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RESEARCH REPORT EE 541

NANOSECOND STUDY

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Technical Report No. 20

30 September 1962

School of Electrical Engineering
CORNELL UNIVERSITY
Ithaca, New York

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LINEAR BEAM MICROWAVE TUBES

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Published under Contract No. AF30(602)-2573
Rome Air Development Center, Griffiss Air Force Base, New York

There are two elements that together determine the response of a microwave tube to short pulses. One of these is the electron beam (with the nonlinear characteristic being the most important factor), and the second is the interaction circuit. For a klystron, the interaction circuit is usually a resonant cavity; therefore the response of a resonant cavity to short pulses is of interest in estimating the short-pulse capability of klystrons. This section summarizes the results of a study of the response of a resonant cavity to a pulse with a Gaussian envelope.

It is assumed that the klystron cavity can be approximated by the equivalent circuit shown in Figure 1, that is, by a parallel resonant circuit. This circuit is driven by a current generator, i(t), which produces a r-f signal at frequency ω_o with a Gaussian envelope:

$$i(t) = R_e \left[I_o \, e^{-\nu t^2} \, e^{j\omega_o t} \right] . \qquad (1)$$

The voltage across this circuit, v(t), is the signal whose characteristics are sought. The departure of the envelope of v(t) from a Gaussian shape is a measure of the distortion introduced by the finite bandwidth associated with the resonant cavity. The voltage, v(t), can be determined in a straightforward manner using Fourier transform techniques.

The admittance of the parallel resonant circuit; $Y(\omega)$ is given by

$$Y(\omega) = G + j\omega C \left(1 - \frac{1}{\omega^2 LC} \right) . \qquad (2)$$

