AD NUMBER

AD404829

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JAN 1963. Other requests shall be referred to Office of Naval Research, One Liberty Center, 875 North Randolph Street, Arlington, VA 22203-1995.

AUTHORITY

ONR ltr, 28 Jul 1977

SEL-62-142

A Theoretical Study of Hot Electrons in Metal Films

C. A. Bates*

January 1963

Technical Report No. 216-1

Prepared under Office of Naval Research Contract
Nonr 225(24), NR 373 360
Jointly supported by the U.S. Army Signal Corps, the
U.S. Air Force, and the U.S. Navy (Office of Naval Research)

* Visiting scholar from the University of Nottingham, England

ELECTRON DEVICES LABORATORY
STANFORD ELECTRONICS LABORATORIES
STANFORD UNIVERSITY · STANFORD, CALIFORNIA

ASTIA AVAILABILITY NOTICE

Qualified requesters may obtain copies of this report from ASTIA. Foreign announcement and dissemination of this report by ASTIA is limited.

SEL-62-142

A THEORETICAL STUDY OF HOT ELECTRONS IN METAL FILMS

by

C. A. Bates

(Visiting scholar from the University of Nottingham, England)

January 1963

Technical Report No. 216-1

Prepared under
Office of Naval Research Contract Nonr-225(24), NR 373 360
Jointly supported by the U.S. Army Signal Corps,
the U.S. Air Force, and the U.S. Navy
(Office of Naval Research)

Electron Devices Laboratory
Stanford Electronics Laboratories
Stanford University   Stanford, California

ABSTRACT

An analysis of the motion of hot electrons when passing through a thin metal film is given, with particular reference to the cold-cathode emitter and photoelectric-type devices. Electron-electron collisions are considered to be responsible for the slowing down processes within the film and sufficient collisions are assumed to occur so that the motion is diffusive. The differential-scattering cross section is assumed to be almost spherically symmetrical in the center-of-mass system of coordinates, and thus the equations deduced by Wolff (which include electron multiplication) may be used. Methods of solving the integrodifferential equation are given. The effects of the Exclusion Principle are incorporated and a partial-differential equation, involving the electron-electron mean free path and the energy loss per collision as energy-dependent parameters, is deduced. This equation is solved in detail using boundary conditions which allow for a large percentage ($\sim$ 90 percent) of the hot electrons to be reflected from the film surfaces. It is found that, for diffusive motion, the total current emitted is approximately proportional to the inverse of the film thickness.

An analysis of electron tunneling through an insulator is also made in relation to the cold-cathode emitter.

CONTENTS

TABLES

ILLUSTRATIONS

ACKNOWLEDGMENT

The author wishes to thank Prof. H. Heffner, who originally suggested the topic, for numerous stimulating discussions. In addition, he also benefited from several talks with Prof. J. L. Moll and Dr. J. J. Quinn.

I. INTRODUCTION

The expression "hot" electron is used to describe those electrons present in a solid which have an energy much above that of the average electron present in the material and are thus not in thermal equilibrium with the lattice. In a metal the energy may be about twice that associated with those electrons at the Fermi level, whereas in a semiconductor their energies would be larger than that corresponding to the lower limit of the conduction band. The properties of hot electrons differ from those of ordinary electrons in the solid, as they have a much shorter lifetime and undergo different energy-loss mechanisms. Any description of their motion within a material is necessarily different from that of other electrons.

There has been considerable interest recently in the cold-cathode emitter as a device in which hot electrons are injected into a thin metal film and allowed to diffuse across the film. The theoretical problems involved are very complicated and the present report contains the results of investigations aimed at understanding more fully the physical principles inherent in such devices. The approach is phenomenological and no attempt will be made to derive the parameters from first principles. In addition, the description to be used is classical in several instances, although quantum mechanical modifications are fully discussed.

Hot electrons may be injected into a film using either electrical or photoheating methods. Technically the latter method is more straightforward but one advantage of the electrical heating method is that hotter electrons may be produced. Chapter II describes the tunnel emission of the electrons into the film through a triangular-shaped potential barrier presented by the insulator, and also describes the geometry of the cold-cathode device. A brief survey of the motion of hot electrons in metal films is given in Chapter III and the problems present are outlined. In addition, a summary of other recent theoretical discussions is given. Chapter IV is largely concerned with the background to the central theme of the present investigation, namely, the diffusive motion of the hot electrons through a metal film. It is applied to the problem of photoelectrons generated within the metal film (Chapter V) and followed by considering the injection of hot electrons into the film (Chapter VI). The diffusion equation is solved taking into account reflections at the film surface. Interpretation of the calculation for the electron yield is given in Chapter VII and is discussed in relation to experimental work.

II. GENERATION OF HOT ELECTRONS IN A COLD-CATHODE DEVICE

Consider a sandwich composed of a thin dielectric film of thickness a (~ 100 A) deposited on a base metal with another thin metal film of thickness b (~ a) deposited on the dielectric. With no potential applied, the energy band structure and Fermi level may be sketched as shown in Fig. 1. On assuming infinite extensions of the films in the y-z plane,

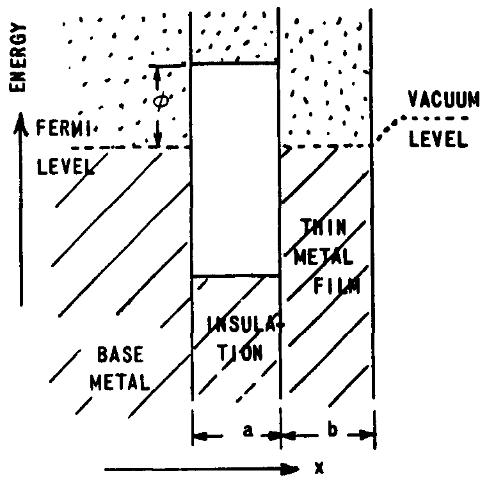


FIG. 1. BAND STRUCTURE OF THE COLD-CATHODE EMITTER.

application of a positive potential to the outer metal film alters the band structure to that shown in Fig. 2. If the two films are sufficiently thin, electrons will tunnel across the dielectric and proceed through the metal film without appreciable loss in energy. Since many of the electrons will arrive at the surface with sufficient energy to overcome the work function, they will be emitted into the vacuum region beyond and collected. An investigation of this problem falls mainly into two categories: the first concerns the theory of electron tunneling through an insulator in the presence of an electric field and the second describes the motion of the hot electrons through the metal film. A discussion of tunneling is given in the present chapter, and succeeding chapters analyze the hot-electron diffusive motion.

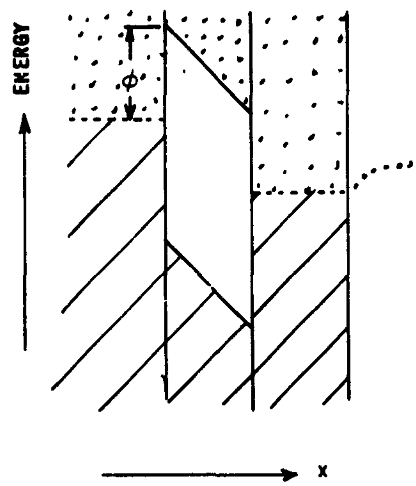


FIG. 2. BAND STRUCTURE OF THE COLD-CATHODE EMITTER ON APPLICATION OF A POSITIVE POTENTIAL TO THE METAL FILM.

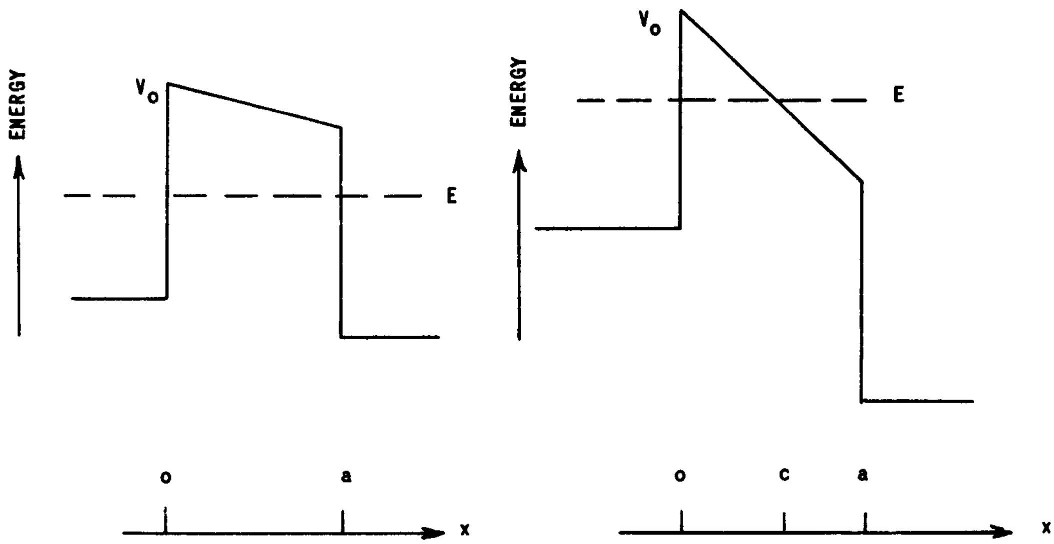
A. THE ELECTRIC TUNNEL EFFECT

An investigation of quantum mechanical tunneling of electrons through thin layers of insulating materials has been reported by Fisher and Giaever [Ref. 1], and explained using the theory of Holm [Ref. 2]. The latter publication presents a theory for the tunneling process which is applicable to all values of applied electric fields existing across the insulating layer. Previous theories were directed only at very weak or very strong fields. The number of electrons tunneling across the insulator, and hence, its effective resistance, was found by multiplying the approximate transmission coefficient quoted by Rojansky [Ref. 3] by an appropriate energy-distribution function and integrating over all available electron energies. (The Theory of tunneling as presented by Kane [Ref. 4] is not applicable here, as such calculations refer to tunneling through forbidden regions of K-space and not real space.)

The aim of the present calculation is to derive the spectrum of the hot electrons incident on the metal film. This depends mainly on the density-of-states curve N(E) for the base metal, the applied electric field, and the energy lost by the electrons when traveling through the conduction band of the insulator. As the formulas quoted by Holm are not readily applicable to such a calculation, it was decided to examine the problems involved from first principles. The probability that a single electron of definite kinetic energy penetrates the potential barrier is calculated by solving the Schrödinger Equation for different values of applied field. This expression is then modified by N(E) and the energy-lost parameters for the insulator in order to calculate $\mu(E')$.

1. <u>The Theory of Tunneling</u>

If it is assumed that the resulting electric field $\mathcal{E}$ in the insulator is uniform, the shape of the potential barrier is shown in Fig. 3. When an electron enters the insulator at $x = 0$, it has an energy $E = mv_x^2/2$ where v_x is the x-component of velocity and is accelerated across the film by the applied potential $\mathcal{V}$. However, an alternative and more useful description is obtained by assuming that the



(a) Weak and intermediate electric fields

(b) Strong fields

FIG. 3. POTENTIAL BARRIER THROUGH WHICH AN ELECTRON HAVING KINETIC ENERGY E PASSES.

electron has energy E throughout the film, but is moving in a potential V given by

$$V = V_o - \frac{\hbar^2}{2m} \lambda x \tag{2.1}$$

where V_o is a constant as shown, $\lambda = \mathcal{E}(2em/\hbar^2) = (\mathcal{V}/a)(2em/\hbar^2)$. Assuming that the electron suffers no loss in energy in this region, the classical Hamiltonian is given by:

$$\mathcal{H} = \frac{1}{2} mv_x^2 + V_o - \frac{\hbar^2}{2m} \lambda x \tag{2.2}$$

Thus, the corresponding Schrödinger equation for a wave function ϕ describing the motion of an electron having eigenvalue E_o is:

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + \left(V_o - \frac{\hbar^2}{2m}\lambda x\right)\phi = E_o\phi \tag{2.3}$$

Putting

$$\alpha = \sqrt{\frac{2m(V_o - E_o)}{\hbar^2}}, \tag{2.4}$$

Equation (2.3) reduces to:

$$\frac{d^2\phi}{dx^2} = (\alpha^2 - \lambda x)\phi \tag{2.5}$$

The solution to (2.5), given in series by Forsyth [Ref. 5] and convergent for finite x, is:

$$\phi = a_o(1 + Q^2B + Q^2BQ^2B + \dots) + a_1(x + Q^2Bx + Q^2BQ^2Bx \dots)$$

where a_o and a_1 are arbitrary constants; $B = (\alpha^2 - \lambda x)$; and Q denotes the operation of integrating from 0 to x all terms succeeding it. Each term in each series may be obtained from its predecessor by multiplying by B and then integrating twice in succession from 0 to x. Substituting for B gives

$$\phi = a_o\left(1 + \frac{\alpha^2x^2}{2!} + \frac{\alpha^4x^4}{4!} + \frac{\alpha^6x^6}{6!} + \dots\right) + a_1\left(x + \frac{\alpha^2x^3}{3!} + \frac{\alpha^4x^5}{5!} + \dots\right)$$

$$+ a_o\left(-\frac{\lambda x^3}{3!} + \frac{4\lambda^2x^6}{6!} - \frac{28\lambda^3x^9}{9!} + \dots\right) + a_1\left(-\frac{2\lambda x^4}{4!} + \frac{10\lambda^2x^7}{7!} - \frac{80\lambda^3x^{10}}{10!} + \dots\right)$$

$$= \left(\frac{a_o}{2} - \frac{a_1}{2\alpha}\right)e^{-\alpha x} + \left(\frac{a_o}{2} + \frac{a_1}{2\alpha}\right)e^{\alpha x} - a_o\left(-\frac{\lambda x^3}{3!} + \dots\right) + a_1\left(-\frac{2\lambda x^4}{4!} + \dots\right) \tag{2.6}$$

Before continuing, it may be noted that Eq. (2.6) is often called the Airy Equation. It is discussed in several texts where the term involving a_0 is called the first Airy function and that involving a_1 is called the second Airy function. Extensive mathematical tables of the functions exist for wide ranges of ϕ (see Smirnov [Ref. 6], for example). As we are primarily concerned with the case of an electron traveling across the film in a positive x-direction, we set

$$a_0 + \frac{a_1}{\alpha} = 0 \tag{2.7}$$

Thus, the wave function for such an electron becomes:

$$\phi(+x) = a_0 \left[e^{-\alpha x} + \left\{ \left(-\frac{\lambda x^3}{3!} + \frac{4\lambda^2 x^6}{6!} - \frac{28\lambda^3 x^9}{9!} \ldots \right) \right.\right.$$

$$\left.\left. -\alpha \left(\frac{2\lambda x^4}{4!} + \frac{10\lambda^2 x^7}{7!} - \frac{80\lambda^3 x^{10}}{10!} \ldots \right) \right\} \right] \tag{2.8}$$

representing an exponentially damped wave on which is superimposed a ripple { }.

In order to obtain an estimate of the importance of the exponential decay of ϕ compared to the ripple, corresponding terms in the series expansion of $e^{-\alpha x}$ have been compared with these in the ripple. For example, the coefficient of x^3 in $e^{-\alpha x}$ is $(-\alpha^3 a_0/3!)$ and in the ripple it is $(-a_0 \lambda/3)$. Their ratio (α^3/λ) is proportional to $(V_0 - E_0)^{3/2}/e\mathcal{V}$ and is always much greater than one in the case of weak or intermediate fields as shown in Fig. 3a. Similarly, ratios of other terms are of the same order, and thus $e^{-\alpha x}$ dominates ϕ. As $\mathcal{V}$ is increased, the importance of the ripple increases.

Consider the case of a very strong field as shown in Fig. 3b. At $x = c$, $V_0 - \hbar^2/2m\lambda c = E_0$ (i.e., $c = \alpha^2/\lambda$), admitting a solution

$$\phi(\pm x) = a_0 \pm a_1 x \tag{2.9}$$

For $c \leq x \leq a$, Eq. (2.9) still applies, but the contribution from the ripple now dominates ϕ. This is to be expected as the electron can exist in this region on classical grounds and the problem is similar to that of a free particle having a sinusoidal type of wave function. In this region, the probability of an electron tunneling through the potential barrier is independent of barrier width (apart from small secondary effects relating to the nonuniform potential).

For small values of $\mathcal{V}$ or E, it may be possible to consider only the first term of each series in the ripple. However, the available values for E are limited by the physical properties of the metal and insulator (e.g., the energy gap of the insulator and the separation in energy between the Fermi level of the metal and the conduction band of the insulator). Neglect of each second term in the ripple series is possible to within 10 percent if $\mathcal{V}a^2 \sim 10^{-11}$ v m^2. The relevant values of $\mathcal{V}$ and of a that satisfy these conditions are given below:

$\mathcal{V}$ (volts)	10	1	0.1	0.01
a (angstroms)	1	3.3	10	33

In examples of practical importance relating to the cold-cathode emitter, it is thus necessary to consider many terms in the series for both very weak or very strong fields.

One of the objects of the calculation was to determine whether or not it is possible to substitute a potential barrier having a sloping top with either another square barrier of a different constant potential less than V_0 and width a, or a square barrier of constant potential equal to V_0 but width greater than a. No satisfactory results have so far been obtained in either case, due mainly to the complex nature of the ripple.

