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NATURE AND OBSERVATION OF HIGH-LEVEL TURBULENCE ESPECIALLY IN CLEAR AIR

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FOREWORD

This report was prepared for the Navy Weather Research Facility by the Department of Atmospheric Science, Colorado State University. It is one of a series of reports of work done at CSU under contract for NWRF Task 15, "Operational Jet Stream Research".

NWRF contract reports are normally given limited distribution, since they usually describe detailed methods and results of specific research projects or exploratory feasibility studies which are not of general interest to operational activities. This report, on the other hand, presents a comprehensive review of our present knowledge of clear-air turbulence (CAT), including a survey of the literature and an extensive bibliography. In view of the present widespread interest in CAT, and its significance to jet aircraft operations, the report has been reprinted by NWRF in order to give it wide distribution to operational units of the Naval Weather Service and to Naval Aviation units.

Dr. Elmar R. Reiter, Associate Professor of Atmospheric Science at CSU, was the principal investigator and the author of this report.

This publication has been currently reviewed and approved for reprinting on April 18, 1963 by the undersigned.

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NATURE AND OBSERVATION OF HIGH-LEVEL TURBULENCE ESPECIALLY IN CLEAR AIR

by:

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NATURE AND OBSERVATION OF HIGH-LEVEL TURBULENCE, ESPECIALLY IN CLEAR AIR

Abstract:

This is a "state-of-the-art" report on clear-air turbulence (CAT) research. Knowledge of micro-structural details of flow patterns in the free atmosphere is of importance to aircraft and missile designers. It also is of interest in considering passenger comfort.

There may be at least three different causes for bumpiness experienced by an aircraft flying horizontally:

(i) convective currents

(ii) gravity waves on interfaces

(iii) temperature discontinuities intercepted by a supersonic aircraft

For vertical take-off vehicles excessive shears of alternating sign in the vertical wind or momentum profiles should be considered as possible sources of vibration.

So far, observations of high-level turbulence have been made by aircraft and by balloon-borne gust sondes. Especially aircraft measurements present a problem inasmuch as the response characteristics of the vehicle usually make it difficult to evaluate the actual atmospheric gust input.

With our present theoretical modelling capabilities we may be able to study wave perturbations in the free atmosphere from detailed measurements of wind and temperature profiles. Such measurements will have to be orders of magnitude more accurate however, than the ones provided by the synoptic rawinsonde and radiosonde network.

1. INTRODUCTION:

Ever since the development of fast-flying jet aircraft there has been an increasing concern about clear-air turbulence (CAT). The importance of research in this field may be stressed by various considerations:

(1) First of all the aircraft engineer would like to have as detailed information as possible on gust loads to be expected with specific types of aircraft under design. From these data he will be able to compute stresses as well as aerodynamic behavior under average and extreme atmospheric conditions.

(2) Of no little concern to airlines is the "passenger comfort". It can be considered a function inversely related to frequency and amplitude of accelerations, and to length of time exposed to turbulent conditions. Crew-performance is also decreasing with similar correlations. This factor, however, may be of more concern in missile operations than in regular aircraft flights (Clark 1962).

(3) A particular phase of turbulence research is devoted to missile operations. While aircraft are following a quasi-horizontal track, missiles are traversing the atmosphere along a quasi-vertical trajectory. The atmospheric conditions, which might set off vibrations in the traveling vehicle will therefore be quite different in the two instances, and they will have to be considered separately.

2. OBSERVATIONAL EVIDENCE OF CAT

Quite to the contrary of earlier assumptions, experience from aviation and research shows that the stratosphere is not nearly as "stratified" as its name might lead us to believe. Although some statistical evidence gained from U-2 data indicates that the frequency of severe gust encounter decreases as we go beyond the tropopause (Rhyne and Steiner, 1962) we can by no means be certain of the absence of turbulence at flight levels of supersonic transport aircraft.

Fig. 1 contains a summary of turbulence frequencies obtained from various sources. The solid line indicates measurements mainly collected by NASA from VGH instruments, which were carried along on commercial transport operations and on research missions (Press and Steiner, 1958; Tolefson, 1956a; Binckley and Funk, 1949). These data cover the altitude range up to about 50,000 ft. An extension into higher altitude ranges was possible with the aid of U-2 data from some 315,000 flight miles over the Northern Hemisphere (Coleman and Steiner, 1960). These have been entered by circles.

According to these statistical findings a frequency maximum of CAT occurrence will have to be expected near tropopause level. This agrees well with

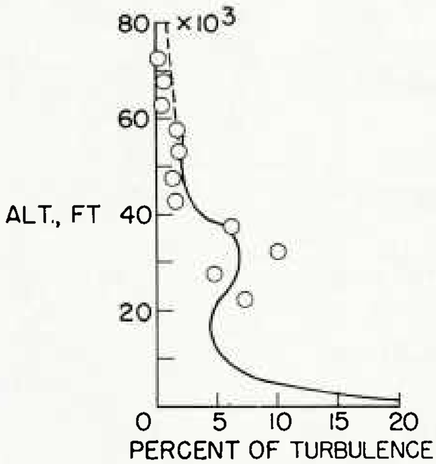


Fig. 1 Turbulence occurrence from U-2 data (circles; Coleman and Steiner 1960), and from data presented by Press and Steiner (1958) (solid line), and by Coleman and Stickle (1961) (dashed line). (Rhyne and Steiner 1962).

Table I. Characteristics

1 Source	2 Altitude Range	3 Alt. Range of Frequency Max.	4 Intensity of Turbulence
1. Bannon (1951a, b)		28000 to 32000 ft.	
2. Bindon (1951)	24000 ft.		
3. Clem (1957)	25000 to 45000 ft.	40000 to 44000 ft. Secondary max. at 34000 over NE USA	light CAT moderate severe
4. Clodman (1953)	18000 to 40000 ft.	No significant altitude dependency over So. Canada betw. 18000 and 38000 ft.	all intensities moderate and severe severe
5. Cunningham (1958)			
6. Estoque (1958)	150 mb 200 mb 250 mb 300 mb 350 mb	250 mb (Europe, Turkey, USA, Japan)	10 5 2 ft/sec 10 5 2 ft/sec 10 5 2 ft/sec 10 5 2 ft/sec 10 5 2 ft/sec
7. Heath-Smith (1955)		Decreasing to 25000 ft. above increasing again	
8. Hislop (1951)	25000 ft. 35000 ft.		4 ft/sec 8 ft/sec 12 ft/sec
9. Hyde (1954)	to 49000 ft. to 36000 ft. 22000 to 28000 ft.		light or moderate severe extremely severe
10. Kuettner (1952a)	40000 ft.		
11. Murray (1953)	400 to 200 mb		≥ 0.1 g ≥ 0.2 g ≥ 0.3 g
12. Pinus (1957)		Decrease to middle troposph., above increase by 15 to 20% to tropopause	
13. Press et al. (1953)			

of Various CAT Observations

5	6	7	8	9	10
Frequency of Occurrence	Extreme Intensity	Extent of Turbulent Region			
		Horizontal		Vertical	
		Mean	Extreme	Mean	Extreme
	0.7 g				
	3 g				
19% of all flights 12% 2%		15 to 60 km, longer than wide		500 to 2000 ft.	
1: 35 km 1: 85 km 1: 440 km		$\sim$90 km	$\geqq$ 450 km	3000 ft.	100 to 15000 ft.
		Areas with strong turbulence usually are larger and thicker			
		50% 30 km	6.9 to 328 km	$\sim$ 1500 ft.	500 to 3000 ft.
0.02 0.16 2.0% 0.09 0.75 9.0 0.15 1 .2 15 0.11 0.88 11 0.08 0.72 9.0 of km flown					
	0.8 g or 20 ft/sec				
1 gust per 13 km 1 gust per 97 km 1 gust per 650 km	1.5 g $\simeq$ 35 ft/sec	75 to 100 km		900 m	
28% of all flights 7 % of all flights 1 % of all flights	2.5 to 4 g (estimated)				
	+3 g, -2.5 g (sail plane) (21 m/sec)				
37.4% of all flights 4.9% of all flights 0.2% of all flights	0.4 g				
		50% $<$ 80 km	$>$ 800 km	50% $<$ 2000 ft.	100 to 10000 ft.
		Areas with strong CAT usually are larger and thicker			

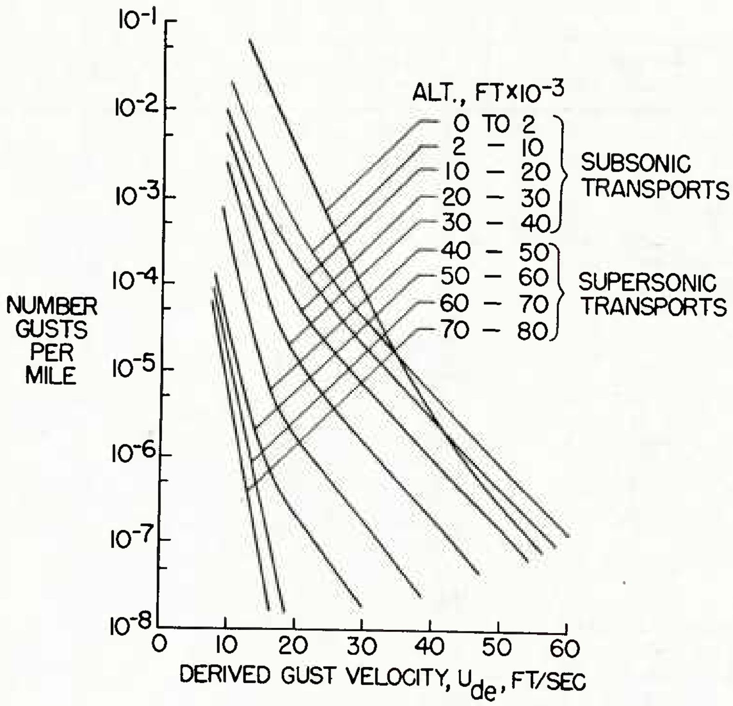


Fig. 2 Estimated overall gust distributions for operations at various altitudes (Rhyne and Steiner 1962).

earlier findings as shown in Table 1 (Reiter 1960a, 1961a). In the stratosphere the U-2 data indicate a pronounced decrease in turbulence frequencies. The same holds for gust intensities. It will have to be borne in mind, however, that the U-2 does not acquire supersonic speeds. It reacts to a different portion of the spectrum of disturbances which might exist at these altitudes, than does a supersonic transport. We therefore will have to apply some caution in generalizing the findings presented in Fig. 1 to other aircraft types. This point will be discussed in more detail in Chapter 3 of this report.

