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"Research on Clustering of Galaxies"

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I. Planned Investigations

According to the contract the following studies on clustering of galaxies have been planned:

1. The existence and evolution of superclusters should be proved. Therefore statistical methods must be applied to the existing catalogue of clusters. The theoretical foundation has been developed by Dr. Just. His results are given in the appendix of this report.

2. The richness-distribution of clusters of galaxies should be the subject of the second part of the investigations. Especially the influence of systematic and random errors on the richness-distribution should be studied.

II. Results

In the following we give the results and the state of the investigations ap to 31 August 1962.

1. Spatial Distribution of Clusters

a. Empirical quasi-correlation according to ABELL's catalogue.

After the theoretical foundation of the method of deriving quasi-correlations it was our aim to get numerical values for these quasi-correlations. It was clear that the numerical investigations could only be done by using big digital computers. Therefore the needed datas (ABELL-number, magnitude, place and richness) of the clusters has been transfered from ABELL's publication on IBM-cards and on paper tapes. For the electronic computer SIEMENS S 2002 a program has been prepared, which allows us to derive the empirical quasi-correlations from the catalogue. In its last form this program needs about 9 hours for one set of the parameters. Because of the low use of in- and output units it is qualified for working at night. The program was in action at the S 2002 - computers in Berlin and Heidelberg. But up to day no certain interpretation of the result can be given. These results, however, are at Dr. JUST's disposal for his continued studies on the same topic in the United States.

b. and c. Theoretical quasi-correlation according to NEWMAN's statistics and comparison of the empirical and theoretical values in order to derive the temporal evolution of the superclusters.

To solve the problems of existence and evolution of superclusters, it is necessary to compare the empirical and theoretical results for the quasi-correlations. This was the task of a second program. It was constructed in such a manner, that it could use the results of the first program (item 1a) directly. The second program exists in a proved and workable form. But because it depends on the results of the first problem, which do not exist in an adequate size, we were not able to get practical results with the second program. Therefore no solution of the problems of the existence and evolution of super-clusters could be given.

2. Richness Distribution of Clusters

a. Modification of ABELL's catalogue according to supposed systematic errors.

ABELL's catalogue was examined in detail according to effect of systematic errors. The results of the investigation has been condensed in our Technical Note No.1 (K.Just und R.Wielen; "Remark on the validity of a test for evolution"). The treatment of the problem has been continued at the S 2002. The result was a program which makes it possible to modify ABELL's catalogue in a rather general kind and to punch it in that form on paper tapes for subsequent use within the programs of supercluster research.

b. Further research on the richness distribution; especially application of more involved statistics.

For the derivation of the richness-distribution of clusters of galaxies and for the application of statistical methods on that problem we have prepared some programs for the S 2002. But their enforcement has been deferred because of a suggested deficiency of the underlying mathematics and because of their high computing times

3. Investigations on Related Topics

Beside the research on superclusters and on the richness distribution we have started and partly finished some other investigations about clusters of galaxies:

a. Research on cluster modells in cosmology.

The investigation of this problem has led to the following publication: K. Just and K. Kraus: "Spherically Symmetric Models in Relativistic and Newtonian Cosmology". This paper has been published with the additional note "Supported by the U S Air Force Office of Scientific Research" and has been already submitted to this office.

b. Density distribution in clusters of galaxies.

The problem of computing the density distribution in clusters has been discussed in the preprint: K. Just and R. Wielen: "On Flattened Clusters of Galaxies", A program has been completed, which makes it possible to derive the spatial distribution from the observed projected density. The results were not of the suggested quality because the fundamental observations (Catalogue of SHANE and Wirtanen) show counting regions of a too extended size for this problem.

III. Difficulties

1. Technical Difficulties.

All investigations could be done only by using big electronic computers. In the beginning we had hoped to get sufficient access to the SIEMENS 2002 in Berlin. But this aim was not attained, although we have made all possible exertions. At the S 2002 we disposed of three hours weekly, on the average, for the research on clustering of galaxies. This time was much too short for our purpose. Therefore in most cases we could only prove but not performe the program at the machine. We have tried to carry out our computation at machines of the same type at other computing centers (Heidelberg, Tübingen), but those actions were of nearly no success.

2. Personal difficulties.

The most important reason for the lack of a conclusion of the research is the fact that Dr. Just as the principal investigator has left Berlin in July 1961. He became assistant professor at the University of Arizona. Thus a planned division for physics of stellar systems could not be etablished in our institute.
Two of the three participating investigators left Berlin too. Therefore it is impossible for our institute to continue the research on clustering of galaxies.

IV. Continuation of the Investigations

Dr. Just continues the investigations on clustering of galaxies in the United States. For that purpose he can dispose of all results, which have been derived during the research on the same topics in Berlin.

Appendix

Institut für Theoretische Physik
der Freien Universität Berlin

Circular Counting Zones in the Cluster Statistics of NEYMAN and SCOTT

by

Kurt JUST

RESEARCH ON CLUSTERING OF GALAXIES

Abstract:

Existence, structure and evolution of superclusters of galaxies have been investigated by using ABELL's catalogue of rich clusters of galaxies. The effect of systematical errors in this catalogue has also been studied. Because of technical and personal difficulties the greater part of the planned investigations could not been carried out. However, the studies are continued by Dr. Just at the University of Arizona.

1. Introduction and Outlook

Those parts of the elaborate theory of NEYMAN, SCOTT, and SHANE, which are most important for our purpose, were summarized best in their paper "Statistics of Images of Galaxies ...." (1956), to which we shall refer by (S) . Originally that theory (NEYMAN, SCOTT 1952) was intended to derive from counts of single galaxies their tendency to concentrate in clusters. In this manner one could even investigate those small clusters of galaxies, which are not discernable individually. For the observer they only form a rather fluctuating background, called the "general field"; but even this was found in accordance with the assumption, that any galaxy belongs to a cluster.

Although their general theory allows to treat the counting of galaxies in arbitrary areas of the sky, NEYMAN and SCOTT (1952, 1959) have applied it only to counts in a net of squares (SHANE, WIRTANEN 1954), since no others are available. But when trying to evaluate simultaneously the counts in $1° \times 1°$ and $10' \times 10'$ squares, they found a discrepancy (S, Figure 3) which seems to require the assumption that the clusters themselves are clustered (S, Figures 5, 6).

The existence of <u>superclusters</u> suggested in this way was further confirmed by ABELL (1958), whose catalog contains the positions of all <u>rich clusters</u> of galaxies, which are clearly discernable individually. Discussing them under the hypothesis that their random distribution is a homogeneous one, he could reject that hypothesis with an overwhelming reliability (ABELL 1958), (Table 12). Then he repeated his test with counting areas of various size and separately for early and late clusters, in order to estimate the apparent size of those aggregates. The result was (ABELL 1958, Tables 13 to 15), that the angular diameter characterizing the phenomenon of superclustering is inversely proportional to the redshift z.

