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Report No.
TDR-169(3560-30)TN-3

ON THE FREE VIBRATION OF THIN CYLINDRICAL SHELLS

V. I. Weingarten

20 December 1962

Prepared For
COMMANDER SPACE SYSTEMS DIVISION
UNITED STATES AIR FORCE
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AEROSPACE CORPORATION
2400 East El Segundo Boulevard
El Segundo, California

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Prepared by V. I. Weingarten
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ABSTRACT

The free vibrations of finite cylindrical shells are investigated. With the aid of a number of simplifying assumptions, a frequency equation based on the known characteristic functions for beams with any combination of boundary conditions is obtained. Experimental results for frequency spectra and mode shapes of a cylinder fixed on one edge and free on the other are in good agreement with both Rayleigh's inextensional theory and the approximate frequency equation. Structural damping coefficients obtained for the test cylinders are compared with those of previous investigations.

NOMENCLATURE

b	Parameter $\left[\frac{12\rho R^4(1-\nu^2)}{Eh^2}\right]$
c^2	Geometry parameter $\left[\frac{h^2}{12(1-\nu^2)R^2}\right]$
D	Flexural stiffness of cylinder wall $\left[\frac{Eh^3}{12(1-\nu^2)}\right]$
E	Young's modulus of cylinder material
f	Frequency (cyc/sec) $\left(f = \frac{\omega}{2\pi}\right)$
$f'g$	Viscous damping coefficient
h	Thickness of the cylinder
ℓ	Length of the cylinder
$m-1$	Number of nodes in axial mode shape
n	Number of circumferential waves
p	Internal or external pressure
R	Radius of cylinder
t	Time
w	Radial deflection
x	Distance along the longitudinal axis of the cylinder
θ	Angle denoting the circumferential location of a point on the cylinder middle surface
ξ	Nondimensional axial coordinate $\left(\frac{x}{R}\right)$
λ_{mn}	Axial wave length parameter $\left(\frac{m\pi R}{L}\right)$
ρ	Density of the shell material
ω	Circular frequency $(2\pi f)$
Ω	Frequency parameter $\left[\frac{12\rho R^4(1-\nu^2)\omega^2}{Eh^2}\right]$
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2}$
$\bar{\nabla}^2$	$\frac{\partial}{\partial \xi^2} + \frac{\partial}{\partial \theta^2}$

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I. INTRODUCTION

The free vibration of a cylindrical shell has interested many investigators. In 1894, Lord Rayleigh (Reference 1) derived an approximate expression for the natural frequencies of vibration of a cylindrical shell based on a separation of the effects of bending and stretching. A later treatment by Love (Reference 2) resulted in a general dynamical theory of shells which included both bending and extensional deformations. Love's equations were first used by Flugge (Reference 3) to obtain a cubic frequency equation for a simply supported cylinder, a result which indicated that there were three frequencies for each nodal pattern. A more detailed investigation made by Arnold and Warburton (References 4 and 5) showed that the three frequencies corresponded to essentially radia, axial and circumferential vibrations with the radial vibration frequency much lower than the other two. Their analysis also showed that the natural frequency may decrease as the number of circumferential waves increases, in contrast with the results of inextensional theory. Arnold and Warburton also investigated the natural frequencies of cylinders clamped at both edges, with the use of the Rayleigh-Ritz method.

Recent investigators have concentrated on simplifying the method of analysis of vibrating cylindrical shells. By means of a number of approximations, Yu (Reference 6) was able to obtain a simple expression for the radial frequencies of a clamped or simply supported cylinder vibrating in a mode consisting of a number of circumferential waves that is large compared to the number of axial waves. Simplified frequency equations were also obtained by Vlasov (Reference 7), Breslavskii (Reference 8), and Reissner (Reference 9) by neglecting the circumferential and axial inertia forces of the shell. Finally, the simplifications of Breslavskii and Yu were combined by Rapoport (Reference 10) to yield frequency equations for a shell with various boundary conditions.

In the present paper, a method similar to Rapoport's has been used. An experimental investigation of the frequency spectra and mode shapes of a clamped-free cylinder was also performed. The experimental data are in good agreement with theory. Structural damping was investigated as a secondary part of the experimental program. Viscous damping coefficients were obtained for each resonance point of the supported-free cylinder and tabulated as a function of wave shape and frequency. The results are compared with those of previous investigations in the Appendix.

II. APPROXIMATE METHOD OF ANALYSIS

The well-known Donnell differential equation of a circular cylindrical shell under an external radial loading p can be written as (Reference 11)

$$D\,\nabla^8 w + \frac{Eh}{R^2}\frac{\partial^4 w}{\partial x^4} - \nabla^4 p = 0 . \qquad (1)$$

This equation can be applied to vibration problems of cylindrical shells when we assume that the circumferential and longitudinal inertia forces are negligible. Then the external loading p can be replaced by the radial inertia force

$$-\rho h\,\frac{\partial^2 w}{\partial t^2} .$$

On substituting this value into Equation (1) and nondimensionalizing the resulting equation, we obtain

$$\overline{\nabla}^8 w + \frac{1}{c^2}\,\frac{\partial^4 w}{\partial \xi^4} + b\,\frac{\partial^2}{\partial t^2}\,\overline{\nabla}^4 w = 0 . \qquad (2)$$

Let us assume w to be of the form

$$w = \left(\sum_k e^{i\lambda_{kmn}\xi}\right)\cos n\,\theta\,\sin\omega t \qquad (3)$$

Equation (3) will satisfy Equation (2) if the coefficients λ_{mn} are the roots of the following equation

$$\Omega = \frac{(\lambda^2_{kmn} + n^2)^4 + \frac{1}{c^2}\,\lambda^4_{kmn}}{(\lambda^2_{kmn} + n^2)^2} , \qquad (4)$$

which is an eighth order equation for λ_{mn} as a function of c, n and Ω. Conversely, we note that if any one of these roots is known, then the frequency parameter Ω is determined.

Since the exact determination of λ_{mn} is quite difficult, a number of simplifying assumptions will be made to obtain approximate values. Let us assume that n^2 is large compared to λ_m^2. Then Equation (4) can be approximated by

$$\lambda_{kmn}^4 = c^2 (\Omega n^4 - n^8) \tag{5}$$

For given values of n, c, and Ω the right-hand side of Equation (5) is a constant. Therefore, four approximate values of λ_{kmn} are of the form

$$\lambda_{mn}, \ -\lambda_{mn}, \ i\lambda_{mn}, \ -i\lambda_{mn} \tag{6}$$

The remaining four values of λ_{kmn} implied by Equation (3) are neglected.