From the observational evidence contained in Fig. 1, the diagram shown in Fig. 2 has been obtained. In constructing this diagram it has been assumed, that storm turbulence would not be encountered above 60,000 ft., which stands to reason. Earlier computations by Hislop (1951) fit the curve for the 30-40,000 ft. layer very well. According to his computations a fleet of 20 "Comets" with about 3000 flying hours or 44 x 10^6 km flown during one year should expect one CAT report of 1.5 g (equivalent to a gust velocity of about 36 ft/sec = 10.5 m/sec) every two weeks, and a report of 2.0 g (50 ft/sec = 15.2 m/sec) once every 4 years. Statistics presented in this form, of course, do not reveal too much. As we will see later, CAT occurrence is strongly correlated with orographic features. It will have to be expected, therefore, that some air traffic routes will experience more turbulence, some less, than indicated by the statistical average.

From the foregoing and from Table 1 it may be seen, that cases of extremely severe turbulence may occur, albeit not very frequently. Accelerations of 0.25 g will be considered uncomfortable when lasting over some length of time. 0.5 g already constitutes heavy turbulence. Cases with 2 to 3 g have been reported, usually in connection with mountain waves when observed in clear air. Turbulence observed in thunder storms may be of the same magnitude. Needless to say that such heavy turbulence may cause the loss of control over the aircraft and may even result in structural damages. Bindon (1951) reports on accelerations of 3 g measured over Canada, causing partial destruction of galvanometers carried on board. Jones (1954, 1955) describes a CAT case on 14 April 1954 near Edinbourgh in the course of which a "Canberra" aircraft flipped over on its back. Another case history, leading to the loss of a B-52 aircraft over north western New Mexico on January 19, 1961, has been analyzed by the author (Reiter 1962a). All these severe cases mentioned occurred under conditions favorable for the formation of standing waves to the lee of mountain or hill ranges.

According to statistical evidence most turbulent regions show a rather small horizontal and vertical extent (see Table I). Cunningham (1958) presents the following frequency distribution after classes of extent:

Table II: Horizontal Extent of Turbulence Regions

Extent (miles)	10	10-19.9	20-29.9	30-39.9	40-49.9
Per Cent Occurrence	31.5	21.4	9.5	10.1	6.5
Extent (miles)	50-59.9	60-69.9	70-79.9	80.89.9	90-99.9
Per Cent Occurrence	4.8	2.4	2.4	0.6	1.2
Extent (miles)	100-149.9	150-199.9	200		
Per Cent Occurrence	6.5	2.4	0.6		

These data are made up by a total of 168 turbulence regions observed during "Project Jet Stream" Flights (Fetner 1956), and obtained over the Continental United States.

In contrast to these results, a study by Clodman et al. (1961) shows, that over the oceans the frequency of CAT occurrence is at least one order of magnitude less than over the continents, being approximately 0.2 per cent of the miles flown (compare with frequencies given by Estoque in Table I). On the other hand, however, the average horizontal extent of turbulence regions of about 100 km observed over oceans exceeds by far what has been reported in Table II. An explanation for this will be attempted in the following chapter.

3. NATURE OF CLEAR-AIR TURBULENCE

In searching for mechanisms that might cause bumpiness in flight, we will have to be aware of the fact, that the aerodynamic and elastic properties of the traveling vehicle in itself may -- under certain circumstances -- set off vibrations which may appear as CAT, although the ambient atmosphere may be only partly responsible for the observed effects. We will return to this problem of aircraft response farther below. Presently we will concern ourselves only with possible atmospheric causes of turbulence. These may be classified into two categories:

(1) Convective motions in an unstable atmosphere.

(2) Perturbation motions in a stable atmosphere.

3.1 Turbulence in an Unstable Atmosphere: This type of turbulence will have to be expected.

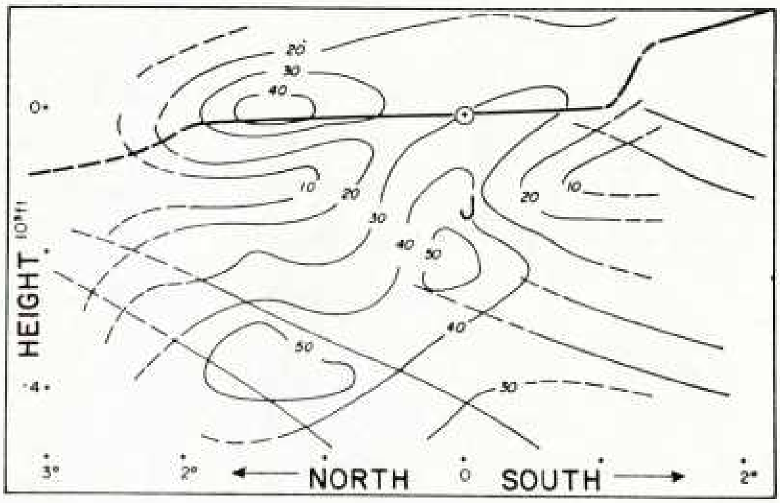
(a) Within the friction layer of the atmosphere near the ground.

(b) Underneath, in or near convective cloud systems.

Although this type of turbulence may occur in clear air, it usually is excluded from clear-air turbulence considerations, mainly because its occurrence is to be expected by the experienced pilot. While the most violent forms of convective turbulence, which occur in or near thunderstorm clouds, may easily be avoided by high-flying aircraft, the turbulence within the friction layer may be of some concern. It will affect any type of aircraft during take-off and landing operations, and therefore will have to be taken into account in structural design and manoeuverability of the vehicle. The problems associated with this type of turbulence are anticipated to be fewer, however, with supersonic aircraft, than with our present subsonic jets (Koch 1961) because of their lower aspect ratio and their relatively small lateral area.

Another turbulent region in the atmosphere extends above the "mesopeak" between a height of 50 and 80 km. It is associated with temperatures strongly decreasing with height. Turbulence in this region has been measured from meteor trails (for literature see Murgatroyd 1957) and sodium vapor trails (Manring 1961, et al. 1961a, 1961b, 1962). In view of the low densities in this altitude range, any atmospheric turbulence will have but a negligible

Fig. 3 Mean distribution of turbulent flying time (CAT) in per cent of total flying time for nine Project Jet Stream flights 1956-1957. (Sasaki 1958).



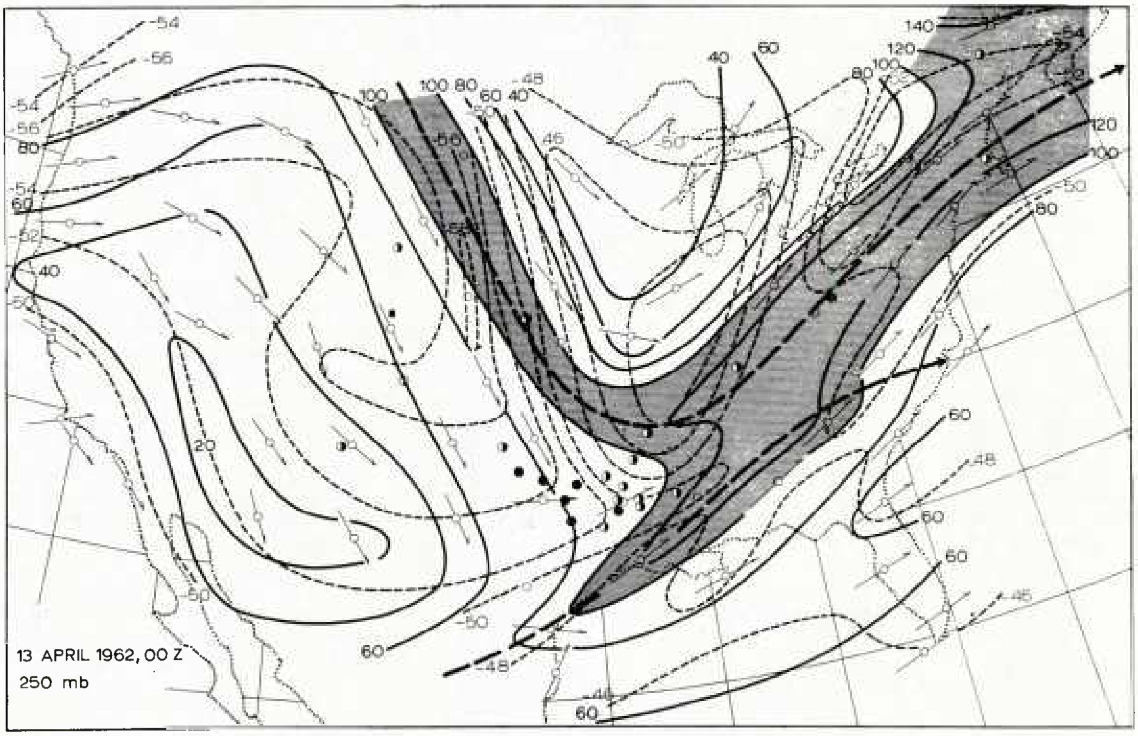


Fig. 4 **250-mb isotachs (areas > 100 knots shaded) and isotherms (°C),** 13 April 1962, 0000 GCT, and moderate and severe cases of CAT observed within ± 6 hours of map time.

