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PULSE PROPAGATION INTO WATER

By

Hans J. Schmitt

March 18, 1963

Technical Report No. 405

Cruft Laboratory
Harvard University
Cambridge, Massachusetts

④ #1.60   ⑤ 234 300

Office of Naval Research
⑮ Contract Nonr 1866(26),
⑯ Proj. NR - 371 015

⑰ - ⑱ NA ⑲ W ⑳ NA

Technical Report

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TR405   PULSE PROPAGATION INTO WATER

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ABSTRACT

The transmission of short unidirectional plane-wave pulses into water and solutions of sodium chloride has been investigated experimentally. The time sequence of the transient response for incident pulses with a duration of several nanoseconds is observed at various depths in the electrolyte. The dispersion of the pulse is mainly due to the ionic conductance of the solution. For very rapid polarizing forces the relaxation associated with the hindered rotation of the water molecules also contributes to the dispersion. Unlike the exponential attenuation of sinusoidal signals in conducting media, for short pulses the maximum response ultimately follows an inverse cube law with increasing depth. The experimental results are compared to the calculated transient response obtained from an approximate theory. The agreement is excellent.

INTRODUCTION

The peculiar dispersion of transient electromagnetic signals in conducting media has been discussed theoretically in several recent papers. Richards [1] analyzes the radiation of short unidirectional pulses by electric and magnetic dipoles immersed in a conducting medium. The asymptotic behavior of the transient response at great distances from the source excited by a Dirac impulse shows a smooth but relatively fast rise followed by a slow drop in the signal level. The maximum of the response proceeds at a slow speed. The speed, moreover, decreases with distance and, coincidentally, is equal to the sound velocity at a distance of about 1 km . The amplitude of the maximum response decreases in a manner inversely proportional to the cube of the distance, while the entire pulse is progressively smoothed out in time resulting from the increase in relative importance of lower frequency components at larger distances. This asymptotic behavior has been directly derived by Zisk [2] using a saddlepoint integration in the transfer integral. Anderson and Moore [3] have investigated the energy frequency spectrum of electromagnetic pulses in a conducting medium.

For communication purposes, for example through sea water, pulse excitation does not appear advantageous, since for any appreciable distance, i. e., more than 1 m , the energy contained in the higher frequency components of the pulse is essentially dissipated. Also the

-1-

long-time spread in the response at greater distances prevents rapid

modulation of the pulse sequence in amplitude or pulse position. Pulse

dispersion in conducting media is also important in the study of shielding

effectiveness for transient electromagnetic signals. Harrison [4]

investigated the pulse transmission through metallic plates and shielding

enclosures using a high-speed computer.

The radiation of pulses, for example under water, is difficult to

investigate experimentally since any physical antenna in itself represents

a dispersive system [5] , [6] , which has to be taken into account for a

comparison with theoretically determined responses. A simpler

situation which can be directly subjected to experimental comparison is the

transmission of normally incid⌐ⁿᵗ plane unidirectional pulses into a

conducting medium. The analogy between TEM-mode propagation and

transmission-line propagation allows the observation of the transient

response inside a coaxial line, as indeed the original analytical solution

of the propagation of transients in lossy media has been specifically

concerned with dispersion in transmission lines [7] , [8] .

It is the aim of this investigation to give a comparison of experimental

observations and analytical results for the dispersion of short pulses

propagating through an air-liquid interface to various depths in water or

aqueous electrolytic solutions of different conductivity. In particular,

the transition from a relatively well-preserved pulse shape at a small

depth in the conducting solution to the ultimate asymptotic form for large

depths is investigated. In aqueous solutions a loss mechanism and an

associated dispersion arise due to the hindered rotation of the water

molecules in addition to the ionic conductivity. Although the relaxation

time, $\tau_R \sim 0.94 \; 10^{-11}$ sec [9] is extremely short compared to the rise

time of the duration of the pulse actually used, the absorption is shown

experimentally in the smoothing out of the initial discontinuity of the

precursor signal. For solutions with negligible conductivity an approximate

analysis indicates how, in principle, the relaxation time can be measured

directly by observation of the transient response.

EXPERIMENTAL PROCEDURE

For the detailed observation of pulse dispersion over relatively

small distances repetitive pulses with durations of 20 nsec and 60 nsec

and a rise time of approximately 0.25 nsec   supplied by a mercury switch

pulse generator have been used. The pulses are led through a 20 dB

matched attenuator and a 3' section of 50 ohm cable (RG8U) to the test

line. The test line consists of an air-filled stainless steel coaxial line

with matched characteristic resistance of 50 ohms and a length of 1

meter. Mounted in a vertical position it is partially or fully filled with

the electrolytic solution through a small inlet near the bottom. The liquid

is contained by a thin Teflon membrane with lateral support sleeves, Fig. 1.

With a thickness of the membrane of 0.025" no degrading of the pulse

shape in the empty line could be observed.
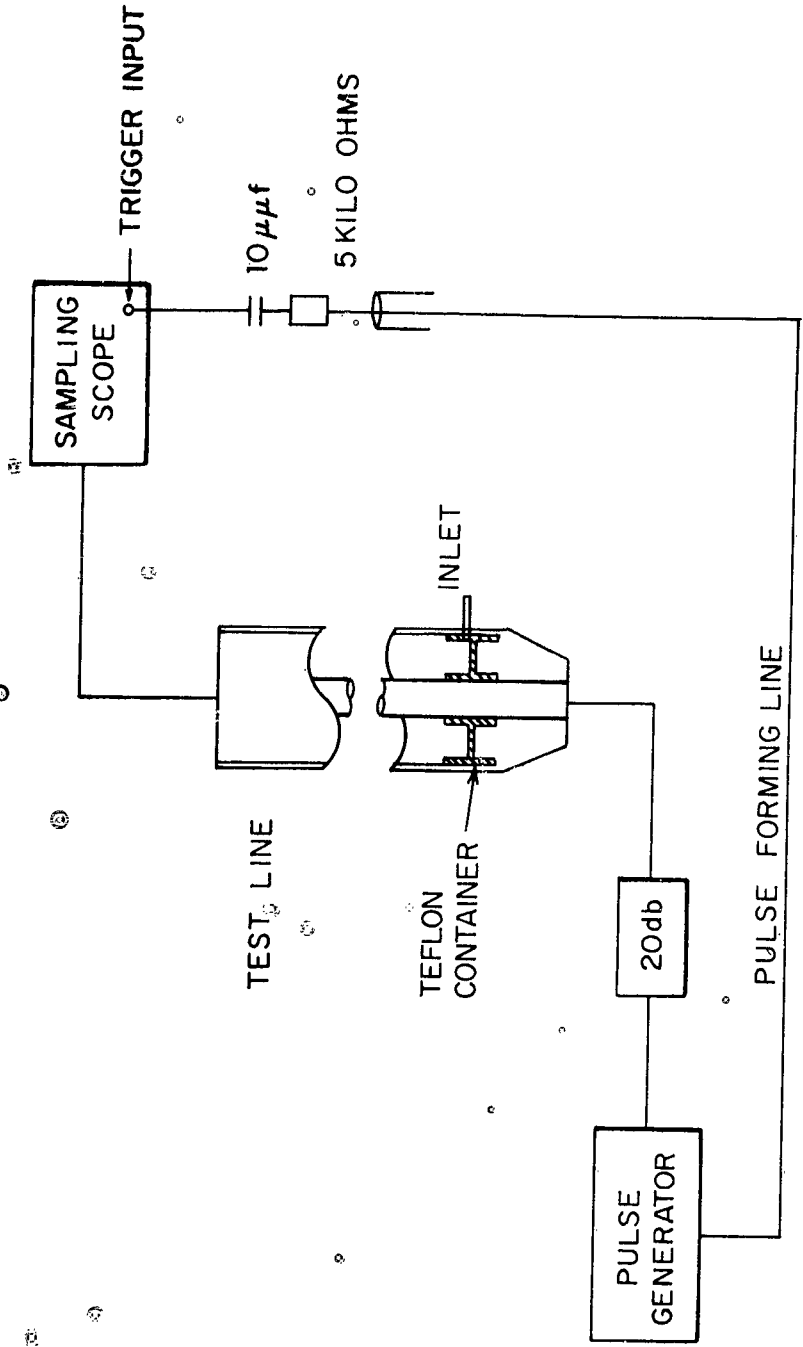
FIG. 1 EXPERIMENTAL SETUP