B. THE TRANSMISSION COEFFICIENT

Cold-cathode emitters are normally designed in such a way that they correspond to the case of strong fields as shown in Fig. 3b. The probability that an electron, initially found in the base metal, tunnels through the barrier and emerges into the conduction band of the insulator at x = c is given by:

$$T_1 = \frac{[\phi^*_{(+x)}\ \phi_{(+x)}]_{x=c}}{[\phi^*_{(+x)}\ \phi_{(+x)}]_{x=0}}$$

The reverse process of electrons tunneling from the conduction band back into the base metal may also occur; thus we define a probability:

$$T_2 = \frac{[\phi^*_{(-x)}\ \phi_{(-x)}]_{x=0}}{[\phi^*_{(-x)}\ \phi_{(-x)}]_{x=c}}$$

The total transmission coefficient T for electrons to leave the base metal for the insulator is thus $(T_1 - T_2)$. However, T_2 may be neglected since there is only the conduction band of the insulator, and thus T_1 represents the total transmission coefficient. From (2.9),

$$[\phi^*_{(+x)}\ \phi_{(+x)}]_{x=c} = [a_o + a_1 c]^2$$

$$= a_o^2(1 - \sqrt{c\lambda} \cdot c)^2$$

using (2.4) and (2.7). Now $[\phi^*_{(+x)}\ \phi_{(+x)}]_{x=0} = a_o^2$ and thus

$$T = (1 - \sqrt{c^3\lambda})^2 = \left[1 - \left(\frac{a}{U e}\right)\sqrt{\frac{2m(V_o - E_o)^3}{\hbar^2}}\right] \qquad (2.10)$$

This shows that the transmission coefficient depends entirely on $(V_o - E)$, and the applied electric field $\mathcal{E}(=U/a)$. While $\mathcal{E}$ and V_o are constant for all electrons, T depends on the energy of the electron considered in a very complicated way. In the above analysis, a potential barrier with sharp edges has been assumed. This is not strictly true, due to the effect of the image force and the rounding off of the edges, which results have been discussed by Sommerfeld and Bethe [Ref. 7].

For $x > c$, the electrons move in the conduction band of the insulator until they reach the plane $x = a$. Suppose a diffusion length L for these electrons is defined by the equation

$$n_1 = n_o \exp\left(- \frac{a - c}{L}\right) \qquad (2.11)$$

where n_1 is the number of electrons in an energy range E to $(E + dE)$ traveling in the positive x direction at the plane $x = a$, and n_o is the number in the same energy range traveling in the same direction at $x = c$. The parameter L allows a certain fraction of the electrons to be scattered from the beam in a simple phenomenological way. The physical principles involved are quite complex--in fact, the problem is almost analogous to the one to be analyzed later concerning the diffusive motion of electrons in metal films. However, as $(a - c)$ is generally considerably smaller than b, the thickness of the metal film, it may be anticipated that the error involved in using (2.11) is likely to be small.

C. HOT-ELECTRON SPECTRUM, $\mu(E')$

The hot-electron energy spectrum incident on the metal film is then of the form:

$$\mu(E') = \begin{bmatrix}\text{Spectrum} \\ \text{at } x = 0\end{bmatrix} \times \begin{bmatrix}\text{Transmission coefficient} \\ \text{through potential barrier}\end{bmatrix} \times \begin{bmatrix}\text{Transmission coefficient} \\ \text{through conduction band} \\ \text{of insulator}\end{bmatrix}$$

$$= N(E')\ F(E', T) \left[1 - \left(\frac{a}{U_e}\right) \sqrt{\frac{2m(V_o - E')^3}{\hbar^2}}\right]^2 \exp\left(- \frac{b - c}{L}\right) \qquad (2.12)$$

where all energies are now measured with respect to an origin at the bottom of the conduction band of the metal film, and $F(E',T)$ is the Fermi function at the temperature T, defined by:

$$F(E'', T) = \frac{1}{\left[\exp\left(- \frac{E'' - E_F}{kT}\right) + 1\right]} \qquad (2.13)$$

The electrons enter the thin metal film with an energy up to eU above the Fermi level. They then proceed through the film and arrive at the surface where those with a perpendicular energy component greater than the electron affinity of the metal can escape. As these electrons are not in thermal equilibrium with the lattice, the formal method of treating electronic conduction in metals is no longer applicable and thus it is necessary to make a detailed analysis of this motion.

III. HOT ELECTRONS IN THIN METAL FILMS

A. INTRODUCTION

The first results concerning hot electrons were obtained by Ryder and Shockley [Ref. 8] and Ryder [Ref. 9] for semiconductors, and the observed deviations from Ohm's law were explained by Shockley [Ref. 10]. Recently, a detailed study of hot-electron emission from silicon p-n junctions was made by Moll, Meyer, and Bartelink [Ref. 11]. On the other hand, little is known about the properties of hot electrons in metals, although tunnel emission of electrons into a thin metal film has been reported by Mead [Ref. 12], Giaever [Ref. 13], and Nicol, Shapiro, and Smith [Ref. 14]. References 13 and 14 largely concern superconductors. The first publication specifically dealing with hot electrons in metal films was by Spratt, Schwarz, and Kane [Ref. 15], but the interpretation of their results has recently been criticized by Hall [Ref. 16]. Mead [Ref. 17] investigated the transport of hot electrons in gold films using tunnel injection and vacuum collection; and Spitzer, Crowell, and Atalla [Ref. 18] obtained values for the mean free paths in gold using photoheating and internal collection. At the present time, several other groups are undertaking experiments using various techniques for production and collection, and it is hoped that the results of such experiments will soon appear.

A theoretical investigation into the problem involves analyzing all the possible mechanisms responsible for abstracting energy from the hot electrons. Generally speaking, these are:

1. Electron-electron collisions (elastic, inelastic, and plasma oscillations),
2. Electron-phonon collisions (acoustical, optical, and polar modes),
3. Electron-lattice imperfections (point, line, and surface defects), and
4. Size and surface effects of the film.

In a typical metal film (gold or aluminum, for example), the most likely mechanisms involve electron-accoustical phonon collisions (optical phonon losses will only be present in metals having more than one atom per unit cell) and electron-electron collisions. The effect of having a very thin film rather than a large piece of metal, as suggested in item 4, influences the energy-loss mechanisms indirectly. It limits the mean free path for the various interactions and modifies the electronic band structure and vibrational spectrum in a manner described by Blatt [Ref. 19]. The occupied states in k-space and the allowed vibrational modes in q-space lie on infinitely thin sheets (in the y-z plane) separated by $2\pi/Na_0$, where N is the number of unit cells across the film and a_0 is the lattice parameter. (The surface atoms of the films are not in the same environment as they would be in a block of metal of infinite size, and hence the sheets are slightly displaced.) At low temperatures, therefore, transitions involving changes in k_x and q_x are forbidden. For films of the size considered here, this effect is so small that it may be neglected.

In order that an accurate theoretical analysis of the problem may be given, it is necessary to examine several points from an experimental angle:

1. How does energy loss depend on temperature? (This indicates the importance of mechanism 2, since mechanism 1 is almost temperature independent.)
2. How does energy loss depend on applied voltage, and thus hot electron energy? (Electron-electron interaction is altered since the scattering function is energy dependent. In addition, the possibility of plasma excitation occurs at the higher energy ranges.)
3. How does the energy loss depend on thickness? (The type of motion experienced by the electron depends on the number of collisions suffered by the electron.)

The implications of these points will now be considered.

B. THEORY

An examination of experimental results reveals that elastic electron-electron collisions are largely responsible for the slowing down of the hot electrons in the metal film since the electron yield is almost temperature independent. This may be verified also by considering the geometry of the film in relation to the incoming hot-electron beam. The electron-phonon collision mean free path will be almost the same as that in the bulk material. Typical figures for the latter, obtained from Dekker [Ref. 20], show that the mean free path for metals is several hundred angstroms; for gold, the figure is 570 A. The total effective mean free path λ_t for hot electrons is of the form:

$$\frac{1}{\lambda_t} = \sum \frac{1}{\lambda_i} \tag{3.1}$$

where λ_i are the individual mean free paths for the various possible collisions. Using the above figures, Eq. (3.1) shows that little error is involved when electron-electron collisions only are considered.

Several important factors must be examined. Whether the motion is "ballistic" or "diffusive" is largely determined by the following:

1. The incoming hot-electron energy spectrum,
2. The film thickness,
3. The mean free path between--and the energy loss occurring at--each electron-electron collision, and
4. The probability of escape at the rear surface.

Essentially the motion is ballistic when the energy of the incident electrons is only slightly greater than the effective work function; when the film is relatively thin compared to the mean free path; and when a large fraction of energy is lost with each collision. As soon as the electron suffers a collision, it is removed from the electron beam and so there is no need to consider how the electron random walks through the crystal. The emitted electron current I depends on the film thickness b and on the incoming electron current I_o according to the equation:

$$I = I_o \, e^{-b/\Lambda} \tag{3.2}$$

where Λ is the diffusion range.

Diffusive motion occurs under the following conditions: 1) when the incoming hot-electron energy is high; 2) when the film is fairly thick so that the electron suffers several collisions within the film; and 3) when the electrons lose a relatively small fraction of their energy at each collision. The range for the hot electron, which depends on the initial electron energy, is approximately given by [Ref. 21]:

$$\Lambda = \lambda\sqrt{\frac{\Delta E}{\xi}} \tag{3.3}$$

where ΔE is the total energy loss, ξ is the energy loss per collision, and λ is the mean free path between collisions.

Ballistic motion was discussed by Quinn [Ref. 22], who calculated the range of hot electrons in metals by considering the energy of interaction of a single excited electron with the sea of conduction electrons by a self-energy approach. Hot electrons are assumed incident on the metal film and are scattered by electron-electron collision. For low-energy hot electrons (i.e., those electrons whose energy E is less than about 20 to 30 percent bigger than the Fermi energy E_F), an analytical expression for the diffusion range has been obtained. For higher energies, one must resort to numerical integration and also take into account the extra contribution from plasmon creation when ω is greater than ω_p. However, the treatment is strictly valid for r_s (the radius of a sphere equal in volume to the volume per electron, measured in units of the Bohr radius) small compared to unity, whereas the results are applied to metals in which r_s is always greater than 2. Thus good quantitative agreement is not expected, but it is anticipated that the qualitative dependence of the mean free path will be correct.

The diffusion, energy loss, and multiplication of secondary electrons within metals was considered by Wolff [Ref. 23]. His calculation assumes that the excited electron interacts with the conduction electrons through a screened Coulomb potential and that the resulting scattering is spherically symmetrical in the center-of-mass system of coordinates. The theory for the diffusion and scattering of neutrons, as developed by Marshak [Ref. 24], was applied to calculate the energy distribution and yield of the hot electrons. The effects of the Exclusion Principle were incorporated in the manner suggested by Goldberger [Ref. 25], and the effects of the motion of the conduction electrons were discussed. The calculation is strictly valid for electrons whose energy is greater than $2E_F$. For such electron energies, however, the excitation of plasma oscillations should be considered, as they play such an important part. Nevertheless, Wolff's predictions were verified by experimental results indicating that a combination of Goldberger's modification and the assumption of spherically symmetrical scattering counteract in some obscure way the effects of plasmon creation. Hall [Ref. 16] showed how Wolff's calculation could be extended to cover the complete range of hot electrons from E_F by suitably modifying the effective scattering cross section. It is important to note that although Wolff's discussion concerns essentially diffusive motion and thus $l(E) = \lambda$, the Goldberger-Hall modification is essentially ballistic, in which case $l(E) = \Lambda$.

An interesting connection between the results of Quinn and Hall has been found by Sze [Ref. 26]. A graph of Λ for hot electrons in gold films as a function of their incident energy was plotted from the figures of Quinn and Hall and it was found that the two curves connect smoothly and agree reasonably with experimental points. Similar curves for aluminum films do not appear to have this property, though they have a similar form. All that can be said is that both theories require knowledge of the values of several parameters and that the figures chosen may be questionable.

An analysis of the diffusive motion of hot electrons in thin metal films has been made here and will be discussed in detail in the following chapters.

IV. DIFFUSIVE MOTION OF HOT ELECTRONS IN METAL FILMS

The diffusive motion of hot electrons within a metal was considered first by Wolff [Ref. 23]. Irrespective of the method of production, the electrons will travel through the metal until they suffer a collision with another electron in the conduction band. At such a collision, the incident electron loses some of its energy which is imparted to the conduction electron. The scattering angle is determined by the dynamics of the system and will be discussed in some detail below. In the subsequent diffusive motion, therefore, the electron will be considered as diffusing through the solid and suffering surface reflections, multiplying and losing energy in the process, until it either drops back into the sea of conduction electrons or escapes from the rear surface of the film. As this description of the hot-electron motion will be used in the present work, the Boltzmann equation as quoted by Wolff may be used, namely:

$$\frac{\partial N}{\partial t}(\underline{r},\underline{\Omega},E,t) + \underline{v}\cdot \text{grad}\, N(\underline{r},\underline{\Omega},E,t) = \frac{-vN(\underline{r},\underline{\Omega},E,t)}{\lambda(E)} + S(\underline{r},\underline{\Omega},E,t)$$

$$+ \int_0^\infty dE' \int \frac{d\underline{\Omega}' v' N(\underline{r},\underline{\Omega}',E',t)}{\lambda(E')} F(\underline{\Omega},E;\underline{\Omega}',E') \qquad (4.1)$$

where:

$N(\underline{r},\underline{\Omega},E,t)$ = number of electrons between the space coordinates $\underline{r}$ and $(\underline{r}+d\underline{r})$, traveling between the directions $\underline{\Omega}$ and $(\underline{\Omega}+d\underline{\Omega})$, and having energies in the range E to $(E+dE)$ at time t,

$\lambda(E)$ = mean free path for an electron, in the direction of motion, having an energy E,

$\underline{v}$ = electron velocity,

$F(\underline{\Omega},E;\underline{\Omega}',E')$ = probability that, given an electron at $(\underline{\Omega}',E')$ one will be found at $(\underline{\Omega},E)$ after scattering. This term is normalized so that those electrons knocked up from the conduction band, as well as those knocked down in energy from E' to E, are included. Thus

$$\int_0^{E'} dE \int d\Omega F(\underline{\Omega},E;\underline{\Omega}',E') = 2$$

$S(\underline{r},\underline{\Omega},E,t)$ = number of electrons created per second within the energy range E to $(E+dE)$ and moving in a direction between $\underline{\Omega}$ and $(\underline{\Omega}+d\underline{\Omega})$.

Equation (4.1) assumes that no inelastic scattering processes are present and that no chemical bonding occurs, indicating that the conduction electrons are treated as if they are free. In addition, it will be assumed that the maximum energy of the hot electrons present is less than that required to excite plasmon oscillations. Before attempting to solve (4.1), it is very important to give a detailed discussion of the scattering function $F(\underline{\Omega},E;\underline{\Omega}',E')$.

A. THE SCATTERING FUNCTION

Wolff [Ref. 23] assumed that the scattering was spherically symmetrical in the center-of-mass system of coordinates, thus inferring that, on the average, the electron loses about half its energy at each collision. The scattering function may then be expanded in the form:

$$F(\underline{\Omega},E;\underline{\Omega}',E') = F(\cos\theta;E,E') = \frac{1}{4\pi}\sum(2\ell+1)\,F_\ell(E,E')\,P_\ell(\cos\theta) \qquad (4.2)$$

where θ is the angle between $\underline{\Omega}$ and $\underline{\Omega}'$. In his subsequent calculation, it was further assumed that only the first two terms $F_0(E,E')$ and $F_1(E,E')$ need be considered and that the effects of the Exclusion Principle could be described in the manner suggested by Goldberger [Ref. 25].

Landau and Lifshitz [Ref. 27, p. 423] wrote an expression for the differential scattering cross section for two electrons having spins of one-half which interact by Coulomb's law. As the colliding electrons need not be in definite spin states, the result is averaged over all possible spin states assuming that they are all equally probable. The formula quoted assumes that the conduction electrons are at rest; even so, it is very complicated.

As previously stated, a self-energy approach to electron-electron interactions was considered by Quinn [Ref. 22]. The effect of the Exclusion Principle and motion of the conduction electrons is taken into account and thus an accurate description of the scattering function may be obtained as follows.

Now, $2|E_I(p)|$ is the total transition rate for real scattering events and is given by:

$$2|E_I(p)| = \mathrm{Im}\left\{\frac{e^2}{\pi^2}\int\frac{d^3k}{k^2\epsilon[k,E(\underline{p})-E(\underline{p}-\underline{k})+i\delta]}\right\} \qquad (4.3)$$

where $\epsilon(k,\omega)$ is the Lindhard [Ref. 28] dielectric constant. The above equation is applicable to an electron of momentum $\underline{p}$ colliding with a conduction electron imparting a momentum $\underline{k}$ to the conduction electron in the process. But,

$$\int F(\cos\theta;E,E')\, dE d\theta = 2E_1(\underline{p})$$

where

$$p = \sqrt{2mE}, \quad p' = \sqrt{2mE'}, \quad \frac{\underline{p}\cdot\underline{p}'}{pp'} = \cos\theta$$

and

$$d^3k = d^3(\underline{p}-\underline{p}') = -2\pi p' \sin\theta\, p'd\theta\, dp'$$

For the energy range considered, $\epsilon_2\,[(\underline{p}-\underline{p}'),\Delta E]$ is given by Quinn [Ref. 22, Eq. (3)]. Substituting $(\underline{p}-\underline{p}')$ for $\underline{k}$ gives:

$$F(\cos\theta;E,E') = \frac{3\pi e^2(\omega_p\omega/v_o^3)\; p'^2 \sin\theta}{|\underline{p}-\underline{p}'|\;\;[|\underline{p}-\underline{p}'|^2 + k_s^2]} \tag{4.4}$$

Equation (4.4) has been expanded in terms of spherical harmonics in order to see the dominant symmetry functions contained by F, but the calculation revealed only a slowly converging series. However, as the above derivation is only strictly valid for energies less than 20 to 30 percent larger than the Fermi energy, it will not be very accurate for the present problem. For larger values of $\underline{p}$, an accurate analytical expression for $E_I(p)$ cannot be obtained. In this case, the percentage momentum transferred will be decreased and thus the scattering angle will be smaller. This is equivalent to reinforcing the lower order terms in the expansion of $F(E,E';\cos\theta)$, thus making the scattering tend toward the isotropic scattering envisaged by Wolff.

