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Continuous Forcing of an On-Off Control System

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SUMMARY OF RESULTS

The objective of this research project has been to conduct analytical investigations of nonlinear systems in the areas of automatic control and mechanics. We have been principally concerned with the study of ordinary differential equations of the piecewise-linear type (such as govern relay or "on-off" control systems) in the presence of time-dependent excitations or input signals. A second phase of the work has been a study of a class of nonlinear wave equations.

Piecewise-Linear Systems: This work has proceeded in several directions. The equation

$$d^2x/dt^2 + x + x/|x| = f(t) \qquad (1)$$

where $f(t)$ is a periodic function of t, has been of particular interest. It has been studied first in [1] *, with $f(t) = A \sin \omega t$ (A and ω constants). Periodic solutions of harmonic and subharmonic types (having periods $2\pi/\omega$ and $2\pi n/\omega$, respectively) are derived in exact form, when $\omega > 1$. The results are extended, in [2], to the cases $\omega \leq 1$; here also the existence of other types of periodic response (ultraharmonic and ultra-subharmonic) are investigated.

The stability of the periodic solutions established in [1] has also been studied. By use of theorems due to Aizerman and Gantmacher those solutions which on heuristic grounds appear to be unstable, have indeed been proved to be unstable. While this (variational) approach yields no conclusion concerning the apparently stable solutions, which fall into the so-called "critical" case, the latter have at last been proved to be stable, by applying a new result by Moser on Hamiltonian systems. These stability results are presently being written up; they comprise Mr. Gustavson's doctoral dissertation [3].

Equation (1) is again considered in [4], where now $f(t)$ is an arbitrary continuous periodic function of t. Periodic responses of harmonic and subharmonic type are obtained in exact form. In addition it is shown that for certain classes of periodic inputs various superposition properties hold. Of course, this is not to be expected in a

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* Numbers in brackets refer to list of "Reports" below.

nonlinear system -- and indeed it is not possible with continuous nonlinearities. Thus, piecewise-linear systems may display both nonlinear features (subharmonic oscillations) and linear features (superposition). An auxiliary theorem, on phase relations in a forced harmonic oscillator, is proved in [5]. Finally, the superposition principle established in [4] is extended in [6] ; it is shown to apply to any system of piecewise-linear differential equations of the relay type with periodic inputs.

Nonlinear Mechanics: This phase of the project has been concerned with wave propagation in media characterized by wave equations of the form

$$u_{tt} - u_{xx} = f(u,u_x,u_t). \qquad (2)$$

Some general properties of wave solutions of (2), for certain classes of functions $f(u,u_x,u_t)$, have been developed in [7] .

DOCTORAL CANDIDATE

Mr. Fred Gustavson, whose research was for several years supported by the subject contract and grant, is now having his doctoral dissertation (see [3] of "Reports") typed up in final form. He is expected to receive his degree in June, 1963.

REPORTS

1. Fleishman, B. A., "Harmonic and Subharmonic Response of an On-Off Control System to Sinusoidal Inputs," J. Franklin Inst. 270 (2), 99-113 (1960).

2. Gustavson, F., "Various Types of Forced Oscillations of an On-Off Control System." Not yet published.

3. Gustavson, F., "The Stability of Periodic Response of an On-Off Control System to Sinusoidal Inputs," doctoral dissertation.

4. Fleishman, B. A., "Periodic Response and Superposition in an On-Off Control System," J. Math. Anal. & Appl. 5(2), 306-315 (1962).

5. Fleishman, B. A., "A Generalization of the Phase Relations in a

Forced Harmonic Oscillator," J. Franklin Inst. 272 (5), 360-5 (1961).

6. Fleishman, B. A., "Limited Superposition in Piecewise-Linear Systems," to appear in "J. Math. Anal. & Appl."

7. Fleishman, B. A., "Wave Propagation in Non-Simple Media," article in the book "Nonlinear Differential Equations and Nonlinear Mechanics", Academic Press, 1963.