The deflection function w can now be written approximately as

$$\begin{aligned} w = {} & (c_1 \sin \lambda_{mn} \xi + c_2 \cos \lambda_{mn} \xi + c_3 \sinh \lambda_{mn} \xi \\ & + c_4 \cosh \lambda_{mn} \xi) \cos n \theta \sin \omega t \end{aligned} \tag{7}$$

which gives a longitudinal deflection shape similar to that of the vibrating beam. Approximate values of λ_{mn} are now obtained by substituting Equation (7) into the appropriate boundary condition equations for a vibrating beam and solving the resulting determinant. The characteristics roots λ_{mn} obtained by the above procedure are identical to the vibrating beam characteristic roots. A tabulation of these values, for various combinations of boundary conditions, can be found in Reference (12) and in many text books (for example, References 13 and 14). The frequency parameter Ω for a given n is now obtained by substituting the real values of the beam characteristic roots λ_{mn} into Equation (4) and solving directly for Ω.

Although the method outlined is based on heuristic reasoning, its justification is that the results obtained from it are in good agreement with experimental results. As an initial check, results of the approximate frequency equation for a cylinder with clamped ends are compared in Table 1 with experimental frequencies obtained by Koval (Reference 15) and with the results of another approximate equation obtained by Arnold and Warburton (Reference 5). In general, the results

Table 1. Frequency Comparison of Experimental Results with the "Approximate" and Arnold and Warburton Theories.*

m \ n		3	4	5	6	7	8	9	10	11	12	13	14
1	Exp.	1025	700	(545) (559)	525	(587) (598)	720	885	(1090) (1100)	1310	1560	1850	2140
	Approx.	1431	872	629	565	617	739	905	1101	1323	1569	1837	2127
	A-W	1220	796	594	541	595	718	883	1079	1301	1547	1815	2105
2	Exp.	-	1620	1210	980	(838) (875)	900	995	(1135) (1145)	1365	(1555) (1600)	1865	2160
	Approx.	-	2084	1460	1118	964	949	1034	1186	1384	1616	1877	2162
	A-W	-	2086	1453	1106	952	938	1024	1178	1376	1609	1870	2156
3	Exp.	-	-	-	1650	1395	1350	(1260) (1295)	1325	(1460) (1470)	(1680) (1700)	(1900) (1930)	2210
	Approx.	-	3434	2503	1911	1551	1366	1319	1380	1520	1717	1955	2228
	A-W	-	3130	2332	1812	1490	1322	1284	1348	1489	1687	1925	2197
4	Exp.	-	-	-	-	1960	1865	-	1690	1730	1830	2020	2260
	Approx.	-	-	-	2800	2268	1928	1746	1695	1751	1888	2088	2335
	A-W	-	-	-	2845	2294	1942	1752	1697	1750	1888	2087	2334

*Frequency measured in cyc/sec

Cylinder properties: Material - steel
Radius - 3 inches
ℓ/a - 4
a/h - 300

of the present approximate theory are greater than those obtained by Arnold and Warburton but are in about as good agreement with experimental results. From Table 1 we see that the agreement between the present approximate theory and experiment becomes better as the number of circumferential waves increases, in accordance with the assumptions of the theory.

III. EXPERIMENTAL INVESTIGATION

A series of tests were performed on cylinders with one end clamped and the other free, as an additional check of the approximate method outlined in the previous section. Since a detailed discussion of the test setup and test procedure can be found in Reference 16, only a brief outline will be given here.

The two test specimens used were made of 1020 steel with dimensions as given in Table 1. The cylinders were formed over an 8-inch diameter mandrel and the seam was formed by a butt weld. The cylinders were then spun to eliminate eccentricities. One end of the cylinders was clamped in an aluminum plate which contained a trough filled with cerrobend, a low melting point alloy. The other end of the cylinder was free. The cylinder was supported by a shaft attached to the end plate as shown in Figure 1.

An electromagnet was used to excite the specimen. The test procedure consisted of varying the frequency of the electromagnet by means of an oscillator until a resonant frequency was reached. The frequency was accurately measured by an electronic counter. A microphone, which could traverse the cylinder axially and circumferentially, measured the response of the shell. The microphone output and the geometrical position of the microphone were recorded on an x-y plotter to yield a graphical plot of the longitudinal and circumferential mode shape for a given resonance frequency. A typical record is shown in Figure 2.

IV. COMPARISON OF CLAMPED-FREE CYLINDER TEST RESULTS WITH THEORY

The numerical and experimental results of the present investigation are given in Table 2. For the first longitudinal mode of vibration ($m=1$), the results of the approximate theory and of Rayleigh's inextensional theory are compared with the experimental results in Figures 3 and 4. Numerical results of the

Table 2. Tabulation of Experimental Values of Frequency Spectra, Node Locations, and Viscous Damping Coefficients.