Define,

$$Y_o = \sqrt{\frac{C}{L}} \quad ,$$

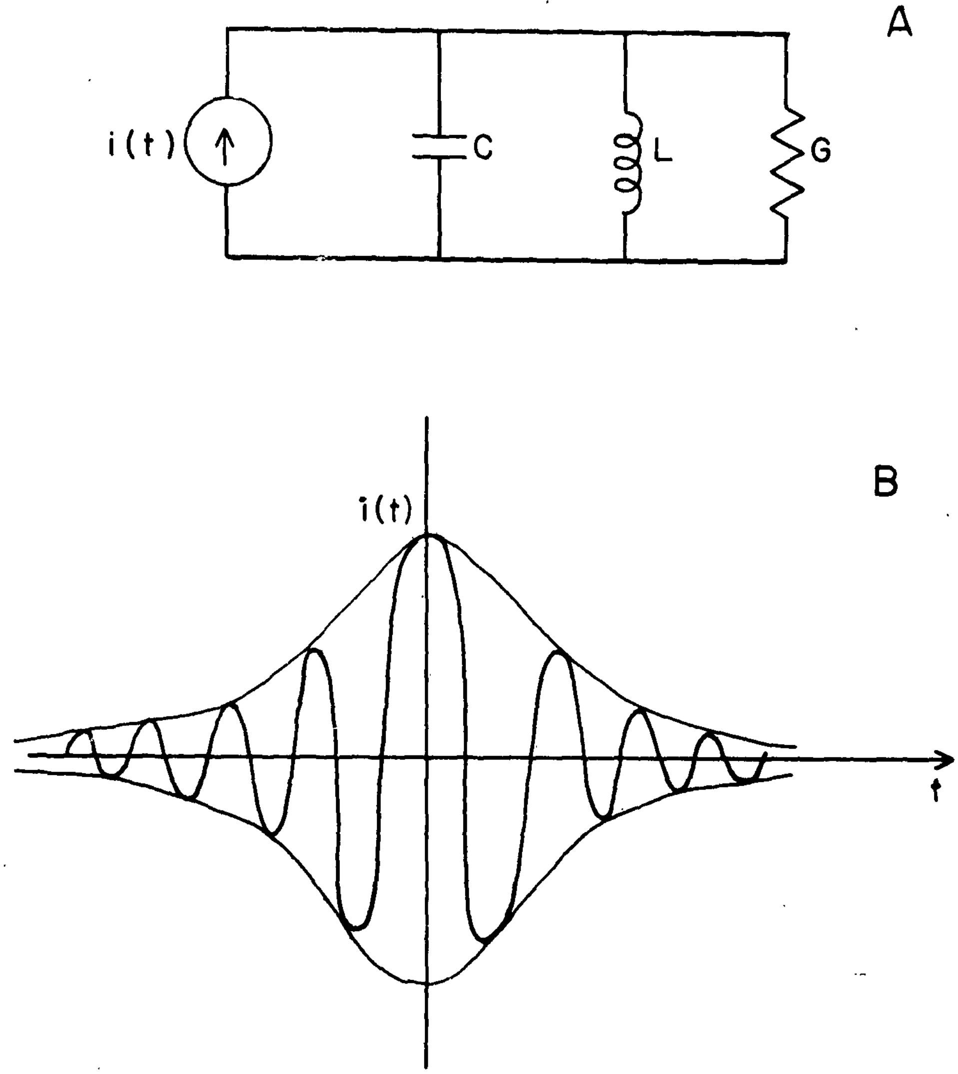

Figure 1. (a) Equivalent Circuit for Resonant Cavity Driven by a Current Pulse. (b) Current Pulse with Gaussian Envelope.

$$\omega_r^2 = \frac{1}{LC} \quad ,$$

$$Q = \frac{\omega_r C}{G} \quad ; \tag{3}$$

then

$$\frac{Y(\omega)}{Y_o} = \frac{1}{Q} + j\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right) . \tag{4}$$

For simplicity, the resonant frequency of the cavity, ω_r, is taken equal to the carrier frequency of the input signal, ω_o. This will almost always be at least approximately true in practice.

Using the Fourier transform of the input current pulse, the Fourier transform of the output voltage pulse is

$$\frac{V(\omega)}{V_o} = \sqrt{\frac{\pi}{\nu}} \; \frac{e^{-\frac{(\omega - \omega_o)^2}{4\nu}}}{\frac{1}{Q} + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \quad , \tag{5}$$

where

$$V_o = \frac{I_o}{Y_o} = \sqrt{\frac{L}{C}}\; I_o \quad . \tag{6}$$

The output voltage as a function of time is

$$v(t) = \frac{R_e \omega_o}{4} \sqrt{\frac{\pi}{\nu}} \; V_o \left\{ \left(1 - \frac{j}{2Q\sqrt{1 - \frac{1}{4Q^2}}} \right) \right.$$

$$\left. \left[1 + \operatorname{erf}\left\{ \sqrt{\nu}\, t - \frac{\omega_o}{4Q\sqrt{\nu}} - j\,\frac{\omega_o}{2\sqrt{\nu}} \left(1 \mp 1 - \frac{1}{4Q^2} \right) \right\} \right] \right.$$

$$
e^{-\frac{\omega_o t}{2Q} - \frac{\omega_o^2}{2Q\nu}\left(1 + \sqrt{1 - \frac{1}{4Q^2}}\right) - j\sqrt{1 - \frac{1}{4Q^2}}\ \omega_o t + j\frac{\omega_o^2}{4Q\nu}\left(1 + \sqrt{1 - \frac{1}{4Q^2}}\right)}
$$

$$
+ \left(1 + \frac{j}{2Q\sqrt{1 - \frac{1}{4Q^2}}}\right)\left[1 + \operatorname{erf}\ \nu t - \frac{\omega_o}{4Q\ \nu}\left(\sqrt{1 - \frac{1}{4Q^2}} - 1\right)\right]
$$

$$
e^{-\frac{\omega_o t}{2Q} + \frac{\omega_o^2}{2\nu}\left(\sqrt{1 - \frac{1}{4Q^2}} + \frac{1}{4Q^2} - 1\right) + j\sqrt{1 - \frac{1}{4Q^2}}\ \omega_o t - j\frac{\omega_o^2}{4Q\nu}\left(\sqrt{1 - \frac{1}{4Q^2}} - 1\right)} \quad . \tag{7}
$$