The test line is terminated in a reflection-free transition to a 3'

section of RG 8U cable leading directly to the input of a Tektronix 611

sampling scope. The time base of the scope is triggered from the end

of the pulse-forming section of coaxial line by connecting the inner conductor

through a resistor of 5 kiloohms in series with a capacitor of $10\,\mu\mu$F to the

trigger input. The advantage of this triggering procedure is that the initial

rise of the pulse is preserved and no unnecessary degrading junctions are

introduced into the transmission path of the signal. Reflections from the

air-water interface are absorbed without reflection in the 20 dB attenuator.

In this scheme the transmission through a slab of liquid is

measured. If the observation is restricted to the first transmitted pulse

only, the signal observed on the oscilloscope is proportional to

$$V \stackrel{\wedge}{=} T\,V\,(t,z) \tag{1}$$

where $V(t,z)$ represents the voltage at the depth z in a semi-

infinite medium and T the transmission coefficient of the interface

between the liquid and air. For a liquid of complex relative dielectric

constant $\epsilon_r = \epsilon_r' + j\epsilon_r''$ and a conductivity σ the plane wave

transmission coefficient is

$$T \;=\; \cfrac{2}{1 + \cfrac{1}{\sqrt{\epsilon_r' - j(\epsilon_r'' + \dfrac{\sigma}{\omega\,\epsilon_o})}}} \;. \tag{2}$$

In the frequency range relevant to the experiment, the real part of

the dielectric constant of water is $\epsilon_r' = 78.2$. The loss term varies

with frequency. Hence, T lies between 1.8 and 2.0 . The additional dispersion in this transmission is negligible and for the correlation of the observed response to the results calculated for a semi-infinite liquid medium a value of 1.8 has been assumed throughout. The elimination of multiple reflections is automatically achieved for solutions with sufficiently high conductivity in view of the relatively large absorption in the conducting medium. For extremely low conductivity the dispersion is small enough so that for the chosen pulse length the secondary reflections are separated in time.

Electrolytic solutions were prepared by dissolving sodium chloride in distilled water. All measurements were taken at room temperature (25° C). The conductivity values, σ , are calculated from tabulated values of loss tangents measured at 10^5 c/sec [10] . The following values are considered

distilled water	$\sigma \sim 10^{-4}$ [mho]
0.01 molar	$\sim$ 0.1
0.03 "	.315
0.1 "	1.0
0.3 "	2.75
0.5 "	4.33

Sea water corresponds approximately to a 0.5 molar solution of sodium chloride.

EXPERIMENTAL RESULTS AND DISCUSSION.

The original unidirectional pulse with a duration of 20 nsec after

transmission through the empty test line is shown in Fig. 2. The rise time

observed is essentially that of the oscilloscope, approximately 0.35 nsec.

Negligible ringing is observed at the leading edge. It results from internal

reflections in the pulse generator and coaxial junctions. The trailing edge for

both the pulse shown and the longer pulse (60 nsec) reveals a small rounding-off,

similar to the effect which arises if a small capacitance is connected in parallel

to a transmission line. Such capacitive effects occur due to junctions of coaxial

systems with different geometry and the behavior of electromagnetic fields in

the junction zone. [11].

(1) Relaxation Dispersion

For the test line filled with distilled water to three different levels,

the transient response is shown in Fig. 3. In distilled water the

absorption over distances considered here, i.e., up to about 1 meter, is

negligible. The sequence of pulses arising from multiple reflections in

the water slab is clearly shown in the exposure with the longer time scale.

The exposures with the shorter time scale of 1 nsec per major division

show the leading edge of the first pulse transmitted only. While for

a non-conducting non-dispersive medium the rise time of the incident

FIG. 2 TRANSMISSION OF 20 nsec PULSE THROUGH EMPTY TEST LINE.

1 $\frac{nsec}{div}$ AND 20 $\frac{nsec}{div}$
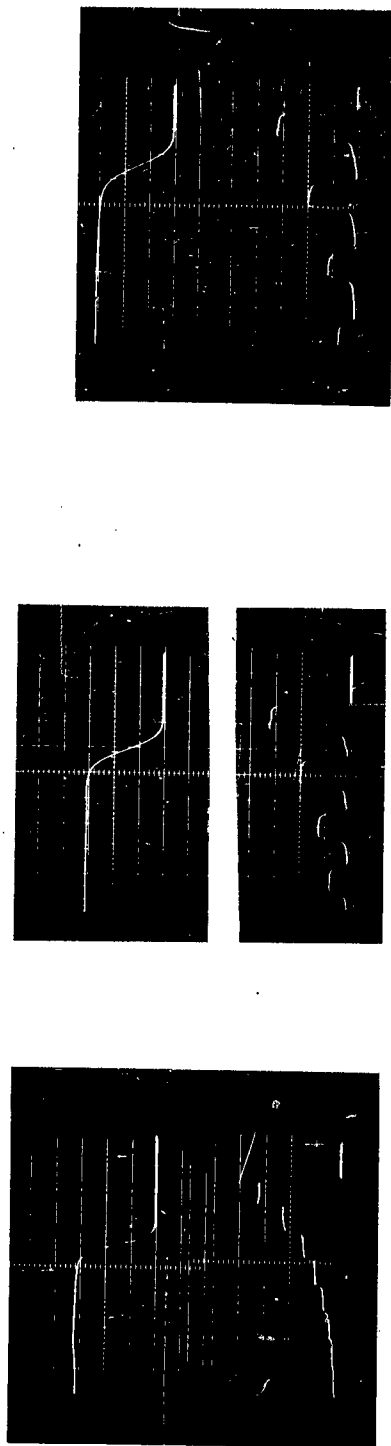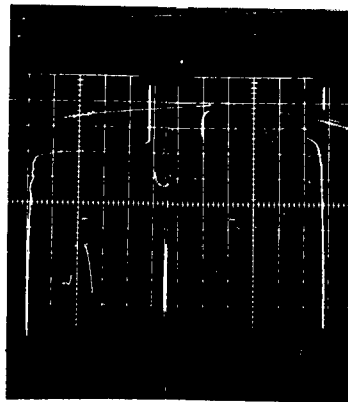
(a)

(b)

(c)

FIG. 3 TRANSMISSION OF 20 nsec PULSE INTO DISTILLED WATER. INPUT VOLTAGE 0.48 V, TIME SCALE 1 $\frac{nsec}{div}$ AND 20 $\frac{nsec}{div}$, SENSITIVITY 50 m $\frac{V}{div}$