		Mat - 1020 Steel		h - 0.010 inches		a/h = 400		a/ℓ = 0.448			
		Inextensional Theory	Experimental Data	Approximate Theory Cl.-Free	Approximate Theory S.S.-Free						
m	n	f(cyc/sec)	f(cyc/sec)	f(cyc/sec)	f(cyc/sec)	$c\mu$	S_1	S_2	S_3	S_4	S_5
1	3	46	400	422	54	1.06	0	-	-	-	-
	5	140	239	219	151	0.73	0	-	-	-	-
	7	280	304	310	296	0.75	0	-	-	-	-
	8	368	376	396	387	0.79	0	-	-	-	-
	10	577	595	610	605	1.40	0	-	-	-	-
	11	700	713	737	733	1.19	0	-	-	-	-
	12	831	844	876	872	2.03	0	-	-	-	-
	13	978	992	1027	1023	-	0	-	-	-	-
2	7		837	742	590	-	0	0.75	-	-	-
	8		693	666	561	1.22	0	0.68	-	-	-
	9		612	665	596	1.10	0	0.7	-	-	-
	12		855	931	908	-	0	0.40	-	-	-
	13		992	1071	1053	2.00	0	0.46	-	-	-
	14		1161	1228	1213	2.13	0	0.51	-	-	-
	15		1375	1399	1386	1.98	0	0.71	-	-	-
	16		1373	1584	1573	1.65	0	0.56	-	-	-
	17		1716	1782	1772	2.26	0	0.70	-	-	-
3	6		1905	2070	1850	3.00	0	0.50	0.86	-	-
	11		1081	1098	1021	1.46	0	0.44	0.85	-	-
	13		1219	1227	1182	2.17	0	0.35	0.76	-	-
	14		1314	1349	1313	1.77	0	0.39	0.85	-	-
	16		1640	1668	1642	1.83	0	0.48	0.87	-	-
	17		1816	1855	1833	2.57	0	0.45	0.84	-	-
4	6		3250	3245	3105	3.05	0	0.35	0.66	0.80	-
	7		3083	2689	2517	2.95	0	0.25	0.64	0.91	-
	8		2208	2266	2131	1.97	0	0.38	0.63	0.88	-
	12		1598	1541	1160	1.70	0	0.27	0.66	0.94	-
	14		1513	1600	1543	1.63	0	0.25	0.65	0.82	-
	15		1679	1700	1652	1.83	0	0.32	0.56	0.84	-
5	6		1060	4216	4207	2.80	0	0.23	0.45	0.60	0.80
	14		1985	1974	1910	1.88	0	0.10	0.40	0.66	0.89
	17		2184	2220	2174	1.80	0	0.15	0.40	0.62	0.89
		Mat 1020 Steel		h = 0.040 inches		a/h = 100		a/ℓ = 0.448			
1	2	68	320	910	96	1.12	0	-	-	-	-
	3	186	332	186	218	0.900	0	-	-	-	-
	4	350	402	470	387	0.860	0	-	-	-	
	5	560	559	639	605	0.485	0	-	-	-	-
	6	815	795	892	872	0.365	0	-	-	-	-
	7	1120	1081	1203	1187	0.345	0	-	-	-	-
	8	1473	1114	1565	1551	0.387	0	-	-	-	-
	9	1870	1791	1976	1963	0.38	0	-	-	-	-
	10	2310	2211	2436	2123	0.82	0	-	-	-	-
2	5		1186	1132	1167	0.965	0	0.710	-	-	-
	6		1156	1335	1168	0.88	0	0.700	-	-	-
	7		1311	1168	1361	0.300	0	0.710	-	-	-
	8		1582	2118	1675	0.550	0	0.710	-	-	-
	11		2678	3057	3017	1.12	0	0.220	-	-	-
	13		3762	4212	1157	-	0	0.110	-	-	-
	14		4350	1865	4829	-	0	0.130	-	-	-
3	2		5782	6106	6056	-	0	0.325	0.540	-	-
	3		4225	4719	4501	3.55	0	0.310	0.760	-	-
	5		2547	2850	2591	3.25	0	0.370	0.810	-	-
	6		2162	2101	2178	1.22	0	0.420	0.800	-	-
	7		2007	2227	2047	1.24	0	0.380	0.780	-	-
	8		2081	2284	2113	1.00	0	0.130	0.840	-	-
	9		2316	2518	2406	-	0	0.380	0.700	-	-
	10		2630	2877	2785	1.01	0	0.420	0.820	-	-
	12		3742	3853	3781	1.12	0	0.230	0.720	-	-
	14		5014	5082	5017			-	-	-	-
	15		5316	5777	5711	-	0	0.380	0.810	-	-
	16		6001	6523	6462	3.01	0	0.375	0.790	-	-
4	2		6883	6937	7104	3.73	0	0.225	0.525	0.890	-
	3		5713	5941	5962	3.36	0	0.230	0.140	0.790	-
	4		4766	4977	4897	3.30	0	0.175	0.550	0.780	-
	6		3377	3593	3433	3.25	0	-	-	-	-
	7		3033	3240	3077	1.37	0	0.22	0.51	0.83	-
	8		2871	3112	2957	1.18	0	0.27	0.56	0.85	-
	9		2949	3181	3044	1.00	0	0.29	0.56	0.85	-
	10		3159	3423	3297	1.38	0	0.25	0.51	0.86	-
	11		3495	3792	3678	1.36	0	0.28	0.57	0.87	-
	12		3936	4261	4156	2.01	0	0.27	0.56	0.85	-
	13		4429	4811	4712	1.71	0	0.30	0.56	0.85	-
	16		6310	6815	6756	-	0	0.28	0.53	0.87	-
5	8		1069	1012	3916	1.17	0	0.21	0.47	0.65	0.88
	10		1117	1114	1009	1.41	0	0.20	0.43	0.68	0.90
	11		3743	1126	4291	-	0	0.20	0.45	0.65	0.88
	16		6728	7288	7172	2.70	0	0.23	0.40	0.56	0.88

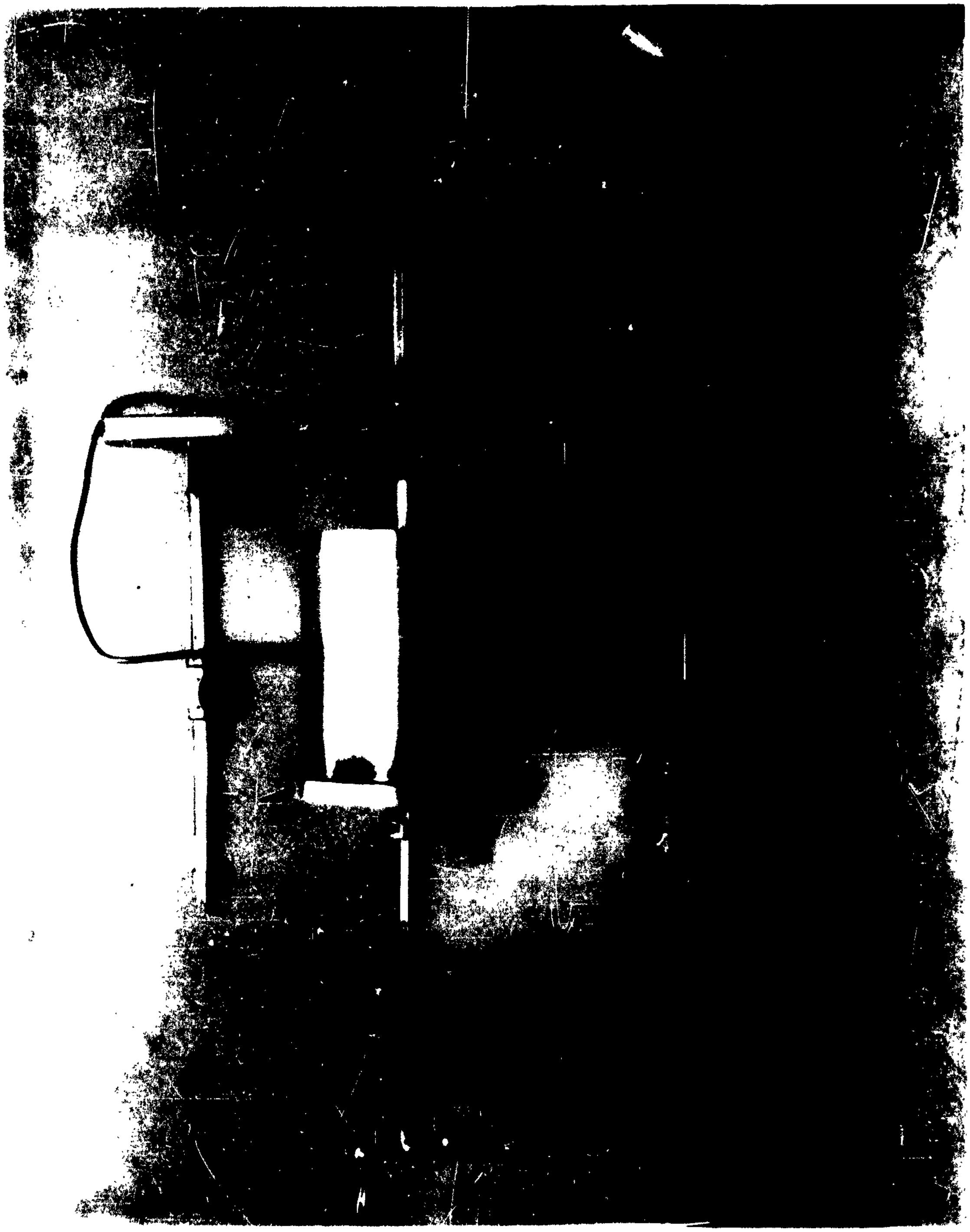

Figure 1. Over-all View of Test Setup.

