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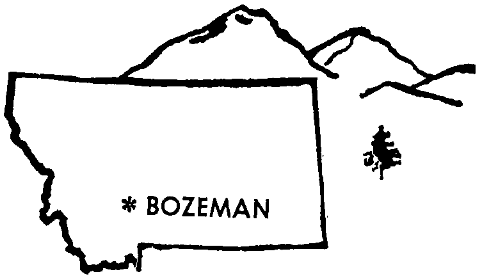

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# On Dynamic Switching in One-Dimensional Iterative Logic Networks

BY W. L. KILMER

March, 1963

Project 5632 Task 563202

ELECTRONICS RESEARCH LABORATORY
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Montana State College
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# On Dynamic Switching in One-Dimensional Iterative Logic Networks*

William L. Kilmer

Research Laboratory of Electronics
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## ABSTRACT

A SITN is a cascade of identical finite automata such that the $i^{th}$ automaton receives an $x_i$ input from the outside world and a $y_i$ input from its left neighbor, and produces a $z_i$ output to the outside world and a $y_{i+1}$ output to its right neighbor. We prove three main theorems: 1) For every integer k there is a cell definition such that a corresponding SITN either can or cannot switch from equilibrium to a cycling condition following a single $x_i$ change according as $n \leqslant k$ or $n > k$, respectively; 2) there do not exist algorithms to tell whether or not a given cell definition admits of a SITN that can start from equilibrium and following a single $x_i$ change either a) switch into a cycling condition, or b) put out a $z_i = 1$ during a switching transient; and 3) there do not exist algorithms to tell whether or not a given SITN cell definition must have every switching transient following a single $x_i$ change from equilibrium either a) die out a bounded number of cells to the right of the change, or b) extend all the way to the SITN boundary. All theorems are proved constructively on finite-state diagrams, and 2) and 3) hinge on an embedding of Minsky's Post Tag system results into such diagrams. We conclude with several iterative network equivalence demonstrations.

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## I. Introduction

We consider a concatenation of n identical logic cells as shown in Fig. 1. The $i^{th}$ cell has associated with it the following: 1) a memory state variable $s_i$, with domain of values $a_1, a_2, \ldots a_m$; 2) an external (i.e., outside world) input variable $x_i$, with domain $b_1, b_2, \ldots b_p$; 3) lateral input and output variables $y_i$ and $y_{i+1}$ respectively, each with domain $c_1, c_2, \ldots c_q$; and 4) an external output variable $z_i$, with domain $d_1, d_2, \ldots d_r$. We assume that the functions $z_i(x_i, y_i, s_i)$ and $y_{i+1}(x_i, y_i, s_i)$ are realized with zero time delay across the cells; and that the function $s_i(x_i, y_i, s_i)$ is realized with unit time delay within the cell. At time $t = 0$ the $y_1$, $x_i$, $s_i$ variables are all assigned arbitrary values from their respective domains, and for all $t > 0$ the values of $y_1$ and all the $x_i$ remain fixed. We denote such cell systems *SITNs* (for Sequential ITerative Networks).

A SITN is said to be in *equilibrium* at $t > 0$ if and only if all of its $y_i$, $s_i$ values remain fixed from t on. A SITN is said to be *cycling* at $t > 0$ if and only if its over-all configuration of $y_i$, $s_i$ values at t first recurs at $t + T$, for $T > 1$. If a SITN is in equilibrium at $t = -1$, and at $t = 0$ some $y_1$ and/or $x_i$ values change, the corresponding sequence of $y_i$, $s_i$ value changes is called a *transient* just in case the SITN reaches equilibrium at some $t > 0$. Otherwise the SITN enters a cycle.

The main purpose of this paper is to present some results on the problem of determining, for an arbitrary SITN cell definition, the various ways in which corresponding SITNs can switch from equilibrium to equilibrium, and equilibrium to cycle, following single $x_i$ value changes. We also apply these results to non-SITN models discussed in Kilmer (1961), (1962A), (1962B) in order to extend the present theory of switching dynamics for iterative systems.

We claim that the results of this paper furnish new insights into the classical long-range order problems[1] of statistical mechanics, sociology, nonlinear control theory, neurophysiology,[2] and genetics.

[1] Those problems that involve the derivation of long-range order patterns from short-range order relations.

2. In particular, the operational problem of the reticular formation in vertebrate nervous systems. This formation is always the central command and control center in such systems.

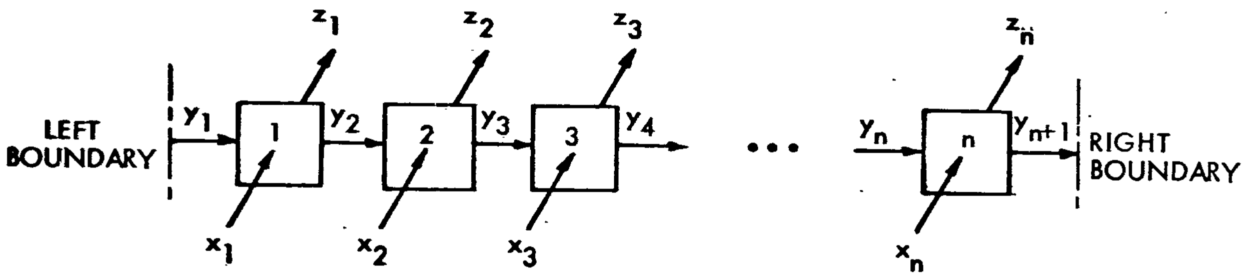


FIG. 1. SCHEMATIC DIAGRAM OF A SITN.

## II. On the Relationship of SITN Size to Possible Cycle-Entry

Our concern in this Section is to give a constructive proof of Theorem 1.

Theorem 1: For every positive integer k there exists a SITN cell definition such that any corresponding n-celled SITN which is in equilibrium at $t = -1$, and which has a single $x_i$ value change at $t = 0$, either can or cannot possibly enter a cycle at some $t \geqslant 0$ according as $n > k$ or $n \leqslant k$ respectively.

Proof: Consider the partially complete[3] SITN memory state diagram for $x_i = b_1$ shown in Fig. 2. The $c_i/c_j$ label on each arrow there indicates that if a cell with x input equal to $b_1$ has the memory state value given at the tail of an arrow and if its y input value is $c_i$, its corresponding y output value is $c_j$ and its next memory state value is the one given at the head of the arrow.

Now assume an n-celled SITN in equilibrium at $t = -1$ as follows: all $x_i = b_1$; $s_i = a_1$ for i odd and $s_i = a_2$ for i even; and $y_1 = c_1$. This equilibrium is an obvious consequence of the self-returning arrows out of $a_1$ and $a_2$. Then suppose that at $t = 0$, $x_1$ changes to $b_2$. Figure 3 shows that this causes $y_2$ to change immediately from $c_2$ to $c_1$. Figure 2 accordingly indicates the successive SITN variable value changes listed in Fig. 4. Each square's entry in Fig. 4 contains the column variable's value at the corresponding row time. Any column left blank above a certain row denotes that the column's topmost entry persists from that row time on. Note that the $a_j$ values assumed by each $s_i$ in Fig. 4 always have (maximum j) = i. Hence the (k+1)st cell is the leftmost one which can ever have its $s_i$ variable take on the $a_{k+1}$ value, and from that time on cycle around the $a_{k+1}$, $a_{k+2}$ loop. Since this is the only cycle admitted by Fig. 2, the figure provides the essentials of a proof of Theorem 1.

