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NO. 070

ELECTRONIC EQUIPMENT INTERFERENCE
CHARACTERISTICS -- RADAR TYPES

Report Nr. 3
Contract Nr. DA 36-039 SC-89102
Third Quarterly Progress Report
15 November 1962 to 14 February 1963
United States Army Electronics
Research and Development Laboratory
ARF Project Nr. E178
Armour Research Foundation
of Illinois Institute of Technology
Chicago 16, Illinois

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ELECTRONIC EQUIPMENT INTERFERENCE

CHARACTERISTICS -- RADAR TYPES

Report Nr. 3
Contract Nr. DA 36-039 SC-89102
Technical Requirement Nr. SCL-4237
Date: 22 April 1960
DA Task 3B24 01 001 01
Third Quarterly Progress Report
15 November 1962 to 14 February 1963

The object of this program is an investigation to determine the characteristics of U. S. Army radar equipment deemed necessary for the prediction and minimizing of radio interference.

by:
M. W. Scheldorf
M. Sherman

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ELECTRONIC EQUIPMENT INTERFERENCE
CHARACTERISTICS -- RADAR TYPES

Third Quarterly Report
Contract DA 36-039 SC-89102
ARF Project E178
15 November 1962 - 14 February 1963

I. PURPOSE

The purpose of this program is to investigate and develop measurement procedures that will define the interference characteristics of Army Search Type Radars. This investigation is based on the USAELRDL's recognition of the need for approaches to the solution of the crowding of the electromagnetic spectrum and the resultant unintentional interference problems which result. This program constitutes a continuation of effort carried out by Armour Research Foundation under Contract Nos. DA 36-039 SC-78022 and DA 36-039 SC-87176. These programs laid the ground work for establishing system parameters to be measured, measurement techniques and data format for radar emanation and susceptibility characteristics. The results obtained throughout these programs indicated that significant improvements in radar system measurements could be obtained by additional investigation in this area.

II. ABSTRACT

A matrix representation of a radar system has been formulated which characterizes the radar's transmitting and receiving interference properties under multimode conditions.

III. CONFERENCES

Date:	December 11, 1963
Place:	Armour Research Foundation, Chicago, Illinois
Personnel:	W. Kesselman, USAELRDL J. T. Ludwig, ARF

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The results of the two mode antenna measurements and the requirements for the instrumentation van were discussed.

IV. FACTUAL DATA

A. Introduction

During this period an equivalent circuit representation was formulated for the RF system of the radar under the conditions of multimode propagation. This analysis can be applied to relate the transmitter and receiver measurements to the over-all radar interference characteristics.

The experimental investigations that were made on a two mode antenna system during this period will be reported in the next quarterly report.

B. General Multimode Matrix Representation

The in-the-guide measurement procedure divides the radar system into two parts, (1) the antenna and (2) everything beyond the antenna, and separate measurements are made on each part. In the transmitting case, everything beyond the antenna consists of the magnetron and the waveguide structure connecting the magnetron to the antenna. In the receiving case it consists of the waveguide connecting the antenna to the receiver. The following analysis can be applied to any multimode microwave structure. The theoretical

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basis for this multimode representation is given in the appendices. A review of the multimode matrix notation and techniques is given below.

A waveguide in which N modes can propagate is equivalent to N waveguides in each of which only one propagating mode can exist. The basis for this statement is the orthogonality of the waveguide modes which ensures the absence of cross terms involving the fields of two or more modes in the expression for the power flow. An arbitrary discontinuity at which reflection, transmission, and mode conversion of the N propagating modes occurs is equivalent to a junction with N input and N output waveguides. The description of such a junction in terms of incident and reflected waves is:

$$\begin{bmatrix} b^m \\ p \end{bmatrix} = \begin{bmatrix} S^{mn} \\ pq \end{bmatrix} \begin{bmatrix} a^n \\ q \end{bmatrix} \quad (1)$$

where

$$\begin{bmatrix} b^m \\ p \end{bmatrix} \text{ is the column matrix of the N reflected waves.}$$

$$\begin{bmatrix} a^n \\ q \end{bmatrix} \text{ is the column matrix of the N incident waves.}$$

$$\begin{bmatrix} S^{mn} \\ pq \end{bmatrix} \text{ is the N x N matrix relating the incident and reflected waves.}$$

The superscripts are mode identification numbers. Each number corresponds to a specific mode. The subscripts are the physical port identification numbers. For example, b_2^1 is the reflected wave in mode 1 at port 2; a_q^n is the incident wave in mode n at port q. The elements of the scattering matrix, S_{pq}^{mn} , relate the reflected waves to the incident waves; that is, S_{12}^{34} is equal to b_1^3/a_2^4 when all ports are matched and the only incident wave

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is a_2^4 . The wave amplitudes are normalized so that $1/2 a_q^n a_q^{*n}$ is the average incident power in the n^{th} mode and correspondingly for the reflected waves.

To make evident the input and output nature of the multimode junction, the scattering matrix in Eq. 1 may be subdivided into four submatrices as follows:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2)$$

The submatrix $[S_{11}]$ contains elements that relate the incident and reflected waves in port 1; similarly for $[S_{22}]$ and port 2. The submatrices $[S_{12}]$ and $[S_{21}]$ contain the transfer coefficients of the junction and relate the incident waves at one port to the reflected waves at the other port.

The method of determining the resultant scattering matrix of two cascaded multimode networks from the scattering matrices of the individual networks is described in Appendix B. Figure B1 shows two cascaded multimode networks. Each propagating mode is represented by an individual port. The scattering matrices of the networks are $[S]$ and $[T]$ and the resultant scattering matrix of the cascaded networks is $[U]$. The submatrices of the resultant matrix, expressed in terms of the submatrices of $[S]$ and $[T]$, are:

$$U_{11} = S_{11} + S_{12} (1 - T_{11} S_{22})^{-1} T_{11} S_{21} \quad (3a)$$

$$U_{12} = S_{12} (1 - T_{11} S_{22})^{-1} T_{12} \quad (3b)$$

$$U_{21} = T_{21} (1 - S_{22} T_{11})^{-1} S_{21} \quad (3c)$$

$$U_{22} = T_{22} + T_{21} (1 - S_{22} T_{11})^{-1} S_{22} T_{12} \quad (3d)$$

The brackets denoting matrix quantities have been omitted for clarity. The preceding analysis will now be used to analyze radar transmitter and receiver characteristics at frequencies where higher order modes can propagate in the RF system.