The retention of only a few terms is reasonable on physical grounds since a high-order spherical harmonic has a very complicated form. If they occur, they will be smeared out partly by the random motion of the conduction electrons and also through succeeding collisions. High values for the reflection coefficients R_1 and R_2 also reinforce ψ_0 and, to a lesser extent, ψ_1 in any partial wave expansion of the density function and thus reduce the significance of higher order terms in $F(\cos\theta;E,E')$.

It is felt that the inclusion of the Exclusion Principle and electron multiplication, and the relative simplicity of the resulting equations when spherically symmetrical scattering in the center-of-mass coordinate system is assumed, give a reasonably accurate description of the diffusion motion of hot electrons in metal films.

B. THE DIFFUSION EQUATIONS

When hot electrons are injected onto or produced in a direction normal to the film surface, the diffusion problem is one-dimensional and has azimuthal symmetry. If x is the distance from the surface and θ_N is the angle that the velocity of the electron makes with the normal, we can write:

$$\left.\begin{aligned} N(\underline{r},\underline{\Omega},E,t) &= N(x, \cos\theta_N, E) = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)\, N_\ell(x,E)\, P_\ell(\cos\theta_N) \\ &\text{and} \\ S(\underline{r},\underline{\Omega},E,t) &= S(x, \cos\theta_N, E) = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)\, S_\ell(x,E)\, P_\ell(\cos\theta_N) \end{aligned}\right\} \quad (4.5)$$

(In the present problem, only $S_0(x,E)$ occurs in the summation.)

Now, ψ_ℓ is defined by:

$$\psi_\ell = \frac{vN_\ell}{\lambda(E)} \quad (4.6)$$

From the previous section, only the first two terms for $\ell = 0$ and $\ell = 1$ are required, and thus (4.1) splits into a pair of simultaneous integrodifferential equations in the steady state--the p_1 approximation of Weymouth [Ref. 29]. These are

$$\left.\begin{aligned} \lambda(E)\frac{\partial\psi_1}{\partial x} + \psi_0 &= 2\int_0^\infty \psi_0(x,E')\, F_0(E,E')\, dE' + S_0(x,E) \\ &\text{and} \\ \frac{\lambda(E)}{3}\frac{\partial\psi_0}{\partial x} + \psi_1 &= 2\int_0^\infty \psi_1(x,E')\, F_1(E,E')\, dE' \end{aligned}\right\} \quad (4.7)$$

The electron currents in the positive and negative x directions, for electrons having an energy E, are:

$$i_{\pm x}(E) = \int_{\pm 1}^{0} ev\, N(x, \cos\theta_N, E) \cos\theta_N d(\cos\theta_N)$$

$$= \frac{\lambda e}{2\pi}\left[\frac{1}{2}\psi_0(x,E) \pm \psi_1(x,E)\right] \qquad (4.8)$$

where e is the electronic charge and may be obtained from solving (4.7) with the approximate boundary conditions.

One method of solution follows that originally given by Marshak [Ref. 24] for neutrons, if it may be assumed that $\psi_0(x,E)$ does not vary appreciably with energy. Then we can write:

$$\psi_0(x,E') = \psi_0(x,E) - (E' - E)\frac{\partial\psi_0(x,E)}{\partial E}$$

and (4.9)

$$\psi_1(x,E') = \psi_1(x,E)$$

where we assume that ψ_1 is an order smaller than ψ_0. This simplification is only valid providing that ψ, and thus F, is almost isotropic; that the electrons suffer a reasonable number of collisions within the film; and that the film is of reasonable thickness. Assuming that the criterion of "reasonable" is appropriate in our case--this will be discussed again later--we can make the substitutions:

$$\int_0^\infty dE'\, F_1(E,E') = \langle \cos\theta \rangle_{av}$$

and (4.10)

$$\int_0^\infty dE'\,(E' - E)\, F_0(E,E') = \zeta \left(= \begin{array}{c}\text{average energy lost}\\ \text{per electron collision}\end{array}\right)$$

For the particles of equal mass

$$\langle \cos\theta \rangle_{av} = \frac{2}{3} \tag{4.11}$$

However, with the present problem, $F_1(E,E')$ is likely also to include small contributions from higher order terms $F_\ell(E,E')$ and thus the accuracy of this value is likely to be in doubt.

Substituting (4.9) and (4.10) in (4.7), we obtain:

$$\psi_0 + \lambda \frac{\partial \psi_1}{\partial x} = 2\psi_0 - 2\zeta \frac{\partial \psi_0}{\partial E} + S_0(x,E)$$

$$\psi_1 + \frac{\lambda}{3}\frac{\partial \psi_0}{\partial x} = 2\psi_1 \langle \cos\theta \rangle_{av} \tag{4.12}$$

From the second of the two Eqs. (4.12), we have:

$$\psi_1 = -\frac{\lambda}{3}\frac{1}{\left(1 - 2\langle \cos\theta \rangle_{av}\right)}\frac{\partial \psi_0}{\partial x} \tag{4.13}$$

Letting

$$X = \frac{\lambda^2}{3\left(1 - 2\langle \cos\theta \rangle_{av}\right)} \tag{4.14}$$

Substituting (4.13) in the first of Eqs. (4.12) gives:

$$X \frac{\partial^2 \psi_0}{\partial x^2} + \psi_0 = 2\zeta \frac{\partial \psi_0}{\partial E} - S_0(x,E) \tag{4.15}$$

The diffusion Eqs. (4.12) and (4.15) will be solved for two special cases. First, the problem of generating photoelectrons within the interior of the metal film will be considered in the following chapter, and second, the motion of hot electrons injected into the film by a cold-cathode device will be discussed in later chapters.

V. PHOTOELECTRONS GENERATED WITHIN THE FILM

Suppose monochromatic light of frequency ν falls on a very large but thin metal film of thickness b. At a point x within the film, a photon of energy $h\nu$ will knock an electron, that is initially in the conduction band of the metal at energy E_o, into a state of energy $(E_o + h\nu)$. Normally, ν will be sufficiently large that the energy level $(E_o + h\nu)$ is above the Fermi level E_F, and thus the electron is "hot." The electron then moves in an arbitrary direction, until its motion is interrupted by scattering processes characterized by a mean free path λ in the direction of motion. Thus, at any point x' in the metal, electrons will be moving in both the positive and negative x directions giving rise to measurable electron currents i_r and i_f emitted from the rear and front surfaces of the film. respectively (see Fig. 4).

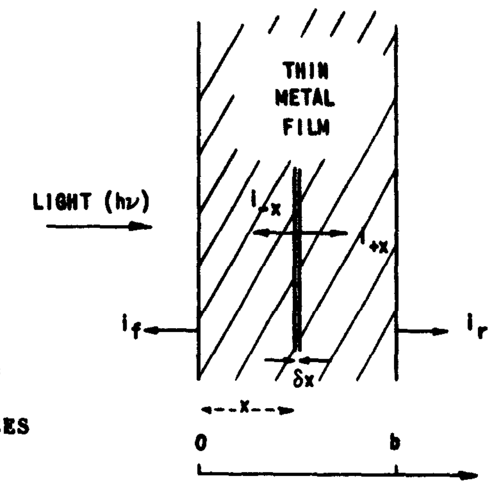


FIG. 4. LIGHT FALLING ON METAL FILM GIVING RISE TO CURRENTS i_f AND i_r FROM THE FRONT AND REAR SURFACES OF THE FILM.

In order to calculate i_r and i_f, it will be assumed that no electrons are generated on the surface of the film and that the escape probability for an electron approaching the metal surface is unity.

A. SIMPLE THEORY

If α_1 is the absorption coefficient of the light, and α_e is the absorption coefficient of the electron along the x-axis, then the number of electrons generated within a portion δx of the film at the point x is $N_o \exp(-\alpha_1 x)\, \delta x$. The number of these electrons reaching the rear surface of film is:

$$n_r = \left(N_o \; e^{-\alpha_1 x} \; \delta x \right) e^{-\alpha_e (b - x)}$$

while the number reaching the front surface is:

$$n_f = \left(N_o \; e^{-\alpha_1 x} \; \delta x \right) e^{-\alpha_e x}$$

Now

$$i_r = \int_0^b n_r \, dx$$

and

$$i_f = \int_0^b n_f \, dx$$

and therefore,

$$\frac{i_r}{i_f} = \frac{(\alpha_e + \alpha_1)}{(\alpha_e - \alpha_1)} \cdot \left(\frac{e^{-\alpha_e b} - e^{-\alpha_1 b}}{e^{-(\alpha_e + \alpha_1) b} - 1} \right) \tag{5.1}$$

This expression has been used by Hall [Ref. 16] and Spitzer, Crowell, and Atalla [Ref. 18] to estimate limits to the mean free paths of photo-excited electrons in gold and aluminum. In order to determine how α_e is related to the mean free path, a more detailed analysis is required. The following sections contain details of the calculation connecting α_e and λ, if it may be assumed that the electron motion is diffusive.

B. THE DIFFUSION EQUATION

Consider the density-of-states curve $N(E)$ vs E for the metal at a temperature of absolute zero. The photoelectric effect promotes electrons from the conduction band to the empty states within the energy range E_F to $(E_F + h\nu)$ from the range E_F to $(E_F - h\nu)$ as shown in Fig. 5. (For light of wavelength $\sim$ 6000 A, the maximum energy that an electron can have is 2.04 ev above E_F. Such an energy could also have been obtained by raising the temperature by $\sim 2000^o$K. This gives some meaning to the term "hot" electrons.)

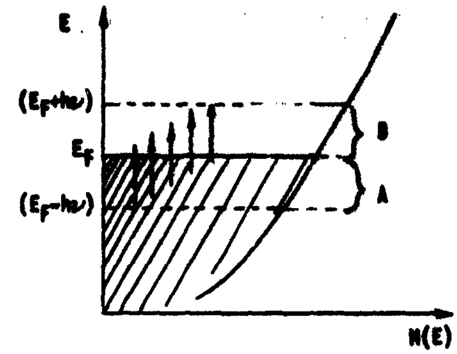


FIG. 5. DENSITY OF STATES CURVE. Light excites electrons from region A to region B.

Let us suppose that an equal number of electrons are generated in all directions at the point x by this process. Then:

$$S(\underline{r},\Omega,E,t) = N(E)\ F(E,T)\ [1 - F(E+h\nu,T)]\ e^{-\alpha_1 x} \tag{5.2}$$

where the Fermi distribution function is

$$F(E',T) = \frac{1}{\left[\exp\left(-\frac{E'-E_F}{kT}\right)+1\right]} \tag{5.3}$$

From (4.5) and (5.2), we can write

$$S_o(x,E) = N(E)\sqcap\ e^{-\alpha_1 x} \tag{5.4}$$

where $\sqcap$ takes into account the Fermi distribution function (5.3), such that

$$\left.\begin{array}{ll} \sqcap = 0 & \text{for } E < E_F \text{ and for } E > (E_F+h\nu) \\ \text{and} & \\ \quad = 1 & \text{for } E_F \leq E \leq (E_F+h\nu) \end{array}\right\} \tag{5.5}$$

(The limits for the integration take into account all values for E' before the effects of the Exclusion Principle are incorporated.)

Thus, by solving the Eqs. (4.12), it is possible to calculate $i_{\pm x}(E)$ as a function of x and E. Two different methods for doing this follow.

C. SOLUTION BY LAPLACE AND MELLIN TRANSFORMATIONS

In order to solve (4.7), we follow the procedure adopted in Wolff [Ref. 23] by Mellin-transforming both equations with respect to E and then Laplace-transforming with respect to x. We also make the following assumptions:

1. $F_\ell(E,E)$ is defined as

$$F_\ell(E,E) = \frac{2}{E'} P_\ell \left(\sqrt{\frac{E}{E'}} \right) \tag{5.6}$$

2. The mean free path $\lambda(E)$ is independent of energy and will be called λ,

3. The motion of the conduction electrons is small and merely smears out the scattered electron beam, and

4. The functions ψ_0 and ψ_1 are both nonzero within the energy range E_F to $(E_F + h\nu)$ only. This occurs in the source term due to the presence of $\sqcap$ and in the scattering term because of the Exclusion Principle. The effect of the latter was discussed by Wolff following the work of Goldberger [Ref. 25]. Figure 2 of Wolff shows that $F_0(E,E')$ is zero for $E < E_F$ and for $E > E'$, but has a constant value in between. The number of electrons having an energy greater than $(E_F + h\nu)$ is small because they must have suffered several collisions with other electrons having energies $(E_F + h\nu)$ moving in the correct direction. Thus, the maximum E' available is normally $(E_F + h\nu)$.

Let

$$\psi_{\substack{0\\1}}(y,s) = \int_0^\infty dx\, e^{-yx} \int_0^\infty dE\, E^s\, \psi_{\substack{0\\1}}(x,E)$$

and

$$\psi_{\substack{0\\1}}(s) = \int_0^\infty dE\, E^s\, \psi_{\substack{0\\1}}(x=0,E) \tag{5.7}$$

Also,

$$S(y,s) = \int_0^\infty dx\, e^{-yx} \int_0^\infty dE\, E^s\, N(E)\, e^{-\alpha_1 x} = \frac{\int_0^\infty dE\, E^s\, N(E)}{(y+\alpha_1)} = \frac{I}{(y+\alpha_1)} \tag{5.8}$$

Then application of the transformations to (4.12) gives:

$$\frac{(s-1)}{(s+1)}\psi_0(y,s) + y\lambda\,\psi_1(y,s) = \lambda\mathcal{X}_1(s) + S(y,s)$$

and (5.9)

$$\frac{y\lambda}{3}\psi_0(y,s) + \frac{(s-1/2)}{(s+3/2)}\psi_1(y,s) = \frac{\lambda}{3}\mathcal{X}_0(s)$$

Equations (5.9) solved simultaneously give:

$$\psi_0(y,s) = \frac{\begin{vmatrix} \lambda\mathcal{X}_1 + S, & y\lambda \\ \frac{1}{3}\lambda\mathcal{X}_0, & \frac{(s-1/2)}{(s+3/2)} \end{vmatrix}}{\left[\frac{(s-1)(s-1/2)}{(s+1)(s+3/2)} - \frac{1}{3}\lambda^2 y^2\right]} = \frac{D_0}{D}$$

and (5.10)

$$\psi_1(y,s) = \frac{\begin{vmatrix} \frac{(s-1)}{(s+1)}, & \lambda\mathcal{X}_1 + S \\ \frac{1}{3}y\lambda, & \frac{1}{3}\lambda\mathcal{X}_0 \end{vmatrix}}{\left[\frac{(s-1)(s-1/2)}{(s+1)(s+3/2)} - \frac{1}{3}\lambda^2 y^2\right]} = \frac{D_1}{D}$$

Inverting the Laplace transform gives:

$$\psi_{\substack{0\\1}}(x,s) = \frac{1}{2\pi i}\int_c \psi_{\substack{0\\1}}(y,s)\, e^{yx}\, dy$$

and thus involves calculating the residues of D_0, D_1, and D. (The calculation closely follows that of Weymouth [Ref. 29].)