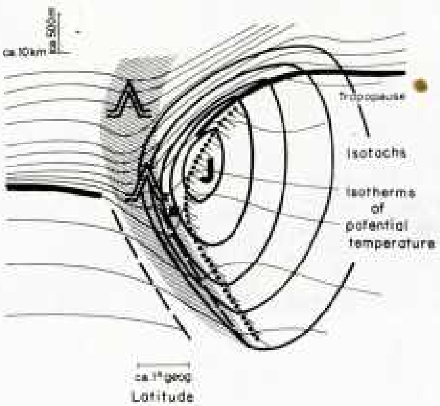


Fig. 5 Schematic cross-section through the jet stream (J): potential temperatures (thin lines), and isotachs (semi-heavy lines). The boundaries of the stable baroclinic frontal zone are marked by heavy dashed lines, the tropopause by heavy solid lines. The shaded area within the "isentrope trough" indicates the region in which the occurrence of moderate to severe CAT (Λ) is most likely. The dotted line represents the northern boundary of extensive cirrus-cloud sheets.

effect on vehicles.

3.2 <u>Turbulence in a Stable Atmosphere</u>: It is believed, that this type of turbulence is responsible for most CAT cases observed in the upper troposphere and in the stratosphere (Reiter 1962b, 1962c, Reiter and Hayman 1962).

Fig. 3 shows the distribution of turbulent flying time in per cent of total flying time around the jet stream for nine research flights of Project Jet Stream (Sasaki 1958). From this diagram it may be seen, that most of the turbulence is observed below and above and -- in looking down-stream -- to the left of the jet stream core within stable and baroclinic zones (Reiter 1960b, 1962d). This is also evident from Fig. 4, which contains a typical turbulence distribution around a jet stream as reported over teletype by commercial and military aircraft: Most of the cases of moderate and severe turbulence occur either in a region of strong horizontal temperature gradients at the 250-mb level (the larger part of the turbulence cases stem from the vicinity of this level) -- indicating the intersection of a sloping stable zone with this pressure surface -- or they occur over mountainous terrain.

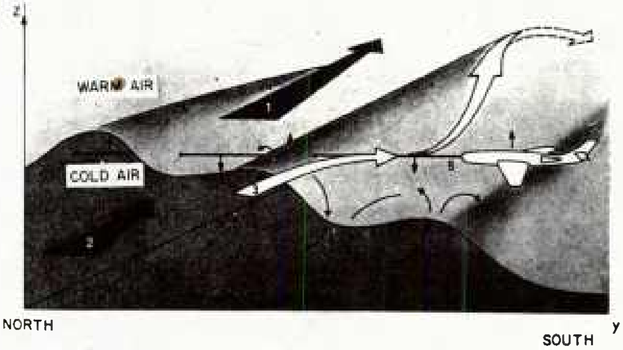
This leads to the theoretical jet-stream model shown in Fig. 5 (Reiter 1961b, 1962d): Moderate and severe cases of CAT seem to be concentrated in stable regions near the jet stream, which are located in or near a depression in the isentropic surfaces, the so-called "isentrope trough". It is marked by shading in Fig. 5.

With this evidence in mind we will have to look now for a physical process which may result in bumpiness in flight. This can be found easily in gravity-wave type disturbances on interfaces. Cloud studies by Conover (1960) as well as aircraft measurements (Reiter 1960a) reveal the existence of helical vortices in air particle trajectories as they travel through such disturbances. As we shall see in Chapter 4, these wave motions may be visible at times in a rather spectacular way in cirrus-cloud patterns. In Chapter 5 it will be shown, that gravity waves in a shearing current do possess the correct wave-length, to which an aircraft of present design would respond with typical CAT accelerations.

Fig. 6 contains a schematic plot of the conditions that would be encountered by an aircraft traversing such a gravity-wave train. Some evidence which points toward a certain anisotropy of CAT (Clodman et al. 1961) can be explained by the fact, that the accelerations experienced by an aircraft will not only depend on the absolute accelerations present in the ambient atmosphere, but also on the angle at which a wave train is intercepted. Thus, a change in course often changes the intensity of CAT measured by the aircraft.

This is an interesting aspect from the point of view of the statistical theory of turbulence, because there small-scale turbulence usually is assumed to be isotropic. On the other hand, a gravity-wave motion from a hydrodynamic point of view could be considered "laminar" as long as the waves do not show dynamic instability which would result in an

Fig. 6 Schematic cross-section through a region with CAT, (corresponding to the location of the black square in Fig. 5). There is only a slight temperature difference between cold air and warm air, the latter flowing somewhat faster than the former (indicated by the length of arrows 1 and 2). The wave crests intersect the wind direction at an angle α, which may be as large as 90°. The air-particle trajectories (white arrow 3) follow a wave-like pattern with small upward components in the crests and downward components in the valleys (small arrows 4). If an aircraft (5) intersects the wave pattern at suitable intervals it will experience CAT. Hardly any turbulence should be experienced when flying parallel to the wave pattern.



increase in amplitude, and in a break down of the flow pattern.

From the foregoing it appears, that we will have to keep a watchful eye on stable regions in the atmosphere which, at the same time, show a vertical wind shear. (This is equivalent to their being "baroclinic"). We know of three processes which tend to stabilize an atmospheric layer (see Appendix I).

(a) differential sinking motion;

(b) differential temperature advection;

(c) decrease of absolute vorticity.

Referring again to Fig. 4 we find, that the streamline and isotherm configuration suggests strong sinking motion in the confluent region between the two "jet fingers" over northern Texas and Oklahoma. Off hand, this would qualify mechanism (a) to be effective. Young and Corwin (1962) report on the occurrence of CAT in a weakly rising current off the U. S. Northeast Coast, near the inflection point upstream from a high-pressure ridge. Since in these cases the vorticity of the current seemed to be decreasing, mechanism (c) might have been prevalent. More detailed research will be needed however, to make any definite statements on this point.

If stable and baroclinic zones, due to their susceptibility to gravity wave formation, were responsible for a major portion of CAT reports, the statistical results on horizontal temperature gradients and

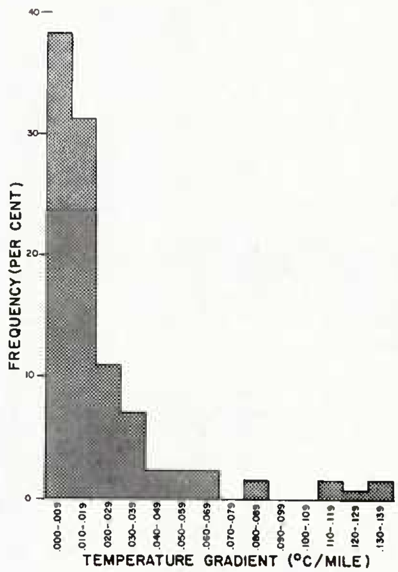


Fig. 7 Correlation between CAT frequency and horizontal temperature gradient (after Cunningham 1958).

CAT occurrence obtained by Cunningham (1958) would seem contradictory (Fig. 7). He finds that most cases of CAT occur when there is hardly any temperature gradient along the turbulent flight leg. We may however argue in the following way: An aircraft traversing a stable, baroclinic layer in a direction normal to the layer will measure a perceptible change in the ambient temperature; it will however be in the close vicinity of this layer for such a short time, that the chances for its experiencing CAT are slim. An aircraft, traveling parallel to such a layer will measure only small temperature gradients, it will however be exposed to wave disturbances in this layer for a longer time.

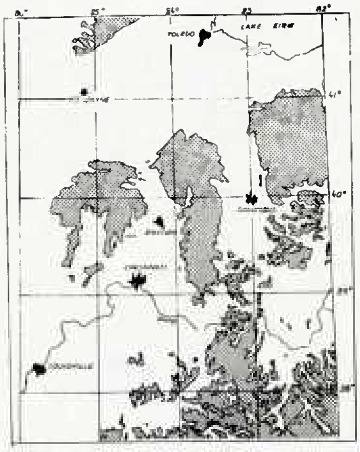


Fig. 8 Surface topography near Wright-Patterson AFB, Ohio. Elevations 1000-2000 ft. indicated by shading. (Clodman et al. 1961).

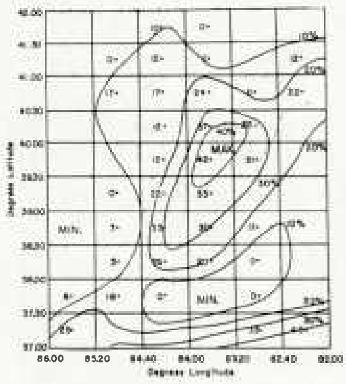


Fig. 9 Percentage occurrence of turbulence near Patterson AFB. (Clodman et al. 1961).

