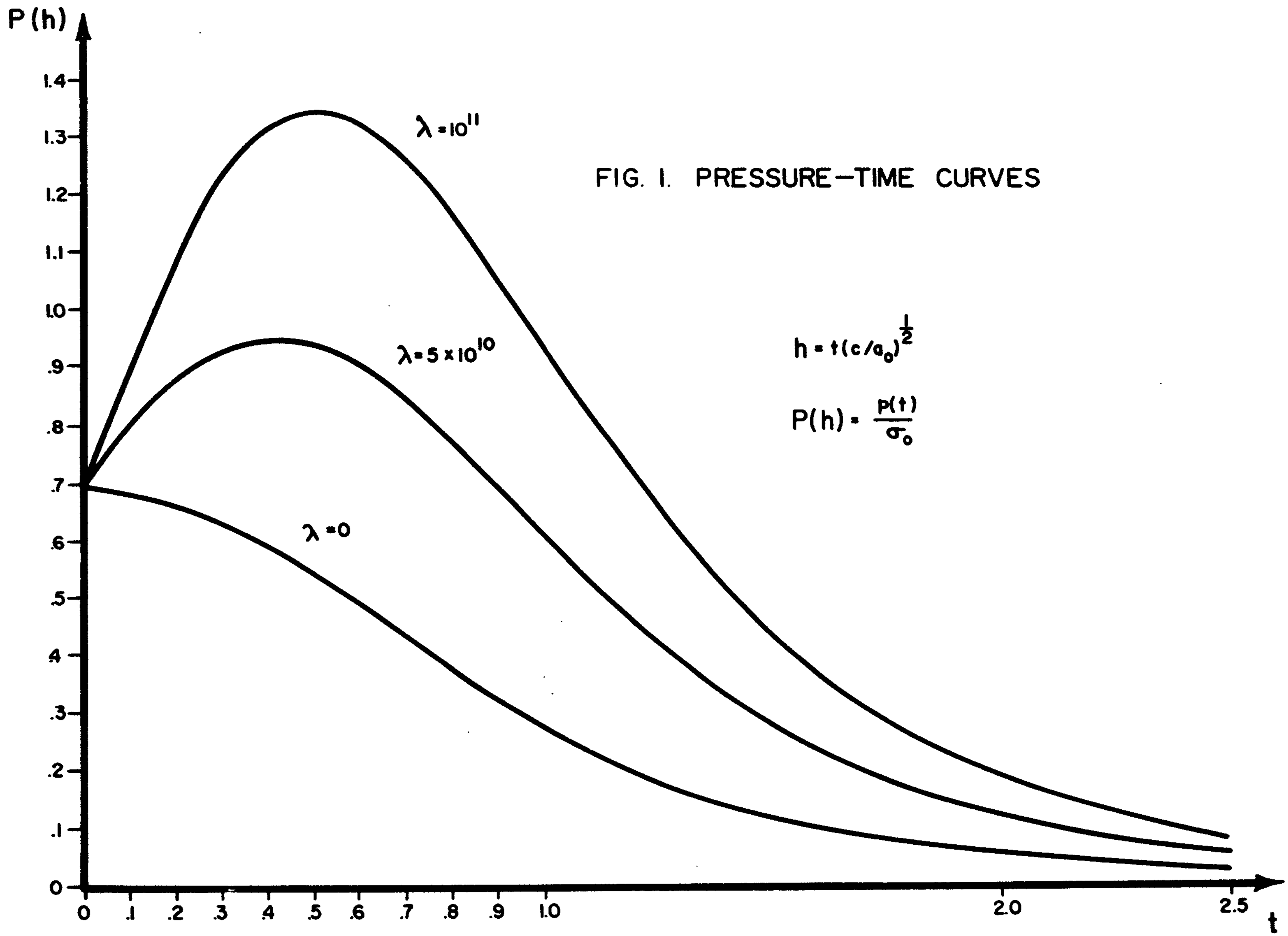

FIG. 1. PRESSURE—TIME CURVES

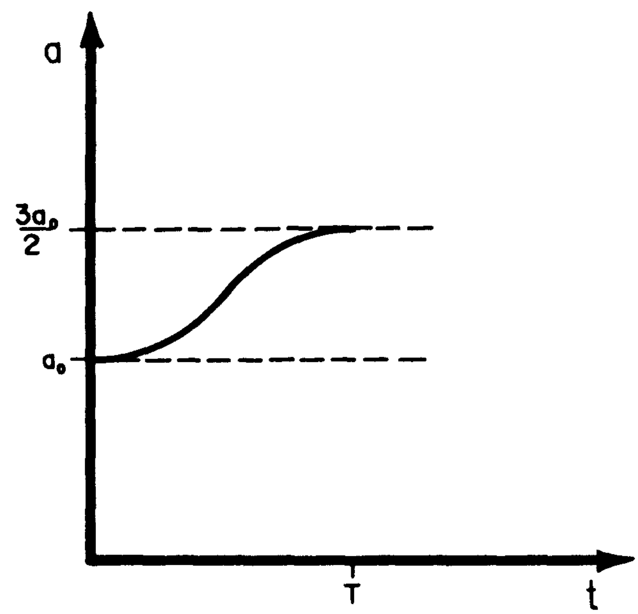