Residue in D_0 occurs when $S\to\infty$; i.e., when $y = -a_1$ [see (5.8)]. Thus, the contribution to $\psi_0(\mathcal{X},s)$ is:

$$\frac{(s-1/2)}{(s+1/2)} \cdot \frac{I\, e^{-a_1 x}}{\left[\frac{(s-1)(s-1/2)}{(s+1)(s+3/2)} - \frac{1}{3}\lambda^2 a^2\right]} \qquad (5.11)$$

Residue in D occurs when $y = \pm\beta$, where

$$\beta = \frac{\pm\sqrt{3}}{\lambda}\left(\frac{(s-1)(s-1/2)}{(s+1)(s+3/2)}\right)^{1/2} \tag{5.12}$$

and gives a contribution to $\psi_0(x,s)$ of:

$$\frac{3}{2\beta\lambda^2}\left[\left(\ell_0 \mathcal{X}_1 + \frac{I}{(a_1 \pm \beta)}\right)\left(\frac{s-1/2}{s+3/2}\right) \mp \frac{\beta}{3}\lambda^2 \mathcal{X}_0\right] e^{\pm\beta x} \tag{5.13}$$

and to $\psi_1(x,s)$ of:

$$\frac{3}{2\beta\lambda^2}\left[\frac{1}{3}\lambda \mathcal{X}_0 \frac{(s-1)}{(s+1)} \mp \frac{1}{3}\beta\lambda\left(\lambda \mathcal{X}_1 + \frac{I}{(a_1 \pm \beta)}\right)\right] e^{\pm\beta x} \tag{5.14}$$

Residue in D_1 again occurs when $S \to \infty$; i.e., when $y = -a_1$, and gives a contribution to $\psi_1(x,s)$ of:

$$\frac{-\frac{1}{3} a_1 \lambda I e^{-a_1 x}}{\left[\frac{(s-1)(s-1/2)}{(s+1)(s+3/2)} - \frac{1}{3}\lambda^2 a_1^2\right]} \tag{5.15}$$

From (5.11) and (5.13) we get:

$$\psi_0(x,s) = \frac{1}{2\gamma}\left[\frac{2\beta I}{(\beta^2 - a_1^2)} e^{-a_1 x} + \left(\lambda \mathcal{X}_1 - \gamma \mathcal{X}_0 + \frac{I}{(a_1 + \beta)}\right) e^{\beta x} + \left(\lambda \mathcal{X}_1 + \gamma \mathcal{X}_0 + \frac{I}{(a_1 - \beta)}\right) e^{-\beta x}\right] \tag{5.16}$$

and from (5.14) and (5.15) we get:

$$\psi_1(x,s) = \frac{1}{2\lambda}\left[\frac{2a_1 I}{(\beta^2 - a_1^2)} e^{-a_1 x} - \left(\lambda \mathcal{X}_1 - \gamma \mathcal{X}_0 + \frac{I}{(a_1 + \beta)}\right) e^{\beta x} + \left(\lambda \mathcal{X}_1 + \gamma \mathcal{X}_0 + \frac{I}{(a_1 - \beta)}\right) e^{-\beta x}\right] \tag{5.17}$$

where

$$\gamma = \frac{(s-1)}{(s+1)} \cdot \frac{1}{\beta} \tag{5.18}$$

One boundary condition is that the net electron current entering the film at $x = 0$ is zero, i.e.,

$$[i_{+x}(E)]_{x=0} = 0$$

Therefore, from (4.8),

$$\left[\frac{1}{2}\psi_0(x,E) + \psi_1(x,E)\right]_{x=0} = 0 \tag{5.19}$$

Taking the Mellin and Laplace transforms of (5.19) and putting $x = 0$ gives

$$\frac{1}{2}\mathcal{X}_0(s) + \mathcal{X}_1(s) = 0 \tag{5.20}$$

But

$$\psi_{\substack{0\\1}}(o,s) = \mathcal{X}_{\substack{0\\1}}(s) \tag{5.21}$$

Substituting (5.20) and (5.21) in (5.16) and (5.17) when $x = 0$, gives

$$\mathcal{X}_1 = \frac{-I}{(\lambda+2\gamma)(\beta+a_1)}$$

and

$$\mathcal{X}_0 = \frac{-2I}{(\lambda+2\gamma)(\beta+a_1)} \tag{5.22}$$

Substituting (5.22) and (5.16) and (5.17) gives:

$$\left.\begin{aligned} \psi_0(x,s) &= \frac{I}{\gamma(\beta^2-a_1^2)}\left[\beta\, e^{-a_1x} - \frac{(\lambda\beta+2\gamma a_1)}{(2\gamma+\lambda)}\, e^{-\beta x}\right] \\ &\text{and} \\ \psi_1(x,s) &= \frac{I}{\gamma(\beta^2-a_1^2)}\left[a_1\, e^{-a_1x} - \frac{(\lambda\beta+2\gamma a_1)}{(2\gamma+\lambda)}\, e^{-\beta x}\right] \end{aligned}\right\} \tag{5.23}$$

It will be noticed that the coefficient of $e^{\beta x}$ is zero. This is consistent with the idea that $\psi_0 \to 0$ as $x \to \infty$ and occurs because we have assumed that there is no reflection of light or electrons from the film boundary at $x = b$, that is, the equations are ideally valid for a semi-infinite slab of material.

The current flow in the positive x direction at the point x is:

$$i(+x) = \int_{E_F}^{E_F+h\nu} i_{+x}(x,E)\, dE$$

$$= \frac{\lambda e}{2\pi} \int_{E_F}^{E_F+h\nu} \left[\frac{1}{2}\psi_0(x,E) + \psi_1(x,E)\right] dE \text{ [from (4.8)]}$$

$$= \frac{\lambda e}{2\pi} \int_0^{\infty} \left[\frac{1}{2}\psi_0(x,E) + \psi_1(x,E)\right] dE$$

$$= \frac{\lambda e}{2\pi}\left[\frac{1}{2}\psi_0(x,s) + \psi_1(x,s)\right]_{s=0} \tag{5.24}$$

When $s = 0$, then $\beta = 1/\lambda$ [from (5.12)] and $\gamma = -\lambda$ [from (5.18)] giving:

$$i(+x) = \frac{\lambda e \cdot \int N(E)\, dE}{2\pi}\left[\frac{(1/2 - \lambda a_1)}{(1 - \lambda^2 a_1^2)}\right]\left(e^{-x/\lambda} - e^{-a_1 x}\right) \tag{5.25}$$

Therefore,

$$\frac{i_r}{i_f} = \frac{1}{2} \cdot \frac{(1/2 - \lambda x_1)}{(1 - \lambda^2 a_1^2)}\left(e^{-b/\lambda} - e^{-a_1 b}\right) \tag{5.26}$$

This last equation has a form similar to that of Eq. (5.1) obtained by the simple theory.

However, it would seem difficult to use the above method of finding i_r/i_f if we assume that λ is a function of E; and a more accurate expression for the scattering function is used instead of the expression given in (5.6). Also, an expression for the electron current flow as a function of E, $[i_{\pm x}(E)]$, would only be obtained by performing the inverse Mellin transform on (5.23), and so far it has been found impossible to do this. In view of these difficulties, another method for solving (4.7) has been investigated, following the procedure used by Marshak [Ref. 24].

D. SOLUTION BY SERIES

Using (5.4) and the substitution

$$\phi_0 \, e^{E/2\zeta} = \psi_0, \tag{5.27}$$

the diffusion Eq. (4.15) becomes

$$\frac{\partial^2 \phi_o}{\partial x^2} - \frac{2\zeta}{X} \frac{\partial \phi_o}{\partial E} = -N(E) \, e^{-\alpha_1 x} \, e^{-E/2\zeta} \tag{5.28}$$

where the solution is understood to be valid in the range defined by ⎍.

The general solution of the complementary function of (5.28) is:

$$\phi_0^I = \sum_{A,h} A \, e^{hx} \, e^{(X/2\zeta)h^2 E} \tag{5.29}$$

and the corresponding particular integral is:

$$\phi_0^{II} = - \frac{N'(E) \, e^{-\alpha_1 x} \, e^{-E/2\zeta}}{(X \, \alpha_1^2 + 1)} \tag{5.30}$$

where

$$N'(E) = \left[N(E) + \frac{2\zeta}{(X \, \alpha_1^2 + 1)} \frac{\partial N(E)}{\partial E} \right] \tag{5.31}$$

Now $\phi_0 = \phi_0^I + \phi_0^{II}$ and thus, using (5.27), we get:

$$\psi_0 = \sum_{A,h} A \, e^{hx} \, e^{(X \, h^2+1)E/2\zeta} - \frac{N'(E) \, e^{-\alpha_1 x}}{(X \, \alpha_1^2 + 1)} \tag{5.32}$$

But we have the boundary condition given by (5.19) for $x = 0$. Using (4.13) to derive ψ_1 [from (5.32)] and this boundary condition, we get:

$$\sum A = N'(E)\frac{\left(\frac{1}{2}+\frac{X\alpha_1}{\lambda}\right)}{(X\,\alpha_1^2+1)}\;\frac{1}{\sum\left(\frac{1}{2}+\frac{Xh}{\lambda}\right)\left(e^{(Xh^2+1)E/2\zeta}\right)} \tag{5.33}$$

But $\sum A$ is independent of E and therefore, from (4.11) and (4.14),

$$\sum h^2 = -\frac{1}{X} = \frac{1}{\lambda^2} \tag{5.34}$$

Also, as $x \to \infty$, $\psi_0 \to 0$, and therefore all h must be negative.

1

Substituting (5.33) and (5.34) in (5.32) and (4.13) gives:

$$\psi_0 = \frac{N'\left(\frac{1}{2}-\lambda\alpha_2\right)}{(1-\lambda^2\alpha_1^2)}\left[\frac{\sum\limits_h e^{hx}\,e^{\eta E}}{\sum\limits_i\left(\frac{1}{2}+\lambda h\right)\,e^{\eta E}} - e^{-\alpha_1 x}\right]$$

and

$$\psi_1 = \frac{N'\left(\frac{1}{2}-\lambda\alpha_1\right)}{(1-\lambda^2\alpha_1^2)}\left[\frac{\lambda\sum h\,e^{hx}\,e^{\eta E}}{\sum\left(\frac{1}{2}+\lambda h\right)\,e^{\eta E}} + \alpha_1\,\lambda\,e^{-\alpha_1 x}\right] \tag{5.35}$$

where

$$\eta = \frac{h^2 X}{2\zeta}$$

From (4.8)

$$i_{+x}(E) = \frac{\lambda e}{2\pi}\cdot N'\,\frac{\left(\frac{1}{2}-\lambda\alpha_1\right)}{(1-\lambda^2\alpha_1^2)}\left[\frac{\sum\limits_h\left(\frac{1}{2}+\lambda h\right)\,e^{hx}\,e^{\eta E}}{\sum\limits_h\left(\frac{1}{2}+\lambda h\right)\,e^{\eta E}} - e^{-\alpha_1 x}\right] \tag{5.36}$$

Consider only one term in the sum; then from (5.34) $h = -1/\lambda$; therefore,

$$i_{+x} = \frac{\lambda e}{2\pi} \cdot N' \left[\frac{\left(\frac{1}{2} - \lambda a_1\right)}{(1 - \lambda^2 a_1^2)}\right] \left[e^{-x/\lambda} - e^{-a_1 x}\right] \tag{5.37}$$

which has exactly the same form as (5.25) derived in Sec. C, when we integrate (5.37) over E, since the only energy-dependent term is N'. Similarly:

$$\frac{i_r}{i_f} = \frac{1}{2} \frac{\left(\frac{1}{2} - \lambda a_1\right)}{(1 - \lambda^2 a_1^2)} \left[e^{-b/\lambda} - e^{-a_1 b}\right]$$

which has the same form as (5.26).

The question which then arises is what we should do with all the other solutions contained in (5.36). Under the assumptions made, however, it may be argued that there should be no more solutions on physical grounds, as we have considered values for ψ_0 and ψ_1 only between the limits E_F and $(E_F + h\nu)$. Within this energy range, both the source function $S(x,E)$ and the scattering function--this also implies ζ and X--have been taken to be independent of E and so (5.28) contains no energy-dependent term other than $\partial\psi_0/\partial E$ which must thus be zero.

The diffusion theory reported here could easily be adapted to include a surface generation of electrons by replacing the zero occurring in Eq. (5.19) by a constant. In addition, a more refined calculation must incorporate the probability P(E) of an electron of energy E escaping from a surface having a work function ϕ, where:

$$P(E) = \left[1 - \left(\frac{\phi}{E}\right)^{1/2}\right] \tag{5.38}$$

according to Wolff.

E. DISCUSSION

The relationship derived above uses an isotropic scattering function and assumes $\langle\cos\theta\rangle_{av} = 2/3$. As it is impossible to calculate the error involved in doing so, little can be said about the exactness of the result obtained. However, it is very interesting to find that exactly the same result is obtained by two methods which apparently contain unrelated assumptions. Comparison of theory with experiment will not be made until a more refined calculation, which includes boundary reflections, has been made.

One or two points concerning the physical principles involved in the generation of photoelectrons are pertinent here. An elementary analysis of the dynamics involved in the excitation of an electron by a photon reveals that the electron excited must initially be bound to an atom in the solid in order that both energy and momentum may be conserved. As the photon momentum $(h\nu/c)$ is very small, the momentum vector $\underline{k}$ of the excited electron is unchanged. Thus the conduction electrons, having been given energy by a collision with a photon, do not change their direction of motion. As there is no electric field present, the conduction electrons will be moving in every direction at random and thus the source function will contain $S_0(x,E)$ terms only. However, the error involved in this assumption is likely to depend on the metal used. An accurate description may be obtained from using the tight binding approximation (see Slater and Koster [Ref. 30]) when the conduction electrons are described by wave functions of the form:

$$\psi = \frac{1}{\sqrt{N!}} \sum_{r_n} e^{i\,\underline{k}\cdot\underline{r}_n}\, \phi_{n,\ell,m}$$

where $\phi_{n,\ell,m}$ is a hydrogen-like wave function. When such electrons are promoted, the source function will depend on the symmetry properties of $\phi_{n,\ell,m}$--for example, whether it consists of basic s-type or p-type atomic orbitals. The photoelectron yield is thus expected to depend in some involved way on the local character of the conduction electrons of the metal film. Information on this could probably be obtained by shining polarized light onto the metal film at different angles to the surface, and measuring the yield across the film as a function of the angle of incidence.

VI. GENERAL THEORY FOR THE DIFFUSIVE MOTION OF HOT ELECTRONS INJECTED IN THE FILM

It will be assumed that hot electrons are injected into a thin metal film and undergo scattering, diffusion, and reflection before they are emitted from the rear surface of the film. Solutions of the second-order partial-differential Eq. (4.15) will be obtained using ζ and X as parameters which depend on energy. The reflection coefficients are functions of energy, but an average value will be taken in order that a solution can be obtained. An expression for the electron current at a point x within the film having an energy between E and $(E + dE)$ will be calculated when such an approximation is valid. The yield depends on the incoming hot-electron spectrum $\mu(E')$, pertinent energies, and film thickness. The relationship existing between the diffusion range Λ and mean free path λ will also be discussed.

A. SOLUTION TO THE DIFFUSION EQUATION

Consider monoenergetic hot electrons injected into the metal film with energy E' at $x = 0$. Their subsequent diffusive motion may be described by the equation [see (4.15)]:

$$X \frac{\partial^2 \psi_0}{\partial x^2} + \psi_0 = 2\zeta \frac{\partial \psi_0}{\partial E} \tag{6.1}$$

Putting

$$\psi_0 = \phi_0 \, e^{\int (dE/2\zeta)} \tag{6.2}$$

gives

$$\frac{\partial^2 \phi_0}{\partial x^2} = \frac{2\zeta}{X} \frac{\partial \phi_0}{\partial E} \tag{6.3}$$

Now $\phi_0(x,E)$ may be written as $\mathcal{E}(E)\,\mathcal{X}(x)$, in which case (6.3) becomes

$$\frac{\ddot{\mathcal{X}}}{\mathcal{X}} = \frac{2\zeta}{X} \frac{\dot{\mathcal{E}}}{\mathcal{E}} = h_n^2 \tag{6.4}$$

where h_n^2 is the separation constant and may be positive or negative depending on whether h_n is real or imaginary. As the other parts to the equation are necessarily real, h_n cannot be complex.

Solutions of (6.4) are:

$$X = \left(C_n' \, e^{h_n x} + D_n' \, e^{-h_n x} \right)$$

and

$$E = G_n e^{\int (h_n^2 \lambda^2 dE / 2\zeta)}$$

giving

$$\psi_0 = e^{\int (dE/2\zeta)} \sum e^{\mathcal{E}_n} \left(C_n \, e^{h_n x} + D_n \, e^{-h_n x} \right) \tag{6.5}$$

from (6.2), where

$$\mathcal{E}_n = h_n^2 \int \left(\frac{X}{2\zeta} \right) dE \tag{6.6}$$

On entering the film, the electrons will move at random but in such a manner that their directions at any instant of time are determined by the various scatterings suffered with electrons in the conduction band and reflections from the two film surfaces. Classically, an electron will pass through the potential barrier existing at the surface when its component of momentum perpendicular to that surface has an equivalent energy greater than the work function. The reflection coefficient R for a surface having a work function φ is given by:

$$R = \left\{ \begin{array}{ll} \left(\frac{\varphi}{E} \right)^{1/2} & \text{for} \quad E > \varphi \\ 1 & \text{for} \quad E \leq \varphi \end{array} \right\} \tag{6.7}$$

where all energies are measured with respect to an origin at the bottom of the conduction band. However, the expression (6.7) when $E > \varphi$ should be modified, since those electrons which may be emitted classically may be reflected quantum mechanically. The latter occurs because the amplitude of the oscillating wave functions, as derived from the Schrödinger equation in a region of space where the energy E is greater than the potential function V, depends on the magnitude of (E - V)--see Eq. (2.8). For a sharp discontinuity in the potential function, there will be a discontinuity in the wave function which is equivalent to a partial reflection of the incident wave function. As this reflection factor depends on the sharpness of the energy discontinuity, it must also depend on the surface geometry of the film. Since little is known about this surface geometry, it cannot be accurately calculated.

