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(4) $4.60 (5) 524 000

(14) LMSC-A062400
(11) 30 Oct[redacted]62,

(7) NA
(8-9) NA
(12) 43p
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(6) HIGH ALTITUDE WIND RESPONSE OF MISSILE SYSTEMS,

(10) by C. J. Van Der Maas,
~~Research Specia~~list

LOCKHEED MISSILES & SPACE COMPANY

FOREWORD

The author is grateful to Mr. Muller of the Aeronautical Systems Division, Air Force Systems Command, for his cooperation in furnishing the Lockheed Missiles & Space Company with a magnetic tape containing wind soundings compiled by AviDyne Research, Incorporated. Grateful appreciation is also due to Messrs. B. Brougher and B. Laing of the Lockheed Missiles & Space Company for their considerable efforts in writing an IBM 7090 program for the statistical analysis of large bodies of wind sounding data.

ABSTRACT

As part of continuing studies performed by the Lockheed Missiles & Space Company, a survey of current practices to evaluate the critical rigid body response of vertically-rising missiles to high-altitude winds was conducted. As a result, a parametric approach was developed which permits optimization of the design of a missile system. The launch probability can be optimized with respect to the mission of the vehicle and the structural weight traded off against payload capability or flight performance.

The study resulted in the isolation of two definitive parameters: maximum wind velocity and integrated area under the wind profile. Within statistical limits, these two parameters define the rigid body response to large scale winds. The probability that a certain response will be exceeded depends upon the joint probability function of maximum wind velocity and wind profile area. Parametric plots have been prepared to show this relationship.

Statistical analysis of actual wind soundings has been initiated and preliminary joint distributions of the primary variables were obtained. These distributions agree qualitatively with the predictions and their application to several vehicles resulted in parametric response diagrams ~~similar to those published previously~~. The overall technique was verified using actual wind soundings.

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ILLUSTRATIONS

INTRODUCTION

The maximum airload bending condition usually is critical in the design of vertically-rising missile systems. In traversing the atmosphere, the vehicle often encounters strong high-altitude winds which cause large angles-of-attack or sideslip. Since the dynamic pressure is significant at the same time, high bending moments are induced.

Because of the random nature of high-altitude winds, great difficulties are being experienced in defining the forcing function for the missile system response such that the structure is adequate within well-defined limits of probability. Numerous approaches have been published but nearly all of them are of the go/no-go type, resulting in a singular response making system optimization very difficult to achieve.

The generally accepted procedure for evaluating wind-induced structural loads is to approximate the extreme vehicle response using wind profiles of estimated probabilities. A wind profile is a graph of wind velocity as a function of altitude (Fig. 1). The earliest procedure was to analyze the radiosonde data statistically for horizontal wind velocity only. The profiles resulting from this procedure depict average wind velocities and average-plus-n-number-standard-deviations extreme winds. This process smoothes out the rate of change of wind velocity and thereby causes these profiles to be unsuitable for maximum response analyses. A more recent approach is to compute the shears for each sounding and then to evaluate these statistically. Since the results are limited to shears associated with a specified probability, it is necessary that the profiles be rounded out with velocity statistics obtained by the former procedure.

A schematic wind profile is presented in Figure 2. It is seen that there are six fairly important parameters: (1) maximum wind velocity, (2) critical

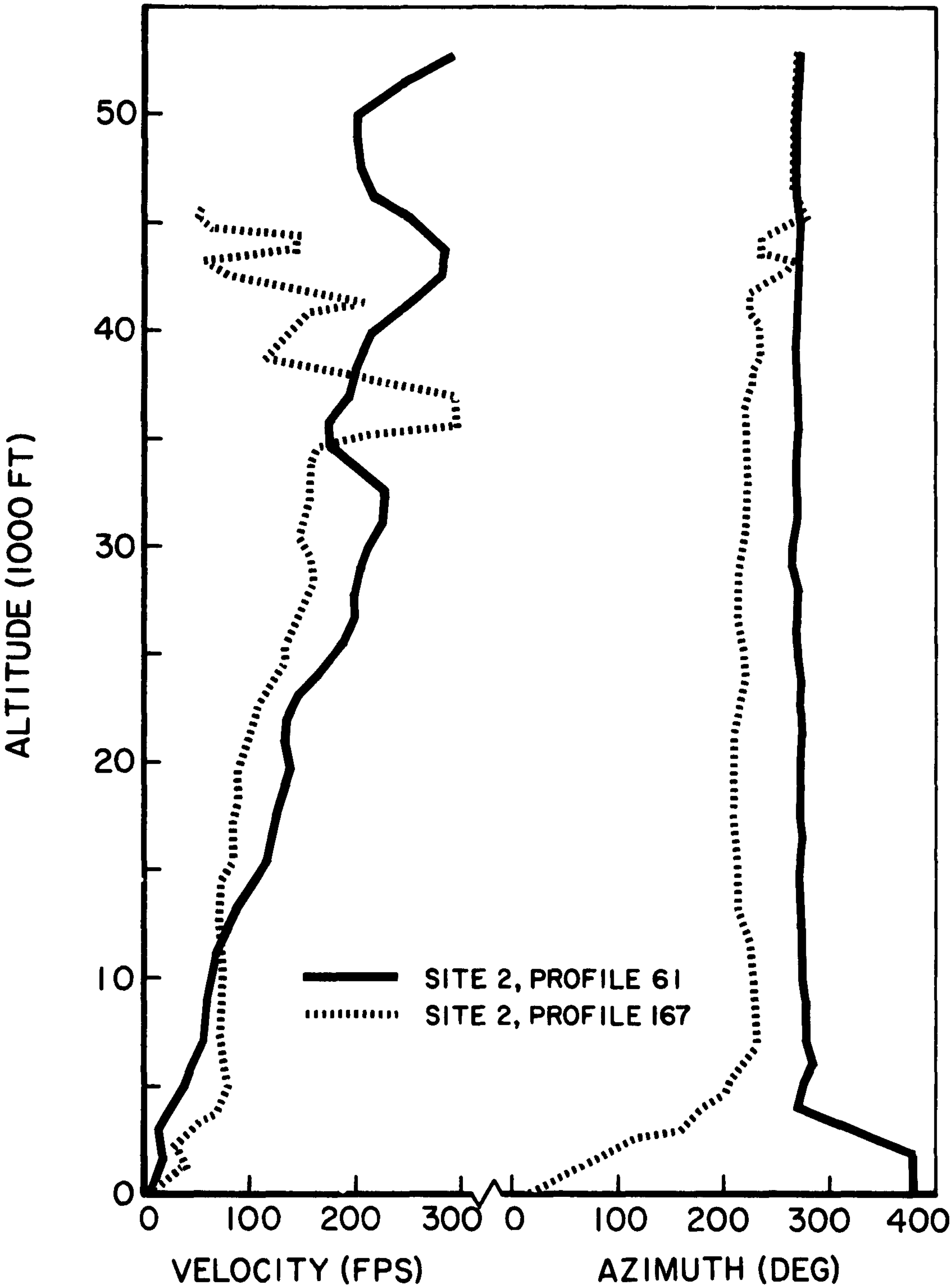


Figure 1 Typical Wind Profiles

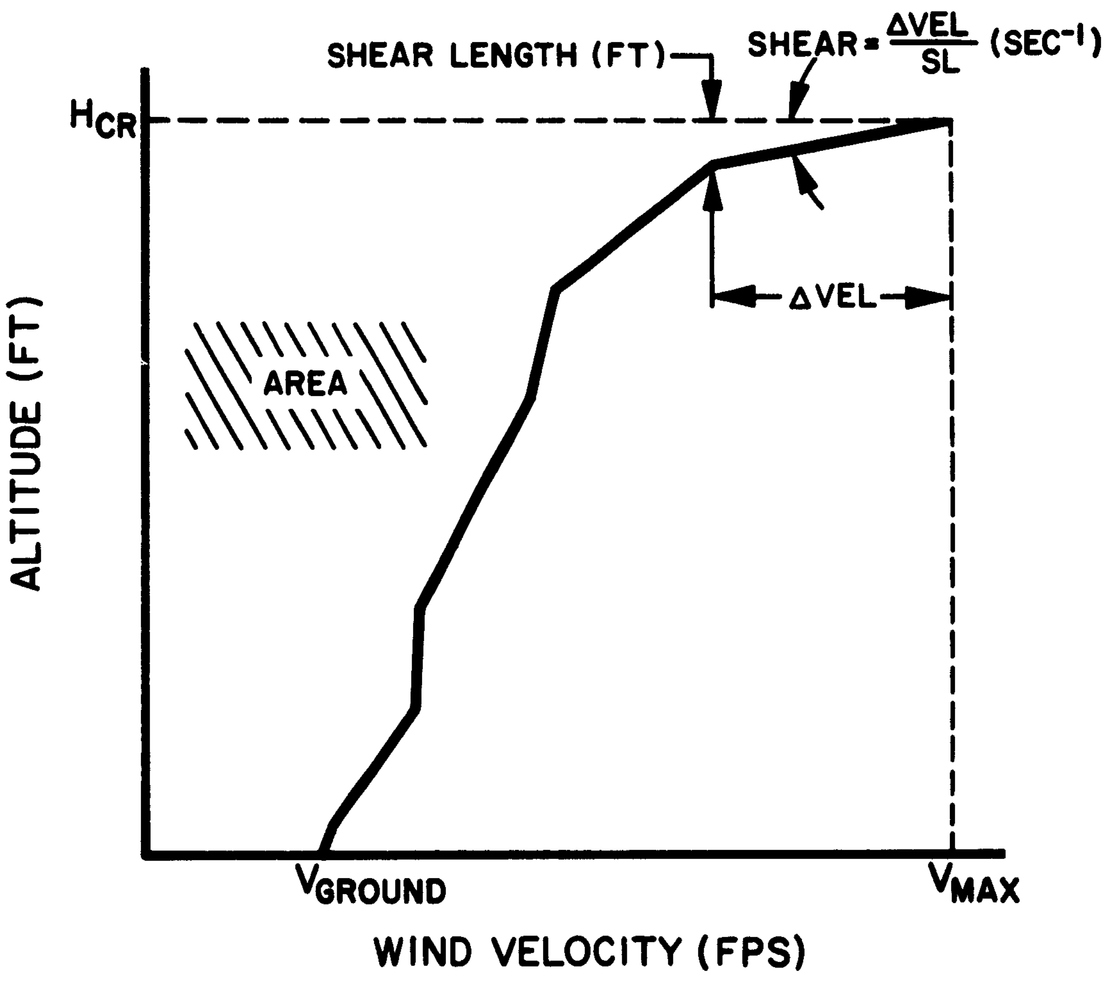


Figure 2 Wind Profile Parameters

altitude, (3) shear, (4) shear length, (5) ground wind, and (6) wind profile area. In the process of deriving one- or five-percent design wind profiles*, only the independent probabilities of maximum wind velocity and shear are commonly considered. General agreement exists that 300 fps and 250 fps are one- and five-percent maximum wind velocities, respectively, for the Continental United States. However, quantitative agreement on one-percent shears is poor as is shown in Figure 3.

In summary, attempts should be made to improve present techniques in the following areas:

a. Instead of the go/no-go type criteria, the total response capability of a missile system should be considered so that negative margins of safety may be corrected intelligently or a better than expected capability of the missile system may be properly utilized.

b. At the time of launch, it should be possible to relate the severity of an actual wind directly to the vehicle design limits.

c. Use should be made of design parameters which are not as widely varying as the shears shown in Figure 3, but instead are truly definitive parameters.