Thus the absolute sizes of the super-clusters would be the same at all epochs, showing a <u>characteristic length</u> of about $4 \cdot 10^7$ parsec. This also follows from the "index of clumpiness" as defined for galaxies by ZWICKY (1953) or by NEYMAN, SCOTT, SHANE (1954), and calculated by ABELL (1958, Tables 17,18) for the clusters of his catalog. It rejects completely the opinion of ZWICKY (1959), that the observed tendency of superclustering is <u>nothing</u> but an effect of intergalactic <u>obscuration</u>, although nobody denies that such an obscuration might also have some influence.

Since, however, the single clusters show a <u>temporal evolution</u> (JUST 1959), one might suspect that also their aggregates are not in a completely steady state. But the mentioned methods of ABELL will not to suitable to estimate more than the coefficient c of the expansion

$$\tau \cdot \sigma = c\,(1 + \gamma \cdot \tau + \delta \cdot \tau^2 + \dots), \qquad (1)$$

where σ shall be the characteristic length of superclustering and τ the distance from us, measured by the time of light travel. If one may get further at all, the best method known will be that of serial quasi-correlations developed by NEYMAN and SCOTT (1952); therefore we shall modify this method for our purpose.

Of course one can never estimate in (1) the coefficient δ, since a sufficiently complete survey of indidually discernable clusters will be rather limited in depth of space. But already γ would answer the important question, to what extent the superclusters are participating in the cosmic expansion. If their average size would be constant with respect to our local (atomic) measures, we shall have $\gamma = \frac{1}{2}$, while its constancy with respect to the expanding universe would mean $\gamma = -\frac{1}{2}$. This statement, which holds under rather general assumptions of cosmology, will be proved in another paper, where also the actual estimate of γ from ABELL's catalog will be given.

2. Fundamental Assumption

In analogy to those of NEYMAN, SCOTT, and SHANE (S) our basic assumptions shall be:

1. Each cluster belongs to a <u>super-cluster</u>, which however shall not be characterized by the unobservable total number of its members, but by its "<u>richness</u>" N ,defined as the number of those clusters, which themselves have a richness $n \geq 50$ (Appendix 1).
2. The richness N of a super-cluster is a random variable having the time-dependent frequency distribution

$$\varphi(N|\tau) \qquad \text{with} \qquad \sum_{N=1}^{\infty} \varphi(N|\tau) = 1 \,, \tag{2}$$

which is obtainable from a probability generating function

$$G_N(\xi|\tau) \doteq \mathcal{E}\{\xi^N\} = \sum_{N=1}^{\infty} \xi^N \varphi(N|\tau) \tag{3}$$

(by $\mathcal{E}\{\,\}$ we denote an exp ctation value).

3. Given the center of a super-cluster by its celestial coordinates B, Λ, and the epoch $t = t_0 - k\cdot\tau$ of light-emission, the probability density of its members at β, λ shall be

$$F(\beta, \lambda | B, \Lambda, \tau) = \text{const} \cdot \sigma^{-2} f(\vartheta/\sigma) \,, \tag{4}$$

where the "angular distance" ϑ is a certain combination the four celestial coordinates (Appendix 2):

$$\tfrac{1}{2}\vartheta^2 = 1 - \sin\beta \sin B - \cos\beta \cos B \cos(\lambda - \Lambda) . \tag{5}$$

The parameter σ shall arbitrarily depend on the epoch t :

$$\sigma = \sigma(\tau) \qquad \text{with} \qquad k\cdot\tau = t_0 - t \,, \tag{6}$$

where t_0 denotes our epoch, and h the reciprocal HUBBLE parameter:

$$h \approx 10^{10} \text{ years} . \tag{7}$$

4. The random distribution of the <u>super-clusters</u> in space shall be strictly <u>homogeneous</u>, such that we neglect the possibility of third order clustering (Appendix 3).

5. Thanks to our convention to consider as members of a super-cluster only the clusters with <u>richness</u> $n \geq 50$ (Appendix 1), we may assume that in the regions selected by ABELL (1958, § II b) nearly <u>all</u> those objects are really observed. This yields $\theta = 1$ for the probability of "visibility" instead of the expression (15) of (S). Therefore we may replace the formulae (23,24) of that paper by

$$p_i(B,\Lambda,\tau) = \int_{\omega_i} F(\beta,\lambda \mid B,\Lambda,\tau) \cos\beta \, d\beta \, d\lambda \tag{8}$$

for $i = 1, 2$, but $p_3 = 0$,

if we also agree to consider only <u>disjoint regions</u> ω_1, ω_2 of observation. Since F is already a "projected" density (Appendix 2), ω_1 and ω_2 are no sp tial regions, but areas on the celestial sphere.

6. The joint probability $P(\nu_1, \nu_2 \mid \omega_1, \omega_2)$ to observe exactly ν_1 clusters in the area ω_1 and ν_2 in ω_2 shall follow from a generating function

$$G_{\nu_1\nu_2}(\xi_1, \xi_2 \mid \omega_1, \omega_2) = E\{\xi_1^{\nu_1} \xi_2^{\nu_2}\} = \sum_{\nu_1,\nu_2=1}^{\infty} \xi_1^{\nu_1} \xi_2^{\nu_2} P(\nu_1, \nu_2 \mid \omega_1, \omega_2), \tag{9}$$

which is given by (22) of (S):

$$\begin{aligned} \ln G_{\nu_1\nu_2}(\xi_1,\xi_2) &= \text{const} \cdot \int z_{12} \cos B \, dB \, d\Lambda \, dV(\tau), \\ z_{12} &= 1 - G_N(1 + (\xi_1 - 1)p_1 + (\xi_2 - 1)p_2 \mid \tau). \end{aligned} \tag{10}$$

Here G_N means the generating function defined by (3), while $dV(\tau)$ is that volume of <u>comoving space</u>, where the light observed now was emitted between τ and $\tau + d\tau$, thus being given by (Appendix 4):

$$dV(\tau) = \mathrm{const} \cdot \left(1 + 2\tau + \tfrac{4}{3}\tau^2\right)\tau^2 \, d\tau \qquad (11)$$

3. Remarks on some Distributions

Of course the two functions $G_N(\xi|\tau)$ and $f(\vartheta/\sigma)$ needed to specify the stochastic model cannot be determined from the observations in detail. We can only hope to estimate very few arbitrary parameters (actually only two) of a plausible assumption. Now it would be difficult to find such an assumption for (3), but fortunately we shall only need the lowest moments:

$$\begin{aligned} \mu_0 &\equiv \mathcal{E}\{1\} = G_N(1|\tau) = 1, \\ \mu_1 &\equiv \mathcal{E}\{N\} = G_N'(1|\tau) , \\ \mu_2 &\equiv \mathcal{E}\{N^2-N\} = G_N''(1|\tau) , \end{aligned} \tag{12}$$

and also these only in intermediate calculations.