We fill in the details of our proof in Fig. 5. Figures 5 and 3 clearly indicate that the only possible equilibrium $s_i$ values are $a_1$ and $a_2$. Thus it is easily seen that regardless of the sequence of $x_i$ values along any corresponding SITN in equilibrium, if any single $x_i$ value change is to cause the SITN to enter a cycle, it must do so essentially in accordance with Fig. 4. Hence at least

[3] In the obvious sense that out of each $s_i$ memory state value there should be an output arrow for each possible $y_i$ value.

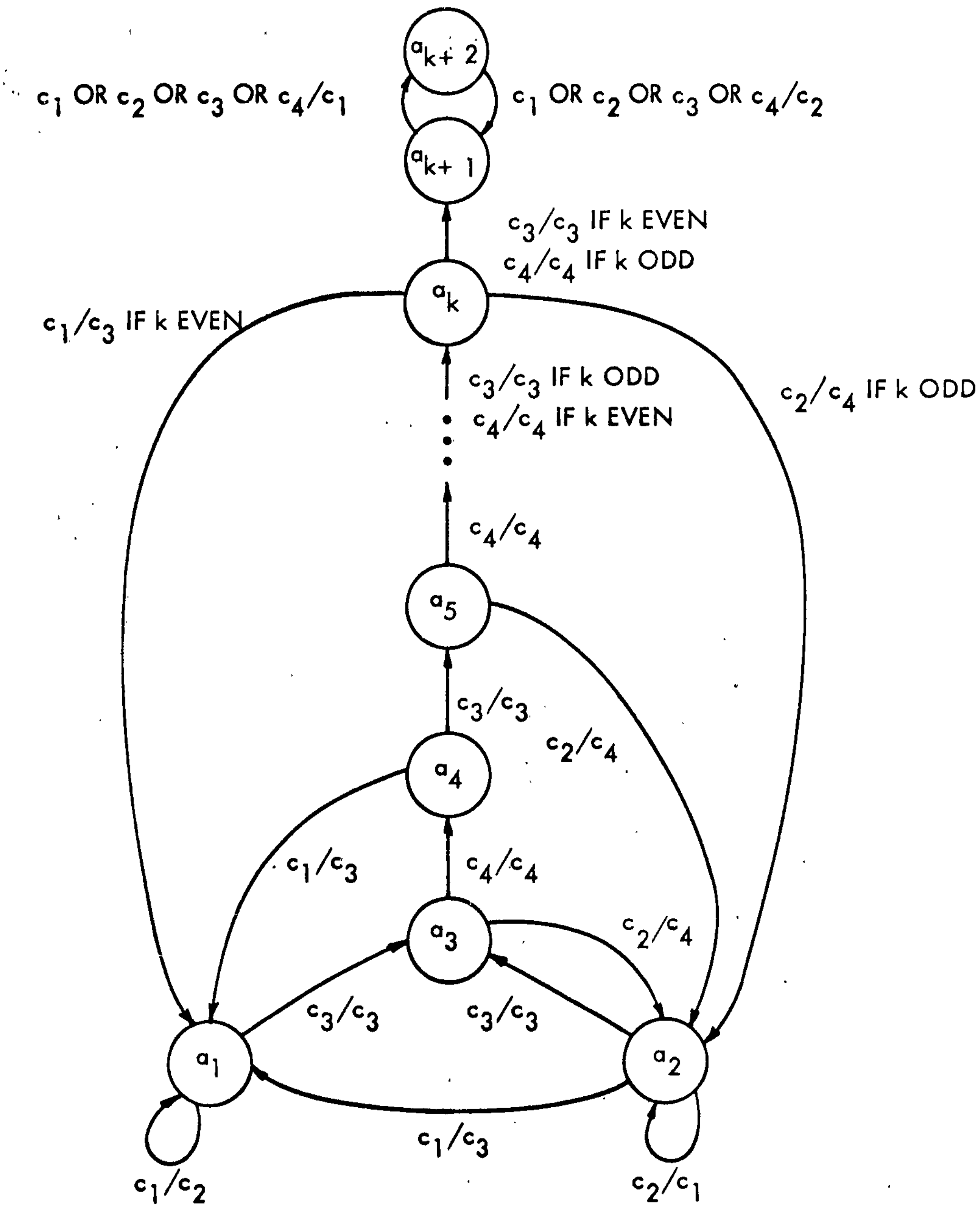


FIG. 2 PARTIALLY COMPLETE SITN MEMORY STATE DIAGRAM FOR $x_i = b_1$ USED IN THE PROOF OF THEOREM 1.

ALL THE REST OF THIS
MEMORY STATE DIAGRAM
(i.e., ALL BUT THE $c_1$ INPUT
ARROW OUT OF $a_1$ AND THE $c_2$
INPUT ARROW OUT OF $a_2$)
IS THE SAME AS THAT
FOR $x_i = b_2$

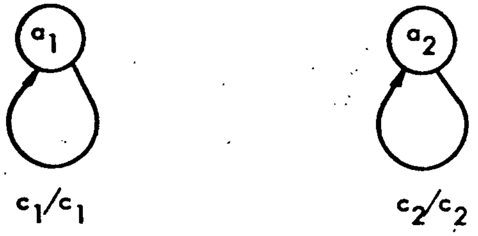
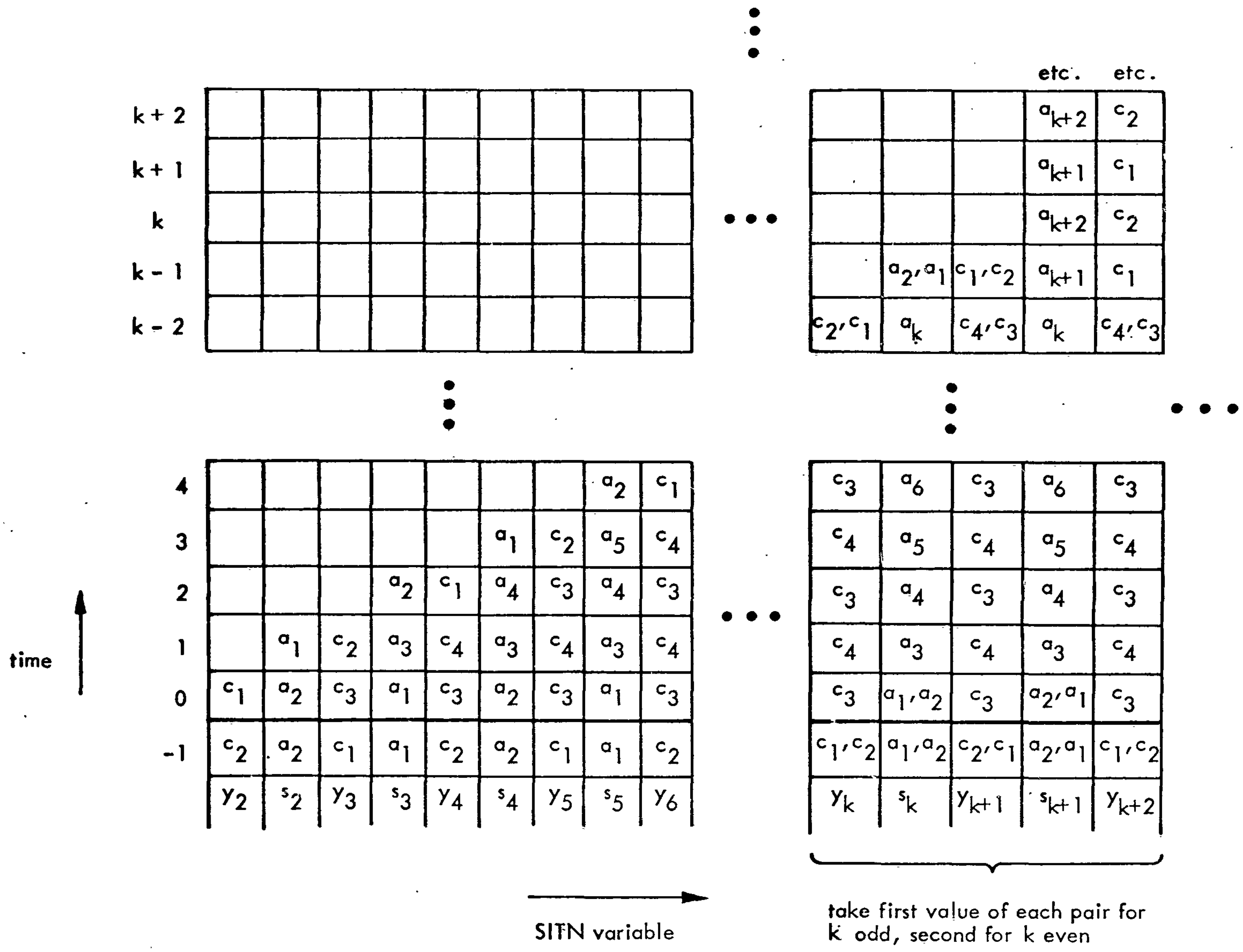


