UNCLASSIFIED

AD NUMBER

AD405303

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; APR 1963. Other requests shall be referred to Air Force Research Development Command, Attn: Rand Project, Washington, DC 20330.

AUTHORITY

AFRDC/Rand ltr, 31 Mar 1966

THIS PAGE IS UNCLASSIFIED

MEMORANDUM
RM-3609-PR
APRIL 1963

Scale -2-

A COMPUTING PROGRAM FOR DETERMINING CERTAIN STATISTICAL PARAMETERS ASSOCIATED WITH POSITION AND VELOCITY ERRORS FOR ORBITING AND RE-ENTERING SPACE VEHICLES

R. T. Gabler, S. J. Belcher and G. D. Johnson

PREPARED FOR:
UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation
SANTA MONICA • CALIFORNIA

405303

(4) NA
(5) 739300

MEMORANDUM
(14) RM-3609-PR
APRIL 1963

(6) A COMPUTING PROGRAM FOR DETERMINING CERTAIN STATISTICAL PARAMETERS ASSOCIATED WITH POSITION AND VELOCITY ERRORS FOR ORBITING AND RE-ENTERING SPACE VEHICLES,

(7) NA
(8) NA
(9) NA

R. T. Gabler, S. J. Belcher and G. D. Johnson

(11) Apr 63
(12) 47p (13) NA

This research is sponsored by the United States Air Force under Project RAND—contract No. AF 49(638)-700 monitored by the Directorate of Development Planning, Deputy Chief of Staff, Research and Development, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force.

(15) AF 49(638)700

16-19 NA

(20) RAND

The RAND Corporation
1700 MAIN ST • SANTA MONICA • CALIFORNIA

PREFACE

For most applications of satellites and re-entering space vehicles, one must be concerned with the accuracy with which position and velocity can be determined and predicted while on orbit and at the time of earth impact. ~~This Memorandum describes~~ a computing program for estimating, in terms of confidence regions, the on-orbit and impact errors of such vehicles.

In estimating impact errors, guidance errors are combined with orbital prediction errors. The analytically determined sensitivity coefficients are used in this program as a means of error propagation. Their expression as functions of orbital parameters may make them useful for other purposes, such as estimating performance requirements of tracking and prediction systems.

SUMMARY

This Memorandum describes a computing program for determining errors in position and velocity on a satellite orbit. Error coefficients are computed from analytic formulas. These may be used in the further computation of systematic and random errors in the prediction of satellite position and velocity. The computing program handles the propagation of variance-covariance and the determination of confidence regions for position and velocity estimates.

ACKNOWLEDGMENT

The help of W. L. Sibley in checking out certain parts of the program, and in suggesting additions, is gratefully acknowledged.

CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS*

a	semi-major axis of elliptical orbit
d	central angle between ascending node and radius vector from earth's center at time t.
E	eccentric anomaly (elliptical orbit parameter) at time t
E_1	eccentric anomaly (elliptical orbit parameter) at time t_1
e	eccentricity of elliptical orbit
G	universal constant of gravity
i	inclination angle of orbital plane
K	with subscript - error sensitivity coefficient or partial derivative (Other capital letters with subscripts are also used to designate these coefficients or partial derivatives).
k, k_e	$\sqrt{GM}$ - Gauss constant for orbital motion
k_e	.07436574 for displacement in earth radius units and time in minutes
l, m, n, p	with subscripts - direction cosines
M	mass of the earth or of the larger body in the restricted two body problem
P	point (on orbit) for which an error estimate is desired
P_1	point (on orbit) for which the initial evaluation has been made
r	radial distance from center of mass of larger body in a two-body system
$\dot{s}$	magnitude of velocity in orbit
t	time for which the estimate is made
t_1	time of the initial estimate
v	true anomaly (orbital parameter) at time t

*Capital letters with and without subscripts are used to designate matrices, and the symbolism is defined in the text.

v_1	true anomaly (orbital parameter) at time t_1
x_o, y_o, z_o	coordinates associated with input variances
x, y, z	coordinate of point on orbit at time t
x_1, y_1, z_1	coordinates of point on orbit at time t_1
$\dot{x},\dot{y},\dot{z}$; $\dot{z}_1,\dot{y}_1,\dot{z}_1$, etc.	velocity components
β	angle between horizontal direction and velocity increment
γ	angle between horizontal direction and total velocity vector
Δ_1, Δ_2, Δ_3	variances added to x, y, z components of velocity errors due to guidance system
λ	with subscript - eigenvalue of covariance matrix
ϕ	angle between orbitrary reference line and radius from earth center to vehicle at time t
ϕ_1	(same) at time t_1
χ^2	statistical parameter associated with a particular distribution function
Ω	angle which the nodal line makes with the reference direction, generally through the point of Aries
ω	angle between the nodal line and the radius vector at perigee

I. INTRODUCTION

In first-order error propagation in any system, a necessary step is the determination of partial derivatives which are error sensitivity coefficients. When it is feasible, there is an advantage in having these coefficients expressed as analytic functions. The coefficients concerned with the propagation of position and velocity errors into position errors (and other related coefficients) for Keplerian orbits were given in an earlier RAND paper.[1] For the present Memorandum this work has been extended to include velocity errors at a terminal point as well as position errors on the earth's surface for impact trajectories.

These coefficients are used in the propagation of variance-covariance for position and velocity errors in orbits. The resulting variance-covariance matrices are used to determine confidence regions for position and velocity errors at selected points on an orbit. The introduction of guidance errors for orbits that are impulsively changed permits an assessment of errors for points on a new trajectory including errors at impact with the earth's surface, when the new trajectory intersects the earth's surface.

This Memorandum is intended to provide sufficient information for possible future users of the computing program to assemble the proper input data and interpret the output data. It should also provide the equations and background information for re-programming for another computer.

II. ERROR SENSITIVITIES

The error sensitivities are coefficients in the first-order error equations and are obtained by partial differentiation of the equations of motion expressed in a particular coordinate system.

The second-order differential equations characterizing two-body motions can be solved to give the position and velocity of each body as a function of time. When these are solved to give the motion of an infinitesimally smaller body moving about the center of a body of great mass, the usual form taken is that of parameters describing the shape of the path (a conic section) and an equation (generally transcendental) relating time and angular position. When the "total energy" is negative, the path is an ellipse and time and angle are related through Kepler's equation.

COORDINATE SYSTEMS

Figure 1 shows polar coordinates in the plane and a graphical relationship between true anomaly v and eccentric anomaly E. Figure 2 gives more detailed position and velocity coordinates in the planes and Fig. 3 shows the three-dimensional picture. In all representations $P_1(x_1,y_1,z_1)$ is the point where observations are made, and $P(x,y,z)$ is the point for which predictions are made. In path prediction from initial position and velocity, it is essential to note that perigee and apogee are initially undetermined, requiring angular position, ϕ, to be measured from an arbitrary reference.