(a) z = 0.35 m, (b) z = 0.7 m, (c) z = 0.918 m

pulse should be preserved, it is seen that the leading edge is smoothed out progressively with increasing distance. This effect arises from the relaxation dispersion associated with the hindered rotation of the water molecules. For a simple relaxation effect, the complex dielectric constant of a non-conducting medium is represented by [12]

$$\epsilon_r = \epsilon_\infty' + \frac{S}{1 - j\omega\tau_R} \tag{3}$$

where $S = \epsilon_S' - \epsilon_\infty'$ defines the total decrease in the dielectric constant from ϵ_S' measured statically and ϵ_∞' measured far beyond the relaxation frequency. The response observed at some depth z below the surface at $z = 0$ to a Dirac pulse incident from air, (velocity of light c) is

$$V(t, z) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-j\omega t + j \frac{\omega}{c} z \sqrt{\epsilon_\infty' + S/(1 - j\omega\tau_R)}}}{1 + \sqrt{\epsilon_\infty' + S/(1 - j\omega\tau_R)}} d\omega \tag{4}$$

Since the actual pulse has a finite rise time, the integration should be extended only up to frequencies of approximately 1000 Mc/sec., the limit of the oscilloscope. Hence, $\omega\tau_R \ll 1$ and the integral may be simplified by neglecting the dispersive term in the denominator, while retaining it in first order in the exponent,

$$V(t, z) = \frac{1}{\pi [1 + \sqrt{\epsilon_S'}]} \int_{-\infty}^{+\infty} e^{-j\omega\tau - \frac{\omega^2}{4\lambda}} d\omega = \frac{2}{1 + \sqrt{\epsilon_S'}} \sqrt{\frac{\lambda}{\pi}} e^{-\lambda\tau^2}$$

$$\tag{5}$$

where $\tau = t - \dfrac{z}{c}\sqrt{\epsilon_S'}$ is the delay in a non-dispersive medium, and

$$\lambda = \frac{1}{2} \frac{c\sqrt{\epsilon_S'}}{z S \tau_R}.$$

The response to a Dirac pulse is proportional to the time derivative of the response to a step function. If the incident signal is given by

$$V^i(t, 0) = \begin{cases} 0 & t < 0 \\ V_o & t > 0 \end{cases}$$

or by the leading part of a sufficiently long pulse, the response is

$$V_S(t, z) = \frac{2 V_o \sqrt{\dfrac{\lambda}{\pi}}}{1 + \sqrt{\epsilon_S'}} \int_{f(\lambda)}^{\tau} e^{-\lambda \tau'^2} d\tau' \tag{6}$$

The lower limit can be determined by inspection of the behavior at large times. The integration results in an error function

$$V_S(t, z) = \frac{1}{1 + \sqrt{\epsilon_S'}} (\phi(\sqrt{\lambda}\tau) + 1) \tag{7}$$

It is seen in Fig. 3 that the response to a step indeed resembles the behavior as given by Eq. 7. The steepest rise for step excitation occurs in this approximation at $\tau = 0$ and is numerically given by Eq. 5 after multiplication with the amplitude factor V_o. The gradient at $\tau = 0$ decreases in a manner proportional to the square root of the depth

of observation.   The evaluation of the rise times observed experimentally

gives an average value of the relaxation time $\tau_R \sim 1.24 \; 10^{-11}$ sec ,

about 30 % larger than the published value.   For an accurate determination,

the integral 4 has to be solved.   Although interesting for a discussion

of transient propagation in media showing relaxation dispersion, it is not

attempted here.

It should be noted that in all later results on the transient response

in conducting liquids, the initial precursor is not a discontinuous step but

shows, in principle, a smooth transition similar to that found for distilled water.

(2) Conductivity Dispersion

The experimental results for pulse dispersion in salt water are

reproduced in Figs. 4-8 :   For the lowest concentration, a 0.01 molar

solution of NaCl with $\sigma \sim 0.1$ mho, Fig. 4, only the response to a 20 nsec

pulse is shown, since the response to a longer pulse would be perturbed due

to insufficient attentuation between multiple reflections over shorter distances.

Indeed, the effect of first multiply-reflected pulse is still slightly visible in

the observation of the 60 nsec pulse for the next higher concentration ($\sigma = 0.315$)

over the smallest depth measured, Fig. 6a .   All other responses represent

essentially the directly transmitted pulse only.
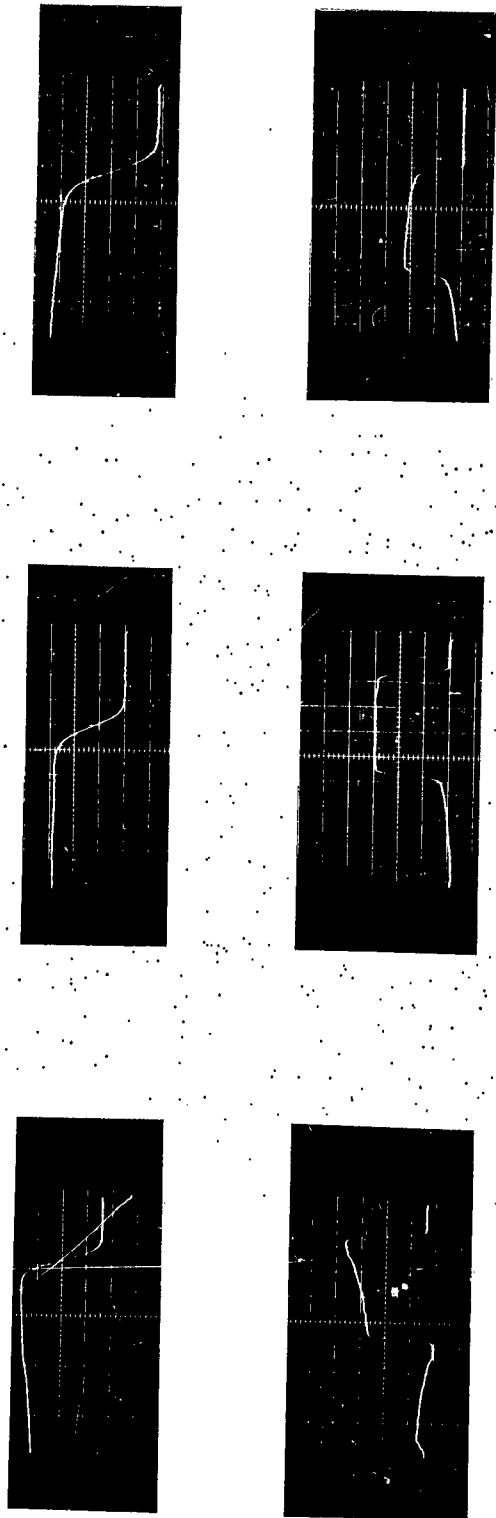
(a)
                   (b)
                   (c)

FIG. 4   TRANSMISSION OF 20 nsec PULSE INTO 0.01 MOLAR NaCl SOLUTION. INPUT VOLTAGE 0.48 V, TIME SCALE 1 nsec/div AND 5nsec/div

