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# STATISTICAL DESIGN TECHNIQUES

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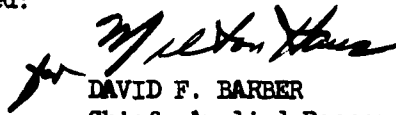
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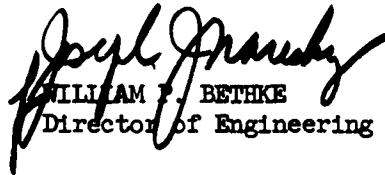
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## STATISTICAL DESIGN TECHNIQUES

### INTRODUCTION

The reliability of electronic equipment depends on both the reliability (inherent and applied) of its parts, and its design tolerances. Although it is difficult to calculate the relative contribution of each to overall equipment reliability due to a lack of valid data, component part deterioration (the cause of tolerance failures) appears to be at least equivalent to the catastrophic type of failure (Ref #1 "AGREE"). The design engineer realizes, in theory, that the ultimate reliability of his design is dependent upon the tolerance of the design to component part variations. However, all too often the design is accomplished on a rather vague basis without consideration of the statistical qualities which can lead to an optimum tolerance design. The application of statistics to design is not a new concept, however its application until recent years has been relatively neglected. It is the purpose of this discussion to trace the development of the application of statistical techniques from their first simple fundamentals to the sophisticated procedures which are being investigated for near future application.

Prior to starting on any discourse perhaps it should be mentioned that many papers, reports, etc., especially of the type which are for the most part non-computer oriented, are in general based upon similar concepts although each individual paper or report differs from any other in various particulars and areas of emphasis. Also since the progress of statistical design will be discussed from its first serious development - to its contemporary stage - to its near future stage, this report is divided into three sections. It is interesting to notice the shotgun approach of the first stage narrowing to a noticeable degree as the second stage is entered, and narrowing still more for entrance into the third stage. The fact that there are three definable stages in statistical design is evident from a study of the literature. There does exist an overlap, however, both in methodology and time, and hence some of the items listed below may belong to two developmental stages rather than a single one. In these instances the methodology was put in the earlier stage of development.

#### I. FIRST DEVELOPMENT

It is difficult to pinpoint any specific time at which the consideration of statistics in circuit design was established. In all likelihood it was utilized unconsciously at one time or another by every design engineer. The first report of note, however, was published in 1942 (Ref #2 - Wilkinson "The Combination of Probability Curves in Engineering"), and provided techniques based upon the rudiments of statistics for the determination of performance probability functions. In late 1948 (Ref #3 - Schwartz "Statistical Methods in the Design and Development of Electronic Systems"), a paper was published

which dealt with procedures to determine overall system tolerances. The author described a World War II Navy beacon system to which statistical methods could have been applied with considerable advantage, but were not. The general statistical procedure described was dependent upon mean values and standard deviations. Under the assumptions of this particular exercise, a conclusion was reached that a bandwidth of 10mc (of the airborne responder) would be unacceptable only 1% of the time. (Hence, one of the earliest applications of statistical design to systems was defined).

Statistical procedures for circuit design received little usage, however, until around 1955 when a number of significant efforts such as the following were undertaken:

1. Reference #4 - Hinrichs, "A Statistical Method for Analyzing the Performance Variations of Electronic Circuits". This effort describes an analytical method for determining the probability density functions of the steady state performance characteristics of simple linear circuits from the statistical distributions of the circuit parts' characteristics, and had been applied only to independent variables. The method consists of finding the central moments of the probability density function of the performance characteristic from the moments of the parts distributions. The required density function is then found from its central moments. The method (as described in the report) is limited to the investigation of performance characteristics expressible as independent algebraic forms and therefore to simple linear circuits. The procedure is rather complex, tedious, and time consuming for hand computation. (For example, in the report the first 6 moments of the performance characteristic were used as guidelines. In order to determine just the 5th moment involved the addition or multiplication, or both, of twenty separate summations, most of which require the summation of products.) It is stated, however, that computers could be utilized to this end.