C. Matrix Representation Applied to Radar Systems

I. Introduction

The principles involved in the following discussion are applicable to any radar system. As a specific illustration, however, reference will be made to M-33 Acquisition Radar which is currently being tested. A block diagram of the RF system of the M-33 Radar is shown in Fig. 1. The measurement technique to determine interference characteristics has been to separate the radar at terminals XX in Fig. 1 and make tests on the individual parts.

The purpose of this section is two-fold: (1) to characterize the interference characteristics of the separate parts of the radar in matrix form, and (2) to determine the over-all interference characteristics from the separate measurements.

2. Multimode Antenna

In the present program the important properties of an antenna which is capable of propagating more than one mode at its waveguide output are:

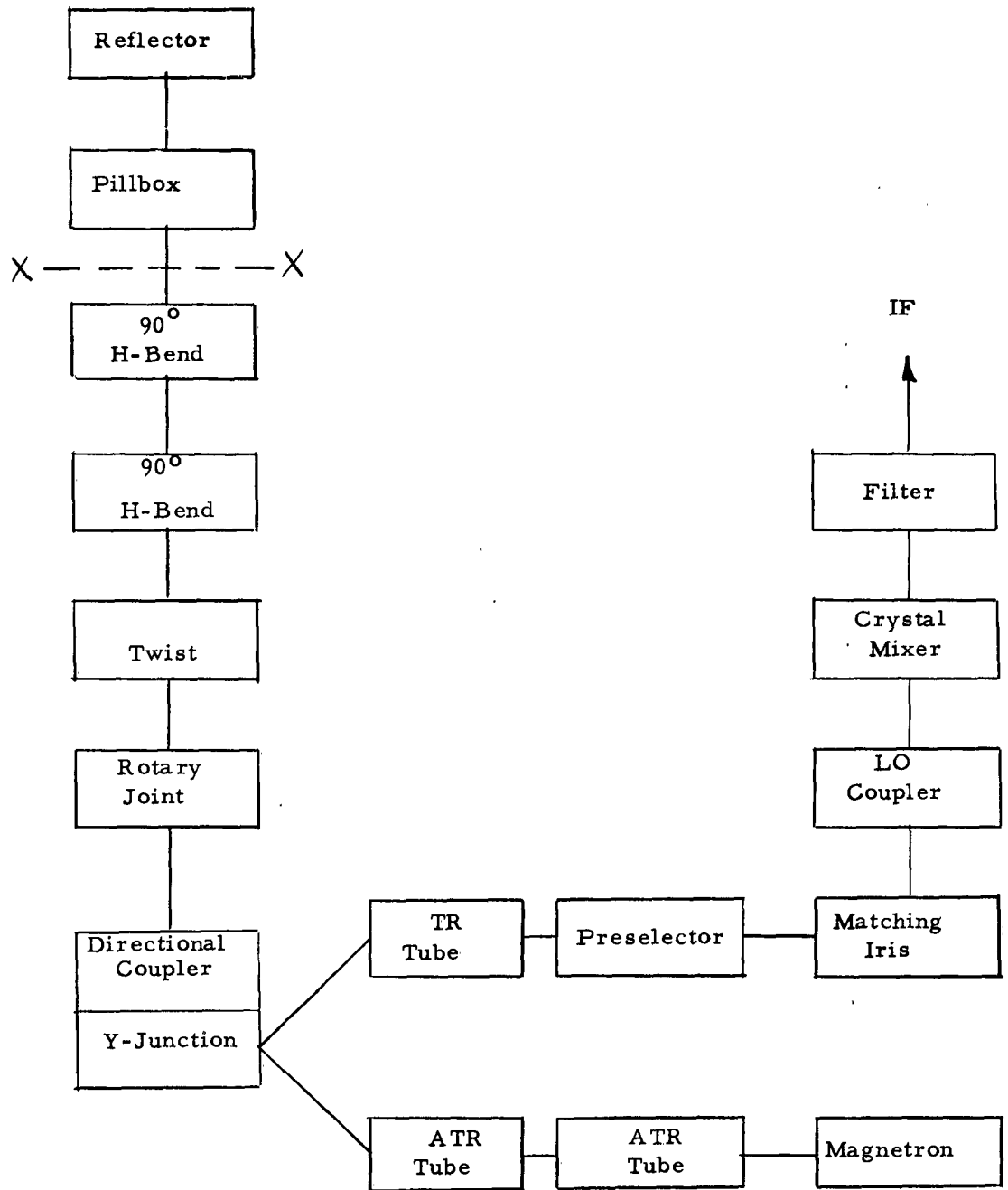


FIG. 1 RF SECTION OF M-33 ACQUISITION RADAR

1. The ratio of power in the different modes at the output waveguide of a receiving antenna when a plane wave is radiated at the antenna.
2. The radiation pattern of an antenna which is excited at its input by more than one propagating mode.

A matrix which describes these multimode antenna properties has been formulated in Appendix C. The matrix relation for an N mode system is:

$$\begin{bmatrix} b_1^0(\theta, \phi) \\ b_2^n \end{bmatrix} = \begin{bmatrix} \text{---} & | & k_1 [G_V^n(\theta, \phi)] \\ \text{---} & | & \text{---} \\ k_2 \lambda_0 [G_V^n(\theta, \phi)] & | & [S_{22}^{11}] \end{bmatrix} \begin{bmatrix} a_1^0(\theta, \phi) \\ a_2^n \end{bmatrix} \quad (4)$$

where

(θ, ϕ) are the spherical coordinates (antenna at origin) giving the propagation direction of the plane wave with respect to the antenna.