There is quite a bit of evidence which supports the hypothesis of CAT being to a large extent a gravity-wave phenomenon. Some of it will be pointed out in the discussion of cloud photogrammetry (Chapter 3). The perturbation theory gives mathematical support to the occurrence of wave disturbances on stable interfaces (Haurwitz 1941, Clodman et al. 1961). In the simplest form of this theory perturbations usually are assumed to be small, and we try to arrive at conditions of atmospheric density stratification and vertical wind shear, which would indicate an amplification of these disturbances.

In reality however we will find regions, which provide an appreciable input of perturbation energy into the atmosphere. Such regions may be mountain or hill ranges, even coast lines, under favorable low-level wind conditions, and areas of convective activity. Thus we might devide CAT-cases into two categories:

(1) CAT with large perturbation-energy input (over hills, etc.).

(2) CAT with small perturbation-energy input (over oceans).

Apparently, whenever the perturbation-energy input is large, the atmosphere does not require as much dynamic instability to produce "CAT"-waves, than with small disturbances. Let us assume, that the general statistical characteristics of atmospheric stratification and wind shear in the upper troposphere and stratosphere are the same over oceans and continents. In this case mountainous regions would experience CAT waves more frequently than ocean areas. Many of these cases would constitute dissipating disturbances in a dynamically stable atmosphere. This would explain the observed fact, that on the average CAT is more patchy and of smaller extent -- however by far more frequent -- over continents than over oceans.

A study by Clodman et al. (1961) clearly shows the influence of relatively small terrain features upon the occurrence of CAT. The dark areas in Fig. 8 indicate terrain in the vicinity of Wright Patterson AFB, rising 1000 to 2000 ft. above MSL, while the unshaded regions are 0 to 1000 ft. above MSL. The per cent occurrence of CAT for the same area is shown in Fig. 9. A frequency maximum is well established over the range of hills east of Columbus, Ohio. Specifically, this turbulence seems to occur, when the low-level (850 mb) winds blow normal to the central ridge of these hills (Figs. 10 and 11). The turbulence pattern also depends on low level wind speeds and on the turning of wind with height. If the former are large ($>$ 18 kts) and the latter is small (less than 30° between 850 and 500 mb) conditions for gravity-wave formation improve.

Clodman's findings indicate, that standing gravity waves are one of the main reasons for CAT

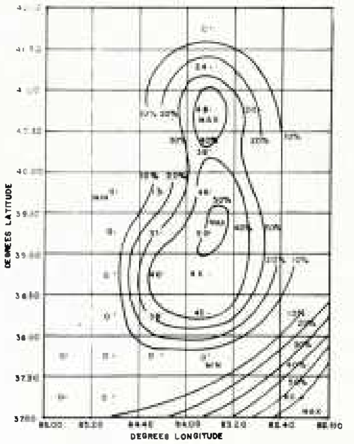


Fig. 10 Percentage of turbulence occurrence in the Patterson area with 850 mb wind direction normal (± 30°) to the central ridge shown in Fig. 8. (Clodman et al. 1961).

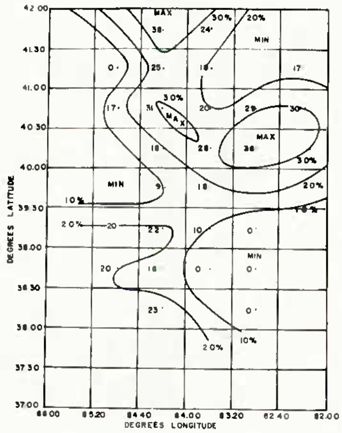


Fig. 11 Percentage turbulence occurrence in the Patterson area with the 850-mb wind direction more than 30° from the direction normal to the central ridge shown in Fig. 8 (Clodman et al. 1961).

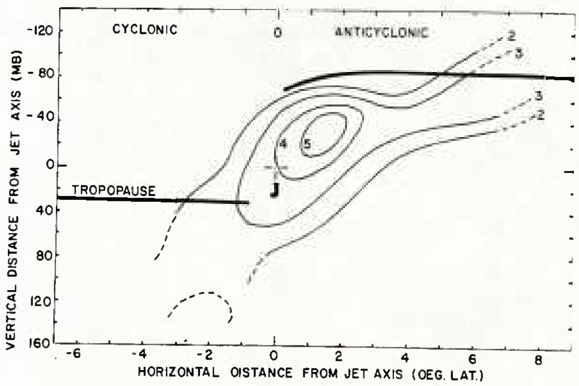
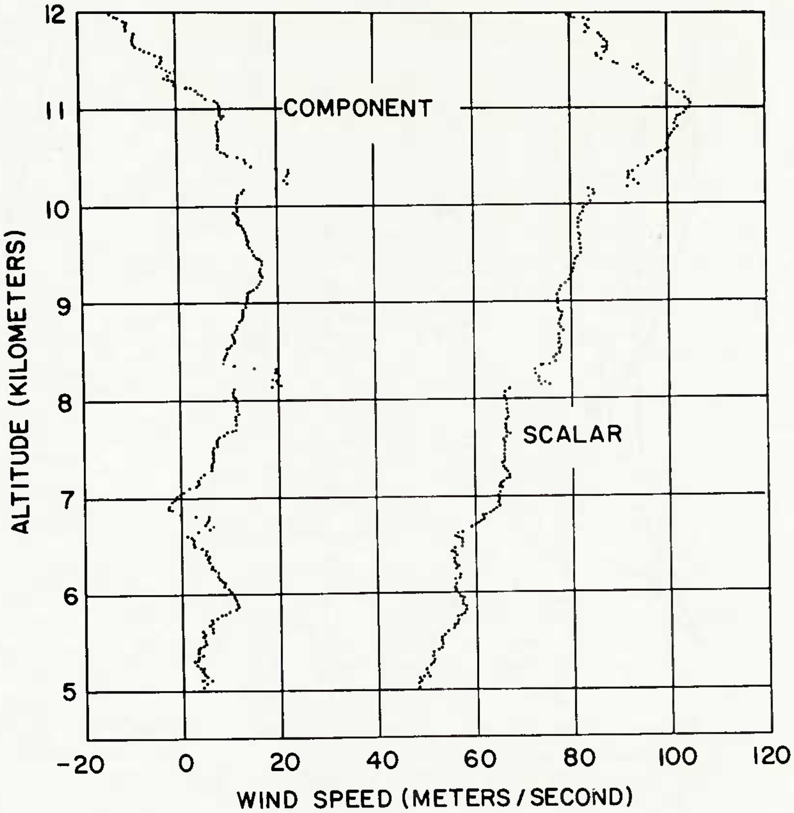


Fig. 12 Per cent occurrence of high-level turbulence about the jet stream over the North Atlantic Ocean in vertical "boxes" of 40 mb x 120 n. mi. (after Clodman et al. 1961).

occurrence over mountains. Over level terrain, as well as over oceans, traveling wave disturbances might be the dominating feature. In this connection it is interesting to note the average distribution of CAT-cases over the oceans as computed by Clodman et al. (Fig. 12), and to compare it with the pattern in Fig. 3, which characterizes continental conditions. In the latter case most of the turbulence occurs in or near the jet stream fronts above and below -- and to the left of -- the jet core, i.e. in regions of rather deep inversions and therefore of high stability, and with strong wind shears (see Table III). Over the oceans the anticyclonic side of jet streams seems to prevail in turbulence reports. In this region one usually finds very shallow stable layers with only small temperature discontinuities, and therefore with rather low values of stability. It seems that these layers are more susceptible to CAT formations when only small initial values of perturbation energy are available, than the deep stable layers of the jet core.

3.3 Turbulence Problems in Supersonic Transport Operations: The information contained in Figs. 1 and 2 for supersonic transport flight levels has mainly been obtained from U-2 aircraft. Since this is a subsonic airplane, it will react to a different portion of the perturbation spectrum than a vehicle traveling at supersonic speeds. In order to estimate the impact of atmospheric gusts upon vehicles which are still in the planning stage, we would have to know more about the atmospheric structure on a horizontal scale of 200 to 2000 m. None of our present aircraft-instrument packages provide

Fig. 13 Wind speed profiles over Wallops Island, 30-sec averaged winds measured by the smoke trail photographic technique. (Scoggins 1962a).



adequate information in this range of resolution.

A further problem arises from the fact that temperature discontinuities affect the speed of sound and thereby the Mach number at which an aircraft is flying. Some of this detailed structure of the upper atmosphere is slowly emerging from acoustic and other research (see Webb, 1962), but we are still far from being able to estimate all effects which this structure might have, and from understanding the processes which lead to the formation of this microstructure.

3.4 Turbulence Problems in Missile Operations: To arrive at an estimate of the response of missiles to atmospheric conditions, so-called "design-wind profiles" have been constructed from upper-level wind statistics (Van Der Maas, 1962; Sissenwine 1954, 1958; Tolefson 1956; Williams and Bergst, 1958). These profiles -- as for instance the "one per cent profile" -- give the maximum wind speed, and maximum shears above and below the maximum wind level, which are likely to be exceeded in only 1 per cent of all cases.

This statistical approach provides the missile designer with valuable information on specifications which are made necessary by atmospheric conditions: It does not allow a prediction of missile behavior in specific launchings, however. It might be pointed out here that most of the data which went into the above-mentioned statistics were extracted from routine upper-wind observations from the standard AN/GMD-1 and AN/GMD-2 rawinsonde systems. We are presently learning a good deal more about vertical wind shears in the atmosphere measured with more sophisticated instrumentation which provides a much higher resolution than was available before. Data obtained with the FPS-16 radar/sperical balloon technique, and especially wind profiles obtained from smoke-trail photography (Fig. 13) reveal details which have not been known before (Scoggins 1962a, b; Tolefson 1962).