2. Reference #5 - Hinrichs, "A Second Statistical Method for Analyzing the Performance Variations of Electronic Circuits". This effort describes further results of a study to determine the probability density function of an electronic circuit performance function from component part data. The analytical method described utilizes a Taylor Series to represent the actual circuit performance and is more general than the moment method described above, because limitations as to independent form are removed. The method in the report was at that time only applied to steady state performance of simple linear circuits. The methodology of this procedure follows combined computer and manual algebraic sets of operating procedures and tends to become tedious and long even with computer assistance.

3. Reference #6 - Pugsley, "The Influence of Component Tolerances on the Design and Specifications of a Complex Electronic Equipment". This paper gives an account of techniques which could be utilized to determine the frequency of tolerance failure due to

unfavorable combinations of tolerances. The approach taken is an analytical one based on the results of combining component parts having rectangular deviations in values, component parts having normal deviations in values and component parts in a system having a combination of the two distributions. The assumption taken in the report is that each component part contributes linearly to the overall performance independently of the deviations of other components. This procedure also is tedious and is very complex for combinations of distributions which are of various types, especially if the expression for the performance characteristic contains a number of variables.

4. Reference #7 - Marini, "The Evaluation and Prediction of Circuit Performance By Statistical Techniques". This paper describes a method for predicting the respective values of the mean and the variance of the distribution of a circuit characteristic (Y), by using regression analysis to determine empirically a relationship between Y and the specification characteristics,  $X_1, X_2, \dots, X_n$ , of the parts used in the circuit. The determining relationship and the distribution of the specification part characteristics as assumed are used to calculate the mean and variance values. The general method is to assume that the equation which relates the circuit characteristics to the part characteristics can be expressed as a linear combination of known functions of part characteristics plus a variable, E, which will represent the variation in Y caused by variation in operating part characteristics, experimental error, etc. From regression analysis coefficients of the linear expansion are calculated and the mean and variance determined for Y. With these parameters a statistically defined range of operation can be defined for the performance characteristic Y. This procedure always requires experimentation and is tedious, time consuming, and prone to human error. (Ref #8)

5. Reference #10 - Whiteman, "Reliability Starts with Design". This paper points out that in designing a reliable piece of equipment, the designer is primarily concerned with the average value of the data that will result when the design is ultimately tested rather than the spread of values around the average. This spread is, however, also of paramount importance. It is suggested in the paper that the designer include an "error" equation in his designs such that the relative spread may be determined. A complete form of the variance is suggested for this error function. However in general due to the makeup of this function, a simplified form of variance measure is exact and may be used for linear functions of independent variables. For other functions of mutually independent variables, the simplified form is sufficiently close if the standard deviations are small. This approach is similar to that of Reference #3 and also Reference #11 and bears similarity also to Reference #16, #17, and #2 and the same drawbacks apply to all these approaches.

6. Reference #12 - Xavier, "Utilization of Component Part Reliability Information in Circuit Design". This paper describes how component part reliability information may be utilized to

calculate performance spread. It relies on histogram studies of part value variations both initial and with time. Every possible combination of parameter values is used to determine all possible performance characteristic variations and a histogram of final performance variation results. The main drawback to this system is that if the variables determining the performance characteristic are larger than three or if the histogram of each component part's variation contains more than several bars, the number of possible combinations which result make calculation prohibitive.

In all of the models of statistical design discussed above it must be noted that each depends on the concept of mean and variance. Most depend on the relationship of the central limit theorem to normality. All but one (regression analysis) require an expression of system performance (a transfer function) before any steps can be taken and all are time consuming. In addition each requires knowledge of component part variation both initial and with time.

The Worst Case design concept first officially appeared around 1955 (Ref #13). The concept of Worst Case criteria can be stated as follows: A component must perform its function by exhibiting each and every specified parameter within tolerance when all of its part parameters are at their worst case values. The worst case values are defined as those values which are not mutually exclusive and lie within the tolerance limit, but tend to affect an operating parameter in the most adverse possible manner (Ref #14).