For $f(y) = f(\vartheta/\sigma)$ we may assume with NEYMAN and SCOTT (Appendix 2):

$$f(y) = \text{const} \cdot e^{-\frac{1}{2}y^2} ; \tag{13}$$

or perhaps take according to EMDEN (1907) the equilibrium configuration of a gravitating gas sphere with a polytropic index between the adiabatic and the isothermal one. The [illegible] between these extremes are so similar (JUST, [illegible]N 1961), that we may choose a particular one by asking for mathematical simplicity. This leads to the only case [illegible], where EMDEN's equation has an explicit solution, and which yields the projected density (Appendix 2):

$$f(y) = \text{const} \cdot \left(1 + \tfrac{1}{5} y^2\right)^{-2} . \tag{14}$$

[illegible] the integral (10) extends over the whole celestial sphere, but practically only the regions ω_1, ω_2, and their immediate surroundings will contribute: From (8) and (4) with (13) or (14) we get $\mu \longrightarrow 0$, if the point

B , Λ moves far from ω_ℓ , while (10) with the first line of (12) yields:

$$\mathfrak{Z}_{12} \longrightarrow 1 - G_N(1/\tau) = 0 \qquad \text{for } p_1, p_2 \rightarrow 0. \quad (13)$$

In τ the integral (10) shall extend over those epochs, which are included in the sky survey considered. More correctly this ought to hold for that integration (Appendix 2), which led to the projected density (4), because we had to sum over the observable members of the super-clusters, while the integral (10) over their centers should include all epochs. But the dependence of (4) and therefore (8) on τ will be weak, and τ itself cannot be measured without much uncertainty. Hence both procedures are practically equivalent, while that adopted here is easier than the rigorous one.

4. The Method of Quasi-Correlations

To compare the theoretical result (10) with the observations and thus to estimate its parameter σ, we use the formulae (26, 27, 28) of (S). Since we have $p_3 = 0$ thanks to our use of disjoint regions ω_1, ω_2 we may write in our notation:

$$\overline{V}_i \overset{\text{def}}{=} \mathcal{E}\{\nu_i\} = q \cdot \mu_1 R(p_i^2), \quad q = \text{const}, \tag{16}$$

and

$$\begin{aligned} \sigma_{ik} &\overset{\text{def}}{=} \mathcal{E}\{(\nu_i - \overline{\nu}_i)(\nu_k - \overline{\nu}_k)\} \\ &= q(\mu_1 \delta_{ik} + \mu_2) \cdot R(p_i p_k) \end{aligned} \tag{17}$$

with

$$R(f) \overset{\text{def}}{=} \iint f \cdot \cos B \, dB \, d\Lambda \, dV(r), \tag{18}$$

where q is the spatial density of super-clusters. To eliminate from this expressions the hardly determinable parameters μ_1, μ_2 and q, we define as in (29) of (S) the theoretical "quasi-correlation":

$$T \overset{\text{def}}{=} \frac{\sigma_{12}}{\sigma_{11} - \overline{\nu}_1} = \frac{R(p_1 p_2)}{R(p_1 p_1)} . \tag{19}$$

An empirical counterpart of this may be calculated as follows:

1. We select a sequence of $K \gg 1$ congruent regions Ω_χ, the common shape of which is arbitrary, and which are distributed anyhow (systematically or at random) over the field of the sky survey.
2. Denoting by Ω the common area of the regions Ω_χ, that of the whole survey by $\mathcal{L}$, and its total number of clusters by C, we get as empirical counterpart of the expectation value $\overline{\nu}_1$ defined in (16):

$$\bar{\nu} \overset{\text{def}}{=} \Omega \cdot \bar{n} \qquad \text{with} \qquad \bar{n} = C/\Omega \;, \tag{20}$$

the last being the average number of clusters in a unit of solid angle.

3. Denoting by $\nu_\varkappa$ the number of clusters observed in the region $\Omega_\varkappa$, we calculate as empirical counterpart of σ_{11}:

$$S_{11} \overset{\text{def}}{=} K^{-1} \sum_{\varkappa=1}^{K} (\nu_\varkappa - \bar{\nu})^2 . \tag{21}$$

4. Around each $\Omega_\varkappa$ we select a sequence of L congruent regions $\omega_\varkappa^\lambda$, such that each of the $K \cdot L$ pairs $\Omega_\varkappa, \omega_\varkappa^\lambda$ is congruent to every other of them.

5. Denoting by ω the common area of the regions $\omega_\varkappa^\lambda$, we define in analogy to (20):

$$\bar{\mu} \overset{\text{def}}{=} \omega \cdot \bar{n} \;. \tag{22}$$

6. If $\mu_\varkappa^\lambda$ is the number of clusters observed in the region $\omega_\varkappa^\lambda$, the emperical counterpart of σ_{12} is

$$S_{12} = (KL)^{-1} \sum_{\varkappa=1}^{K} (\nu_\varkappa - \bar{\nu}) \sum_{\lambda=1}^{L} (\mu_\varkappa^\lambda - \bar{\mu}) . \tag{23}$$

7. With (20) to (23) we get as counterpart of (19) the <u>empirical</u> quasi-correlation

$$E = S_{12} \cdot (S_{11} - \bar{\nu})^{-1} \;. \tag{24}$$

5. Counts in Circular Regions

If the common shape of the regions $\mathcal{R}_\chi$ is irregular, we can relate to each of them only one $\omega_\chi^\lambda = \omega_\chi^1$, because no other figure $\mathcal{R}_\chi, \omega_\chi^\lambda$ with the same $\mathcal{R}_\chi$ would be congruent to the pair $\mathcal{R}_\chi, \omega_\chi^1$. If $\mathcal{R}_\chi$ is a region allowing C different congruent mappings onto itself, we have up to C congruent pairs $\mathcal{R}_\chi, \omega_\chi^\lambda$ (Figure 1).

The special case of squares arranged in a regular net (without gaps) is the only one used up to now; a very similar arrangement would be a net of sexangles. These nets have the welcome property, that each central counting region $\mathcal{R}_\chi$ can also serve as a ω_χ^λ related to several other $\mathcal{R}_\chi$. But on a large part of the sphere such a "net" without too much distortion does not exist.

Therefore we cannot use for our purpose any "net" at all; but this gives us the freedom to choose as a central region $\mathcal{R}_\chi$ the simplest possible, namely that inside a circle. Then any complete sequence of possible ω_χ^λ fills the whole zone between two circles; and the simplest arrangement of those actually used is that, which covers a zone exactly once (Figure 2 with the internal borders).