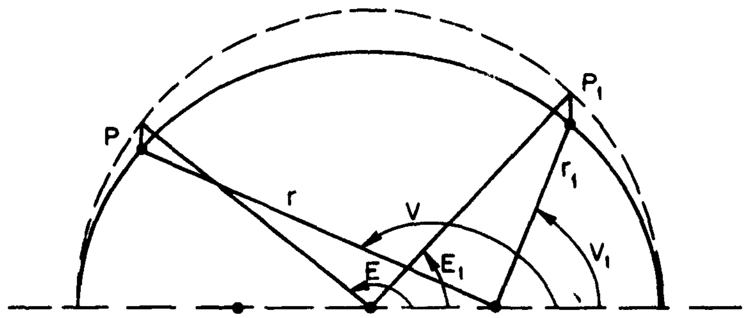


Fig. 1 — Polar coordinates in trajectory plane

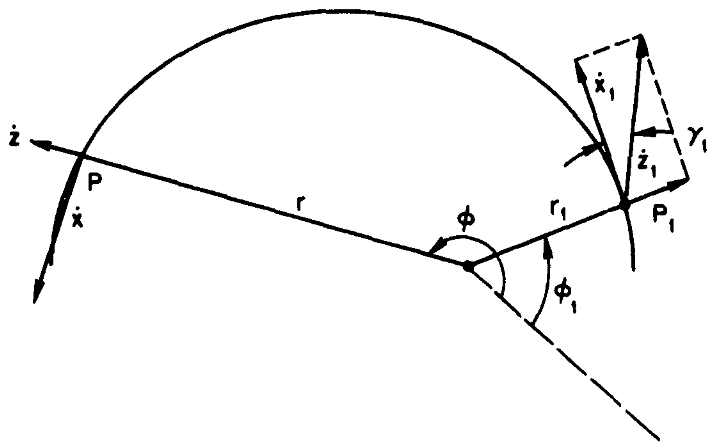


Fig. 2 — Detailed position and velocity coordinates in the plane

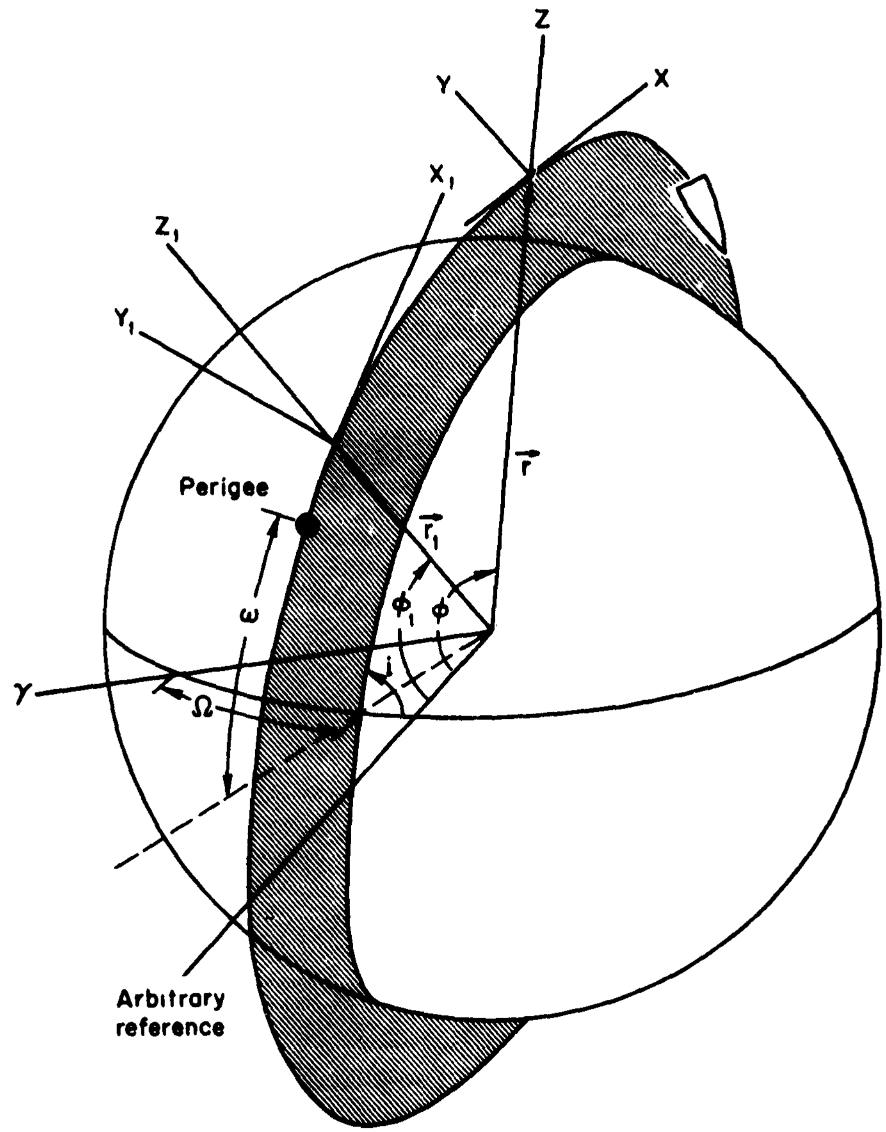


Fig. 3 — Three dimensional picture of trajectory

OUTLINE OF DERIVATIONS

The two parameters describing the elliptical path are a, the semi-major axis, and e, the eccentricity. In terms of initial position and velocity (see Fig. 2) these can be expressed as

$$\frac{1}{a} = \frac{2}{r_1} - \frac{\dot{r}_1^2 + (r_1 \dot{v}_1)^2}{k^2}$$

$$e^2 = 1 - \frac{\dot{v}_1^2 r_1^4}{k^2 a}$$

$$k = \sqrt{GM}$$

With zero time reference at perigee or perihelion, etc., the relation between time and angular position is $\frac{k}{a^{3/2}} t = E - e \sin E$, known as Kepler's equation. The eccentric anomaly E is related to true anomaly v by $\cos E = \frac{\cos v + e}{1 + e \cos v}$, an ambiguous expression unless we include

$$o < E < \pi \text{ for } o < v < \pi$$

$$\pi < E < 2\pi \text{ for } \pi < v < 2\pi$$

Auxiliary relations used are:

$$r = \frac{a(1 - e^2)}{1 + e \cos v}$$

$$\dot{r} = \frac{k\, e \sin v}{\sqrt{a(1 - e^2)}}$$

$$\dot{v} = \frac{k\sqrt{a(1 - e^2)}}{r^2}$$

$$\cos E = \frac{a - r}{ae}$$

$$\cos v = \frac{a(1 - e^2) - r}{er}$$

$$\tan \gamma = \frac{e \sin v}{1 + e \cos v}$$

$$\cos \gamma = \frac{1 + e \cos v}{1 + e^2 + 2e \cos v}$$

$$\dot{s} = k\sqrt{\frac{2}{r} - \frac{1}{a}}$$

For earth satellite orbits and ballistic missiles, k equals K_e equals .07436575 for time in minutes, and distance in earth radius units equals 3444 n mi.

Using the constraint

$$k(t - t_1) = a^{3/2}\left[E - E_1 + e(\sin E_1 - \sin E)\right] = \text{constant},$$

partial differentiation yields the first-order error expressions for errors in a and e, and for position and velocity in the plane at any time t or for any true anomaly v.

The initial position and velocity vectors determine the plane defined by a unit vector $\vec{n}_1$. Errors in components of this vector are determined. From these, errors in position and velocity perpendicular to the plane and errors in angles i and Ω defining the location of the plane follow.

ERROR EQUATIONS

The first-order error equations for which the coefficients have been determined analytically by partial differentiation are listed below. Labeling of these partial derivatives is quite arbitrary and grew out of work which extended over some time.

$$\frac{da}{a} = K_1\, d\dot{r}_1 + K_2\, d\dot{\phi}_1 + K_3\, dr_1$$

$$\frac{de}{e} = K_4\, d\dot{r}_1 + K_5\, d\dot{\phi}_1 + K_6\, dr_1$$

$$de = K_4'\, d\dot{r}_1 + K_5'\, d\dot{\phi}_1 + K_6'\, dr_1$$

$$dr = K_{13}\frac{da}{a} + K_{14}\frac{de}{e} + K_{15}\, dr_1$$

$$dr = K_7\, d\dot{r}_1 + K_8\, d\dot{\phi}_1 + K_9\, dr_1$$

$$d\phi = d\phi_1 + K_{16}\frac{da}{a} + K_{17}\frac{de}{e} + K_{18}\, dr_1 + K_{19}\, dr$$

$$d\phi = d\phi_1 + K_{10}\, d\dot{r}_1 + K_{11}\, d\dot{\phi}_1 + K_{12}\, dr_1$$

$$dn_1x_1 = K_{20}\, d\dot{y}_1 + K_{21}\, dy_1$$

$$dn_1z_1 = K_{22}\, dy_1$$

$$di = K_{23}\, dn_1x_1 + K_{24}\, dn_1z_1$$

$$di = KJ\, d\dot{y}_1 + K_K\, dy_1$$

$$d\Omega = K_S\, dn_1x_1 + K_R\, dn_1z_1$$

$$d\Omega = K_L\, d\dot{y}_1 + K_M\, dy_1$$

$$dy = K_N\, di + K_O\, d\Omega$$

$$dy = K_P\, d\dot{y}_1 + K_Q\, dy_1$$

$$d\dot{s} = D_1\, d\dot{r}_1 + D_2\, d\dot{\phi}_1 + D_3\, dr_1$$

$$dv = H_1\, d\dot{r}_1 + H_2\, d\dot{\phi}_1 + H_3\, dr_1$$

$$dy = W_1\, d\dot{r}_1 + W_2\, d\dot{\phi}_1 + W_3\, dr_1$$

In the following error equations, the coefficients evaluated later are for the particular case of earth satellites or ballistic missiles when units of displacement are n mi, units of velocity are ft/sec.

$$dx = K_A \, d\dot{x}_1 + K_B \, d\dot{z}_1 + K_C \, dx_1 + K_D \, dz_1$$

$$dy = K_E \, d\dot{y}_1 + K_F \, dy_1$$

$$dz = K_G \, d\dot{z}_1 + K_H \, d\dot{x}_1 + K_I \, dz_1$$

$$d\dot{x} = N_1 \, d\dot{z}_1 + N_2 \, d\dot{x}_1 + N_3 \, dz_1$$

$$d\dot{y} = P_1 \, d\dot{y}_1 + P_2 \, dy_1$$

$$dz = L_1 \, d\dot{z}_1 + L_2 \, d\dot{x}_1 + L_3 \, dz_1$$

$$d\dot{s} = F_1 \, d\dot{z}_1 + F_2 \, d\dot{x}_1 + F_3 \, dz_1$$

$$d\gamma = Q_1 \, d\dot{z}_1 + Q_2 \, dx_1 + Q_3 \, dz_1 \text{ (in degrees)}$$

$$dR = T_1 \, d\dot{x}_1 + T_2 \, d\dot{z}_1 + T_3 \, dx_1 + T_4 \, dz_1$$

dR = error in range on the earth's surface (in plane of trajectory).

ERROR COEFFICIENTS

The expressions for coefficients in the first-order error equations which are partial derivatives are listed below. In some cases, these coefficients are given as functions of others which are also listed:

$$K_1 = \frac{2a \, \dot{r}_1}{k^2}$$

$$K_2 = \frac{2a \sqrt{a(1 - e^2)}}{k}$$

$$K_3 = \frac{2a}{r_1^2}$$

$$K_4 = \frac{\dot{r}_1 \, a(1 - e^2)}{k^2 \, e^2}$$

$$K_4' = \frac{\sqrt{a(1 - e^2)} \sin v_1}{k}$$

$$K_5 = \frac{\sqrt{a(1 - e^2)}}{k\, e^2} \left[a(1 - e^2) - \frac{r_1^2}{a}\right]$$

$$K_5' = \frac{e + e \cos^2 v_1 + 2 \cos v_1}{\dot{v}_1}$$

$$K_6 = \frac{a(1 - e^2)\,(e + \cos v_1)}{e\, r_1^2\, (1 + e \cos v_1)}$$

$$K_6' = \frac{a(1 - e^2)\,(e + \cos v_1)}{r_1^2\, (1 + e \cos v_1)}$$

$$K_7 = K_1 K_{13} + K_4 K_{14}$$

$$K_8 = K_2 K_{13} + K_5 K_{14}$$

$$K_9 = K_3 K_{13} + K_6 K_{14} + K_{15}$$

$$K_{10} = K_1 K_{16} + K_4 K_{17} + K_7 K_{19}$$

$$K_{11} = K_2 K_{16} + K_5 K_{17} + K_8 K_{19}$$

$$K_{12} = K_3 K_{16} + K_6 K_{17} + K_{18} + K_9 K_{19}$$

$$K_{13} = r - \frac{r_1^2 \sin E}{r \sin E_1} - \frac{3}{2}\, \frac{a^{1/2}\, k(t - t_1)\, e \sin E}{r}$$

$$K_{14} = ae \left[\frac{r_1 \cos E_1 \sin E}{r \sin E_1} + \frac{ae(\sin E - \sin E_1) \sin E}{r} - \cos E\right]$$

$$K_{15} = \frac{r_1 \sin E}{r \sin E_1}$$

$$K_{16} = \frac{1 + e \cos v_1}{e \sin v_1} - \frac{1 + e \cos v}{e \sin v}$$

$$K_{17} = \frac{r \cos v + 2ae}{r \sin v} - \frac{r_1 \cos v_1 + 2ae}{r_1 \sin v_1}$$