Worst Case design may be achieved either experimentally or analytically (Ref #15). Perhaps one of the best known means to apply the former is to devise "Schmoo" diagrams (profiles). These diagrams are obtained by finding the operational limits as direct functions of component variations commonly used to determine the tolerance of a circuit to component change. The circuit design is then repeatedly modified such that when all parts are at their nominal values the circuit performance falls into the center of the "Schmoo" diagram. A second empirical means for performing a Worst Case design is an instrument similar to the AIL (Ref #18) Circuit Design Reliability tester. Utilizing such equipment it is possible to "wire" in and out of a given circuit all possible combinations of high and low end point limit values of each part. This is done automatically, applying tens of thousands of combinations in a few minutes.

The analytical basis for Worst Case design is based upon determining mathematically which of the circuit variables contribute most to performance degradation, set those variables at their extreme values and modify the circuit performance function until minimum performance requirements are reached. This methodology is repeated until all parts are capable of worst case operation without degraded circuit performance.

Many things may be said pro and con concerning Worst Case



analysis (Ref #14). Worst case analysis calculations are relatively simple, (because end point calculations do not involve statistical or nonlinear theory) but tells the engineer nothing about the mean value, or spread which will occur in his design. Worst case analysis makes no distinction between the probability of mean and extreme outputs. The specification of the limit part values is somewhat arbitrary and the designer never knows how much reliability he has bought by using one or another set of extreme values (Ref #19 and #6). The above is borne out by a rigorous analysis performed in Reference #15. It is the contention of the author that Worst Case design often can result in larger complexities and higher stresses.

It is not fair, however, to damn Worst Case design in its entirety. Reference #14 (Ashcraft, "Design by Worst Case Analysis") shows that a Worst Case design can be modified as follows: In cases where use of worst case values is felt to be the direct cause of an undesirable increase in power dissipation, circuit complexity, or physical size, adjustment (narrowing) of worst case values should be considered. A technique for this adjustment is shown. The technique is based on the fact implied in Reference #16 (Dreste, "Circuit Design Concepts for High Reliability") that a magnitude of  $3\sigma$  deviations of system performance (that is 99.8% of all possible performance values) is still achievable even if the tolerance bands on part variations are relaxed a given amount from their original arbitrary chosen values and Worst Case design reimplemented on these narrower limits.

## II. PRESENT STAGE

In addition to 1955 being for all practical purposes the beginning of the era of Mathematical and Worst Case design, it was also the approximate time when circuit design by computer began to make headway. One of the earlier types of applications of computers to circuit design can be found in References #2 and #3. In these instances, the computer was merely required to compute moments of a distribution and numerically evaluate derivatives. In later work, however, many of the mathematical techniques of circuit design including Worst Case design were implemented by computer. This approach had the advantage of cutting down appreciably (and at the same time doing with less chance of error) the time necessary for all types of design.

Computer circuit analysis procedures were developed utilizing the principle of moments (Ref #20). The moment method of network analysis bears this name because it makes use of the mean (the first moment about the origin) and the variance (the second moment about the mean) of the frequency distributions of the circuit input and part parameters to predict the mean and variance of the circuit performance distribution. (It should be noted that this approach is the computer implementation of concepts discussed in Section I). If a frequency distribution is normal, these two parameters

alone describe it completely. If it is not normal, the mean and the variance can still be computed but additional quantities are needed to describe its skewness and peakedness. Parameter distributions encountered in practice are rarely exactly normal; consequently, because this procedure disregards such additional quantities the method is incapable of including their effects in the analysis. It may, however, be considered as adequately accurate for many, if not most, cases. This procedure, like similar procedures described in Section I, requires an equation of the performance characteristic, (in only matrix form however), values of the standard deviations of component parts and approximations to the variance function. This procedure is accurate when the change from the nominal part value is small when compared to the nominal value of the part value.