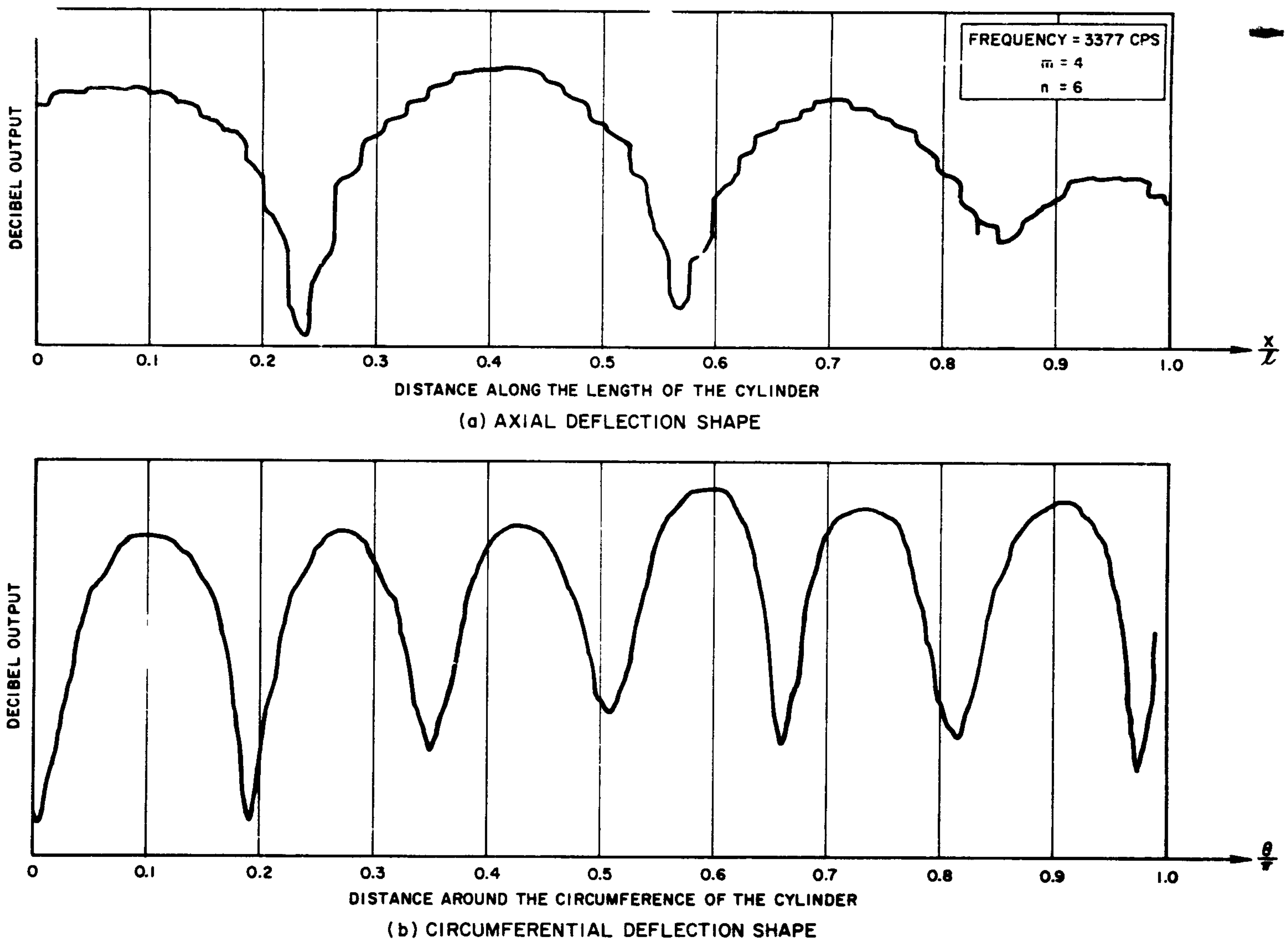

Figure 2. A Typical Mode Shape Record.