In the following material it will be shown that the large variation among design wind profiles is caused principally by the fact that the statistically evaluated shear is a secondary variable which does not sufficiently describe the atmospheric forcing function to completely define the vehicle response. Extensive response analyses have shown that the wind profile area in conjunction with the maximum wind velocity, while not describing every physical detail of the actual atmospheric disturbances, appear to define those characteristics of atmospheric winds that affect the vehicle response most significantly and consistently. Maximum wind velocity and wind profile area turn out to be primary forcing function parameters which, over their total range, define a vehicle response closely related to that due to actual winds. With these primary parameters isolated, it becomes possible to

*See Reference 1 for bibliography

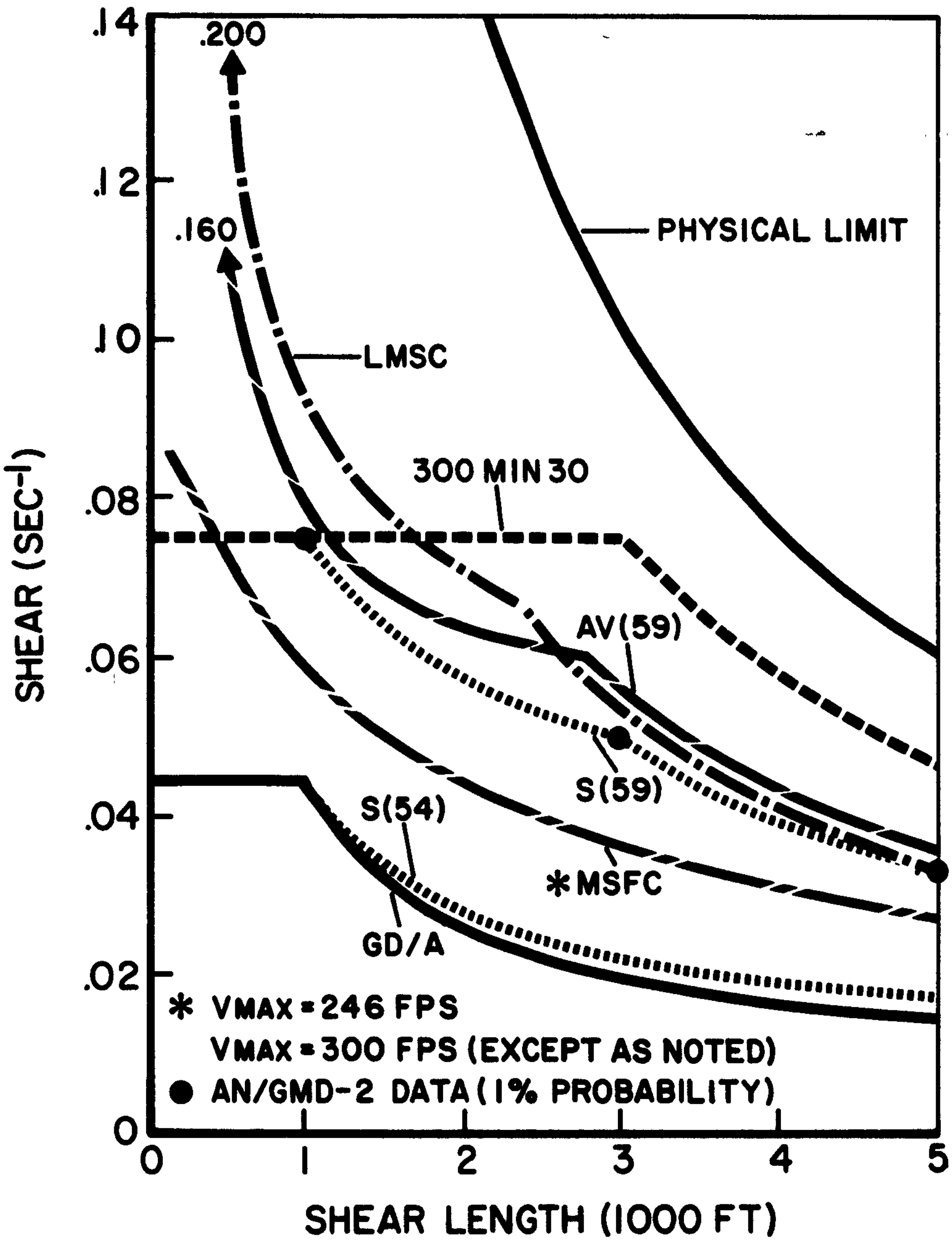


Figure 3 Shear and Shear Length for Literature Profiles

evaluate the missile response parametrically with a high degree of flexibility. The launch probability can be optimized with respect to the vehicle mission and the structural weight traded off against payload capability or flight performance.

VEHICLE RESPONSE

From the foregoing it is apparent that the selection of the maximum airload bending condition is hard to accomplish if the problem is approached from the wind sounding data only. Actually, the maximum response of a vehicle is the result not only of high shears but also of the effects of the winds beneath the altitude range of high shear. The attitude of the vehicle upon entering the range of maximum shear is dependent upon its response at lower altitudes. Therefore, the only way to resolve the question of how to define the wind forcing function appears to be the study of response of different vehicles to a variety of wind profiles and the isolation of those parameters which significantly affect the vehicle response.

The response to a large number of literature wind profiles* was obtained by means of an IBM 7090 six-degree-of-freedom trajectory flight loads program which incorporated the characteristics of several existing missiles as well as advanced designs. The scope of this program is summarized in Figure 4. The analysis is limited to rigid body motions since these constitute the large-scale effects. Moreover, only limited data are available on atmospheric disturbances capable of exciting flexible bending modes. The analyses were conducted for three relative wind directions (head, side, and tail), disregarding the structural and performance limitations of the missile. In some headwind cases this necessitated the removal of the engine stops† from the program.

*See Reference 1 for bibliography

†Restraints built into the engine gimbal mechanism which limit the engine deflections to a predetermined maximum value.

- 6 DEGREES OF FREEDOM
- CG POSITION, VELOCITY, AND ACCELERATION
- MOMENTS AND PRODUCTS OF INERTIA (INCL. DERIVATIVES)
- AERODYNAMIC CHARACTERISTICS
- AERODYNAMIC AND JET DAMPING
- AUTOPILOT STEERING EQUATIONS (OUTER LOOP)
- ATTITUDE COMMANDS
- ENGINE THRUST AND MOMENTS
- WIND PROFILE AND GUST

Figure 4 6D Trajectory Flight Loads Program

To isolate the principal parameters relating winds aloft and vehicle response, extensive parameter studies were conducted. To properly describe the response, the vector sum of αq and βq was determined to be definitive. This parameter correlates closely with the maximum bending moment in the missile structure and since it is the resultant of the responses in the pitch and yaw planes the correlation is with resultant bending moments. It evolves that the maximum response to each wind profile is defined by a single value of this parameter. It may occur at any altitude and does not necessarily coincide with maximum wind velocity.

In the search for definitive wind profile parameters numerous variables were investigated. The vehicle response generally increases with maximum wind velocity but this relationship has such a large band width that a second parameter is required to define the response more closely. All the conventional wind profile parameters shown in Figure 2 were studied (wind shear, shear length, ground wind, velocity increment during shear, and critical altitude). Although variations in each one of these variables have a more or less significant effect upon the missile response, the variation of a single variable is correlated with the response only if all or most of the other variables are held constant. This appears to be due to the fact that each one of these variables defines only a small portion of the wind profile. Inspection of the schematic wind profile (Figure 2) reveals that the integrated effects of a wind profile can be described by the maximum wind velocity in conjunction with the wind profile area,

$$A = \int_0^{H_{cr}} V \, dH \ \ ft^2/sec,$$

where H_{cr} is the altitude of maximum wind velocity. Within statistical limits, all profiles with the same maximum wind velocity fall on a single curve of maximum response versus wind profile area (Figure 5).

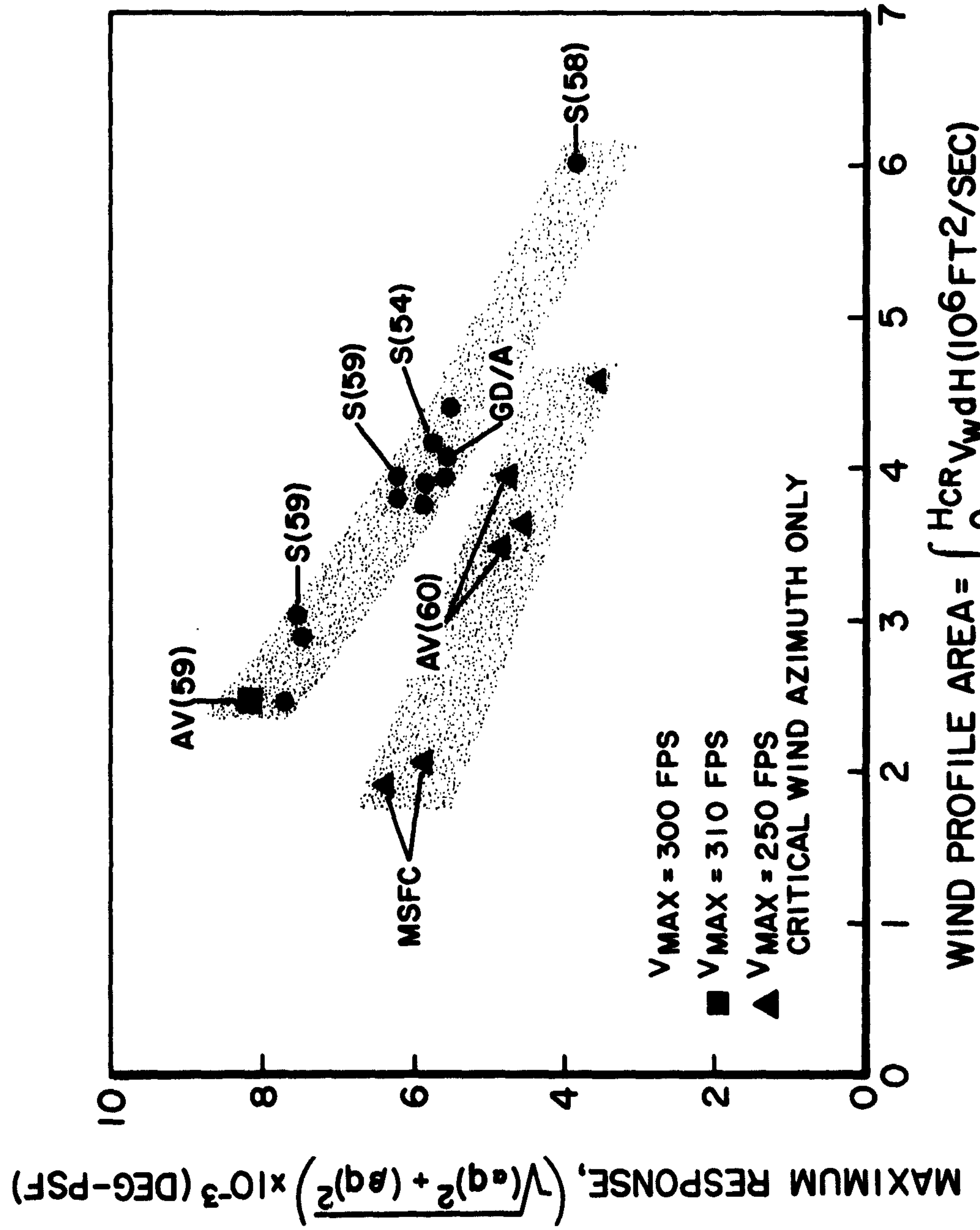


Figure 5 Rigid Body Response for Literature Profiles

In order to investigate these relationships still further a variety of synthetic profiles with maximum wind velocity of 300 fps was constructed and the vehicle "flown" through them on the computer. The vehicle response for these profiles (which range widely up to excessively severe) is plotted in Figure 6. Besides the obvious conclusion that the data trend is identical to that of Figure 5, several other observations can be made:

a. Variations of critical altitude induce response changes which are parallel with or on the conservative side of the general data trend.

b. Increasing ground winds cause a reduced response but also a shift of the curve into the critical direction.

c. An envelope for constant maximum wind velocity can be constructed that defines the maximum attainable response as a function of wind profile area.

The possibility of establishing a family of curves for a range of ground wind probabilities was investigated. However, this approach was abandoned since the present state-of-the-art is such that combined probabilities for ground wind and high-altitude shear cannot be determined with any degree of reliability. In the following, therefore, ground wind is considered as a random variable which contributes to the data scatter.

The foregoing procedure to obtain a response envelope is entirely too complex and laborious to be used in the design and launch control of missile systems. Instead, the envelope should be approximated by a minimum number of simple profiles with areas across the range of physical occurrence. In the absence of probability distributions of wind profile area, judgment had to be exercised. As a result, the following basic profiles were selected as analysis tools to develop wind response envelopes*:

a. MIN30 – minimum area profiles with critical altitude of 30,000 feet

*For detailed justification, see Reference 1

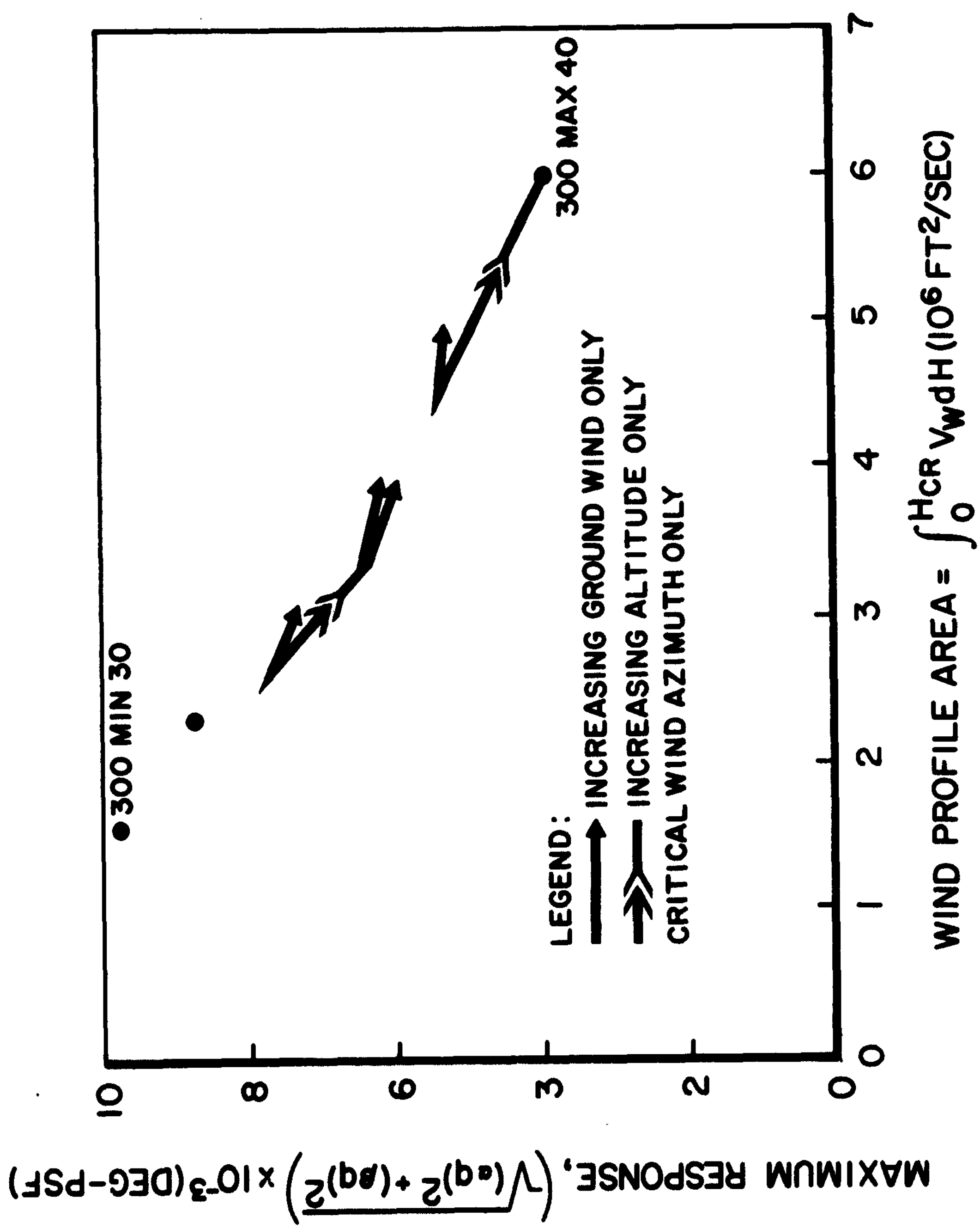


Figure 6 Rigid Body Response for Study Profiles

b. P30 and P40 – probable area profiles with critical altitudes of 30,000 feet and 40,000 feet respectively

c. MAX40 – maximum area profiles with critical altitude of 40,000 feet

Each one of these basic profiles was expanded into a series, with maximum wind velocity as the argument, so that they cover the total estimated range of maximum wind velocity and wind profile area.

Several missile systems were "flown" through the basic profile series and thus exposed to the total estimated range of maximum wind velocity and wind profile area. The results for one of these vehicles, including several literature wind profiles, are shown in Figure 7. It is seen that the constant maximum wind velocity curves are envelopes to the literature wind profiles. The diagram of Figure 7 maintains its basic configuration regardless of the missile system; only the relative magnitudes vary.

From Figure 7 it is apparent that the response of a missile system is bounded by the design maximum wind velocity (300 fps in Figure 7), minimum area wind profiles, and maximum area wind profiles. The most likely response appears to be bounded by the P30 and P40 series. From this it follows that the wind profile area can be eliminated by plotting the response as a function of maximum wind velocity only. This results in a wedge-shaped band with maximum wind velocity as the independent variable as shown in Figure 8. The band is bounded by the response associated with minimum and maximum wind profile areas while the most likely response is centrally located as expected. The significance of this diagram lies in the following:

a. The independent variable, maximum wind velocity, is a direct variable (directly associated with the physical data) rather than a derived variable like the wind profile area.

b. The diagram may be directly associated with probability of occurrence by visualizing a probability scale in the third dimension. The bell-shaped curve would have its mode between the response associated with P30 and P40 and reduce to near-zero at the response associated with minimum and maximum wind profile areas.

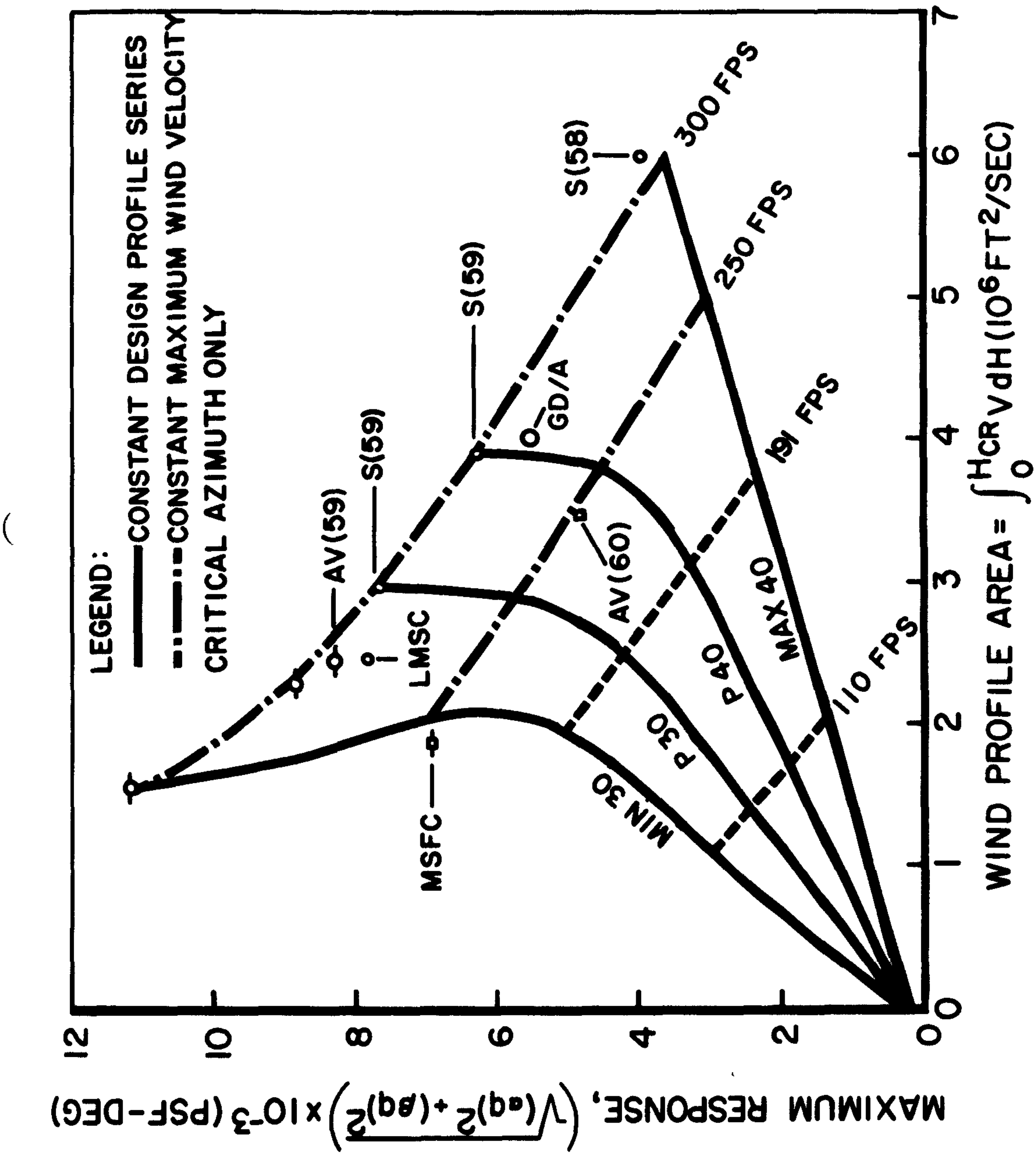


Figure 7 Rigid Body Response Diagram (Interim)

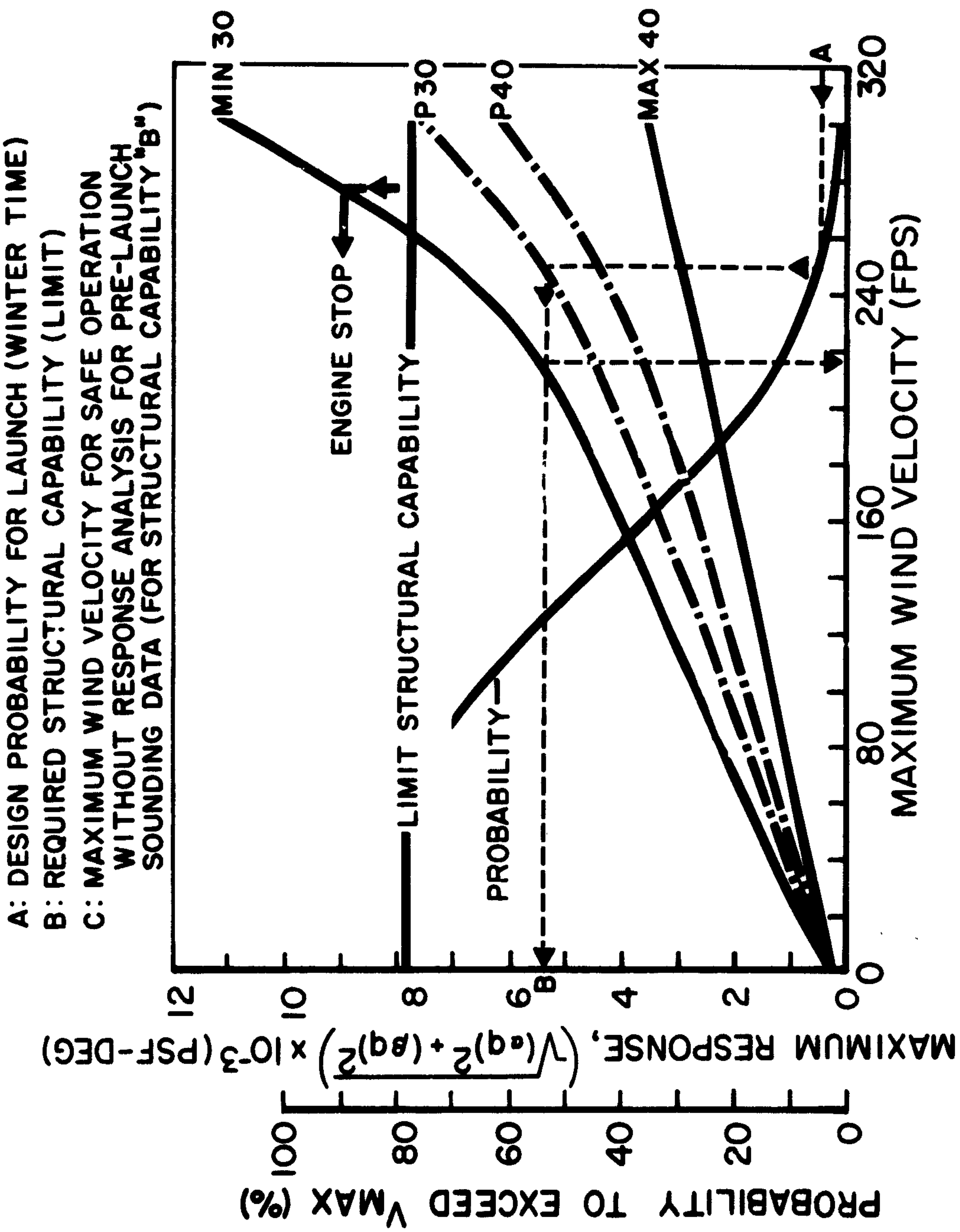


Figure 8 Design Response Diagram (Interim)

The design response diagram of Figure 8 does not negate the significance of the wind profile area as a definitive parameter. To the contrary, only by virtue of the area-parameter can it be stated that, for a specific maximum wind velocity and regardless of the wind profile, the missile system response should not exceed the minimum area response and will occur with the greatest likelihood between the responses associated with P30 and P40. The design response diagram also includes:

a. The probability to exceed the maximum wind velocity[2]. (Note that the probability curve is associated only with the most likely response, never with the minimum area response. This derives from the observations made about probability of occurrence in (b) above).

b. Any capability limitations of the missile system such as structural, engine gimbal stops, etc.

The design diagram may be put to many uses. For a completed missile system, the launch probability and the safe maximum wind velocity (without detailed analysis of prelaunch wind soundings) may be determined. In the early stages of design required limit capabilities may be obtained based upon launch probabilities desired by the customer. For operational missile systems the diagram's usefulness is that it facilitates a prelaunch check to determine whether limit capability may be exceeded. A detailed trajectory analysis to ascertain safe launch conditions is required only when the maximum wind velocity obtained from wind soundings is between the wind velocity associated with the most likely and extreme response.

In the foregoing material, only those wind azimuths which induce the highest response are considered. Therefore, the diagrams represent the maximum attainable response regardless of the launch azimuth and prevailing wind directions. Similar diagrams may be developed for specific relative wind directions; the configuration of the diagrams will remain the same but the magnitude of the response will be less or equal, never greater. Such an approach is desirable if a missile system is specifically intended for launch from a particular site, such as AMR, that has a decidedly predominent wind direction. Weight savings or performance increases may be effected in this manner.