FIG. 3 SITN MEMORY STATE DIAGRAM FOR $x_i = b_2$ USED IN THE PROOF OF THEOREM I.



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| k + 2 | | | | | | | | | |
| k + 1 | | | | | | | | | |
| k | | | | | | | | | |
| k - 1 | | | | | | | | | |
| k - 2 | | | | | | | | | |

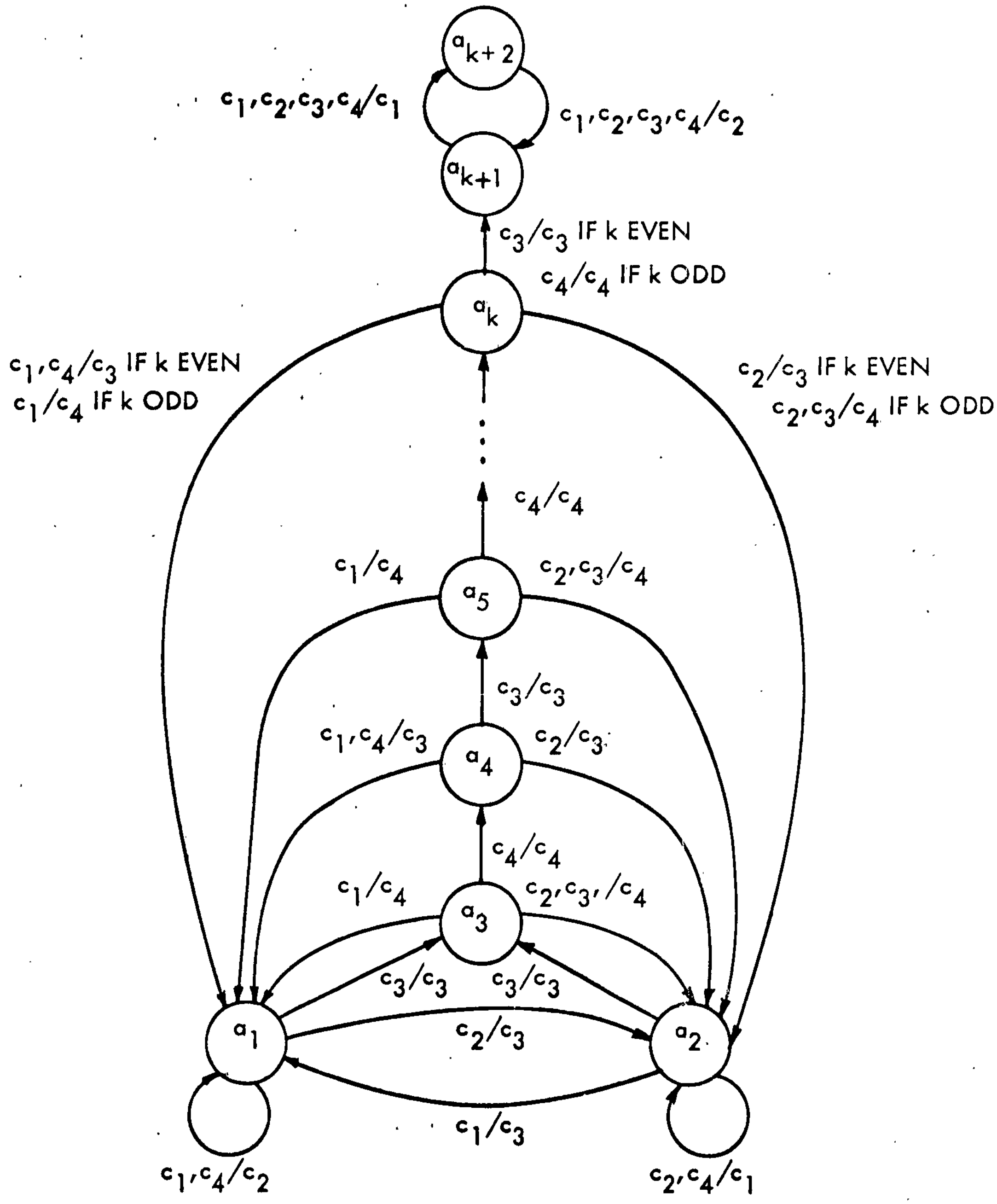
| | | | | | |
|---|---|---|---|---|---|
| k + 2 | | | | $a_{k+2}$ | $c_2$ |
| k + 1 | | | | $a_{k+1}$ | $c_1$ |
| k | | | | $a_{k+2}$ | $c_2$ |
| k - 1 | | $a_2, a_1$ | $c_1, c_2$ | $a_{k+1}$ | $c_1$ |
| k - 2 | $c_2, c_1$ | $a_k$ | $c_4, c_3$ | $a_k$ | $c_4, c_3$ |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | | | | | | | | $a_2$ | $c_1$ |
| 3 | | | | | | $a_1$ | $c_2$ | $a_5$ | $c_4$ |
| 2 | | | | $a_2$ | $c_1$ | $a_4$ | $c_3$ | $a_4$ | $c_3$ |
| 1 | | $a_1$ | $c_2$ | $a_3$ | $c_4$ | $a_3$ | $c_4$ | $a_3$ | $c_4$ |
| 0 | $c_1$ | $a_2$ | $c_3$ | $a_1$ | $c_3$ | $a_2$ | $c_3$ | $a_1$ | $c_3$ |
| -1 | $c_2$ | $a_2$ | $c_1$ | $a_1$ | $c_2$ | $a_2$ | $c_1$ | $a_1$ | $c_2$ |
| | $y_2$ | $s_2$ | $y_3$ | $s_3$ | $y_4$ | $s_4$ | $y_5$ | $s_5$ | $y_6$ |

| | | | | |
|---|---|---|---|---|
| $c_3$ | $a_6$ | $c_3$ | $a_6$ | $c_3$ |
| $c_4$ | $a_5$ | $c_4$ | $a_5$ | $c_4$ |
| $c_3$ | $a_4$ | $c_3$ | $a_4$ | $c_3$ |
| $c_4$ | $a_3$ | $c_4$ | $a_3$ | $c_4$ |
| $c_3$ | $a_1, a_2$ | $c_3$ | $a_2, a_1$ | $c_3$ |
| $c_1, c_2$ | $a_1, a_2$ | $c_2, c_1$ | $a_2, a_1$ | $c_1, c_2$ |
| $y_k$ | $s_k$ | $y_{k+1}$ | $s_{k+1}$ | $y_{k+2}$ |

take first value of each pair for k odd, second for k even

Fig. 4. Successive variable values from t = -1 on in the SITN used to prove Theorem 1.