$$K_{18} = \frac{-(1 + e \cos v_1)}{er_1 \sin v_1}$$

$$K_{19} = \frac{1 + e \cos v}{er \sin v}$$

$$K_{20} = \frac{-1}{r_1 \dot{\phi}_1}$$

$$K_{21} = \frac{\dot{r}_1}{r_1^2 \dot{\phi}_1}$$

$$K_{22} = -\frac{1}{r_1}$$

$$K_{23} = -\cos(\omega + v_1)$$

$$K_{24} = -\sin(\omega + v_1)$$

$$K_A = \frac{r K_{11} K_y}{r_1} \qquad K_y = .0098745519$$

$$K_B = r K_{10} K_y$$

$$K_C = \frac{r}{r_1}$$

$$K_D = r K_{12}$$

$$K_E = K_P K_Y$$

$$K_F = K_Q$$

$$K_G = K_7 K_y$$

$$K_H = \frac{K_8 K_y}{r_1}$$

$$K_I = K_9$$

$$K_J = K_{20} K_{23}$$

$$K_K = K_{21} K_{23} + K_{22} K_{24}$$

$$K_L = K_{20} K_S$$

$$K_M = K_{21} K_S + K_{22} K_R$$

$$K_N = -r K_{24}$$

$$K_O = r K_{23}$$

$$K_P = -K_{20} r \sin(v - v_1)$$

$$K_Q = -K_{21} r \sin(v - v_1) - K_{22} r \cos(v - v_1)$$

$$K_R = -\frac{K_{23}}{\sin i}$$

$$K_S = \frac{K_{24}}{\sin i}$$

$$C_1 = \frac{k}{2a\sqrt{\frac{2}{r} - \frac{1}{a}}}$$

$$C_2 = \frac{-k}{r^2\sqrt{\frac{2}{r} - \frac{1}{a}}}$$

$$D_1 = C_1 K_1 + C_2 K_7 = F_1$$

$$D_2 = C_1 K_2 + C_2 K_8$$

$$D_3 = C_1 K_3 + C_2 K_9$$

$$D_4 = \frac{e^2 + e \cos v}{1 + e^2}$$

$$D_5 = \frac{\sin v}{(1 + e^2 + 2 e \cos v)}$$

$$F_2 = \frac{D_2}{r}$$

$$F_3 = \frac{D_3}{K_y}$$

$$G_1 = \frac{1 + e \cos v}{e \sin v}$$

$$G_2 = \frac{2 ae + r \cos v}{r \sin v}$$

$$G_3 = \frac{1 + e \cos v}{e r \sin v}$$

$$H_1 = G_1 K_1 + G_2 K_4 + G_3 K_7$$

$$H_2 = G_1 K_2 + G_2 K_5 + G_3 K_8$$

$$H_3 = G_1 K_3 + G_2 K_6 + G_3 K_9$$

$$Q_1 = D_4 H_1 + D_5 K_4' \quad x \quad .000164281$$

$$Q_2 = \frac{D_4 H_2 + D_5 K_5'}{r_1} \quad x \quad .000164281$$

$$Q_3 = (D_4 H_3 + D_5 K_6') \quad x \quad .0166365$$

$$T_1 = K_A - \frac{K_H}{\tan \gamma}$$

$$T_2 = K_B - \frac{K_G}{\tan \gamma}$$

$$T_3 = K_C$$

$$T_4 = K_D - \frac{K_I}{\tan\gamma}$$

$$L_1 = D_1 \sin\gamma + (D_4 H_1 + D_5 K_4') \dot{s} \cos\gamma$$

$$L_2 = \frac{D_2 \sin\gamma + (D_4 H_2 + D_5 K_5') \dot{s} \cos\gamma}{r_1}$$

$$L_3 = \frac{D_3 \sin\gamma + (D_4 H_3 + D_5 K_6') \dot{s} \cos\gamma}{K_y}$$

$$N_1 = D_1 \cos\gamma - (D_4 H_1 + D_5 K_4') \dot{s} \sin\gamma$$

$$N_2 = \frac{D_2 \cos\gamma - (D_4 H_2 + D_5 K_5') \dot{s} \sin\gamma}{r_1}$$

$$N_3 = \frac{D_3 \cos\gamma - (D_4 H_3 + D_5 K_6') \dot{s} \sin\gamma}{K_y}$$

$$P_1 = - K_{20}(r\dot{\phi} \cos\Delta v + \dot{r} \sin\Delta v)$$

$$\Delta v = v - v_1$$

$$P_2 = \frac{K_{22}(r\dot{\phi} \sin\Delta v - r \cos\Delta v) - K_{21}(r\dot{\phi} \cos\Delta v + \dot{r} \sin\Delta v)}{K_y}$$

$$J_1 = D_1 \sin\gamma + (D_4 H_1 + D_5 K_4') \dot{s} \cos\gamma$$

$$J_2 = D_2 \sin\gamma + (D_4 H_2 + D_5 K_5') \dot{s} \cos\gamma$$

$$J_3 = D_3 \sin\gamma + (D_4 H_3 + D_5 K_6') \dot{s} \cos\gamma$$

$$W_1 = D_4 H_1 + D_5 K_4'$$

$$W_2 = D_4 H_2 + D_5 K_5'$$

$$W_3 = D_4 H_3 + D_5 K_6'$$

III. PROPAGATION OF VARIANCE-COVARIANCE

This program starts with a variance-covariance matrix for position and velocity errors at some point on a nominal orbit. The orbit is specified by position and velocity at the point and/or its osculating Keplerian parameters. If necessary this variance-covariance matrix is transformed to a new coordinate system and also transformed with respect to units. Transformations to new coordinate systems or from one column vector to another are obtained by the matrix multiplication

$$C = A\ B\ A^T$$

where, for example, B is the variance-covariance matrix associated with column vector x, and C is the variance-covariance matrix for errors in a column vector y, and A is a sensitivity matrix

$$\begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \cdots\cdots \\ \frac{\partial y_2}{\partial x_1} & \cdots\cdots\cdots \\ \cdots & \cdots\cdots\cdots \end{bmatrix}$$

The input variance-covariance matrix may come from a number of sources, such as a differential correction routine which has been used to process actual tracking data and has the variance-covariance matrix for initial condition error estimates as a by-product of the orbit determination process. We have used for our source a program which simulates the errors in radar tracking of a satellite and computes the statistical parameters associated with least squares polynomial fitting of short arcs of the trajectory. We are now using a more general program[2] which generates variance-covariance matrices resulting from the use of a wide variety of tracking data from as many as 12 different trackers in orbit determination.

This input matrix represents the variances and covariances for errors in initial conditions consisting of three components of position and three of velocity. Since it is convenient to use a coordinate system associated with the plane of the trajectory for the determination of error sensitivities, a coordinate transformation of the input variance-covariance matrix is usually necessary. Since the sensitivity coefficients as given in Section II are functions of the Keplerian parameters a, e, i, ω, and v_1, it is also necessary to compute these parameters from the initial conditions, x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$.

COORDINATE TRANSFORMATION TO PLANE OF TRAJECTORY

Given the initial conditions x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$ in an arbitrary inertial coordinate system, the transformation of the initial variance-covariance matrix B to the coordinate system associated with the plane of the trajectory as given in Figs. 1, 2, and 3 is obtained as

$$C = \begin{bmatrix} A & o \\ o & A \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} A^T & o \\ o & A^T \end{bmatrix}$$

where

$$A = \begin{bmatrix} m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \\ p_1 & p_2 & p_3 \end{bmatrix}$$

with the elements of A determined from

$$r_o = \sqrt{x_o^2 + y_o^2 + z_o^2} \qquad \dot{s}_o = \sqrt{\dot{x}_o^2 + \dot{y}_o^2 + \dot{z}_o^2}$$

$$\ell_1 = \frac{\dot{x}_o}{\dot{s}_o}, \quad \ell_2 = \frac{\dot{y}_o}{\dot{s}_o} \qquad \ell_3 = \frac{\dot{z}_o}{\dot{s}_o}$$

$$p_1 = \frac{x_o}{r_o}, \quad p_2 = \frac{y_o}{r_o} \qquad p_3 = \frac{z_o}{r_o}$$

$$N_1 = p_2\ell_3 - p_3\ell_2 \qquad n_1 = \frac{N_1}{\sqrt{N_1^2 + N_2^2 + N_3^2}}$$

$$N_2 = p_3\ell_1 - p_1\ell_3 \qquad n_2 = \frac{N_2}{\sqrt{N_1^2 + N_2^2 + N_3^2}}$$

$$N_3 = p_1\ell_2 - p_2\ell_1 \qquad n_3 = \frac{N_3}{\sqrt{N_1^2 + N_2^2 + N_3^2}}$$

$$m_1 = n_2p_3 - n_3p_2$$
$$m_2 = n_3p_1 - n_1p_3$$
$$m_3 = n_1p_2 - n_2p_1$$

THE OSCULATING KEPLERIAN PARAMETERS

To obtain a, e, i, ω, v_1 from the initial conditions x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$, one may use,

$$r_o = \sqrt{x_o^2 + y_o^2 + z_o^2}$$

$$\dot{r}_o = \frac{x_o\dot{x}_o + y_o\dot{y}_o + z_o\dot{z}_o}{r_o}$$

$$\dot{s}_o = \sqrt{\dot{x}_o^2 + \dot{y}_o^2 + \dot{z}_o^2}$$

$$a = \frac{1}{\frac{2}{r_o} - \frac{\dot{s}_o^{\,2}}{k_e^{\,2}}}$$

$k = .07436574$ (when displacements are in earth radius units and time is in minutes)

$$e = \sqrt{\frac{r_o^{\,2}\,\dot{r}_o^{\,2}}{k_e^{\,2}\,a} + \left(1 - \frac{r_o}{a}\right)^2}$$

$$v_1 = \tan^{-1} \frac{e \sin v_1}{e \cos v_1}$$

where

$$e \sin v_1 = \frac{\dot{r}_o \sqrt{a(1 - e^2)}}{k_e}$$

$$e \cos v_1 = \frac{a(1 - e)^2 - r_o}{r_o}$$

To determine i and ω, compute

$$n_{ix} = \frac{y_o \dot{z}_o - \dot{y}_o z_o}{D}$$

$$n_{iy} = \frac{z_o \dot{x}_o - \dot{z}_o x_o}{D}$$

$$n_{iz} = \frac{x_o \dot{y}_o - \dot{x}_o y_o}{D}$$

$$D = r_o \sqrt{\dot{s}_o^{\,2} - \dot{r}_o^{\,2}}$$

$$\sin i = + \sqrt{n_{1x}^2 + n_{1y}^2}$$

$$0 < i < 180^\circ$$

$$\cos i = n_{1z}$$

$$i = \tan^{-1} \frac{+\sqrt{n_{1x}^2 + n_{1y}^2}}{n_{1z}}$$

$$\sin d = \frac{\pm \sqrt{z_o^2 \left(n_{1x}^2 + n_{1y}^2\right) + x_o^2\, n_{1x}^2 + y_o^2\, n_{1y}^2 + 2x_o\, y_o\, n_{1x}\, n_{1y}}}{r_o \sin i}$$