$a_1^0(\theta, \phi)$ is the amplitude of the free space wave traveling towards the antenna in the θ, ϕ direction.

$b_1^0(\theta, \phi)$ is the amplitude of the free space wave propagating away from the antenna in the θ, ϕ direction.

$\begin{bmatrix} a_2^n \end{bmatrix}$ is the column matrix of the N incident modes at the waveguide side of the antenna.

$\begin{bmatrix} b_2^n \end{bmatrix}$ is the column matrix of the N reflected waves at the waveguide side of the antenna.

$\begin{bmatrix} G_V^n(\theta, \phi) \end{bmatrix}$ is a row matrix, the elements of which are the complex voltage gains for each mode.

k_1, k_2 are real proportionality factors.

$\begin{bmatrix} S_{22}^{11} \end{bmatrix}$ is the standard multimode scattering submatrix at the waveguide side of the antenna.

The element in the first row and first column has been omitted since it is irrelevant to the interference analysis. It relates the incident wave in the θ, ϕ direction to the wave scattered back in the same direction.

From this matrix the following conclusions can be drawn (see Appendix C):

1. The modal content of the output terminals of a multimode antenna is a function of the direction of the incoming plane wave.
2. When a plane wave is incident on the antenna, the ratio of modal amplitudes generated at the output terminals of the antenna (into a matched load) is in direct proportion to the single mode voltage gains of the antenna.
3. The radiated field of a multimode antenna is a linear superposition of the radiated fields of the individual modes and thus depends upon the relative amplitudes and phases of the individual modes.

The last two statements apply to an antenna in which the load (receiving state) or the generator (transmitting state) is matched to the waveguide feed. The effects of mismatches are discussed in the next section.

3. Transmitting Radar

In the transmitting state the magnetron output contains harmonics and other spurious frequencies as well as the fundamental. For the frequencies at which multimode propagation can occur, the modal output of the magnetron will depend upon its physical characteristics. This modal content will be changed as it travels through the ATR tubes, the Y-junction and the remainder of the RF

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system (Fig. 1), the latter being common to both the transmitter and the receiver. If the radar is separated at terminals XX and the modal content at a single spurious frequency is determined, then the radiation pattern at this frequency can be related to the measured modal content. An equivalent circuit for the radar in the transmitting state is shown in Fig. 2. Using the flow graph technique in Appendix B, the expression for the radiated plane wave amplitude in terms of the measured modal content of the separated system (into a matched load) is

$$b_1^0(\theta, \phi) = k_1 \left[G_V^n(\theta, \phi) \right] \left[(1 - [T_{11}] [S_{22}])^{-1} \right] \left[a_2^n \right] \quad (5)$$

where

$b_1^0(\theta, \phi)$ is the radiated plane wave amplitude.

$\begin{bmatrix} a_1^n \\ a_2^n \end{bmatrix}$ is a column matrix which has the measured complex wave amplitudes as elements.

$[S_{22}]$ is the antenna "reflection" submatrix at its waveguide terminals.

$[T_{22}]$ is the "reflection" submatrix of the radar system at the terminals where the modal content was measured.

$[G_V^n(\theta, \phi)]$ is the row matrix of modal voltage gains.

k_1 is a proportionality factor.

The matrix $\left[(1 - T_{11} S_{22})^{-1} \right]$ accounts for the multiple reflections between the two sections at which the radar was separated and the measurements were taken.

4. Receiving Radar

In the receiving state, a signal, other than the desired signal, which is incident upon the receiving antenna can also transfer energy into the RF system. At frequencies where higher order modes can propagate the antenna will launch

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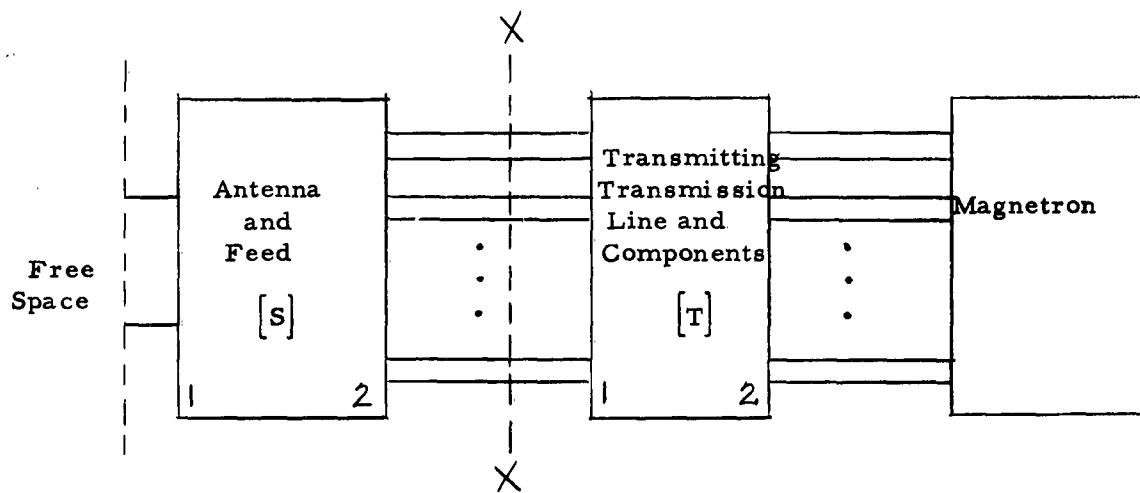