FIG. 2. EXPANSION OF INNER BOUNDARY

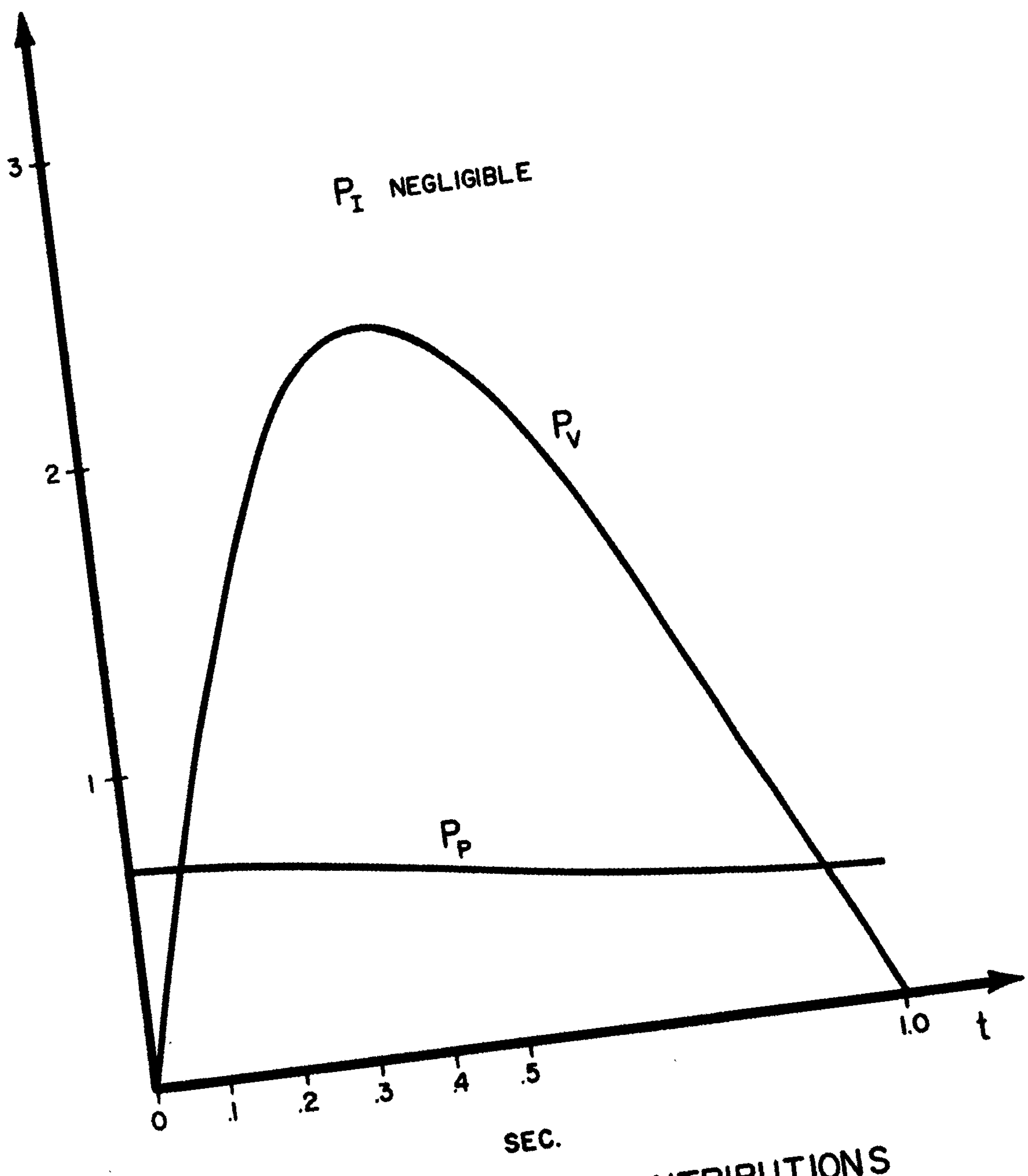

FIG. 3. PRESSURE CONTRIBUTIONS WHEN T=1 SEC.