$$\sin d > o \text{ if } z_o > o$$

$$\sin d \leq o \text{ if } z_o \leq o$$

$$\cos d = \frac{y_o\, n_{1x} - x_o\, n_{1y}}{r_o \sin i}$$

$$\omega = d - v_1$$

TRANSFORMATION OF VARIANCE-COVARIANCE MATRIX FROM INITIAL POINT TO ANOTHER TRAJECTORY POINT

If C_1 represents the variance-covariance matrix for errors in the initial point and G the sensitivity matrix of partial derivatives which relates errors for another orbital time to the initial condition errors, then the variance-covariance matrix for position and velocity estimates at the new orbital time is given by

$$H = G\, C_1\, G^T$$

where the elements of G are determined from the error coefficient expressions and more specifically

$$G = \begin{bmatrix} K_C & o & K_D & K_A & o & K_B \\ o & K_F & o & o & K_E & o \\ o & o & K_I & K_H & o & K_G \\ o & o & N_3 & N_2 & o & N_1 \\ o & P_2 & o & o & P_1 & o \\ o & o & L_3 & L_2 & o & L_1 \end{bmatrix}$$

CONFIDENCE REGIONS

The H matrices resulting from transformations of this kind represent the error situation for other trajectory points in terms of variances and covariances. For certain purposes a further description in terms of confidence regions is desirable. It is possible to define an α per cent confidence region (generally an ellipsoid) for position errors, or another for velocity errors. This is the region in which estimates would fall α per cent of the time if the experiment were repeated a very large number of times. If we consider a partitioning of the H matrix into

$$H = \begin{bmatrix} J_1 & J_2 \\ J_2 & J_3 \end{bmatrix}$$

then J_1 represents the variance-covariance matrix for position errors and J_3 the variance-covariance matrix for velocity errors.

The quadratic form defining a confidence ellipsoid for position errors is given by[3]

$$\chi^2_{1-\alpha} = x^T J_1 x$$

where $\chi^2_{1-\alpha}$ is the 1-α level of the χ^2 distribution for three degrees of freedom. For α = 50 per cent, χ^2 = 2.366 and for α = 95 per cent, χ^2 = 7.815. Accordingly, the eigenvalues of the J_1 matrix determine the size of the semi-axes of the confidence ellipsoid and the eigenvectors and/or the associated rotation matrix determines the relative orientation of the confidence ellipsoid. If the eigenvalues are respectively λ_1, λ_2, λ_3 then the semi-major axes are given by

$$d_s = \sqrt{2.366\ \lambda_s} \qquad s = 1,2,3$$

for the 50 per cent confidence ellipsoid, and

$$d_s = \sqrt{7.815\ \lambda_s} \qquad s = 1,2,3$$

for the 95 per cent confidence ellipsoid.

Operating in an identical manner with J_3 determines the confidence region for velocity errors. The six-dimensional confidence region (hyperellipsoid) for the combined position and velocity estimates is obtained in an analogous manner, with

$$d_s = \sqrt{5.348\ \lambda_s} \qquad s = 1,..\ 6$$

for the 50 per cent confidence region,

and

$$d_s = \sqrt{12.592\ \lambda_s} \qquad s = 1,\ ..\ 6$$

for the 95 per cent confidence region.

INCORPORATION OF GUIDANCE ERRORS

When a trajectory is changed by impulsive velocity components, errors due to the guidance system may be introduced. If there is no correlation with prediction errors, the variances in impulsive velocity components are simply added to those due to the prediction process. In gen ral when correlation exists, the guidance errors are incorporated by transforming to a covariance matrix for a nine element vector, introducing guidance error variances, and then transforming back to a 6×6 variance-covariance matrix. The transformation which we use recognizes the possibility of a relationship between the predicted position error in the plane and in-plane velocity component error when a stellar referenced stabilized platform is used.

The transformation required is:

$$M = L\,H\,L^T$$

where H is the variance-covariance matrix for errors at the trajectory point before the impulsive velocity increment is added and

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \ell_{71} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \ell_{91} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\ell_{71} = \frac{- V_\beta \sin \beta}{r}$$

$$\ell_{91} = \frac{- V_\beta \cos \beta}{r}$$

See Fig. 4 for definition of β and V_β. This is for the stellar referenced platform and applies to circular orbits. For ideal compensation, β and V_β have particular values. For other cases, other known correlations could be introduced in an analogous manner.

It has been shown by Frick[(4)] and others that the relationship between the impulsive velocity increment and the in-place predicted position error for circular orbits can result in compensation of in plane position error to first order for a particular velocity increment when the range to impact is fixed. Figure 4 shows schematically how this is accomplished. This range error compensation occurs when $\frac{\partial \theta}{\partial \beta} = 1$, and the particular values of β and V_β are given by solving first for the required velocity components as follows,

$$\dot{S}_x^{\,4} - \frac{\dot{S}_o/k_e \sin^2 \Delta V}{r_1(r_1 - \cos \Delta V)} \dot{S}_x^{\,3} + \frac{1 - \cos \Delta V}{r_1^{\,2} (r_1 - \cos \Delta V)} \dot{S}_x^{\,2}$$

$$- \frac{(1 - \cos \Delta V)^2}{r_1^{\,3} (r_1 - \cos \Delta V)} = 0 \qquad \text{for } \dot{S}_x$$

and

$$\dot{S}_z = \frac{(1 - \cos \Delta V)}{r_1 \sin \Delta V} \frac{1}{\dot{S}_x} - \left(\frac{r_1 - \cos \Delta V}{\sin \Delta V} \right) \dot{S}_x$$

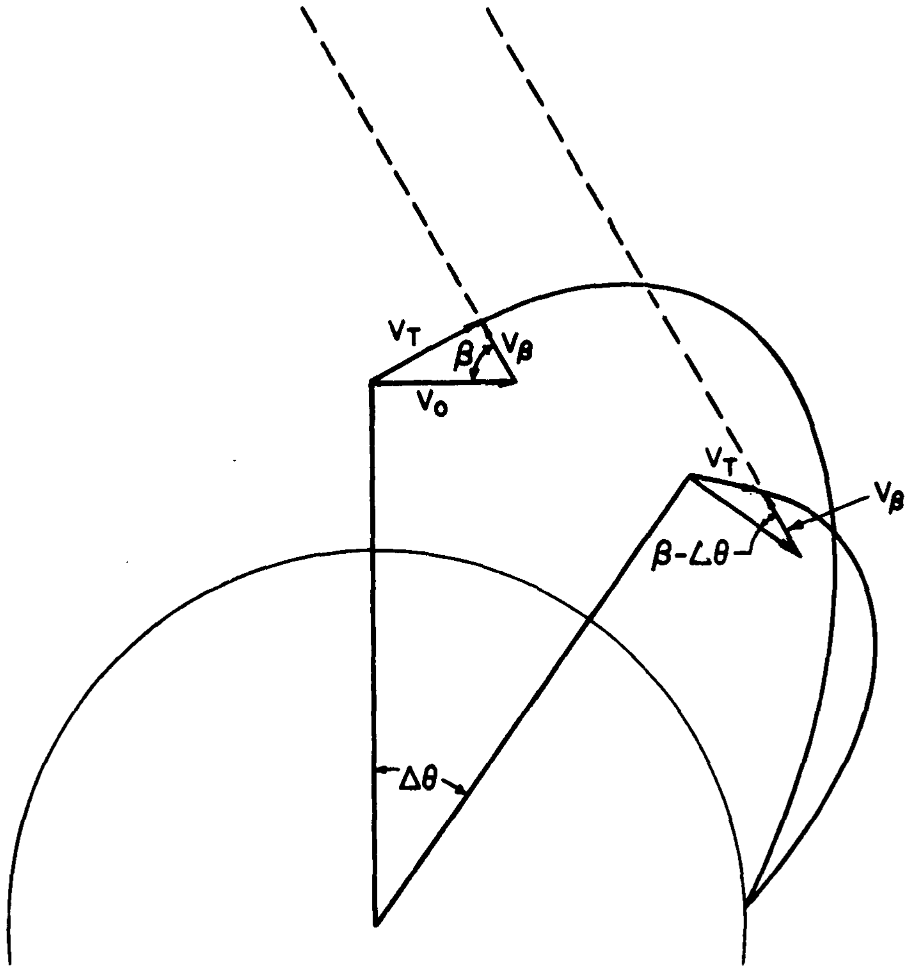


Fig. 4—Scheme for error compensation by stellar inertial reference

$$\dot{S}_\beta = \sqrt{\left(\frac{\dot{S}_o}{k_e} - \dot{S}_x\right)^2 + \dot{S}_z^{\,2}}\; k_e$$

$$V_\beta = \dot{S}_\beta \times \frac{20.92608 \times 10^6}{60} \text{ ft/sec}$$

$$\beta = \tan^{-1} \frac{\dot{S}_z}{\frac{\dot{S}_o}{k_e} - \dot{S}_x}$$

where r is the radius of the initial orbit and ΔV is the change in true anomaly identical with the angular range to impact. Another interesting case is the minimum impulse path occuring when $\frac{\partial\theta}{\partial\beta} = 0$. The value of V_β and β can be obtained by solving

$$\dot{S}_x^{\,4} - \frac{\frac{\dot{S}_o}{k_e} \sin^2 \Delta V}{1 + r_1^{\,2} - 2r_1 \cos \Delta V} \quad \dot{S}_x^{\,3} - \frac{(1 - \cos \Delta V)^2}{r_1^{\,2}\,(1 + r_1^{\,2} - 2r_1 \cos \Delta V)}$$

for $\dot{S}_x$

$$\dot{S}_z = \left(\frac{1 - \cos \Delta V}{r_1 \sin \Delta V}\right) \frac{1}{\dot{S}_x} - \left(\frac{r_1 - \cos \Delta V}{\sin \Delta V}\right) \dot{S}_x$$