$s_i$ DOMAIN: $a_1, a_2, \cdots a_{k+2}$
$x_i$ DOMAIN: (IN FIGS. 5 AND 3) : $b_1, b_2$.
$y_i$ DOMAIN : $c_1, c_2, c_3, c_4$.

FIG. 5. COMPLETE SITN MEMORY STATE DIAGRAM TOGETHER WITH FIG. 3) USED IN THE PROOF OF THEOREM 1

k + 1 cells are always required to the right of any single $x_i$ value change if a Fig. 5-type SITN is to enter a cycle under the conditions of Theorem 1.

Q. E. D.

## III. Some Unsolvable Problems on Cycle Entry, Equivalence, and Transient Character

In this Section we prove two unsolvability theorems, using essentially one SITN memory state diagram and Minsky's (1961) result that universal Turing machines can be represented by Post tag systems. We state both theorems before proving either.

Theorem 2: There does not exist a recursive procedure to determine of an arbitrary SITN cell definition[4] whether or not any corresponding SITN in any arbitrary equilibrium at $t = -1$ can have a single $x_i$ value change at $t = 0$ cause it to:

i) enter a cycle at some $t \geqslant 0$,

or ii) pass through a transient which causes a 1 output on some $z_i$ at some $t \geqslant 0$.

The ii) part of this Theorem pertains to the existence of certain SITN equivalence tests [cf. Hennie, (1961)].

We now consider SITNs which if in equilibrium at $t = -1$ and subjected to single $x_i$ value changes at $t = 0$, admit only transient responses (i.e., no cycle entries at any $t \geqslant 0$). We call such SITNs transient SITNs. In case a transient-SITN cell definition is such as to insure that all single $x_i$ changes from equilibrium cause transients involving $y_i$ value changes all the way to the right boundary of every corresponding SITN, we call the cell definition boundary transient. And in case a transient-SITN cell definition is such as to insure that no single $x_i$ change from equilibrium can cause transients involving $y_i$ value changes more than a bounded (hence calculable) number of cells to the right of the $x_i$ change in any corresponding SITN, we call the cell definition bounded transient.

Theorem 3: There does not exist a recursive procedure to determine of an arbitrary transient-SITN cell definition whether or not it is:

i) boundary transient,

or ii) bounded transient.

[4] Such as given in Figs. 3 and 5 for example.

Our first step in proving Theorems 2 and 3 is to define a Post tag system. Let A be a finite set of letters $a_1, a_2, \ldots a_m$; and let W be an associated set of words, such that for each i, $W_i$ is a fixed string or word of letters of A. Let P be some integer, and define the following process applied to some initially given string S of letters of A: Examine the first letter of the string S. If it is $a_i$, remove the first P letters of S, and then adjoin the word $W_i$ to the end of the remainder. Perform the same operations, defined a production, on the resulting string, and repeat so long as there are P or more letters left in each resulting string. If at some point there are fewer than P letters left in the resulting string, the process is said to terminate at that string. We call A, W, S, P, and the process just defined a Post tag system. Minsky, (1961), showed that the problem of determining for any given Post tag system whether or not the corresponding process ever terminates is recursively unsolvable. We will now embed this result into a SITN memory state diagram.

We replace the letters $a_1, a_2, \ldots, a_m$ (but not the symbols S, $W_i$) of a given Post tag system by the $y_i$ values $c_1, c_2, \ldots, c_m$ respectively, and then complete the $y_i$ domain by adding three special values, $\phi$, $\omega_1$, and $\omega_2$. The latter two are interpreted as marker values, and $\phi$ is interpreted as the null value. Next we specify $a_o$, $a_1$, $a_2$, $a_3$, and $a_4$ as the only possible equilibrium $s_i$ values, and $b_1$, $b_2$ as the $x_i$ domain. Figures 6 and 7 then give the essential outline of a SITN memory state diagram sufficient to prove Theorems 2 and 3.

Our notation in these Figures is as follows: C denotes any $y_i$ value; $\sim\omega_1$, $\sim\omega_2$, and $\sim\phi$ denote any $y_i$ values but $\omega_1$, $\omega_2$, and $\phi$ respectively; $y_i$ values raised to the $j^{th}$ power denote j successive repetitions of those values; $\ell(T)$, for any string of letters T, denotes the number of letters in T; and TU, T and U both strings, denotes the string consisting of the letters of T followed by the letters of U in order.

Let us now assume, in order to explain Figs. 6 and 7, that we have a corresponding SITN in equilibrium at t = −1 as follows: all $x_i$ values are $b_1$, $s_1$ is $a_o$, $s_2$ is $a_1$, and all other $s_i$ are $a_3$. Then suppose that at t = 0 there is a single $x_i$ value change from $b_1$ to $b_2$ in the first cell. This causes $y_2$ to change from $\phi$ (i.e. null) to $\omega_1$. Subsequently $s_2$ passes from $a_1$ down through $a_5$ to $a_2$, causing $y_3$ to put out the string $\omega_2 S$ before settling down at $\phi$.