A third procedure of circuit design analysis (Ref #8) which can be interpreted as a modified type of Worst Case design is the Parameter Variation Method. This method of analysis is directed at the designer who after completing his preliminary design, would like to have an analysis of his circuit with specific considerations to the types of parts that must be used for successful circuit operation. An analysis of the results of this procedure may be used to determine parameter drift limits and to determine which parameters are interdependent and the extent of interdependence. The procedure is dependent on an equation of circuit performance, a knowledge of the nominal values of all parameters, the range of variations the analyst wishes to investigate for each parameter, the analyses of a large number of "Schmoo" plots (the results of the computer method) and verification by breadboard. This procedure is not a prediction technique relative to performance but a design tool to determine appropriate component part tolerance.

An additional helpful computer method to aid design (Ref #8) (although like the above, not a procedure determining overall performance characteristics) is the "Vinil" procedure. This method of analysis is used primarily by computer designers to match the input-output characteristics of computer building blocks (e.g., flip-flops, inverters, etc.). The name VINIL was obtained from the nature of this method of analysis where  $V_{IN}$  is swept from its minimum to its maximum end of life value and the output parameter of interest  $I_L$  is plotted for each sweep increment. Like the other procedures mentioned above, this procedure requires circuit equations, end of life values and analysis of computer results.

A powerful tool in the hands of the design engineer is the computer implementation of a poles and zeros distribution (Ref #9). Time-invariant linear systems are completely described by their responses to a unit impulse input. This response is often called the system weighting function and its Laplace transformation is the system transfer function. The gain and phase response of such a linear system have simple geometric interpretations in terms of the positions of the zeros and poles of the complex transfer function. However, the

relationship between the roots and coefficients of a polynomial is complicated and when these coefficients in turn are functions of circuit element values, the primary relationships between the roots and the circuit element values have no simple analytical forms. This computer procedure generates and plots root locations directly from circuit element values. The application of this procedure requires much the same information as the computer approaches previously mentioned.

A relatively new computer procedure for analysis of circuit design is described in References #21 and #22. The procedure has as its basis the hypothesis that component parts drift non-monotonically with time (a component part may tend to increase in value for a time then begin to decrease). A Markovian Chain process is the heart of the procedure. This concept or its utility has not been fully verified at this time and hence has not as yet been put to general use.

The most powerful tool for system performance prediction has been found to be the application of Monte Carlo procedures (Ref #9 and #32). (This procedure of course is also computer oriented.) This process is rigorous in that it requires no specialized assumptions as to the distribution of the deviations of component part value, and it is extremely close in form to reality. It is capable of virtually simulating the performance and determining the performance distribution curve of samples of thousands of models of any given circuit. In addition, there are procedures available to determine to any value of confidence (dependent upon the number of computer runs) that the simulated result represents the true picture of circuit performance (Ref #9). Utilizing this procedure both initial performance and performance as a function of component part drift with time can be statistically determined. To utilize this method requires circuit equations and reliable knowledge of the distribution of change in value of component parts both initial and with time. At present the main drawback to the use of Monte Carlo has been the lack of reliable data on component part drift. However, recent progress has been made in this area with the addition of a new Section VIII to the RADC Reliability Notebook (Ref #17). This Section contains the first effort to depict change in component part value as a function of time.

A comprehensive evaluation of the area of electronic circuit and system performance prediction resulted in the finding (Ref #9), that in all reality there is no prediction procedure superior to all others in every circumstance. Although on the whole a Monte Carlo procedure is most effective (and a strictly analytical procedure is almost impossible), there are times when a regression procedure is necessary to determine the circuit equation for a Monte Carlo analysis, and there are circumstances which arise where a moment or a Taylor series approach are adequate. Indeed, there are even techniques using modified or cut down Monte Carlo procedures that are

sufficiently accurate. The applicability of any of these alternate choices depend upon resources available, the nature of the circuit, and the data available on part drift.

The above Section II is representative of what is available now to design reliable circuits and to predict their performance characteristics as a function of time. Let us now summarize and see the steps necessary to machine analysis and hence the complexity it involves.

1. Equivalent circuits must be drawn for each circuit. This involves, at least for the AC analysis some assumptions as to performance characteristics of active devices. This problem can be met by hypothesizing various transistor equivalents in the design by analyzing the design by computer, and by verifying with breadboard until the equivalent circuit analysis corresponds to a reasonable degree with the breadboard findings.