Assuming that the electron does not lose any energy on reflection, the following boundary conditions are obeyed:

$$\left.\begin{aligned} i_{+x}(E) &= R_1\, i_{-x}(E) \qquad \text{at} \quad x = 0 \\ &\text{and} \\ i_{-x}(E) &= R_2\, i_{+x}(E) \qquad \text{at} \quad x = b \end{aligned}\right\} \tag{6.8}$$

where the suffix attached to R relates to the value of φ at the surface according to (6.7). Using (4.8) and (4.13), the boundary conditions (6.8) become:

$$\left.\begin{aligned} c_1\psi_0 &= -\frac{\partial\psi_0}{\partial x} \qquad \text{at} \quad x = 0 \\ &\text{and} \\ c_2\psi_0 &= \frac{\partial\psi_0}{\partial x} \qquad \text{at} \quad x = b \end{aligned}\right\} \tag{6.9}$$

where

$$c = \frac{\frac{1}{2}\,(1 - R)}{\lambda\,(1 + R)} \tag{6.10}$$

a function of E through R and λ. In order to proceed, we must make the further assumption that c is approximately constant over the range of electron energies considered. The problem then becomes similar to that of heat conduction through a solid bounded by parallel planes when heat is radiated from the ends into a medium at zero temperature (see Carslaw and Jaeger [Ref. 31]). Substituting (6.9) in (6.5) gives

$$\frac{D_n}{C_n} = \frac{(h_n + c_1)}{(h_n - c_1)} = e^{2h_n b}\,\frac{(h_n - c_2)}{(h_n + c_2)} = \Gamma_n \tag{6.11}$$

Thus

$$\psi_0 = e^{\int (dE/2\zeta)} \sum_n C_n e^{\mathcal{E}_n} \left(e^{h_n x} + \Gamma_n e^{-h_n x} \right) \tag{6.12}$$

where the values for h_n are determined from (6.11) and will be discussed later and where the C_n's are the only unknowns. These may be determined by expressing the incident hot-electron current in the form:

$$I_o = \frac{\lambda e}{2\pi} K(E') \, \delta(x - 0) \tag{6.13}$$

1. Determination of C_n

From (6.13),

$$i_{+x}(E') = \frac{\lambda e}{2\pi} K(E') \, \delta(x - 0)$$

$$i_{-x}(E') = 0$$

From (4.8)

$$\frac{1}{2} \psi_0(x, E') = \psi_1(x, E')$$

and therefore

$$\psi_0(x, E') = K(E') \, \delta(x - 0) \tag{6.14}$$

Suppose

$$X_n = e^{h_n x} + \Gamma_n e^{-h_n x} \tag{6.15}$$

Then multiplying (6.12) by X_m and integrating with respect to x over the film at an energy E', and using (6.14), gives:

$$\int_0^b X_m K(E') \, \delta(x - 0) \, dx = \exp\left[\int_{(E')} \left(\frac{dE}{2\zeta} \right) \right] \sum_n C_n \exp\left[\mathcal{E}_n \atop (E') \right] \int_0^b X_n X_m \, dx \tag{6.16}$$

We now must evaluate $\int_0^b X_n X_m \, dx$. From (6.15)

$$\frac{d^2 X_{n,m}}{dx^2} = h_{n,m}^2 X_{n,m}$$

and therefore

$$(h_m^2 - h_n^2) \int_0^b X_n X_m \, dx = \int_0^b \left(X_n \frac{d^2 X_m}{dx^2} - X_m \frac{d^2 X_n}{dx^2} \right) dx$$

$$= \left(X_n \frac{dX_m}{dx} - X_m \frac{dX_n}{dx} \right)_0^b$$

But, from (6.9),

$$\left.\begin{aligned} &\frac{\partial X_n}{\partial x} + c_1 X_n = 0 \quad \text{at} \quad x = 0 \\ \text{and} \\ &\frac{\partial X_n}{\partial x} - c_2 X_n = 0 \quad \text{at} \quad x = b \end{aligned}\right\} \qquad (6.17)$$

whatever the value for n. Thus

$$(h_m^2 - h_n^2) \int_0^b X_n X_m \, dx = 0$$

and therefore,

$$\int_0^b X_n X_m \, dx = 0 \qquad \text{when} \quad m \neq n \qquad (6.18)$$

meaning that all X_r are orthogonal. To obtain $\int_0^b X_n^2 \, dx$ we note that:

$$h_n^2 \int_0^b X_n^2 \, dx = \int_0^b X_n \frac{d^2 X_n}{dx^2} \, dx = \left(X_n \frac{dX_n}{dx} \right)_0^b - \int_0^b \left(\frac{dX_n}{dx} \right)^2 dx \qquad (6.19)$$

From (6.15),

$$\frac{dX_n}{dx} = h_n \left(e^{h_n x} - \Gamma_n e^{-h_n x} \right)$$

and therefore,

$$h_n^2 X_n^2 = \left(\frac{dX_n}{dx} \right)^2 + 4 h_n^2 \Gamma_n \tag{6.20}$$

Integrating gives

$$h_n^2 \int_0^b X_n^2 \, dx = \int_0^b \left(\frac{dX_n}{dx} \right)^2 dx + 4 h_n^2 \Gamma_n b \tag{6.21}$$

Combining (6.19) and (6.21) gives:

$$2 h_n^2 \int_0^b X_n^2 \, dx = \left(X_n \frac{dX_n}{dx} \right)_0^b + 4 h_n^2 \Gamma_n b \tag{6.22}$$

Now, from (6.17),

$$X_n \frac{dX_n}{dx} = \begin{cases} -c_1 X_n^2 & \text{at } x = 0 \\ +c_2 X_n^2 & \text{at } x = b \end{cases}$$

Therefore

$$\frac{dX_n}{dx} = \begin{cases} -c_1 X_n & \text{at } x = 0 \\ +c_2 X_n & \text{at } x = b \end{cases} \tag{6.23}$$

Using (6.20),

$$X_n^2 = \begin{cases} \dfrac{4h_n^2\,\Gamma_n}{(h_n^2 - c_1^2)} & \text{at } x = 0 \\[2ex] \dfrac{4h_n^2\,\Gamma_n}{(h_n^2 - c_2^2)} & \text{at } x = b \end{cases}$$

Using (6.23),

$$\left(X_n \frac{dX_n}{dx} \right)_0^b = 4h_n^2\,\Gamma_n \left[\frac{c_2}{(h_n^2 - c_2^2)} + \frac{c_1}{(h_n^2 - c_1^2)} \right]$$

Substituting into (6.22) gives:

$$\int_0^b X_n^2\,dx = 2\Gamma_n \left[b + \frac{c_2}{(h_n^2 - c_2^2)} + \frac{c_1}{(h_n^2 - c_1^2)} \right] \tag{6.24}$$

From (6.18), (6.24), (6.16), and (6.11), we can get:

$$C_n = K(E')\left(\frac{h_n}{h_n + c_1} \right) \frac{\exp\left[-\int_{(E')} \left(\frac{dE}{2\zeta} \right) \right] \exp\left[-\mathcal{E}_{n\,(E')} \right]}{\left[b + \frac{c_2}{(h_n^2 - c_2^2)} + \frac{c_1}{(h_n^2 - c_1^2)} \right]} \tag{6.25}$$

2. <u>Evaluation of h_n</u>

(i) Put $h_n = \alpha_n$ when α_n is real. Then α_n are the roots of the equation contained by (6.11), namely:

$$y_1 = e^{2\alpha_n b} = \frac{(\alpha_n + c_1)(\alpha_n + c_2)}{(\alpha_n - c_1)(\alpha \;\; - c_2)} \tag{6.26}$$

Consider c_1 and c_2 unequal; (6.26) may be solved graphically by plotting y_1 as a function of α_n as shown in Fig. 6.

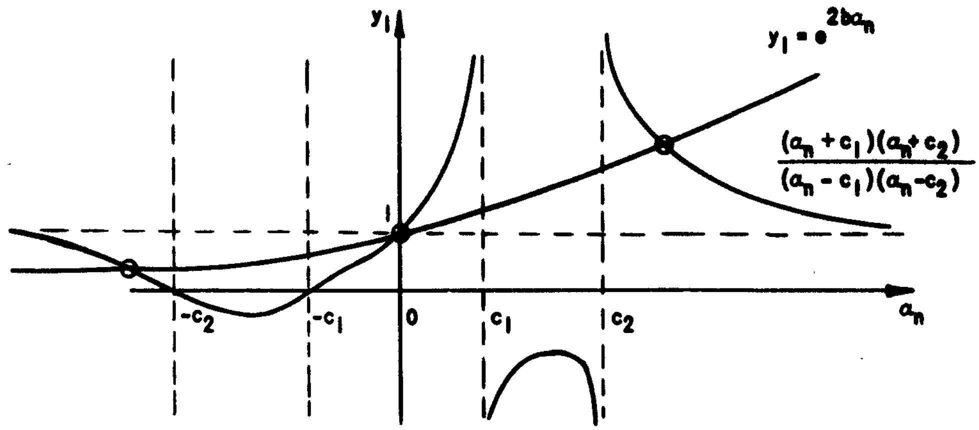


FIG. 6. TYPICAL GRAPHS USED TO DETERMINE THE ROOTS α_n.

Suppose the two nontrivial roots of (6.26) are $(c_2 + p)$ and $-(c_2 + q)$. On substituting these values into the equation, it is found that $p = q$, showing that the two roots are $\pm\alpha_n'$.

(ii) Put $h_n = i\beta_n$ where β_n is real. Then β_n are the roots of the equation

$$e^{2i\beta_n b} = \frac{(i\beta + c_1)(i\beta + c_2)}{(i\beta - c_1)(i\beta - c_2)}$$

By equating real and imaginary parts, it may be shown that the roots exist and are contained by:

$$y_2 = \sin 2\beta_n b = \frac{2\beta_n(c_1 + c_2)(c_2c_1 - \beta_n^2)}{(\beta_n^2 + c_1^2)(\beta_n^2 + c_2^2)} \qquad (6.27)$$

Inspection of this equation shows that if a root $+\beta_n'$ exists then so does $-\beta_n'$. Graphical solutions of (6.27) may be obtained by plotting y_2 against β_n for positive β_n (Fig. 7).

Further simplification of the roots is impossible unless special simplifications are made between b, c_2, and c_1. These will be discussed in the following chapter.

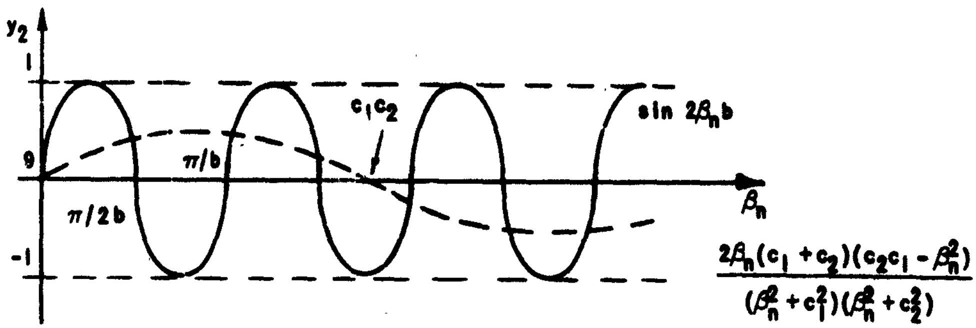


FIG. 7. TYPICAL GRAPHS USED TO DETERMINE THE ROOTS β_n.

B. EVALUATION OF THE ELECTRON CURRENT DENSITY

Substituting (6.25) into (6.12) gives:

$$\psi_0(x,E) = \sum_n H_n \frac{h_n}{(h_n + c_1)} \left(e^{h_n x} + \Gamma_n \, e^{-h_n x} \right)$$

where

$$H_n = \frac{K(E') \exp\left[- \int_E^{E'} \left(\frac{dE}{2\zeta} \right) \right] \exp\left[-h_n^2 \int_E^{E'} \left(\frac{XdE}{2\zeta} \right) \right]}{\left[b + \frac{c_1}{(h_n^2 - c_1^2)} + \frac{c_2}{(h_n^2 - c_2^2)} \right]} \tag{6.28}$$

and thus

$$\left(\frac{2\pi}{\lambda e} \right) i_{\pm x}(E) = \sum_n H_n \frac{h_n}{(h_n + c_1)} \left[\left(\frac{1}{2} \pm \lambda h_n \right) e^{h_n x} + \Gamma_n \left(\frac{1}{2} \mp \lambda h_n \right) e^{-h_n x} \right] \tag{6.29}$$

It may be noticed that the age τ of the electron as defined by Fermi [Ref. 32] in the form $\tau = -\int_E^{E'} (XdE/2\zeta)$ enters into the calculation in a natural way. Also the factor $\exp[-\int_E^{E'} (dE/2\zeta)]$ arises because electron multiplication has been assumed.

Equation (6.29) may be simplified by grouping together the contributions from two roots $\pm h_n'$. The current density then involves the following terms:

$$\left[\frac{h_n'}{h_n'+c_1} + \Gamma(-h_n') \cdot \frac{-h_n'}{(-h_n'+c_1)}\right] e^{h_n' x}\left(\frac{1}{2} \pm \lambda h_n'\right)$$

$$+\left[\Gamma(h_n') \cdot \frac{h_n'}{(h_n'+c_1)} + \frac{-h_n'}{(-h_n'+c_1)}\right] e^{-h_n' x}\left(\frac{1}{2} \mp \lambda h_n'\right)$$

From (6.11), this may be simplified to:

$$2h_n\left[\frac{e^{h_n x}\left(\frac{1}{2} \pm \lambda h_n'\right)}{(h_n' + c_1)} + \frac{e^{-h_n x}\left(\frac{1}{2} \mp \lambda h_n'\right)}{(h_n' - c_1)}\right] \qquad (6.30)$$

For the case when $h_n' = i\beta_n'$, (6.30) may be simplified further to:

$$\frac{2\beta_n}{(c_1^2 + \beta_n^2)}\left[\beta_n(1 \mp 2\lambda c_1) \cos \beta_n x - (c_1 \pm 2\lambda\beta_n^2) \sin \beta_n x\right]$$

Thus

$$\left(\frac{2\pi}{\lambda e}\right) i_{\pm x}(E) = 2H(\alpha_n')\ \alpha_n' \cdot \left[\frac{\left(\frac{1}{2} \pm \lambda\alpha_n'\right)}{(\alpha_n' + c_1)} e^{\alpha_n' x} + \frac{\left(\frac{1}{2} \mp \lambda\alpha_n'\right)}{(\alpha_n' - c_1)} e^{-\alpha_n' x}\right] + 2 \sum_{\beta_n > 0}$$

$$\left\{ \frac{H(\beta_n) \cdot \beta_n}{(c_1^2 + \beta_n^2)}\left[\beta_n(1 \mp 2\lambda c_1) \cos \beta_n x - (c_1 \pm 2\lambda\beta_n^2) \sin \beta_n x\right]\right\}$$

$$(6.31)$$

When the roots β_n are large, the contribution to (6.31) is proportional to

$$\frac{2H(\beta_n)}{\beta_n}\left[\beta_n(1 \pm 2\lambda c_1) \mp 2\lambda\beta_n^2\right] \to \mp 4\lambda H(\beta_n)\ \beta_n$$

Now

$$H(\beta_n) \sim \exp\left[\beta_n^2 \int_E^{E'} \left(\frac{X dE}{2\zeta} \right) \right]$$

and, since X is negative, $H(\beta_n) \cdot \beta_n \to 0$ as $\beta_n \to \infty$.

The current density emitted from the rear surface of the film is obtained by taking the positive sign at $x = b$ in (6.31) and multiplying the result by the transmission coefficient $(1 - R_2)$; that is,

$$\left(\frac{2\pi}{\lambda e} \right) i_r = \left(\frac{2\pi}{\lambda e} \right) (1 - R_2)\ i_{+b}(E) = \frac{8H(a_n')\ \lambda c_2 a_n'^2\ e^{ab}}{(a_n' + c_1)\ (a_n' + c_2)}$$

$$+ 8 \sum_{\beta_n > 0} \frac{H(\beta_n)\ \lambda c_2 \beta_n^2}{(\beta_n^2 + c_1^2)(1 + 2\lambda c_2)} \left[\begin{aligned} &(1 - 2\lambda c_1) \cos \beta_n b \\ &- \left(\frac{c_1}{\beta} + 2\lambda\beta_n \right) \sin \beta_n b \end{aligned} \right] \tag{6.32}$$

using (6.11) and where the coefficients are given by (6.28).

Thus the total current emitted from the rear of the film from an incident monoenergetic beam is:

$$\int_{\varphi_2}^{E'} i_r\ dE$$

and, from an incoming beam having incident spectrum $\mu(E')$, is:

$$I_r = \int_0^{\infty} dE'\ \mu(E) \int_{\varphi_2}^{E'} dE\ i_r \tag{6.33}$$

Similarly, the current density emitted from the front surface of the film is given by:

$$\left(\frac{2\pi}{\lambda e} \right) i_f = \left(\frac{2\pi}{\lambda e} \right) (1 - R_1)\ i_{-0}(E) = \frac{8H(a_n')\ \lambda c_1 a_n'^2}{(a_n'^2 - c_1^2)} + 8 \sum_{\beta_n > 0} \frac{H(\beta_n)\ \lambda c_1 \beta_n^2}{(c_1^2 + \beta_n^2)}$$

(6.34)

The total current from the front of the film from an incident mono-energetic beam is:

$$\int_{\varphi_1}^{E'} i_f \, dE$$

and, from an incoming beam of incident spectrum $\mu(E')$, is:

$$I_f = \int_0^{\infty} dE' \, \mu(E') \int_{\varphi_1}^{E'} dE \, i_f \tag{6.35}$$

Complete expressions for I_f and I_r may thus be obtained by substituting (6.32) and (6.34) in (6.33) and (6.35), respectively. Further simplification is possible only if we remove the restriction that x, λ, and ζ are energy dependent, and this will be done now.

C. SIMPLIFICATION

Let us suppose that λ_o and ζ_o represent the average values of λ and ζ over the hot-electron energy range considered. For a given experimental situation they are constants, but for different materials, $\mu(E')$ and φ, they will change. Suppose further we assume that $\chi = -\lambda_o^2$ in accordance with our assuming (4.11) to be accurate. Performing the various integrals on the energy-dependent exponential terms in H_n gives a contribution of

$$\frac{-2\,\zeta_o}{(h_n^2\lambda_o^2 - 1)} \exp\left[\frac{(\lambda_o^2 h_n^2 - 1)}{2\zeta_o}\,\varphi_2\right] \int_0^{\infty} \mu(E')\,K(E')\,dE' \tag{6.36}$$

to I_r, and a contribution of

$$\frac{-2\,\zeta_o}{(h_n^2\lambda_o^2 - 1)} \exp\left[\frac{(\lambda_o^2 h_n^2 - 1)}{2\zeta_o}\,\varphi_1\right] \int_0^{\infty} \mu(E')\,K(E')\,dE' \tag{6.37}$$

to I_f. Now

$$\int_0^{\infty} \mu(E')\,K(E')\,dE' = \langle E' \rangle \, I_o \tag{6.38}$$

since it is a measure of the total current arriving at the front surface of the film. Application of the above calculation to the cold-cathode device and the photoelectric experiment will now be made.