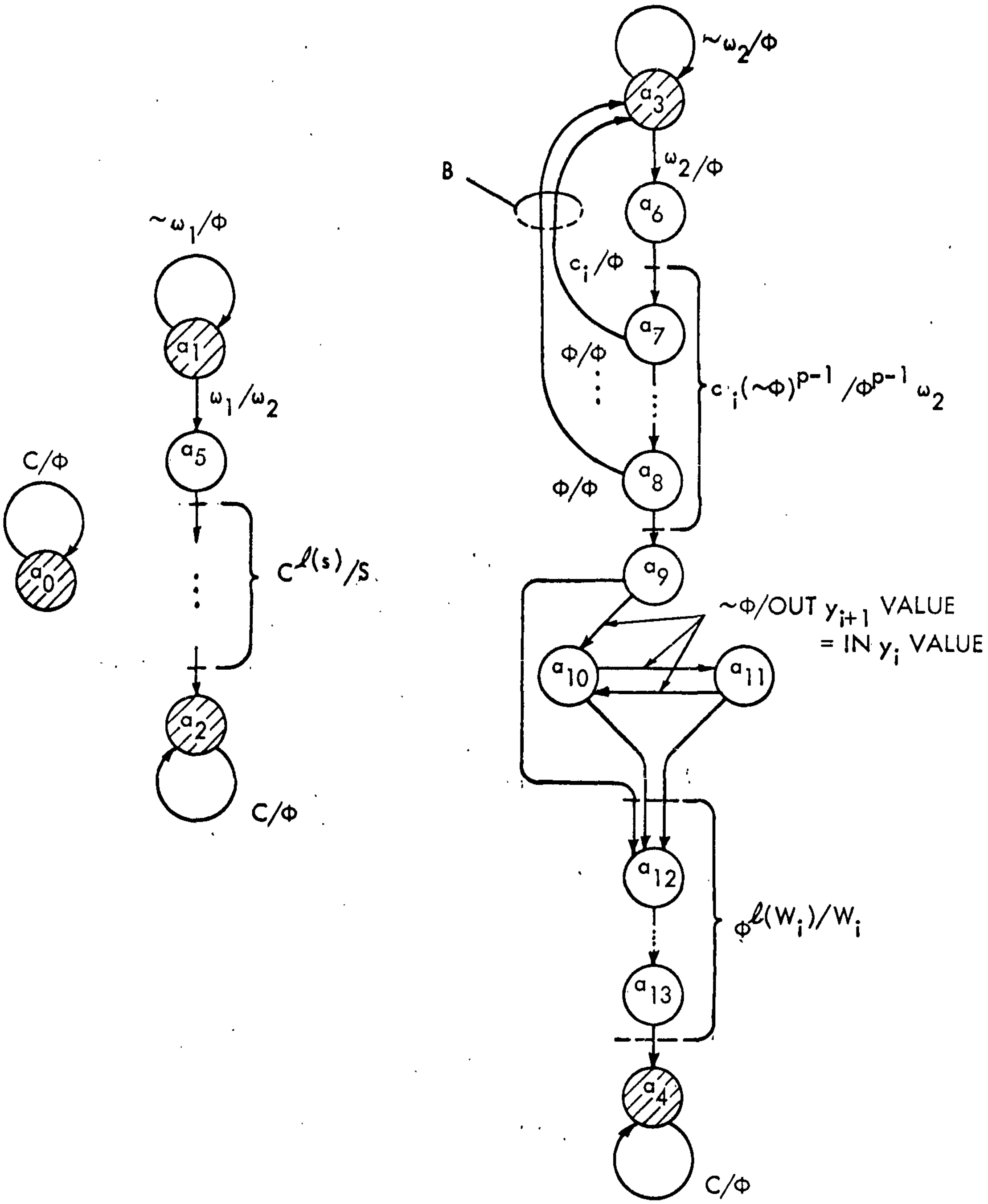
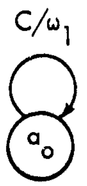


FIG. 6. THE PARTIALLY COMPLETE SITN MEMORY STATE DIAGRAM FOR $x_i = b_1$ USED IN THE PROOF OF THEOREMS 2 AND 3.



THE REST OF FIG. 7 IS THE SAME AS THE CORRESPONDING PART OF FIG. 6, EXCEPT $a_1$ IS INTERCHANGED WITH $a_2$, AND $a_3$ IS INTERCHANGED WITH $a_4$.

FIG. 7. THE SITN MEMORY STATE DIAGRAM FOR $X_i = b_2$ USED IN THE PROOF OF THEOREM 2 AND 3

Next we show that $y_4$ accordingly puts out essentially the result of the first production in the corresponding Post tag system. To see this, we note that $s_3$ is taken from $a_3$ to $a_6$ by $y_3 = \omega_2$; from $a_6$ to $a_7$ by the next value of $y_3$ (i. e., the first letter of S, assumed $c_i$), and from $a_7$ to $a_9$ by the next P − 1 values of $y_3$ (i. e., the next P − 1 letters of S, whatever they might be). $y_4$'s value remains $\phi$, or null, during all of these changes. Following them, however, the $(P+1)^{st}$ non-$\phi$ value of $y_3$ (i. e., the $P^{th}$ letter of S) produces $\omega_2$ out at $y_4$. Then the sequence consisting of $y_3$'s $(P+2)^{nd}$ non-$\phi$ value to its last non-$\phi$ value (i. e., the $(P+1)^{st}$ to the last letter of S) produces itself out at $y_4$. Finally, when $y_3$ settles down at $\phi$, $s_4$ leaves $a_{10}$ or $a_{11}$ (whichever state it is in) and causes $y_4$ to put out the string $W_i$, corresponding to the first letter of S. After that $y_4$ also settles down at $\phi$. Thus the first production in the Post tag system,

$$S = \underbrace{c_i T_1}_{\text{P letters}} \underbrace{T_2} \rightarrow T_2 W_i,$$

is represented by the $y_3 \rightarrow y_4$ transformation across the $3^{rd}$ SITN cell, $\omega_2 S \rightarrow \omega_2 T_2 W_i$ (preceding and succeeding $\phi$ values not shown).

More generally, the $y_i \rightarrow y_{i+1}$ transformation across the $i^{th}$ SITN cell can be made to represent the $(i-2)^{nd}$ tag system production as follows: For each i in the tag system alphabet of letters, $\{c_i\}$, we add to Fig. 6 a portion of $s_i$ state diagram which is exactly the $c_i^{th}$ counterpart of the portion already there from $a_7$ to $a_{13}$. This is primarily to enable the first $c_j$ value in each incoming $y_i$ sequence to direct $s_i$ to a portion of over-all state diagram that ends $y_{i+1}$'s non-$\phi$ sequence with the right $W_j$. The $W_j$ are produced in one of the $a_{12}$-$a_{13}$-type portions of augmented $s_i$ state diagram, and are SITN representations of completions of corresponding tag production steps. The purpose of adding $a_7$-$a_{11}$ as well as $a_{12}$-$a_{13}$-type portions of $s_i$ state diagram is threefold: 1) the $a_7$-$a_8$ portions enable the $i^{th}$ cell to effectively remove (i. e., replace by $\phi$) the $2^{nd}$ to $P^{th}$ $c_j$ values of each incoming $y_i$ sequence. This begins the SITN representation of each corresponding tag production; 2) If there are ever fewer than P $c_j$ values, the $a_7$-$a_8$ portions allow $y_{i+1}$ to remain fixed at $\phi$.In each such case one Bbundle arrow in Fig. 6 is traversed; and 3) The $a_9$-$a_{11}$-type portions enable the $i^{th}$ cell to simply pass the $(p+1)^{st}$

to last $c_j$ values of each $y_i$ sequence. Thus they represent intermediate tag production steps, preparatory to $W_j$ adjoinments.

Hence if the corresponding tag system productions terminate at the $i^{th}$ string, $y_{i+2}$ is the leftmost SITN value that is left unchanged in the associated network transient.