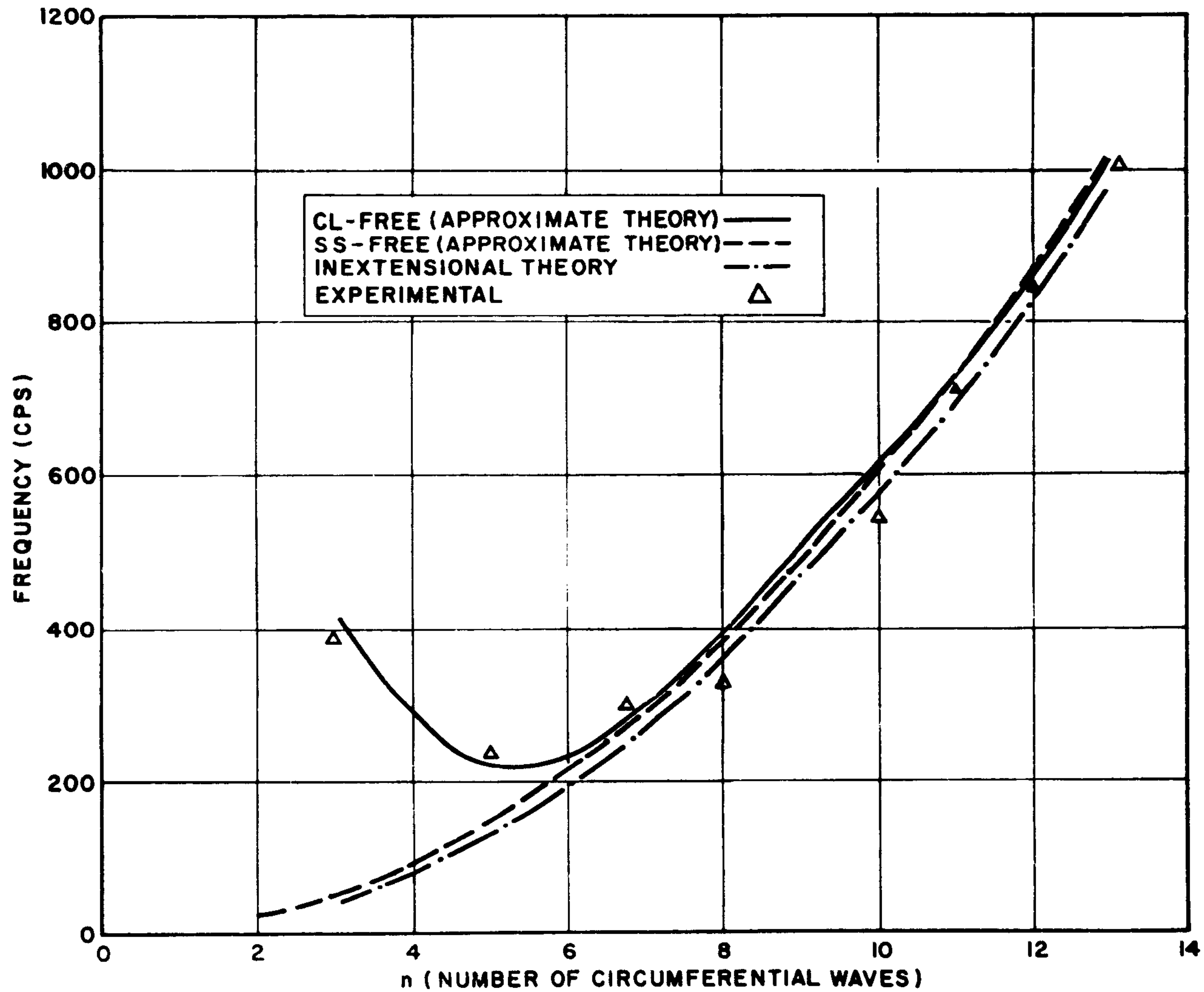

Figure 3. Graphical Comparison of Experimental Results with Theory for m = 1 of a Clamped-Free Shell (a/h = 400).

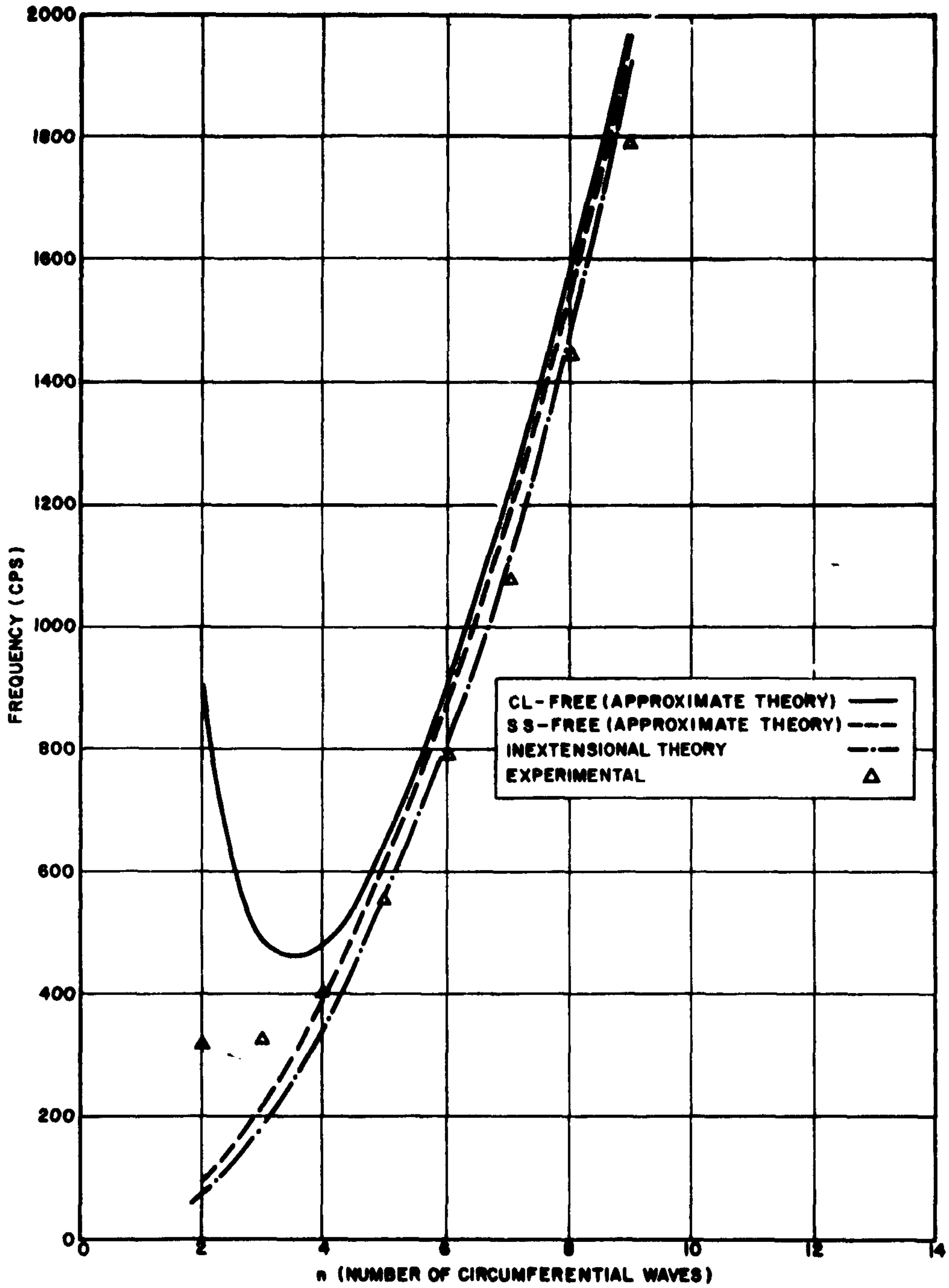

Figure 4. Graphical Comparison of Experimental Results with Theory for m = 1 of a Clamped-Free Shell (a/h = 100).

approximate theory for both a simply supported-free cylinder and a clamped-free cylinder are plotted. All the theoretical curves are very close to each other for the larger values of n. It is interesting to note that the experimental values follow the approximate curve for the clamped-free cylinder very closely for all values of n for the cylinder with a radius thickness ratio of 400. The experimental results for the cylinder with a radius thickness ratio of 100, on the other hand, fall between the theoretical curves for the clamped-free cylinder and the simply supported-free cylinder for low values of n ($n \leq 4$). The reason for this discrepancy is suspected to be imperfect clamping and is being investigated further by means of additional tests and by a more accurate theory.

A comparison, in Figures 5 and 6, of the results of the approximate equation with experimental results for $m \geq 2$ indicates good agreement. The results also indicate that the value of n at which the minimum frequency occurs depends upon the axial wave length. As m increases, the value of n corresponding to the minimum frequency also increases. This phenomenon was first noted by Arnold and Warburton for cylinders that were simply supported or clamped at both ends.

The position of the experimentally determined axial node locations are designated as S_i in Table 2. The position of clamped-free and simply supported-free beam node points are tabulated in Table 3. The variation of the position of the experimental node points for different values of n is greatest at $m = 2$ and decreases as the number of axial waves increases. The tabulated results of the average experimental axial node positions correspond fairly well with the beam node positions but do not coincide exactly, a result probably due to the fact that the beam functions are not exact solutions of the equations for the vibrations of cylindrical shells.