FIG. 2. EQUIVALENT TRANSMITTING CIRCUIT

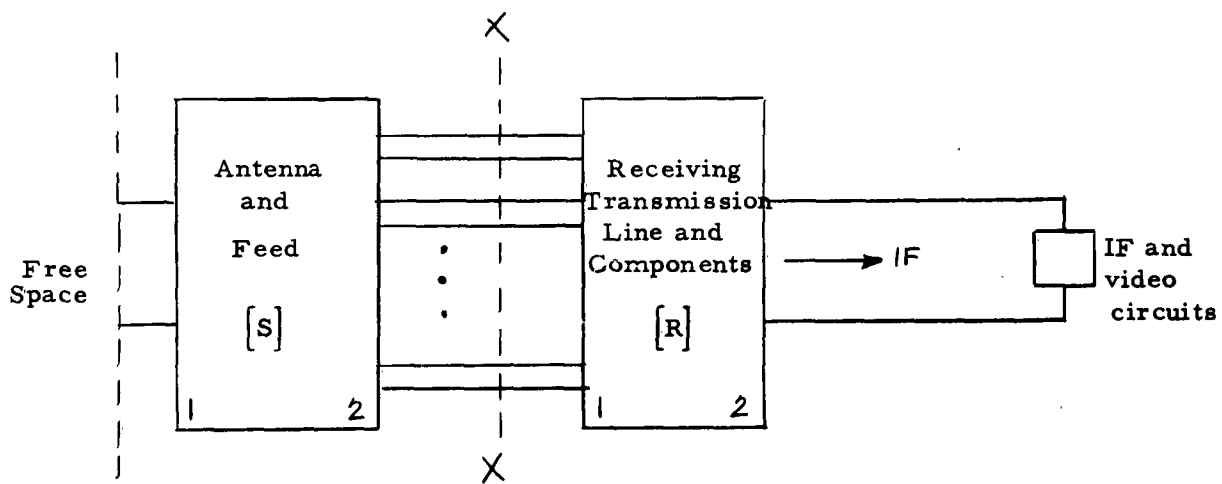


FIG. 3. EQUIVALENT RECEIVING CIRCUIT

the energy in the different modes. The modal content will be changed as it travels down the RF system. The percentage of this energy that is converted to the intermediate frequency and causes a spurious response on the radar scope depends upon the frequency of the signal and the modal content of the energy at the mixer.

A matrix description of this process can be formulated with the aid of the equivalent receiving circuit in Fig. 3. The radar is again separated at terminals XX and the spurious response measurements are made at this point. The over-all spurious response to a plane wave is

$$b_{1F}(\theta, \phi) = k_2 \lambda_o \left[G_V^n(\theta, \phi) \right] \left[\left(1 - \begin{bmatrix} R_{11} \end{bmatrix} \begin{bmatrix} S_{22} \end{bmatrix} \right)^{-1} \right] \begin{bmatrix} R_{21} \end{bmatrix} a_1^0(\theta, \phi) \quad (6)$$

where

$a_1^0(\theta, \phi)$ is the incident plane wave.

b_{1F} is a measure of the strength of the spurious response.

$\begin{bmatrix} R_{21} \end{bmatrix}$ is the "transmission" matrix between terminals XX and the receiver output.

$\begin{bmatrix} S_{22} \end{bmatrix}$ is the antenna "reflection" submatrix at its waveguide terminals.

$\begin{bmatrix} R_{11} \end{bmatrix}$ is the "reflection" submatrix of the receiving system at terminals XX.

$\begin{bmatrix} G_V^n(\theta, \phi) \end{bmatrix}$ is the row matrix of modal voltage gains.

$k_2 \lambda_o$ is a proportionality factor.

The spurious response is equal to the product of the matrix transfer functions of the two separate sections of the radar system, modified by a matrix which accounts for the multiple reflections between the two separate sections.

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V. CONCLUSIONS

A mathematical model for describing radar interference generation and susceptibility characteristics under multimode conditions has been formulated and applied to the in-the-guide measurements. The analysis relates the measurements on the separate sections of the radar to the over-all interference characteristics and forms a basis for the types of measurements that should be made and the reliability that can be placed on a limited number of measurements. Future experimental investigations will be concerned with evaluating approximations that can be used to simplify the exact mathematical model.

IV. PROGRAM FOR NEXT INTERVAL

It is expected that effort during the next period will be devoted to the following areas:

1. Continued development of mode launchers and mode couplers for antenna and receiver testing.
2. Dominant mode spurious response measurements at different receiver tuned frequencies.
3. Initiation of receiver intermodulation and desensitization tests.
4. Additional measurements on multimode antennas.

VII. IDENTIFICATION OF TECHNICAL PERSONNEL

The following ARF staff members have worked on this program during this period:

	<u>Man-Hours</u>
J. T. Ludwig, Research Engineer, Acting Manager	116
M. W. Scheldorf, Senior Research Engineer	381
R. D. Standley, Research Engineer	28
M. Sherman, Assistant Engineer	386
W. A. Grew, Technician	363

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	<u>Man-Hours</u>
W. Lancaster, Technician	255
S. Tumarkin, Technician	98

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
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M. Sherman

M. Sherman, Assistant Engineer

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APPROVED:

J. E. McManus

J. E. McManus, Assistant Director
Electronics Research

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APPENDIX A

SCATTERING MATRIX REPRESENTATION OF A MULTIMODE JUNCTION

The formal basis for the scattering matrix representation of microwave junctions may be found in the literature [1 - 4]. The extension to waveguides capable of propagating more than one mode and the notation employed in this case will be briefly reviewed in this appendix.

Consider a junction excited by a single waveguide (or any generalized transmission line) along which N modes may propagate. To completely describe the conditions in a given waveguide it is necessary to introduce a voltage and current for each mode in the guide. The total transverse field at a specified transverse section (called the terminal pair) is obtained by summing over the fields of the individual modes.

$$\begin{aligned} E_x &= \sum_{n=1}^N e_n f_x^{(n)}(x, y) \\ E_y &= \sum_{n=1}^N e_n f_y^{(n)}(x, y) \\ H_x &= \sum_{n=1}^N i_n g_x^{(n)}(x, y) \\ H_y &= \sum_{n=1}^N i_n g_y^{(n)}(x, y) \end{aligned} \tag{A1}$$

$f_x^{(n)}(x, y)$ is a real function of the coordinates which describes the distribution of the x -component of the electric field in the n^{th} mode over the terminals. Similar statements apply to the other functions. The normalizing parameters e_n and i_n are so chosen that

$$\int \left(f_x^n g_y^n - f_y^n g_x^n \right) dx dy = -1 \text{ for all } n \quad (A2)$$

Since the transverse electric and magnetic fields for one mode are orthogonal to the transverse electric and magnetic fields, respectively, of any other modes it can be shown that (1)

$$\int \left(f_x^n g_y^m - f_y^m g_x^n \right) dx dy = 0 \text{ for } n \neq m \quad (A3)$$

This choice of parameters e_n and i_n result in the same connection between terminal energy quantities that is obtained in the single mode guide. Hence, this terminal-parameter description of a waveguide with N propagating modes is equivalent to the description of N single mode waveguides. A junction or discontinuity with one input and one output waveguide, in each of which N modes may propagate, is equivalent to a junction with N input and N output terminal pairs, and the network theory of 2N ports may be applied to this multimode case. Thus, the wave formalism commonly used in the scattering matrix representation of a two-port network can be extended to the case of a microwave junction with N propagating modes.