It would lead us too far afield if we attempted here to deal with problems of general missile response. Let us only consider atmospheric parameters, which might cause "turbulence" in flight, experienced as vibrations of the missile.

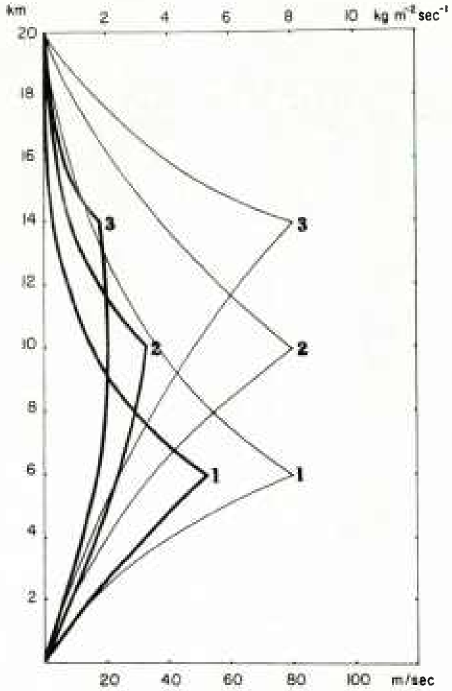


Fig. 14 Hypothetical profiles of wind speed (light lines, lower scale), and of momentum (heavy lines; upper scale) using standard-atmosphere densities.

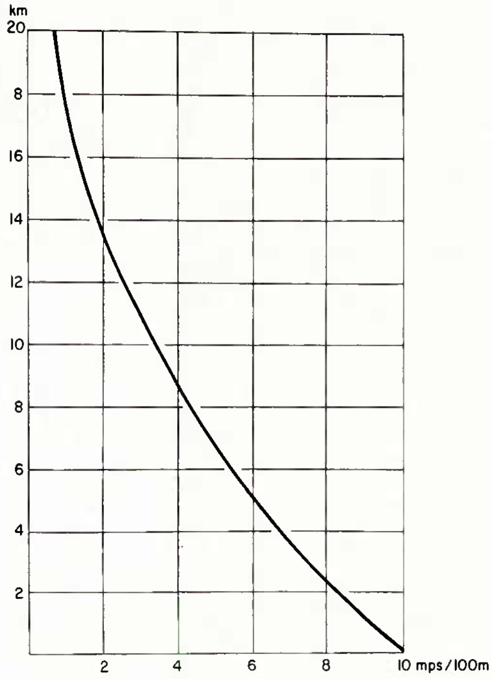


Fig. 15 Effective momentum shear of 10 mps/100 m vertical wind shear.

Naturally, the atmospheric motions which are mainly horizontal, will act differently upon a vertically rising vehicle than on an aircraft which "floats" horizontally. While in the latter case the wind speed usually suffices to describe the influence of the atmosphere, in the former case the vertical momentum profile gives a more adequate picture of atmospheric input. The design-characteristics of certain re-entry vehicles may show a different response to different wind-directions. It might be advisable, therefore to consider the ρu and ρv mementum components, rather than the total atmospheric momentum ρV.

Fig. 14 shows three hypothetical vertical wind profiles together with the momentum profiles obtained from the standard atmosphere densities. It is evident that the vertical momentum shear decays rapidly with altitude. Let us assume as an approximation, that the torque exerted by the atmosphere upon a vertically rising missile is proportional to the momentum shear, defined as $\overline{\rho}\Delta u/\Delta z$, where $\overline{\rho}$ is the mean density of the shearing layer. Fig. 15 shows the effective momentum shear of a 10 mps/100 m vertical wind shear: According to this a wind shear of 10 mps/ 100 m at an altitude of 20 km is equivalent to a shear of only 0.9 mps/100m at sea level. Again this curve holds for standard atmosphere densities, from which appreciable deviations may occur under specific weather patterns.

While shears acting over deeper atmospheric layers will have to be counteracted by the guidance system, shallow shearing layers may, together with aerodynamic properties of the vehicle, set off resonance vibrations within the missile or its components. In order to evaluate the atmospheric input into such vibrations properly we ought to consider a momentum-time profile rather than the momentum-height profiles of Figs. 14 and 15. AN/GMD-1 and AN/GMD-2 radar measurements of wind profiles may not meet the accuracy requirements for such input computations. In Appendix II it will be shown that these accuracy requirements become less stringent, as the missile rises into the atmosphere.

4. METHODS OF OBSERVATION OF CLEAR-AIR TURBULENCE

4.1 Aircraft Measurements: Since CAT primarily affects aircraft it is only natural that most of CAT research measurements are conducted by airplanes. There are, however, several shortcomings, which shall be outlined here.

Assuming a sharp-edged gust and a rigid air-foil, we have

$$\frac{\delta L}{W} = \frac{\rho}{2} \frac{A}{W} \frac{dC_L}{d\alpha} \omega V \qquad 4.1$$

where δL is the additional lift produced by the gust, W is the weight of the aircraft, ρ the air density, A the wing area, C_L the lift coefficient, α the angle of attack, ω the gust velocity and V the horizontal velocity of the aircraft (for literature see Krumhaar 1958). There are more sophisticated treatments on aircraft response available, which incorporate various shapes of gusts and elastic properties of the aircraft. The above case suffices, however, to point out the following:

The response of the aircraft depends on air density, thus on flight altitude (accelerations decrease with increasing altitude); on the weight of the aircraft (sensitivity to CAT increases with fuel burn-off); on the type of aircraft $(dC_L/d\alpha)$, and on the airspeed V (a reduction in airspeed should reduce the accelerations experienced in CAT). It is apparent from Equ. 4.1 that the complexity of aircraft response -- especially when considering elastic properties which are omitted in this equation -- makes it difficult to estimate the atmospheric input into the experienced accelerations. Furthermore, the observed accelerations are characteristic for this particular type of aircraft. Extrapolation to other vehicles is possible only, if the whole power-spectrum of atmospheric gusts in the CAT range is known.

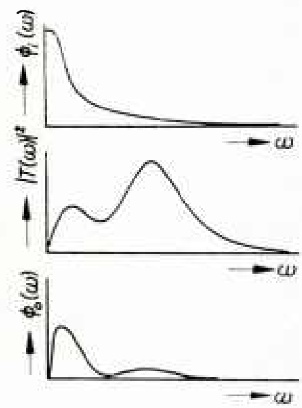


Fig. 16 Illustration of Equ. 4.2 (after Houbolt and Kordes 1954).

The relationship between atmospheric input and aircraft reaction is given by

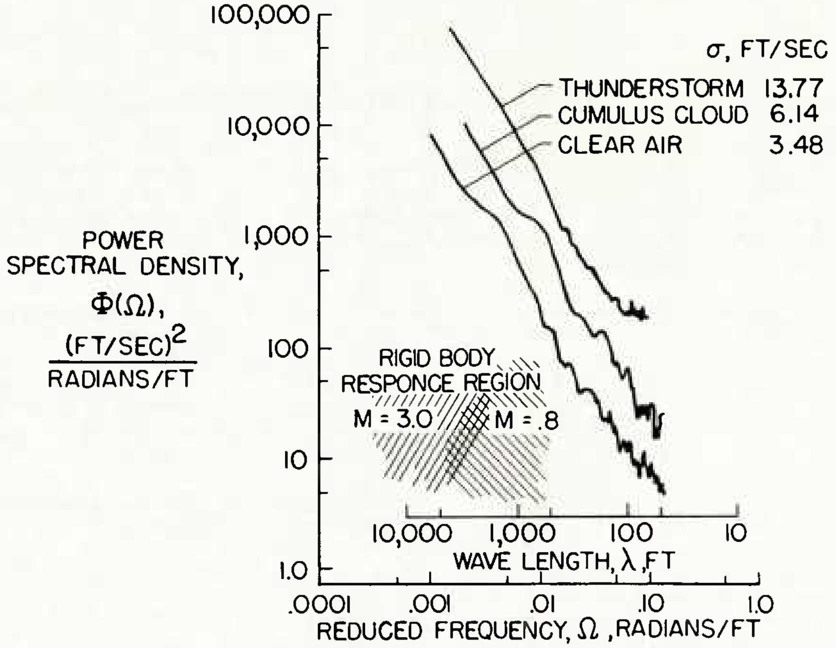
$$\phi_o = |T(\omega)|^2 \cdot \phi_i(\omega) \qquad 4.2$$

where $\phi_i(\omega)$ is the power spectrum of atmospheric input (e. g. of vertical accelerations in the surrounding air), $\phi_o(\omega)$ is the resulting output power spectrum (e. g. the vertical accelerations of the aircraft) and $T(\omega)$ is the frequency response function, which is controlled by the physical (mostly the elastic) properties of the aircraft (Houbolt and Kordes 1954; see also Krumhaar 1958; Panofsky and Press 1962). The effects of this equation are outlined schematically in Fig. 16. Similar considerations would apply to missile response, the only difference being, that the power spectrum of vertical momentum profiles would enter the computations, rather than vertical acceleration spectra measured during horizontal flight.

A few power spectra obtained from aircraft measurements are given in Fig. 17 (Rhyne and Steiner 1962; see also MacCready 1962). The data cover a wave-length range of about 50 to 4000 ft. The "reduced frequency" $\Omega = \frac{\omega}{V}$ is measured in radians per ft. and essentially removes the effect of air speed V. ω is the gust frequency measured by the aircraft.