(a) $z = 0.35$ m   (b) $z = 0.7$ m.   (c) $z = 0.918$ m
    SENSITIVITY   SENSITIVITY   SENSITIVITY
       20m V/div   10m V/div   5m V/div AND 10m V/div
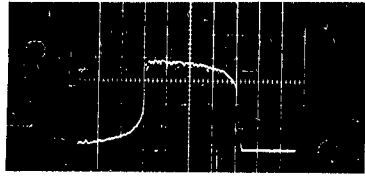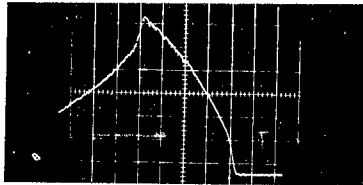
(a)
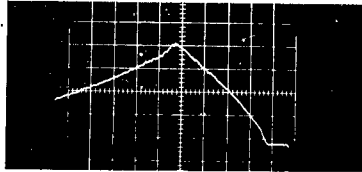
(b)
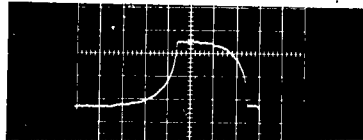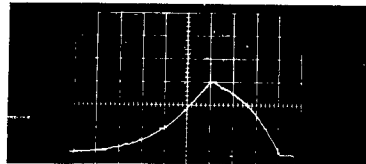
(c)

FIG. 5 TRANSMISSION OF 20 nsec PULSE INTO 0.03 MOLAR NaCl SOLUTION. INPUT VOLTAGE 4.8 V

(a) $z = 0.35m$, 5 $nsec/div$ 50 mV/div

(b) $z = 0.7m$, " , 10 mV/div

(c) $z = 0.918m$, " , "

(d) $z = $ " , 20 $nsec/div$, "
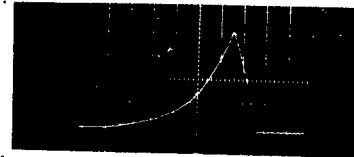
(d)

FIG. 6 SAME AS FIG. 5, PULSE DURATION 60 nsec

(a) $z = 0.35m$, 20 $nsec/div$, 50 mV/div

(b) $z = 0.525m$; " , "

(c) $z = 0.7m$, " , 20 mV/div

(d) $z = 0.918m$, " , "

(a)

(b)

(c)

(d)
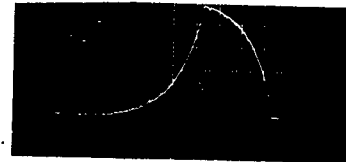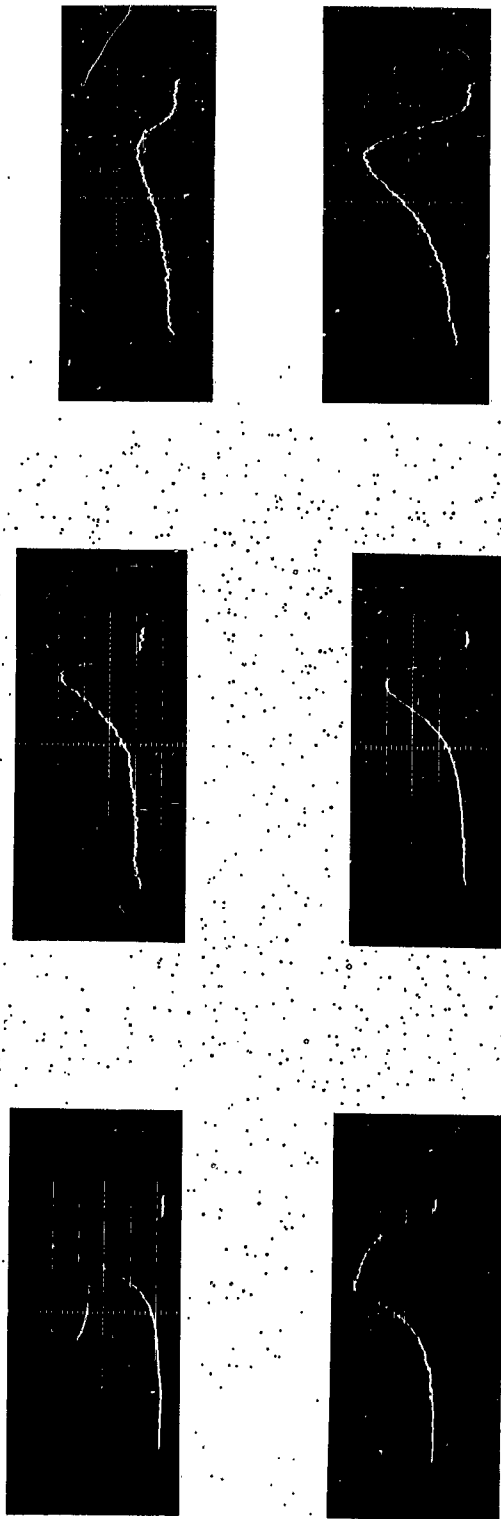
FIG. 7 TRANSMISSION OF PULSES INTO 0.1 MOLAR NaCl SOLUTION. INPUT VOLTAGE 4.8 V

	z	TIME SCALE	20 nsec PULSE (UPPER)	60 nsec PULSE (LOWER)
			SENSITIVITY	
(a)	0.35m	20 nsec/div.	10 mV/div	20 mV/div
(b)	0.70m	50 nsec/div	2 mV/div	5 mV/div
(c)	0.918m	50 nsec/div	2 mV/div	2 mV/div
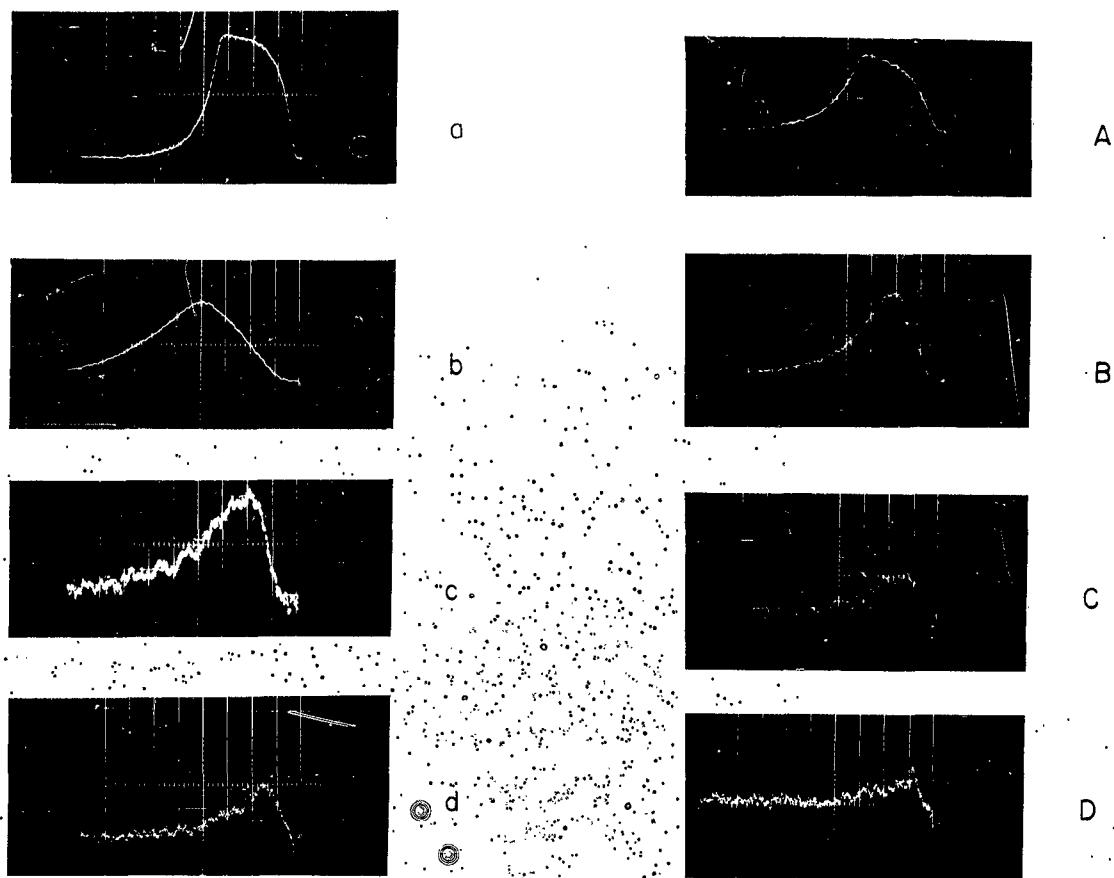
FIG. 8 TRANSMISSION OF 60 nsec. PULSES INTO ELECTROLYTES. INPUT VOLTAGE 4.8 V