Let a_q^n be a complex number representing the amplitude and phase of the transverse electric field of the incident in the n^{th} mode at the q^{th} port. Let b_p^m be the corresponding measure of the emergent wave. It is assumed that a_q^n and b_p^m are normalized in such a way that $1/2 a_q^n a_q^{*n}$ is the average incident power in the n^{th} mode and correspondingly for b_p^m . The scattering matrix representation of the multimode junction (N propagating modes in each port) in terms of the incident and reflected waves is:

$$\begin{bmatrix} b_p^m \end{bmatrix} = \begin{bmatrix} S_{mn} \\ S_{pq} \end{bmatrix} \begin{bmatrix} a_q^n \end{bmatrix} \quad (A4)$$

where

$\begin{bmatrix} b \\ p \end{bmatrix}^m$ is the column matrix of the N reflected waves.

$\begin{bmatrix} a \\ q \end{bmatrix}^n$ is the column matrix of the N incident waves.

$\begin{bmatrix} S \\ pq \end{bmatrix}^{mn}$ is an $N \times N$ matrix relating the incident and reflected waves.

The superscripts are mode identification numbers and the subscripts are the physical port identification numbers. For example, b_2^1 is the reflected wave in mode 1 at port 2; a_q^n is the incident wave in mode n at port q . The elements of the scattering matrix, S_{pq}^{mn} , relate the reflected waves to the incident waves as in the usual scattering matrix notation; that is, S_{12}^{34} is equal to b_1^3/a_2^4 when all ports are matched and the only incident wave is a_2^4 .

The scattering matrix is necessarily a square matrix; its order is given by

$$N = \sum_p M_p \quad (A5)$$

where

N is the order of the square matrix

M_p is the number of propagating modes in the p^{th} port.

To make evident the input and output nature of the multimode junction, the scattering matrix in Equation A4 may be subdivided into four submatrices as follows:

$$\begin{bmatrix} b_1^m \\ b_2^m \end{bmatrix} = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{bmatrix} a_1^n \\ a_2^n \end{bmatrix} \quad (A6)$$

The submatrix $[S_{11}]$ contains elements that relate the incident and reflected waves in port 1; similarly for $[S_{22}]$ and port 2. Submatrices $[S_{12}]$ and $[S_{21}]$

contain the transfer coefficients of the junction and relate the incident waves at one port to the reflected waves at the other port.

A few examples will serve to elucidate the preceding discussion. Consider a two-port microwave junction with two propagating modes in each port. From Equation A5 the order of the scattering matrix is four. The matrix equation for the incident and reflected waves is:

$$\begin{bmatrix} b_1^1 \\ b_1^2 \\ b_2^1 \\ b_2^2 \end{bmatrix} = \begin{bmatrix} S_{11}^{11} & S_{11}^{12} & S_{12}^{11} & S_{12}^{12} \\ S_{11}^{21} & S_{11}^{22} & S_{12}^{21} & S_{12}^{22} \\ S_{21}^{11} & S_{21}^{12} & S_{22}^{11} & S_{22}^{12} \\ S_{21}^{21} & S_{21}^{22} & S_{22}^{21} & S_{22}^{22} \end{bmatrix} \begin{bmatrix} a_1^1 \\ a_1^2 \\ a_2^1 \\ a_2^2 \end{bmatrix} \quad (A7)$$

The submatrices are:

$$\begin{aligned} [S_{11}] &= \begin{bmatrix} S_{11}^{11} & S_{11}^{12} \\ S_{11}^{21} & S_{11}^{22} \end{bmatrix} & [S_{12}] &= \begin{bmatrix} S_{12}^{11} & S_{12}^{12} \\ S_{12}^{21} & S_{12}^{22} \end{bmatrix} \\ [S_{21}] &= \begin{bmatrix} S_{21}^{11} & S_{21}^{12} \\ S_{21}^{21} & S_{21}^{22} \end{bmatrix} & [S_{22}] &= \begin{bmatrix} S_{22}^{11} & S_{22}^{12} \\ S_{22}^{21} & S_{22}^{22} \end{bmatrix} \end{aligned}$$

Next consider a coaxial-to-waveguide transition. The order of the scattering matrix is three, and the equation for this junction is:

$$\begin{bmatrix} b_1^0 \\ b_2^1 \\ b_2^2 \end{bmatrix} = \begin{bmatrix} S_{11}^{00} & S_{12}^{01} & S_{12}^{02} \\ S_{21}^{10} & S_{22}^{11} & S_{22}^{12} \\ S_{21}^{20} & S_{22}^{21} & S_{22}^{22} \end{bmatrix} \begin{bmatrix} a_1^0 \\ a_2^1 \\ a_2^2 \end{bmatrix}$$

The submatrices are:

$$\begin{aligned} [S_{11}] &= S_{11}^{00} & [S_{12}] &= \begin{bmatrix} S_{12}^{01} & S_{12}^{02} \\ S_{22}^{11} & S_{22}^{12} \end{bmatrix} \\ [S_{21}] &= \begin{bmatrix} S_{21}^{10} \\ S_{21}^{20} \\ S_{21}^{21} \end{bmatrix} & [S_{22}] &= \begin{bmatrix} S_{22}^{11} & S_{22}^{12} \\ S_{22}^{21} & S_{22}^{22} \end{bmatrix} \end{aligned}$$

The scattering matrix has several general properties of importance (1) which are listed below.