4.2 Cloud Observations: Power spectrum analysis of gust-loads on aircraft treats CAT as a stochastic problem without explanation of its physical causes. It has been pointed out in Chapter 2,

Fig. 17 Typical power spectra of turbulence for three weather conditions (Rhyne and Steiner 1962).



that gravity waves on stably stratified interfaces with a vertical wind shear would offer a very plausible explanation for many CAT observations in the upper troposphere and stratosphere. Under favorable conditions cirrus layers may develop along such interfaces in the upper troposphere. Wave formation in these cirrus clouds then would give an indication of the wave length and orientation of perturbation motions in the field of flow at these levels. For practical purposes it may be assumed that the energy contribution towards de-stabilization from the latent heat of water vapor is negligibly small at these high altitudes, so that cirrus waves may be taken as indicators of perturbations in cloud-free air.

A field program has been instituted at Colorado State University, which measured the wave disturbances at cirrus level in the jet-stream region by photogrammetric means, (Reiter 1962b, Reiter and Hayman 1962). Fig. 18 contains the evaluation in a horizontal x-y plane of two pairs of stereophotographs, obtained from 2 camera sites to the northeast of Fort Collins, and approximately 4 1/2 miles apart. The left-hand portion of the figure shows the outlines of a cirrus-cloud sheet on 13 April 1962, 1443 MST. Small wave details are indicated by heavy lines. The numbers give the elevation of certain cloud features above MSL. The right-hand portion of the figure shows the same cloud at 1445 1/2 MST. The 2 1/2-minute displacement is indicated by the length of the arrows.

In this particular case the small wave disturbances have a length of about 80 to 450 m. They are oriented almost normal to the general direction of flow. This agrees well with the relatively weak wind shears observed over Denver at these levels (Fig. 19). According to a theoretical investigation by Sekera (1948), a strong wind shear across an interface will produce wave trains which are almost parallel to the direction of the mean current.

Fig. 19 also contains an evaluation of the Scorer-Parameter

$$l^2 = \frac{\frac{g}{\theta}\frac{\partial\theta}{\partial z}}{V^2} \qquad 4.3$$

where θ is the potential temperature, and V is the wind speed. The vertical distribution of this parameter in the atmosphere may give some indications as to the formation of standing lee waves, as shall be pointed out in Chapter 5. Since usually short gravity waves are generated over a mountain ridge simultaneously with long lee waves, conditions favorable to the formation of the latter may be considered instrumental in the occurrence of CAT over these regions.

This opens the possibility of utilizing satellite observations in CAT research, and ultimately in CAT forecasting. Although small wave patterns

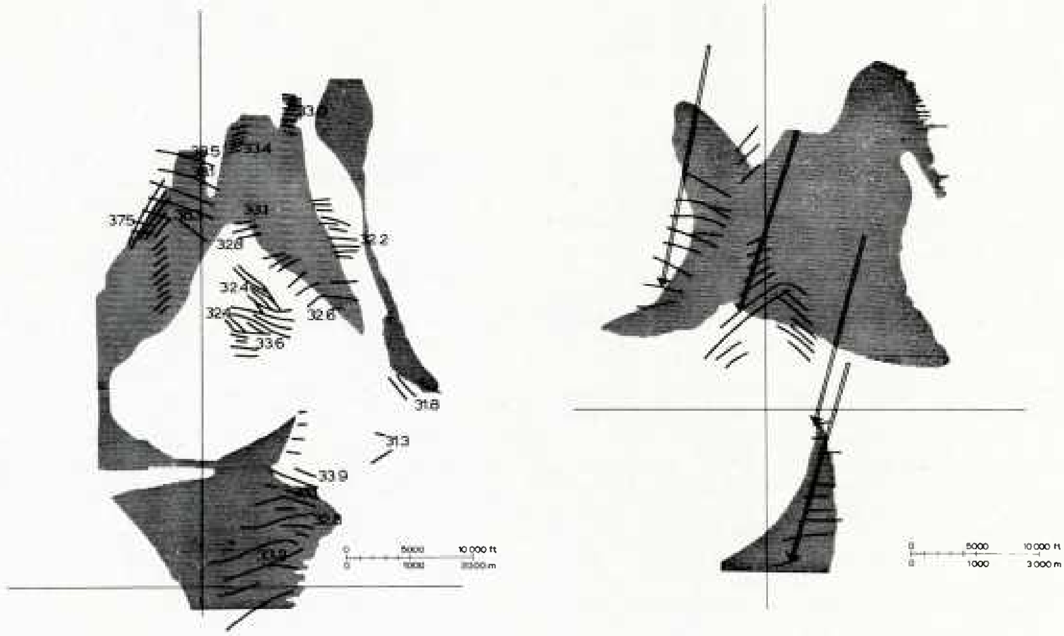


Fig. 18 Projection into horizontal x-y plane of waves at cirrus level obtained from stereo pairs of photographs. Left part of diagram: 13 April 1962, 1443 MST; right part: 1445 1/2 MST. Wave trains are indicated by lines. The numbers give the height of the cloud field in thousands of feet. The straight double lines with arrows mark the 2 1/2-minute displacement of certain characteristic features in the cloud pattern.

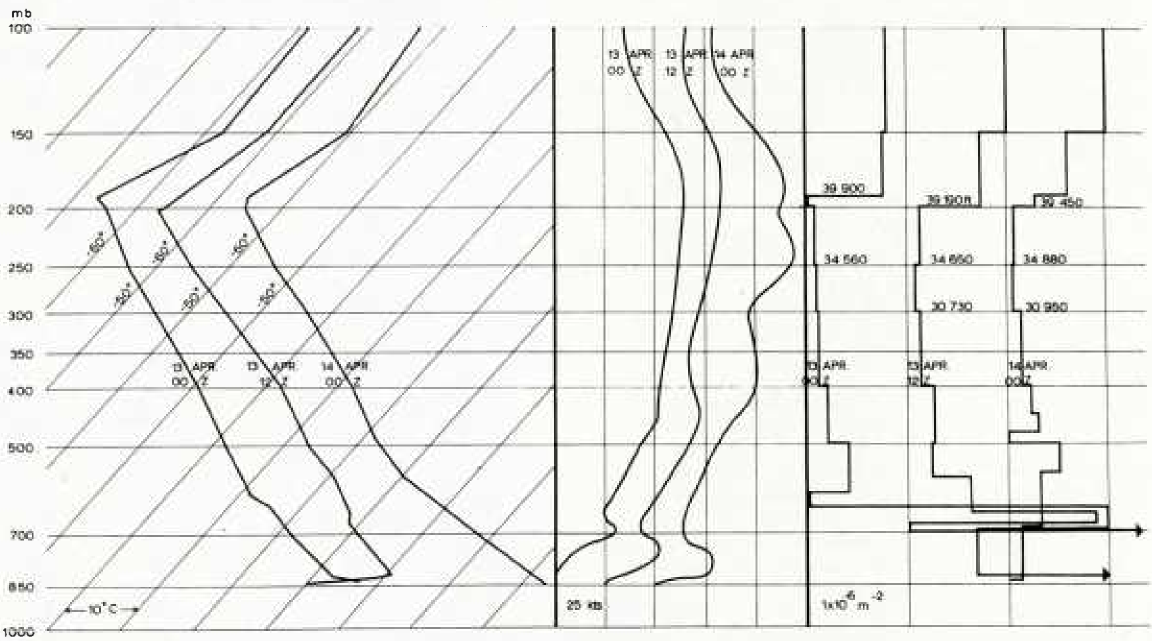


Fig. 19 Left side: Denver, Colo., soundings of 13 April 1962, 0000 GCT and 1200 GCT and of 14 April, 0000 GCT, on a USAF Skew T log p Diagram; consecutive soundings have been displaced to the right by a coordinate unit of 10^{o} C. Center: Denver wind profiles for same dates; consecutive profiles have been displaced to the right by a coordinate unit of 25 Knots. Right side: Scorer parameter l^2 for same soundings; consecutive profiles have been displaced to the right by a coordinate unit of $10^{-6}\,m^{-2}$.

which are responsible for CAT occurrence, lie well below the resolution threshold of present satellite data, large-scale lee-wave phenomena which usually have a wave-length of about 20 km may be observed directly. They might give a hint as to regions, in which an excessive perturbation energy input into the atmosphere could set off CAT-waves. More research in this field is needed before an operationally useful CAT model can be obtained.

4.3 Artificial Tracers: Indications on upper-level turbulence may be obtained from the behavior of condensation trails of aircraft (Clodman 1958). Smoke-trails have also been used for such studies (Durst 1948). As has been shown in Fig. 13, detailed vertical wind profiles may be obtained from smoke trails released from rockets. Similar methods, using sodium vapor, may be employed to study the wind structure and diffusion regimes at heights from about 80 to 150 km (Manring 1961, et al. 1961a, b, 1962).

Two of the basic difficulties in the application of these techniques to CAT research are:

(a) The aircraft or rocket itself disturbs the air flow in the atmospheric layer under consideration. One therefore will have to be careful in the interpretation of waves detected along contrails, etc.

(b) Small-scale turbulence and diffusion processes make the contrails or smoke trails spread out and dissipate. Due to these superimposed small-scale effects it may sometimes become difficult to identify perturbation motions on a scale corresponding to CAT.

In spite of these shortcomings a research aircraft, which operates in the vicinity of ground-based photogrammetric cameras, and which is equipped to release a smoke-trail whenever it encounters CAT, might help to gain valuable information on the physical causes and the structure of turbulence zones.