KEY a – d, 0.3 MOLAR NaCl SOLUTION
A – D, 0.5 MOLAR NaCl SOLUTION

	z [m]	TIME SCALE $[\text{nsec}/\text{div}]$	SENSITIVITY $[\text{mV}/\text{div}]$
a	0.175	20	10
A	0.175	20	10
b	0.35	20	5
B	0.35	50	2
c	0.7	100	0.6
C	0.7	200	0.6
d	0.918	200	0.6
D	0.918	500	0.6

A qualitative appraisal of the different pulse responses observed

shows that for low conductivity and a small depth of observation the

initial response is a rapid rise and thereafter a slow decrease until the

pulse ends, Fig. 4a. (Time increases to the left in the figures). As the

depth of observation is increased, the top of the pulse flattens, Fig. 4b ,

and with further increases in depth ultimately starts rising as in Fig. 4c.

For a larger conductivity, Fig. 5, $\sigma = 0.315$, the same transition takes

place at a much reduced depth of observation. For the same depth as in

Fig. 4a, it is seen in Fig. 5a, that the pulse top already starts rising. If,

for the higher conductivity, the point of observation is moved deeper into

the electrolyte the initial sharp step decreases in amplitude, Fig. 5b and

5c , and also Fig. 6a-d for the longer test pulse. After the termination of

the pulse a slow drop in voltage is observed.

With increasing conductivity, Fig. 7 and Fig. 8 , this initial

transition occurs at even shorter distances; in a depth of a few centimeters.

In the range of depth considered in the experiment, the transient response

no longer resembles the original pulse. With increasing distance, e. g.

for a depth of more than 50 cm in sea water, an asymptotic pulse shape

is observed which shows an initial smooth rise and a slightly slower decay

of the signal level in time. With increasing distance and increasing

conductivity the entire pulse is progressively spread out in time. In sea

water, for example, a 60 nsec pulse is spread to about 1.5 μ sec

in a distance of roughly 1 meter.

A point of recent discussion Eqs. 1 - 3 has been the fact that the maximum amplitude of the transient response to short unidirectional pulses decreases only as $1/z^3$ as compared to the much more rapid exponential attenuation of continuous wave signals. While, of course, each Fourier component of the original pulse spectrum is exponentially attenuated, the slower damping of the transient response is a result of dispersion and the increasing importance of lower frequency components at large distances.

In the case of a plane-wave transition into the liquid, the transient response at z for an incident step function signal of amplitude V_o is

$$V_S(z, t) = \frac{2j\, V_o}{2\pi} \int_{-j\infty}^{+j\infty} \frac{e^{pt - p\frac{z}{c}\sqrt{\epsilon_r' + \frac{\sigma}{p\epsilon_o}}}}{p\left[1 + \sqrt{\epsilon_r' + \frac{\sigma}{p\epsilon_o}}\right]}\, dp \quad . \tag{8}$$

Relaxation losses have been neglected in Eq. 8. In view of the large value of $\epsilon_r' = 78.2$ the first term in the bracket in the denominator may be neglected too. With the substitution of $p = x - \dfrac{\sigma}{2\epsilon_o\epsilon_r'}$ it follows from Eq. 8 that

$$V_S(z, t) = \frac{-2j\, V_o}{2\pi\sqrt{\epsilon_r'}}\, e^{-\frac{\sigma}{2\epsilon_o\epsilon_r'}t} \int_{\frac{\sigma}{2\epsilon_o\epsilon_r'} - j\infty}^{\frac{\sigma}{2\epsilon_o\epsilon_r'} + j\infty} \frac{e^{xt - \frac{z}{c}\sqrt{\epsilon_r'}\sqrt{x^2 - (\frac{\sigma}{2\epsilon_o\epsilon_r'})^2}}}{\sqrt{x^2 - (\frac{\sigma}{2\epsilon_o\epsilon_r'})^2}}\, dx \quad .$$

$$\tag{9}$$

This integral is listed in $[1.3]$. The approximate transient response to an incident step function signal at a depth z is

$$
V_S(z, t) \begin{cases} \dfrac{2 V_o}{\sqrt{\epsilon_r'}}\, e^{-\frac{\sigma}{2\epsilon_o \epsilon_r'} t}\, J_o\left(j\frac{\sigma}{2\epsilon_o \epsilon_r'}\sqrt{t^2 - \left(\frac{z}{c}\sqrt{\epsilon_r'}\right)^2}\right) & t > \frac{z}{c}\sqrt{\epsilon_r'} \\[3mm] 0 & t < \frac{z}{c}\sqrt{\epsilon_r'} \end{cases}
$$

$$(10)$$

The transient response calculated from Eq. 10 is plotted in Fig. 9 as a function of the depth of observation for the three highest conductivities used in the experiment. The transient response of a pulse of duration T_o sec. follows from Eq. 10 in view of the linearity of the system and causality:

$$
V_p(z, t) = V_S(z, t) \qquad\qquad t < \frac{z}{c}\sqrt{\epsilon_r'} + T_p
$$

$$
= V_S(z, t) - V_S(z, t\text{-}T) \qquad t > \frac{z}{c}\sqrt{\epsilon_r'} + T_p \qquad (11)
$$