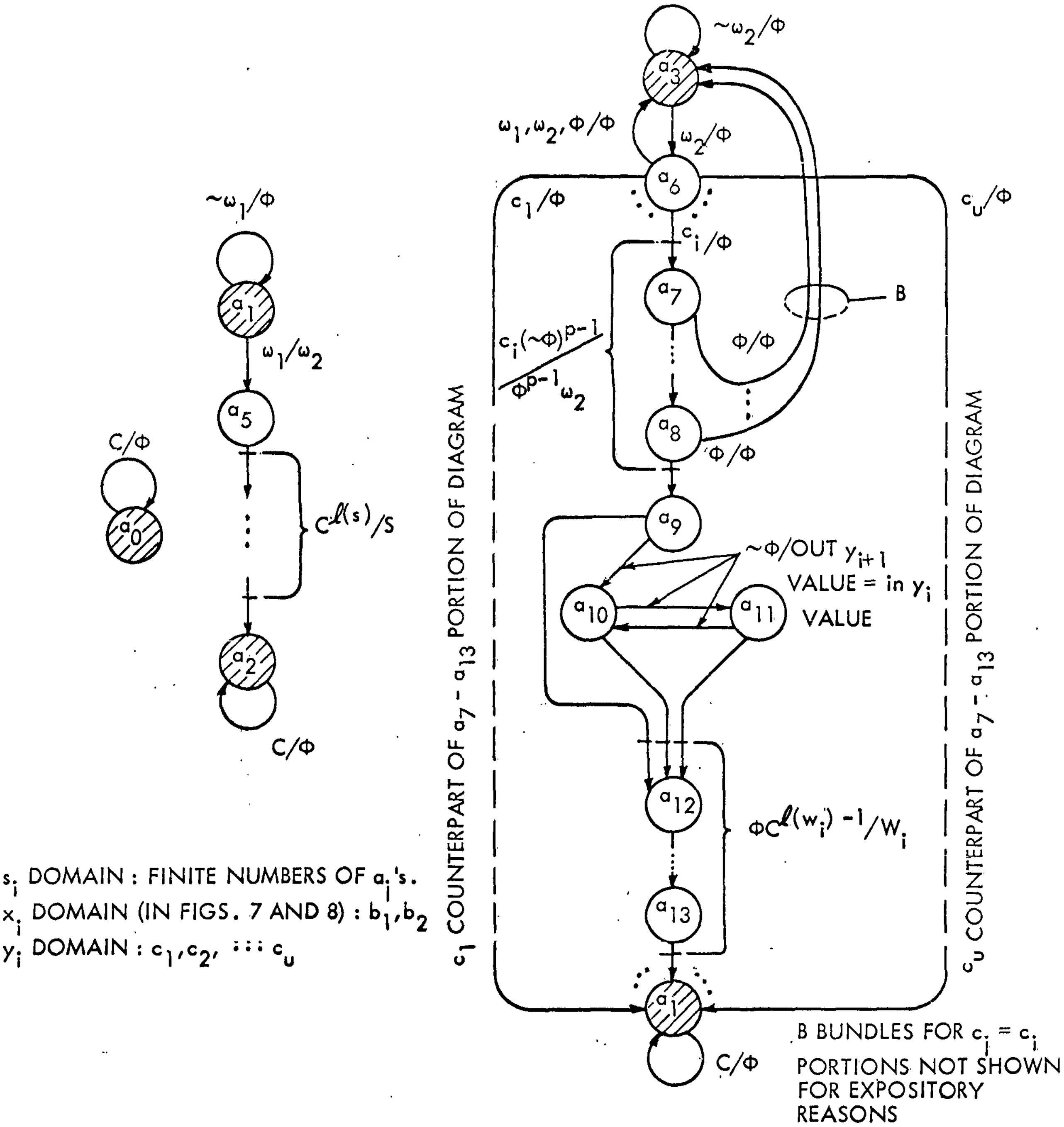
We now finish our proof of Theorem 3 by filling in the missing details of Fig. 6 in Fig. 8. We leave it to the reader to check in Figs. 7 and 8 that only one type of transient can involve $y_i$ value changes more than one cell to the right of a single $x_i$ perturbation of a corresponding SITN at equilbrium. And that transient type is the one discussed above. Since the problem of determining whether or not the tag system productions corresponding to such a transient ever terminate is recursively unsolvable, so also is the problem of determining whether or not the transient itself is bounded or boundary. This completes our proof of Theorem 3.

The proof of Theorem 2 follows almost trivially. We prove part ii) by modifying the B bundle in Fig. 8 as follows: Instead of directing this bundle into $a_3$, we direct it into a new state, $a_{14}$, as shown in Fig. 9. Then we specify that $z_i = 0$ for all $s_i$ values except $a_{14}$, in which case $z_i = 1$. Theorem 2 ii) follows immediately by noting that it is yes if and only if the transient in the proof of Theorem 3 is bounded. But this question is recursively unsolvable.

We prove Theorem 2 i) by modifying Fig. 9 as shown in Fig. 10. Then states $a_{14}$ and $a_{15}$ comprise the only cycle that is accessible under the conditions of Theorem 2. Hence Theorem 2 i) is yes if and only if Theorem 2 ii) is yes, which is recursively unsolvable.

Q.E.D.

As a passing point, we note that by identifying the right and left boundary signals in the SITNs of Theorem 3, we can get a result much like Theorem 1, but with the inequalities reversed. Although this point has considerable interest, we will not develop it further here.



$s_i$ DOMAIN : FINITE NUMBERS OF $a_i$'s.

$x_i$ DOMAIN (IN FIGS. 7 AND 8) : $b_1, b_2$

$y_i$ DOMAIN : $c_1, c_2, \vdots\vdots\vdots\ c_u$

B BUNDLES FOR $c_j = c_i$

PORTIONS NOT SHOWN FOR EXPOSITORY REASONS

FIG. 8. COMPLETE SITN MEMORY STATE DIAGRAM TOGETHER WITH FIG. 3) USED IN THE PROOF OF THEOREM. 3

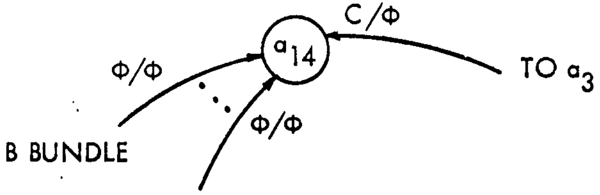


FIG. 9 . CHANGE IN FIG. 8 FOR PROVING THEOREM 2ii)

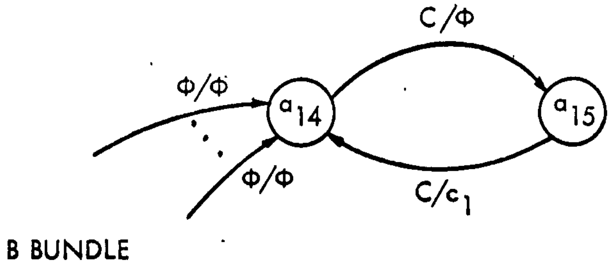


FIG. 10. MODIFICATION OF FIG 9 FOR PROVING THEOREM 2i

## IV. Applications to Non-SITN Models

In this Section we apply our results to some non-SITN models discussed in Kilmer (1961), (1962A), (1962B). Our method is to develop a chain of equivalences from one of those models to SITNs.

First, we define the network model shown in Fig. 11. The large square boxes there represent identical combinational logic cells, each having zero switching delay, and the small rectangles represent unit delay elements. Cellular $\alpha$, $\beta$, and x inputs are constant during each unit time interval, so the network operates synchronously. Each cell's $\alpha_i$ and $\beta_i$ lateral inputs and $x_i$ external input take on values ranging over finite $\alpha_i$, $\beta_i$, and $x_i$ domains respectively. Correspondingly, each cell's $\alpha_i$ and $\beta_i$ lateral outputs and $z_i$ external output range over finite domains of values as determined by the truth table comprising the network's cell definition. We require only that the number of network cells be finite, and define such networks BITNs (for Bilateral ITerative Networks).

In Fig. 12 we show a reconception of Fig. 11. There the i^th cell maps $\alpha_i \rightarrow \beta_{i-1}$ under the influence of $x_i$ and a left-coupling parameter, $C^{\ell i}$; and also maps $\beta_i \rightarrow \alpha_{i+1}$ under the influence of $x_i$ and a right-coupling parameter, $C^{ri}$, all with zero delay. The coupling idea is to let $C^{\ell i}$ be a function of $\beta_i$ such that all those $\beta_i$ values which exert the same influence in every $\alpha_i \rightarrow \beta_{i-1}$ mapping cause the same $C^{\ell i}$ value. Similarly for $C^{ri}$ and $\alpha_i$ values.

