FTD-TT-62-1928

TRANSLATION

DISTRIBUTION OF BRIGHTNESS OVER THE DISK OF THE MOON

By

A. Yezerskaya and V. I. Yezerskiy

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO

FTD-TT-62-1928/1+2

UNEDITED ROUGH DRAFT TRANSLATION

DISTRIBUTION OF BRIGHTNESS OVER THE DISK OF THE MOON

By: A. Yezerskaya and V. I. Yezerskiy

English Pages: 11

Source: Russian periodical, ANSSR, Astrofizicheskii sovet. Komissiya po fizike planet. Izvestiya, No. 3, 1961, pp. 68-73

T-71
SOV/35-62-0-6-41/64

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

FTD-TT-62-1928/1+2+4 Date 1 May 1963

P

DISTRIBUTION OF BRIGHTNESS OVER THE DISK OF THE MOON

V. A. Yezerskaya and V. I. Yezerskiy
Astronomical Observatory of Kharkov University

Photometric investigation of the distribution of brightness over the disk of the Moon has shown that along the planetocentric meridians the brightness remains constant within the limit of error and along the equator the intensity of brightness increases continuously from the center of the disk to the limb. On the basis of these data a functional equation is found for the function expressing brightness:

$$B = f\left(\frac{\cos i}{\cos \varepsilon}, \alpha\right).$$

The basic properties of the reflection of light from the surface of the Moon, which were first established by N. P. Barabashov [1], N. P. Barabashov and A. V. Markov [2,3], as is well known, lead to the following two premises: 1) any portion of the Moon's surface regardless of its position on the disk reaches maximum brightness when the phase angle is at a minimum ($\alpha \approx 0^{\circ}$); at full moon the sections having the same albedo also have the same brightness. These properties were interpreted [1,2,3] as the result of pittedness and roughness of the Moon's surface.

Subsequently the photometric peculiarities of the lunar surface as well as their interpretation have been studied in a number of works [4-13] which confirmed and developed these conclusions.

The results of photometric investigation of the lunar surface are usually presented as the dependence of the brightness of various surface areas on the phase angle (phase function or phase curve) as well as on the angle of incidence and angle of reflection.

A comparison of phase curves obtained for a significant number of lunar surface areas located at different parts indicates that the shape of the phase curve is determined by selenographic longitude and depends very little on selenographic latitude. This property is intimately associated with the distribution of brightness over the disk of the Moon. This article deals with the peculiarities of this distribution.

First of all, data on the distribution of brightness along the central meridian of the Moon for different phase angles was obtained from a catalogue [10]. For this purpose details were selected which have selenographic longitudes $\lambda \approx 0$. The difference in details with respect to albedo was considered reduced to a common value of brightness equal to unity for $\alpha \approx 0^{\circ}$. Roughly considereing that for each value of phase angle the indicated details are distributed on the central meridian relative to the equator of intensity, i.e., that $l = 0^{\circ}$, we obtained final data on the distribution of brightness along the central meridian for various phase angles (Table 1).

These data indicate that brightness varies little along the central meridian and, taking into account the inaccuracy of maintaining necessary conditions, they can to a certain approximation be considered constant.

TABLE 1

N_{cat}	132	87	59	19	58	45
λ	0,°0	+5,0	0,0	+2,0	−2,0	+2,0
ρ \ a	−70,°0	−32,0	0,0	+14,0	+17,0	+33,0
−70°	—	0,16	0,18	0,16	0,14	0,16
60	0,20	0,24	0,26	0,23	0,25	0,23
50	0,28	0,32	0,31	0,31	0,28	0,32
40	0,40	0,10	0,42	0,39	0,37	0,41
30	0,52	0,47	0,47	0,49	0,47	0,50
20	0,64	0,57	0,60	0,61	0,58	0,61
−10	0,78	0,78	0,75	0,78	0,75	0,78
0	1,00	1,00	1,00	1,00	1,00	1,00
+10	0,79	0,79	0,83	0,87	0,80	0,89
20	0,60	0,61	[illegible]	0,72	0,64	0,75
30	0,45	0,47	0,58	0,55	0,51	0,60
40	0,32	0,34	0,43	0,41	0,40	0,45
50	0,21	0,23	0,31	0,28	0,28	0,32
+60°	—	0,14	0,18	0,15	0,19	0,20

In order to study this peculiarity more completely, photometric measurements were made of several photographs of the Moon from the series obtained at the Kharkov Astronomical Observatory which served for the compiling the catalogue [10].