VII. INTERPRETATION

A. THE COLD-CATHODE EMITTER

The front surface of the metal film is joined to the insulator and thus R_1 must be nearly unity. (If the electrons were to enter the insulator, they would do so either in a filled or forbidden band, and thus the only possibility would be for them to tunnel back into the base metal. This probability is extremely small, as discussed in Chapter II-B, and so it may be ignored.) Putting $R_1 = 1$ in (6.10) gives $c_1 = 0$ and $\Gamma_n = 1$.

Estimates of c_2 depend entirely on the value assigned to R_2. In typical metals, the maximum energy that an electron can have before exciting plasma oscillations is approximately $2E_F$. For typical metals, $(\varphi - E_F) \sim 1$ ev and thus, from (6.7), R_2 is always greater than $\sqrt{1/2}$. Averaging over all energies and remembering that electrons having energies nearest to φ_2 dominate, suggests putting R_2 equal to 0.9, giving:

$$c = c_2 = \frac{1}{38\lambda_o} = \frac{0.0263}{\lambda_o} \tag{7.1}$$

The roots α_n' are obtained from solving graphically Eq. (6.26) with $c_1 = 0$, namely:

$$e^{2\alpha_n' b} = \frac{(\alpha_n' + c)}{(\alpha_n' - c)} \tag{7.2}$$

In order to simplify (7.2) it is convenient to relate the film thickness to electron mean free path by

$$b = N\lambda_o \tag{7.3}$$

where N is an unknown number related to the number of collisions experienced by an electron passing through the film. (In a given experiment, b is assumed to be known, while λ_o and N are unknowns.) Typical values for N have been chosen and the corresponding roots obtained are shown in Table 1.

TABLE 1. CALCULATION OF FACTORS OCCURRING IN THE COLD-CATHODE EMITTER

	FACTORS			
N	α'	$\lambda_o^2 \alpha_n'^2$	$[b + c/(\alpha_n'^2 - c^2)]$	$[\alpha_n'/(\alpha_n' + c)]\ e^{\alpha b}/2F_1$
5	$\frac{0.075}{\lambda_o} = 2.85c$	0.006	$2\lambda_o N(1.03)$	0.54
10	$\frac{0.052}{\lambda_o} = 2.05c$	0.003	$2\lambda_o N(1.09)$	0.52
25	$\frac{0.035}{\lambda_o} = 1.385c$	0.001	$2\lambda_o N(1.34)$	0.53
	$\lambda_o^2 \beta_1^2$	$\lambda_o^2 \beta_2^2$	$\lambda_o^2 \beta_5^2$	$[b - c/(\beta_n^2 + c^2)]$
5	0.1	0.4	2.5	$N\lambda_0$
10	0.025	0.1	0.625	$N\lambda_0$
25	0.004	0.016	0.100	$N\lambda_0$
	$\{\ \}_{\zeta_o = 1/2}$	$[\ ]_{\zeta_o = 1/2}$	$\{\ \}_{\zeta_o = 1}$	$[\ ]_{\zeta_o = 1}$
5	-0.40	0.14	-0.20	0.34
10	-0.48	0.04	-0.45	0.07
25	-0.39	0.14	-0.46	0.07

In a similar manner, roots β_n are obtained by solving graphically:

$$y_2 = \sin 2\beta_n b = \frac{-2\beta_n c}{(\beta_n^2 + c^2)} \tag{7.4}$$

Figure 8 shows the form of (7.4) for typical values of N. The minimum in $-2\beta_n c/(\beta_n^2 + c^2)$ occurs at $\beta_n = c$, where it has the value -1; and the roots occur approximately where

$$\beta_n = \frac{n\pi}{2b} = \frac{n\pi}{2N\lambda_o} \tag{7.5}$$

where n is an integer [Eq. (7.5) is more accurate for the larger values of n, but is normally also sufficiently accurate for n = 1].

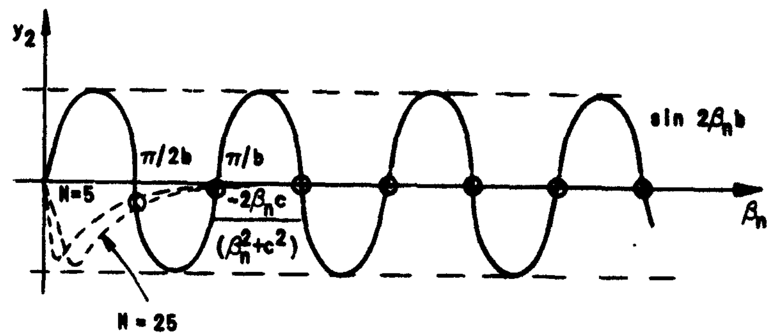


FIG. 8. GRAPHS FOR THE EVALUATION OF β_n IN THE COLD-CATHODE EMITTER.

I_r is obtained by substituting (6.36) in (6.32) giving:

$$\frac{I_r}{I_o} = \frac{9}{19}\frac{\lambda_o\zeta_o}{\langle E'\rangle}\left\{\frac{\alpha_n'}{(\alpha_n'+c)}\frac{\exp\left[\frac{(\alpha_n'^2\lambda_o^2-1)}{2\zeta_o}\varphi_2\right]e^{\alpha_n b}}{\left[b+\frac{c}{(\alpha_n'^2-c^2)}\right](1-\alpha_n'^2\lambda_o^2)}\right.$$

$$-\sum_{\beta_n>0}\frac{2\lambda\beta_n\sin\beta_n b\exp\left[\frac{-(\beta_n^2\lambda_o^2+1)}{2\zeta_o}\varphi_2\right]}{(\beta_n^2\lambda_o^2+1)(1+2\lambda c)\left[b-\frac{c}{(\beta_n^2-c^2)}\right]}$$

$$\left.+\sum_{\beta_n>0}\frac{\cos\beta_n b\exp\left[\frac{-(\beta_n^2\lambda_o^2+1)}{2\zeta_o}\varphi_2\right]}{(\beta_n^2\lambda_o^2+1)(1+2\lambda c)\left[b-\frac{c}{(\beta_n^2+c^2)}\right]}\right\} \qquad (7.6)$$

The various factors appearing in (7.6) have been calculated for typical values of N and appear in Table 1. Inspection reveals that further simplification of the equation is possible since $\alpha_n'^2\lambda_o^2 << 1$,

$$\left.\begin{aligned} &\left[b - \frac{c}{(\beta_n^2 + c^2)}\right] \sim N\lambda_o \\ \text{and} \\ &\left[b + \frac{c}{(\alpha_n^2 - c^2)}\right] = 2N\lambda_o(F_1) \end{aligned}\right\} \tag{7.7}$$

where F_1 is very close to unity. Thus

$$\frac{I_r}{I_o} = \frac{8}{19}\frac{\lambda_o\zeta_o \exp\left(-\frac{\varphi_2}{2\zeta_o}\right)}{(N\lambda_o)\langle E'\rangle}\left[\frac{1}{2F_1}\left(\frac{\alpha_n'}{\alpha_n' + c}\right)\exp(\alpha_n' b) + \left\{\sum_{\beta_n>0}\frac{\exp\left[-(\beta_n^2\lambda_o^2)\frac{\varphi_2}{2\zeta_o}\right]\left[\cos\left(\frac{n\pi}{2}\right) - \left(\frac{n\pi}{N}\right)\sin\left(\frac{n\pi}{2}\right)\right]}{(\beta_n^2\lambda_o^2 + 1)(1 + 2\lambda c)}\right\}\right] \tag{7.8}$$

The factor contained by the large brackets [ ] is a very complicated function of N, b, ζ_o, and φ_2. However, for given initial conditions, it does not vary greatly with N. This has been demonstrated in a particular case: consider a gold film similar to that used by Mead [Ref. 17] in which φ_2 = 10 ev. The values of { } and [ ] have been calculated for two different values of ζ_o, namely 1/2 ev and 1 ev [in which case $(\varphi_2/2\zeta_o)$ has the values 10.4 and 5.2, respectively]. It must be emphasized that the values obtained are very rough and involve taking the sum of two converging series, each consisting of partially canceling terms. Thus it is reasonable to take [ ] approximately constant and of order 0.1. From (7.8)

$$\frac{I_r}{I_o} \propto \frac{\lambda_o\zeta_o \exp\left(-\frac{\varphi_2}{2\zeta_o}\right)}{\langle E'\rangle}\cdot\frac{1}{b} \tag{7.9}$$

predicting that a graph of (I_r/I_o) against $(1/b)$ should be a straight line whose slope depends on λ_o, ζ_o, φ_2, and $\langle E' \rangle$ as indicated by (7.9). Departures of the experimental points from such a straight line may be attributed to errors involved in assuming that [ ] is a constant, indicating that attempts at obtaining self-consistent values of [ ] as a function of $1/b$ should be made. In this way, a more accurate value for the slope should be attainable.

By obtaining a series of results as a function of the energy range of the incident beam and also of the work function at the emitting surface, detailed information should be obtained about the variation of the actual mean free path λ and energy loss per collision ζ as functions of the energy of the primary electron.

The only set of results available at present to which diffusive motion might apply are those by Mead [Ref. 17, Fig. 1]. If his results are replotted in the form of ratio of emitted current to input current as a function of $1/b$, a straight line is obtained for the two input energy spectra considered. It is found that the ratio of the slopes (for high-energy electrons to low-energy electrons) is given by:

$$\frac{0.435}{0.130} = \frac{\lambda_o^h \zeta_o^h \exp\left(-\frac{\varphi_2}{2\zeta_o^h}\right)}{\lambda_o^\ell \zeta_o^\ell \exp\left(-\frac{\varphi_2}{2\zeta_o^\ell}\right)}$$

where the indices h and ℓ refer to high- and low-energy incident electrons. Now $\zeta_o^h > \zeta_o^\ell$ and thus any change in λ_o with energy is likely to be outweighed by ζ_o variations with energy. The only method of getting more accurate information would involve additional experiments measuring the total yield for a given incident-electron energy beam as the barrier height φ_2 is varied.

B. THE PHOTOELECTRIC PROBLEM

When hot electrons are produced by shining light on the metal film, the two surfaces will behave similarly as far as hot electrons are concerned and thus the reflection coefficients will be equal. Thus $c_1 = c_2 \sim 1/38\,\lambda$, taking $R = 0.9$ as before. Using this relationship, the nonzero root is found from (6.26) in the form:

$$y_1 = e^{\alpha_n' b} = \frac{(\alpha_n' + c)}{(\alpha_n' - c)} \qquad (7.10)$$

Now $(\alpha_n' + c/\alpha_n' - c) = y_1$ is a rectangular hyperbola having asymptotes

$$y_1 = 1 \quad \text{and} \quad \alpha_n' = c$$

In a similar manner, the roots β_n are found by solving:

$$y_2 = \cos \beta_n b = \pm \frac{(c^2 - \beta_n^2)}{(c^2 + \beta_n^2)} \qquad (7.11)$$

The graphs have the form shown in Fig. 9 and thus

$$\beta_n = \frac{n\pi}{b} = \frac{n\pi}{N\lambda_o} \quad \text{where } n = 1, 2, 3, \ldots \qquad (7.12)$$

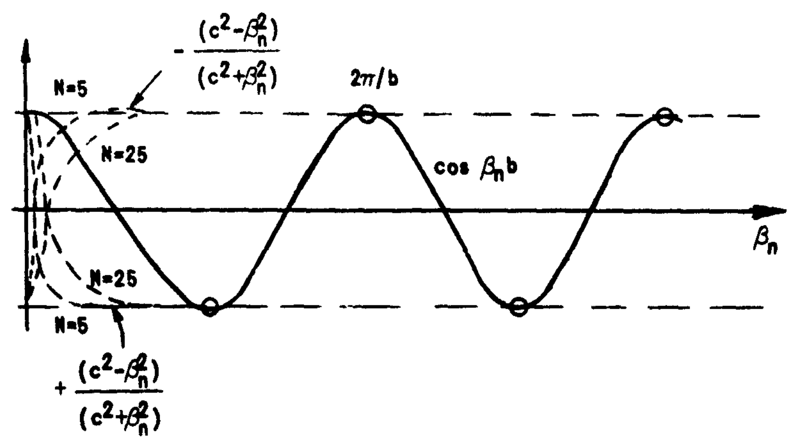


FIG. 9. GRAPHS FOR THE EVALUATION OF β_n IN THE PHOTOELECTRIC DEVICE.

From (6.32),

$$\frac{I_r}{I_o} = 16\lambda_o^2\, c\zeta_o \exp\left(-\frac{\varphi}{2\zeta_o}\right)\left[\left(\frac{\alpha_n'}{\alpha_n'+c}\right)^2 \frac{\exp\left[(\alpha_n^2\lambda_o^2)\frac{\varphi}{2\zeta_o}\right]\exp(\alpha b)}{(1-\alpha_n^2\lambda_o^2)\left[b+\frac{2c}{(\alpha_n^2-c^2)}\right]} + \sum_{\beta_n>0}\right.$$

$$\left.\left\{\left(\frac{\beta_n}{\beta_n+c}\right)^2 \frac{\exp\left[-(\beta_n^2\lambda_o^2)\frac{\varphi}{2\zeta_o}\right]\begin{bmatrix}(1-2\lambda c)\cos\beta_n b\\ -\left(\frac{c}{\beta_n}+2\lambda\beta_n\right)\sin\beta_n b\end{bmatrix}}{(1+\beta_n^2\lambda_o^2)\left[b-\frac{2c}{(\beta_n^2+c^2)}\right](1+2\lambda c)}\right\}\right] \tag{7.13}$$

and, from*(6.34),

$$\frac{I_f}{I_o} = 16\lambda_o^2\, c\zeta_o \exp\left(\frac{-\varphi}{2\zeta_o}\right)\left[\left(\frac{\alpha_n'}{\alpha_n'-c}\right)^2 \frac{\exp\left[(\alpha_n^2\lambda_o^2)\frac{\varphi}{2\zeta_o}\right]}{(1-\alpha_n^2\lambda_o^2)\left[b+\frac{2c}{(\alpha_n'^2-c^2)}\right]}\right.$$

$$\left. + \sum_{\beta_n>0}\left\{\left(\frac{\beta_n}{\beta_n+c}\right)^2 \frac{\exp\left[-(\lambda_o^2\beta_n^2)\frac{\varphi}{2\zeta_o}\right]}{\left[b-\frac{2c}{(\beta_n^2+c^2)}\right](1+\beta_n^2\lambda_o^2)}\right\}\right] \tag{7.14}$$

Table 2 contains values of α_n' and the various factors derived from it for typical values of N. Inspection reveals that both (7.13) and (7.14) may be further simplified as $\alpha_n^2\lambda_o^2 << 1$,

$$\left[b + \frac{2c}{(\alpha_n'^2 - c^2)}\right] = 2\lambda_o N(F_2)$$

$$\left[b - \frac{2c}{(\beta_n^2 + c^2)}\right] \approx N\lambda_o$$

TABLE 2. CALCULATION OF FACTORS OCCURRING IN THE PHOTOELECTRIC DEVICE

N	FACTORS			
	α_n'	$[b + 2c/(\alpha_n'^2 - c^2)]$	$[b - 2c/(\beta_n^2 + c^2)_{n\geq 1}]$	$\beta_n^2/(\beta_n^2 + c^2)_{n>1}$
5	$\frac{0.104}{\lambda_o} = 3.95c$	$2\lambda_o N(1.02)$	$\lambda_o N$	1
10	$\frac{0.075}{\lambda_o} = 2.85c$	$2\lambda_o N(1.07)$	$\lambda_o N$	1
25	$\frac{0.049}{\lambda_o} = 1.85c$	$2\lambda_o N(1.13)$	$\lambda_o N$	1
	$\lambda_o^2\beta_1^2$	β_o	$M = 1/2F_2\ e^{\alpha b}[\alpha_n'/(\alpha_n' + c)]^2$	$P = 1/2F_2[\alpha_n'/(\alpha_n' - c)]^2$
5	0.4	4.0c	0.36	0.90
10	0.1	1.75c	0.35	1.17
25	0.016	1.5c	0.37	1.71

and

$$\left(\frac{\beta_n}{\beta_n + c}\right)^2 \approx 1 \qquad \text{for} \quad n \geq 1$$

From (7.13), therefore,

$$\frac{I_r}{I_o} = \frac{8}{19}\,\frac{\lambda_o\zeta_o\ \exp\left(\frac{-\varphi}{2\zeta_o}\right)}{(\lambda_o N)}\left[M + R\sum_{n\geq 1}(-1)^n\,Q_n + Q_o\left\{R\cos(\beta_o b) - \frac{\left(\frac{c}{\beta_o} + 2\lambda\beta_o\right)}{(1 + 2\lambda c)}\sin(\beta_o b)\right\}\right] \tag{7.15}$$

and from (7.14),

$$\frac{I_f}{I_o} = \frac{8}{19}\,\frac{\lambda_o\zeta_o\ \exp\left(\frac{-\varphi}{2\zeta_o}\right)}{(\lambda_o N)}\left[P + \sum_{n\geq 1} Q_n + Q_o\right] \tag{7.16}$$

where

$$
\left.
\begin{aligned}
M &= \frac{e^{\alpha_n' b}}{2F_2}\left(\frac{\alpha_n'}{\alpha_n' + c}\right)^2 \quad \text{(approximately constant and equal to 0.36)} \\
P &= \frac{1}{2F_2}\left(\frac{\alpha_n'}{\alpha_n' - c}\right)^2 \\
Q_n &= \sum_{n \geq 1} \frac{\exp\left[-(\beta_n^2 \lambda_o^2)\,\frac{\varphi}{2\zeta_o}\right]}{(1 + \beta_n^2 \lambda_o^2)} \\
&\text{and} \\
Q_o &= \frac{\beta_o^2}{(\beta_o^2 + c^2)}\,\frac{1}{\left[1 - \frac{2c}{(\beta_o^2 + c^2)}\cdot\frac{1}{\lambda_o N}\right]}
\end{aligned}
\right\} \quad (7.17)
$$

For photoelectrons, the average energy loss per collision must be small since the electrons have an energy nearer to the Fermi energy than those generated in the cold-cathode device. Estimates of the two factors [ ] appearing in (7.15) and (7.16) have again been made for various values of $(\varphi/2\zeta_o)$ and appear to vary only slightly with N. Thus both I_r and I_f appear to depend on $(1/b)$.