Fig. 11-to-Fig. 12-transformations are easily made 1-to-1. To illustrate, assume in Fig. 13 that the $\alpha_i\beta_i\alpha_{i+1}$ portion of the left-hand table is identical for all $x_i$ values. Then $\beta_i$ maps into $\alpha_{i+1}$ in the same way regardless of $x_i$'s value and whether $\alpha_i$'s value is 0 or 2. Therefore let $C^{ri}(\alpha_i)$ be $R_1$ for $\alpha_i = 0$ or 2, and $R_2$ for $\alpha_i = 1$. Similarly for the right-hand table, assume that the $\beta_i\alpha_i\beta_{i-1}$ portion of the table is identical for all $x_i$ values. Then let $C^{\ell i}(\beta_i)$ be $L_1$ for $\beta_i = 0$, and $L_2$ for $\beta_i = 1$ or 2. Our method should be clear by now, so we omit the remaining details.

Henceforth we denote Fig. 12 renditions of BITNs, BITN*s. And if the $C^{\ell}$ domain has only one element, we denote the corresponding networks R-BITN*s (for right-coupled BITN*s). Now consider the R-BITN* shown in Fig. 14. We note that each light dashed rectangle there encloses a structure

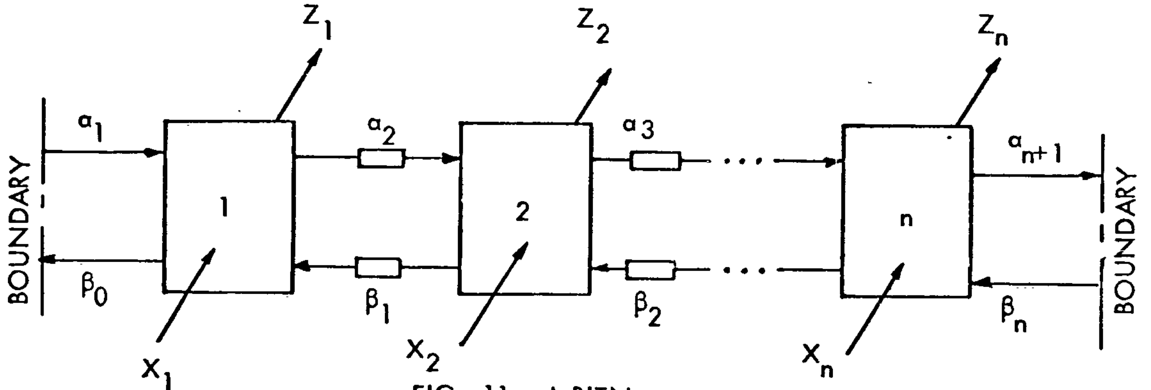


FIG. 11. A BITN.

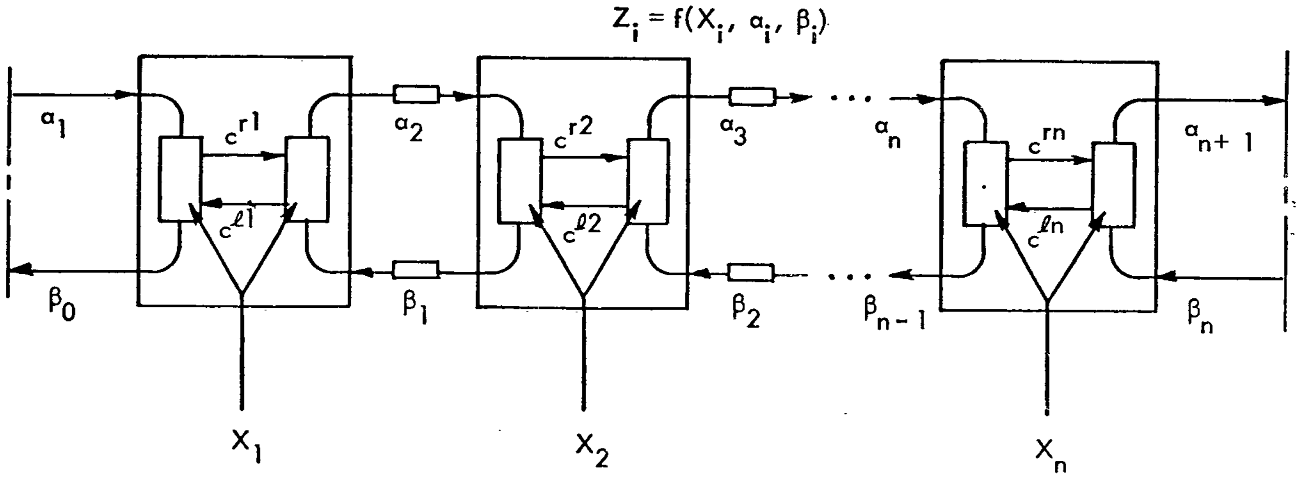
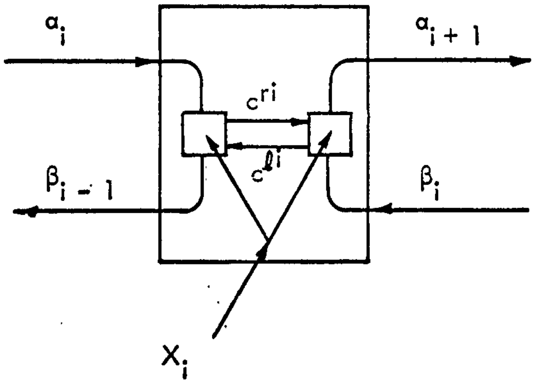


FIG. 12. A RECONCEPTION OF FIG. 11



| $X_i$ | $\alpha_i$ | $\beta_i$ | $\alpha_{i+1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 2 |
| 0 | 0 | 2 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 2 |
| 0 | 1 | 2 | 1 |
| 0 | 2 | 0 | 1 |
| 0 | 2 | 1 | 2 |
| 0 | 2 | 2 | 0 |
| 1 | 0 | 0 | 1 |
|  | ⋮ |  |  |

| $X_i$ | $\beta_i$ | $\alpha_i$ | $\beta_{i-1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 2 | 0 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 2 |
| 0 | 1 | 2 | 1 |
| 0 | 2 | 0 | 2 |
| 0 | 2 | 1 | 2 |
| 0 | 2 | 2 | 1 |
| 1 | 0 | 0 | 1 |
|  | ⋮ |  |  |

FIG. 13. AN OUTLINE FOR A FIG. 11-TO-FIG. 12 TRANSFORMATION

which closely approximates a SITN cell. We will show that the network in Fig. 14 is, in fact, equivalent to a SITN.

Suppose in Fig. 14 that $C^{r1}$ maps $\beta_1$ into $\alpha_2$ at t. Then this $\alpha_2$ produces $C^{r2}$, which maps $\beta_2$ into $\alpha_3$ at t + 1. This $\alpha_3$ in turn produces $C^{r3}$, which maps $\beta_3$ into $\alpha_4$ at t + 2, and so forth. Thus if one knows $C^{r1}$ at t, t + 2, t + 4, ..., and one knows $\beta_1$ into cell 1 at t, $\beta_2$ into cell 2 at t + 1, ..., and $\beta_n$ into cell n at t + n − 1 for a R-BITN*, one has sufficient information to establish exactly half of its $\alpha$ and $\beta$ values during each successive time interval. Hence the listed set of $\beta$'s and associated $C^r$'s is called the R-BITN*'s correspondence set at t. (We note that such a set is generally quite distinct from the analogous "initial condition set.")