Measurements were made with an MF-4 microphotometer without the use of a recording attachment and consisted of the following: prints on which the central meridian and equator of intensity were drawn were made from selected photographs of the Moon. The sections subject to photometry were marked on the central meridian and the equator of intensity. Measurement was conducted along the central meridian from the center northward and along the equator of intensity from the center to the limb. These same sections were also marked on prints obtained from photographs with a minimum phase angle $\alpha = 1^{\circ}5$. The preliminary selection and labeling of sections marked for photometry made it possible to carry out guidance and photometric reading with sufficiaccuracy. The determination of brightness of the same sections at full moon made it possible to allow for the difference in albedo and to obtain data on the distribution of brightness both along the central meridian and the equator of intensity.

Figure 1 shows data on the distribution of brightness along the central meridian based on the measurement of photographs with phase angles $\alpha = +50^\circ.2$. The mean square deviation of brightness shown on this graph are small and the standard deviation was only $\sigma = \pm 0.06$. The same result was obtained when measuring brightness along the central meridian on a photo with $\alpha = \pm 42^\circ.0$, wherein the standard deviation was $\sigma = \pm 0.03$. Measurements were also made on the same photograph along meridians with longitudes $l = 27^\circ.0$ and $43^\circ.0$, whose positions are shown in Fig. 2. It turned out that in all cases the brightness along a meridian remains constant within the limits of error.

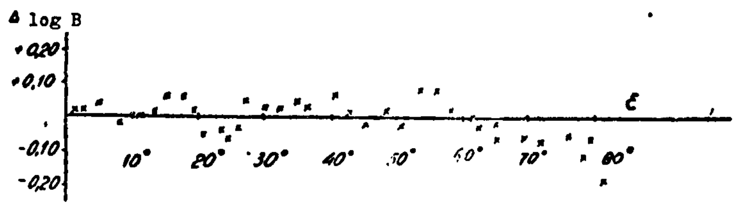

Fig. 1. Distribution of brightness along the central meridian of the disk of the Moon according to measurements of a photograph with phase-angle $\alpha = +50^\circ.2$.

Data on the distribution of brightness along the equator of intensity are presented in Figs. 3, 4, and 5 where the brightness of the center of the disk is taken equal to unity.