Finally removing the internal borders we get the same picture, as if we would have considered the circular regions from the beginning. Thus our derivation appears as a detour, but we have given it in order to show:

1. why we shall practically use the zones not with the same area as the central regions, but with nearly the same extent in radial direction (Figure 2),

2. that the counting in circular regions is quite natural and not too different from that in a net of squares.

But now we may consider the <u>whole zone</u> surrounding each central region $\Omega_\varkappa$ as a single region $\omega_\varkappa^1 = \omega_\varkappa$ (Figure 2 <u>without</u> the internal boundaries). Then (23) simplifies to

$$S_{12} = K^{-1} \sum_{\varkappa=1}^{K} (\nu_\varkappa - \bar{\nu})(\mu_\varkappa - \bar{\mu}) , \tag{25}$$

where the actual number $\mu_\varkappa$ of clusters and its expectation value $\bar{\mu}$ belong to the zone $\theta_1 \leq \theta < \theta_2$ around the $\varkappa$-th central region $\theta < \theta_0$. With (21) and (25) the empirical quasi-correlation (24) finally reads:

$$E = \frac{\sum_\varkappa (\nu_\varkappa - \bar{\nu})(\mu_\varkappa - \bar{\mu})}{\sum_\varkappa \{(\nu_\varkappa - \bar{\nu})^2 - \bar{\nu}\}} . \tag{26}$$

6. The Dependence on Parameters

Of course the quasi-correlation (26) and its theoretical counterpart (19) depend on the angular distances θ_0, θ_1, θ_2 between the three bordering circles and their common center (Figure 2). These are the intrinsic parameters defining our pairs of circular counting regions. They are constant within the sums of (26), which range over all points on the sky chosen as centers of those congruent figures. Thus the result E will depend on $\theta_0, \theta_1, \theta_2$ as also on the depth of the sky survey, defined by the limiting values $\underline{m}$ and $\overline{m}$ of ABELL's measure m of distance (Appendix 4):

$$E = E(\theta_0, \theta_1, \theta_2 ; \underline{m}, \overline{m}). \tag{27}$$

Of course E will also depend on the chosen centers of the counting regions; but we hope, that this dependence will be unimportant, if they are distributed with constant density (see § 8,1.) and their number K exceeds a reasonable value.

The theoretical expression (19) to be compared with (27) must of course be calculated with the same parameters $\theta_0, \theta_1, \theta_2$, and the temporal limit $\underline{\tau}$, $\tilde{\tau}$ corresponding to $\underline{m}, \overline{m}$:

$$\underline{\tau} \stackrel{\text{def}}{=} \tau(\underline{m}), \qquad \overline{\tau} \stackrel{\text{def}}{=} \tau(\overline{m}). \tag{28}$$

Here $\tau(m)$ must be the inverse of the following function (Appendix 4):

$$m(\tau) = 5 \lg \tau + a + b \cdot \tau + c \cdot \tau^2, \tag{29}$$

$$a = 21{,}01, \quad b = 4{,}71, \quad c = 3{,}5 .$$

But T will also be a functional of the function $\sigma = \sigma(\tau)$, which was introduced in 6) to measure the apparent size of an average super-cluster at the epoch τ. If we use for σ the first approximation in the sense of (1):

$$\sigma(\tau) = c(\tau^{-1} + \gamma) , \tag{30}$$

then the dependence of T on this is expressed by its constants c and γ; thus we have finally:

$$T = T(\theta_0, \theta_1, \theta_2 ; \underline{\tau}, \overline{\tau} | c, \gamma). \tag{31}$$

It must be emphasized that the distance θ from the center P_χ of a counting region has nothing to do with the distance ϑ from the center C_i of the super-cluster: Empirically the C_i are completely <u>unknown</u>, while the P_χ are <u>arbitrary</u>. In the theory a <u>single</u> P will be made the pole of the coordinate system, while we must <u>average</u> C over the whole sphere.

7. The Numerical Problems

Taking as the pole $\beta = \frac{\pi}{2}$ of the coordinate system the center $\theta = 0$ of one pair of circular regions (Figure 2), we have these given by

$$0 \le \tfrac{\pi}{2} - \beta < \theta_0 \qquad \text{and} \qquad \theta_1 \le \tfrac{\pi}{2} - \beta < \theta_2 . \tag{32}$$

The integration (8) with (4):

$$p_i(B, \Lambda, \tau) = \text{const} \cdot \sigma^{-2} \int_{\omega_k} \cos\beta \, d\beta \, f(\vartheta/\sigma) \tag{33}$$

must now run over the first of the regions (32) - a polar cap - in order to yield p_1, and over the second one - a surrounding latitude zone - to give p_2.

At (8) we have excluded the possibility of overlapping regions of counting, so we must require

$$0 < \theta_0 \leq \theta_1 < \theta_2 \quad . \tag{34}$$

But mathematically the two integrale p_1 and p_2 will follow by

$$p_1 = f(\theta_0 | B, \sigma), \qquad p_2 = f(\theta_2 | B, \sigma) - f(\theta_1 | B, \sigma) \tag{35}$$

from <u>the same</u> function:

$$f(\theta | B, \sigma) \equiv \mathrm{const} \cdot \sigma^{-2} \int_0^{2\pi} d\psi \int_0^{\theta} \sin\theta \, d\theta \, f(\vartheta/\sigma), \tag{36}$$

where we have according to (5):

$$\tfrac{1}{2}\vartheta^2 = 1 - \cos\theta \sin B - \sin\theta \cos B \cos\psi \quad . \tag{37}$$

Comparing (35) with (33) we recognize that the dependence on ω_1 and ω_2 is now expressed by $\theta_0, \theta_1, \theta_2$, while Λ has disappeared, and τ is contained in σ. The two cases of (18), which are needed in (19), may therefore be written

$$\begin{aligned} R(p_1 p_2) &= \int G(\theta_0, \theta_1, \theta_2 | \sigma(\tau)) \, dV(\tau) \\ R(p_1 p_1) &= \int G(\theta_0, 0, \theta_0 | \sigma(\tau)) \, dV(\tau) \end{aligned} \tag{38}$$

with the integral

$$\begin{aligned} G(\theta_0, \theta_1, \theta_2 | \sigma) &= \mathrm{const} \cdot \int_{-1}^{1} d(\cos B) \times \\ &\times f(\theta_0 | B, \sigma) \left\{ f(\theta_2 | B, \sigma) - f(\theta_1 | B, \sigma) \right\} \quad . \end{aligned} \tag{39}$$

Finally inserting the limits in time and the assumption (30) for the apparent size of a super-cluster, we get from (19) with (38):

$$T = \frac{R(\theta_0, \theta_1, \theta_2 ; \mathfrak{T}, \bar{\tau} | c, r)}{R(\theta_0, 0, \theta_0 ; \mathfrak{T}, \bar{\tau} | c, r)} \tag{40}$$

with the numerator given by

$$R = \int_{\tau'}^{\tau''} G(\theta_0, \theta_1, \theta_2, \ldots \tau + c\gamma)\, dV(\tau) , \tag{41}$$

and the denominator following by an easy substitution.