Figures 2 and 3 present the output voltage envelope as a function of time for two different pulse lengths and with Q as a parameter. In Figure 2, the length of the input current pulse is 100 r-f cycles between the -10 db points. For a Q of 10, the output voltage pulse envelope is nearly identical with the current pulse envelope. For a Q of 100, the voltage pulse is delayed in time by approximately 25 cycles, but its shape is still approximately Gaussian. Thus the amount of distortion of the envelope introduced is not large for $Q \lesssim 100$ in this case.

Figure 3 presents the results for a pulse length of 10 r-f cycles between the -10 db points. In this case, the increase in the pulse delay as the Q is increased is clearly evident. Further, although the $Q = 10$ curve appears to be nearly Gaussian, at $Q = 50$ or 100, the curves are far from Gaussian, having long tails. At $Q = 20$, the asymmetry of the curve is becoming evident; therefore $Q = 20$ may be taken as a rough dividing line between acceptable and excessive distortion.

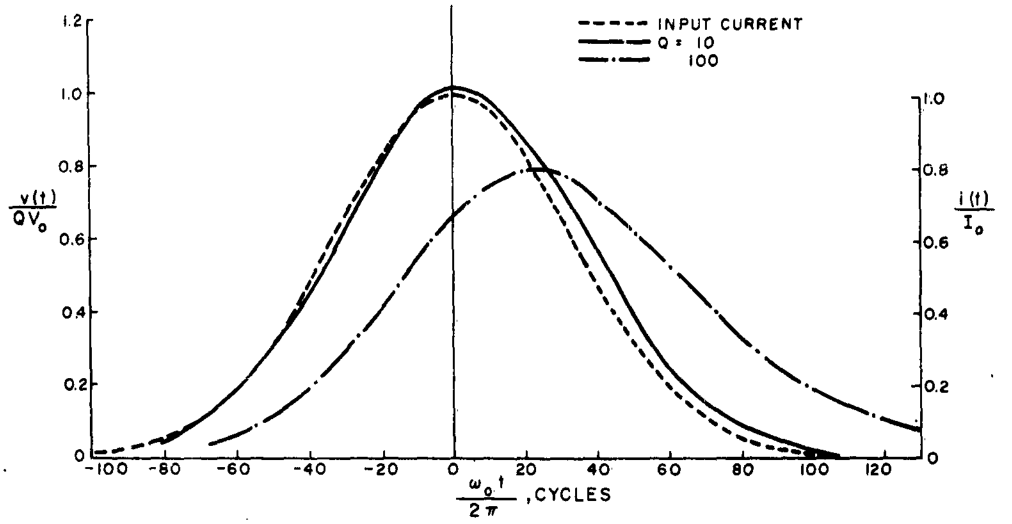

Figure 2. Response of Single Tuned Circuit to a Gaussian Current Pulse. (100 cycles between -10-db points.)