V. CONCLUSIONS

Experimental and theoretical results for clamped-free and clamped-clamped cylinders are in good agreement for larger values of n (say, $n > 4$). It appears therefore, that the approximate frequency equation can be used for arbitrary boundary conditions in this region. For $n > 4$ and $m = 1$, the Rayleigh inextensional theory gives reasonable results for the clamped-free cylinders. The

Table 3. Position of Clamped-Free and Simply Supported-Free Beam Node Points.

m	S_1	S_2	S_3	S_4	S_5
1	0	-	-	-	-
2	0	0.73/0.79	-	-	-
3	0	0.45/0.51	0.85/0.87	-	-
4	0	0.31/0.35	0.61/0.65	0.89/0.91	-
5	0	0.23/0.27	0.47/0.49	0.71/0.73	0.93/0.93

Key = S.S.-Free/Cl-Free

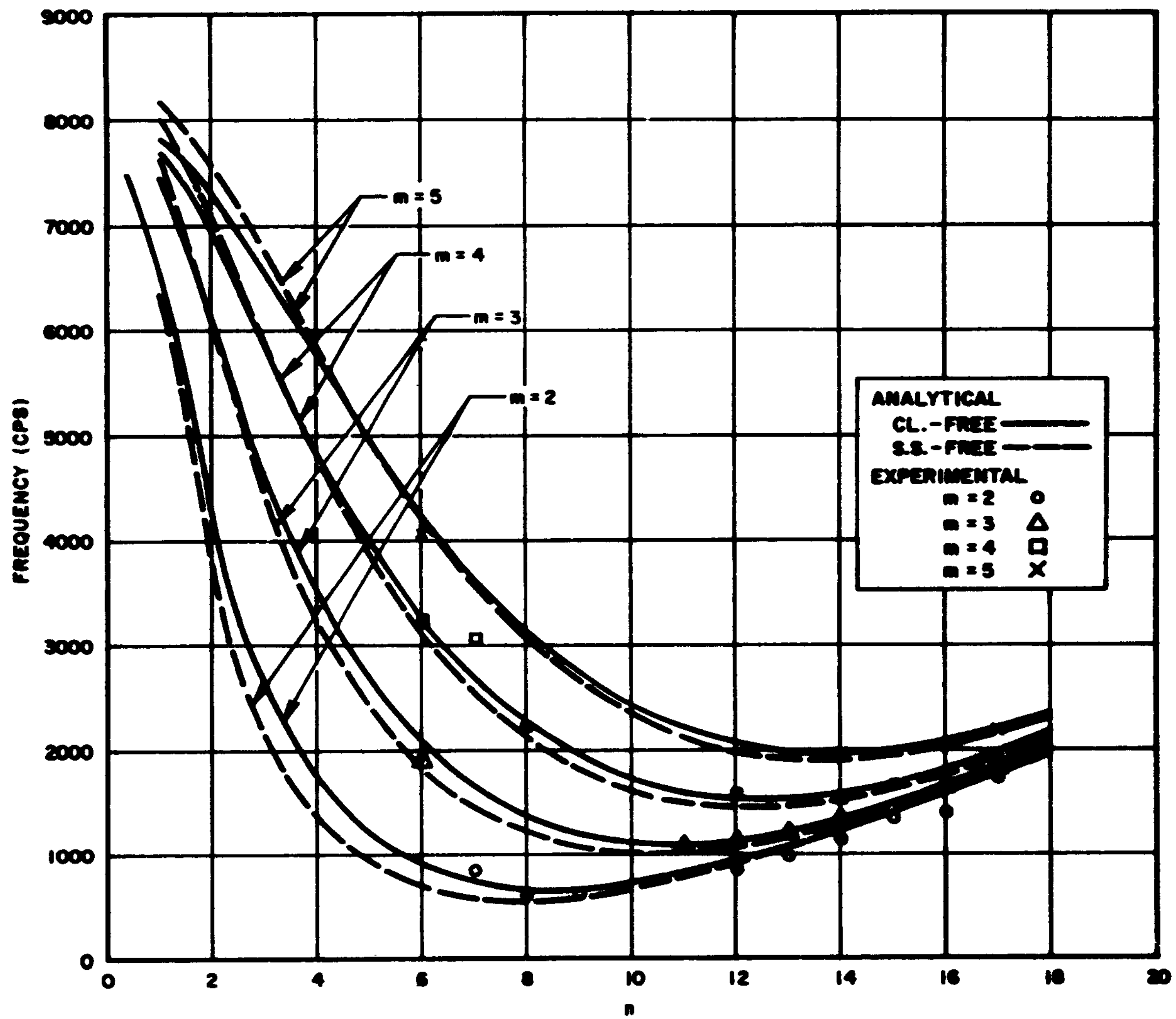

Figure 5. Graphical Comparison of Experimental Results with Theory for m > 1 of a Clamped-Free Shell (a/h = 400).