APPLICATION AND VERIFICATION

Up to this point the development of a wind response analysis procedure was based upon synthetic wind profiles which were obtained by various methods of statistical analysis of radiosonde data. It remains, therefore, to apply the procedure using realistic probability distributions and to verify it with actual wind sounding data.

Through the courtesy of the Aeronautical Systems Division, a magnetic tape was obtained which was compiled by AviDyne Research, Inc. as part of work performed under Contract AF33(616)-8027 with Aeronautical Systems Division. This tape contains 200 wind soundings for each one of eleven geographical sites. The sites are:

Montgomery, Alabama;	Long Beach, California
Caribou, Maine	Tripoli, Libya
Fort Worth, Texas	Keflavik, Iceland
International Falls, Minnesota	Bitburg, Germany
Denver, Colorado	Kadena, Okinawa
Seattle, Washington	

The 200 wind soundings associated with a site were selected at random from all the wind soundings during the winter months from November through March over a time span of five years.

During the development of the wind response analysis procedure it was shown that both maximum wind velocity and wind profile area are statistical variables, each one being distributed statistically. From the configuration of the response diagram it may be concluded with reasonable certainty that the distributions of maximum wind velocity and wind profile area are interdependent. It follows that an adequate statistical definition may be obtained only if the joint distribution of maximum wind velocity and wind profile area is evaluated. Since this is a three-dimensional function, the sample of 200 wind soundings available for each single site is inadequate for statistical definition at the one-percent level. Therefore, the wind soundings from all eleven sites were combined and the analyses performed on one world-wide sample of 2200 wind soundings. Although the analysis results may not apply to any one particular site, the approach does serve to show the application and verification of the wind response analysis procedure. Work is already underway to collect sufficiently large wind sounding samples in the proper format for selected launch sites.

Because of the specialized nature of the statistical analysis of wind soundings, its detailed discussion is presented in the Appendix. Some of the results, however, are presented here because they are important tools in the further development of the wind response analysis procedure.

The diagram of Figure 9 represents the confidence limits of wind profile area as an independent statistical variable. These may be interpreted to mean that a specific percentage of all wind profile areas is on or above the confidence line associated with this percentage. The confidence level (percent) is associated with wind profile area only and does not include probabilities of maximum wind velocity or critical altitude. It may be seen that the configuration of Figure 9 is very similar to that anticipated*.

The confidence limits for the joint occurrence of maximum wind velocity and minimum wind profile area are presented in Figure 10. These should be read to mean that of all the maximum wind velocity and wind profile area combinations, a specific percentage is on or below and to the right of the confidence line associated with this percentage. The confidence level is associated with maximum wind velocity and wind profile area only and does not include probabilities of critical altitude. It may be seen that the highest possible maximum wind velocity on the one-percent level is 285 fps and that high maximum wind velocities are associated with large wind profile areas. Note

* Compare with MIN30 and MAX40 in Figure 8.

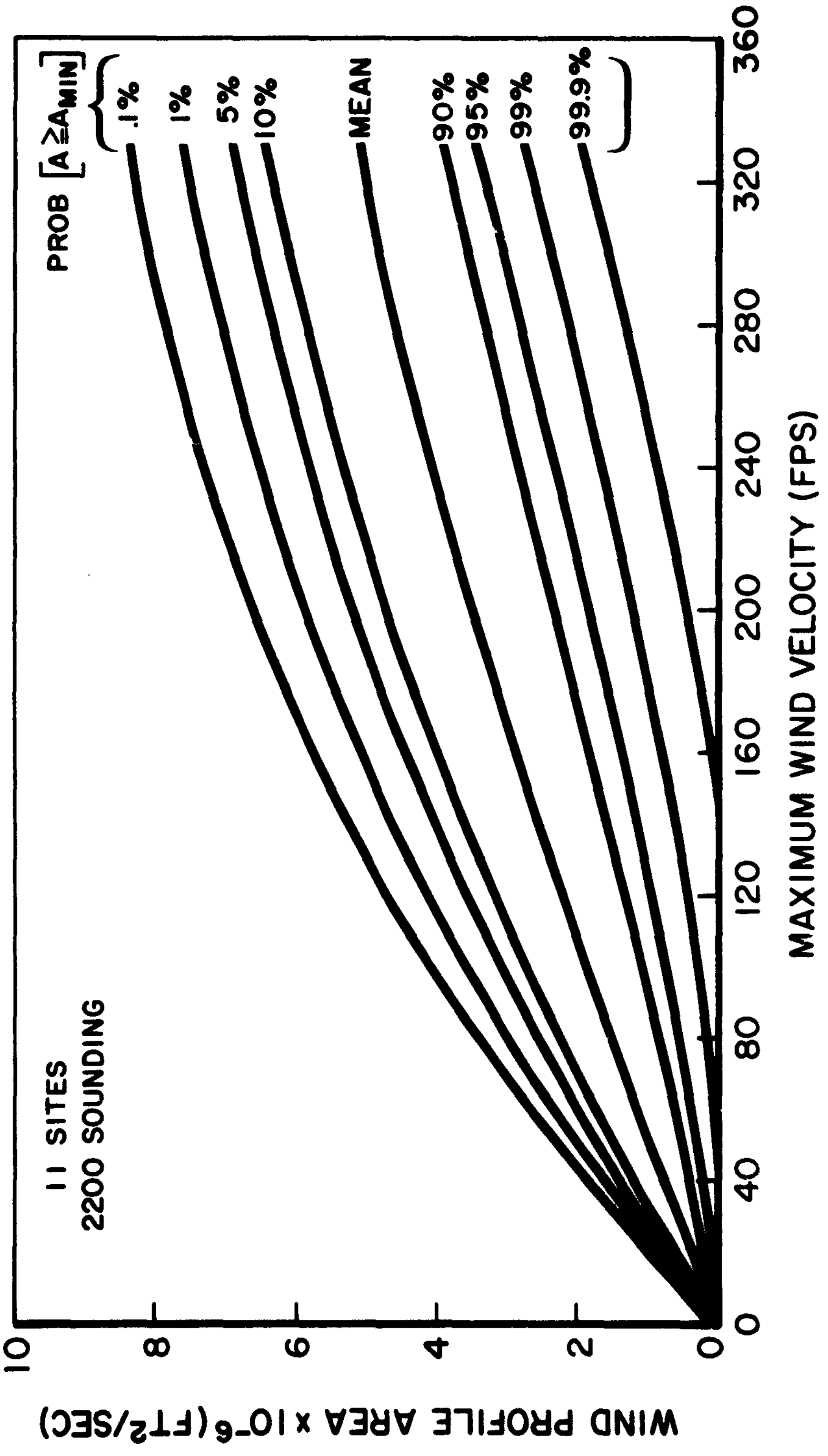


Figure 9 Confidence Limits for Wind Profile Area (Independent)

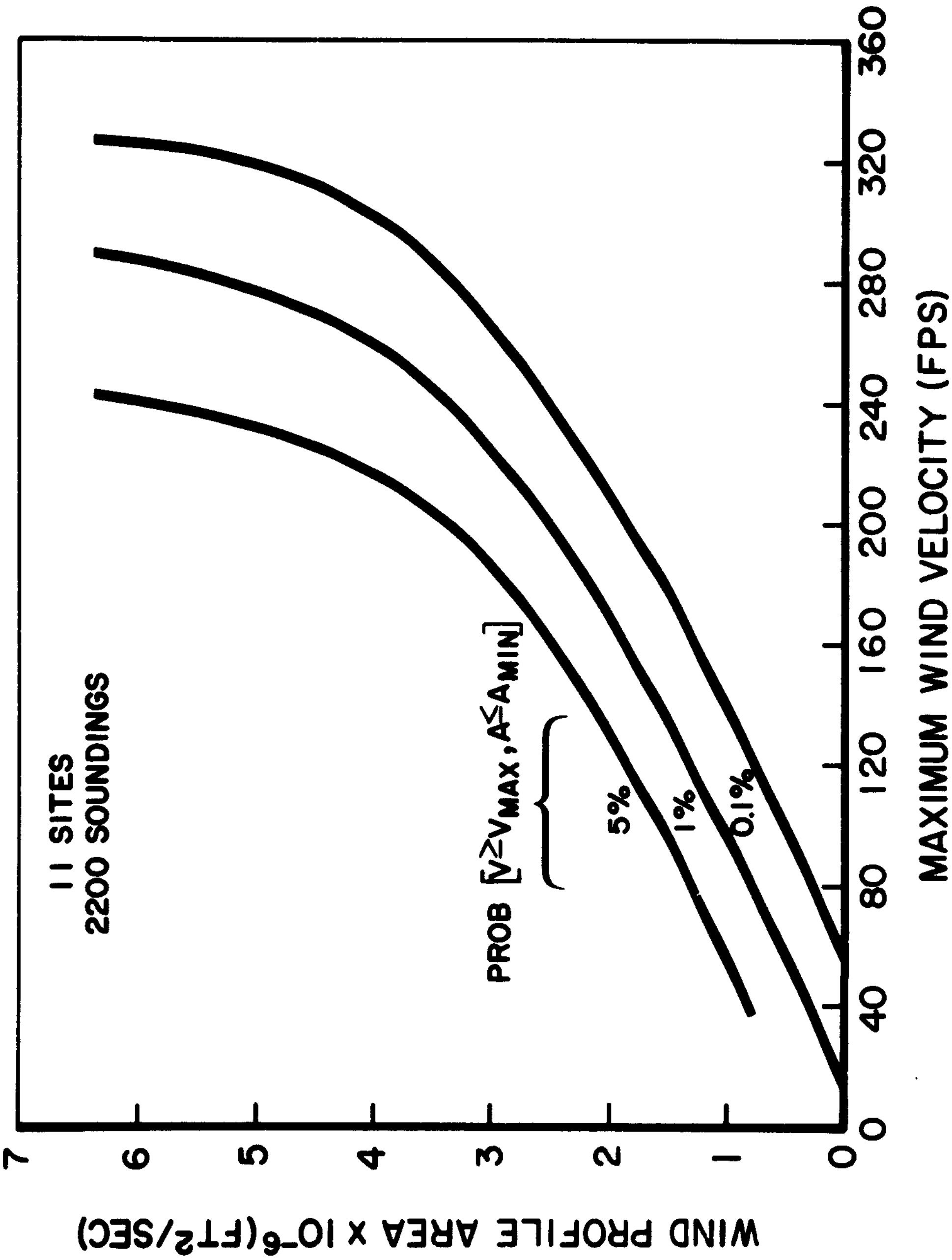


Figure 10 Confidence Limits for the Joint Occurrence of Maximum Wind Velocity and Minimum Area

that these confidence limits are not based on any assumptions but result strictly from statistical analysis of the raw data.

Confidence limits for the joint occurrence of maximum wind velocity and critical altitude are shown in Figure 11. Their interpretation is that of all the maximum wind velocity and critical altitude combinations, a specific percentage is on or below and to the right of the minimum critical altitude confidence line associated with this percentage. This confidence level does not include probabilities of wind profile area. It may be seen that there is an extremely low probability for the occurrence of a 300-fps maximum wind velocity below 32,000 feet and above 45,000 feet. Again these confidence limits result strictly from statistical analysis of the raw data.

In developing the interim diagrams of Figures 7 and 8, the vehicle response was obtained over the range of maximum wind velocity (0 to 300 fps) for estimated minimum, probable, and maximum wind profile areas. The same procedure still holds, except that actual data are now available on the range of wind profile areas for specific maximum wind velocities (Figure 9). In order to obtain the response envelope for 300-fps maximum wind velocity, the missile response is determined for a set of wind profile areas which are the intercepts of the confidence limits with 300-fps maximum wind velocity. By repeating this for lower values of maximum wind velocity, several response envelopes are obtained. Simultaneously, confidence limits of the response may be had by fairing radials through those responses which are related to the same wind profile area confidence line (Figure 12).