$$\dot{S}_\beta = \sqrt{\left(\dot{S}_o/k_e - \dot{S}_x\right)^2 + \dot{S}_z^{\,2}}\quad k_e$$

$$V_\beta = \dot{S}_\beta \times \frac{20.92608 \times 10^6}{60} \text{ ft/sec}$$

$$\beta = \tan^{-1} \frac{\dot{S}_z}{\frac{\dot{S}_o}{k_e} - \dot{S}_x}$$

To incorporate guidance errors into the M matrix we add variance terms to the diagonal elements in the lower right hand corner, thus

$$M_{77} \text{ becomes } M_{77} + \Delta_1$$

$$M_{88} \text{ becomes } M_{88} + \Delta_2$$

and

$$M_{99} \text{ becomes } M_{99} + \Delta_3$$

If covariances in the guidance errors are appreciable and known, these may be added also. The resulting M_1 matrix is transferred to a 6 x 6 by

$$P = N M_1 N^T$$

with

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

<u>PROPAGATION OF VARIANCE IN NEW TRAJECTORY</u>

Since the sensitivity coefficients are expressed analytically in terms of the osculating o oital parameters, these parameters for the new orbit must be determined. These follow from

$$\dot{r}_1 = \dot{r} + V_\beta \sin \beta/u \qquad u = \frac{20.92608 \times 10^6}{60}$$

$$\dot{s}_1 = \sqrt{\dot{r}_1^{\,2} + \left(r\dot{V} - \frac{V\beta \cos \beta}{u}\right)^2}$$

$$a = \frac{1}{\frac{2}{r} - \frac{\dot{s}_1^{\,2}}{k_e^{\,2}}}$$

$$e = \sqrt{\frac{r^2 \dot{r}_1^{\,2}}{k_e^{\,2} a} + (1 - r/a)^2}$$

$$v_1 = \tan^{-1} \left(\frac{e \sin v1}{e \cos v_1}\right)$$

$$\text{where } e \sin v_1 = \frac{\dot{r}_1 \sqrt{a(1 - e^2)}}{k_e}$$

$$e \cos v_1 = \frac{a\ (1-e^2) - r_1}{r_1}$$

$$v = v_1 + \Delta v$$

The value of v does not change and ω is arbitrary for the purpose. These new trajectory parameters now constitute the input to a new computation of error sensitivity coefficients. If one is concerned about the variance-covariance, for errors or confidence regions for a general point on the new trajectory the procedure is identical with that described on pages 19-20. However, if the errors in a tangent plane at the earth's surface at the point of intersection or impact are to be described, the transformation requires a different set of sensitivity coefficients. The 2 x 2 variance-covariance matrix for errors at the impact point is given by

$$R = Q\,P\,Q^{T}$$

where Q is the sensitivity matrix

$$Q \equiv \begin{bmatrix} T_3 & 0 & T_A & T_1 & 0 & T_2 \\ 0 & K_F & 0 & 0 & K_E & 0 \end{bmatrix}$$

and the sensitivity coefficients are appropriate for the new trajectory and the impact point. A confidence region which is now an ellipse is defined by the quadratic form

$$\chi^2_{1-\alpha} = \begin{bmatrix} x, y \end{bmatrix} R \begin{bmatrix} x \\ y \end{bmatrix}$$

with $\chi^2_{1-\alpha} = 1.386$ for $\alpha = 50$ per cent confidence

$\chi^2_{1-\alpha} = 5.99$ for $\alpha = 95$ per cent confidence

The eigenvalues and/or eigenvectors for the R matrix then determine the size and direction of the semi-axes of the confidence ellipse.

IV. LAYOUT OF THE PROGRAM

The routine was coded using FAP and FORTRAN for the IBM 7090 computer. It contains the following:

Hand-coded subroutines:

PAST7	- computes the orbit change
KEP	- computes error coefficients
AEI	- computes a, e, i, ω, v_1
MATMPY, MATMP8	- computes product of two matrices (6x6) and (9x9), respectively
STEP2	- pre(post)-multiplier of the input covariance matrix
SCALE	- scales a matrix to avoid overflow in EIGEN
ARCSIN, ARCCOS	- computes $\cos^{-1}$ and $\sin^{-1}$ from Hastings approximation
AVG6	- averages elements of a real symmetric matrix
XERA	- checks for BEGIN flag
RTSXD	- computes β and V_β on orbit change (see options for this part)

Library routines:

SHARE EIGEN	- computes eigenvalues and eigenvectors
RAND X006	- $\tan^{-1}$ of double argument
SQRTF, COSF, SINF	- standard FORTRAN library

plus the master routine

PROGRAM DESCRIPTION

Step 1: Start with a given covariance matrix, B(6x6), that gives the errors in the initial conditions of a nominal orbit specified by x_o , y_o , z_o , $\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$. These conditions are combined using a subroutine called AEI to give the alternate specification of the orbit by a, e , i , ω , v_1 . The eigenvalues of the matrix B are found and printed. If these are not all positive, the matrix is not meaningful and the program will halt later on.

Step 2: A transformation matrix A_1 is found (subroutine STEP2) such that the given covariance matrix is transformed into a coordinate system associated with the plane of the trajectory.

Step 3: This new matrix is now called C where

$$C = A_1 B A_1^T .$$

Step 4: Convert the units of C to nautical miles and ft/sec. to give the matrix C_1; i.e.,

if

$$C = \begin{bmatrix} D & E \\ E & F \end{bmatrix}$$

then

$$C_1 = \begin{bmatrix} Dxk_1 & Exk_2 \\ Exk_2 & Fxk_3 \end{bmatrix}$$

where $k_1 = (3444)^2$; $k_2 = \left(\frac{3444 \times 20.926}{60 \times 10^{-6}} \right)$; $k_3 = \left(\frac{20.926 \times 10^6}{60} \right)^2$

Step 5: Input the Δv's (change in true anomaly) to be considered. Compute the error coefficients for the orbit using subroutine KEP. The formulas are contained in Sec. II. Although all of these coefficients are not used in the program, they may be printed out at the option of the user by setting the correct value of KPRINT (see input requirements). The sensitivity matrix G is computed using the proper error coefficients. The matrix G gives the propagated errors in x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$, for a given v.

Step 6: For each G compute

$$H = GC_1G^T$$

for the errors in position and velocity.

Step 7: Various submatrices of H are then used to give confidence ellipsoids for which semi-axes, angles of rotation, eigenvalues and eigenvectors are computed and printed.

This completes the first part of the program. If one wishes to go on to a new trajectory, the value of KFLAG is appropriately set on input and computation proceeds.

Step 8: There are three options for input of data on the new trajectory.

a. specify β , V_β , Δ_1 , Δ_2 , Δ_3 , Δv , KLM = 1 .

b. minimum impulse path,

KLM = 2, specify Δ_1 , Δ_2 , Δ_3 , Δv , machine computes β and V_β.

c. range compensation path,

KLM = 3, specify Δ_1 , Δ_2 , Δ_3 , Δv , machine computes β and V_β .

Then, having a value of β and V_β , an L(6x9) matrix is determined so that the M(9x9) matrix can be found

$$M = LHL^T .$$

Step 9: M_1 is formed by changing three elements of the M matrix as follows:

m_{77} to $m_{77} + \Delta_1$

m_{88} to $m_{88} + \Delta_2$

m_{99} to $m_{99} + \Delta_3$

where Δ_1 , Δ_2 , Δ_3 , are inputs. (Not cumulative)

Step 10: A new transformation, matrix N(6x9), is found so that the following transformation can be made:

$$P = NM_1N^T$$

where

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Step 11: New values of a , e , i , ω , v_1 , and new values of the error coefficients (which will be printed if KPRINT IS SET $\neq$ 0) are found.

Step 12: A new sensitivity matrix Q(6x2) is computed giving the errors in range and y, and a covariance matrix R(2x2) is determined for these errors,

$$R = QPQ^T .$$

Step 13: The confidence ellipsoid semi-axes are computed and printed, which completes the problem.

OPTIONS

KPRINT = 0 No error coefficient printout.

$\neq$ 0 Error coefficients printed.

KFLAG < 0 Change to new trajectory (ies).

$\geq$ 0 Go to new case - no change of trajectory.

KLM = 1 Specify β , V_β , Δ_1 , Δ_2 , Δ_3 , on new trajectory.

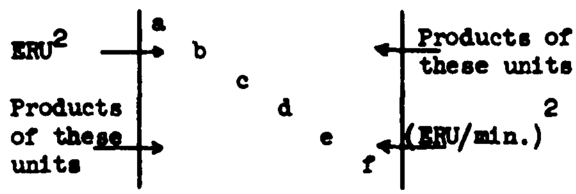
= 2 Specify Δ_1 , Δ_2 , Δ_3 , Δ_v ; compute β , V_β , minimum impulse path. $\beta < 0$.

= 3 Specify Δ_1 , Δ_2 , Δ_3 , Δ_v ; compute β , V_2 , range compensation path. $\beta > 0$.

KMN = 0 Compensation desired. **Only reasonable for $\beta > 0$ trajectories.**

$\neq$ 0 No compensation desired.

INPUT

Card No.	Description	Format
1	KPRINT { = 0 Error coefficients printed; = 0 Error coefficients not printed }	I 2
2	x_o , y_o , z_o , (Earth radius units)	3E18.8
3	$\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$, (Earth radius units/min)	3E18.8
4-15	Covariance matrix, B; diagonals are variances (or standard errors), off diagonals are cross product terms; i.e.,	3E18.8
		
16	KØUNT5 - Count of the number of Δv's (true anomalies) to be input.	I12
17 ⋮ 17+KØUNT5-1	Δv's in degrees, 1 per card	F12.0
17+KØUNT5	KFLAG { < 0 Change trajectory. $\geq$ 0 Don't change trajectory; go to new case. Next card is the L+KØUNT card. } Let L = 17+KØUNT5+1	I12
L	KØUNT, KMN KØUNT - Number of new trajectories to be computed. KMN = 0 compensation desired; ≠ 0 no compensation desired	2I12
L+1	KLM - As explained in writeup - step 8	I 12

Card No.	Description	Format
L + 2	Δ_1 , Δ_2 , Δ_3 , Δ_v	6E12.8
	Δ_1 , Δ_2 , Δ_3 , - Elements to add to M matrix to get M_1 matrix.	
	Δ_v - Quantity to add to V_1 to get V.	
L + 3	if KLM = 1; β, V_β	6E12.8
	if KLM = 2 or 3 next card like L + 1	
	Repeat cards L + 1, L + 2 (and L + 3 if KLM = 1) KØUNT-1 times.	
L+KØUNT	"BEGIN" punched in cols. 1-5 - Signifies the start of a new case.	A6

OUTPUT

FIRST PAGE

1. Characteristic roots of B, the input covariance matrix. These must all be positive or B is not a valid matrix for this problem. The first three are in ERU^2 and the last three in $(ERU/min)^2$.