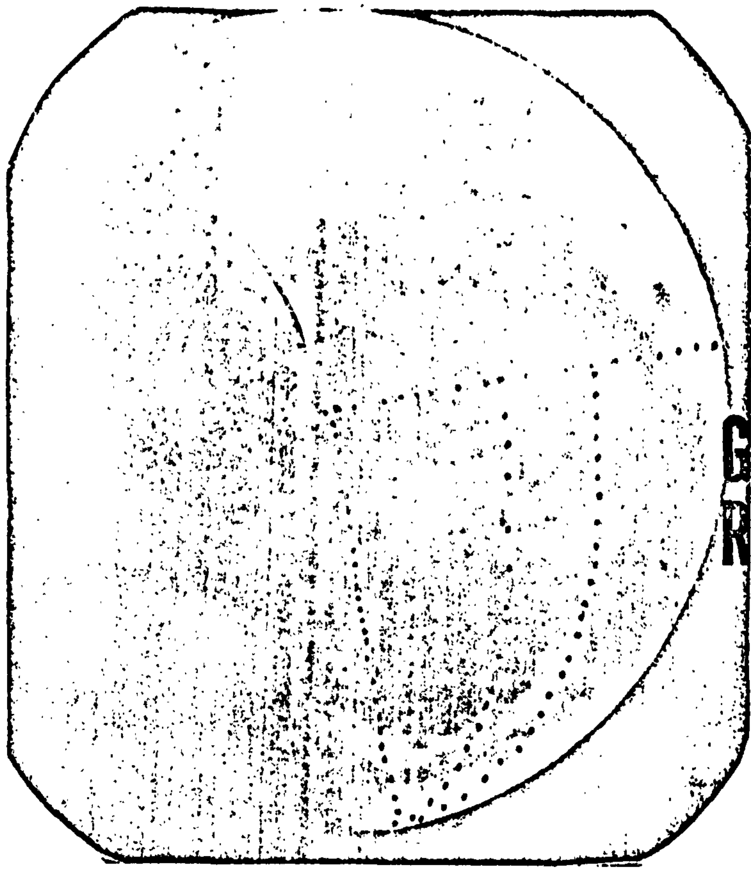

Fig. 2. Location of measured sections on the disk of the Moon at a phase-angle $\alpha = +42^\circ.0$.

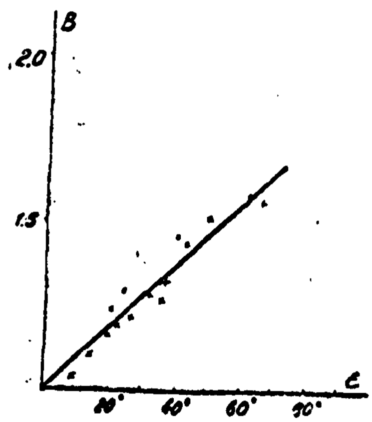

Fig. 3. Distribution of brightness along the equator of intensity for phase-angle $\alpha = +42^\circ.0$.

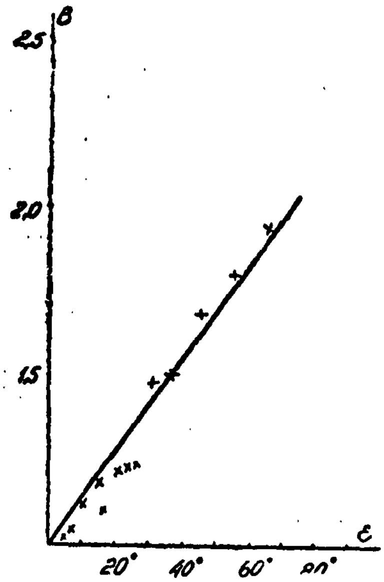

Fig. 4. Distribution of brightness along the equator of intensity for phase-angle α = +50°.2.

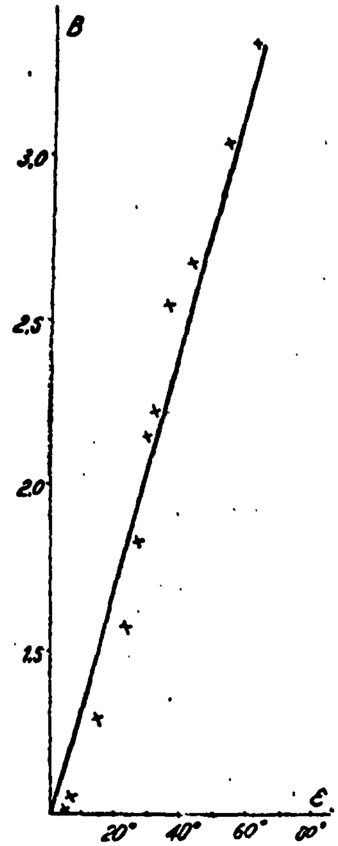

Fig. 5. Distribution of brightness along the equator of intensity for phase-angle α = +37°.6.

In all cases the brightness increases with an increase in distance from the center of the disk measured by angle of reflection ε; dependence is linear with a slope which increases with phase-angle.

Naturally the question arises as to what these features of the distribution of brightness over the disk of the Moon signify. It is easy to realize that they are in fact only another expression of earlier established photometric properties of the lunar surface, but in addition they may prove useful for indicatometric measurements and for theoretical study of the structure of the microrelief of the lunar surface.

First let us examine what conclusions follow from the assumption that brightness maintains a constant value along a meridian.