1. The scattering matrix of a linear bilateral junction is always symmetrical.

$$[S] = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} = [S]^T = \begin{bmatrix} [S_{11}]^T & [S_{21}]^T \\ [S_{12}]^T & [S_{22}]^T \end{bmatrix}$$

2. The scattering matrix of a lossless linear, bilateral, junction unitary.

$$[S]^{-1} = [S^*]^T$$

3. The scattering matrix of a junction with losses satisfies the following condition:

$$\det [I - [S^*][S]] \geq 0$$

and similiary for the determinants of each of the principle minors.

APPENDIX B

SCATTERING MATRIX REPRESENTATION OF TWO CASCADED MULTIPOINTS

It has been shown in Appendix A that a junction or discontinuity with one input and output waveguide, in each of which N modes may propagate, is equivalent to a junction with N input and N output ports. Therefore, the properties of cascaded multimode junctions can be deduced from the theory of cascaded multipoints.

The formulation of the resultant scattering matrix of two cascaded multipoints from the scattering matrices of the individual multipoints will now be derived. Using the notation in Appendix A and Figure B1 and by virtue of

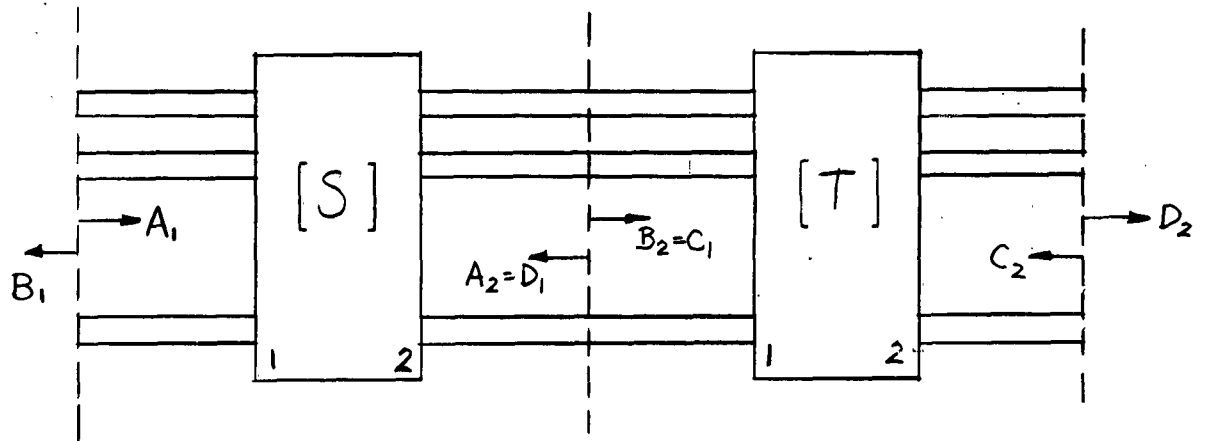


FIG. B1. TWO CASCADED MULTIPOINTS

the superposition principle and the definition of the scattering matrix, the following matrix equations describe the cascaded junction (with $C_2 = 0$):

$$A_2 = T_{11} B_2$$

$$B_2 = S_{21} A_1 + S_{12} A_2$$

$$D_2 = T_{21} B_2$$

The waves are represented by a column matrix which has one element for each port. The brackets denoting matrix quantities has been omitted for clarity. The solution of these equations yields:

$$U_{21} = D_2 A_1^{-1} = T_{21} \left(1 - S_{22} T_{11} \right)^{-1} S_{21} \quad (B1)$$

$$U_{11} = B_1 A_1^{-1} = S_{11} + S_{12} \left(1 - T_{11} S_{22} \right)^{-1} T_{11} S_{21} \quad (B2)$$

A similar set of linear equations can be written assuming $A = 0$.

The solution of this system yields:

$$U_{12} = B_1 C_2^{-1} = S_{12} \left(1 - T_{11} S_{22} \right)^{-1} T_{12} \quad (B3)$$

$$U_{22} = D_2 C_2^{-1} = T_{22} + T_{21} \left(1 - S_{22} T_{11} \right)^{-1} S_{22} T_{12} \quad (B4)$$

The resultant scattering matrix of the cascaded junction is:

$$[U] = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \quad (B5)$$

where the submatrices of U are defined in terms of the submatrices of S and T in Equations B1 - B4.

The resultant scattering matrix of two cascaded multiports can also be derived by signal flow graph analysis (5). This method simplifies the the mechanics of solving the system of linear equations and more clearly exhibits the physical phenomenon of multiple reflections. A flow graph diagram for the cascaded networks in Fig. B1 is shown in Fig. B2. A_1 and B_1 are the column matrices of the incident and reflected waves, respectively, at the left side of the cascaded network; similarly for C_2 and D_2 at the right side of the network. The branch from B_2 to C_1 has a directed "gain" of 1 (the identity

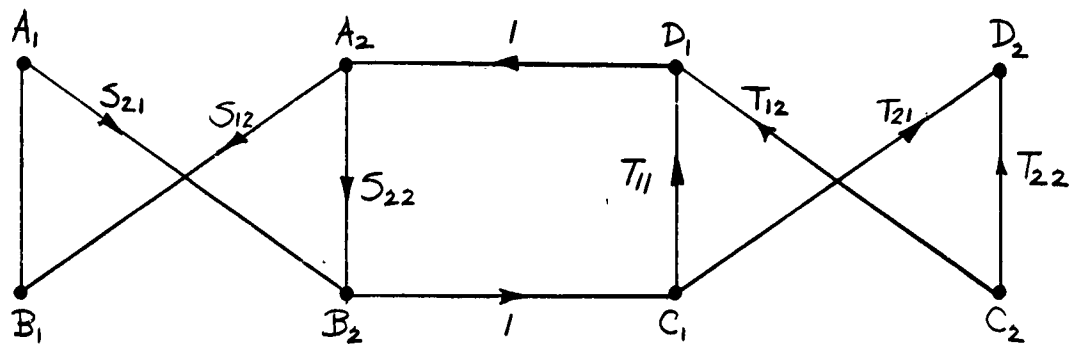


FIG. B2. SIGNAL FLOW GRAPH OF TWO CASCADED MULTIPOINTS

matrix) and similar for D_1 and A_2 . This represents the waveguide connection of the separate multiports and has a "gain" of 1 since it is assumed that the waveguides themselves are matched. If the waveguides were not matched (e. g., two different size waveguides) this connection would be represented by an additional scattering matrix.