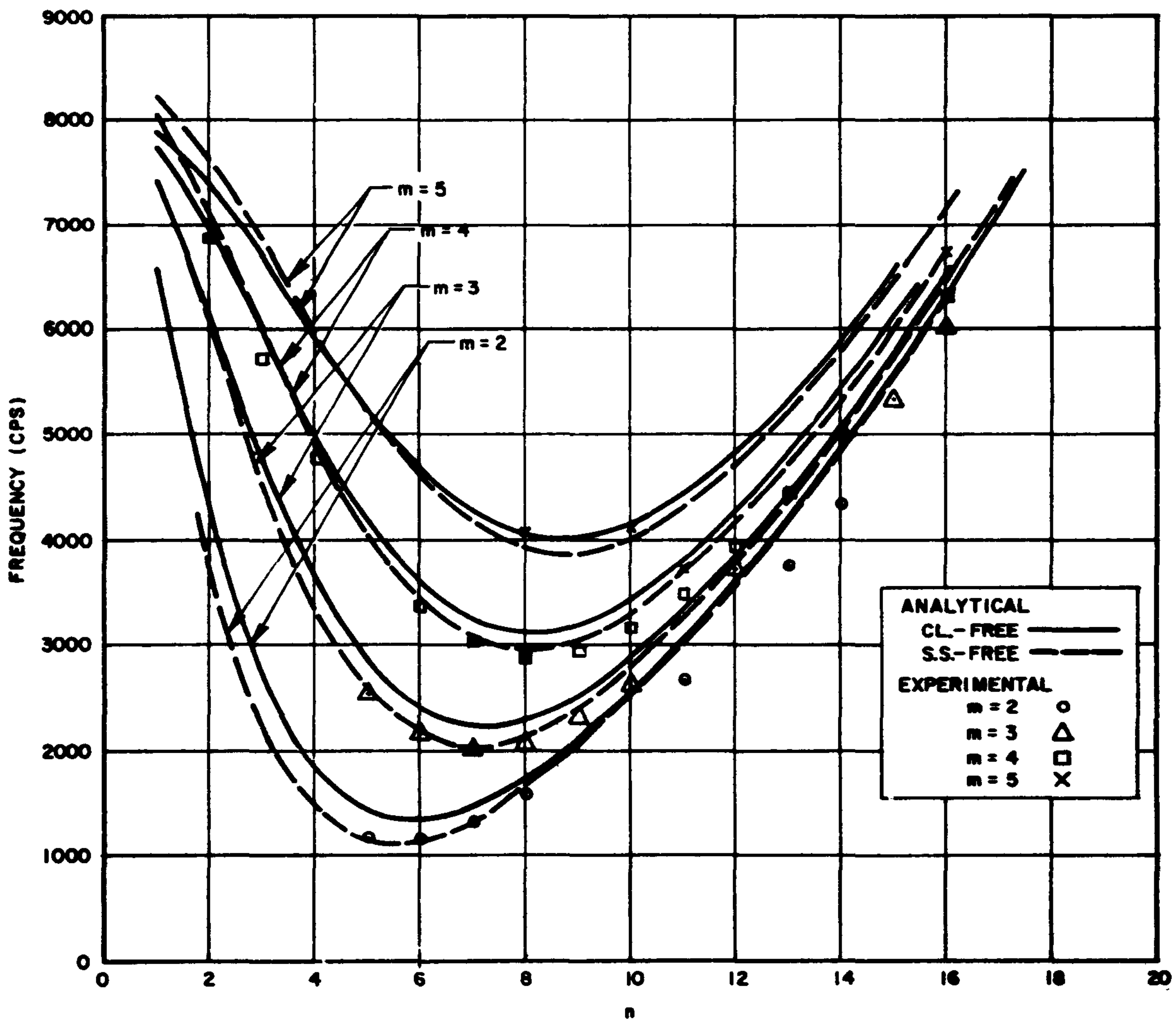

Figure 6. Graphical Comparison of Experimental Results with Theory for m > 1 of a Clamped-Free Shell (a/h = 100).

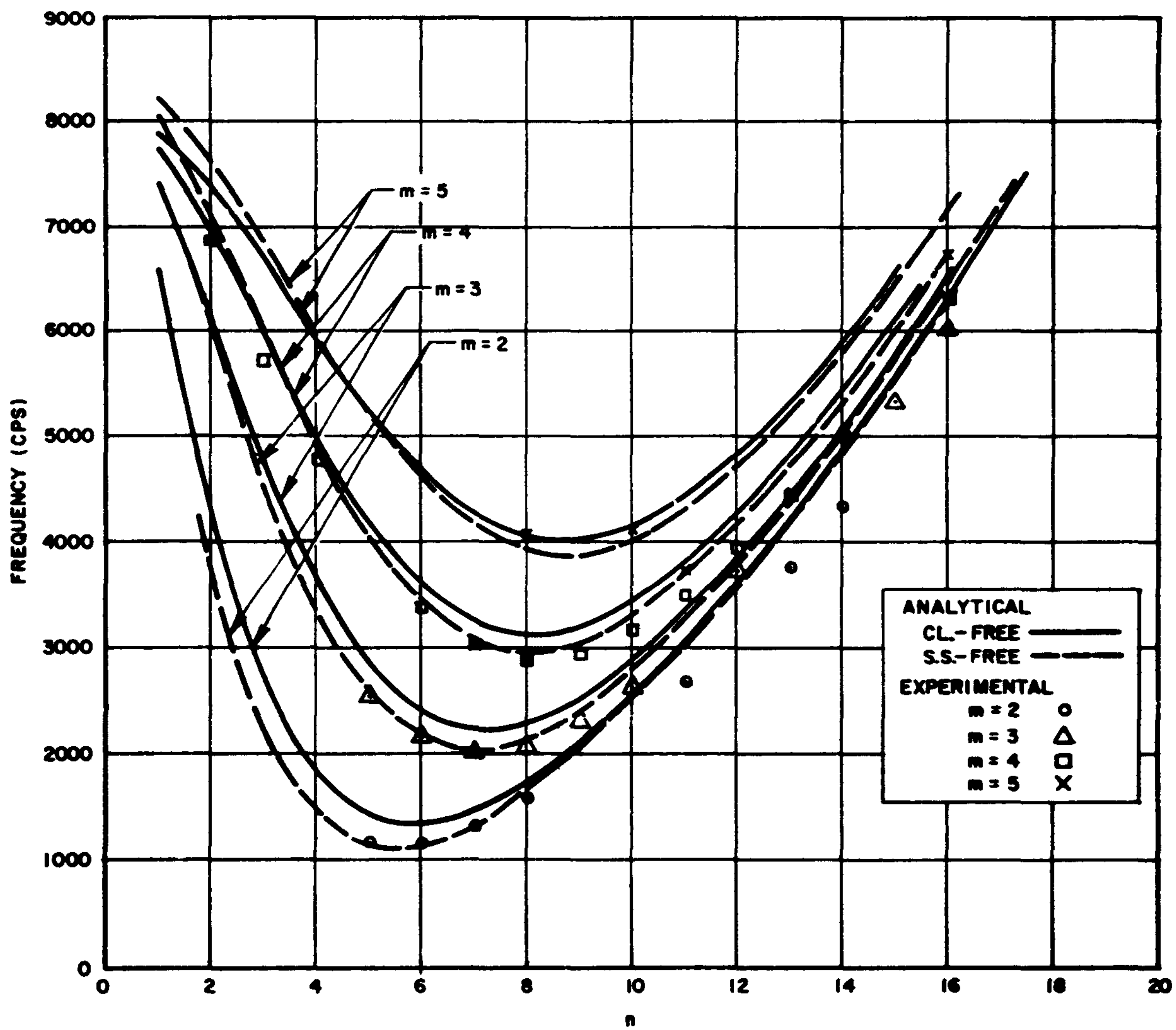

Figure 6. Graphical Comparison of Experimental Results with Theory for m > 1 of a Clamped-Free Shell (a/h = 100).

experiments also indicate that although the beam functions may not be the true deflection shape, they are near enough to the true shape so that when used in conjunction with the frequency equation they give a very close approximation to the experimental data. Some anomalies in the agreement between theory and experiment were obtained for modes characterized by $n < 4$. These are being investigated further at the present time.

VI. ACKNOWLEDGEMENT

The author is indebted to Dr. Paul Seide for his advice; and to Mr. Anthony DiGiacomo for performing the experimental investigation.

APPENDIX

STRUCTURAL DAMPING

In addition to the frequency spectra, over-all structural damping coefficients were experimentally obtained. These coefficients were obtained by passing the response signal coming from the microphone through an ac-dc log converter and recording the output on an oscilloscope camera. The output was calibrated by a db meter and the decay curves obtained by disconnecting the electromagnet from the circuit while the cylinder was at resonance. A typical decay curve appears in Figure 7.