From the well-known relationships between the angle of incidence i, the angle of reflection ε and the planetocentric coordinates ψ, l, it follows that the relationship

$$\frac{\cos i}{\cos \varepsilon} = \frac{\cos(\alpha - l)}{\cos l}$$

is independent of ψ and for constant values of α and l, i.e., along some meridian, remains constant.

On the strength of this the brightness may be represented as some function of the argument $\frac{\cos i}{\cos \varepsilon}$, and also α, i.e.,

$$B = f\left(\frac{\cos i}{\cos \varepsilon},\ \alpha\right).$$

First of all let us direct our attention to the fact that when $\alpha = 0^{\circ}$ and $i = \varepsilon$, that is at full moon, the brightness, as was established earlier, is constant over the entire disk (if difference due to albedo is eliminated).

Further, for the equator of intensity $l = \varepsilon$

$$\frac{\cos i}{\cos \varepsilon} = \frac{\cos(\alpha - l)}{\cos l} = \cos\alpha + \sin\alpha \tan\varepsilon$$

and consequently when α is constant the brightness may be represented as a dependence on ε which takes place when determining the distribution of brightness along the equator of intensity.

The application of the principle of optical reciprocity introduced into photometry by Minnaert [14] makes it possible to obtain a functional equation for the function $f\left(\frac{\cos i}{\cos \varepsilon},\ \alpha\right)$. In fact, brightness may be represented by means of the coefficient of brightness $r(i, \varepsilon, \alpha)$, and illuminance

$$B(i, \varepsilon, \alpha) = E_0 \cos i \; r(i, \varepsilon, \alpha) = f\left(\frac{\cos i}{\cos \varepsilon}, \alpha\right),$$

$$B(\varepsilon, i, \alpha) = E_0 \cos \varepsilon \; r(\varepsilon, i, \alpha) = f\left(\frac{\cos \varepsilon}{\cos i}, \alpha\right).$$

Since, in agreement with the principle of optical reciprocity

$$r(i, \varepsilon, \alpha) = r(\varepsilon, i, \alpha),$$

$$\frac{f\left(\frac{\cos i}{\cos \varepsilon}, \alpha\right)}{f\left(\frac{\cos \varepsilon}{\cos i}, \alpha\right)} = \frac{\cos i}{\cos \varepsilon}.$$

Having designated $\frac{\cos i}{\cos \varepsilon} = x$, we obtain a functional equation in the form

$$f(x, \alpha) = x f\left(\frac{1}{x}, \alpha\right). \tag{1}$$

The resultant condition, as may be demonstrated, allows a wide class of solutions.

Supposing α = const, we perform the substitution $x = e^y$:

$$f(e^y) = e^y f(e^{-y}) \qquad \psi(y) = \frac{e^{y/2}}{f(e^y)}$$

$$\frac{e^{-\frac{y}{2}}}{f(e^{-y})} = \frac{e^{\frac{y}{2}}}{f(e^y)}.$$

$\psi(y) = \psi(-y)$, i.e., condition (1) signifies simply that the function ψ is even.

One of the partial solutions of equation (1) is of interest since it was obtained by a semi-empirical method.

Let us assume that $f(x,\alpha) = f_1(x)f_2(\alpha)$. It is obvious that the function $f_1(x) = \frac{x}{1+x}$ satisfies condition (1) and, finally, the brightness may be represented in the following way

$$B(i, \varepsilon, \alpha) = \frac{\cos i}{\cos i + \cos \varepsilon} f_2(\alpha). \tag{2}$$

This expression differs from the formula proposed and explained by V. G. Fesgenkov [15] only by the absence of a multiplier on cos ε which, nevertheless, agrees with the principle of optical reciprocity.

According to Fessenkov the physical significance of the mentioned formula indicates that diffusion takes place in a medium which is a porous combination of coarse inhomogeneities capable of scattering light mainly in the reverse direction and of partially shadowing one another.

Expression (2) is only one of the possible solutions of condition (1) taking into account that brightness is constant along the meridian.