The resultant scattering matrix can be determined visually by using the standard flow graph rules. As an illustration, the "gain" or transfer function from node A_1 to node C_2 (which is equal to $U_{21} = D_2 A_1^{-1}$) will be derived. The pertinent nodes and branches are redrawn in Fig. B3. The closed loop $B_2 C_1 D_1 A_2$ accounts for the effects of multiple reflections and is equivalent

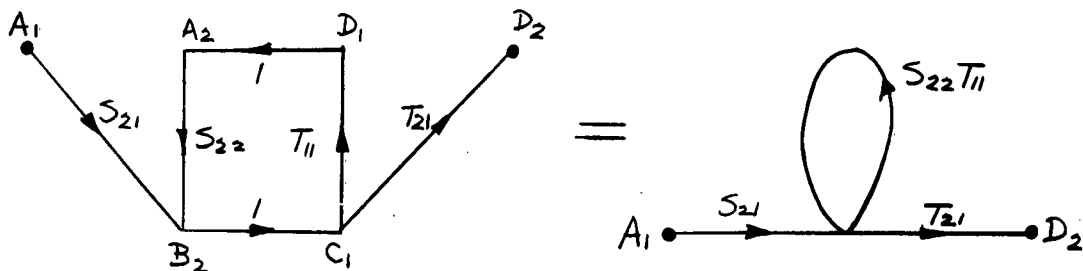


FIG. B3. TRANSFER FUNCTION FROM A_1 TO D_2

to feedback in a control system circuit. The transfer function U_{21} can be written by inspection as,

$$U_{21} = T_{21} \left(1 - S_{22} T_{11} \right)^{-1} S_{21} \quad (B6)$$

which is identical to the previously determined transfer function.

The resultant scattering matrix U is symmetrical when the separate multiports are linear and bilateral. Thus,

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} = U^T = \begin{bmatrix} U_{11}^T & U_{21}^T \\ U_{12}^T & U_{22}^T \end{bmatrix} \quad (B7)$$

APPENDIX C

MULTIMODE ANTENNA ANALYSIS

The purpose of this appendix is to characterize an antenna, capable of propagating higher order modes at its waveguide terminals, in a manner which exhibits its relevant interference properties. These properties are:

1. The ratio of power in the different modes at the output waveguide of a receiving antenna when a plane wave is incident upon the antenna.
2. The radiation pattern of an antenna which is excited by more than one mode at its waveguide terminals.

A one mode transmitting and receiving system is first analyzed as a coupled four-terminal impedance network. The antenna characteristics are then written in matrix form which is extended to include multimode propagation.

Consider the one mode system in Fig. C1(a) and its equivalent representation in Fig. C1(b). The magnitude of the coupling impedance from transmitter to receiver is (6):

$$|Z_{21}|^2 = \frac{R_{r1} R_{r2} G_1}{\pi r^2} \cdot \frac{\lambda_o^2}{4\pi} G_2 \quad (C1)$$

where

- Z_{21} = transmitter-receiver coupling impedance
- R_{r1} = transmitting antenna input resistance
- R_{r2} = receiving antenna load resistance
- r = distance between transmitting and receiving antennas
- λ_o = free space wavelength
- G_1 = transmitting antenna power gain

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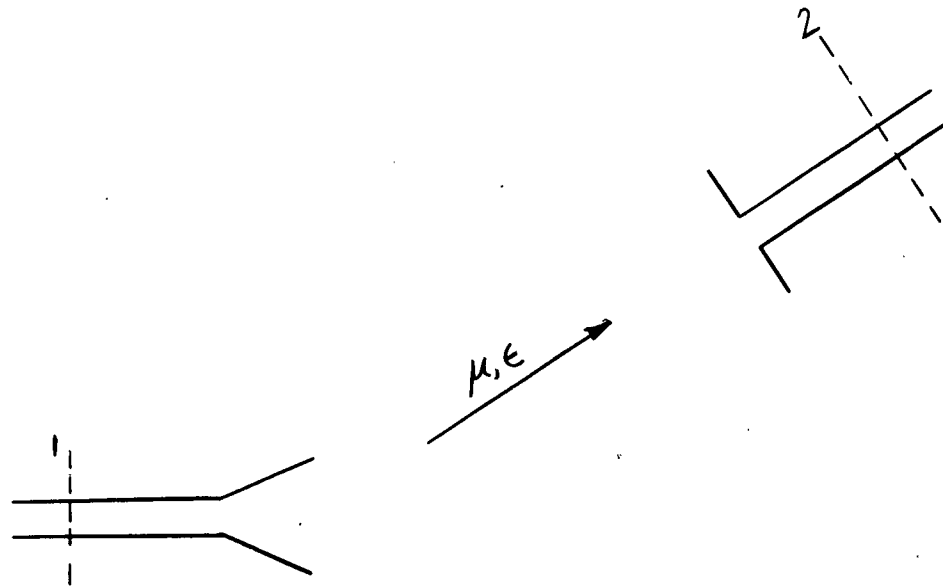