At this point, we have a rigid body response diagram similar to that of Figure 7. In order to verify the validity of the overall procedure, all wind soundings with maximum wind velocity between 280 and 320 fps were sorted out. (There are 21 of them.) The vehicle response to each one of them was evaluated for critical launch azimuth and the results superimposed on Figure 12. The maximum wind velocities were conservatively taken as the highest wind velocity attained over the total altitude range of the sounding data regardless whether the maximum response occurred at this point. The

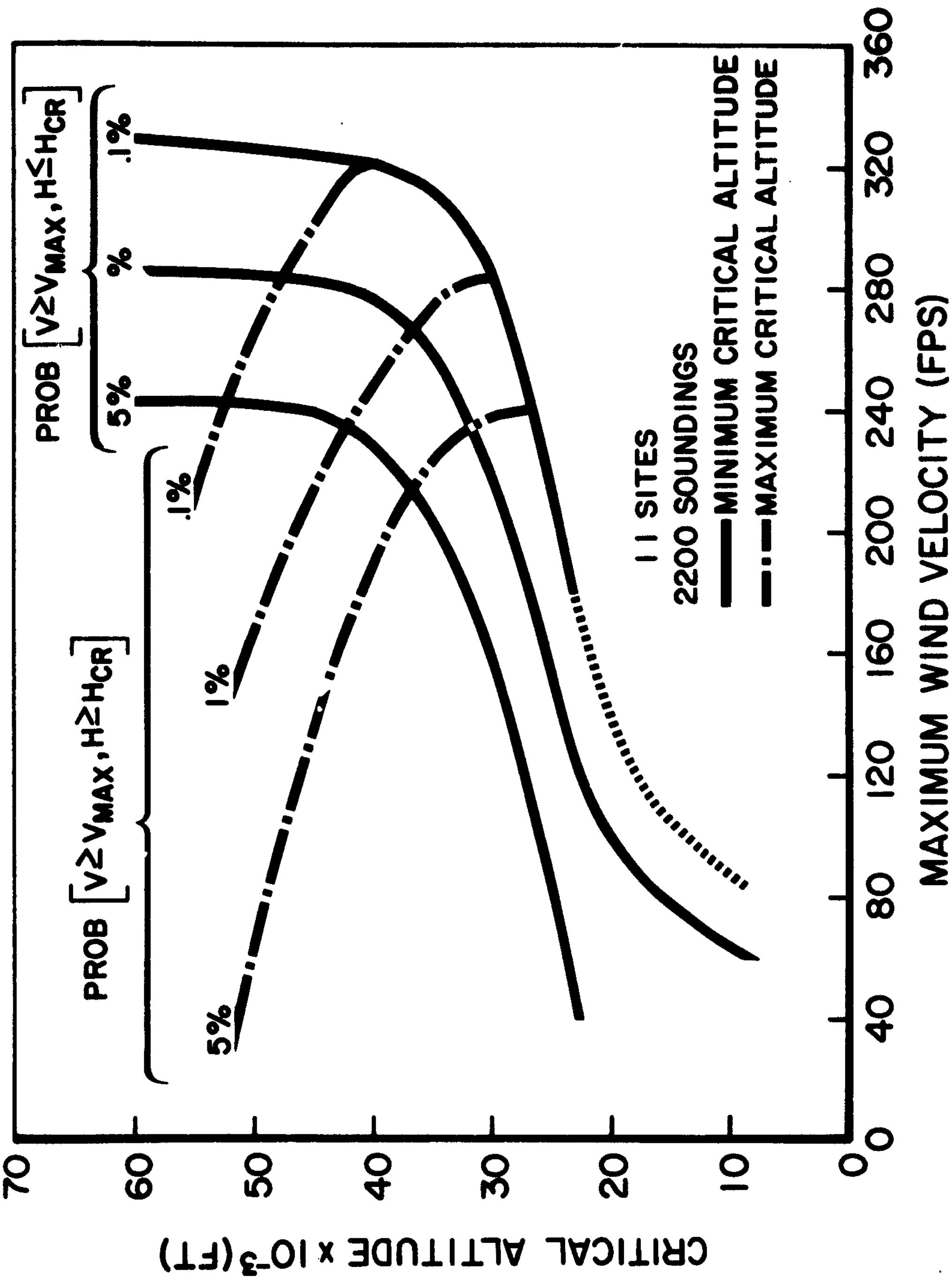


Figure 11 Confidence Limits for the Joint Occurrence of Maximum Wind Velocity and Critical Altitude

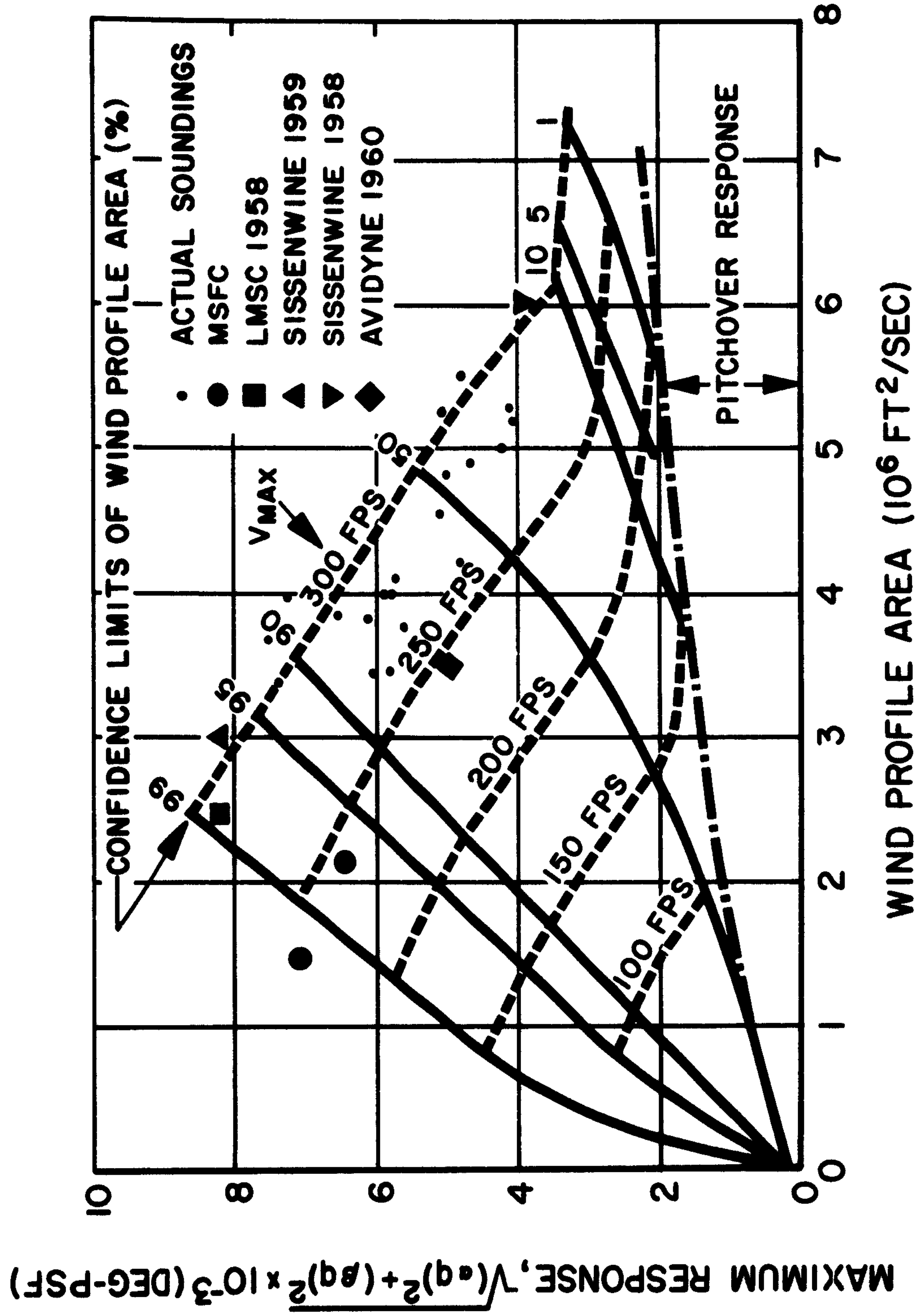


Figure 12 Rigid Body Response Diagram Based Upon 2200 AviDyne Wind Soundings

wind profile areas were obtained by integration of the soundings up to the altitude of this maximum wind velocity. It happens occasionally that a sounding contains two or more wind spikes with the maximum response occurring at a wind velocity less than the maximum. Since with the present procedure the maximum response is associated with the maximum wind velocity at whatever altitude each one occurs, the effect is to increase the band-width of the response and to extend the response diagram to larger values of wind profile area. A refinement which will be attempted in the future is to statistically analyze individual wind spikes rather than whole soundings. Such a procedure would reduce the data scatter and be consistent with the basic approach to consider the wind forcing function independent of the missile response. It is seen that the correlation between the 300 fps response envelope and the response for the individual soundings is good. It is expected to be even better when individual geographical sites are analyzed and wind spikes are considered rather than whole soundings. The vehicle response for several literature wind profiles is also shown in Figure 12.

The final design response diagram may now be prepared as shown in Figure 13. This diagram is similar to that of Figure 8 except that it is based upon actual wind distributions. The probabilities shown in Figures 12 and 13 are associated with wind profile area only and do not include probabilities of occurrence of maximum wind velocity and critical altitude. In order to permit an assessment of the actual launch probability, the response for the joint confidence limits of maximum wind velocity and wind profile area was determined (Figure 10). The results are shown, in Figure 13, as confidence limits to reach or exceed the associated response and maximum wind velocity simultaneously on the five-, one-, and one-tenth-percent levels. Since these confidence levels do not include the probability of occurrence of critical altitude, the actual joint launch probabilities are less than those shown.

In order to obtain the extreme response for a specific combination of maximum wind velocity and wind profile area, two assumptions had to be made. These involve critical altitude and ground wind, both of which are required to

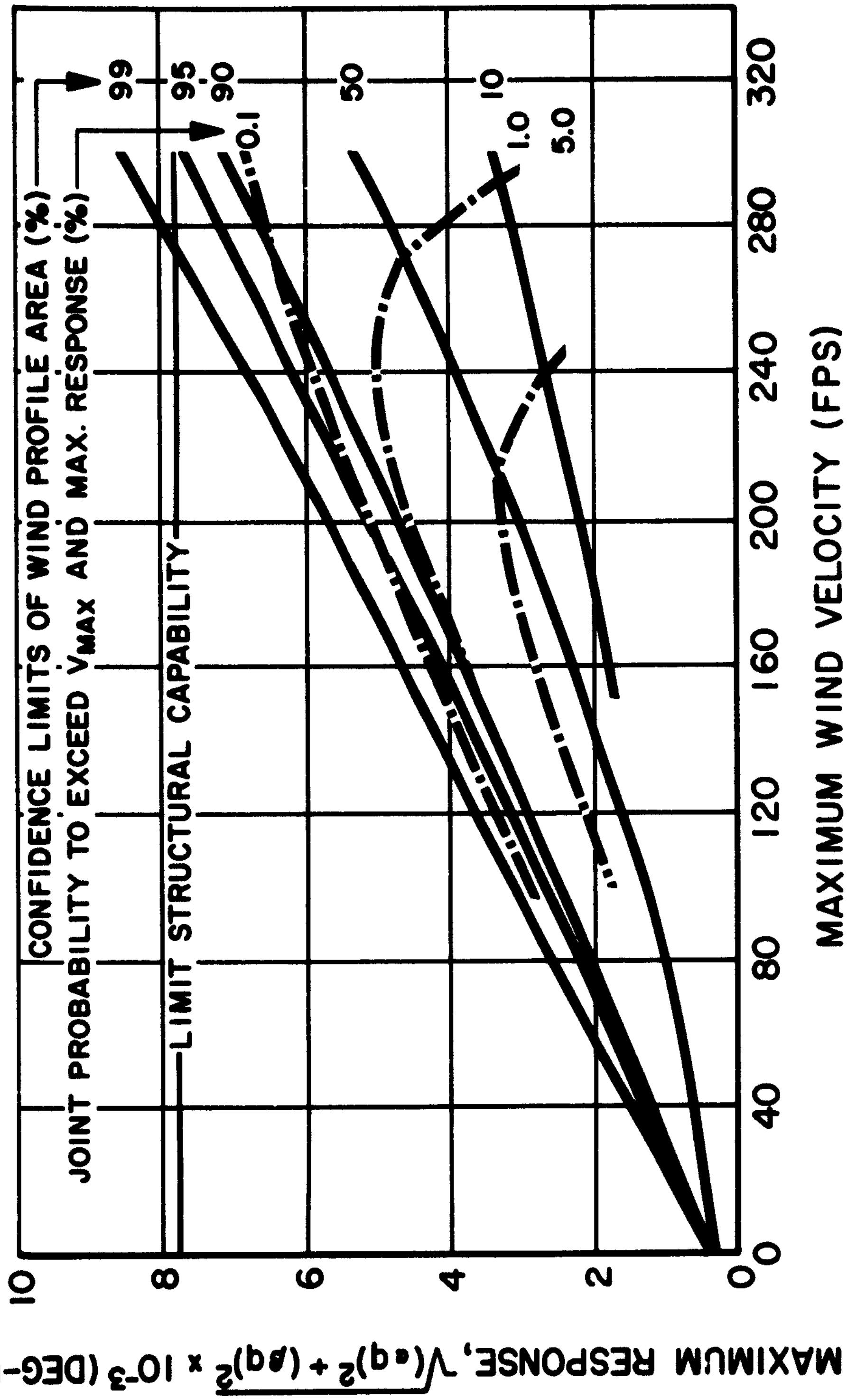


Figure 13 Design Response Diagram Based Upon 2200 AviDyne Wind Soundings

obtain a certain wind profile area at a specific maximum wind velocity. It was conservatively assumed that minimum wind profile areas are associated with the one-tenth-percent confidence limit for minimum critical altitude in Figure 11. Simultaneously, the moderately conservative assumption was made that ground wind is as high as compatible with the wind profile area (Figure 6). In summary, the joint probabilities shown in Figure 13 are conservative estimates of the actual launch probabilities.