2. Compiling the circuit equations in matrix form.

3. Incorporating the matrix, and input data as to part variations into a computer program.

4. Debugging the program.

5. Running the program.

6. Analysis of results.

It has been estimated that the time necessary to analyze an average circuit including programming and debugging is approximately 15 man hours spread over one week. Moreover, high level personnel must be utilized throughout this analysis.

### III. ADVANCED TECHNIQUES

There are several approaches which have been or might be developed to improve the entire area of statistical design and prediction of circuit performance. These are:

1. Development of special computer techniques such that a non-skilled individual could, given a circuit schematic, perform relatively simple operations (in a very short time) and give the result directly to a computer for a full Monte Carlo operation. In fact this approach does exist now for DC analysis (developed approximately 6 months ago by IBM). The same procedure may be utilized for AC analysis were it not for the problems associated with actual transistor equivalent circuits described previously. Approaches have been suggested from several sources on means to approach this problem and determine such equivalents.

2. Development of analogs of circuit component parts which may be configured in any design form. Such a plan could conceivably allow a complete Monte Carlo analysis to take place without the need for either a large digital computer, circuit analysis, or programming of any sort.

See attached Table I for condensed table of statistical design techniques and guidelines.

TABLE I  
CONDENSED TABLE

	<u>Use</u>	<u>Information &amp; Resources needed for use</u>	<u>Comments</u>
<u>1</u> Regression	To determine an empirical expression for circuit performance when none is available analytically.	Breadboarding to a significant extent.	In every method of circuit analysis an equation of circuit performance is necessary. This procedure generates an equation empirically.
<u>2</u> Analytical means (This includes most procedures included in Section I)	To determine gross distribution of the performance characteristic in terms of its mean and standard deviation (and sometimes other moments of the distribution - see Reference #4)	Expression of circuit performance. Knowledge of the standard deviation of the variation for each part versus time.	In most cases this procedure is based upon fundamentals of the central limit theorem and the assumptions involved with its application. In general, not as exact a prediction means as a Monte Carlo technique but gives adequate results. Can be used when computers not available.
<u>3</u> Worst Case Design	To design a circuit such that when all component part values are at their maximum allowable tolerance limits the circuit will continue to perform.	Breadboarding, or an expression for circuit performance. Knowledge of the spread values on each part.	This procedure is over cautious and not statistically or probabilistically defined. Drift failures, however, may be eliminated if information concerning spread is accurate. This procedure, since it depends only on maximum changes in component part values can be utilized with a minimum of information on component part drift, when compared to other techniques available.

TABLE I (continued)

4 Monte Carlo	To determine the distribution of a circuit performance characteristic.	Expression for circuit performance (necessary only in matrix form.) Knowledge of the drift distribution of each part value vs time. Availability of a computer.	This procedure has ease of application. It is non-parametric in that no assumptions need be made concerning the shape of distribution, etc. It is the most accurate of the prediction and analysis techniques available.
5 Moment Analysis (by computer)	To determine gross distribution of the performance characteristic in terms of its mean and standard deviation.	Same as #2 above.	Same as #2 above. When a new circuit is first being used, this procedure could first be used as a guide in adjusting parameters - then the Monte Carlo procedure could be used for the final analysis.
6 Worst Case (by computer)	Same as #3 above.	Same as #3 above - plus a computer.	Same as #3 above.
7 Poles and Zeros	To determine the distribution of pole locations.	Expression for circuit performance. Knowledge of the distribution of each part value vs time.	This procedure is useful for analysis of oscillators.
8 Parameter Variation Method	To determine parameter drift limits and to determine which parameters are interdependent and the extent of interdependence.	Expression for circuit performance.	This is a design tool (not a prediction technique) to determine appropriate component part tolerance.

2  
Vinil

TABLE I (continued)

To match input-output characteristics.	Expression for circuit tolerance. Maximum and minimum component part values vs. time.	This is a design tool (not a prediction technique) for computer building blocks (flip-flops, inverters, etc.)
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