The result (40) actually depends on the parameters indicated in (31), among which the five θ and τ must be the same as in the empirical result (26) to (28). On the other hand the two <u>unknows</u> c and γ must be the same for all values of the θ and τ. Thus we shall estimate them in a following paper by looking for the best fit of (40) to (27) with many different choices of the independent parameters $\theta_0, \theta_1, \theta_2 ; \tau' ; \tau''$.

Unfortunately, in (36) with (37), only the integration over ψ can be performed explicitly. With (13) it yields by a well known definition of the BESSEL functions:

$$f(\theta \mid B, \tau) = \mathrm{const} \cdot \omega \int_0^{\theta} e^{-k} J_0(il)\, d(\cos\theta) \; ; \tag{42}$$

with (14) we receive:

$$f(\theta \mid B, \tau) = \mathrm{const} \cdot \omega \int_0^{\theta} k\,(k^2 - l^2)^{-\frac{3}{2}}\, d(\cos\theta) , \tag{43}$$

where we have used in both cases the abbreviations:

$$k \overset{\text{def}}{=} \frac{c}{2} + \omega\,(1 - \sin B \cos\theta)$$

and (44)

$$l \overset{\text{def}}{=} \omega \cos B \sin\theta$$

with

$$\omega \overset{\text{def}}{=} e^{-a} = (\tau/c)^2 (1 + \gamma\tau)^{-2} . \tag{45}$$

8. Summary

In spite of the similarity shown in § 5 we have the following diversities between our arrangement and that based on a net of squares:

1. Since a systematic distribution of more than 24 points on a sphere cannot be homogeneous, we rather prefer to distribute the centers of our circular regions at random, but with a constant probability density.
2. Since the regions, from which the numbers ν_χ and μ_χ are counted, have arbitrary and independent areas, the quasi-correlations (19) and (26) can have any order of magnitude, while those from pairs of the same area are smaller than one.
3. Besides the new freedom to choose the centers of our counting regions, the influence of which we hope to get small enough, we have the welcome possibility to vary the three parameters $\theta_0, \theta_1, \theta_2$ independently. With a net of squares one could only vary the distance of the two correspondi fields (S, Figures 1 and 2), perhaps also their area (S, Figure 3); therefore the method was called that of "serial" quasi-correlations.

Another difference between our method for superclusters and that for clusters is due to the observations available here and there; and two further diversities are following from these and from our arrangement of counts:

4. The parameter γ to be estimated here besides $c = \lim_{\tau \to 0} \tau \cdot \sigma(\tau)$ concerns the variation of $\tau \cdot \sigma(\tau)$ in time; the second parameter $m_1 - M_0$ there (S, Table I)

determined the "visibility" of the objects. But this concept is not necessary here (§ 2,5), while our hope to estimate γ by means of the additional freedom in the temporal limits $\underline{\tau}$ and $\bar{\tau}$ is due to the fact that the observations available here (ABELL 1958) are <u>classified</u> in depth of space.

5. A counting in so many regions of circular shape, many of which will even overlap, cannot be achieved without a computing machine, and only from a catalog of <u>positions</u>. For the big <u>clusters</u> this fortunately does exist (ABELL 1958), while for sufficiently many <u>galaxies</u> we only know a catalog of counts in squares (SHANE, WIRTANEN 1954).

6. With only few independent parameters the comparison of theory and observation could proceed graphically (S, Figure 1 to 3); thanks to the many parameters to be varied here a <u>numerical</u> method will be preferable.

Appendix 1: The richness of clusters

The richness n of a cluster was defined by ABELL (1958, § II f 3) as that number of its galaxies, the apparent magnitudes m of which exceed that of the third brightest galaxy by less than two. Denoting by m_ν the apparent magnitude of the ν-th brightest member of a cluster, we have thus

$$m_n = m_3 + 2 . \qquad A\ (1)$$

In order to make the result rather independent of the area of counting, ABELL also applies a "field subtraction". Therefore n is defined more exactly by

$$n \equiv n' - n'' \quad \text{with} \quad m_{n'} = m_{n''} = m_{3'} + 2 , \qquad A\ (2)$$

where ν' numbers the galaxies within a certain circle around the center of the cluster, and ν'' those in a distant field of the same area.

The n defined in this way depends on the initial steepness of the luminosity function in a cluster. As far as it can be determined at all, it is thus a good measure for its total content. In contrast to the number of all visible galaxies, which is preferred by Zwicky (1960), it is namely independent of the distance from us, and of the various conditions of the exposure.

In our present context it is only important, that ABELL feels sure to have catalogued (in certain fields of the sky and between certain limits of the distance from us) almost all the clusters with richness $n \geq 50$.

If this is correct, also our investigation of super-clusters will be free from an observational bias, since we are basing it only on that "homogeneous sample" of clusters (ABELL 1958, § III a).

Appendix 2: The projected density

Although the observable appearance of the "super-clusters" is that of irregular clouds rather than organized structures (ZWICKY 1959), we shall for our present purpose consider them as spherically symmetric. This assumption is somewhat justified by the fact that also a model with spherically symmetric clusters of galaxies (SCOTT, SHANE, SWANSON 1954) yields such a cloudy appearance, from which only the very largest clusters are discernable individually.

Thus the bad immediate appearance of the super-clusters is no argument against a well defined organisation, although this hypothesis ought to be tested in as many ways as possible. But if organized at all, the super-clusters might still deviate from spherical symmetry, like the clusters themselves do (JUST, WIELEN 1961). Hence also this symmetry is an additional hypothesis, but without it the statistical theory of NEYMAN and SCOTT would be too complicated.

Thus we assume that the normalized spatial distribution ϱ of the clusters forming a super-cluster depends only on their distance ξ from its center:

$$\varrho(\xi) = \mathrm{const} \cdot s^{-3} F(\xi/s) \quad \text{with} \quad s = \mathrm{const.} \qquad \text{A (3)}$$

Here $F(x)$ is a function like

$$F(x) = e^{-\frac{1}{2}x^2} \quad \text{or} \quad F(x) = \left(1+\tfrac{1}{3}x^2\right)^{-\frac{5}{2}}, \qquad \text{A (4)}$$

the first being that used by NEYMAN and SCOTT, the second the only elementary function among those found by EMDEN (1907) for the equilibrium density of a self-gravitating gas sphere with a polytropic equation of state (JUST, WIELEN 1961).