CONCLUSIONS AND RECOMMENDATIONS

An elaborate investigation of high altitude winds and their effect upon vertically-rising missile systems resulted in the isolation of two definitive parameters: maximum wind velocity and integrated area under the wind profile. Within statistical limits, these two parameters completely define the rigid body vehicle response. For constant maximum wind velocity, the response is a monotonously decreasing function of wind profile area. All "design" wind profiles are discrete cases of this general relationship. For the class of missile systems studied, the vehicle response to large-scale winds aloft needs to be determined for a rigid body only.

The probability that a certain response will be exceeded depends jointly upon the probability functions of maximum wind velocity and wind profile area. The data contained on a magnetic tape obtained from Aeronautical Systems Division were analyzed to obtain various statistical distributions. The wind response analysis procedure was applied, using several of these distributions, resulting in a design response diagram which defines the total response capability of a missile system including realistic statistical confidence limits associated with wind profile areas. Included also are joint confidence limits on the five-, one-, and one-tenth-percent probability levels which are associated with maximum wind velocity and wind profile area simultaneously. The latter confidence limits show which portion of the total response capability is within the realm of a certain probability.

The design diagram may be used for a variety of purposes during all phases of design, including a check on the missile system's capability to withstand winds measured prior to launch. A major advantage of the proposed procedure is its flexibility, which permits the designer to trade off launch probability against cost in weight and dollars. Unlike the case with most existing wind criteria, it is now possible to back off from an initial launch probability while retaining a reasonably accurate knowledge of performance gains and the resulting launch probability.

Some miscellaneous conclusions of interest are:

a. The relationship between vehicle resonse and probability of occurrence is nonlinear.

b. If a missile system is being designed for launch from one base only and if this is a base with a predominant wind direction, then significant performance increases may be accomplished by accounting for this bias in the construction of the design diagram.

This investigation has resulted in a systematic and parametric approach for the determination of loads due to winds aloft. The procedure was applied using actual statistical data and its validity was verified with actual wind soundings. However, the body of data used consists of soundings from eleven world-wide sites. This means that the quantitative results are not directly applicable to a specific launch site. To remedy this situation, work is in progress on a translator which permits the transfer of standard Weather Bureau data cards to magnetic tape in a format acceptable to the IBM 7090 statistical analysis program. Once this program is completed, a large body of sounding data obtained at a particular launch site may be collected and analyzed by the procedures described herein. This would result in design response diagrams specifically applicable to a certain missile system at one particular launch site. It is anticipated that considerable performance gains may be realized in this manner, where the concept of performance includes structural, flight, launch probability, and reliability.

Time did not permit a more than cursory study of wind azimuth. However, this is an important parameter for two reasons: (1) its statistical distribution varies drastically from one site to another, and (2) changes in relative wind azimuth can affect the vehicle response considerably. Because of the present lack of detailed information, this study was limited to the relative wind azimuth (head, side, or tail wind) which induces the highest response. When large bodies of data are collected for specific launch sites, however, the detailed effects of wind azimuth should certainly be considered.

APPENDIX
STATISTICAL ANALYSIS OF WIND SOUNDINGS

Through the courtesy of the Aeronautical System Division, Air Force Systems Command, a magnetic tape was obtained which contains 200 wind soundings for each one of eleven geographical sites. In order to obtain adequate statistical definition, all analyses discussed below were performed on the combined total of 2,200 wind soundings for all eleven sites.

An IBM 7090 program was prepared which accepts input from the tape. It performs the following operations on the data:

a. For each wind sounding, the maximum wind velocity and associated critical altitude are selected; subsequently, the wind profile is integrated from zero to critical altitude.

b. For each sounding, the mean and standard deviations of the wind azimuth are computed over the total altitude range.

c. The sums, sums of squares, and sums of products of maximum wind velocity, wind profile area, critical altitude, and wind azimuth are computed for each site.

d. The joint cumulative frequency distributions of maximum wind velocity and wind profile area, maximum wind velocity and critical altitude, and maximum wind velocity and wind azimuth, are determined for either one particular site or all sites combined.

e. The independent cumulative frequency distributions of wind profile area, critical altitudes, and wind azimuth are obtained; each one is for specified class intervals of maximum wind velocity.

f. Finally, the total cumulative frequency distributions are determined for maximum wind velocity, wind profile area, critical altitude, and wind azimuth.

In evaluating the results of the machine computations, particular attention was focused on all possibilities for simplication. Examples are (1) whether a distribution could justifiably be considered normal or (2) whether a joint distribution was independent and thus could be obtained by multiplication of the total distributions. It was found that the magnetic tape data being truncated at 50,000 feet altitude renders the altitude distributions invalid from about 48,000 feet altitude and above.

The total distribution of wind profile area (independent of maximum wind velocity and critical altitude) is definitely more peaked than the normal distribution. However, within class intervals of maximum wind velocity, the assumption that the distribution of wind profile area is normal appears to be justified (Figure 14). Within each class interval of maximum wind velocity the mean and standard deviations of the wind profile areas were computed. The means of the wind profile areas appear to vary consistently with maximum wind velocity (Figure 15) while the standard deviations of the wind profile areas are related to the means (Figure 16). Using the curves fitted through the means and standard deviations, as well as the assumption of normal distribution of wind profile area within class intervals of maximum wind velocity, confidence limits for the wind profile area may be computed (Figure 9). These confidence limits may be interpreted to mean that a specific percentage of all wind profile areas is on or above the confidence line associated with this percentage. The confidence level (percent) is associated with wind profile area only and does not include probabilities of maximum wind velocity or critical altitude. It may be seen that the configuration of Figure 9 is very similar to that anticipated (compare with MIN30 and MAX40 in Figure 8).

In order to obtain probability levels for the simultaneous occurrence of wind profile area and maximum wind velocity, joint cumulation frequencies were computed treating wind profile area and maximum wind velocity as dependent variables. There results a three-dimensional body (Figure 17) which, in this particular instance, defines the probability to exceed maximum wind velocity and not to exceed wind profile area simultaneously. This joint probability function was tested to determine whether the variables are independent or, in other words, whether the joint distribution could be approximated by the product of the total distributions of maximum wind velocity and wind profile area. It was established that maximum wind velocity and wind

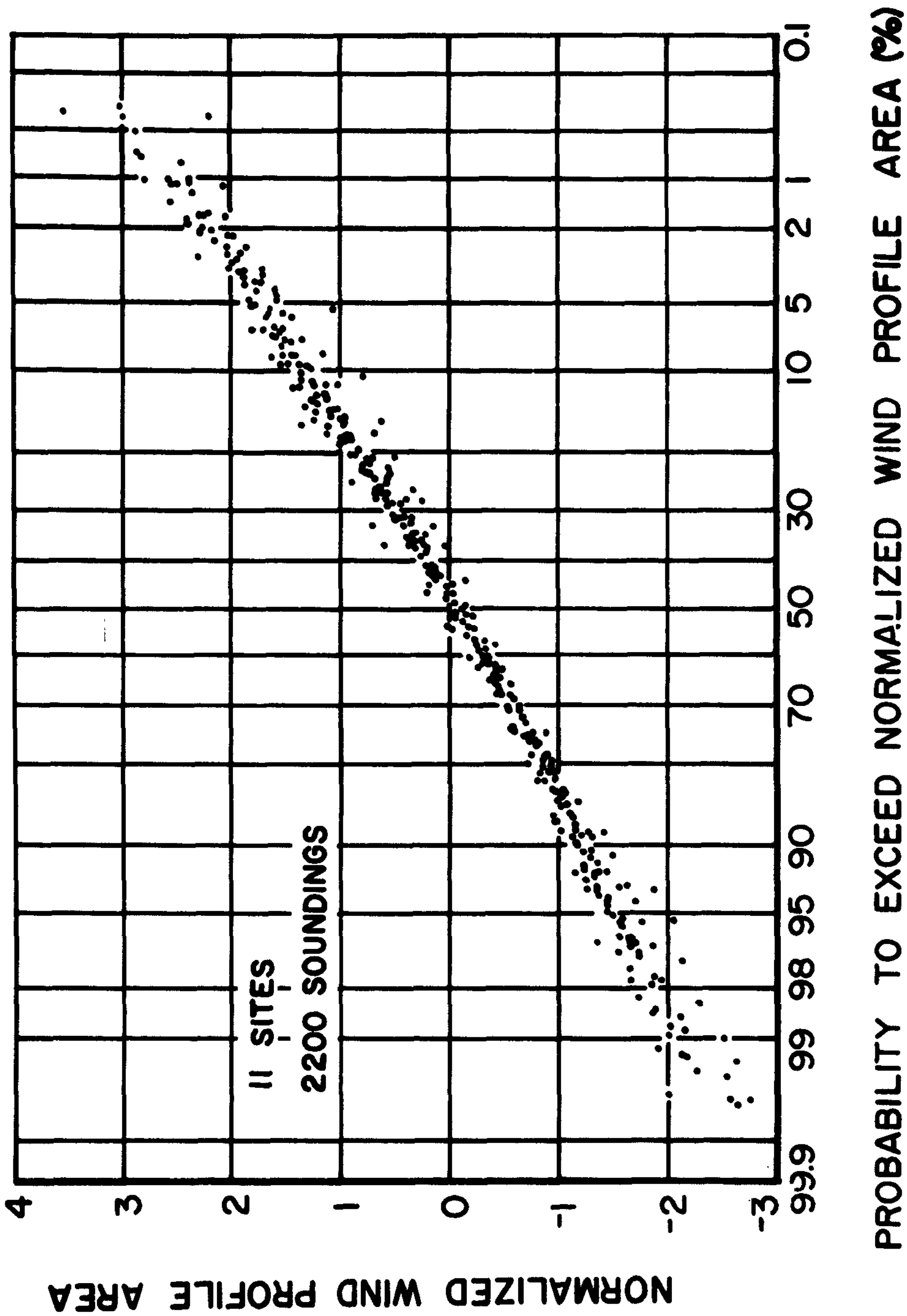


Figure 14 Independent Distributions of Wind Profile Area Within Class Intervals of Maximum Wind Velocity