Obviously any two independent R-BITN* correspondence sets, say at t and t + 2k + 1 for some integer k, respectively, are analyzed separately, yet in the exact same way, in order to determine their respective response. Each set is also analyzed independently of the unit time delay between cells. Hence the R-BITN* in Fig. 14 is equivalent to the SITN in Fig. 15 under the conditions that[5]:

1) the small rectangles beneath each cell in Fig. 15 represent unit delays; and

2) in Fig. 15 $C^{ri}$ and $\beta_i$ into cell i at time t map into $C^{r(i+1)}$ and $\beta_i$ out of cell i at time t, just as in Fig. 14 $C^{ri}$ and $\beta_i$ of the correspondence set at t map into $C^{r(i+1)}$ and $\beta_i$ of the correspondence sets at t and t + 2, respectively.

From this discussion, we readily see Theorem 4.

Theorem 4: For each SITN result in Theorems 1, 2, and 3, there are exactly analogous results for R-BITN*s, BITN*s, and BITNs.

We remark that Theorems 3 and 4, with more or less immediate proof modifications, give strengthened versions[6] of Hennie's (1961) Theorems 10, 10.1, 10.2, 11, 11.1, and 15. Also since the proof of Theorem 3 embodies a SITN representation of a universal Turing machine [cf. Minsky, (1961)], the result

[5] Hennie, (1961), has developed a class of equivalence results that are related to, but essentially distinct from, those derived above.

[6] Because we start from equilibrium instead of arbitrary initial conditions.

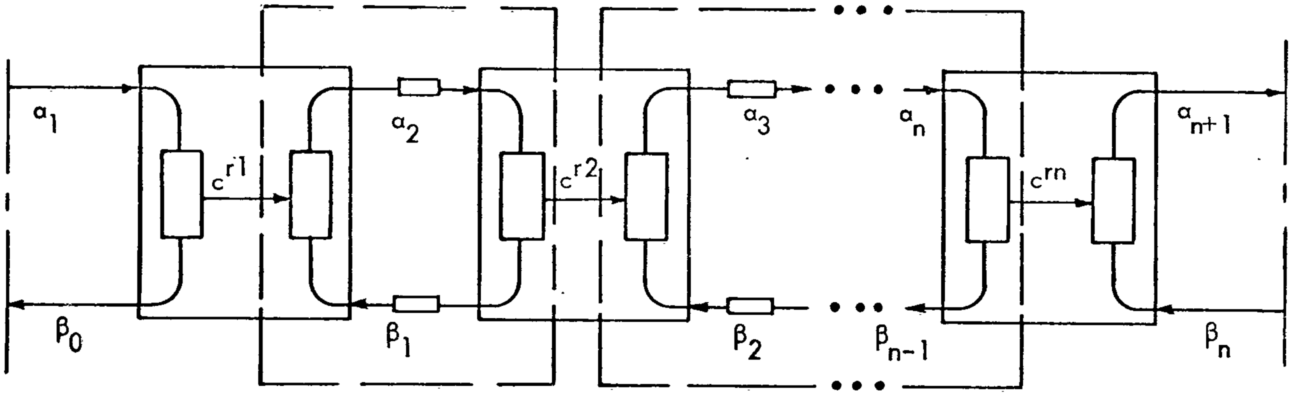


FIG. 14. AN R-BITN*

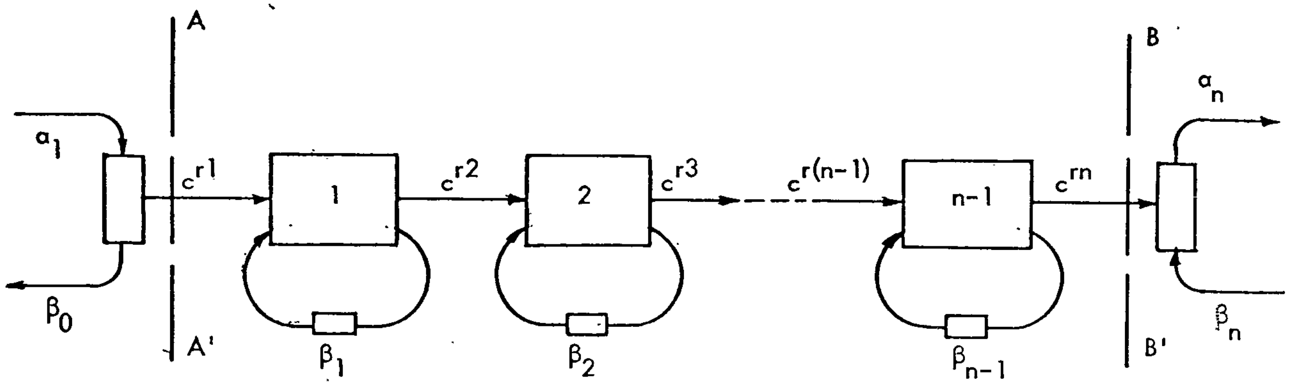


FIG. 15. A SITN EQUIVALENT OF AN R-BITN*

clarifies several computing capacity problems alluded to in Hennie, (1961). Finally, we claim that the present paper distributes the proof burden for Theorems 3 and 4 in such a manner as to substantially illuminate the basic nature of Hennie's previous work.

## V. Conclusions

Kilmer, (1961), and Winograd, (1962), essentially closed out the main switching transients problems for BITNs which have either $\alpha_i$ or $\beta_i$ lines missing. Hennie's previous work, (1961), extends these results to canonical decompositions of over-all memoryless BITNs (i.e., BITNs which have 1-to-1 over-all equilibrium $\{x_1, x_2, \ldots x_n\}$ input – $\{z_1, z_2, \ldots z_n\}$ output relations). Kilmer, (1962B), discusses the unsolvable nature of steady-state cycling problems in BITNs, BITN$^*$s, and R-BITN$^*$s. And this paper shows the essential unsolvability of the main transients problems in BITNs and SITNs. Thus future work must be directed at developing sufficiency conditions for desired transient behavior in such networks.[7]

In closing we note the curious duality between Theorem 1 of this paper and Theorem 2 of Kilmer, (1962A). The latter states: For every positive integer k, there exists a BITN cell definition such that every corresponding n-celled BITN is or is not over-all combinational accoridng as n is or is not $\leq$ k.

Now the curious thing about these theorems is that both of their (constructive) proofs seem to require cell complexities that are directly proportional in some sense to k. For Theorem 1 this proportionality is between k and the size of the $s_j$ domain; and for Theorem 2 it is between k and the number of rows in the corresponding cellular truth table definition. The author is not sure what this really means in terms of recursive function theory, if indeed anything, but it certainly suggests a Cantor diagonalization approach.

[7] (Note added in proof) At least one such set of conditions has already been derived, and it appears that others are forthcoming.

## Acknowledgment

The author wishes to acknowledge Dr. Warren McCulloch, of the Research Laboratory of Electronics, M. I. T., as the one who demonstrated to him the importance of iterative network research to the neurophysiological study of reticular formations in vertebrate nervous systems. At present, the author's main motivations and guidelines are based on this awareness.

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