At first disregarding the expansion and curvature of cosmic space, we have to calculate from A(3) the projected density $\tilde{f}(r)$ as usually:

$$\tilde{f}(r) = \int_{-\infty}^{\infty} \varsigma(\xi)\, dl \qquad \text{with} \qquad l^2 = \xi^2 - r^2 \; . \qquad A\ (5)$$

Here l is a distance measured along the line of sight, r its smallest distance from the center of the supercluster.

With A(3) to A(5) an easy calculation yields

$$\tilde{f}(r) = S^{-2} f(y) \qquad \text{with} \qquad y = r/S \; , \qquad A\ (6)$$

where $f(y)$ is given by (13) or (14). Now denoting by η the distance between the observer and the center of the super-cluster, by σ its apparent radius, we have

$$r = \eta \cdot \vartheta \qquad \text{and} \qquad S = \eta \cdot \sigma \; ,$$

hence $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ A (7)

$$y = \vartheta / \sigma \; .$$

Here ϑ is the angle between the lines of sight to the center and to a particular member of the super-cluster. If in any system of celestial coordinates these have the latitudes β, B and the longitudes λ, Λ we have exactly

$$\cos\vartheta = \sin\beta \sin B + \cos\beta \cos B \cos(\lambda - \Lambda) \; , \qquad A\ (8)$$

which reduces to (5) for a small ϑ.

Now to account for the curvature and expansion of cosmic space, we may still neglect the first within each super-cluster, and the second during that time, in which the light travels thru it. Therefore our derivations up to A(6) remain valid, if we interpret ξ, r, l as those comoving coordinates, which are equal to the corresponding EUKLIDean distances of to-day.

The remaining influence of expansion and curvature is that on our distance η from the super-cluster. This will be observed exactly by giving to η the correct one among the many different definitions of spatial distance in cosmology (distance by apparent size). The dependence of this η and of σ on time shall be discussed in a following paper. Here we are content with the result, that by A(6) to A(8) the assertions (4,5) and (13,14) are justified with ϑ and σ having the well defined meaning of an <u>angular distance</u> on the sky and the <u>apparent radius</u> of a super-cluster.

Appendix 3: The hierarchy of clustering

Compiling the average diameters d of various celestial objects and their mean distances D from another, we get very roughly our Table 1. Then comparing the logarithms of the ratio D/d and of d itself, we get the Figure 3, where the two lines are drawn under the assumption, that the ratios D/d from Table 1 (drawn as circles) are wrong by the factor $0{,}1$ or 10.

Objects	d	D
Larger Planets	$5 \cdot 10^{-9}$	10^{-4}
Planetary Systems	$6 \cdot 10^{-4}$	2
Globular Clusters	$2 \cdot 10^{2}$	$3 \cdot 10^{4}$
Large Galaxies	10^{5}	$2 \cdot 10^{6}$
Rich Clusters	10^{7}	$3 \cdot 10^{7}$
Super-Clusters	$2 \cdot 10^{8}$	10^{8}

Table 1: The diameters d and the mutual distances D of some objects, correct up to factors between $0{,}2$ and 5, and given in the unit 1 year $\approx 10^{18}$ cm.

Such an uncercainty must surely be admitted, since not only the intrinsic variations and the observational errors are very large, but also the definition of an "average diameter" and a "mean distance" is rather arbitrary. But even then our Figure 3 shows the clear tendency of the ratio D/d to become smaller for larger systems. If this tendency goes on, the possible clusters of super-clusters will overlap considerably.

Already the super-clusters are so badly discernable individually, that their mere existence is still now denied by ZWICKY (1960). By statistical methods, however, their existence could be established beyond any doubt, especially by the proof (ABELL 1958, § III e) that their true diameters are roughly the same at all distances from us. Of course this might also be possible for the clusters of super-clusters, if sufficiently many observations would be available. But one may doubt, whether these observations can ever be made.

The present data about the individually discernable clusters of galaxies (ABELL 1958) might surely be improved in detail, but their extent in depth of space is nearly all, what can be achieved with existing instruments. For a preliminary study of the super-clusters they are just sufficient under low pretension; but the third order clusters are at least ten times as large in their linear dimensions, hence they would require a much deeper survey.

The actual situation is still worse, because with very few exceptions the super-clusters cannot be recognised individually. Therefore their tendency to form third order clusters cannot be investigated by the method applicable to themselves. Instead one ought to look for discrepancies between the observations and the statistical model used here, which might then be overcome by dropping our assumption of a uniform distribution of the super-clusters.

Such a method would correspond to an attempt of investigating the super-clusters not by a catalog of clusters, but from counts of galaxies. Although prepared theoretically by NEYMAN and SCOTT (S, § 11), it was not yet performed. Besides its mathematical complexity and the difficulty to evaluate the counts from very large fields of the sky, it has more parameters to be estimated than our method.

Therefore an empirical study of third order clusters will be impossible at least for the next decades, although their existence may well be expected theoretically (see also ULAM 1954,) Besides this our discussion shows that the cosmologically important homogenity in very large parts of the universe is compatible with an hierarchy of clusters up to any order. It only requires that the observed decrease of the ratio D/d (Figure 3) goes on from step to step, such that the clusters of higher order will practically overlap completely.

Such an hierarchy must be well distinguished from a properly "hierarchic model of the universe" like that proposed by KLEIN (1956). This would of course break the trend of our Figure 3; but it can never be rejected by the observations, if one always assumes that the "gap" between "our" part of the universe and the others begins beyond the region already overlooked by any means.

Appendix 4: The time of light travel

In any case the relation between the redshift $z = \Delta\lambda/\lambda$ and the apparent magnitude m of some objects with a uniform intrinsic luminosity can be written

$$m = 5 \lg z + D + B \cdot z + \dots , \qquad A\ (9)$$

where all further terms may be neglected in the case $z < 0.2$ concerned here. For m being the photored magnitude of the tenth brightest galaxy in a cluster, ABELL (1957) assumes

$$D = 21.01 \qquad \text{and} \qquad B = 2 . \qquad A\ (10)$$

On the other hand almost any cosmology connects the redshift z as follows with the time-dependent radius R of the universe:

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_o)}{R(t)} , \qquad A\ (11)$$

where t is the cosmic time. This shall here be expressed as in (6) by its present value t_o, the reciprocal HUBBLE-parameter:

$$k \overset{\text{def}}{=} R(t_o)/R'(t_o) , \qquad A\ (12)$$

and a measure τ of the distance from us:

$$t = t_o - k \cdot \tau . \qquad A\ (13)$$

Now the simplest of all the cosmological models, which do not contradict the present observations, is that of EINSTEIN, De SITTER (1932). There the metric is given by

$$ds^2 = R^2(t)(dr^2 + r^2 d\Omega^2) - dt^2 \qquad \text{A (14)}$$

with

$$R(t) = (t/t_0)^{\frac{2}{3}}\, R(t_0) \quad ,$$

where r is a comoving coordinate and $d\Omega^2$ describes the unit sphere. For A(12) with A(13) it yields:

$$h = \tfrac{2}{3} t_0 \ , \qquad \text{hence} \quad t/t_0 = 1 - \tfrac{3}{2}\tau \ . \qquad \text{A (15)}$$

Therefore we get from A(11):

$$z = (1 - \tfrac{3}{2}\tau)^{-\frac{2}{3}} - 1 = \tau(1 + \tfrac{5}{4}\tau + \tfrac{5}{3}\tau^2 + \ldots) \ , \qquad \text{A (16)}$$

and inserted in A(9) with A(10) this proves (29).