The equation of motion of the free vibration of an elastic system (using the same notation as given in Reference 17 can be written as:

$$\ddot{\phi}_n + c_n \dot{\phi}_n + \omega_n^2 \phi_n = 0 \quad (c_n = 2\pi f'_n g)$$

where

- g structural damping coefficient
- $f'g$ viscous damping coefficient
- ϕ_n amplitude of n^{th} mode
- ω_n natural frequency corresponding to ϕ_n

Tabulated values of $f'g$ as a function of mode shape and frequency are given in Table 2. A plot of the "viscous" damping coefficient as a function of the number of circumferential waves is shown in Figure 8. The results indicate a large scatter with $f'g = 2$ being an average value for both cylinders. It is interesting to note that an average value of two for the viscous damping coefficient is close to that found by the author in Reference 16 for a steel cylinder clamped on both ends. Fung, Sechler and Kaplan obtained an average value of $f'g = 6$ for a set of aluminum cylinders. Figure 8 also shows that for $n < 8$, the $m = 1$ and $m = 2$ modes have an average value of one for $f'g$ and the $m = 4$ and $m = 5$ modes an average value of three. The $m = 3$ modes seem to fluctuate between one and three. The peaking effect at a unique frequency found by the author in Reference 16 did not occur in the present investigation. It is clear that much work remains to be done before the structural damping phenomenon is completely understood.

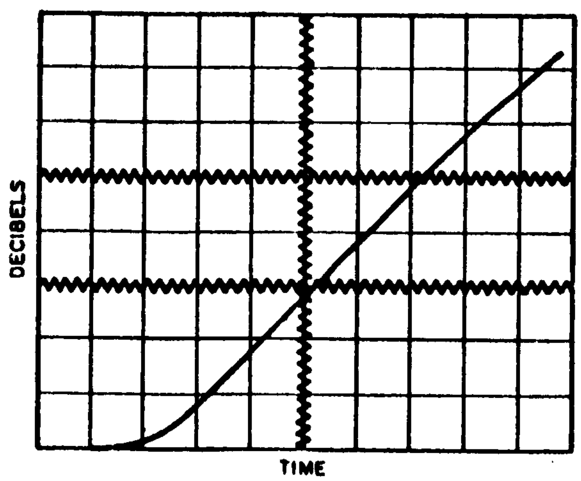

Figure 7. Typical Decay Curve.

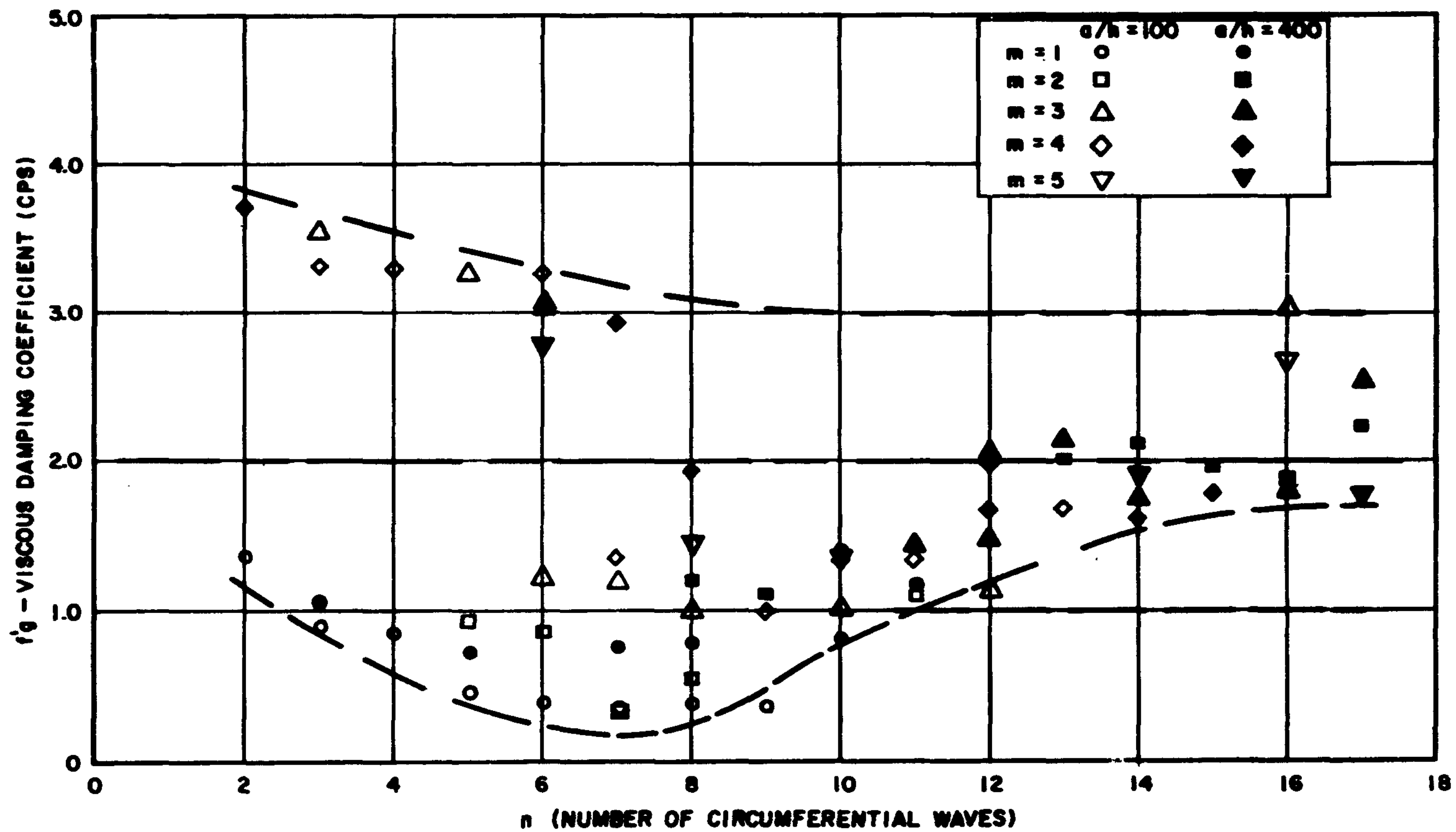

Figure 8. Viscous Damping Coefficient as a Function of Number of Circumferential Waves.

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Unclassified Report

The free vibrations of finite cylindrical shells are investigated. With the aid of a number of simplifying assumptions, a frequency equation based on the known characteristic functions for beams with any combination of boundary conditions is obtained. Experimental results for frequency spectra and mode shapes of a cylinder fixed on one edge and free on the other are in good agreement with both Rayleigh's inextensional theory and the approximate frequency equation. Structural damping coefficients obtained for the test cylinders are compared with those of previous investigations.

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