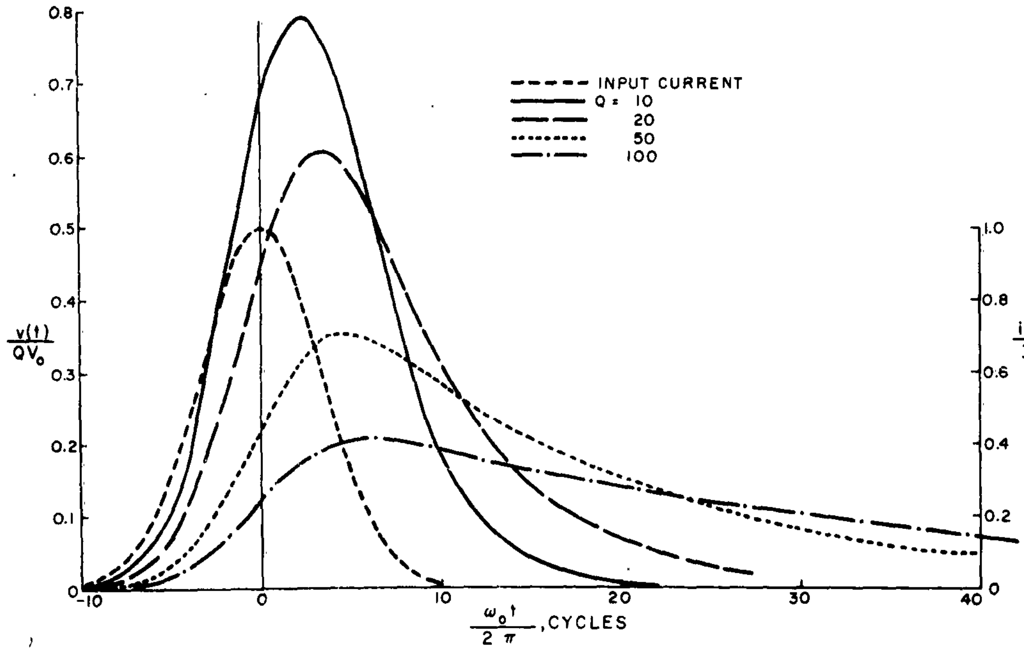

Figure 3. Response of Single Tuned Circuit to a Gaussian Current Pulse. (10 cycles between -10-db points.)

One measure of the distortion of the output voltage pulse is the extent to which the magnitude of its Fourier frequency spectrum departs from that of a Gaussian envelope pulse. The Fourier spectrum of the output voltage can be written from Equation (5) as

$$\sqrt{\frac{\nu}{\pi}}\ \frac{|V(\omega)|}{QV_o} = \frac{e^{-\frac{(\omega - \omega_o)^2}{4\nu}}}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}} \quad . \tag{8}$$

The ideal response is

$$\sqrt{\frac{\nu}{\pi}}\ \frac{|V(\omega)|\ \text{ideal}}{QV_o} = e^{-\frac{(\omega - \omega_o)^2}{4\nu}} \quad . \tag{9}$$

A measure of the distortion as a function of frequency can be defined as

$$D\left(\frac{\omega}{\omega_o}\right) = \frac{\nu}{\pi}\ \frac{-1}{QV_o}\left[|V(\omega)|_{\text{ideal}} - |V(\omega)|\right] \quad ,$$

$$D\left(\frac{\omega}{\omega_o}\right) = e^{-\frac{(\omega - \omega_o)^2}{4\nu}}\left[1 - \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}\right] \quad . \tag{10}$$

Figures 4 and 5 show $D(\omega/\omega_o)$ versus ω/ω_o for 100 and 10 r-f cycles between -10 db points, respectively, with Q as a parameter. From Figure 4 for the 100-cycle case, $D(\omega/\omega_o)$ is negligible for $Q = 10$, and not too large for $Q = 100$. However, for the 10-cycle case of Figure 5, $D(\omega/\omega_o)$ has peaks of appreciable height at $Q = 20$, and these are very large for $Q = 100$. These curves, of course, corroborate the conclusions based on