4.4 Balloon Observations of Turbulence: The existence of turbulent layers above the tropopause has already been surmised in the thirties from the registrations of meteorographs which were attached to high-flying balloons and recovered for evaluation (Junge 1938; Poncelet 1935; Mironovitch and Viaut 1938). Measurements carried out with "gust-sondes" (radiosondes which carried an accelerometer) over the United States showed that turbulence tends to occur in rather shallow layers (Anderson 1957). Moreover, in only 8.6 per cent of all cases turbulence persisted in the same 5000 ft. layer for at least three successive soundings (i. e. over at least 48 hours). This does not necessarily mean, that these successive soundings actually intercepted the identical CAT layer. It shows, however, that turbulence is a rather transient phenomenon. An exception to this was found over Grand Junction, Colorado, where 49 per cent of the soundings showed persistence of turbulence over 3 ascents in the same altitude range (most frequently between 9 and 15 km). Orographically induced perturbations may be the reason for the persistent occurrence of turbulence over this station.

These flights were carried out with regular sounding balloons. It would be highly interesting to see similar experiments conducted with constant-level balloons. In order to determine the wave-length of perturbations in the range up to about 2 km, a micro-barometer recording small pressure variations might render better services than an accelerometer. Such measurements might yield new insight into the actual dissipation of perturbation energy, especially over mountain ranges.

4.5 Scintillation of Stars: Turbulence, especially when occurring as a wave phenomenon along interfaces with stable thermal stratification, will result in small inhomogeneities along the path of light from stars. This results in the well-known "twinkling" of stars (Boutet 1950, Gifford and Mikesell 1953, Keller 1952, Protheroe 1955, Royal Meteor. Soc. 1954). The amplitude of the scintillation may be measured from photographic records. Especially the high-frequency oscillations (150 cps) of star images seem to have a significant correlation with wind speed and wind shear at jet-stream level.

These small density variations not only seem to affect the electro-magnetic waves of visible light, but also short radio waves (Staras 1955, Kazes and Steinberg 1957) and radio waves (Fleisher 1959, Rogers 1957).

Actual correlations between scintillation measurements and turbulence observations from aircraft at jet-stream level are not yet available. An organized research program along these lines combining the efforts of astronomers, meteorologists, and research flight facilities might be a worth-while undertaking.

4.6 Micro-Pressure Variations: Under favorable conditions it might be possible to detect the existence as well as the rate and direction of displacement of gravity-type waves in the free atmosphere from records of sensitive microphones laid out on the ground. Such microphones have been used successfully in studies on the thermal structure and wind distribution in the upper atmosphere. Their possible application to turbulence research near the tropopause level might merit some investigation.

5. THEORETICAL ACHIEVEMENTS

Occurrence of CAT has frequently been correlated with Richardson's criterion

$$Ri = \frac{\frac{g}{\theta}\frac{\partial\theta}{\partial z}}{\left(\frac{\partial V}{\partial z}\right)^2} \qquad 5.1$$

θ is the potential temperature, z the height coordinate, and V the wind speed. Under the assumption that $A_T/A_M = 1$ (a factor which has been neglected in 5.1), i.e. the exchange coefficients of heat (A_T) and of momentum (A_M) are equal to each other, the limiting value of Ri has been assumed to be 1. Smaller values, however, are more likely (see Prandtl 1960). Below this value the turbulence-generating forces of the vertical wind shear would exceed the damping forces of thermal stability, thus turbulent motion would result.

Petterssen and Swinbank (1947) derived a limiting value of Ri = 0.65 for the free atmosphere over England. This agrees well with earlier results given by Fage and Falkner (in: Taylor 1932; see also Schlichting 1960), who found that the ratio of exchange coefficients $A_T/A_M = 2$ for free flow. Thus, if one erroneously assumes $A_T/A_M = 1$, which enters as a factor on the right-hand side of expression 5.1, a limiting value of Ri = 0.5 should result from the experiments, which lies close to the value found by Petterssen and Swinbank. Applying $A_T/A_M = 2$, one again arrives at Ri = 1 (Reiter and Hayman 1962).

Correlations between Ri and CAT usually are rather contradicting (Brundidge 1957, 1958; Lake 1956; Saucier 1956; Colson 1962). The reason for this has to be sought in the fact, that CAT is a small-scale phenomenon, while Ri-values usually are computed over deep atmospheric layers --not to speak about inaccuracies especially in the wind measurements. Wind shears, as revealed by rawinsonde data may be quite erroneous at times, especially in the jet stream region (Reiter 1958), whereas temperatures usually are more reliable, although during the coding process they may be considerably smoothed, too.

The fact, that CAT seems to occur preferably in or near stable layers with vertical wind shear is brought out better by substituting the thermal-wind equation into 5.1 (Reiter 1960a, 1961a). One arrives at

$$Ri^* = \frac{f^2\theta}{g\frac{\partial\theta}{\partial z}\left[\left(\frac{\partial z}{\partial n}\right)_\theta - \left(\frac{\partial z}{\partial n}\right)_p - \frac{\theta}{g}\frac{\partial \dot{V}}{\partial \theta}\right]^2} \qquad 5.2$$

f is the Coriolis parameter; $\left(\frac{\partial z}{\partial n}\right)_\theta$ and $\left(\frac{\partial z}{\partial n}\right)_p$ are the slopes of isentropic and isobaric surfaces. The latter usually may be neglected in comparison with the former. $\dot{V} = dV/dt$ usually is difficult to measure. If we neglect this term, too, we obtain

$$Ri^* = \frac{f^2\theta}{g\frac{\partial\theta}{\partial z}\left(\frac{\partial z}{\partial n}\right)_\theta^2} \qquad 5.3$$

Ri* may now be computed from temperature cross-sections only, without using the less accurately measured wind shears. Fig. 20 shows a correlation between Ri* and the amplitudes of mesostructural

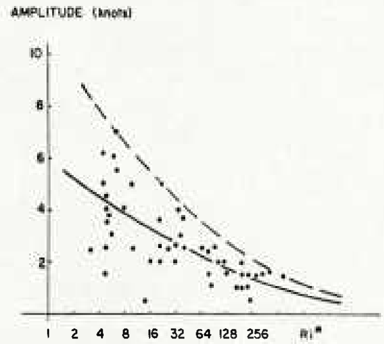


Fig. 20 Correlation of the amplitudes of the mesostructure of wind speed (ordinate) with Ri*-numbers (abscissa).

wind-speed fluctuations with wave-lengths measured normal to the direction of flow of about 50 n. mi. Although these waves are about three orders of magnitude longer than CAT-waves, there still seems to be some physical connection between their occurrence, and atmospheric stability and vertical shear (equivalent to baroclinicity).

Assuming gravity waves in a vertically shearing current to be the main cause for CAT, we may make use of the perturbation theory, which considers the conditions under which small deviations from the average state tend to amplify. Helmholtz (1888, 1889, 1890) was the first to derive the resulting wave formations in a simple two-layer model of the atmosphere, with a discontinuity of wind speed and temperature across the interface. Without going into the details of derivation (see Haurwitz 1941, Reiter 1961a) we obtain the critical wave length L_c, below which the amplitudes of the disturbances will increase exponentially

$$L_c = \frac{2\pi}{g} \frac{(u_0 - u_1)^2 \; T_0 \cdot T_1}{(T_1 + T_0)(T_1 - T_0)} \qquad 5.4$$

u are the wind speeds, T the temperatures. The subscripts 0 and 1 refer to the lower and upper layer, respectively. The following table (Reiter 1960a) gives wind shears and temperature differences across the interface for different initial wave lengths, and for conditions near the tropopause.

Table III: Vertical Wind Shear Δu (m/sec) for Different Temperature Discontinuities and Critical Wave-Lengths at an Interface

ΔT	L_c = 200 m	L_c = 100 m	L_c = 50 m
2°	2.3 m/sec	1.6 m/sec	1.2 m/sec
4°	3.3	2.3	1.6
6°	4.0	2.9	2.0
8°	4.7	3.3	2.3
10°	5.2	3.7	2.6

From this it will be realized, that Helmholtz-wave formation requires conditions which will be met rather frequently in the jet-stream region, especially near shallow stable and baroclinic zones, as they have been found during Project Jet Stream flights (Reiter 1962e).

More refined treatments of wave formation, using three-layer models of the atmosphere with a transition zone instead of a sharp discontinuity have been dealt with by Sekera (1948) and Sasaki (1958).

Clodman et al. (1961), also departing from the perturbation equations, considers the energy transformations in high-level turbulence. The two main sources of turbulent energy are vertical and horizontal shears. The former are taken into account by Richardson's criterion, while the latter in their normal distribution in the free atmosphere usually are not strong enough to constitute a major perturbation energy source. There may, however, be exceptions, where the stream-line pattern is sufficiently distorted so that vertical shear may be converted into horizontal shear on a meso-scale basis. This may be the case in lee waves over hills and mountains. Hydrodynamic instability on occasions may also bring about horizontal shears large enough to be considered as a perturbation energy source.

As has been shown in Chapter 3, CAT frequently occurs over mountain and hill ranges. In more severe cases of this kind it seems to be associated with standing lee waves. Any forecast of levels in the atmosphere, which produce maximum lee-wave amplitudes, or maybe even "rotors", would help to issue CAT warnings over mountainous terrain.