The time sequence of the transient signal calculated from Eqs. 10 and 11 is plotted in Fig. 10 and 11 for $\sigma = 0.315$ mho and $\sigma = 1$ mho. It is clearly seen that the calculated response agrees precisely with the observed wave shapes. A small deviation between the slope of the pulse top for the long exciting pulse in the case of $\sigma = 0.315$ mho and $z = 0.35$ meter is explained by the fact, that here in the experimental observation, the secondary reflection arrives before the direct pulse has ended. This reflection essentially adds a small component to the later part in the direct pulse response.
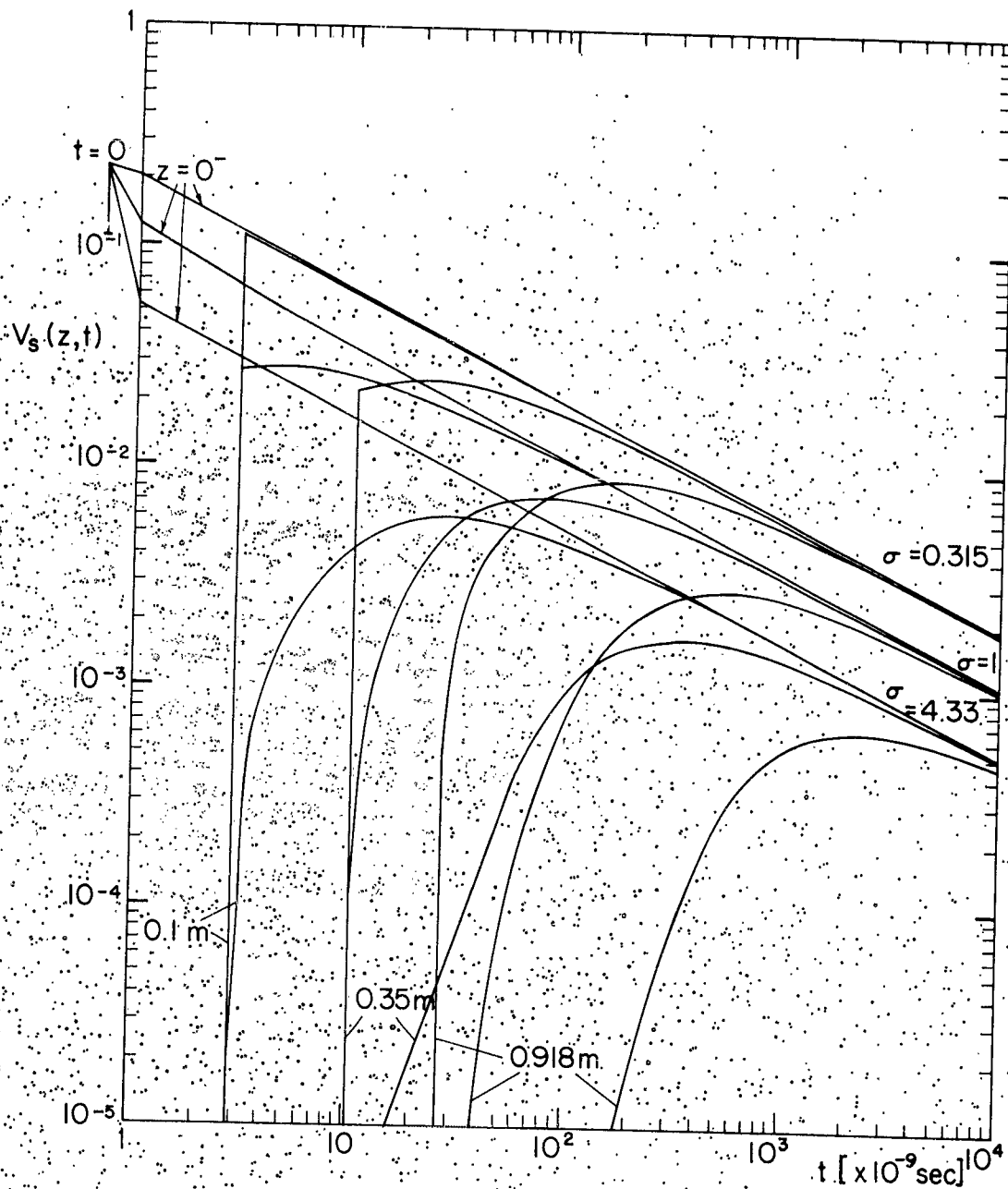
FIG. 9 DISPERSION OF PLANE ELECTROMAGNETIC WAVE WITH
STEP FUNCTION TIME DEPENDENCE IN CONDUCTING MEDIA.
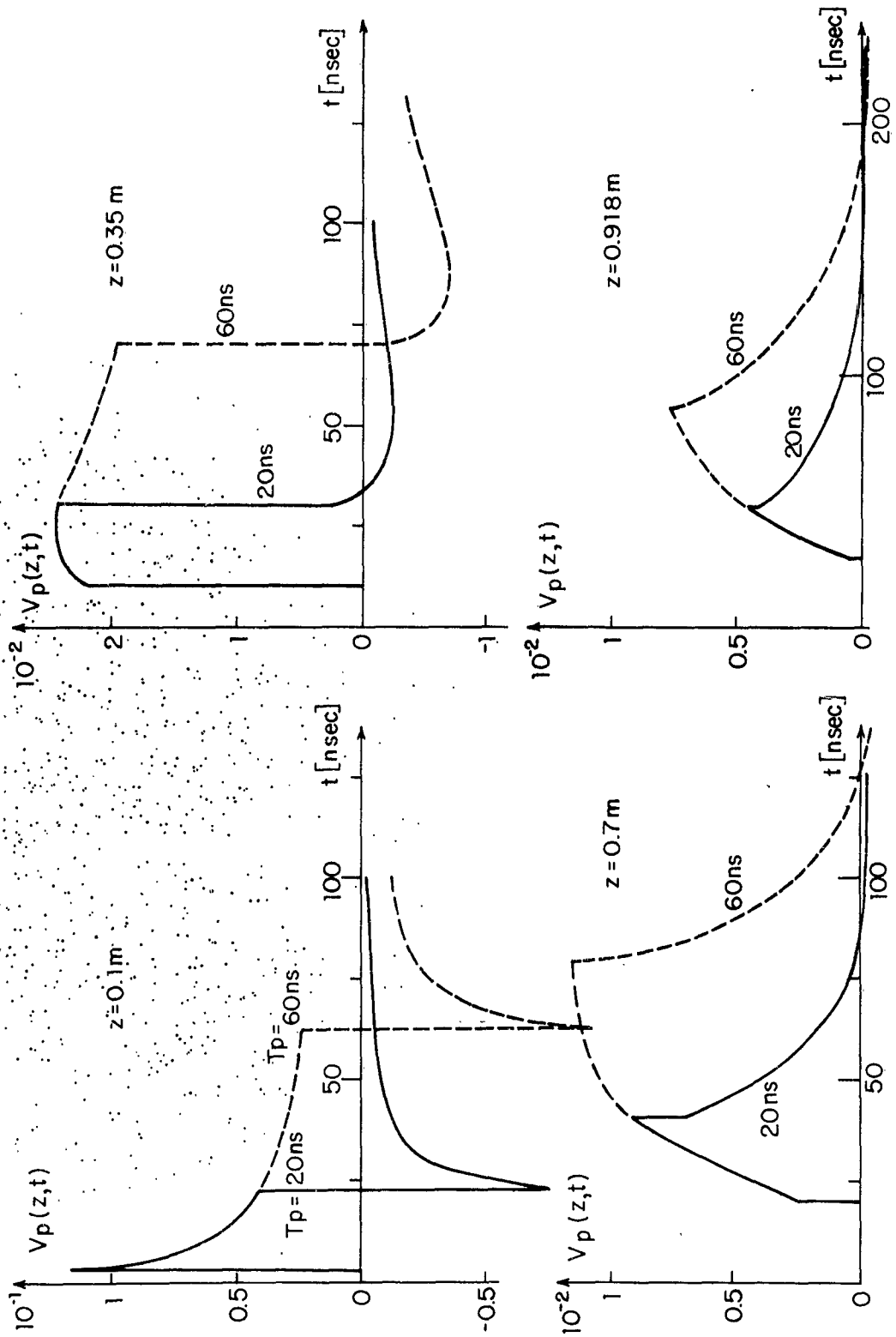
FIG. 10 DISPERSION OF PLANE UNIDIRECTIONAL PULSE IN CONDUCTING MEDIUM. CONDUCTIVITY
σ = 0.315 mho (0.03 MOLAR SOLUTION NaCℓ)
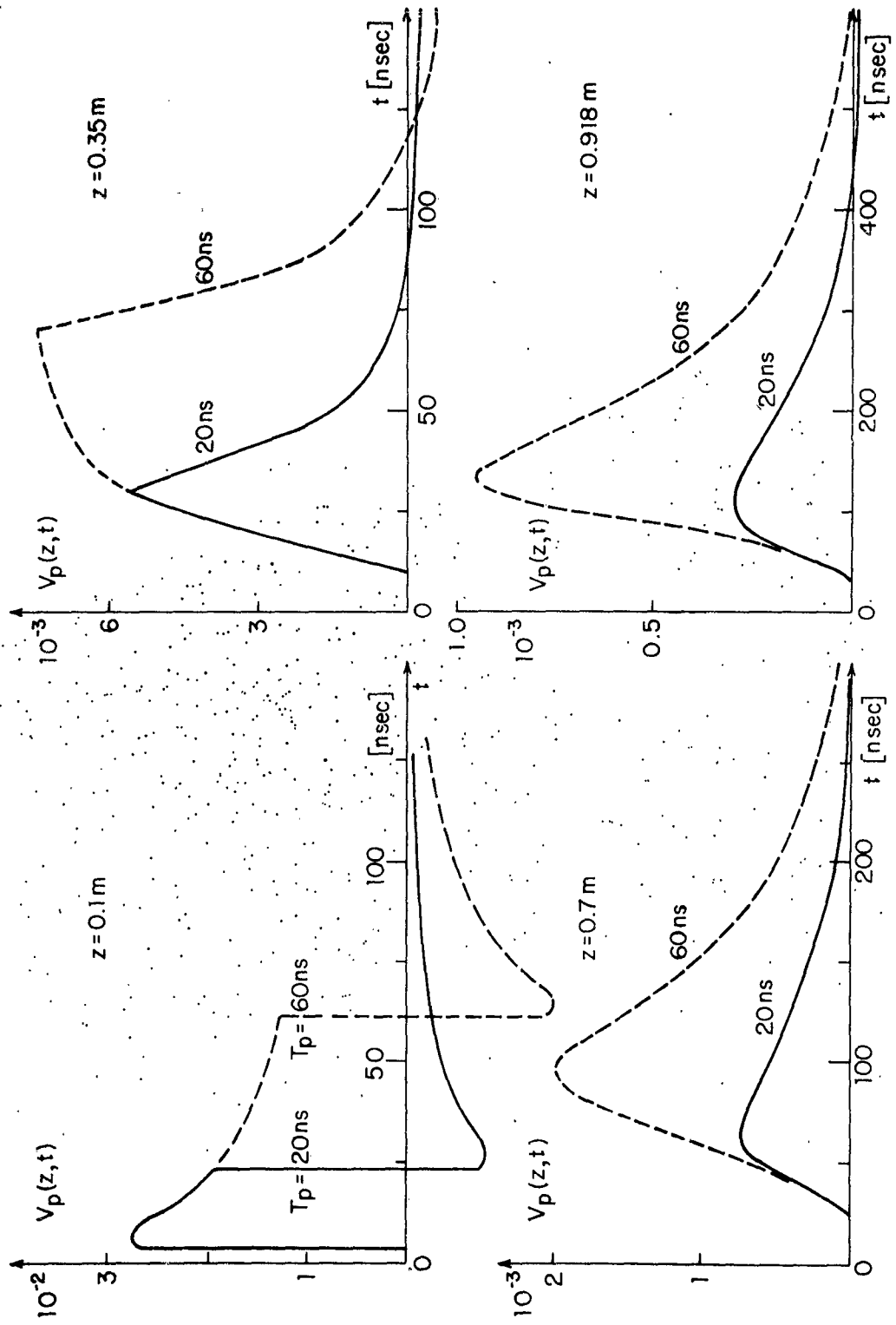
FIG. 11 DISPERSION OF PLANE UNIDIRECTIONAL PULSE IN CONDUCTING MEDIUM CONDUCTIVITY