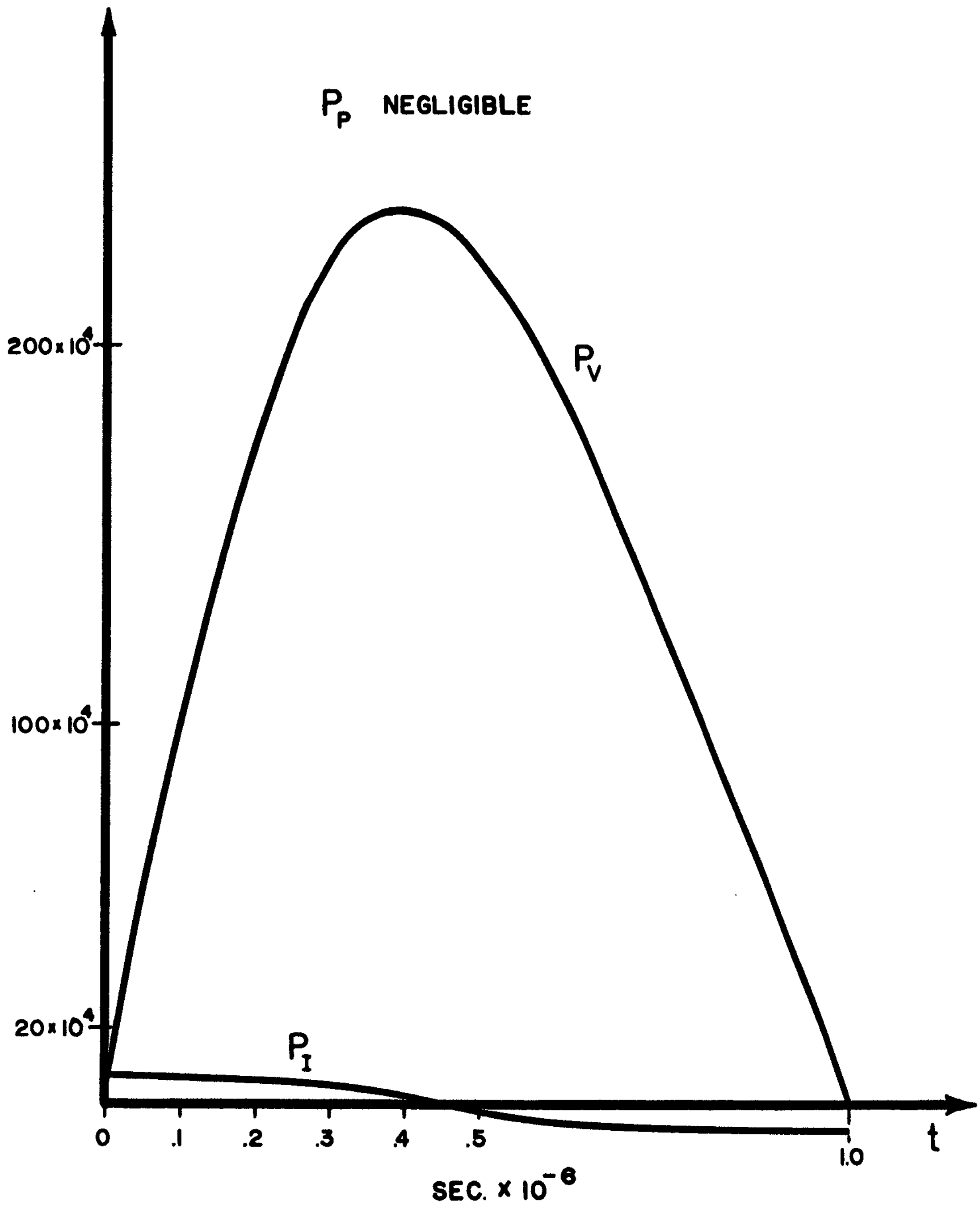

FIG. 4. PRESSURE CONTRIBUTIONS WHEN $T = 10^{-6}$ SEC.