Certain features of the functional dependence of brightness on the specified arguments $\frac{\cos i}{\cos \varepsilon}$ and α may be established on the basis of data from the catalogue [10]. For this reason data were obtained from the catalogue for a number of values of phase-angle α concerning the dependence of brightness on $\frac{\cos i}{\cos \varepsilon}$ or l related by the relationship

$$\tan l = \frac{\frac{\cos i}{\cos \varepsilon} - \cos \alpha}{\sin \alpha}.$$

It was demonstrated that the dependence of brightness on l in the first approximation is linear and may be presented as

$$B = kl^{\circ} + b,$$

where the parameters k and b depend on phase-angle.

The values of the parameters k and b are determined by the method of least squares on the basis of data for 30-40 sections. Here the brightness corresponding to the minimum value of phase-angle was taken as equal to unity.

The data obtained is shown in Table 2 where $\sigma(k)$ and $\sigma(b)$ are the standard deviations of the parameters k and b respectively.

TABLE 2

α	$k\cdot 10^3$	$\sigma(k)\cdot 10^3$	b	$\sigma(b)$
11°,3	0,329	±0,074	0.653	±0,102
17,6	0,288	0,038	0.528	0,054
50,3	0,290	0,027	0.288	0,028
72,0	0,249	0,019	0.113	0,014
90,9	0,194	0,023	0,046	0,001
102,9	0,200	0,025	0,016	0,012
128°,9	0,289	±0,058	−0,114	±0,038

From this data, which in the future must be refined by averaging for a large number of sections and their selection with respect to photometric similarity, it follows that the parameter b decreases monotonically with an increase in phase-angle. It represents a variation in brightness with phase-angle of the sections located on the central meridian. As for the parameter k, it varies relatively little.

If the brightness of the center of the disk is taken equal to one then the gradient of distribution of brightness amounts to k/b and will increase, as is apparent from the data of Table 2, with an increase in phase-angle, which was also obtained when determining the distribution of brightness along the equator of intensity by means of photometric measurement.

REFERENCES

1. N. P. Barabashov, A. N., 217, No. 5207, 1923
2. N. P. Barabashov, and A. V. Markov, A. N. 225, 1926.
3. A. V. Markov, A. N., 221, 1924.
4. E. Opik, Publ. Obs. Tartu, 24, 1, 1924.
5. Ibid. [1], Publ. Khar'kovsloy observatoriy, No. 1, 1927.
6. V. V. Sharonov, Uch. zap. LGU, 31, 1939.
7. V. G. Fessenkov, N. M. Shtaude and P. P. Parenago, Trudy Glavnoy astrofiz. observatoriy, 4, No. 1, 1928.
8. A. Bennet, Aph. I. 88, 1938.
9. N. P. Barabashov and A. T. Chekirda, Publ. Khar'kovskoy observatoriy, 8, 1948.
10. V. A. Fedorets, Trudy Khar'kovskoy observatoriy, 2 (9), 1952.

11. N. N. Sytinskaya and V. V. Sharonov, Uch. zap, LGU. No. 15?, 1952.

12. V. Diggelen. Rech. Astr. Obst. Utrecht, XIV, 2,

13. N. S. Orlova, ATs, No. 156, 1955.

14. M. Minnaert, aph. I. 93, 1941.

15. V. G. Fessenkov, Astr. zh., 37, No . 3, 1960.

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies
HEADQUARTERS USAF	
AFCIN-3D2	1
ARL (ARB)	1
OTHER AGENCIES	
CIA	1
NSA	6
DIA	9
AID	2
OTS	2
AEC	2
PWS	1
NASA	1
ARMY (FSTC)	3
NAVY	3
NAFEC	1
PGE	12
RAND	1
AFCRL (CRXLR)	1

MAJOR AIR COMMANDS	Nr. Copies
AFSC	
SCFDD	1
DDC	25
TDBTL	5
TDBDP	5
AEDC (AEY)	1
SSD (SSF)	2
APGC (PGF)	1
ASD (ASYIM)	1
ESD (ESY)	1
RADC (RAY)	1
AFMTC (MTW)	1