FIG. C1(a). A SYSTEM OF TRANSMITTING AND RECEIVING ANTENNAS

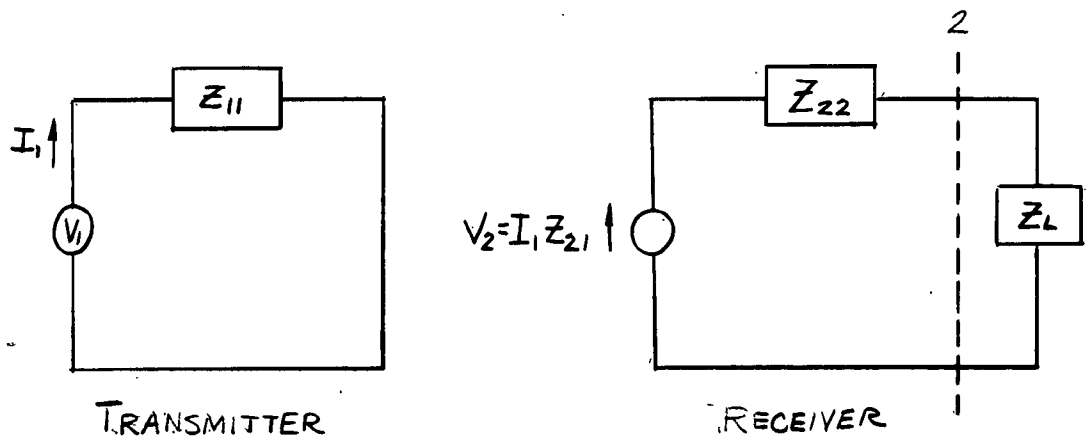


FIG. C1(b). APPROXIMATE EQUIVALENT CIRCUIT NEGLECTING REACTION OF RECEIVER BACK ON TRANSMITTING SYSTEM

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G_2 = receiving antenna power gain.

The second factor in Eq. C1 is the effective receiving area of the antenna with a matched load.

From the above analysis a wave matrix that describes the antenna properties can be formulated. The propagation directions of the waves and the receiving antenna are shown in Fig. C2. The equation for this system

is

$$\begin{bmatrix} b_1^0(\theta, \phi) \\ b_2^1 \end{bmatrix} = \begin{bmatrix} \text{---} & | & k_1 G_V^1(\theta, \phi) \\ \text{---} & + & \text{---} \\ k_2 \lambda_o G_V^1(\theta, \phi) & | & S_{22}^{11} \end{bmatrix} \begin{bmatrix} a_1^0(\theta, \phi) \\ a_2^1 \end{bmatrix} \quad (C2)$$

where

$a_1^0(\theta, \phi)$ is the complex amplitude of the free space wave traveling towards the antenna in the θ, ϕ direction.

$b_1^0(\theta, \phi)$ is the complex amplitude of the free space wave propagating away from the antenna in the θ, ϕ direction.

a_2^1 is the complex amplitude of mode 1 propagating (in the waveguide) towards the antenna.

b_2^1 is the complex amplitude of mode 1 propagating (in the waveguide) away from the antenna.

$G_V^1(\theta, \phi)$ is the complex voltage gain of the antenna for mode 1.

k_1, k_2 are real proportionality factors.

S_{22}^{11} is the usual scattering matrix reflection coefficient at port 2.

The element in the first row and first column has been omitted since it is irrelevant to the interference analysis. It relates the incident wave in the θ, ϕ direction to the wave scattered back in the same direction (along the same ray).

This matrix representation differs from the standard scattering matrix because it does not relate the total fields at both ports. Rather it relates total fields in the wave to the far field along a ray (θ, ϕ direction).

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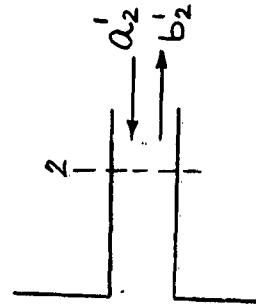
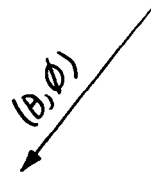
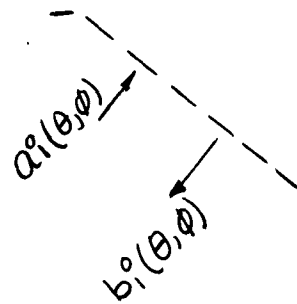


FIG. C2. RECEIVING ANTENNA AND FAR FIELD WAVES IN THE θ, ϕ DIRECTION

where

$[a_2^n]$ is the column matrix of the N incident modes at the waveguide side of the antenna.

$[b_2^n]$ is the column matrix of the N reflected waves at the waveguide side of the antenna.

$[G_V^n(\theta, \phi)]$ is a row matrix, the elements of which are the complex voltage gains for each mode.

$[S_{22}^{11}]$ is the standard multimode scattering submatrix (see Appendix A).

The other symbols have been defined previously:

The extension of Eq. C3-C6 are:

A. Receiving Properties

1. The amplitude of the n^{th} waveguide mode, b_2^n , generated by a plane wave from the θ, ϕ direction, $a_1^0(\theta, \phi)$, is

$$b_2^n = k_2 \lambda_o G_V^n(\theta, \phi) a_1^0(\theta, \phi) \quad (C8)$$

2. The power in the n^{th} waveguide mode is:

$$1/2 |b_2^n|^2 = \frac{k_2^2 \lambda_o^2}{2} |G_V^n(\theta, \phi)|^2 |a_1^0(\theta, \phi)|^2 \quad (C9)$$

B. Transmitting Properties

1. The far field wave amplitude in the θ, ϕ direction, $b_1^0(\theta, \phi)$, excited by the N waveguide modes is

$$b_1^0(\theta, \phi) = k_1 \sum_{n=1}^N a_2^n G_V^n(\theta, \phi) \quad (C10)$$

2. The far field power density in the θ, ϕ direction is

$$1/2 |b_1^0(\theta, \phi)|^2 = \frac{(k_1)^2}{2} \left| \sum_{n=1}^N a_2^n G_V^n(\theta, \phi) \right|^2 \quad (C11)$$

From Eq. C8, when a plane wave is incident on the antenna, the ratio of modal amplitudes generated at the waveguide terminals of the antenna (into a matched load) is in direct proportion to the ratio of the single voltage gains of the antenna.

The radiation pattern of an antenna excited by N modes at its waveguide terminals is expressed in Eq. C10. The radiated field is a linear superposition of the radiated fields of the individual modes and thus depends upon the relative amplitudes and phases of the individual modes.

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