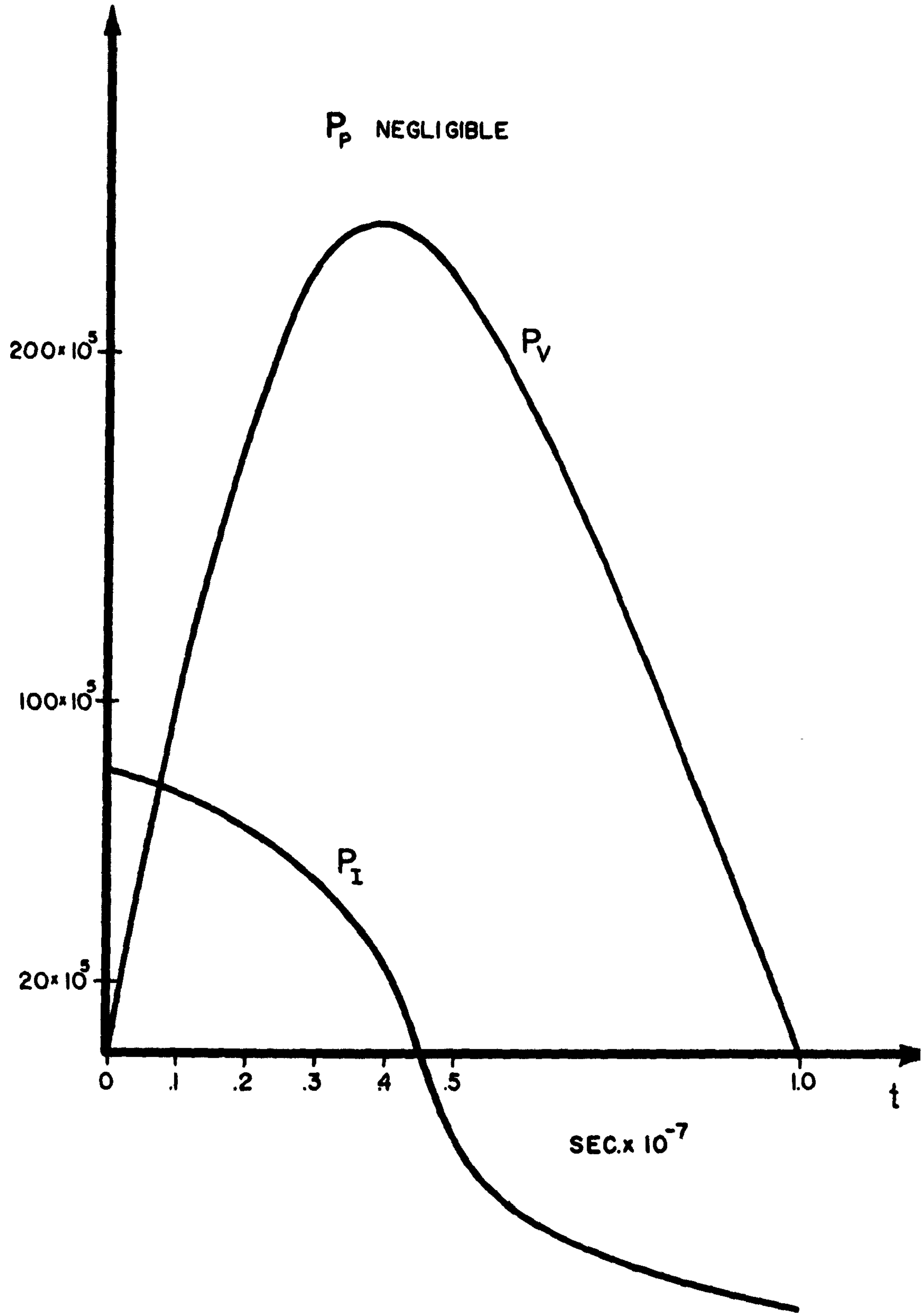

FIG. 5. PRESSURE CONTRIBUTIONS
WHEN $T = 10^{-7}$ SEC.

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