For a volume element dV of the comoving space, and for the radial propagation of light, we obtain from A(14) and A(15):

$$dV = \text{const} \cdot r^2 dr \quad , \qquad \text{A (17)}$$

$$|dr| = R^{-1}(t)\, dt = \text{const} \cdot (1 - \tfrac{3}{2}\tau)^{-\frac{2}{3}} |d\tau|$$

$$= \text{const} \cdot (1 + \tau + \tfrac{5}{4}\tau^2 + \ldots)\, |d\tau| \ .$$

This gives integrated

$$r = \text{const} \cdot (\tau + \tfrac{1}{2}\tau^2 + \tfrac{5}{12}\tau^3 + \ldots) \ ,$$

hence up to the second order in τ :

$$r^2 dr = \tau^2 (1 + \tau + \tfrac{13}{12}\tau^2)(1 + \tau + \tfrac{5}{4}\tau^2)\, d\tau \quad ; \qquad \text{A (18)}$$

such that A(17) proves (11).

If we consider instead of the uniquely defined case A (14) of EINSTEIN, De SITTER (1932) any other of the FRIEDMANN models allowed by the present observations, only the fractions in A(16) and A(18) will slightly change. In (29) this would influence the factors b and c, in (11) only the last term.

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Texte zu den Abbildungen

Figure 1: A central region Ω_x allowing five congruent surrounding regions ω_x^λ in different positions.

Figure 2: One of our pairs of circular counting regions with its three intrinsic parameters.

Figure 3: To compare the mutual distances D of some objects (Table 1) with their diameters d.