Since it is normally possible to measure both (I_r/I_o) and (I_f/I_o) as functions of thickness, more information concerning the factors is forthcoming than in the cold-cathode experiments as

$$
\frac{I_r}{I_f} = \frac{M + R\sum\limits_{n \geq 1} (-1)^n Q_n + Q_o\left[R\cos\beta_o b - \frac{\left(\frac{c}{\beta_o} + 2\lambda\beta_o\right)}{(1 + 2\lambda c)}\sin\beta_o b\right]}{\left[P + \sum\limits_{n \geq 1} Q_n + Q_o\right]} \qquad (7.18)
$$

At the present time, it is not possible to compare the predictions of Eqs. (7.15), (7.16), and (7.18) with experimental data since such results correspond to the cases where the hot-electron motion is unlikely to be diffusive. (Accurate results would involve self-consistent solutions once again.)

Chapter V described the diffusive motion of the electrons generated within the metal film. Normally $a_1 \gg \lambda$ and thus, from (5.26), $i_r/i_f \doteq e^{-b/\lambda}$, a form characteristic of ballistic motion. It would appear, therefore, that the incorporation of a high reflection coefficient is responsible for changing this dependence to $(1/b)$; this result seems quite plausible on physical grounds.

VIII. CONCLUSIONS

The model used to describe the motion of hot electrons within a metal film is one of diffusion. It assumes that most electrons undergo reflection at the film surfaces, that they suffer several collisions to randomize their motion, and that the differential-scattering cross section contains only s-type and p-type angular dependences. The result of introducing electron multiplication into the model produces the factor $\exp(-\varphi/2\zeta_o)$ in (7.8), where $\varphi = \varphi_2$; and also in (7.15) and (7.16). It also makes $\langle \cos\theta \rangle_{av}$ take the value 2/3 in (4.11). If this effect was completely absent, the predicted variation of yield with work function of the emitting surface would not arise and, also, X would equal $+\lambda^2$. On the other hand, partial electron multiplication would change $\exp[-(\varphi_2/2\zeta_o)]$ to $\exp[-(\varphi_2/t\zeta_o)]$, where t lies between 1 and 2 and depends on the energy of the incident hot-electron beam, and also on X.

In a given device, it would be very desirable to decide unambiguously whether the motion of the hot electrons is ballistic or diffusive. It would appear easy to do so at first sight, as ballistic motion is characterized by an exponential decay law $e^{-b/\Lambda}$, whereas an approximate 1/b variation for diffusive motion is predicted here. However, it was seen that the results of Mead, for example, gave straight-line dependences in both cases, although this might not be the case if experimental scatter of the points could be reduced. Thus, at present, such differentiation between the two types of motion is inconclusive.

The theory presented in the preceding chapters should be applicable to other problems in physics. The motion of hot electrons in semiconductors is an example--in germanium and silicon the electrons lose energy by collisions with optical phonons and their motion is certainly diffusive [Bartelink, Ref. 33]. The only modifications necessary would be to remove the terms introduced by electron multiplication and alter the values used for the parameters. The theory should also be applicable to the slowing down of low-energy neutrons in a material. The mathematics involved also resembles that of heat conduction and thus may presumably be interpreted for the case of a thin slab of material.

REFERENCES

1. J. C. Fisher and I. Giaever, *J. Appl. Phys., 32,* 1961, p. 172.
2. R. Holm, *J. Appl. Phys., 22,* 1951, p. 569.
3. V. Rojansky, *Introductory Quantum Mechanics*, Prentice-Hall, Inc., New York, 1938, par. 34-38.
4. O. E. Kane, *J. Appl. Phys., 32,* 1961, p. 83.
5. A. R. Forsyth, *A Treatise on Differential Equations*, McMillan and Co., Ltd., London, 1921, pp. 121, 565.
6. A. D. Smirnov, *Tables of Airy Functions and Special Confluent Hypergeometric Functions*, Pergamon Press, New York, 1960.
7. A. Sommerfeld and H. Bethe, *Handbuch der Physik von Geiger und Scheel*, Verlag, Julius Springer, Berlin, 1933, 2 Aufl. Vol. 11/2, pp. 450.
8. E. J. Ryder and W. Shockley, *Phys. Rev., 81*, 1951, p. 139.
9. E. J. Ryder, *Phys. Rev., 90*, 1953, p. 766.
10. W. Shockley, *Bell Syst. Tech. J., 30*, 1951, p. 990.
11. J. L. Moll, N. I. Meyer, and D. J. Bartelink, *Phys. Rev. Lett., 7,* 1961, p. 87.
12. C. A. Mead, *Proc. IRE, 48,* 1960, p. 359.
13. I. Giaever, *Phys. Rev. Lett., 5*, 1960, pp. 147 and 464.
14. J. Nicol, S. Shapiro, and P. H. Smith, *Phys. Rev. Lett., 5*, 1960, p. 461.
15. J. P. Spratt, F. Schwarz, and W. M. Kane, *Phys. Rev. Lett., 6,* 1961, p. 341.
16. R. N. Hall, *Solid State Electronics, 3,* 1961, p. 320.
17. C. A. Mead, *Phys. Rev. Lett., 8,* 1962, p. 57.
18. W. G. Spitzer, C. R. Crowell, and M. M. Atalla, *Phys. Rev. Lett., 8,* 1962, p. 57.
19. F. J. Blatt, *Solid State Physics, 4,* 1957, p. 199.
20. A. J. Dekker, *Solid State Physics,* Prentice-Hall, Inc., New York, 1957.
21. J. L. Moll, "Studies of Microplasmas and High-Field Effects in Silicon," QSR No. 7 (SEL-62-041), Stanford Electronics Laboratories, Stanford, Calif., 1 Dec to 28 Feb 1962.
22. J. J. Quinn, *Phys. Rev., 126,* 1962, p. 1453.
23. P. A. Wolff, *Phys. Rev., 95,* 1954, p. 1415.
24. R. Marshak, *Rev. Mod. Phys., 19,* 1947, p. 185.
25. M. L. Goldberger, *Phys. Rev., 74,* 1948, p. 1269.
26. S. Sze, Project 1659, *Qtr. Res. Rev. No. 1* (SEL-62-109), Stanford Electronics Laboratories, Stanford, Calif., 1 Apr to 30 Jun 1962, p. 62.

REFERENCES (Cont'd)

27. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics (Non-Relativistic Theory)*, Pergamon Press, New York, 1958.

28. J. Lindhard, Kgl. Danske Videnskab. *Mar-fys. Medd, 28*, No. 8, 1954.

29. J. Weymouth, *Phys. Rev., 84*, 1951, p. 766.

30. J. C. Slater and G. F. Koster, *Phys. Rev., 94*, 1954, p. 1498.

31. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, Oxford, 1947.

32. E. Fermi, *Nuclear Physics*, University of Chicago Press, Chicago, Ill., 1956.

33. D. J. Bartelink, "Hot-Electron Emission from Shallow P-N Junctions in Silicon," Rept SEL-62-021, (TR No. 1654-2), Stanford Electronics Laboratories, Stanford, Calif., Apr 1962.

ELECTRON DEVICES DISTRIBUTION LIST

April 1963

GOVERNMENT

Commanding Officer, USAELRDL
Ft. Monmouth, N.J.
1 Attn: SIGRA/SL-PRM
1 Attn: SIGRA/SL-PA
2 Attn: SIGRA/SL-SC
1 Attn: SIGRA/SL-ES, H. Cohen
1 Attn: TDC Bldg. No. 27
1 Attn: H. Brett
1 Attn: Dr. E. M. Reilley, Dir.
Inst. for Explor. Res.
1 Attn: SIGRA/SL-ADT-E
1 Attn: SIGRA/SL-ADT
2 Attn: Dir. Commun. Dept.

U.S. Army Signal Material
Support Agency
Ft. Monmouth, N.J.
1 Attn: SIGMS-ADJ
1 Attn: Engr. Proc edures Br.
Millard Rosenfeld

Electronic Warfare Div., USASCS
Ft. Monmouth, N.J.
1 Attn: H. Allen

Director of Research, USAELRDL
Ft. Monmouth, N.J.
1 Attn: Harold A. Zahl

Marine Corps Liaison Office
1 Ft. Monmouth, N.J.

Corps of Engr. Liaison Office
1 Ft. Monmouth, N.J.
1 Attn: SIGRA/SL-LNE

Army Liaison Office
1 Lexington, Mass.

U.S. Army Engr. R and D Lab
Ft. Belvoir, Va.
1 Attn: Tech. Doc. Ctr.

Signal Corps Liaison Office
Cambridge, Mass.
1 Attn: A. D. Bedrosian

Commanding General
U.S. Army Ordnance Missile Command
Redstone Arsenal, Alabama
1 Attn: ORDXM-RFE, Mr. Lindberg
1 Attn: Technical Library

Commanding Officer, Frk.Rd.Asnel.
Philadelphia 37, Pa.
1 Attn: ORDBA 1520
1 Attn: Library Br., 0270

Commanding Officer
Watertown Arsenal
Watertown, Mass.
1 Attn: OMRO

Ballistics Research Lab
Aberdeen Proving Ground, Md.
1 Attn: V. W. Richard, BML

Commanding Officer
White Sands Missile Range, N.M.
1 Attn: SIGWS-MEW

Commanding General
U.S. Army Electr. Proving Ground
Ft. Huachuca, Arizona
1 Attn: Tech. Library

Chief of Naval Research
Dept. of the Navy
Washington 25, D.C.
1 Attn: Code 427

Commanding Officer
ONR Branch Office
1000 Geary St.
1 San Francisco 9, Calif.

Chief Scientist, ONR Branch Office
1030 E. Greene St.
1 Pasadena, Calif.

Commanding Officer, ONR Branch Off.
John Crerar Library Bldg.
86 E. Randolph St.
1 Chicago 1, Ill.

Commanding Officer, ONR Branch Off.
207 W 24th St.
1 New York 11, N.Y.

New York Naval Shipyard
Material Lab Library, Bldg. 291
Brooklyn, N.Y.
1 Attn: Code 911B, M. Rogofsky

Officer-in-Charge, ONR
Navy No. 100, Box 39, Fleet P.O.
16 New York, N.Y.

U.S. Naval Research Lab
Washington 25, D.C.
6 Attn: Code 2000
1 Attn: Code 2027
1 Attn: Code 5200
1 Attn: Code 5240
1 Attn: Code 5260
1 Attn: Code 5260
1 Attn: Code 5300
1 Attn: Code 5400
1 Attn: Code 5430

Chief, Bureau of Ships
Washington 25, D.C.
1 Attn: Code 691A1
3 Attn: Code 670B

Chief, Bureau of Naval Weapons
Washington 25, D.C.
1 Attn: Code RAAV-6
1 Attn: Code RREN-3
1 Attn: Code RAAV-44

Chief of Naval Operations
Navy Dept.
Washington 25, D.C.
1 Attn: Code Op 94T - Pentagon 4C717
1 Attn: Code Op 352

Commanding Officer and Director
U.S. Navy Electronics Lab
1 San Diego 52, Calif.

U.S. Naval Post Graduate School
Monterey, Calif.
1 Attn: Tech. Reports Librarian

Weapons System Test Division
Patuxent River, Md.
1 Attn: Library

Commander U.S. Naval Missile Ctr
Pt. Mugu, Calif.
1 Attn: N 3232

U.S. Naval Weapons Lab
Dahlgren, Va.
1 Attn: Tech. Library

Commanding Officer
U.S. Naval Air Dev't. Ctr.
Johnsville, Pa.
1 Attn: NADC Library

U.S. Naval Avionics Facility
Indianapolis 18, Indiana
1 Attn: Station Library

Commanding Officer
U.S. Army Research Office
Durham, N.C.
3 Attn: CRD-AAIP

Department of the Army
Washington 25, D.C.
1 Attn: Research Support Div.

Chief, Research and Dev't Div.
Office of the Chief Signal Officer
Washington 25, D.C.
1 Attn: SIGRD-9A

Commanding General
U.S. Army Electronics Command
Ft. Monmouth, N.J.
3 Attn: AMSEL-AD

Chief, Library Branch
Dept. of the Army
Office of the Chief of Engineers
1 Washington 25, D.C.

OASD (R and E); Rm. 3C-128
The Pentagon
Washington 25, D.C.
1 Attn: Tech. Library

Hq., USAF
Washington 25, D.C.
1 Attn: Mr. Harry Mulkey

Chief of Staff, USAF
Washington 25, D.C.
1 Attn: AFDRT-ER

U.S. Army Signal Liaison Office
Aeronautical Systems Division
Wright-Patterson AFB, Ohio
1 Attn: ASDL-9

Commander, ASD
Wright-Patterson AFB, Ohio
1 Attn: ASNPRO
1 Attn: ASRNCF-1
1 Attn: ASRNEB, Mr. D. R. Moore
1 Attn: ASRNCS-3
1 Attn: ASRNRD
1 Attn: ASRNCC-1
1 Attn: ASAPRL
1 Attn: ASRNEM-2

Commander, AF Inst. of Technology
Wright-Patterson AFB, Ohio
1 Attn: MCLI-Library

Executive Director
AF Office of Scientific Res.
Washington 25, D.C.
1 Attn: SRYA

School of Aerospace Medicine
Brooks AFB, Ohio
1 Attn: SMAP

Air Proving Ground
Eglin AFB, Fla.
1 Attn: APGTRI, Tech. Library

Commander
Holloman AFB, New Mexico
1 Attn: MDR

AF Special Weapons Center
Kirtland AFB, New Mexico
1 Attn: SWOI

Director, Air University Library
Maxwell AFB, Ala.
1 Attn: CR 4582

AF Missile Test Center
Patrick AFB, Florida
1 Attn: AFMTC Tech. Library, MU-135

Commanding General, RADC
Griffiss AFB, Rome, N.Y.
2 Attn: RCRW
1 Attn: RAWCL
1 Attn: Document Library, RAALD
1 Attn: RAWED/L. Suss
1 Attn: RAIS/J. Pallik
1 Attn: RAWEC, T. J. Domurat
1 Attn: RAWE, Haywood Webb

Commander, AFCRL
Bedford, Mass.
1 Attn: CRTOTT-2, Electronics
1 Attn: CRXL, Scientific Lib.
1 Attn: Elec. Res. Lab. (CRR)
1 Attn: CRRCPV
1 Attn: CRRCP, Phys. Elec. Br.

Commander, AF Ballistics
Missile Division
Los Angeles, Calif.
1 Attn: WDLPM-1-TDC

Hq., AFSC Andrews AFB
Washington 25, D.C.
1 Attn: SCEI
1 Attn: SCTAE

Director, R and D
Hq., USAF
Washington 25, D.C.
1 Attn: AFRDR-NU (Lt. Col. Booth)

Assistant Secy. of Defense(R and D)
Washington 25, D.C.
1 Attn: Technical Library

Office of Director of Defense
R and E
1 Washington 25, D.C.

Office of Assistant Sec'y. of Def.
Washington 25, D.C.
1 Attn: AE

Department of Defense
Washington 25, D.C.
1 Attn: Code 121A, Tech. Lib.

Institute for Defense Analyses
1666 Connecticut Ave.
Washington 6, D.C.
1 Attn: W. E. Bradley

Director
Weapons System Evaluation Group
1 Washington 25, D.C.

Central Intelligence Agency
Washington 25, D.C.
1 Attn: OCR Standard Dist.

U.S. Coast Guard
Washington, D.C.
1 Attn: EEE

Advisory Group on Electron Devices
New York 13, N.Y.
2 Attn: Harry Sullivan

Commander, ASTIA
Arlington 12, Va.
20 Attn: TISIA

Advisory Group on Reliability of
Electronic Devices
The Pentagon
1 Washington 25, D.C.

Office of Technical Services
1 Washington 25, D.C.

NASA, Langley Res. Ctr.
Hampton, Va.
1 Attn: C. L. Fricke, IRD-CRS

Commanding Officer
Diamond Ordnance Fuse Labs
Washington 25, D.C.
1 Attn: T. M. Liimatainen
2 Attn: ORDTL 930, Dr. R. Young
1 Attn: ORDTL-450-638-R. A. Comyn

Director
U.S. National Bureau of Standards
Washington 25, D.C.
1 Attn: Sec. 14.1, G. Shapiro

U.S. Department of Commerce
National Bureau of Standards
Boulder, Colo.
1 Attn: Central Radio Prop. Lab.
1 Attn: Library
2 Attn: Miss J. V. Lincoln, Chief
Radio Warning Service Sec.

Director
National Security Agency
Ft. George G. Meade, Md.
1 Attn: R 31
1 Attn: R 304, W. R. Boenning
2 Attn: C3/TDL, Rm. 2c087
1 Attn: R 42
1 Attn: C15

Chief
U.S. Army Security Agency
2 Arlington 12, Va.