$\sigma = 1$ mho (0.1 MOLAR NaCℓ SOLUTION)

For greater depths or larger conductivities the shape of the transient response due to an incident step function signal is given by the asymptotic form of Eq. 10

$$V_S(z, t) \sim \frac{2}{z} \frac{e^{-\frac{\sigma}{4c^2 \epsilon_o} \frac{z^2}{t}}}{\sqrt{\frac{\pi \sigma t}{\epsilon_o z^2}}} \tag{12}$$

The signal ultimately decreases in a manner inversely proportional to distance. For an incident pulse with a duration $T_p \ll$ rise time of the step function response, the corresponding approximation is

$$V_p(z, t) \sim \frac{2T_p}{z^3 \sigma^2 \sqrt{\frac{\pi}{\epsilon_o}}} \frac{e^{-\frac{\sigma}{4c^2\epsilon_o} \frac{z^2}{t}} \left(\frac{\sigma}{4c^2\epsilon_o} \frac{z^2}{t} - \frac{1}{2} \right)}{\left(\frac{t}{\sigma z^2} \right)^{\frac{3}{2}}} \tag{13}$$

Asymptotically, the signal decreases in a manner proportional to z^{-3}

$$z^3 V_p(z, t) \sim \frac{T}{\sigma^2} f\left(\frac{t}{\sigma z^2} \right) .$$

This response is shown in Fig. 12 and is a good approximation of the experimental result for $\sigma = 4.33$ mho ($\sim$ sea water). It is noteworthy that for pulses the response changes polarity and approaches the steady state from the negative side. A comparison of calculated maxima of the transient response and measured values for both the short pulse of
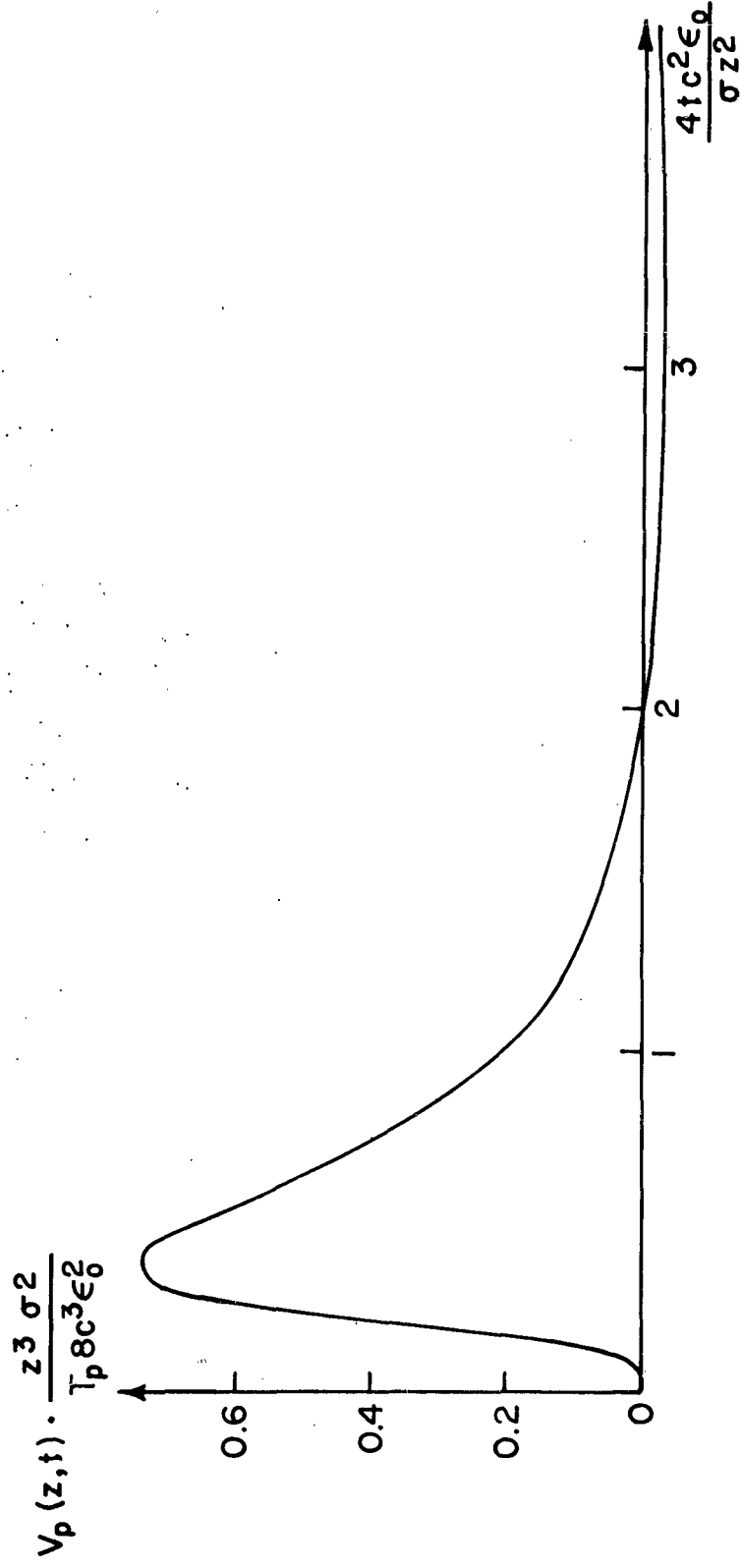
FIG. 12 DISPERSION OF PLANE UNIDIRECTIONAL PULSE IN CONDUCTING MEDIUM. ASYMPTOTIC FORM FOR HIGH CONDUCTIVITY AND LARGE DEPTH OF OBSERVATION.
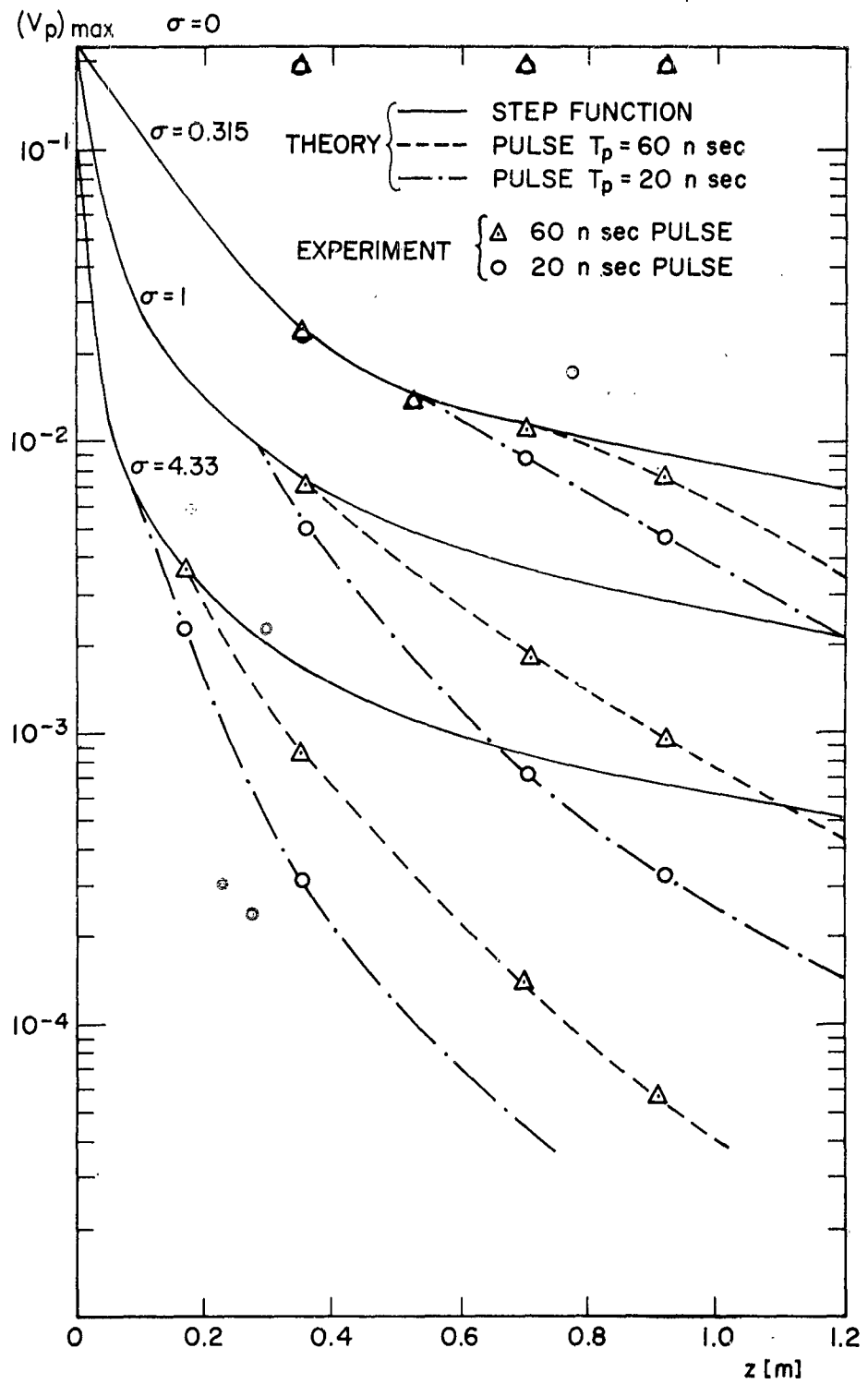
FIG. 13 MAXIMUM RESPONSE OF TRANSIENT PULSES
PROPAGATING INTO CONDUCTING MEDIUM

20 nsec and the longer pulse of 60 nsec is shown in Fig. 13 . The agreement
is excellent.

CONCLUSIONS

The experimental observation of pulse propagation into conducting media
and the good agreement obtained with an approximate theory substantiates the
arguments previously made against the feasibility of pulse communication
under water. The inherent dispersion and the slow speed of propagation are
disadvantages in the rapid transfer of information, and energy contained in the
higher frequency components is wasted for communication over greater distances.

The results obtained are also directly applicable to problems arising
in shielding against radio interferences. The expression 10 , if multiplied
by a factor two, gives directly the electric field strength behind a conducting
wall illuminated by a plane wave with a unidirectional step in electric field
strength. The same applies to Eq. 11 for an incident pulse and the corresponding
approximations 12 and 13 . The condition to be imposed is that multiple
reflections should be negligible, or they may be taken into account individually
and added to form the response for a finite slab.

The agreement between experimentally observed pulse dispersion due
to relaxation processes with the approximate theoretical result is not entirely
satisfactory. It should be possible to extend this analysis, particularly with
respect to relaxation processes that have critical times comparable to the
pulse duration and the pulse rise time.

ACKNOWLEDGMENTS

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