Extensive treatments of the theory of mountain waves have been offered by Lyra (1940, 1943), Queney (1941, 1947) and Scorer (1949, 1951a, b, 1953a, b, 1954, 1955, 1958, and Klieforth 1959). Without going into the details of this theory we might mention, that the formation of lee waves seems to be controlled by the Scorer-Parameter

$$l^2 = \frac{g\beta}{V^2} - \frac{1}{V}\frac{\partial^2 V}{\partial z^2} \qquad 5.5$$

where g is the acceleration of gravity, V the horizontal wind speed, $\beta = \frac{1}{\theta}\frac{\partial \theta}{\partial z}$ the vertical stability, and z the vertical coordinate.

Let us assume, that the cross-section of a mountain range of infinite lateral extent is given by the vertical elevation function

$$\zeta = \frac{hb^2}{b^2 + x^2} \qquad 5.6$$

h is the height of the range, b is the "half width", i.e. the width over which the height of the terrain decreases to half of the height of the range; x is the horizontal coordinate measured normal to the mountain range.

Corby and Wallington (1956) arrived at the vertical displacement of a stream line at the level

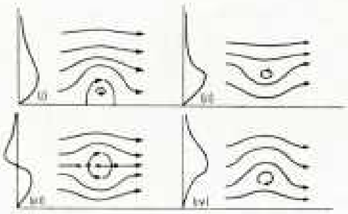


Fig. 21 Formation of rotors in lee-waves with large amplitudes. ζ -profiles are entered along the left-hand sides of the diagrams. Rotors are to be expected, where ζ shows an extreme value. (III) indicates a so-called "double-yolked" rotor. (Scorer and Klieforth 1959).

z from its undisturbed state

$$\zeta_z = 2\pi h b e^{-kb} \left(\frac{V_1}{V_z}\right) \psi_{z,k} \cdot \left(\frac{\partial \psi_{1,k}}{\partial k}\right)^{-1} \sin kx \qquad 5.7$$

k is the lee-wave number, and ψ satisfies the equation

$$\frac{\partial^2 \psi}{\partial z^2} + (l^2 - k^2)\psi = 0 \qquad 5.8$$

Subscripts 1 indicate conditions at the surface, subscripts z at level z.

From Equ. 5.8 we may conclude that lee-wave amplitudes tend to be large if

(1) the mountains are high and narrow(large h, small $\frac{1}{e^{kb}}$)

(2) the surface winds V_1 are high

(3) the vertical wind profiles do not show too rapid a decrease of wind speed with height.

According to Scorer, lee waves will form, if the parameter l^2 shows a decrease with height -- preferably due to a decrease in stability rather than an increase in wind speeds. For practical purposes, levels with l^2 = minimum may be identified as those levels which have the greatest lee-wave amplitudes. Several such levels may be present simultaneously if the profiles of l^2 show corresponding maxima and minima.

In the right portion of Fig. 19 the parameter l^2 has been plotted for the three soundings shown in this diagram. Conditions outlined above are met on 13 April, 1200 Z between 30730 and 34650 ft. A secondary minimum probably was present between the latter of these two levels and 39190 ft. These values are in agreement with the heights of wave clouds plotted in Fig. 18.

At certain levels ζ_z may indicate a reversal in sign, or a rapid change. At levels where ζ_z attains extreme values rotors tend to form, especially if the wave amplitudes on either side of this level are large (Fig. 21). Because of the increased vertical shears and the reduced thermal stability in certain parts of the rotor, it may be associated with CAT.

6. OUTLOOK FOR FUTURE RESEARCH

Our knowledge of the meso- and micro-structure of flow patterns in the free atmosphere, especially above the tropopause, still is rather poor. The need of well-organized and well-equipped measurement programs, preferably at flight levels of supersonic transport aircraft has not yet diminished. U-2 flights might help substantially to bridge this gap. There will, however, be some need of turbulence data from super-sonic aircraft, too, in view of their different flight characteristics.

Measurement programs using other methods of data collection than aircraft should be pursued in order to obtain more information on the genuine micro-structure of the atmosphere, without the large disturbances which a flying aircraft will create itself. Balloons and smoke traces will probably be among the primary choices in this direction of research. Such micro-structural details may be of particular importance in missile design and operation.

Case studies of CAT occurrence so far were limited to correlations of turbulence location with atmospheric parameters measured as close as possible to the time of occurrence. We might gain some additional information on the physical causes of CAT, if the development and previous history of flow patterns bearing CAT were studied.

In the field of instrumentation, there is still need for a compact accelerometer, which measures and records the three components of gustiness separately and simultaneously.

In summarizing we may state, that turbulence research in the free atmosphere has come a long way, especially when considering the fact, that measurements cannot be duplicated very well in controlled experiments. Our laboratory is the free atmosphere itself with its infinite resources of parameter combinations. This, however, makes the field interesting and challenging, and with the combined efforts of physics, aerodynamics, mathematics and meteorology we will steadily probe for more complete and more satisfying solutions of the problem.

Appendix I: Stabilizing Processes in the Atmosphere

1. Considering adiabatic motions $(\frac{d\theta}{dt} = 0)$ we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial \theta}{\partial p}\right) = - \frac{\partial \mathbf{V}}{\partial p} \cdot \nabla \theta - \mathbf{V} \cdot \frac{\partial}{\partial p}(\nabla \theta) \qquad \text{I.1}$$

The first term on the right hand side of this equation contains the influence of horizontal differential temperature advection due to vertical wind shears, and of differential sinking motions $(\partial\omega/\partial p)$. The second term describes the horizontal advection of air masses of different stability, and the effect of the curvature of the vertical potential temperature profiles under vertical motions $(\omega \cdot \frac{\partial^2 \theta}{\partial p^2})$.

2. Considering motions under conservation of potential vorticity $[\frac{d}{dt}(Q_z \cdot \frac{\partial \theta}{\partial p}) = 0]$ we obtain

$$\frac{dQ_z}{dt} \cdot \frac{\partial \theta}{\partial p} + Q_z \frac{d}{dt}\left(\frac{\partial \theta}{\partial p}\right) = 0 \qquad \text{I.2}$$

and

$$\frac{\partial}{\partial t} \cdot \left(\frac{\partial \theta}{\partial p}\right) = - \frac{1}{Q_z} \frac{\partial \theta}{\partial p}\left(\frac{\partial Q_z}{\partial t} + \mathbf{V} \cdot \nabla Q_z\right) - \mathbf{V} \cdot \nabla\left(\frac{\partial \theta}{\partial p}\right) \qquad \text{I.3}$$

The first term on the right-hand side of this equation describes stability changes due to vorticity changes produced by local variations and by advection, the second term contains again the influence of the advection of air masses with different stability.

Appendix II: Accuracy Requirements of Meteorological Measurements to Determine Turbulence Effects on Vertically Rising Vehicles

Let us suppose, that a missile has a critical vibration frequency ω_1. (Naturally, there may be more than one such frequency). A meteorological measuring system should be designed, which would allow a fair estimate of the atmospheric "turbulence" input in this frequency range for a vertically rising vehicle.

Let the vertical acceleration of the missile be given by

$$\frac{dw}{dt} = n(t)g \qquad \text{II.1}$$

where n(t) is the acceleration as a function of time, given in g-units. g is the numerical value of the acceleration of gravity. The vertical velocity of the vehicle may then be written as

$$w = g\int n(t)\,dt = gf(t) \qquad \text{II.2}$$

where f(t) is a function of time which may be obtained from (II.1) by numerical integration. The height of the missile above ground is given by

$$\Delta z = gf(t)\,\Delta t \qquad \text{II.3}$$

and

$$z = g\int f(t)\,dt = gF(t) \qquad \text{II.4}$$

From II.3 and II.4 we may express Δt in terms of Δz and z

$$\Delta t = G(\Delta z,\ z) \qquad \text{II.5}$$

Let us now designate Δt as the time interval which the missile needs to travel through a layer Δz with (uniform) vertical momentum shear. We may then write

$$\omega = \frac{1}{2\,\Delta t} = \frac{1}{2G(\Delta z,\ z)} \qquad \text{II.6}$$

In order to pick up the atmospheric input at the frequency ω_1 our meteorological measuring system should have at least a resolution of momentum shears of

$$\Delta z_1 = gf(t)\frac{1}{2\,\omega_1} \qquad \text{II.7}$$

Let us consider the following, rather simple example: A missile rises with constant acceleration 8 g. Its vertical speed is then given by $w = 8gt$ and its height above the launching site by $z = 4gt^2$. From this we obtain

$$t = \frac{1}{2}\sqrt{\frac{z}{g}} \qquad \text{II.8}$$

and

$$\Delta t = \frac{\Delta z}{4\sqrt{g.z}} \qquad \text{II.9}$$

Let us assume, that the accuracy requirements in determining the atmospheric input are

$$\omega_1 \pm \omega_1{}^*$$

where $\omega_1{}^*$ is the limit of tolerable error. We arrive at error limits

$$\Delta z^* = \frac{2\sqrt{gz}}{\omega_1{}^*} \qquad \text{II.10}$$

This means, that the accuracy requirements in determining the thickness Δz of vertically shearing layers decrease with height at a rate proportional to the square root of z .

If the atmospheric input at a critical frequency should be estimated to $\pm$ 10 cps, we should be able to estimate the thickness of shearing layers to $\pm$ 62 m at a height of 10 km, to 89 m at height of 20 km.

These requirements are certainly beyond the reach of AN/GMD-1 and AN/GMD-2 equipment. This problem becomes less serious, however, if we consider a threshold value of momentum fluctuations $\Delta M = \rho \Delta u$, below which atmospheric "turbulence" input becomes irrelevant. The accuracy requirements in the measurements of Δu, therefore, become less stringent at a rate of approximately $\frac{1}{\rho}$, as the missile rises into the atmosphere.

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