2. The initial conditions x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ in ERU and ERU/min, respectively.

3. The Keplerian parameters of the orbit.

$$\begin{array}{ll} a & \text{in ERU} \\ e & \text{non-dimensional} \\ \left.\begin{array}{l} i \\ w \\ V_1 \end{array}\right\} & \text{degrees} \end{array}$$

4. The input covariance matrix.

   The upper left-hand corner (3x3) in $(ERU)^2$

   The lower right-hand corner (3x3) in $(ERU/min)^2$

   The upper right and lower left-hand corners are combinations of these units.

5. Transformation matrix, A_1 .

   Non-dimensional, takes B from the initial reference system to a coordinate system associated with the orbital plane.

SECOND PAGE

1. Transformed matrix $C_1 = A1BA1^T$.

   Units are the same as B.

2. C_1 Matrix.

   Units changed from $(ERU)^2$ to $(\text{nautical miles})^2$ and $(ERU/min)^2$ to $(Ft/sec)^2$.

3. True anomaly, v, change in true anomaly, Δv.

   The angle the vehicle moves from perigee.

4. G Matrix; error sensitivity matrix.

   a. First row; error in x .

   b. Second row; error in y .

   c. Third row; error in z .

   d. Fourth row; error in $\dot{x}$.

   e. Fifth row; error in $\dot{y}$.

   f. Sixth row; error in $\dot{z}$.

5. H Matrix = GCG^T ; transformed matrix gives error in position and velocity at a new point on the trajectory specified by Δv. Same units as C_1 .

   NOTE: If it is desired to print out all the error coefficients, they will be printed after Step 2 above, and Step 3 will begin a new page. If there is more than one v , each new v will begin a new page.

N+1st Page , $N = \begin{cases} \text{Number of } \Delta v\text{'s} + 2 \text{ if error coefficients printed.} \\ \text{Number of } \Delta v\text{'s} + 1 \text{ if no error coefficients printed.} \end{cases}$

1. J_1 (n) (n = 1, ... number of Δv's)

   Upper left-hand corner of H .

   Error situation in position (n mi)^2.

2. Roots of J_1 .

3. Semi-axes of confidence ellipsoid,

   i.e., where you would expect to find the object 50 per cent and 95 per cent of the time, respectively.

4. L-Matrix - Eigenvectors of J_1 .

   Rotation matrix to give new coordinate system with no correlation.

5. α , β , γ - angles relating new region to the old.

N+2nd Page

Same as above but for J_3(n) , the lower right-hand corner of H.

N+3rd Page

Same as above but for total H matrix. Now you have a confidence hyperellipsoid. Step 5 is not done.

NOTE: These three pages are repeated n times for the n values of v .

N+3n+1st Page,

1. V_β , β , Δ_1 , Δ_2 , Δ_3 , Δv .

   β = angle (in degrees) between the velocity increment and the original velocity vector.

   V_β = velocity increment (ft/sec)

$$\left.\begin{matrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{matrix}\right\} = \text{Error variances for guidance } (\text{ft/sec})^2$$

Δv = Change in true anomaly on new trajectory (in degrees).

2. Parameters associated with new and old orbits as labeled:

   a , e - same units as Step 3, page 1.

   v_1 , ω , i - radians.

3. Q(k,k) matrix - another sensitivity matrix.

   First row; errors in r .

   Second row; errors in y .

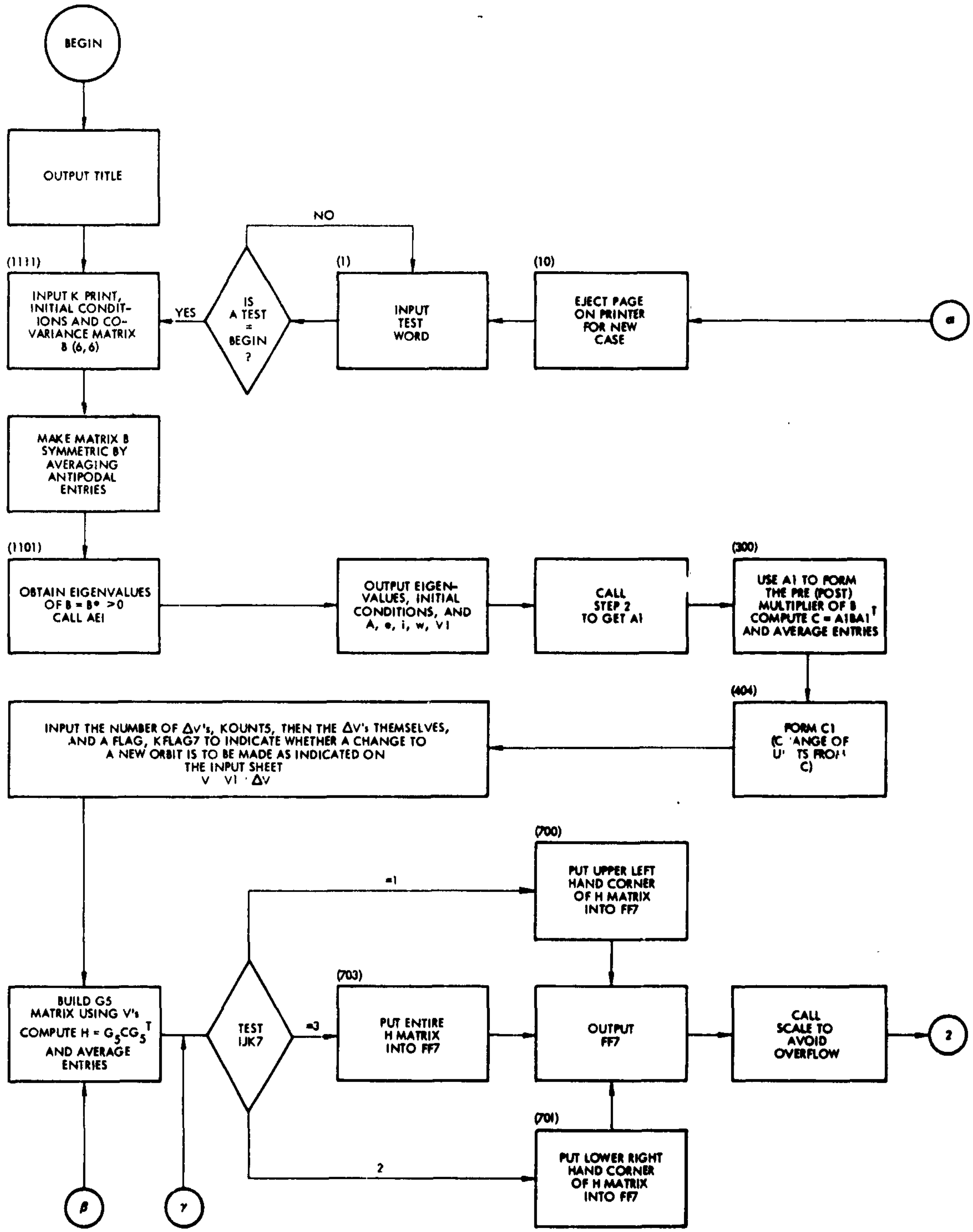
4. R(k,k) - covariance matrix for r and y errors.

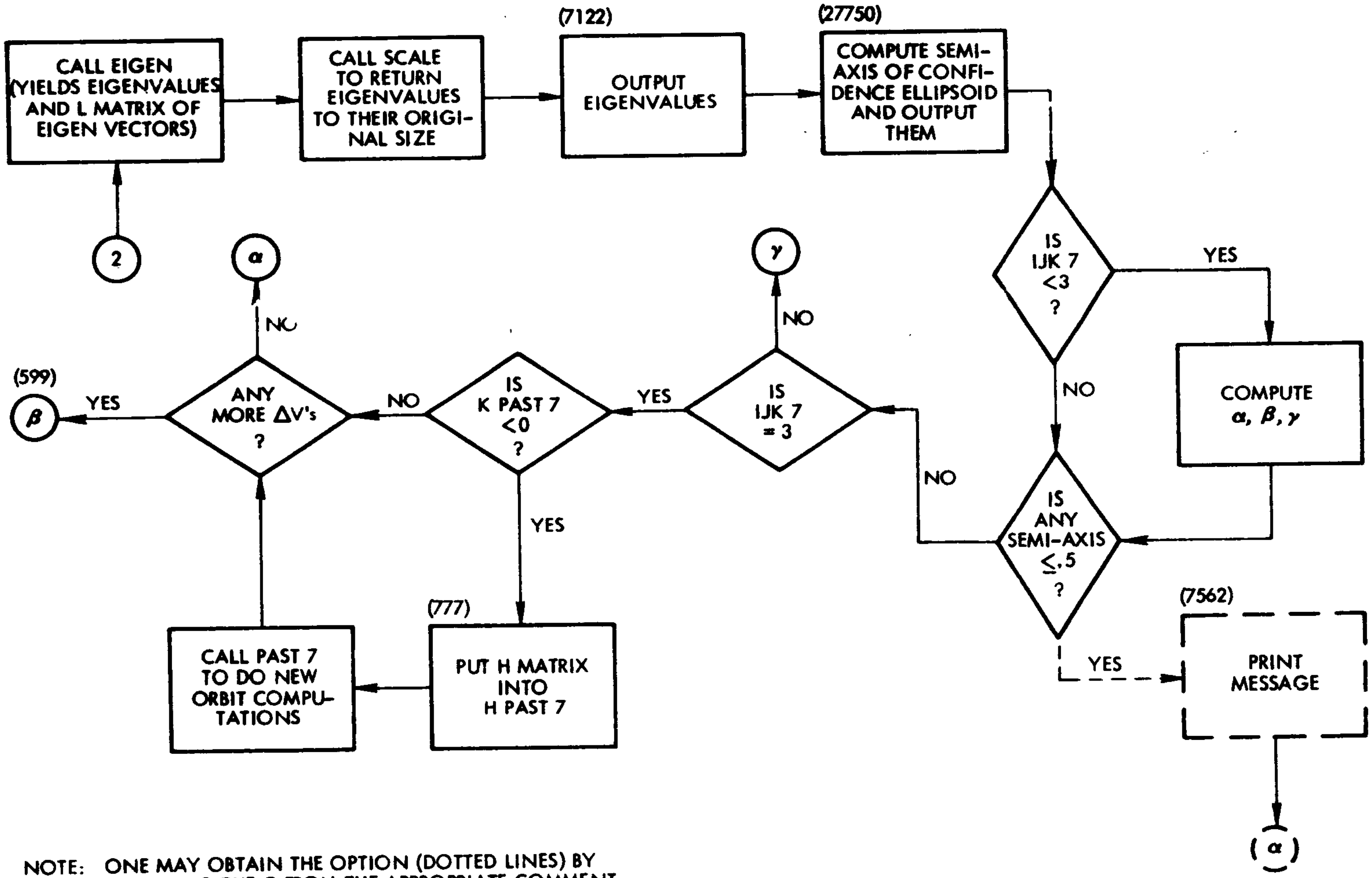
5. Semi-axes for 50 per cent and 95 per cent confidence as before.