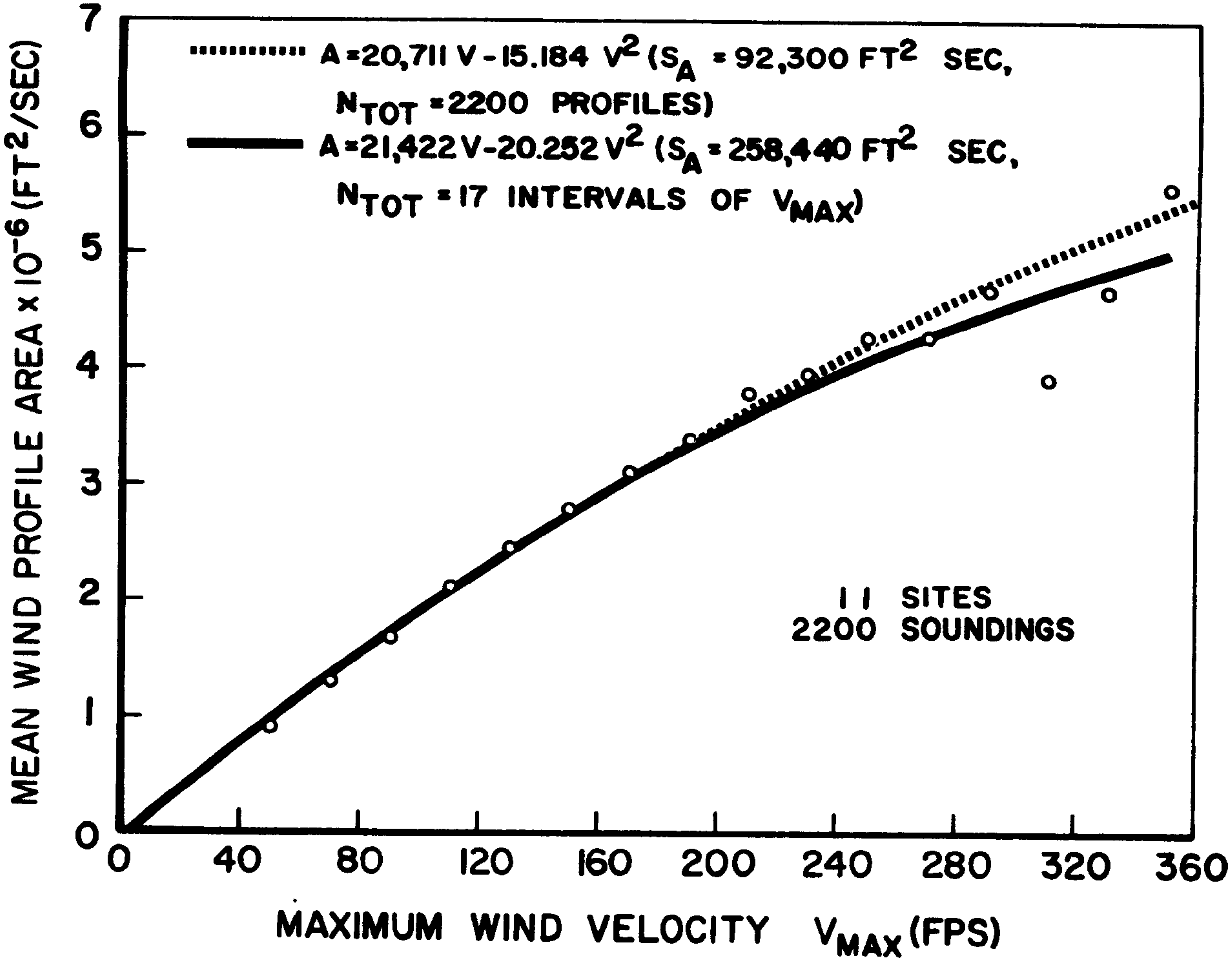


Figure 15 Regression of Mean Wind Profile Area on Maximum Wind Velocity

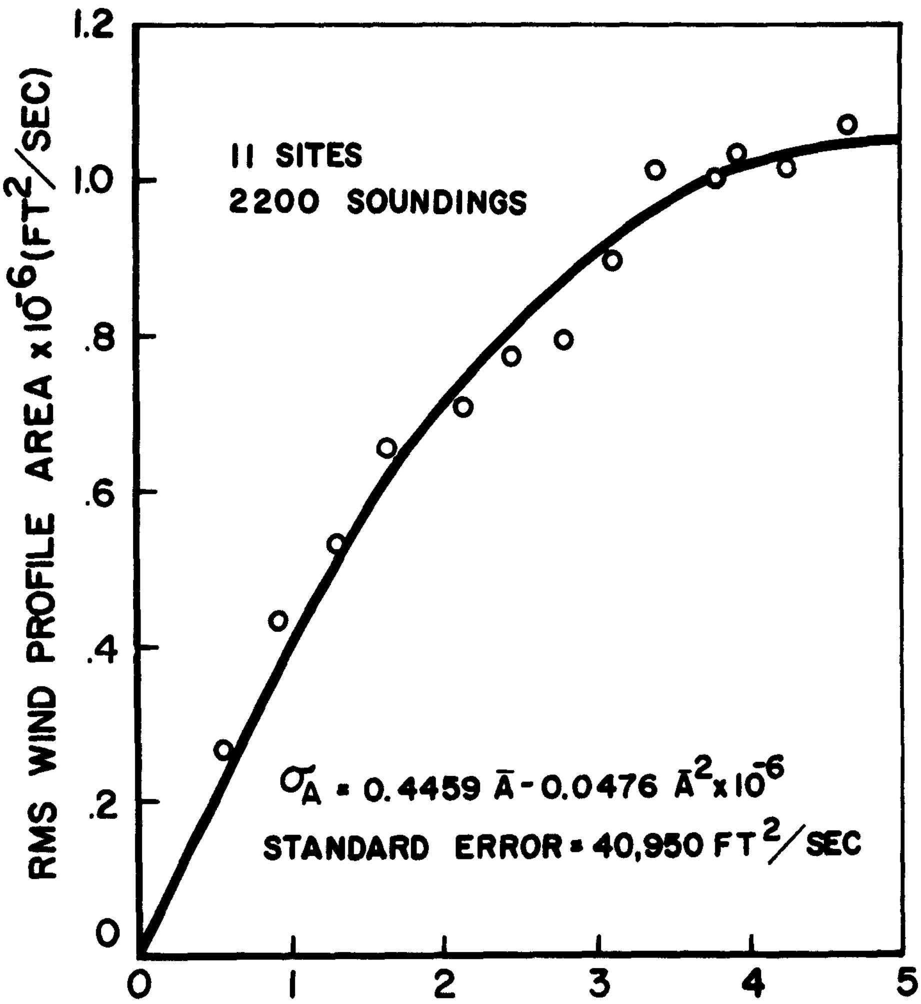
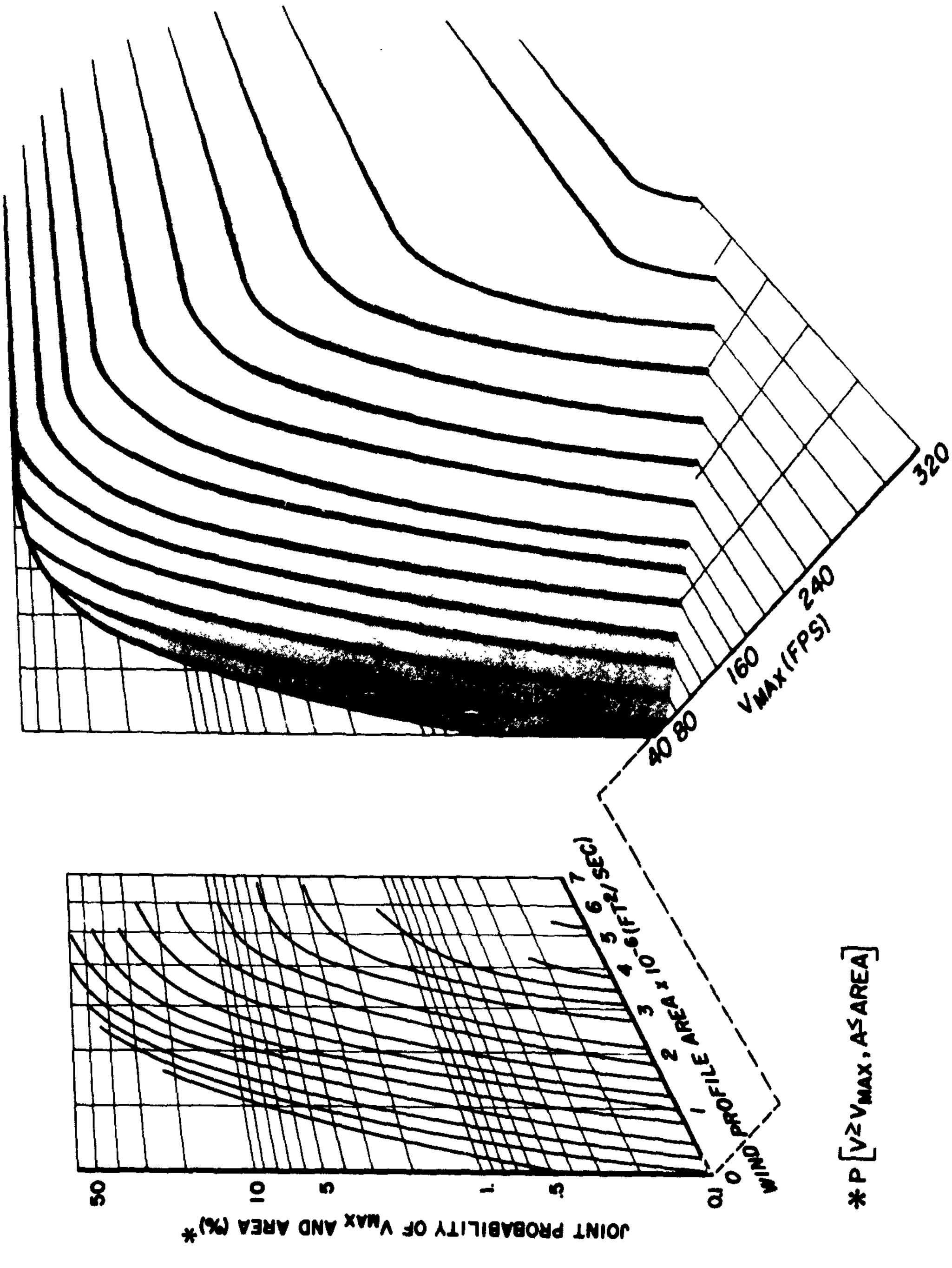


Figure 16 Regression of RMS Wind Profile Area on the Mean



* P $[V \geq V_{MAX}, A \leq AREA]$

Figure 17 Joint Probability of Maximum Wind Velocity and Minimum Wind Profile Area profile area are joint variables, meaning that the joint probability distribution can only be obtained by the technique illustrated in Figure 17.

The joint probability distribution can be sectioned at any level of probability, resulting in a curve which relates maximum wind velocity and wind profile area for a specific joint probability (Figure 10). These confidence limits may be interpreted to mean that of all the maximum wind velocity and wind profile area combinations, a specific percentage is on or below and to the right of the confidence line associated with this percentage. The confidence level is associated with maximum wind velocity and wind profile area only and does not include probabilities of critical altitude. It may be seen that the highest possible maximum wind velocity on the one-percent level is 285 fps. The joint probability distributions and associated confidence limits are not based on any assumptions but result strictly from statistical analysis of the raw data.

The total distribution of critical altitude (independent of maximum wind velocity and wind profile area) is much flatter than the normal distribution. However, within class intervals of maximum wind velocity the assumption that the distribution of critical altitude is normal appears to be justified (Figure 18). The means and standard deviations of critical altitude were computed within class intervals of maximum wind velocity. The means vary little with maximum wind velocity and tests indicate that only a mean critical altitude invariant with maximum wind velocity is statistically justified. The standard deviations of critical altitude appear to vary consistently with maximum wind velocity (Figure 19) and a third order polynomial can be fitted through the data with a high degree of correlation. Using the constant mean, the curve fitted through standard deviations, as well as the assumption of normal distribution of critical altitude within class intervals of maximum wind velocity, confidence limits for the critical altitude may be computed (Figure 20). These confidence limits may be interpreted to mean that a specific percentage of all critical altitudes is on or above the confidence line

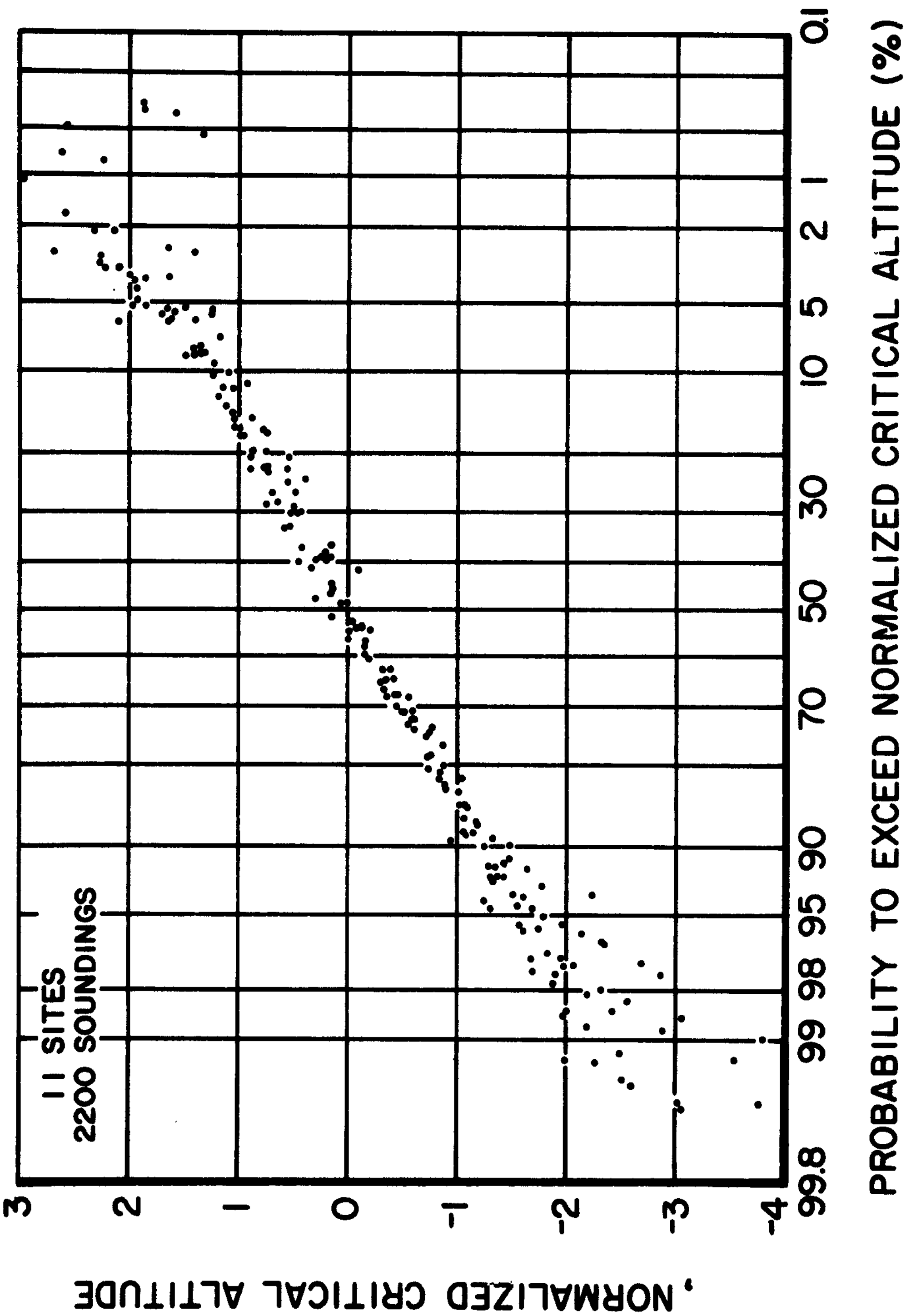


Figure 18 Independent Distributions of Critical Altitude Within Class Intervals of Maximum Wind Velocity