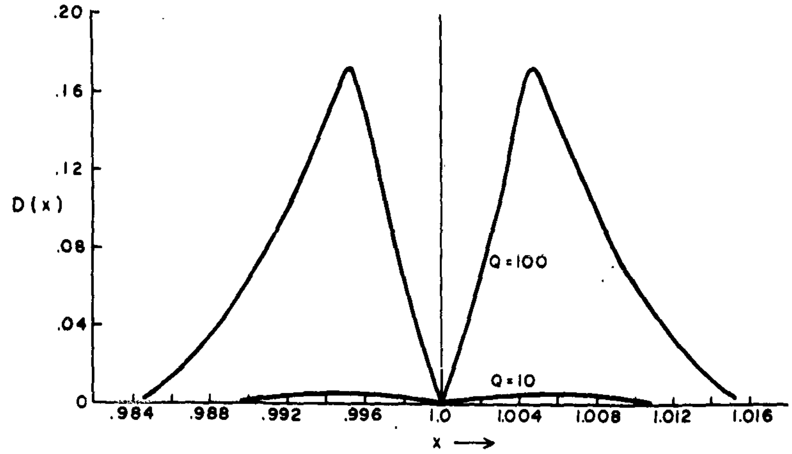

Figure 4. D(x) versus x. (N = 100 r-f cycles to -10-db points.)

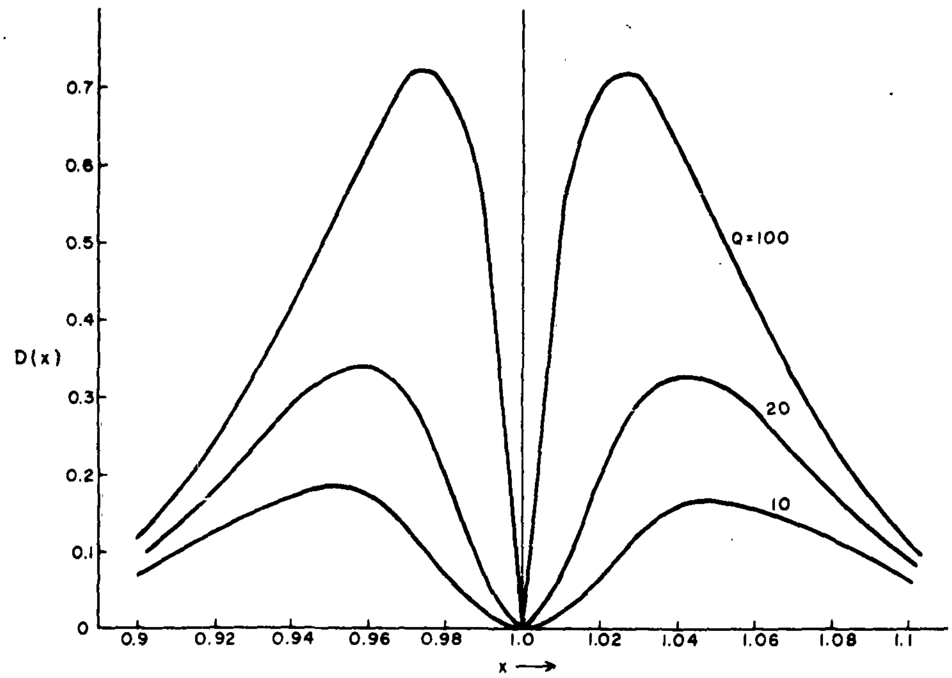

Figure 5. D(x) versus x. (N = 10 r-f cycles to -10-db points.)

the time response curves of Figures 2 and 3 since they are an alternative method of discussing the resonant circuit characteristics.

Two additional observations can be made concerning the $D(\omega/\omega_o)$ curves. The curves are slightly asymmetric about $\omega/\omega_o = 1.0$, the peaks being higher for $\omega < \omega_o$ than for $\omega > \omega_o$. This asymmetry decreases as Q increases, and is negligible for Q values where the distortion is large. Second, the location of the peaks of $D(\omega/\omega_o)$ tend toward $\omega/\omega_o = 1.0$ as Q is increased. A positive value of $D(\omega/\omega_o)$ corresponds, in a sense, to a deficiency in the Fourier spectrum of the output voltage pulse. As the Q is increased, the maximum "deficiency" occurs at a frequency closer to the resonant (and r-f carrier) frequency.