<u>NOTE</u>: If K represents the number of new trajectories to be considered, the above five steps will be output K times. As before, if all the error coefficients are to be printed, they will be printed after Step 2 and Step 3 will begin a new page.

V. FLOW CHARTS

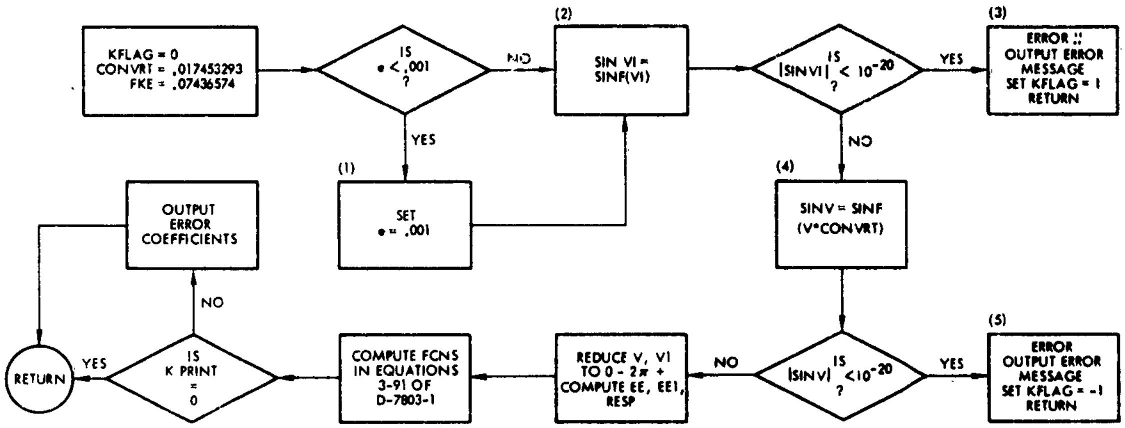
MAIN ROUTINE (STATEMENT NUMBERS IN PARENTHESES)



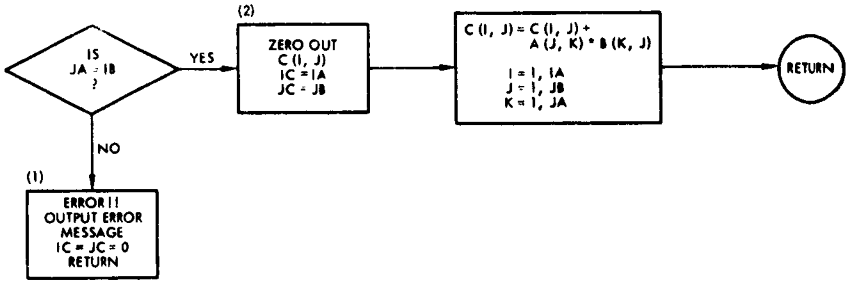


NOTE: ONE MAY OBTAIN THE OPTION (DOTTED LINES) BY REMOVING THE C FROM THE APPROPRIATE COMMENT CARDS AND INSERTING C's ON OTHERS