UNIVERSITIES

University of Arizona
Tucson, Arizona
1 Attn: Dr. D. J. Hamilton

California Institute of Technology
Pasadena, Calif.
1 Attn: Prof. L. M. Field
1 Attn: Documents Library

University of California
Berkeley, Calif.
1 Attn: R. M. Saunders, Chm. EE Dept.
1 Attn: Prof. J. R. Singer
1 Attn: Dr. R. K. Wakerling, Rad. Lab.

University of California
Livermore, Calif.
1 Attn: Tech. Info. Div.

University of California
Los Angeles 24, Calif.
1 Attn: C. T. Leondes, Prof. of Eng.

Laboratories for Applied Sciences
University of Chicago
Chicago 37, Ill.
1 Attn: Library

Cornell University
1 Ithaca, N.Y.

Drexel Institute of Technology
Dept. of EE
Philadelphia 4, Pa.
1 Attn: F. B. Haynes

DePaul University
1036 W. Beldon Ave.
Chicago 14, Ill.
1 Attn: Dr. J. J. Hupert

University of Florida
1 Gainesville, Fla.
1 Attn: Prof. W. E. Lear
1 Attn: M. J. Wiggins, Engr.

George Washington University
Washing 6, D.C.
1 Attn: N. T. Grisamore
Principal Investigator

Georgia Institute of Technology
Atlanta, Georgia
1 Attn: Mrs. H. Crosland, Librarian
1 Attn: ; F. Dixon, Engr. Exp. St.

Harvard University
Cambridge 30, Mass.
2 Attn: Mrs. E. Farkas, Librarian

Illinois Institute of Technology
Chicago 16, Ill.
1 Attn: Dr. Paul C. Yuen

University of Illinois
Urbana, Ill.
1 Attn: Paul D. Coleman
1 Attn: Antenna Lab
1 Attn: Wm. Perkins
1 Attn: Prof. D. Alpert
1 Attn: Library Serials Dept.

Johns Hopkins University
Silver Spring, Md.
1 Attn: A. W. Nagy
1 Attn: N. H. Choksy

Carlyle Barton Lab
Johns Hopkins University
Charles and 34th Streets
Baltimore 18, Md.
1 Attn: Librarian
1 Attn: J. M. Minkowski

Linfield Research Institute
McMinnville, Oregon
1 Attn: G. N. Hickok, Director

Marquette University
Milwaukee, Wisconsin
1 Attn: Arthur C. Moeller

MIT
Research Lab of Electronics
Cambridge 39, Mass.
1 Attn: Document Rm. 26-327
1 Attn: John E. Ward

MIT
Electronic Systems Lab
1 Lexington 73, Mass.
1 Attn: Library A-229
1 Attn: Dr. W. I. Wells

Director
Cooley Electronics Lab, N. Campus
University of Michigan
1 Ann Arbor, Michigan

University of Michigan
Ann Arbor, Michigan
1 Attn: Elec. Phys. Lab.
1 Attn: Prof. J. E. Rowe
1 Attn: Tech. Doc. Service

University of Minnesota
Minneapolis 14, Minn.
1 Attn: Prof. A. Van der Ziel
1 Attn: W. M. Nunn, Jr.

University of Nevada
Reno, Nevada
1 Attn: Dr. R. A. Manhart

New York University
New York 53, N.Y.
1 Attn: R. F. Cotellessa

Northwestern University
Evanston, Ill.
1 Attn: W. S. Toth

University of Notre Dame
South Bend, Indiana
1 Attn: Eugene Henry

Ohio State University
1 Columbus 10, Ohio
1 Attn: Prof. E. M. Boone

Ohio University
Athens, Ohio
1 Attn: R. C. Quisenberry

Oregon State University
Corvallis, Oregon
1 Attn: H. J. Oorthuys

Polytechnic Institute
Brooklyn, N.Y.
1 Attn: Leonard Shaw
1 Attn: Dept. of EE

Polytechnic Institute
Route 110
Farmingdale, N.Y.
1 Attn: Librarian

Princeton University
Dept. of EE
1 Princeton, N.J.

Purdue University
Lafayette, Indiana
1 Attn: Library

Rensselaer Polytechnic Institute
1 Troy, N.Y.

University of Rochester
Rochester 20, N.Y.
1 Attn: Dr. G. H. Cohen

Rose Polytechnic Institute
Dept. of EE
Terre Haute, Indiana
1 Attn: Dr. C. C. Rogers

Rutgers University
Newark 2, N.J.
1 Attn: Dr. Charles Pine

Stanford Research Institute
Menlo Park, Calif.
1 Attn: External Reports G-037

University of Texas
1 Austin, Texas

Texas Technological College
EE Dept.
Lubbock, Texas
1 Attn: Prof. Harold Spuhler

University of Utah
Salt Lake City, Utah
1 Attn: Richard W. Grow

Villanova University
Villanova, Pa.
1 Attn: Thos. C. Gabriele, Asst.Prof.

University of Virginia
Charlottesville, Va.
1 Attn: Alderman Library

University of Washington
Seattle 5, Wash.
1 Attn: A. E. Harrison

Yale University
Engineering Library
New Haven, Conn.
1 Attn: Sloane Physics Lab
1 Attn: Dept. of EE
1 Attn: Dunham Lab

INDUSTRY

Admiral Corp.
Chicago 47, Ill.
1 Attn: E. N. Roberson, Librarian

Airborne Instruments Lab
Deer Park, L.I., N.Y.
1 Attn: J. N. Dyer, VP and Tech. Dir.

Amperex Corp.
Hicksville, L.I., N.Y.
1 Attn: S. Barbasso, Proj. Engr.

Argonne National Lab
Argonne, Ill.
1 Attn: Dr. O. C. Simpson, Director
Solid State Science Div.

Autonetics
Downey, Calif.
1 Attn: Tech. Lib. 3040-3

Bell Telephone Labs
Murray Hill, N.J.
1 Attn: Dr. J. K. Galt
1 Attn: Dr. J. R. Pierce
1 Attn: Dr. M. Sparks
1 Attn: J. W. Fitzwilliam
1 Attn: Mr. J. A. Morton
1 Attn: Dr. R. M. Ryder

Bendix Corp.
Southfield, Detroit, Michigan
1 Attn: A. G. Peifer

Bendix Corp.
North Hollywood, Calif.
1 Attn: Engr. Library

Boeing Scientific Res. Lab
Seattle 24, Wash.
1 Attn: G. L. Hollingsworth

Bomac Laboratories
Beverly, Mass.
1 Attn: Research Library

Baird Atomic
33 University Rd.
Cambridge 38, Mass.
1 Attn: Dr. Hornig

Columbia Radiation Lab
1 New York 27, N.Y.

Convair-A Div. of Gen. Dynamics
San Diego 12, Calif.
1 Attn: Engr. Library

Cook Research Labs
1 Morton Grove, Ill.

Cornell Aeronautical Labs., Inc.
Buffalo 21, N.Y.
2 Attn: Library
1 Attn: D. K. Plummer

Eitel-McCullough, Inc.
San Carlos, Calif.
1 Attn: Res. Library
1 Attn: W. R. Luebke

Electronic Communication, Inc.
1 Timonium, Md.

Emerson Research Labs
Silver Spring, Md.
1 Attn: S. Rattner

Ewen Knight Corp.
East Natick, Mass.
1 Attn: Library

Fairchild Semiconductor Corp.
Palo Alto, Calif.
1 Attn: Dr. V. H. Grinich

Geisler Labs
P.O. Box 252
Menlo Park, Calif.
1 Attn: Librarian

General Electric Co.
Cornell U. Defense Elec. Div.-LMED
Ithaca, N.Y.
1 Attn: Library
VIA: Commander
Wright-Patterson AFB,
Ohio, Attn: WCLGL-4

General Electric Co.
Johnson City, N.Y.
1 Attn: A. C. Warrick

General Electric TWT Product Sec.
Palo Alto, Calif.
1 Attn: Tech. Library
1 Attn: D. H. Goodman, Mgr.
Manufacturing Operations

General Electric Co.
Schenectady, N.Y.
1 Attn: Dr. P. M. Lewis

General Electric Co.
Syracuse, N.Y.
1 Attn: Document Library

General Electric Co.
Auburn, N.Y.
1 Attn: Mr. G. C. Huth

General Electric Co.
Schenectady 5, N.Y.
1 Attn: G. E. Feiker
1 Attn: E. D. McArthur

General Precision, Inc.
Link Division
1451 Calif. Ave.
Palo Alto, Calif.
1 Attn: Miss S. R. Field, Librarian

General Telephone and Electronic
Labs., Inc.
Bayside 60, N.Y.
1 Attn: L. R. Bloom

Gilfillan Brothers
Los Angeles, Calif.
1 Attn: Engineering Library

Goddard Space Flight Center
Greenbelt, Md.
1 Attn: Code 611

Hallicrafters Co.
Chicago 24, Ill.
1 Attn: Wm. Frankart

Hewlett-Packard Co.
1 Palo Alto, Calif.

Hoffman Electronics Corp.
1 Los Angeles 7, Calif.

HRB Singer, Inc.
State College, Pa.
1 Attn: Tech. Info. Ctr.

Hughes Aircraft Co.
Culver City, Calif.
1 Attn: Tech. Documents Ctr.

Hughes Aircraft Co.
Newport Beach, Calif.
1 Attn: Library

Huggins Lab
1 Sunnyvale, Calif.

IBM Corporation
San Jose, Calif.
1 Attn: Dr. Mih Yin
1 Attn: Miss M. Griffen

IBM Corporation
Technical Reports Center
Space Guidance Center
Federal Systems Division
1 Owego, N.Y.

ITT Federal Labs
Nutley 10, N.J.
1 Attn: Librarian, Mr. Ellis Mount
1 Attn: J. J. Golden
1 Attn: A. W. McEwan
1 Attn: J. LeGrand

ITT Labs
San Francisco, Calif.
1 Attn: C. V. Stanley

Jansky and Bailey
Washington 7, D.C.
1 Attn: Mr. J. Renner

Laboratory for Electronics, Inc.
Boston 15, Mass.
1 Attn: Library

LEL, Inc.
Copiague, L.I., N.Y.
1 Attn: Mr. R. S. Mautner

Lenkurt Electric Co.
San Carlos, Calif.
1 Attn: M. L. Waller, Librarian

Librascope
Glendale 1, Calif.
1 Attn: Engr. Library

Litton Industries
San Carlos, Calif.
1 Attn: Tech. Librarian

Lockheed Aircraft Corp.
Marietta, Ga.
1 Attn: Dept. 72-15

Lockheed Aircraft Corp.
Sunnyvale, Calif.
1 Attn: Dr. W. M. Harris

Lockheed Missile and Space Co.
P.O. Box 504
Sunnyvale, Calif.
1 Attn: Dept. 67-33, Bldg. 524
G. W. Price

Lockheed Electronics Co.
Plainfield, N.J.
1 Attn: C. L. Opitz

Loral Electronics Corp.
11844 Mississippi Ave.
1 Los Angeles 25, Calif.

Loral Electronics Corp.
New York 72, N.Y.
1 Attn: Louise Daniels, Librarian

The Martin Co.
SCI-Tech. Library, Mail J 398
1 Baltimore 3, Md.

The Maxson Electronics Corp.
New York 1, N.Y.
1 Attn: M. Simpson

Melpar, Inc.
Falls Church, Va.
1 Attn: Librarian

Microwave Associates, Inc.
Burlington, Mass.
1 Attn: Library

Microwave Associates, Inc.
Burlington, Mass.
1 Attn: Dr. Kenneth Mortenson

Microwave Electronics Corp.
Palo Alto, Calif.
1 Attn: Stanley F. Kaisel
1 Attn: M. C. Long

Minneapolis-Honeywell Regulator Co.
1 Riviera Beach, Fla.

Minneapolis-Honeywell Regulator Co.
Aeronautical Division
1915 Armacost Ave.
Los Angeles 25, Calif.
1 Attn: Tech. Library

Monsanto Research Corp.
Dayton 7, Ohio
1 Attn: Mrs. D. Crabtree

Monsanto Chemical Co.
St. Louis 66, Mo.
1 Attn: Mr. E. Orban, Mgr. of
Inorganic Development

Motorola, Inc.
Chicago 51, Ill.
1 Attn: Jo Goldyn, CMC Librarian
VIA: Motorola, Inc. Security
Officer, Chicago 51, Ill.

Motorola, Inc.
Scottsdale, Arizona
1 Attn: Dr. J. W. Welch, Jr.
1 Attn: Military Electronics Div.
1 Attn: John Cacheris, Mgr.
1 Attn: Paul Stancik

Norden Division of
United Aircraft Corp.
Norwalk, Conn.
1 Attn: Alice Ward, Librarian

Nortronics, Inc.
Palos Verdes Research Park
1 Palos Verdes Estates, Calif.

North American Aviation, Inc.
1 Columbus 16, Ohio

Pacific Semiconductors, Inc.
Lawndale, Calif.
1 Attn: H. Q. North

Philco Corp., W.D.L.
Palo Alto, Calif.
1 Attn: B. Herzog

Philco Corp.
Philadelphia 34, Pa.
1 Attn: F. R. Sherman, Mgr. Ed.

Philco Corp.
Blue Bell, Pa.
1 Attn: Dr. J. R. Feldmeier
1 Attn: C. T. McCoy, Res. Div.

Polarad Electronics Corp.
Long Island City 1, N.Y.
1 Attn: A. H. Sonnenschein

PRD Electronics, Inc.
Brooklyn 1, N.Y.
1 Attn: R. Casiana, Adv. Assit.

Radiation, Inc.
Melbourne, Fla.
1 Attn: Librarian

RCA
Harrison, N.J.
1 Attn: Microwave Tube Dev. Lab.
Bldg. 55
1 Attn: V. Valva

RCA
1 Moorestown, N.J.

RCA Labs
Princeton, N.J.
1 Attn: Harwick Johnson

RCA Labs
New York 13, N.Y.
1 Attn: M. C. Myers, Jr., Mgr.

RCA
Woburn, Mass.
1 Attn: Library

Ramo-Wooldridge
Canoga Park, Calif.
1 Attn: Tech. Info. Services

The Rand Corp.
Santa Monica, Calif.
1 Attn: Helen J. Waldron, Lib.

Raytheon Co.
Burlington, Mass.
1 Attn: Librarian, Microwave and
Power Tube Div.

Raytheon Co.
Bedford, Mass.
1 Attn: Mrs. I. Britton, Lib.

Raytheon Co.
1 Santa Barbara, Calif.
1 Attn: Librarian

Raytheon Co.
Waltham 54, Mass.
1 Attn: Res. Div. Lib.
1 Attn: Dr. H. Scharfman

Revere Copper and Brass, Inc.
Brooklyn, N.Y.
1 Attn: Vincent B. Lane

Sandia Corp.
Albuquerque, New Mexico
1 Attn: Mr. B. R. Allen, Lib.

S.F.D. Labs, Inc.
1 Union, N.J.
1 Attn: Dr. J. Sloom

Socony Mobil Oil Co.
Dallas 21, Texas
1 Attn: Librarian

The STL Tech. Library
1 Redondo Beach, Calif.

Sperry Gyroscope Co.
Great Neck, N.Y.
1 Attn: Leonard Swern (M.S. 3T 105)

Sperry Microwave Electronics Co.
Clearwater, Fla.
1 Attn: J. E. Pippin, Res. Sec. Head

Sperry Electron Tube Div.
1 Gainesville, Fla.

Sperry Gyroscope Co.
Great Neck, L.I., N.Y.
1 Attn: R. L. Wathen
1 Attn: T. D. Sege
1 Attn: K. Barney, Engr. Dept.

Sylvania Electric Products, Inc.
P.O. Box 997
Mt. View, Calif.
1 Attn: Tech. Library
1 Attn: J. B. Caenzle, MTL

Commanding Officer
U.S. Army Signal Electronics
Research Unit
P.O. Box 205
1 Mt. View, Calif.

Sylvania Electronics Products
Waltham, Mass.
1 Attn: Librarian
1 Attn: John Donner
1 Attn: Mr. E. E. Hollis

Technical Research Group
1 Syossett, L.I., N.Y.

Texas Instruments, Inc.
Dallas 22, Texas
1 Attn: M. E. Chun
1 Attn: Dr. W. Adcock

Tektronix, Inc.
Beaverton, Oregon
4 Attn: Dr. Jean F. DeLord, Dir. of Research

TUCOR, Inc.
Wilton, Conn.
1 Attn: Mrs. Marion E. Osband

Varian Associates
Palo Alto, Calif.
1 Attn: Tech. Lib.

Watkins-Johnson Co.
Palo Alto, Calif.
1 Attn: Dr. H. R. Johnson

Westinghouse Elec. Corp.
Baltimore 3, Md.
1 Attn: Eng. Dept.
H. B. Smith, Mgr.
1 Attn: Tech. Info. Ctr.
Mrs. A. E. Battaglia, Supv.
1 Attn: G. Ross Kilgore, Mgr.
Applied Res. Dept.

Westinghouse Elec. Corp.
Elmira, N.Y.
1 Attn: C. Miller
Elec. Tube Div.

Westinghouse Elec. Corp.
Pittsburgh 22, Pa.
1 Attn: Dr. G. C. Sziklai

Westinghouse Elec. Corp.
Pittsburgh 35, Pa.
1 Attn: J. G. Castle, Jr. 401-1B5
2 Attn: R. E. Davis

Zenith Radio Corp.
Chicago 39, Ill.
1 Attn: J. Markin