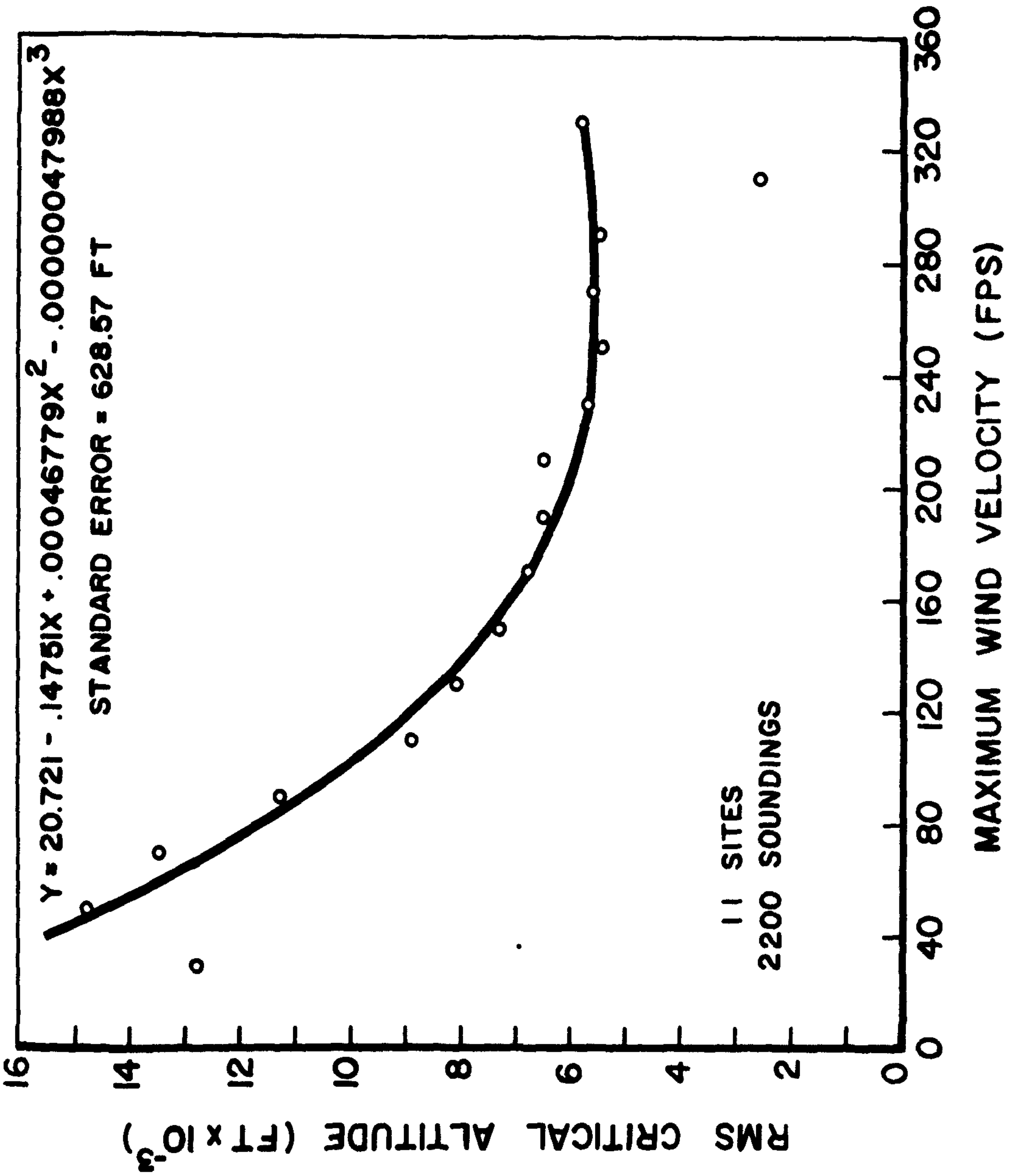


Figure 19 Regression of RMS Critical Altitude on Maximum Wind Velocity

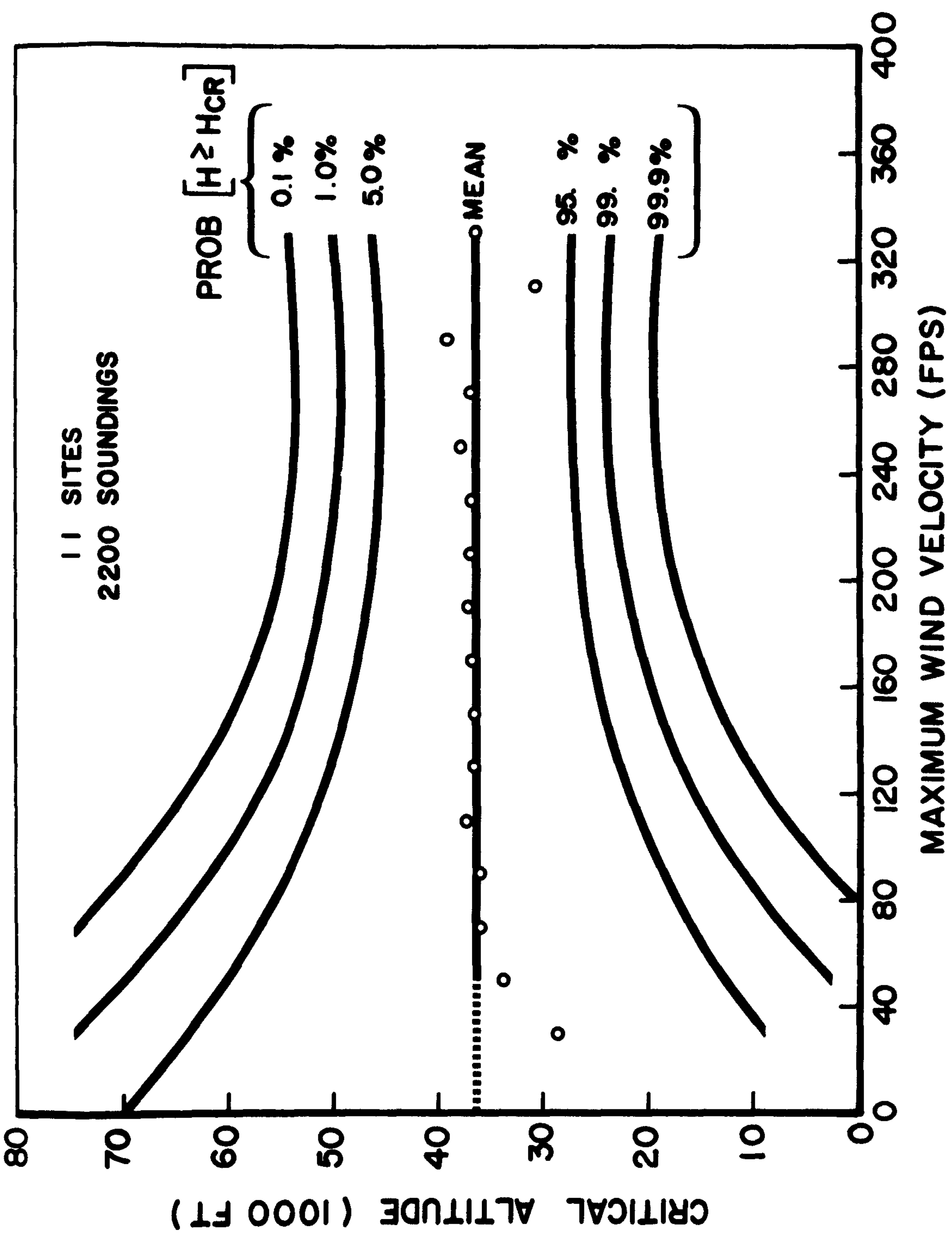


Figure 20 Confidence Limits for Critical Altitude (Independent)

associated with this percentage. The confidence level is associated with critical altitude only and does not include probabilities of maximum wind velocity or wind profile area.

Joint cumulative frequencies were computed treating critical altitude and maximum wind velocity as dependent variables. Tests indicated that maximum wind velocity and critical altitude are joint variables, meaning that the joint probability of occurrence can only be obtained by defining the three-dimensional probability distribution from the raw data. The joint distributions were sectioned at various levels of probability, resulting in confidence curves relating maximum wind velocity and critical altitude at those specific joint probabilities (Figure 11). These confidence limits may be interpreted to mean that of all the maximum wind velocity and critical altitude combinations, a specific percentage is on or below and to the right of the minimum critical altitude confidence line associated with this percentage. The confidence level is associated with maximum wind velocity and critical altitude only and does not include probabilities of wind profile area. It may be seen that there is an extremely low probability that a 300-fps maximum wind velocity will occur below 32,000 feet and above 45,000 feet. Another way to assess the joint distribution of maximum wind velocity and critical altitude is to section the three-dimensional function at constant critical altitudes (Figure 21). It is seen that high maximum wind velocities tend to occur at higher critical altitudes. Note again that the joint distributions and associated confidence limits are not based on any assumptions.

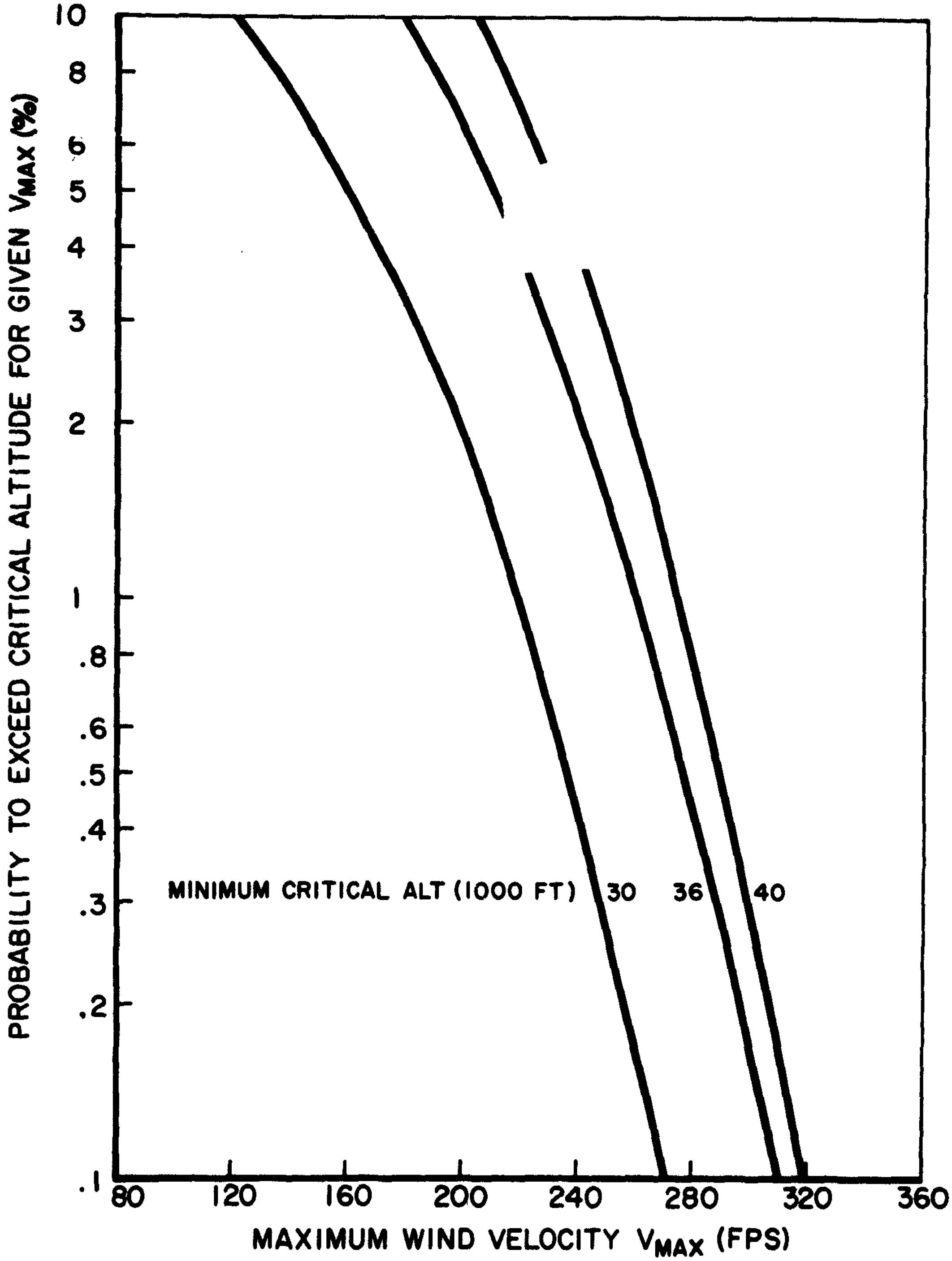


Figure 21 Joint Probability of Minimum Critical Altitude and Maximum Wind Velocity

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