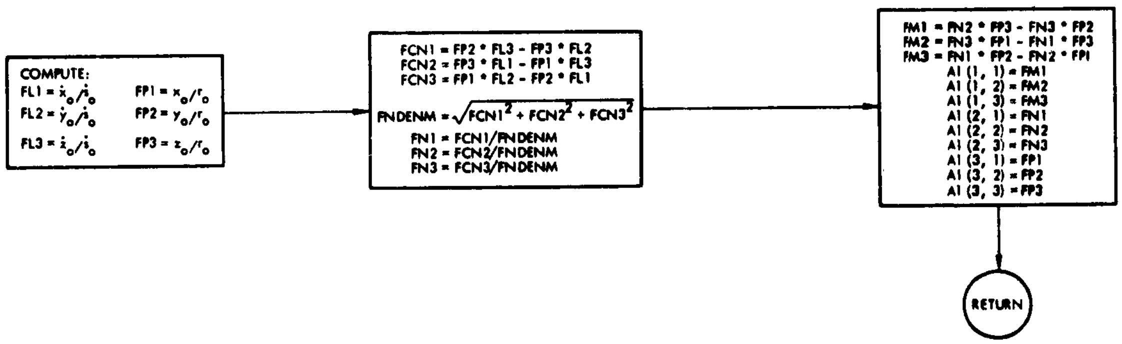
SUBROUTINE KEP - ERROR COEFFICIENTS



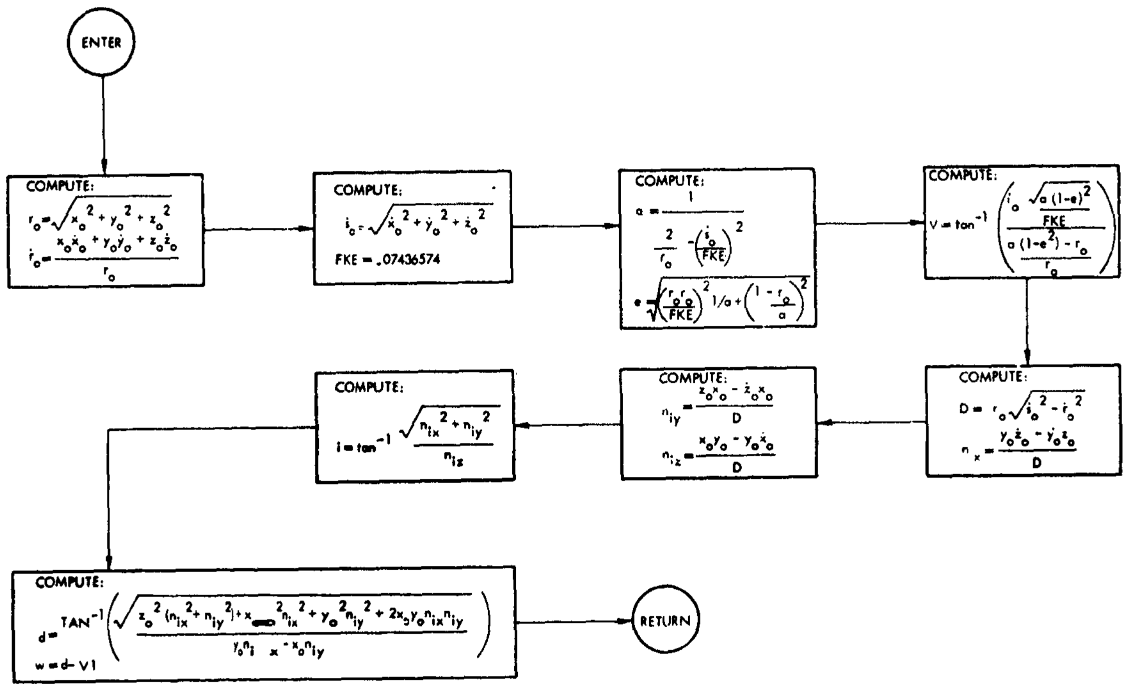
MATMPY OR MATMP8



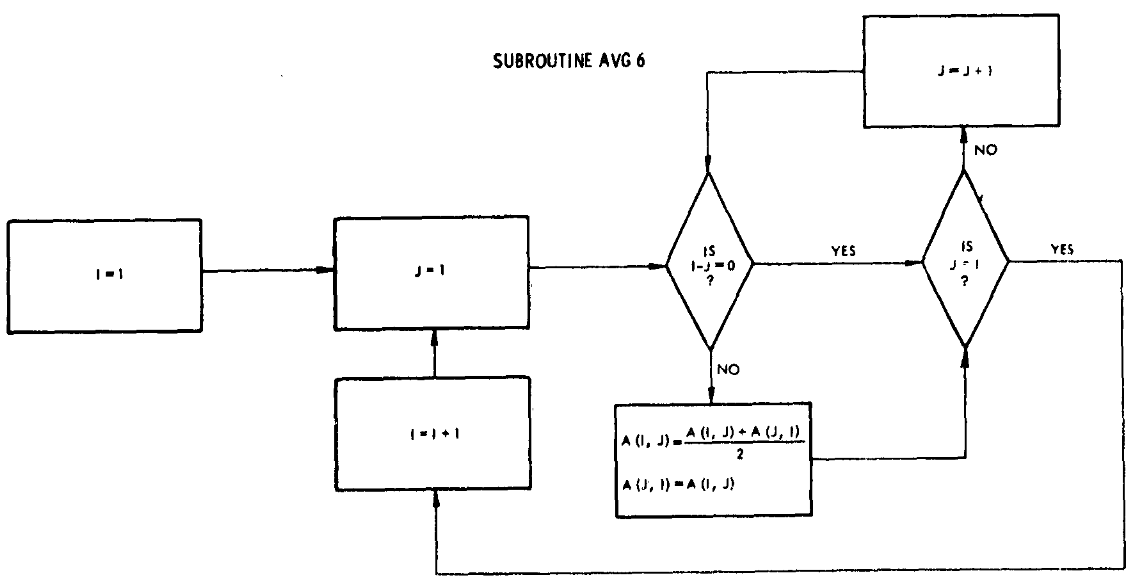
STEP 2 - AI MATRIX



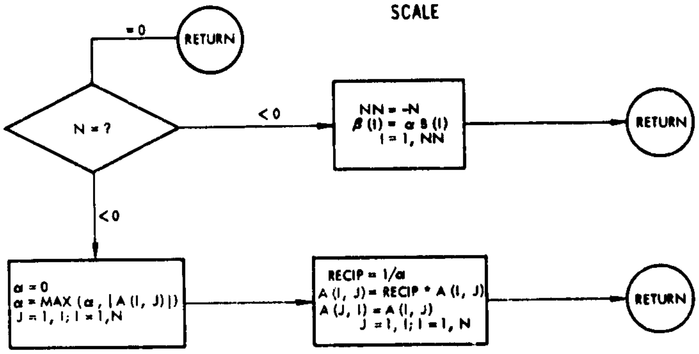
SUBROUTINE AEI, a, e, i, w, v,



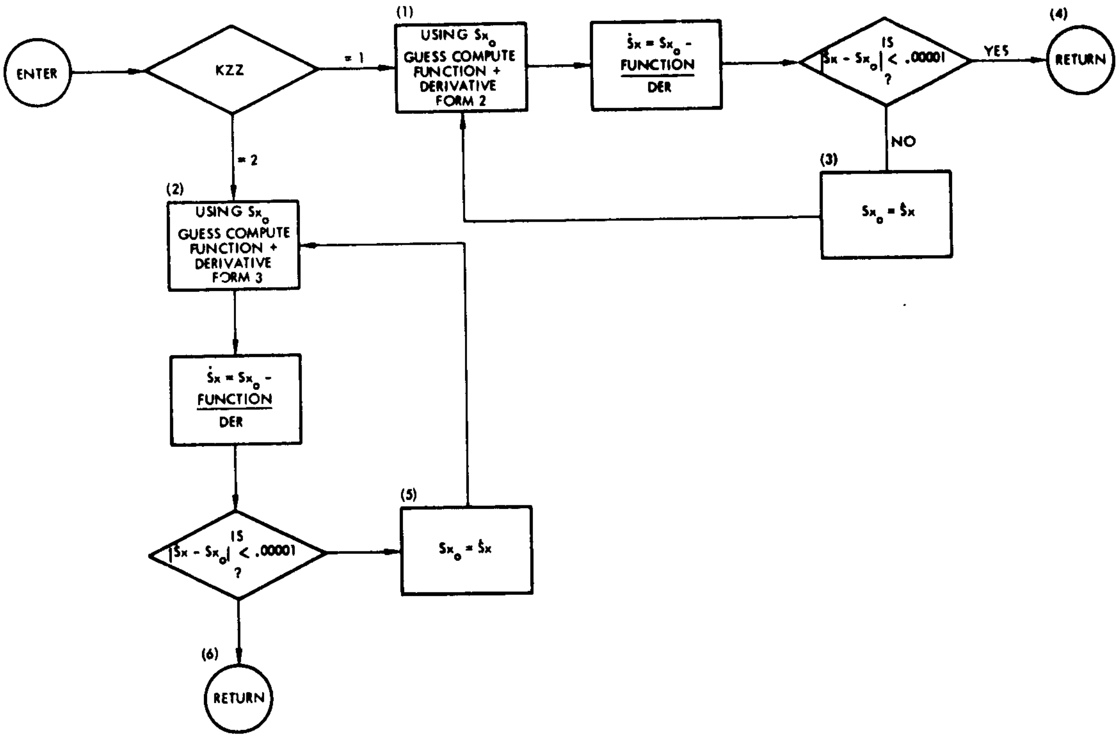
SUBROUTINE AVG 6



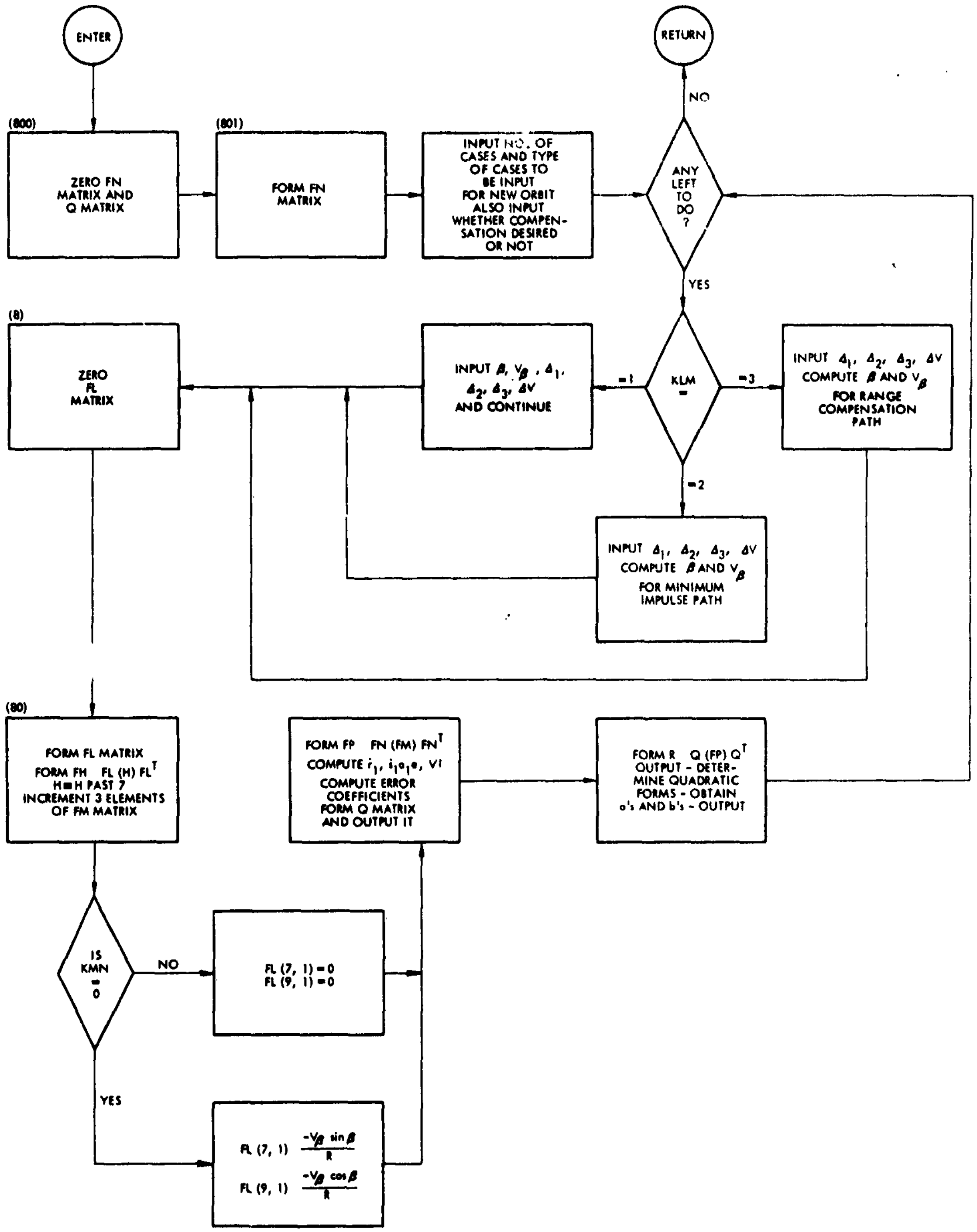
SCALE


RETURN
= 0
N = ?
< 0
NN = -N
β (I) = α B (I)
I = 1, NN
RETURN
< 0
α = 0
α = MAX (α, |A (I, J)|)
J = 1, I; I = 1, N
RECIP = 1/α
A (I, J) = RECIP * A (I, J)
A (J, I) = A (I, J)
J = 1, I; I = 1, N
RETURN

RTSXD


ENTER
KZZ
= 1
(1)
USING Sx_o
GUESS COMPUTE
FUNCTION +
DERIVATIVE
FORM 2
Ṡx = Sx_o - FUNCTION / DER
IS |Ṡx - Sx_o| < .00001 ?
YES
(4)
RETURN
NO
(3)
Sx_o = Ṡx
= 2
(2)
USING Sx_o
GUESS COMPUTE
FUNCTION +
DERIVATIVE
FORM 3
Ṡx = Sx_o - FUNCTION / DER
IS |Ṡx - Sx_o| < .00001 ?
(5)
Sx_o = Ṡx
(6)
RETURN

SUBROUTINE PAST 7



VI. PRESENT LIMITATIONS AND POSSIBLE MODIFICATIONS OF THE PROGRAM

The sensitivity coefficients are here determined for a theoretical Keplerian orbit. Even for cases of moderately high drag the error in these coefficients due to this assumption will not be serious. However, the error in predicted position and velocity components due to error in estimation of drag force represents an additional error component that is not included in this program. An effect which is external to this program is the validity of the input covariance matrix. This program does check on the requirement of positive definiteness of the input covariance matrix for physical realizability, but otherwise imposes no restrictions.

In programming prediction intervals, exact multiples of 180° must be avoided since the sine of v and v_1 occurs in the denominator of certain expressions, rendering them indeterminate. The error in longitude of the node also becomes infinite for zero inclination angle. The case for zero eccentricity or exactly circular orbits is also avoided in the program by making eccentricities never less than .001 in computing error coefficients. In practice this gives values for error coefficients sufficiently close to those for a circular orbit.

It is obvious that a more sophisticated statement of guidance errors including their covariances could be incorporated into the present program without much difficulty. A major modification of the program would be required to compute error sensitivity coefficients for the high drag re-entry case since this would involve integration of the equations of motion and a numerical determination of partial derivatives.

REFERENCES

1. Gabler, R. T. and Helen O'Mara, The Propagation of Errors in Keplerian Orbits, The RAND Corporation, P-1481, August 1, 1958.

2. Swerling, P., A Computer Program for First-Order Error Propagation in Satellite Orbit Prediction, The RAND Corporation, P-1968, April 13, 1960.

3. Mood, A. M., Introduction to Theory of Statistics, McGraw Hill Book Company, Inc., 1950.

4. Frick, R. H., Preliminary Analysis of a Satellite Recovery System, The RAND Corporation, RM-2264, September 19, 1958.