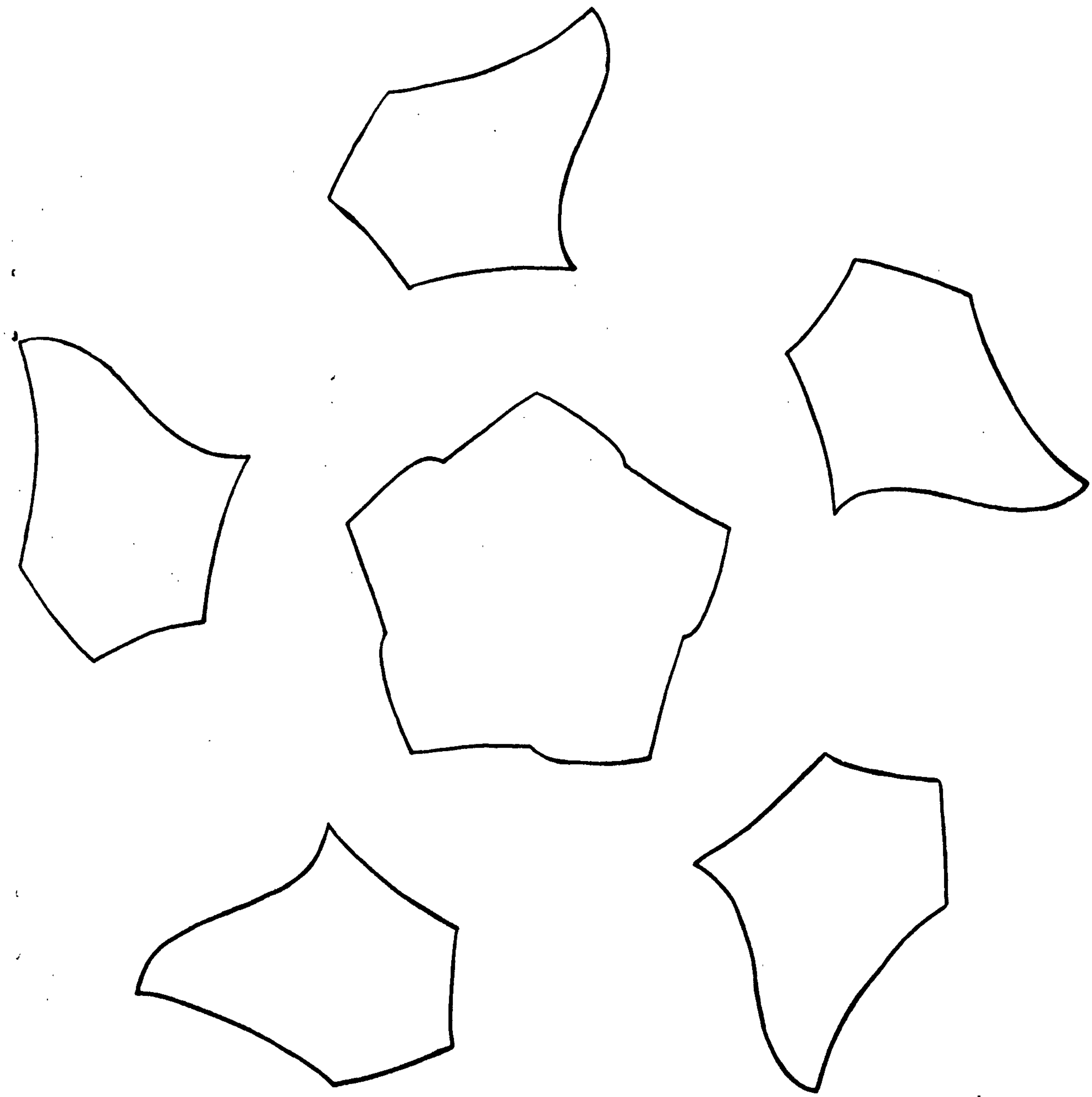


FIGURE 1

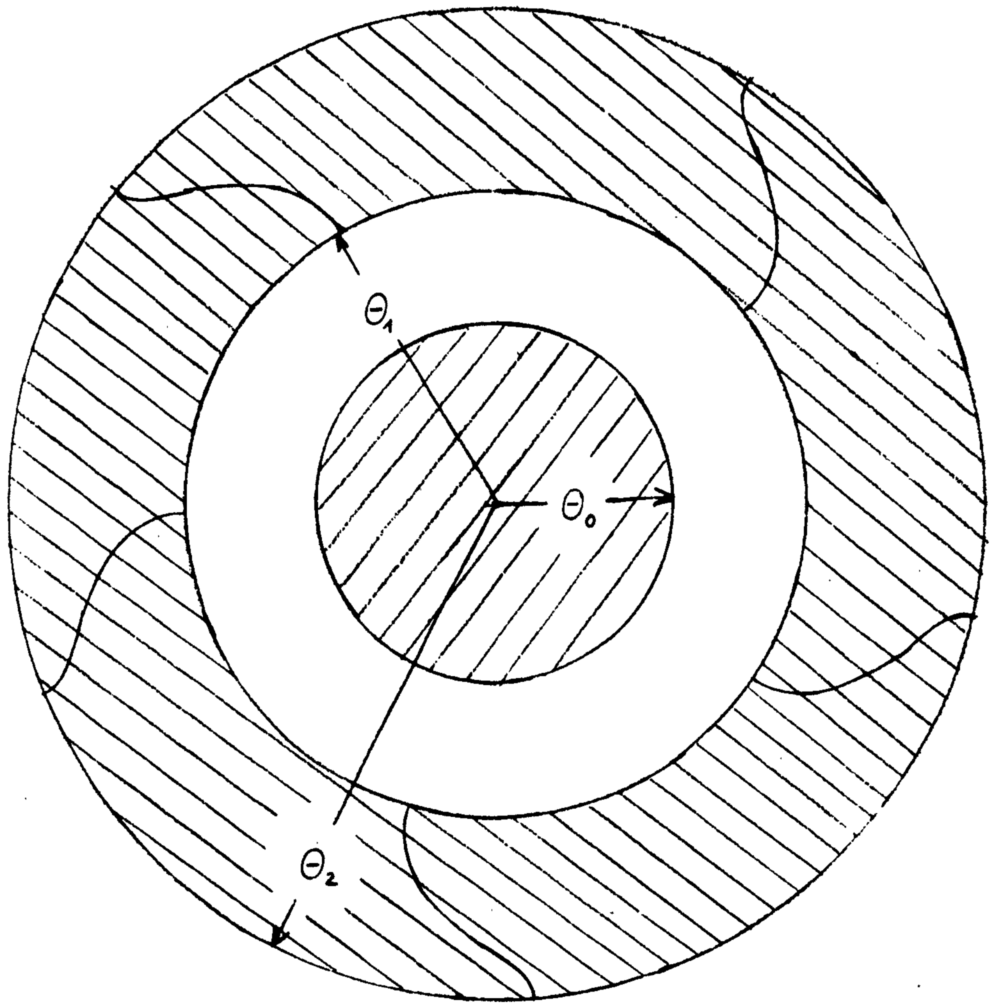
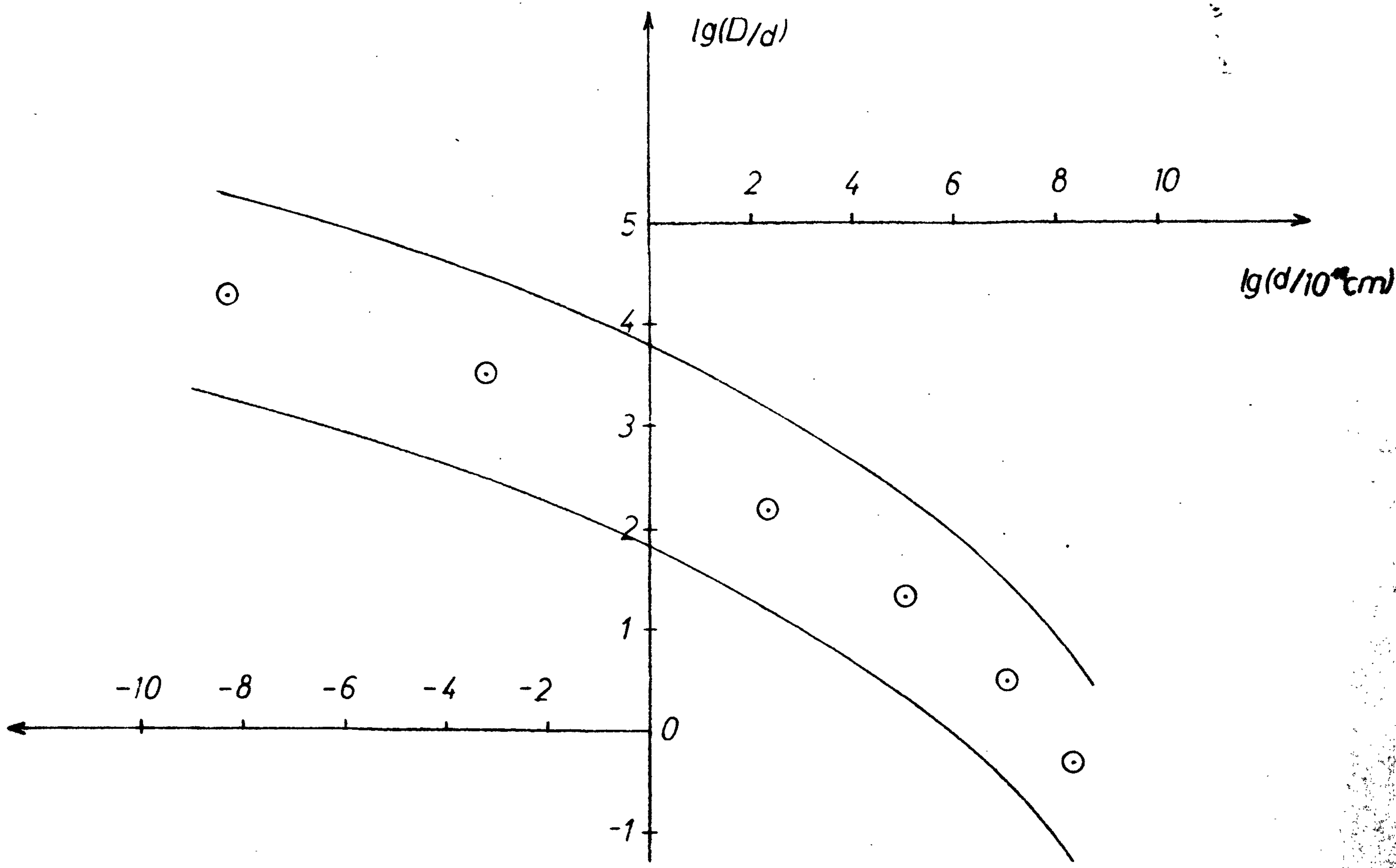


FIGURE 2

FIGURE 3



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